

Characterisation of the Non-Linear Behaviour of Plastics for Finite Element Analysis

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SUMMARY

Tensile, shear and compressive tests have been employed to obtain stress-strain data for plastics for use with elastic-plastic models in finite element analyses of component behaviour. The materials investigated were polycarbonate, polypropylene, a propylene-ethylene copolymer and an acrylonitrile-butadiene-styrene copolymer (ABS). The tests were carried out in Instron machines at different crosshead speeds chosen to give the same effective plastic strain-rate for each mode of deformation. In the tensile tests, non-uniform deformation during yield was monitored with the aid of a videoextensometer. This enabled stresses and strains to be determined in local regions of necking or maximum extension. For polycarbonate, corrections were applied for errors in the calculated stresses arising from a peak in the local strain-rate during the initial neck propagation. Shear data obtained with the Arcan test were corrected for stress and strain distributions in the test specimen on the basis of finite element calculations.

From the tensile data, determinations were made of the 'elastic' modulus (E), Poisson's ratio (ν_e), the variation of yield stress with plastic strain (strain-hardening function) and the flow parameter ψ (which relates to the volume changes during the predominantly shear yielding). The tensile and shear data were also used to determine a parameter λ which provides a measure of the sensitivity of shear yielding to the hydrostatic component of stress.

Values of E , ν_e and the strain-hardening function varied significantly with strain-rate which should thus be specified. Results for polycarbonate, and for polypropylene at strains up to 0.3, were consistent with a pressure modified von Mises yield criterion with a constant λ value around 1.1. The parameters λ and ψ showed large apparent increases with strain for the toughened copolymers. This was ascribed to the onset of cavitation (crazing) under tension and reflected in the inability to accurately predict both shear and compressive behaviour from tensile data with a single value of λ . However, accurate predictions of compressive behaviour from shear data could be made with a strain-independent value of λ (≈ 1.2). It is concluded that the shear yield criteria of existing elastic-plastic models are valid only for stress states having a hydrostatic component that is too small to induce crazing. An additional yield criterion is proposed to allow for the craze yielding which occurs above a critical value of the hydrostatic stress component. A method for applying both shear and craze yield criteria in FE analyses is also indicated.

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Approved on behalf of Managing Director, NPL, by Dr C Lea,
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1 INTRODUCTION

In order that successful predictions can be made of the complex deformation of plastic components under load, there is a need to ensure that finite element (FE) packages contain accurate models of the stress-strain behaviour of polymeric materials. Since there is particular interest in the response of components to impact loads, where the material may rapidly deform to high strain levels, it is important that the non-linear behaviour under multiaxial loads, and the rate-dependence of polymer properties, is satisfactorily accounted for.

The non-linear behaviour of plastics is generally associated with an enhanced stress-relaxation or creep arising from a stress-induced increase in molecular mobility. Much of the strain is recoverable at levels below the yield strain, which is usually taken to correspond to the peak or to the onset of a plateau in a stress-strain curve. However, satisfactory models of this non-linear viscoelasticity have yet to be developed, and most FE packages consider material non-linearity in terms of elastic-plastic models that were developed for metals and adapted for use with plastics and other materials.

A survey previously presented¹ of elastic-plastic models included proposed methods for determining relevant material parameters from tensile and shear stress-strain data on plastic specimens. In this report details are given of the experimental techniques and data analyses to obtain these parameters for different polymers at specified strain-rates. From the results obtained, limitations of the model are discussed and modifications suggested that should improve its overall applicability to plastics. The dependence of material properties on strain-rate, including methods for extrapolating data to strain-rates appropriate to impact loading, will be addressed in a subsequent report.

2 ELASTIC-PLASTIC MODELS

With elastic-plastic models, calculations of stress and strain distributions at low strains (≤ 0.01) are based on the theory of linear elasticity using, for isotropic materials, values of the Young's modulus E and elastic Poisson's ratio ν_e . The onset of non-linearity in a stress-strain curve is then ascribed to plastic deformation and occurs at a stress level regarded as the first yield stress. The subsequent increase in stress with strain is associated with the effects of strain hardening. In this non-linear region the total strain is considered as the sum of a recoverable, elastic component and a plastic component which is non-recoverable. Stress

analysis calculations now involve the use of multiaxial yield criteria, usually expressed in terms of the principal stress components σ_1 , σ_2 and σ_3 . Following von Mises, these criteria often specify that yielding occurs when the second invariant of the deviatoric (shear) stress

$$J_{2D} = \frac{1}{6}[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2] \quad (1)$$

reaches a critical value. However for plastics it is recognised that this value further depends on the hydrostatic component of stress or stress invariant

$$J_1 = \sigma_1 + \sigma_2 + \sigma_3 \quad (2)$$

A relatively simple criterion for plastics^{2,3} assumes that yielding will occur at an effective stress $\bar{\sigma}$, numerically equal to the yield stress σ_T under uniaxial tension, that is linearly related to the critical J_{2D}^* and J_1 values, ie

$$\bar{\sigma} = \sigma_T = AJ_{2D}^* + BJ_1 \quad (3)$$

$$A = \frac{\sqrt{3}(\lambda+1)}{2\lambda}; \quad B = \frac{(\lambda-1)}{2\lambda} \quad (3a)$$

We note that σ_T is a true stress (see Appendix 1) which for a strain hardening material will increase with the plastic component of tensile strain. Equation (3) then defines a yield surface that will expand as σ_T increases with strain. Different stress states with the same effective stress will lie on the same yield surface and have the same effective plastic strain (see below). The parameter λ equals the ratio σ_C/σ_T where σ_C is the yield stress under uniaxial compression associated with the same effective plastic strain as σ_T . The value of λ provides a measure of the sensitivity of yield behaviour to the hydrostatic component of stress. It is related as follows to a parameter $\tan \beta$ which gives the change in shear yield stress with change in hydrostatic pressure and appears in an equivalent version of the above yield criterion due to Drucker and Prager (see ABAQUS technical manual⁴)

$$\tan \beta = 3(\lambda-1)/(\lambda+1) \quad (4)$$

An effective plastic strain $\bar{\epsilon}_p$ has also been defined^{1,3} in the form of equation (3) that is numerically equal to the plastic component of strain ϵ_{pT} under uniaxial tension, ie

$$\bar{\epsilon}_p = \frac{\sqrt{3} (\lambda+1)}{2\lambda (1+v_p)} I_{2D}^{\frac{1}{2}} + \frac{(\lambda-1)}{2\lambda(1-2v_p)} I_1 = \epsilon_{pT} \quad (5)$$

$$I_{2D} = \frac{1}{6} [(\epsilon_{p1} - \epsilon_{p2})^2 + (\epsilon_{p2} - \epsilon_{p3})^2 + (\epsilon_{p3} - \epsilon_{p1})^2] \quad (6)$$

$$I_1 = \epsilon_{p1} + \epsilon_{p2} + \epsilon_{p3} \quad (7)$$

Here ϵ_{p1} , ϵ_{p2} and ϵ_{p3} are the plastic components of the principal strains and v_p is the plastic component of Poisson's ratio. Note that $\bar{\epsilon}_p$, ϵ_{pT} and the ϵ_{pn} ($n = 1,2,3$) are all true strains and that v_p is defined in terms of true strain components (see Appendix 1). With these definitions the sum of principal strains I_1 is zero and $v_p = 0.5$ for a plastic deformation at constant volume.

Strain hardening may now be represented as the dependence of effective stress on effective plastic strain ($\bar{\sigma} = f(\bar{\epsilon}_p)$). Values of v_p are also employed in the calculation of the flow parameter $\tan \psi$ which is the ratio of the change in volumetric plastic strain to the change in deviatoric plastic strain during yield. The magnitude of $\tan \psi$ is determined by the direction of plastic flow and is required by the flow law¹ for relating increments of plastic strain $d\epsilon_{pn}$ to components of stress σ_n . Existing FE packages make no allowance for changes in v_p or $\tan \psi$ with strain, in which case we have

$$\tan \psi = \frac{3(1-2v_p)}{\sqrt{3}} \quad (8)$$

For 'associated flow', $\psi = \beta$ and the direction of flow is perpendicular to the yield strain surface.

If the yielding is insensitive to the hydrostatic stress component then $\sigma_c = \sigma_T$, $\lambda = 1$ and we obtain from equation (3) the von Mises criterion

$$\sigma_T = \sqrt{3} J_{2D}^{\frac{1}{2}} \quad (9)$$

In this case it may be assumed that the yield process occurs at constant volume so that $v_p = 0.5$ and $\psi = \beta = 0$.

3 DETERMINATION OF MODEL PARAMETERS

In this section methods are described that were used to obtain the data required by the elastic-plastic models. It should be noted that each method refers to measurements at a specified strain-rate, or at specified strain-rates, and has been applied to obtain values of the parameters over a wide strain-rate range.

ELASTIC PROPERTIES (E , ν_e)

The Young's modulus and Poisson's ratio were determined from measurements of stress, strain and lateral strain in a tensile test at small strains where the stress-strain plot is linear. Appropriate relations are given in Appendix 1.

3.2 STRAIN-HARDENING FUNCTION ($\bar{\sigma}(\bar{\epsilon}_p) = \sigma_T(\epsilon_{pT})$)

The strain-hardening function was obtained from measurements of yield stress σ_T and plastic strain ϵ_{pT} in a uniaxial tension test at strains ranging from that associated with the onset of non-linearity ($\epsilon \approx 0.01$) up to the failure strain (typically $\epsilon \geq 0.5$). Owing to the high strains involved, true stresses and true strains (as defined in Appendix 1) were determined over the entire strain range. For strains above the linear range, the stress σ is identified with the yield stress σ_T and the true plastic strain component was calculated using equation A7 of Appendix 1.

HYDROSTATIC-STRESS SENSITIVITY PARAMETER λ

The determination of λ , or the related parameter β (equation (4)), requires measurements of the yield behaviour under two different stress states. If stress-strain data are available under both compression and tension, then λ may be obtained directly as the ratio σ_c/σ_T where the yield stresses σ_c and σ_T correspond to the same effective plastic strain. However, owing to difficulties associated with uniaxial compression tests¹, λ values have been determined in this project from a combination of tension and shear tests.

The method is based on the relation

$$\lambda = \frac{\sqrt{3}(\sigma_s/\sigma_T)}{2 - \sqrt{3}(\sigma_s/\sigma_T)} \quad (10)$$

which follows from equation A25 of Appendix 2.

Here σ_s and σ_T are yield stresses in shear and tension having the same effective plastic strain (given by equation A26) and the same effective plastic strain-rate (equation A28). Using relations presented in Appendix 2, the procedure for calculating λ at a given plastic strain $\bar{\epsilon}_p = \epsilon_{pT}$ was as follows (see Fig A1):

- (i) From corresponding σ_T and ϵ_{pT} values on the strain-hardening curve, the secant Young's modulus was calculated from $E_s = \sigma_T / \epsilon_{pT}$.
- (ii) A secant shear modulus $G_s = E_s / 2(1 + \nu_p)$ was evaluated from E_s and the measured ν_p (see A9 and A27).
- (iii) Plastic strains γ_p were calculated as a function of σ_s from the shear data (equation A18) and the value of σ_s identified for which the secant modulus $G_s = \sigma_s / \gamma_p$ was equal to that determined in (ii). This involved modelling the variation of σ_s with G_s using a polynomial function.
- (iv) Noting that the σ_T and σ_s values identified in (i) and (iii) correspond to the same $\bar{\epsilon}_p$, λ was calculated from the σ_s / σ_T ratio using equation (10).

To explore possible variations in λ with strain, the above procedure was repeated for different $\bar{\epsilon}_p$ values in the observed range.

3.4 FLOW PARAMETER ψ

The flow parameter was determined from the tensile data using equation (8) and the measured value of ν_p (equation A9). These calculations were repeated for different ϵ_{pT} values in the non-linear range to ascertain whether ψ varied with strain.

4 EXPERIMENTAL

4.1 MATERIALS

The thermoplastics investigated in this project are listed in Table 1 together with the trade names and suppliers. They comprise an amorphous polymer (polycarbonate), semicrystalline polypropylene, a propylene-ethylene copolymer containing 8% ethylene and an acrylonitrile-butadiene-styrene copolymer (ABS).

Table 1 - Details of Materials

Plastic	Supplier/code
Polycarbonate	BASF, Macrolon 293
Polypropylene	RS
Propylene-ethylene copolymer	BASF, Novolen 2300LL
ABS	BASF, Terluran 967K

The polycarbonate and polypropylene were obtained as sheets of thickness 3 mm. The propylene-ethylene copolymer and the ABS were supplied in the form of pellets from which sheets of thickness 3.2 mm were subsequently compression moulded.

For the tensile tests, standard dumbbell-shaped specimens (ISO 3167) of overall length 150 mm and width of the narrow gauge section 10 mm, were machined from the sheets. For the shear measurements, Arcan V-notched square plates⁵, having a notch separation of 12 mm, were also machined from the supplied sheets. Parallel sided bars of rectangular cross-section were used for the uniaxial compressive tests, the dimensions of these specimens being given in Section 4.4 below.

4.2 TENSILE MEASUREMENTS

Tensile tests were effected using an Instron universal testing machine type 4505. For the determination of Young’s modulus, longitudinal strains were measured using extensometers, developed at the NPL⁶, that were mounted on opposite faces of the specimen. Each extensometer comprised an inductive displacement transducer that contacted the specimen via two knife edges, one attached to the body of the transducer and the other to its core. The gauge length between the knife edges was 50 mm. Measurements of transverse strain, also required for the determination of ν_e , were made using a commercial (Instron) clip-on extensometer mounted across the specimen width. Values of E and ν_e were usually evaluated from data over the strain range 0.0005→0.0025 and were reproducible to within $\pm 1\%$ and $\pm 3\%$ respectively.

For determining the strain-hardening function and associated yield parameters in the non-linear region, strains were determined by means of a Messphysik ME64 videoextensometer. With this method, a video camera (with zoom lens) is employed to generate an image of the

specimen on a computer screen and the distance between gauge marks on the specimen surface is determined from the number of pixels on the screen between the marks. Strains are then calculated from the changes in the measured distance. For longitudinal strain measurements, the gauge marks were usually in the form of parallel black lines drawn across the width of the specimen with a marker pen. The distance between the lines was typically in the range 5-20 mm. For transverse strain measurements, the gauge marks were either in the form of black lines drawn along opposite edges of the specimen or were defined by the edges of the specimen against a black background. Distances between these gauge marks could then be determined along a chosen number of 'measuring axes' equally spaced between the longitudinal gauge marks. With the propylene-ethylene and ABS copolymers it was found necessary to paint the specimen surfaces white between gauge marks (see Figure 1). This served to maintain a uniform contrast as the materials changed colour due to stress-whitening during the deformation.

The tensile data included in this report were obtained at $23 \pm 0.2\text{ }^{\circ}\text{C}$ and at a crosshead speed of 1, 10 or 100 mm/min. Table 2 shows the corresponding range of strain-rates calculated from the recorded displacement versus time data. For each crosshead speed, the strain-rate is seen to increase by a factor of between 2 and 3 as the strain increases from the linear range to that associated with the peak in stress.

Table 2 - Calculated strain-rates in tension and shear tests

Tension			Shear		
Crosshead speed (mm/min)	Strain-rate (s^{-1})		Crosshead speed (mm/min)	Strain-rate (s^{-1})	
	low-strain linear region	at peak in stress		low-strain linear region	at peak in stress
1	0.00015	0.0004	0.2	0.00014	0.0007
10	0.0015	0.004	2	0.0014	0.007
100	0.015	0.04	20	0.014	0.07
			100	0.07	0.35

4.2.1 Data acquisition during non-uniform deformation

At strain levels just above that associated with the peak in stress, the tensile deformation of plastics often becomes non-uniform. For example, over a wide range of strain-rate and temperature, polycarbonate develops a localised neck that subsequently spreads along the length of the specimen. To obtain data representative of the yield behaviour of this material,

tests with the videoextensometer were designed to give stress and strain values over a local region within the neck where the deformation was essentially uniform.

The procedure for polycarbonate recognised that the neck usually originated about 60 mm from one end of the dumbbell specimen where, owing to slight non-uniformities in machining, the width exhibited a minimum value some 0.3% less than that at the centre. The gauge lines for longitudinal strain measurements were thus located in this region separated by a distance of 15 mm, and transverse strains were determined along 10 equally spaced measuring axes (cross-sections) within the gauge lines. Figure 2 shows some results obtained at a crosshead speed of 100 mm/min. From these data we assume that the neck originated around section 6, since for this section the first abrupt increase was observed in the negative transverse strain. A similar effect was subsequently seen for the other sections (exemplified for section 1 in Figure 2) as the neck grew along the gauge length. The variation of the average transverse strain for all sections with the total longitudinal strain (Fig 2) gives a Poisson's ratio between 0.4 and 0.5 over almost all of the strain range.

For each cross-section, the local stress was evaluated from the applied load and the transverse engineering strain ϵ'_t using equation A1. The local strain ϵ for each section was then estimated from the transverse strain ϵ_t using (see equation A4)

$$\epsilon = -\epsilon'_t/\nu \quad (11)$$

Here the local Poisson's ratio ν was assumed to have a value of 0.45 at all strains which is close to the average value determined as above for the gauge section. Figure 3 compares the local stress versus local strain plots calculated for sections 1 and 6 respectively. For section 1, the stress exhibits a marked drop following the peak in the curve, and a strain reversal occurs during the neck formation. However, the stress on section 6 is essentially constant for strains up to about 0.4.

Since cross-section 6 was within the neck region at all times during its formation and growth, the stress-strain curve for this section is assumed representative of the material properties during yield. However, from a plot of the local strain versus the recorded time for section 6 a marked peak in strain-rate was revealed during the initial neck propagation. As illustrated in Figure 4, the maximum strain-rate is about 80 times the value at low strains, so that significant corrections to the stresses were required to obtain valid data for a constant specified strain-rate. From the calculated strain-rates $\dot{\epsilon}$ for section 6, and apparent stresses σ_{app} (Fig 3), corrected stresses σ_{cor} were determined using the empirical formula

$$\sigma_{\text{cor}} = \sigma_{\text{app}} \left[1 - \frac{1}{\sigma_y} \frac{\Delta\sigma_y}{\Delta\log\dot{\epsilon}} (\log \dot{\epsilon} - \log \dot{\epsilon}_{\text{ref}}) \right] \quad (12)$$

where $\dot{\epsilon}_{\text{ref}}$ is a reference strain-rate and $(1/\sigma_y) (\Delta\sigma_y/\Delta\log\dot{\epsilon})$ is the fractional change in peak yield stress σ_y with logarithmic strain-rate. Figure 5 shows the corrected stresses calculated for a reference strain-rate ($\dot{\epsilon}_{\text{ref}} = 0.046 \text{ s}^{-1}$) equal to that at the peak yield stress, using $\sigma_y = 71 \text{ MPa}$ and $\Delta\sigma_y/\Delta\log\dot{\epsilon} = 3.0 \text{ MPa}$. For strains between about 0.15 and 0.3, it will be noted that the corrected stresses are 9% lower than the peak stress.

The propylene-ethylene copolymer often developed a neck during tensile yielding at temperatures around and above room temperature. For tests on this material at 23 °C, specimens were reduced by 0.5% in width over a 5 mm gauge length and transverse strain measurements were made along 4 equally spaced measuring axes between the gauge lines. The yield deformation occurred preferentially within the gauge region, was slightly non-uniform and, for the example shown in Figure 6, the maximum transverse strain was observed at cross-section 2. The local stress and longitudinal strain for this section were thus determined as for polycarbonate. However, in calculating the local strain using equation (11) it was necessary to allow for a substantial decrease in ν with increasing strain. For this purpose the local value of ν for a given local transverse strain was assumed to equal the average ν value for the gauge section at the equivalent average transverse strain.

As shown in Figure 6, the local stress-strain curve determined for cross-section 2 differs only marginally from the average curve for the gauge section. Furthermore, for all strains up to failure, the local strain-rate for section 2 was only 2 to 3 times the value at the strain corresponding to the peak in stress. Hence corrections for strain-rate variations during the test were negligible. By confining the yield largely to the 5 mm gauge section, it appears that the formation and propagation of a neck is inhibited and that the average properties of the gauge section are accurately representative of the material behaviour. In tensile tests carried out at 0 °C, the propylene-ethylene copolymer specimens deformed uniformly up to failure and required no means for inhibiting neck formation.

4.3 SHEAR MEASUREMENTS

In the shear tests, details of which were reported previously⁵, Arcan specimens were loaded in the Instron machine parallel to the (vertical) axis joining the V-notches (see Fig 7). A state of pure shear was thereby produced in regions close to this axis away from the notch roots. Shear stresses were calculated approximately from the applied load F divided by the cross-

sectional area A between notches (notch separation times specimen thickness). Shear strains were obtained from the relative vertical displacement u of two points located on opposite sides of the central axis (Fig 7) and separated by a known horizontal distance d of about 3 mm. This displacement was determined by an extensometer containing two pivoted lever arms each of which contacted the specimen at one of the points shown via a needle point attached at one end⁵. During the specimen deformation, the relative displacement of the needle points (which produces independent rotations of the two lever arms) was monitored through the output of inductive displacement transducers in contact with the opposite ends of the lever arms. To compensate for twisting motions of the specimen, average displacements were determined for opposite faces of the specimen using two extensometers each mounted on the lower grip of the specimen.

The shear tests were performed at the different crosshead speeds given in Table 2, where, for each speed, the calculated strain-rate is seen to increase with increasing strain. The speeds of 0.2, 2 and 20 mm/min were chosen to give shear strain-rates at low strains approximately equal to the tensile strain-rates in the tensile tests for speeds of 1, 10 and 100 mm/min respectively. They also gave shear strain-rates about 1.8 times the tensile strain-rates in the respective tensile tests at strains associated with the peaks in stress (Table 2). It follows that the effective plastic strain-rates in the shear and corresponding tensile tests were closely similar, as required in the determination of λ (section 3.3).

4.3.1 Corrections to shear data

Sources of error in the shear data were previously discussed in some detail⁵ and it was concluded that uncertainties in the shear stresses and strains are substantially larger than those in the corresponding tensile quantities. It has since been found that measured shear moduli G may typically be up to 20% lower than values calculated from tensile data using

$$G = \frac{E}{2(1+\nu_e)} \quad (13)$$

Since accurate values of G are required in calculating the elastic shear strain γ_e (equation A17), and subsequently deriving related quantities such as λ , further studies have been undertaken to ascertain the origin of the discrepancies and provide corrections to the data.

As part of this investigation, Arcan shear tests were carried out on polycarbonate at crosshead speeds of 0.2, 2 and 20 mm/min using different methods of coupling the specimen

grips to the Instron pullrods. For each crosshead speed, shear moduli were determined from the stress-strain data in the strain range 0.003 to 0.006 and were compared with values calculated from tensile data (using equation (13)) over the same strain range and at a corresponding crosshead speed (Table 2). It was noted that the shear data were highly scattered at the lower strains (0.0005-0.0025) frequently used for determining E and ν_e and that the shear and tensile stress-strain curves were each quite linear for strains in the range 0.003-0.006. For all modes of coupling, the measured G values were lower than those calculated from tensile data. However, the discrepancies were minimised by employing universal joints to obtain flexible couplings between the grips and pullrods, which may have resulted from a reduction in in-plane or out-of-plane specimen distortions. With the universal joints, the measured G values were about 11% lower than the calculated values and a similar study of the ethylene-propylene copolymer gave shear moduli about 7% lower than those calculated.

Finite element calculations have shown that the above discrepancies, as well as errors in the high-strain data, can be traced to stress and strain distributions in the loaded Arcan specimen. Results of these calculations have provided correction factors for converting the respective F/A and u/d values to the true shear stress σ_s and true shear strain γ (defined by A15 and A16 of Appendix 1) at the mid-point of the test specimen. In the low-strain, elastic, region we obtained $\sigma_s/(F/A) = 1.03$ and $\gamma/(u/d) = 0.94$ for the propylene-ethylene copolymer and an epoxy adhesive. The correction factor for stress undoubtedly arises from a marked fall-off in shear stress along the vertical centre line of the specimen within about 1 mm of each notch root. The correction factor for shear strain is less well understood but may reflect contributions from bending deformations to the measured displacement u . The combined factors for stress and strain represent a correction of +9.5% to the measured shear modulus which largely accounts for the above discrepancies between the measured and calculated values of G .

With increasing strain, the stress correction factor $\sigma_s/(F/A)$ decreases only slightly, values around 1.015 being obtained for shear strains at which substantial yielding has occurred. However, the strain correction factor $\gamma/(u/d)$ exhibits a marked increase with strain, and values as high as 1.97 were obtained for the propylene-ethylene copolymer for $\gamma \approx 0.3$. This result largely arises from the localised plastic deformation in the vicinity of the centre line, as evidenced by the appearance of a shear band having a width less than the 3 mm separation between the extensometer arms. The non-uniform shear yielding outweighs other effects such as that associated with bending contributions to the measured u , or with the fact that, at high strains, differences will exist between the true shear strain and the measured

engineering strain in any local region. Furthermore the shear banding differs for different materials and will depend on strain-rate and temperature. Finite element calculations of strain correction factors will therefore be required for each shear test or a different technique used to determine local shear strains. It should be emphasised, however, that shear banding is usually associated with a flat region of the stress-strain curve so that relatively large corrections to the shear strain have little effect on the curve. Such corrections have not therefore been applied so far in this work. Corrections to the shear stress and strain values have, however, been made using the factors 1.03 and 0.94 respectively. These factors seem to be independent of the material and test conditions, and give reliable values of G at low strains.

4.4 COMPRESSIVE MEASUREMENTS

Stress-strain data under uniaxial compression were obtained from tests on specimens of two different geometries. At low strains (typically 0-2%) rectangular bars 80 mm long, 12.5 mm wide and about 3 mm thick were loaded along their length between parallel platens in a universal testing machine. Some constraint to buckling was achieved using a jig conforming to ASTM D-3846. The contacts with the specimen faces were spring loaded to allow minimal restraint to lateral expansion under the compressive load. Compressive strains were measured using extensometers with a 25 mm gauge length attached to each face of the specimen. Reliable results using this test were obtained up to strains of about 2% after which specimen buckling occurred.

To obtain data at higher strains, shorter specimens were used with dimensions 12.5 mm long x 12.5 mm wide x about 3 mm thick. The top and bottom faces were machined smooth and accurately parallel. The specimens were loaded in a testing machine between parallel platens whose faces were lubricated with oil. Extensometers were used to measure changes in the platen separation on loading and nominal strain values were then derived from the original specimen length. Errors of 0.002-0.004 in these strain values arose from initial displacements in the regions of the specimen-platen interfaces. Corrections for this effect were derived from a comparison of the data at low strains with results from tests on the longer specimens, where the effect was not observed owing to the attachment of extensometers to the specimen.

Crosshead speeds in the compressive tests were chosen to give strain-rates of about 1, 10 and 100%/min which were consistent with the rates employed in the tensile and shear tests (Table 2). True compressive stresses $\sigma_c = -\sigma$ were calculated from the engineering stresses $\sigma_c' = -\sigma'$ using equation A1, assuming an engineering Poisson's ratio of 0.4 to estimate (+ve)

transverse engineering strains ϵ_t' from the (-ve) longitudinal engineering strains ϵ' . True compressive strains $\epsilon_c = -\epsilon$ were calculated from the engineering strains $\epsilon_c' = -\epsilon'$ using equation A2a.

5 RESULTS AND DISCUSSION

Figures 8-11 show tensile and shear stress-strain curves and the variation of Poisson's ratio with strain for the different materials at 23 °C. Values of parameters determined from the tensile and shear data are also presented in Tables 3 and 4 respectively. Note that all stresses, strains, Poisson's ratios and strain-rates are true quantities as defined in Appendix 1. Furthermore, the selected crosshead speeds gave approximately equal strain-rates in the respective tension and shear tests at low strains and equal effective plastic strain-rates at the strains associated with the stress maxima (section 4.3).

Table 3 - Summary of tensile data at 23 °C

Material	Crosshead speed (mm/min)	E (MPa)	ν_e	$\dot{\epsilon}$ (s ⁻¹) low strain	σ_{Tmax} (MPa)	ϵ_{max}	$\dot{\epsilon}$ (s ⁻¹) at ϵ_{max}
Polycarbonate	100	2400	0.38	0.015	72.5	0.077	0.046
Polypropylene	1	1520	0.42	0.00015	32.5	0.166	0.00033
Propylene-ethylene copolymer	10	1600	0.39	0.0015	27.3	0.082	0.0044
ABS	10	2200	0.40	0.0015	35.1	0.021	0.0045

Table 4 - Summary of shear data at 23 °C

Material	Crosshead speed (mm/min)	G (MPa)	$\dot{\gamma}$ (s ⁻¹) low strain	σ_{Smax}	γ_{max}	$\dot{\gamma}$ (s ⁻¹) at γ_{max}
Polycarbonate	20	870	0.014	43.9	0.128	0.084
Polypropylene	0.2	535	0.00014	20.1	0.301	0.00058
Propylene-ethylene copolymer	2	575	0.0014	20.3	0.285	0.0085
ABS	2	786	0.0014	27.6	0.093	0.0069

The tensile and shear stress-strain curves for polycarbonate (Fig 8) each exhibit a distinct peak in yield stress and subsequent drop ascribed to strain-softening. The Poisson's ratio increases from the elastic value $\nu_e = 0.38$ to a value around 0.48 which is maintained over much of the yield region before decreasing somewhat at the higher strains. This result is consistent with an essentially constant volume during yield. For polypropylene, the tensile and shear curves in Figure 9 have similar shapes for tensile strains up to about 0.2. The decrease in tensile stress at higher strains is attributed to crazing or cavitation as it is accompanied by stress-whitening and a gradual decrease in Poisson's ratio or volume increase (Fig 9). Shear data for the propylene-ethylene copolymer in Figure 10 are similar to those for the propylene homopolymer but the tensile data reflect the onset of cavitation at relatively low strains (0.08) by an abrupt flattening of the stress-strain curve, stress whitening and a marked decrease in Poisson's ratio with increasing strain. An extreme example of the effects of crazing under tension is evident from the results for ABS in Figure 11. Although no necking occurred during the tensile deformation, sharp decreases in tensile stress and Poisson's ratio commenced at a strain as low as 0.02. These effects were again accompanied by stress whitening and cause the tensile stress-strain curve to differ markedly in shape from the shear curve.

From the tensile and shear data, λ values were derived by the method described in section 3.3, and are shown as a function of effective plastic strain in Figure 12. For polycarbonate, λ has a constant value of 1.1 for strains up to 0.3 and then tends to decrease with further increases in strain. The data for polypropylene give $\lambda = 1.16$ for tensile strains up to 0.3 and a gradual increase in λ at higher strains. The λ values for the propylene-ethylene copolymer and for ABS appear to increase considerably with strain from a low-strain value around 1.2.

From the results in Figure 12, $\tan \beta$ values of the Drucker-Prager model were calculated using equation (4). Figure 13 compares these data with values of the flow parameter $\tan \psi$ determined from the tensile data (see section 3.4). In the case of polycarbonate, $\tan \beta$ and $\tan \psi$ are each close to zero over most of the strain range up to failure. Values of these parameters are also quite low for polypropylene at low strains but show a gradual increase at strains above 0.2. For the propylene-ethylene copolymer and ABS, $\tan \beta$ and $\tan \psi$ each appear to increase substantially with increasing strain. Note also that, whereas $\tan \beta$ and $\tan \psi$ exhibit similar trends with increasing strain, the magnitude of $\tan \beta$ usually differs from the magnitude of $\tan \psi$. This would indicate that all of the materials studied, with the exception of polycarbonate, undergo non-associated flow.

5.1 PREDICTION OF SHEAR AND COMPRESSIVE BEHAVIOUR FROM TENSILE DATA

Since most FE packages require, as input, data from tensile tests together with a value for λ (or β), the accuracy with which they may compute multiaxial stress and strain distributions in components can be assessed by considering the predicted behaviour under shear and compression from the measured tensile data and a selected constant λ . In making these predictions, shear stresses (σ_s) and strains (γ) were calculated from tensile stresses (σ_T) and plastic tensile strains (ϵ_{pT}) using

$$\sigma_s = \frac{2\lambda}{\sqrt{3}(\lambda+1)} \sigma_T$$

and

$$\gamma = \frac{4\lambda(1+\nu_p)}{3} \epsilon_{pT} + \frac{\sigma_s}{G}$$

These equations were derived using A25 and A26 of Appendix 2 and adding an elastic strain component (σ_s/G) to γ_p . For predicting the compressive stresses (σ_c) and strains (ϵ_c), the following relations were similarly derived

$$\begin{aligned} \sigma_c &= \lambda \sigma_T \\ \epsilon_c &= \lambda \epsilon_{pT} + \sigma_c/E \end{aligned} \tag{17}$$

Figures 14 and 15 show the results of these predictions for polycarbonate and polypropylene using constant λ values of 1.1 and 1.16 respectively. As these values were obtained from the tensile and shear data for effective strains up to 0.3, the accurate predictions of shear behaviour in this region are as expected. The peak compressive stress predicted for polycarbonate is also close to the measured value which is about 7% higher than the peak tensile stress (corresponding to $\lambda = 1.07$). The discrepancy at higher strains for polycarbonate between the predicted and measured drop in compressive stress (strain softening) is not understood. It could be related to the non-uniform deformation during compressive yielding which has not been corrected for. Although no compressive data have been obtained for polypropylene in this work, the predicted peak value of compressive stress (37.6 MPa) is only slightly higher than the value (36.4 MPa) derived from published data by Duckett and coworkers⁷.

Figure 16 illustrates the predicted shear and compressive behaviour for the propylene-ethylene copolymer using a λ value of 1.85. Since this corresponds to the maximum value obtained from the tension and shear data (Fig 12), the predicted and observed shear stresses agree well in the plateau region of the stress-strain curve. However, the predicted compressive stresses are much higher than the experimental values. Using $\lambda = 1.5$ (Fig 17) the predicted shear and compressive stresses are respectively somewhat smaller and larger than the measured stresses whilst the von Mises value of $\lambda = 1$ (Fig 18) gives predicted shear and compressive stresses much lower than those observed. Similar discrepancies have been obtained for ABS. These are exemplified in Figure 19 for the maximum observed λ value of 3.43 and in Figure 20 for $\lambda = 1.34$, which is the value obtained using the peak tensile yield stress. It will be noted that both the shapes and magnitudes of the predicted curves now differ from the observed shapes and magnitudes.

5.2 LIMITATIONS OF THE MODEL

The parameters obtained for polycarbonate deviate only slightly from those associated with von Mises yielding ($\lambda = 1$, $\tan \psi = 0$). Somewhat larger deviations are observed in the case of polypropylene. However, for each of these materials, the multiaxial yield behaviour at effective strains up to 0.3 is accurately described by the yield criterion of equation (3) with λ constant. At room temperature, and at the specified strain-rates, yielding in these polymers may thus be regarded predominantly as a shear process modified by the hydrostatic component of stress.

For the propylene-ethylene copolymer and ABS, the large apparent increases in λ and $\tan \psi$ with strain (Figs 12 and 13) undoubtedly reflect the multiple crazing which accompanies (or replaces) the shear yielding under tension for these two-phase, rubber-toughened, polymers. As the cavitation requires a dilatational stress component, and does not therefore occur under shear or compression, it serves to lower the tensile yield stress σ_T relative to σ_s and σ_c . This causes the shape of the tensile stress-strain curve to differ from the shapes of the shear and compression curves. It further produces the apparent increase in λ (calculated from the tension and shear data) with increasing strain. Since the elastic-plastic model makes no allowance for the cavitation process, values of λ calculated from the tension and shear data will also differ from the values obtained from tension and compression results. This difference relates to the inability to accurately predict both the shear and compressive behaviour from tensile data with a single λ value (section 5.1). The volume increase (or decrease in v_p) produced by the cavitation also gives rise to the increase in $\tan \psi$ with increasing strain.

Despite the problems associated with cavitation, it should be emphasised that for each material studied equation (3) has been successfully employed in predicting the compressive behaviour from shear data with a constant λ value. This is illustrated for the propylene-ethylene copolymer and ABS in Figures 21 and 22, respectively, where σ_c and ϵ_c were determined using

$$\sigma_c = \frac{\sqrt{3}}{2} (\lambda+1) \sigma_s$$

and

$$\epsilon_c = \frac{\sqrt{3}(\lambda+1)\gamma_p}{4(1+\nu_p)} + \frac{\sigma_c}{E}$$

These equations were obtained from A25 and A26 respectively, of Appendix 2 after adding an elastic strain component (σ_c/E) to ϵ_{pc} . From the success of the predictions in Figures 14, 15, 21 and 22, we then conclude that equation (3) (with λ typically between 1.1 and 1.2) provides an accurate criterion for the yield of plastics if the hydrostatic component of stress is below that required to produce cavitation. If cavitation occurs under tension, then the λ value may be derived from shear and compressive data with the aid of equations (18) and A27. The strain-hardening function $\bar{\sigma}(\bar{\epsilon}_p)$ may also be obtained from either shear or compressive data using (A21 to A24, Appendix 2)

$$\bar{\sigma} = \frac{\sqrt{3}(\lambda+1)}{2\lambda} \sigma_s = \frac{\sigma_c}{\lambda}$$

and

$$\bar{\epsilon}_p = \frac{\sqrt{3}(\lambda+1)}{4\lambda(1+\nu_p)} \gamma_p + \frac{\epsilon_{pc}}{\lambda}$$

where ν_p may be assumed to have a value of 0.5.

5.3 PROPOSED MODIFICATIONS TO THE MODEL

To allow for the effects of crazing, there is a need to modify or extend the yield criterion represented by equation (3). One approach is to develop an additional criterion to equation (3) which is employed when the dilatational stress component is large enough to induce cavitation. Craze initiation in homogeneous glassy polymers is known, in fact, to require a component of hydrostatic tensile stress. Early studies under biaxial loading² suggested that crazing occurred at a critical absolute value of $\sigma_1 - \sigma_2$ (known as the stress bias)

which showed a linear increase with $1/(\sigma_1 + \sigma_2)$. It was later proposed⁸ for triaxial stress states that the crazes initiated at a critical value of $\sigma_1 - \nu(\sigma_2 + \sigma_3)$ which increased linearly with $1/(\sigma_1 + \sigma_2 + \sigma_3)$. This may be expressed in terms of an effective stress for crazing

$$\bar{\sigma}^c = \sigma_T = C[\sigma_1 - \nu(\sigma_2 + \sigma_3)] - D/J_1 \quad (22)$$

where C and D are material parameters and σ_1 is the maximum principal stress. The quantity $\sigma_1 - \nu(\sigma_2 + \sigma_3)$ was, in fact, considered to reflect a critical tensile strain $\epsilon_1 = [\sigma_1 - \nu(\sigma_2 + \sigma_3)]/E$. On the other hand, the stress bias may be regarded as a shear stress although it is not the maximum value as $\sigma_3 = 0$.

If equation (22) is valid, then C and D may be related to a single material parameter whose determination requires stress-strain data under two stress states having different dilatational components large enough to induce cavitation. Biaxial tension tests have been employed in crazing studies and it should be possible to develop a test in which equal tensile stresses are applied orthogonally in the plane of a sheet or wall of a thin tube. If σ_T^* is the yield (crazing) stress determined in an equibiaxial tension test ($\sigma_1 = \sigma_2 = \sigma_T^*$; $\sigma_3 = 0$) then the following relations for C and D are obtained from equation (22)

$$C = \frac{(2\eta - 1)}{2\eta^2(1 - \nu) - 1}; \quad D = \frac{2\sigma_T^{*2}\eta[1 - \eta(1 - \nu)]}{2\eta^2(1 - \nu) - 1} \quad (23)$$

and we also obtain for the effective plastic strain

$$\bar{\epsilon}_p^c = \epsilon_{pT} = \frac{C}{(1 + 2\nu\nu_p)} \left[\epsilon_{p1} - \nu(\epsilon_{p2} + \epsilon_{p3}) \right] - D \left(\frac{\epsilon_{pT}^2}{\sigma_T^2} \right) \frac{(1 - 2\nu_{p'})}{-} \quad (24)$$

In equation (23), $\eta (= \sigma_T^*/\sigma_T)$ is the ratio of yield stresses measured under equibiaxial and uniaxial tension, respectively, at the same effective plastic strain.

For polymeric adhesives, triaxial stress states are obtained by applying a load normal to the plane of a thin adhesive layer in a bonded joint. Our recent investigations with this "butt-tension" test have suggested that an equitriaxial stress state ($\sigma_1 = \sigma_2 = \sigma_3 = \sigma_B$) is established during yield, where σ_B is the applied normal stress. Equation (22) now gives

$$C = \frac{(3\chi-1)}{3\chi^2(1-2\nu)-1}; \quad D = \frac{3\sigma_T^2\chi[1-\chi(1-2\nu)]}{3\chi^2(1-2\nu)-1}$$

where $\chi = \sigma_B/\sigma_T$, σ_B and σ_T being the yield (crazing) stresses under butt tension and uniaxial tension, respectively, measured at the same effective plastic strain (equation 24).

Stress-strain data have been obtained with the butt tension test for a toughened epoxy and a toughened acrylic adhesive each of which exhibited stress-whitening during yield. From the results, we obtained $\chi = 1.0$ for the epoxy and $\chi = 0.8$ for the acrylic material. Using equations (23) and (25) with $\nu = 1/3$, it is found that these χ values correspond to $\eta = 0.87$ and $\eta = 0.80$ respectively.

A theory of crazing has been proposed by Argon^{9,10} in which the initiation and growth mechanisms involve both the deviatoric and hydrostatic stress components. We thus consider, as an alternative to equation (22) a criterion of the form

$$\bar{\sigma}^c = \sigma_T = C J_{2D}^{1/2} - D/J_1$$

where C and D will have different values from the corresponding parameters in equation (22). It should be recognised that the invariance of $J_{2D}^{1/2}$ to a rotation of axes makes it a more convenient function than $\sigma_1 - \nu(\sigma_2 + \sigma_3)$. Furthermore, the dependence of $\bar{\sigma}^c$ on $1/J_1$ (cf equation (22)) has been retained. This is consistent with observations that craze formation requires a hydrostatic tension since when $J_1 = 0$ (pure shear) it occurs only at $J_{2D} = \infty$.

In place of equation (23), we now obtain from (26),

$$C = \frac{\sqrt{3}(2\eta-1)}{2\eta^2-1}; \quad D = \frac{2\sigma_T^2\eta(1-\eta)}{2\eta^2-1}$$

and, in place of equation (25),

$$C = \sqrt{3}(1-3\chi); \quad D = -3\chi\sigma_T^2$$

The effective plastic strain now becomes, from equation (26),

$$\bar{\epsilon}_p^c = \epsilon_{pT} = \frac{C}{(1+\nu_p)} I_{2D}^{1/2} - D \left(\frac{\epsilon_{pT}^2}{\sigma_T^2} \right) \frac{(1-2\nu_p)}{I_1}$$

Using equations (27) and (28), the above χ values obtained for the toughened epoxy and acrylic adhesives now correspond to $\eta = 0.65$ and $\eta = 0.64$ respectively.

Assuming that either equation (22) or equation (26) represents a valid criterion for crazing in plastics and/or the toughened copolymers we write, in general,

$$\bar{\sigma}^c = \sigma_T = CJ_c - D/J_1 \quad (30)$$

where J_c may be identified either with $\sigma_1 - \nu(\sigma_2 + \sigma_3)$ or with $J_{2D}^{1/2}$.

Sections of yield stress surfaces in the 1-2 plane are shown schematically in Figure 23. The criterion given by equation (3) corresponds to an ellipse with major axis at 45° to axis 1. The crazing criterion, equation (30), describes a surface for which $\sigma_1 + \sigma_2 > 0$ and which approaches asymptotically the pure shear line $\sigma_1 + \sigma_2 = 0$. The relative locations of the two yield surfaces will depend on strain, strain-rate and temperature. For certain combinations of these variables the crazing surface will lie outside the shear yield surface (Fig 23a) so that shear yielding will occur for stress states such that

$$\begin{aligned} AJ_{2D}^{1/2} + BJ_1 &= \bar{\sigma} \\ CJ_c - D/J_1 &< \bar{\sigma} \end{aligned} \quad (31)$$

At room temperature, and at the strain-rates so far investigated, this situation applies to polycarbonate over a wide strain range and for polypropylene at tensile strains up to 0.3. It may also apply for the propylene-ethylene and ABS copolymers at strains corresponding to the onset of non-linearity. If the strain-hardening associated with shear yielding exceeds that associated with crazing then, with increasing strain, the shear yield surface will show a relatively large expansion so as to encompass the crazing surface (Fig 23b). In this situation, which applies to polypropylene and the toughened copolymers, crazing will occur when

$$\begin{aligned} AJ_{2D}^{1/2} + BJ_1 &< \bar{\sigma} \\ CJ_c - D/J_1 &= \bar{\sigma} \end{aligned} \quad (32)$$

and shear yielding will again occur for stress states given by equation (31).

It is considered that parameters such as λ and η , which determine the shapes of the yield surfaces, may be taken as constants independent of strain. Changes in the relative locations of the yield surfaces with strain will depend on the form of the functions $\bar{\sigma}(\bar{\epsilon}_p)$ and $\bar{\sigma}^c(\bar{\epsilon}_p^c)$. Figure 24 illustrates a procedure for determining these functions by combining shear (or compressive) data with tensile data obtained at the same effective plastic strain-rate, assuming that crazing is the dominant yield mechanism under tension. The function $\bar{\sigma}(\bar{\epsilon}_p)$ in Figure 24(b) is calculated from the shear or compressive data, using equations (20) and (21) to obtain $\bar{\sigma}$ and $\bar{\epsilon}_p$ respectively. The curve $\bar{\sigma}(\bar{\epsilon})$ in Figure 24(a) is then derived using $\bar{\epsilon} = \bar{\epsilon}_p + (\bar{\sigma}/E)$. It will be evident that $\bar{\sigma}^c(\bar{\epsilon}_p^c)$ is determined by the tensile data $\sigma_T(\epsilon_T)$ at strains originating from the point of departure of the two curves in Figure 24(a). The determination of $\bar{\sigma}(\bar{\epsilon}_p)$ and $\bar{\sigma}^c(\bar{\epsilon}_p^c)$ by the above method is exemplified by results for the propylene-ethylene copolymer in Figures 25 and 26 and for ABS in Figures 27 and 28.

To allow for both shear and craze yielding in FE analyses, it is suggested that two sets of data should be specified. The function $\bar{\sigma}(\bar{\epsilon}_p)$, together with a constant λ ($\approx 1.1 \rightarrow 1.2$) and $\tan \psi = 0$ would serve to characterise the shear yielding. Craze yielding should be characterised by $\bar{\sigma}^c(\bar{\epsilon}_p^c)$ with a constant η and $\tan \psi$ derived using equation (8). During the initial time steps of an FE analysis, stress and strain components for each element will be calculated on the basis of Hooke's law using the given E and ν_e values. At each step the functions $AJ_{2D}^{1/2} + BJ_1$ and $CJ_c - D/J_1$ should be evaluated from the resulting stresses. The conditions represented by relations (31) may be attained at some step for different elements and $AJ_{2D}^{1/2} + BJ_1$ will reach the initial value of $\bar{\sigma}$ in Figure 24(b), corresponding to the onset of shear yielding. For this and the following steps, equations (3) and (5) and the function $\bar{\sigma}(\bar{\epsilon}_p)$ should be used in calculating stress and strain components. Depending on the type of component deformation, the value of $CJ_c - D/J_1$ for some elements may subsequently reach the initial value of $\bar{\sigma}^c$ in Figure 24(b) as crazing is initiated and the conditions (32) are realised. The stress components for other elements may continue to conform to relations (31). Subsequent calculations of stress and strain components will then require a selection of the appropriate yield criterion and strain-hardening function for the different elements.

There is a need for further work to confirm the validity of equations (22) or (26), or to establish an alternative criterion for craze yielding. Procedures should also be considered that allow for the simultaneous occurrence of shear and craze yielding. Limitations of the elastic-plastic model should be recognised with regard to modelling rate effects, the behaviour during unloading and response to an arbitrary loading history. A realistic description of these effects, and of the origins of non-linearity, requires the development of

a non-linear viscoelastic model in which relaxation times under multiaxial loading are related to effective stresses such as $\bar{\sigma}$ and $\bar{\sigma}^c$.

6 CONCLUSIONS

Tensile tests on plastics are most suitable for determining the Young's modulus E and Poisson's ratio ν_e required by the elastic-plastic models in FE analyses of component behaviour. Values of E and ν_e depend on the applied strain-rate which should therefore be specified.

Equation (3) represents a valid yield criterion for those stress states having a hydrostatic component too small to induce cavitation. If crazing does not occur under uniaxial tension, then the strain-hardening function $\bar{\sigma}(\bar{\epsilon}_p)$ is identified with $\sigma_T(\epsilon_{pT})$, and may thus be obtained from tensile data. The parameter λ may be derived from measured tensile and shear yield stresses as described in section 3.3 (eq (10)). When crazing occurs under tension (as for rubber-toughened copolymers), $\bar{\sigma}(\bar{\epsilon}_p)$ may be obtained from either shear or compressive data (equations (20) and (21)) and λ determined from combined shear and compressive data with the aid of equations (18) and A27.

For those stress states having a hydrostatic component large enough to induce cavitation, an alternative yield criterion, equation (22) or (26), is proposed. The hardening function $\bar{\sigma}^c(\bar{\epsilon}_p^c)$ is now derived largely from tensile data obtained under conditions for which the crazing mechanism dominates. The shape of the yield surface may be characterised by a parameter η , defined as the ratio of yield stresses measured under equibiaxial and uniaxial tension.

To allow for both shear and craze yielding in FE analyses, two sets of data should be specified. The hardening function $\bar{\sigma}(\bar{\epsilon}_p)$, together with a constant λ (≈ 1.15) and flow parameter $\tan \psi = 0$, should serve to characterise the predominantly shear yielding. Craze yielding should be characterised by $\bar{\sigma}^c(\bar{\epsilon}_p^c)$ with a constant η and $\tan \psi$ determined from equation (8). At each step in the FE calculations the appropriate yield criterion should be selected depending on the conditions represented by equations (31) and (32).

Further work is required to assess the validity of equations (22) or (26) and the effects of simultaneous shear and craze yielding. The development of a non-linear viscoelastic model should also be considered for describing the effects of non-linearity, strain-rate and load reversal.

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APPENDIX 1. TERMINOLOGY FOR STRESSES AND STRAINS

To account for significant variations in specimen dimensions at high strains, the stress-strain behaviour has been analysed on the basis of true principal stresses and strains for all strain levels. Some relevant definitions and relationships are given in this Appendix.

Uniaxial tension

True normal stress components are defined with reference to the deformed cross-sectional areas. Under uniaxial tension, the true stress is therefore calculated using

$$\sigma = \sigma' / (1 + \epsilon_t')^2 \quad \text{A1}$$

where the nominal (engineering) stress σ' equals the load divided by the unstretched cross-sectional area and ϵ_t' is the transverse engineering strain (fractional change in width or thickness). The true (natural) longitudinal strain ϵ is defined by

$$\epsilon = \ln(1 + \epsilon') \quad \text{A2}$$

where ϵ' is the engineering strain (fractional change in length) and the true transverse strain ϵ_t by

$$\epsilon_t = \ln(1 + \epsilon_t') \quad \text{A3}$$

Using A2 and A3 we define a true Poisson's ratio

$$\nu = -\epsilon_t / \epsilon \quad \text{A4}$$

and note that the Young's modulus E and elastic Poisson's ratio ν_e represent the values of σ/ϵ and ν , respectively, in the linear region at small strains when true stresses and strains tend to the respective engineering quantities.

From the definitions A2 and A3 above, the following relations were employed to calculate the elastic and plastic components of the true strains ϵ and ϵ_t .

True elastic strain,

$$\epsilon_e = \ln\left(1 + \frac{\sigma}{E}\right) \approx \frac{\sigma}{E} \quad A5$$

True transverse elastic strain,

$$\epsilon_{te} = \ln\left(1 - \frac{\nu_e \sigma}{E}\right) \approx \frac{-\nu_e \sigma}{E} \quad A6$$

True plastic strain,

$$\epsilon_p = \epsilon - \epsilon_e \quad A7$$

True transverse plastic strain,

$$\epsilon_{tp} = \epsilon_t - \epsilon_{te} \quad A8$$

True plastic Poisson's ratio

$$\nu_p = -\epsilon_{tp} / \epsilon_p \quad A9$$

A subscript T is usually added to σ and the ϵ components to indicate stresses and strains under tension. Note also that the transverse strains ϵ_t' , ϵ_t , ϵ_{te} and ϵ_{tp} are of negative sign whereas σ and the longitudinal strains are positive.

Uniaxial compression

Equations A1 to A9 are also applicable to the determination of stress and strain components under uniaxial compression. In this case the transverse strains are of positive sign and the stresses and longitudinal strains are negative. However, compressive stresses and longitudinal strains are usually presented as positive quantities ($\sigma_c' = -\sigma'$, $\sigma_c = -\sigma$, $\epsilon_c' = -\epsilon'$, $\epsilon_c = -\epsilon$). Equations A2, A4, A5 and A6 may then be written

$$\epsilon_c = -\ln(1 - \epsilon_c') \quad A2a$$

$$\nu = \epsilon_t / \epsilon_c \quad A4a$$

$$\epsilon_{ec} = -\ln\left(1 - \frac{\sigma_c}{E}\right) \approx \frac{\sigma_c}{E} \quad A5a$$

$$\varepsilon_{te} = \ln \left(1 + \frac{v_e \sigma_c}{E} \right) \approx \frac{v_e \sigma_c}{E} \quad \text{A6a}$$

Generalisation to three dimensions

In three dimensions, true principal (normal) stresses σ_1 , σ_2 and σ_3 are defined with respect to the deformed cross-sectional areas and true principal strains ε_1 , ε_2 and ε_3 are defined as in A2,

$$\varepsilon_n = \ln(1 + \varepsilon'_n) \quad (n = 1, 2, 3)$$

Components of true stress σ_{ij} ($i, j = x, y, z$) and true strain ε_{ij} , with reference to an arbitrary coordinate system 0-xyz, can be written in terms of the principal values σ_n and ε_n as follows

$$\sigma_{ij} = \sum_n \sigma_n a_{in} a_{jn}$$

and

$$\varepsilon_{ij} = \sum_n \varepsilon_n a_{in} a_{jn}$$

Here a_{in} and a_{jn} are the direction cosines between the principal axis n and the axes i and j respectively. In matrix form, we note that equation A12 for the true strains is employed by ABAQUS⁴ in the analysis of high strain behaviour. For a deformation at constant volume, it can be shown that the sum of true principal strains $\varepsilon_1 + \varepsilon_2 + \varepsilon_3$ is zero, from which it follows that $v = 0.5$.

Shear stress and strain

In the case of two-dimensional, in-plane, shearing we take principal axes 1 and 2 to lie in the xy plane and set $\sigma_3 = 0$. Equation A11 then gives

$$\sigma_{xy} = \sigma_{yx} = (\sigma_1 - \sigma_2) \sin\theta \cos\theta$$

where θ is the angle between principal axis 1 and the x direction. True shear strain is taken as the sum $\varepsilon_{xy} + \varepsilon_{yx}$ since, in the small-strain limit, this reduces to the engineering (as opposed to tensor) shear strain. From equation A12, with $\varepsilon_3 = 0$, we obtain

$$\epsilon_{xy} + \epsilon_{yx} = 2(\epsilon_1 - \epsilon_2) \sin\theta \cos\theta \quad A14$$

Pure shear is produced by stresses having a zero hydrostatic component ($\sigma_1 + \sigma_2 + \sigma_3 = 0$) and occurs at constant volume ($\epsilon_1 + \epsilon_2 + \epsilon_3 = 0$). With $\sigma_3 = 0$ we have $\sigma_2 = -\sigma_1$ and define the true shear stress σ_s in pure shear by

$$\sigma_s = \frac{\sigma_1 - \sigma_2}{2} = \sigma_1 \quad A15$$

From A13 we note that σ_s equals the component σ_{xy} for x-y axes oriented at $\theta = \pi/4$ to the principal axes. At this value of θ , it follows from A11 that the normal stresses σ_{xx} and σ_{yy} are each zero. Similarly the true shear strain in pure shear is defined in terms of the true principal strains ($\epsilon_3 = 0, \epsilon_2 = -\epsilon_1$) by

$$\gamma = \epsilon_1 - \epsilon_2 = 2\epsilon_1 \quad A16$$

From A14 and A12 it follows that γ equals $\epsilon_{xy} + \epsilon_{yx}$ for an angle $\theta = \pi/4$ at which the true normal strains ϵ_{xx} and ϵ_{yy} are each zero.

Following the definitions A15 and A16, elastic (γ_e) and plastic (γ_p) components of the true shear strain may be obtained using

$$\gamma_e \approx \sigma_s / G \quad A17$$

and

$$\gamma_p = \gamma - \gamma_e \quad A18$$

No analytical relationships have been obtained between the true shear stress or strain and the engineering shear stress or strain obtained from simple shear tests. However, Section 4.3.1 gives numerical factors from finite element calculations relating the true shear stress and strain to the respective engineering stress and strain determined from Arcan tests.

APPENDIX 2. RELATIONS BETWEEN YIELD STRESSES AND BETWEEN PLASTIC STRAINS FOR SHEAR YIELDING DURING TENSILE, COMPRESSIVE AND SHEAR DEFORMATIONS

Effective stress and effective plastic strain for uniaxial tension

The coefficients A and B in equation (3) are defined as in (3a) so that the effective stress $\bar{\sigma}$ equals the yield stress σ_T under uniaxial tension. This can be confirmed by substituting $\sigma_1 = \sigma_T$ and $\sigma_2 = \sigma_3 = 0$ into equations (1) and (2) which gives $J_{2D}^{1/2} = \sigma_T/\sqrt{3}$ and $J_1 = \sigma_T$. When these relations, together with (3a), are further substituted in equation (3) it follows that

$$\bar{\sigma} = \sigma_T$$

By setting $\epsilon_{p1} = \epsilon_{pT}$ and $\epsilon_{p2} = \epsilon_{p3} = -\nu_p \epsilon_{pT}$ in equations (6) and (7) and substituting the resulting expressions $I_{2D}^{1/2} = \epsilon_{pT}(1+\nu_p)/\sqrt{3}$ and $I_1 = \epsilon_{pT}(1-2\nu_p)$ into equation (5), it can similarly be confirmed that

$$\bar{\epsilon}_p = \epsilon_{pT}$$

Effective stress and effective plastic strain for uniaxial compression

In the case of uniaxial compression tests we have $\sigma_1 = -\sigma_C$, $\sigma_2 = \sigma_3 = 0$ and obtain from equations (1) and (2) $J_{2D}^{1/2} = \sigma_C/\sqrt{3}$ and $J_1 = -\sigma_C$. Equations (3) and (3a) then give

$$\bar{\sigma} = \sigma_C/\lambda \quad \text{A21}$$

Substituting $\epsilon_{p1} = -\epsilon_{pC}$ and $\epsilon_{p2} = \epsilon_{p3} = -\nu_p \epsilon_{p1} = \nu_p \epsilon_{pC}$ into equations (6) and (7) we then obtain $I_{2D}^{1/2} = (1+\nu_p)\epsilon_{pC}/\sqrt{3}$ and $I_1 = -\epsilon_{pC}(1-2\nu_p)$. Equation (5) then gives

$$\bar{\epsilon}_p = \epsilon_{pC}/\lambda$$

Effective stress and effective plastic strain for pure shear

For pure shear loading we have $\sigma_2 = -\sigma_1 = -\sigma_s$ and $\sigma_3 = 0$. Equations (1) and (2) then give $J_{2D}^{1/2} = \sigma_s$, $J_1 = 0$ and it follows from (3) with (3a) that

$$\bar{\sigma} = \frac{\sqrt{3}(\lambda+1)}{2\lambda} \sigma_s$$

The plastic shear strain γ_p is related to the principal components by $\epsilon_{p2} = -\epsilon_{p1} = -\gamma_p/2$ with $\epsilon_{p3} = 0$. Equations (6) and (7) then give $I_{2D}^{1/2} = \gamma_p/2$, $I_1 = 0$ and it follows from (5) that

$$\bar{\epsilon}_p = \frac{\sqrt{3}(\lambda+1)}{4\lambda(1+\nu_p)} \gamma_p \quad A24$$

Derived relationships

Values of σ_T , σ_C and σ_s having the same effective stress $\bar{\sigma}$ lie on the same yield surface and the corresponding plastic strains ϵ_{pT} , ϵ_{pC} and γ_p have the same effective plastic strain $\bar{\epsilon}_p$. With these stress and strain values, the following relations are obtained from equations A19 to A24.

$$\sigma_T = \frac{\sigma_C}{\lambda} = \frac{\sqrt{3}(\lambda+1)}{2\lambda} \sigma_s \quad A25$$

$$\epsilon_{pT} = \frac{\epsilon_{pC}}{\lambda} = \frac{\sqrt{3}(\lambda+1)}{4\lambda(1+\nu_p)} \gamma_p \quad A26$$

and

$$E_s = \frac{\sigma_T}{\epsilon_{pT}} = \frac{\sigma_C}{\epsilon_{pC}} = \frac{2\sigma_s}{\gamma_p} (1+\nu_p) = 2G_s(1+\nu_p) \quad A27$$

As illustrated schematically in Figure A1, it follows from A27 that the coordinates $(\sigma_T, \epsilon_{pT})$ and $(\sigma_C, \epsilon_{pC})$ on plots of yield stress versus plastic strain in tension and compression have a common secant modulus E_s . The coordinate (σ_s, γ_p) on the corresponding shear curve has a secant shear modulus $G_s = E_s/2(1+\nu_p)$. Note that the three curves in Figure A1 have the same effective plastic strain-rate

$$\dot{\bar{\epsilon}}_p = \dot{\epsilon}_{pT} = \frac{\dot{\epsilon}_{pC}}{\lambda} = \frac{\sqrt{3}(\lambda+1)}{4\lambda(1+\nu_p)} \dot{\gamma}_p \quad A28$$

FIGURE CAPTIONS

Fig 1 Illustration of the gauge section and gauge marks on a tensile specimen for determining strains with the videoextensometer.

Fig 2 Plots of local (sections 1 and 6) and average transverse strains against the total (average) longitudinal strain for a 15 mm gauge length in the polycarbonate tensile specimen. A neck formed within the gauge section at location 6 during extension at a crosshead speed of 100 mm/min.

Fig 3 Local stress-strain curves calculated for sections 1 and 6, respectively, of the polycarbonate specimen during extension at a crosshead speed of 100 mm/min.

Variation of strain-rate with strain for section 6 of the polycarbonate specimen during extension at a crosshead speed of 100 mm/min.

Fig 5 Tensile stress-strain data for section 6 of the polycarbonate specimen before ($\square$) and after (o) correcting for the peak in strain-rate during neck propagation. The corrections were made using equation (12) with a reference strain rate $\dot{\epsilon}_{\text{ref}} = 0.046 \text{ s}^{-1}$.

Fig 6 Tensile stress-strain data for the propylene-ethylene copolymer at a crosshead speed of 10 mm/min. (—), average stress versus average strain for the 5 mm gauge section. (o), local stress versus local strain for cross-section 2.

Illustration of the Arcan shear test specimen and grips, showing the locations of contact points for the extensometer lever arms near the centre of the specimen.

Tensile and shear data for polycarbonate. The strain-rate given with each stress-strain curve is for a strain corresponding to the stress maximum.

Fig 9 Tensile and shear data for polypropylene. The strain-rate given with each stress-strain curve is for a strain corresponding to the stress maximum.

Fig 10 Tensile and shear data for the propylene-ethylene copolymer. The strain-rate given with each stress-strain curve is for a strain corresponding to the stress maximum.

Fig 11 Tensile and shear data for ABS. The strain-rate given with each stress-strain curve is for a strain corresponding to the stress maximum.

Fig 12 Dependence of λ on effective plastic strain for polycarbonate (o), polypropylene (◆), the propylene-ethylene copolymer (■) and ABS (Δ).

Fig 13 Variation of $\tan \psi$ (point symbols) and $\tan \beta$ (—) with strain for the different materials as indicated.

Fig 14 Tensile, shear and compressive data (point symbols) for polycarbonate. Continuous lines show the shear and compressive behaviour predicted from tensile data. The predicted curves were based on a λ value of 1.1 obtained from tensile and shear data.

Fig 15 Tensile and shear data (point symbols) for polypropylene. Continuous lines show shear and compressive behaviour predicted from tensile data. The predicted curves were based on a λ value of 1.16 obtained from tensile and shear data.

Fig 16 Tensile, shear and compressive data (point symbols) for the propylene-ethylene copolymer. Continuous lines show shear and compressive behaviour predicted from tensile data using $\lambda = 1.85$.

Fig 17 Tensile, shear and compressive data (point symbols) for the propylene-ethylene copolymer. Continuous lines show shear and compressive behaviour predicted from tensile data using $\lambda = 1.5$.

Fig 18 Tensile, shear and compressive data (point symbols) for the propylene-ethylene copolymer. Continuous lines show shear and compressive behaviour predicted from tensile data using the von Mises criterion ($\lambda = 1$).

Fig 19 Tensile, shear and compressive data (point symbols) for ABS. Continuous lines show shear and compressive behaviour predicted from tensile data using $\lambda = 3.43$.

Fig 20 Tensile, shear and compressive data (point symbols) for ABS. Continuous lines show shear and compressive behaviour predicted from tensile data using $\lambda = 1.34$.

Fig 21 Tensile, shear and compressive data (point symbols) for the propylene-ethylene copolymer. The continuous line shows the compressive behaviour predicted from shear data using $\lambda = 1.2$.

Fig 22 Tensile, shear and compressive data (point symbols) for ABS. The continuous line shows the compressive behaviour predicted from shear data using $\lambda = 1.2$.

Fig 23 Sections of yield surfaces in the 1-2 plane under conditions of strain-rate and temperature such that (a), shear yielding predominates at low strains within the non-linear region and (b), either shear or craze yielding may occur at higher strains depending on the stress state.

Fig 24 Schematic illustration of a method for deriving $\bar{\sigma}(\bar{\epsilon}_p)$ and $\bar{\sigma}^c(\bar{\epsilon}_p^c)$. Functions $\bar{\sigma}(\bar{\epsilon}_p)$ and $\bar{\sigma}(\bar{\epsilon})$ in diagrams (b) and (a) are governed by shear yielding and determined from shear or compressive data as described in the text. Crazing strains $\bar{\epsilon}_p^c$ and the function $\bar{\sigma}^c(\bar{\epsilon}_p^c)$ are then determined as in (a) from the tensile data $\sigma_T(\epsilon_T)$ with the aid of the calculated $\bar{\sigma}(\bar{\epsilon})$.

Fig 25 Stress-strain plots for the propylene-ethylene copolymer at a strain-rate of 0.0044 s^{-1} . (-♦-), tensile data, σ_T versus ϵ_T ; (-■-), effective stress $\bar{\sigma}$ versus effective strain $\bar{\epsilon}$ calculated from shear data with $\lambda = 1.2$ (see text).

Fig 26 Variation of effective stress with effective plastic strain for shear and craze yielding, respectively, in the propylene-ethylene copolymer. Functions $\bar{\sigma}(\bar{\epsilon}_p)$ and $\bar{\sigma}^c(\bar{\epsilon}_p^c)$ were derived from the data in Fig 25 as described in the text.

Fig 27 Stress-strain plots for ABS at a strain-rate of 0.0045 s^{-1} . (-♦-), tensile data, σ_T versus ϵ_T ; (-■-), effective stress $\bar{\sigma}$ versus effective strain $\bar{\epsilon}$ calculated from shear data with $\lambda = 1.2$ (see text).

Fig 28 Variation of effective stress with effective plastic strain for shear and craze yielding, respectively, in ABS. Functions $\bar{\sigma}(\bar{\epsilon}_p)$ and $\bar{\sigma}^c(\bar{\epsilon}_p^c)$ were derived from the data in Fig 27 as described in the text.

Fig A1 Schematic variation of yield stress with plastic strain for uniaxial tension, uniaxial compression and shear. The coordinates indicated on the respective curves have the same effective stress $\bar{\sigma}$ and same effective plastic strain $\bar{\epsilon}_p$.

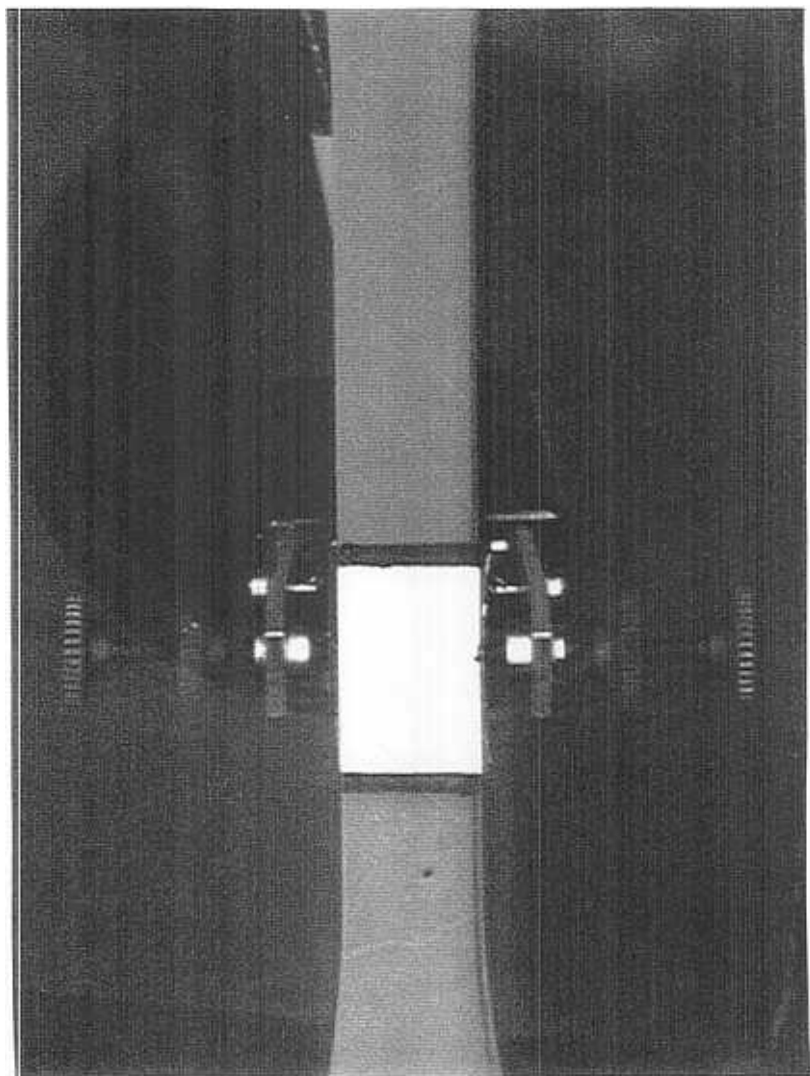


Fig. 1

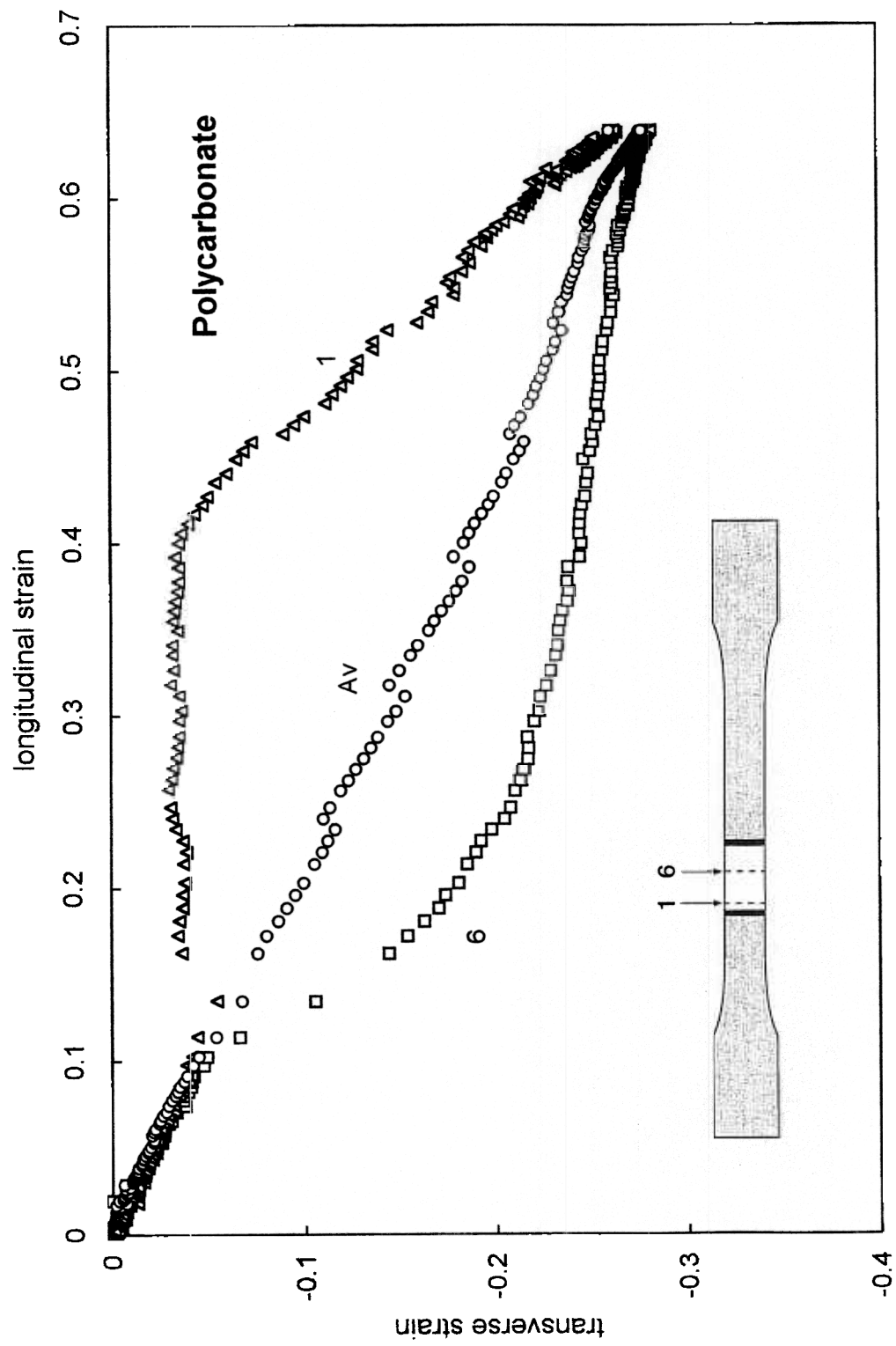
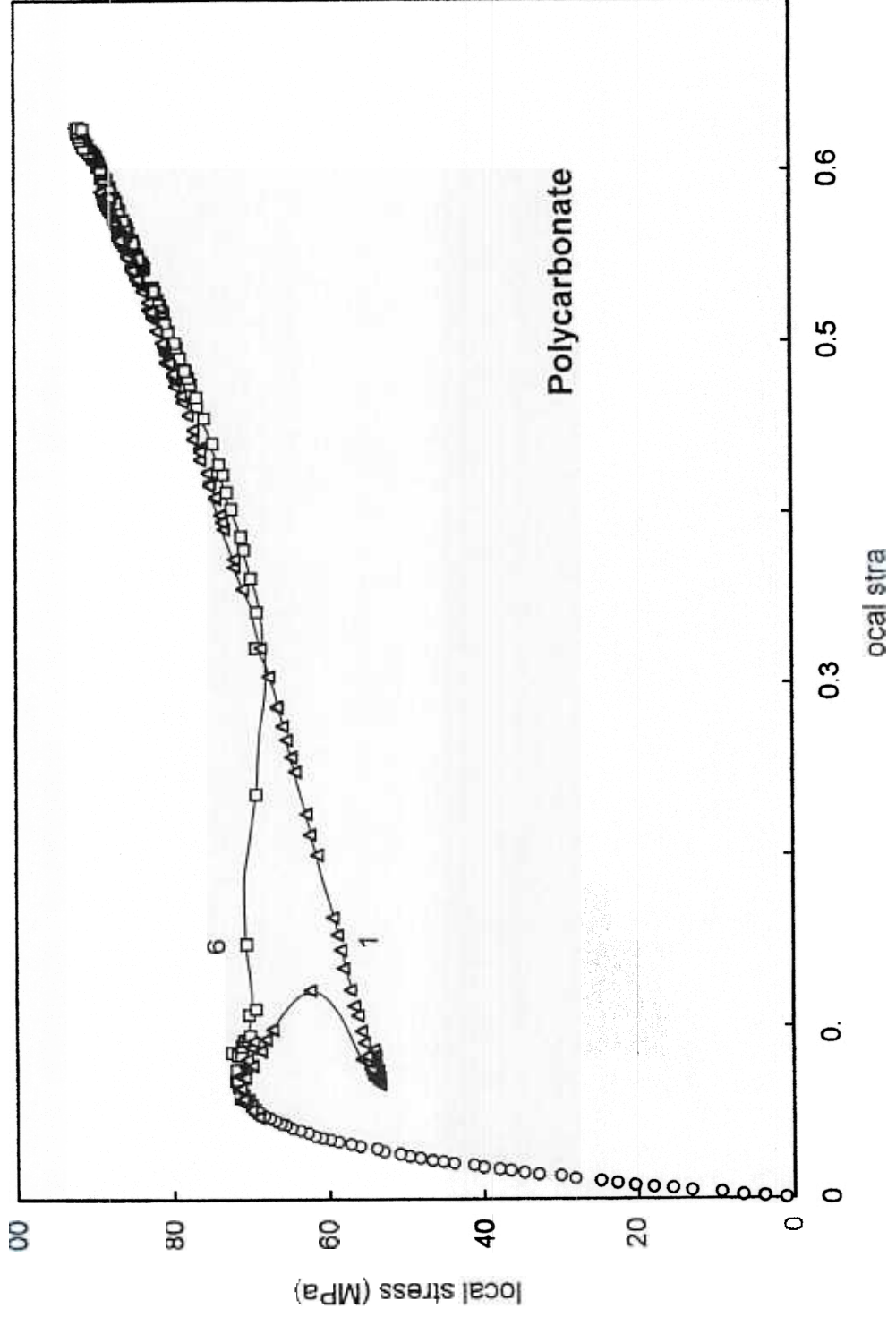


Fig.2



Fig

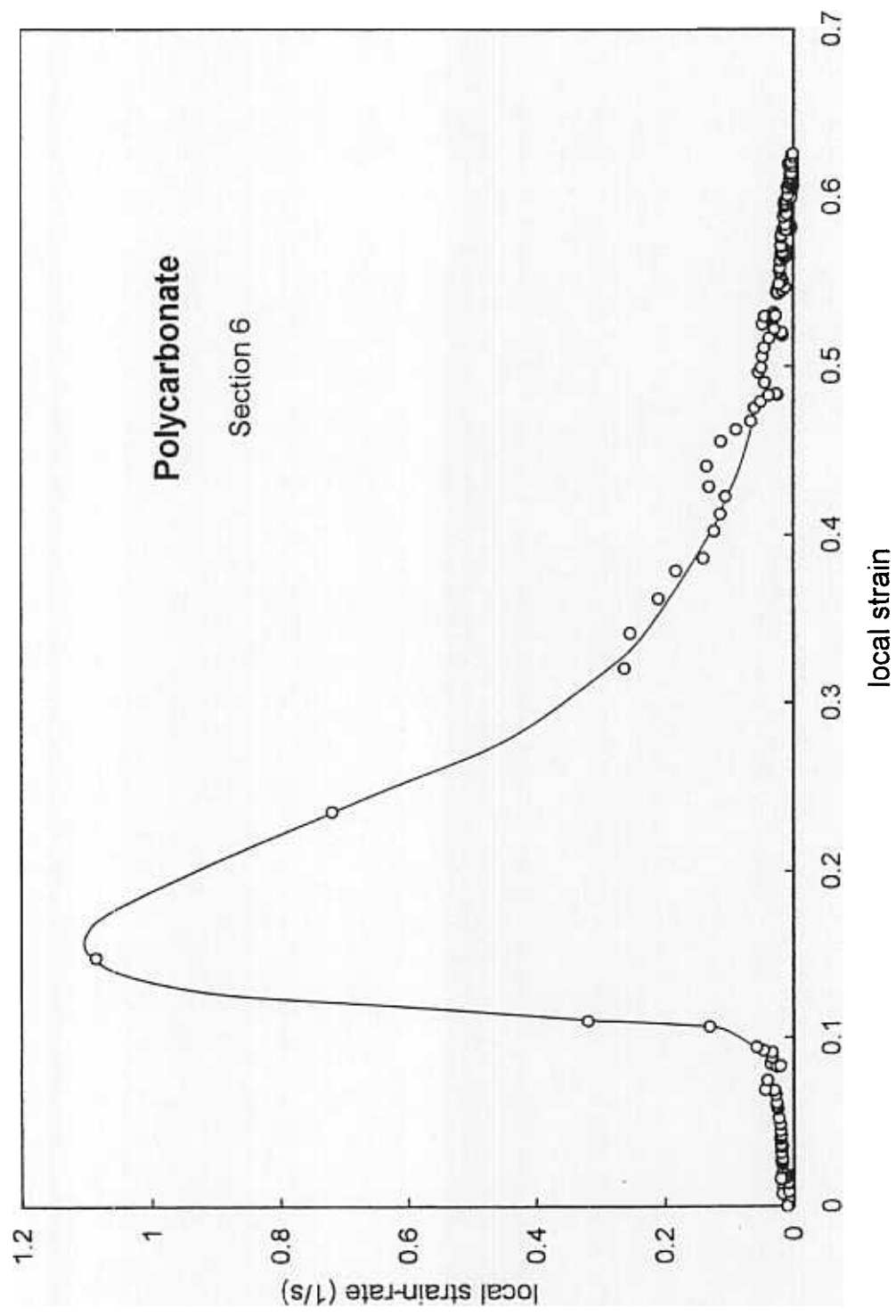
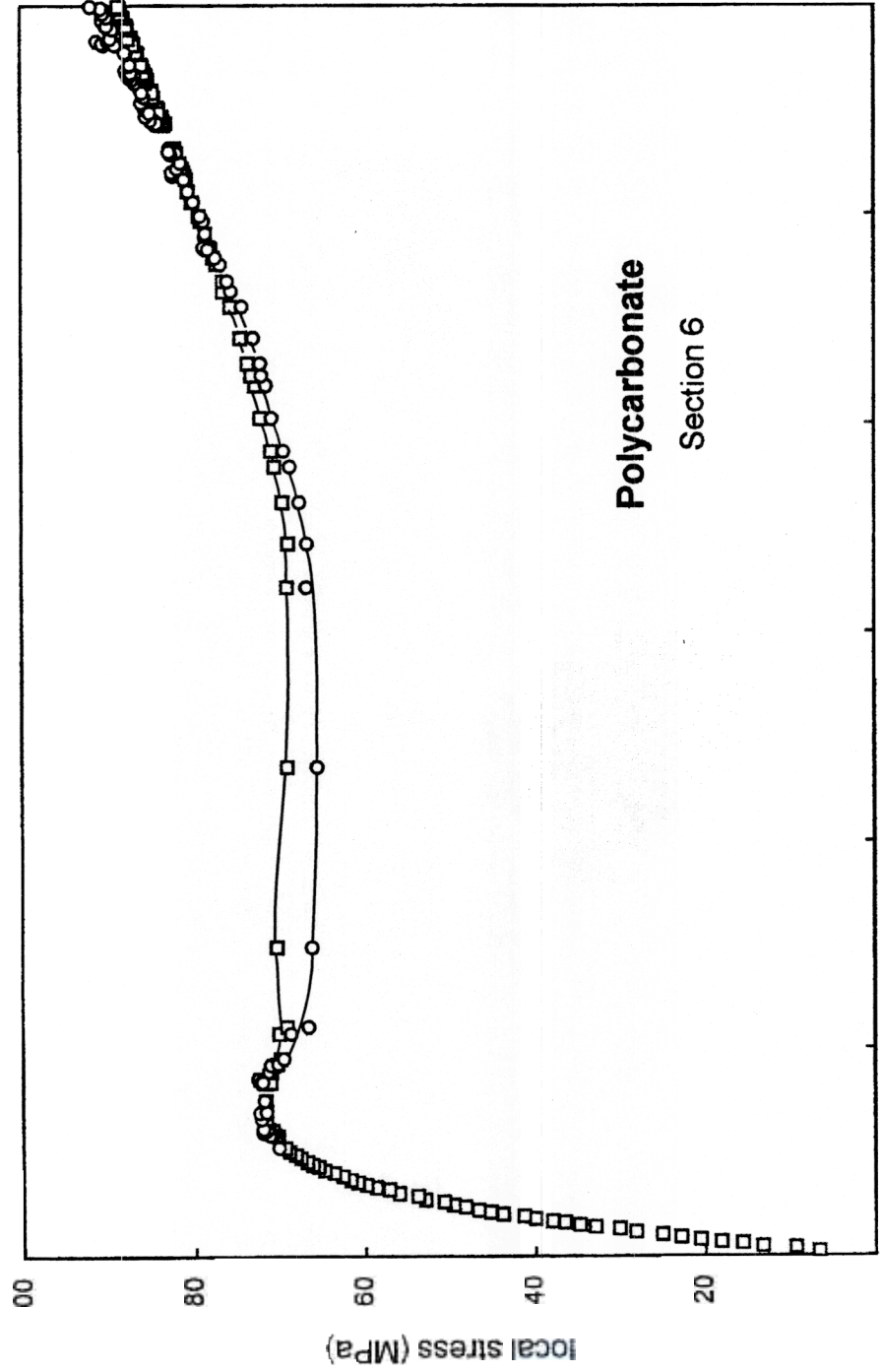


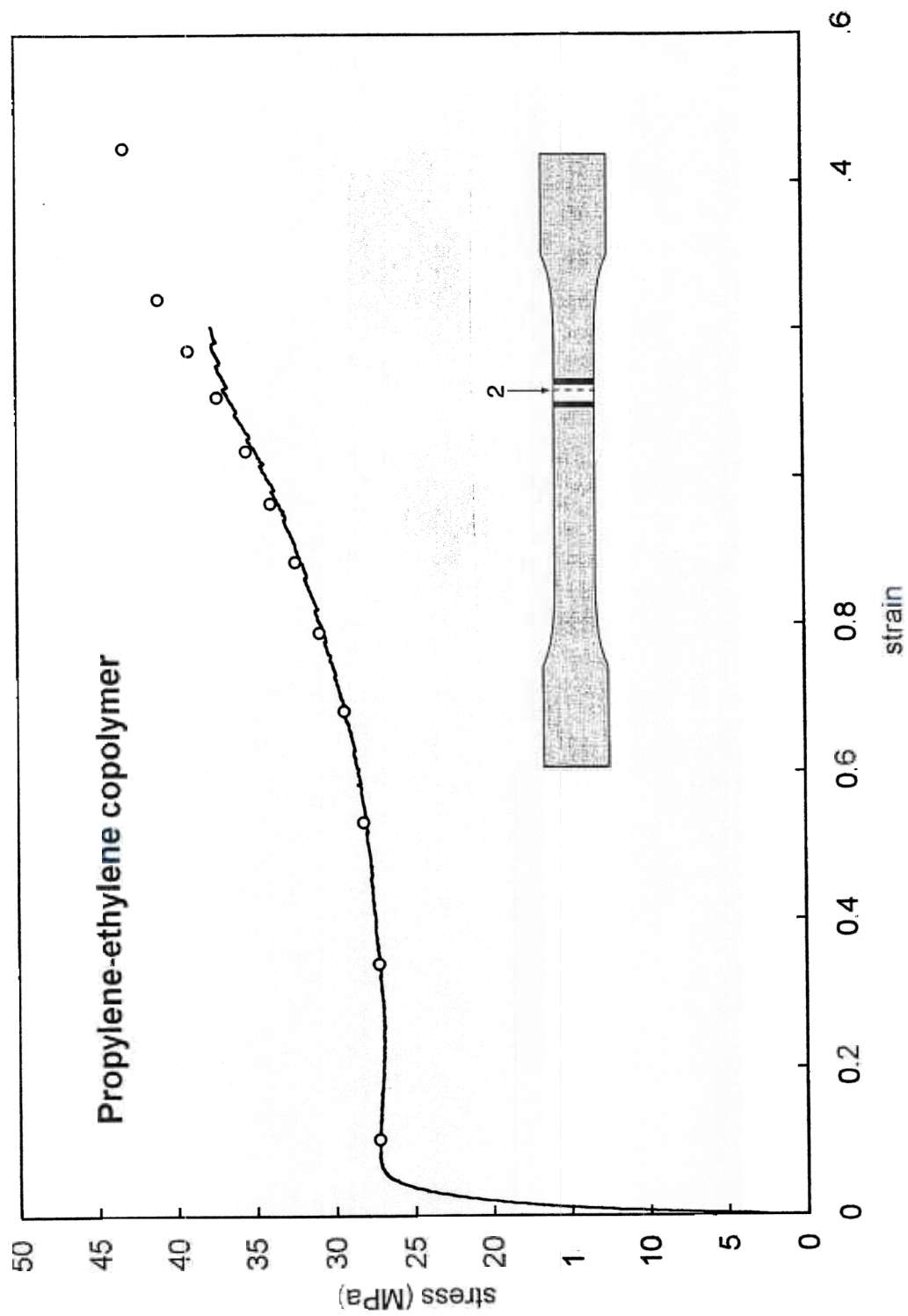
Fig. 4



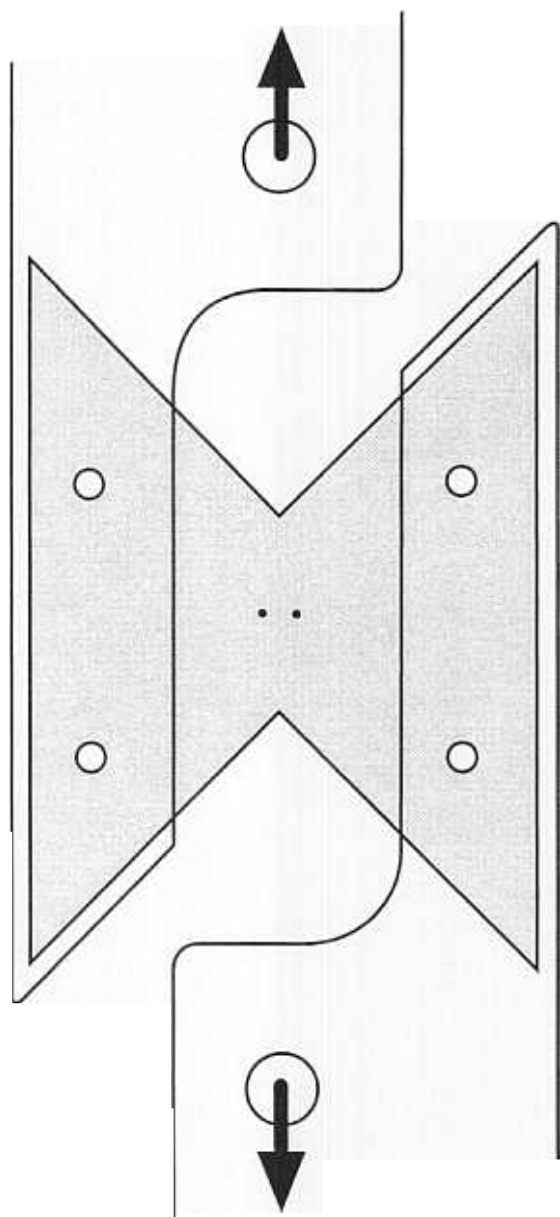
Polycarbonate
Section 6

local strain

Fi



Fig



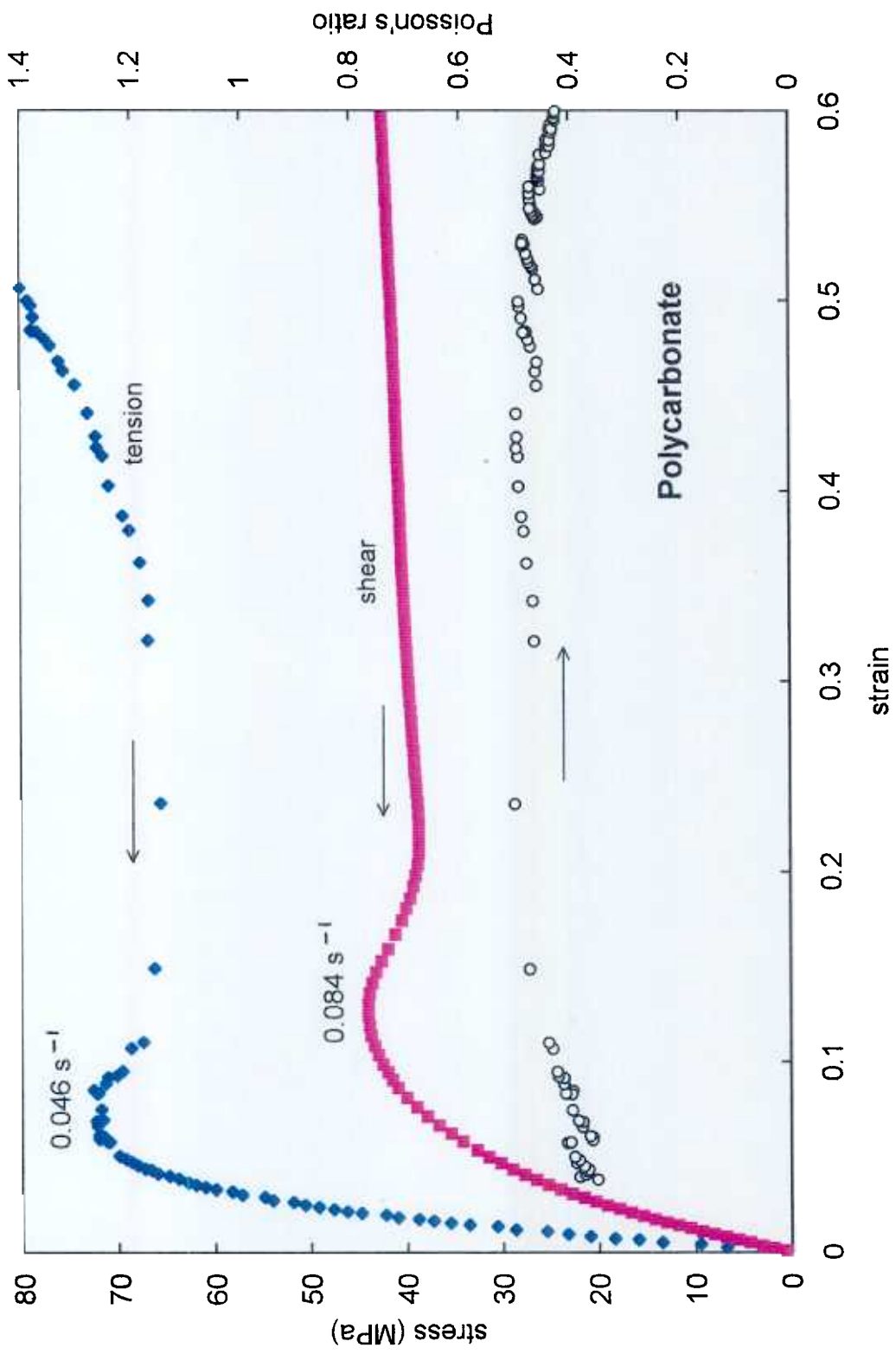


Fig. 8

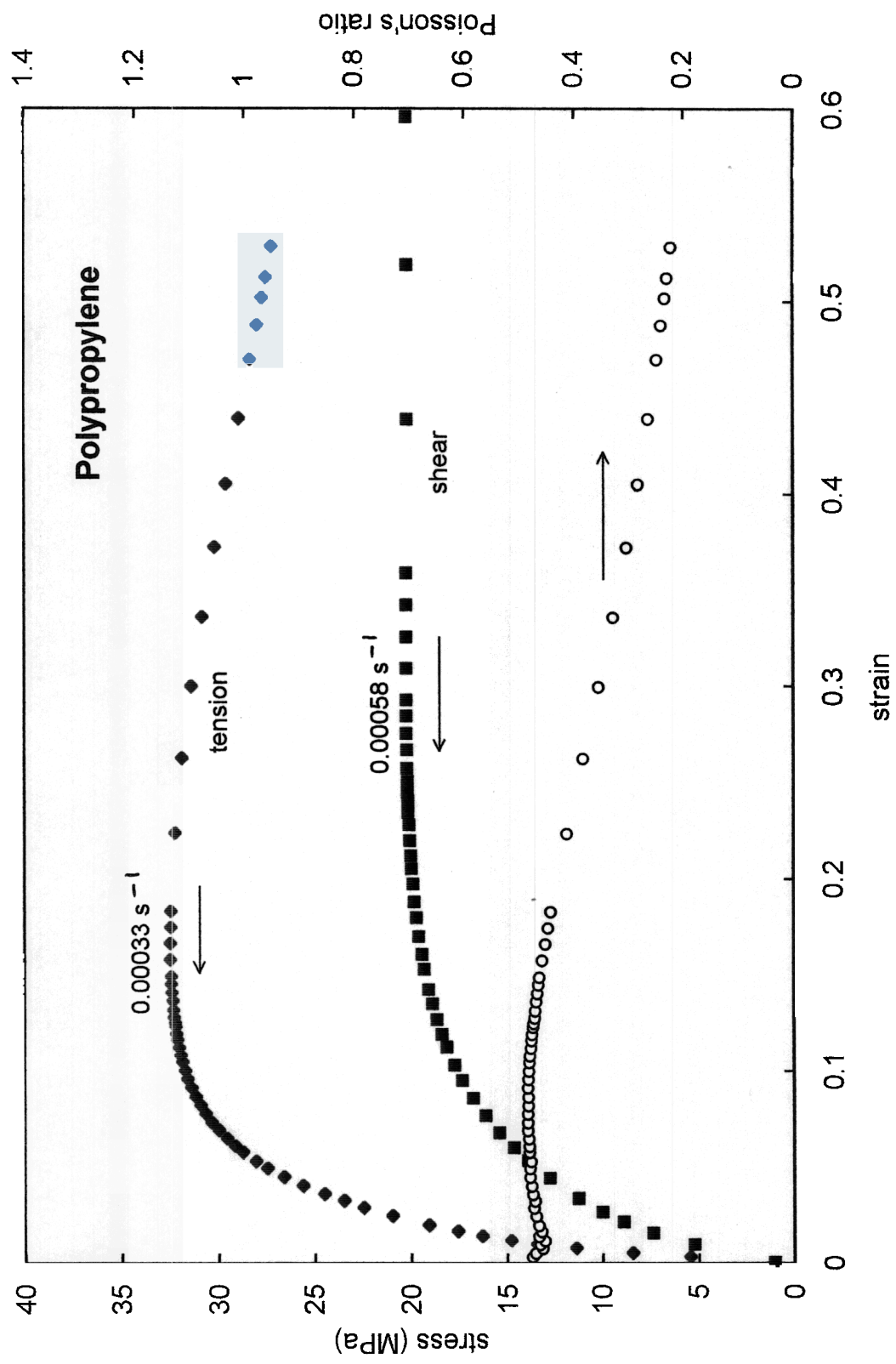


Fig. 9

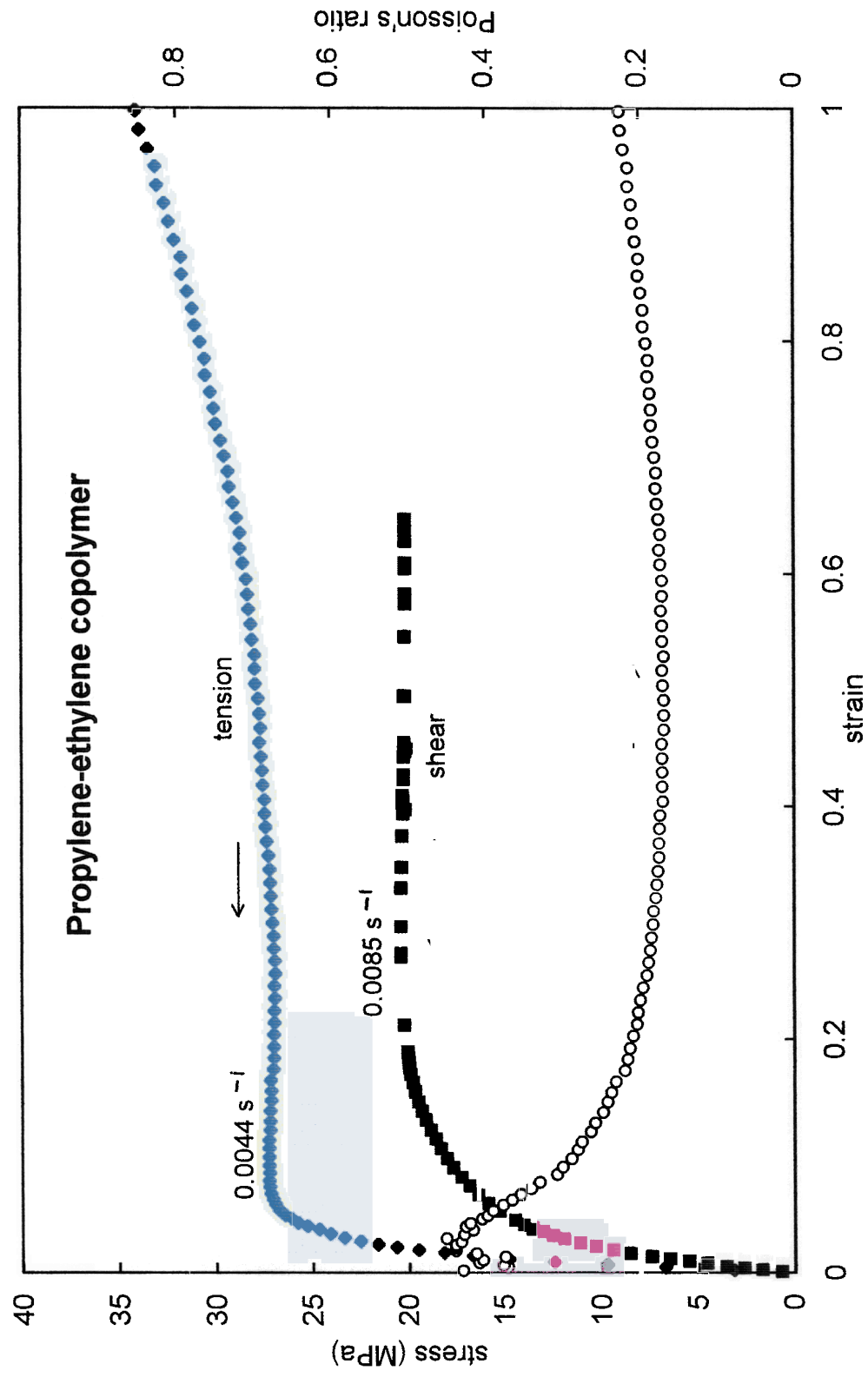


Fig. 10

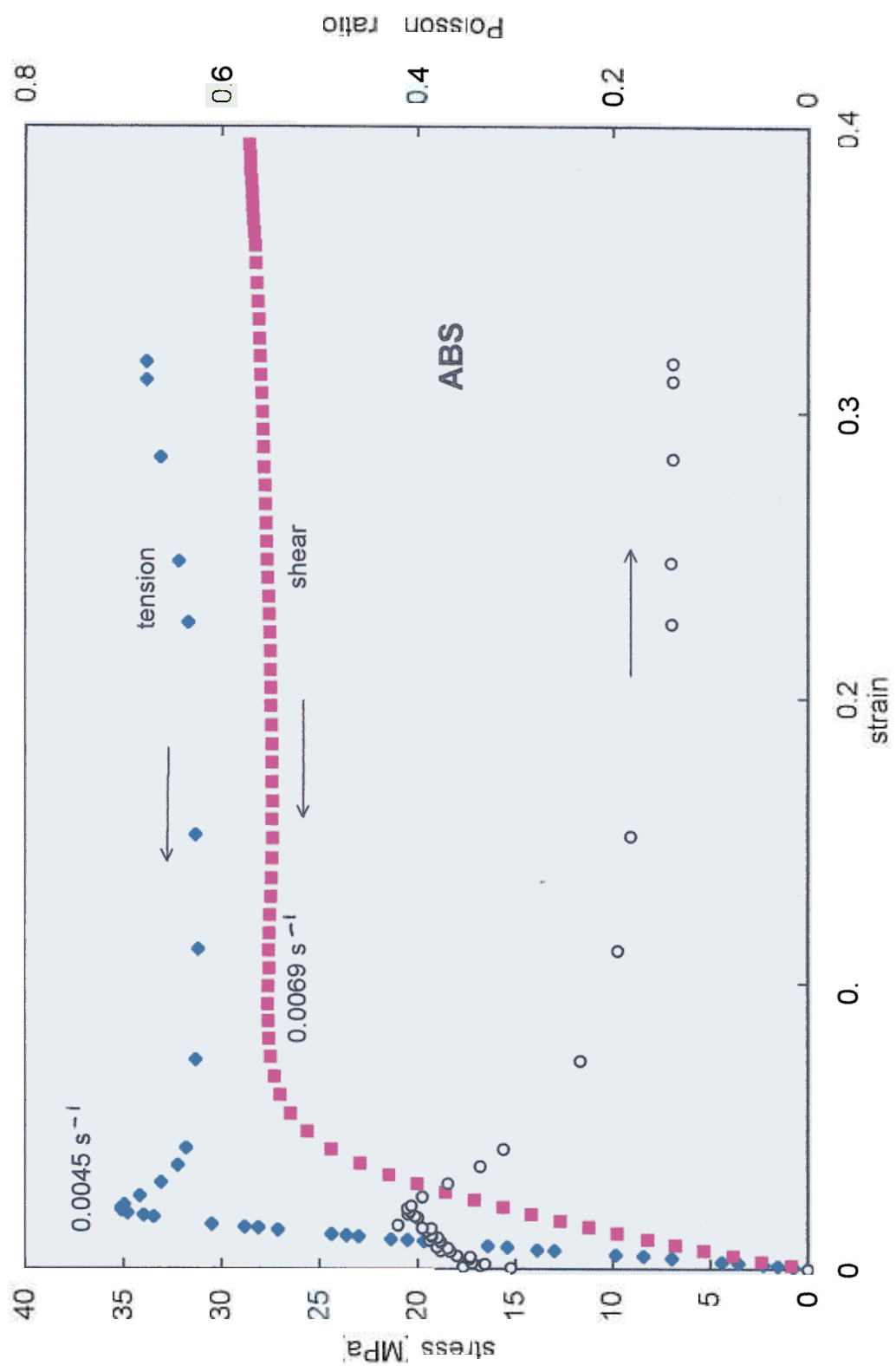


Fig. 11

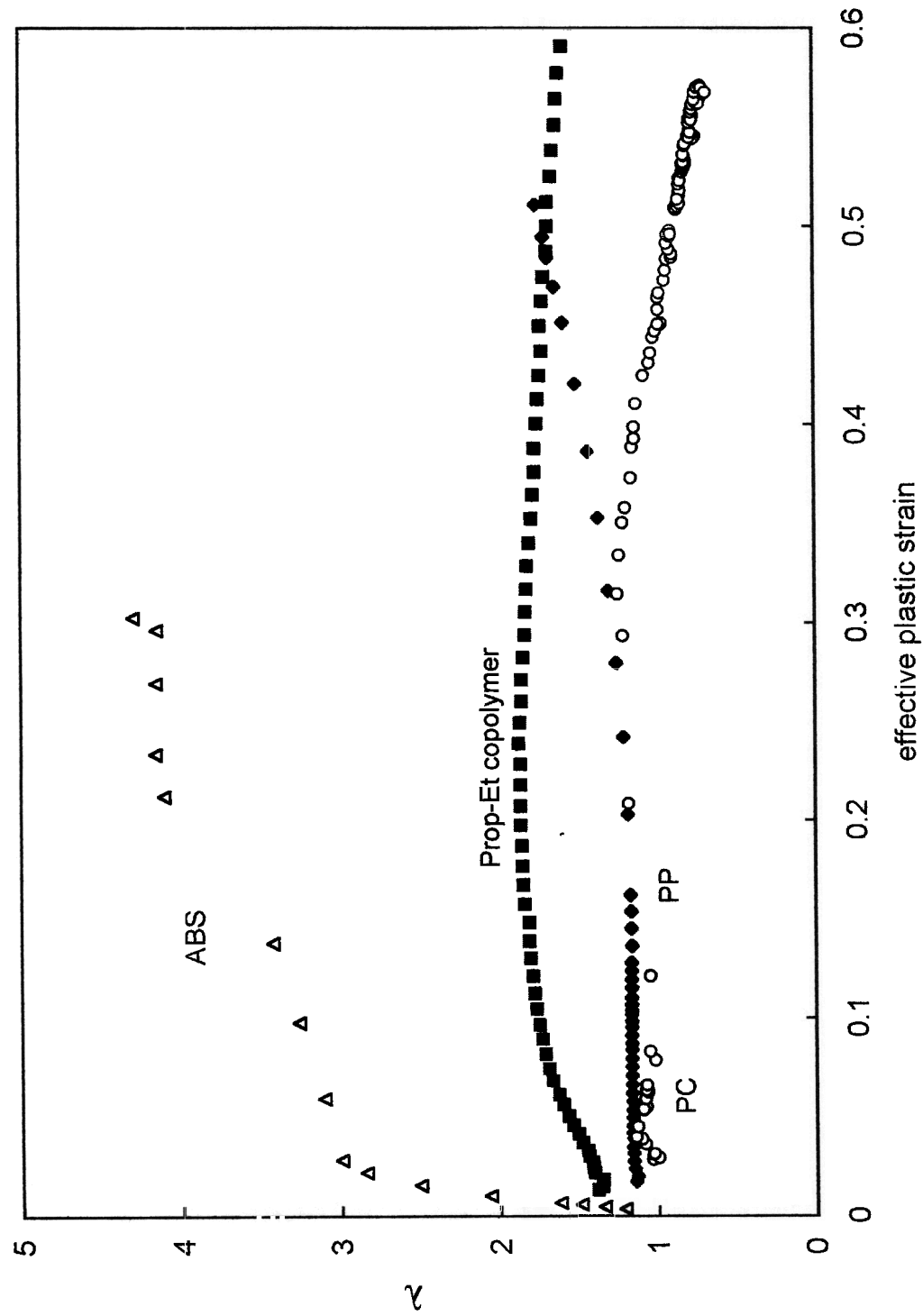


Fig. 12

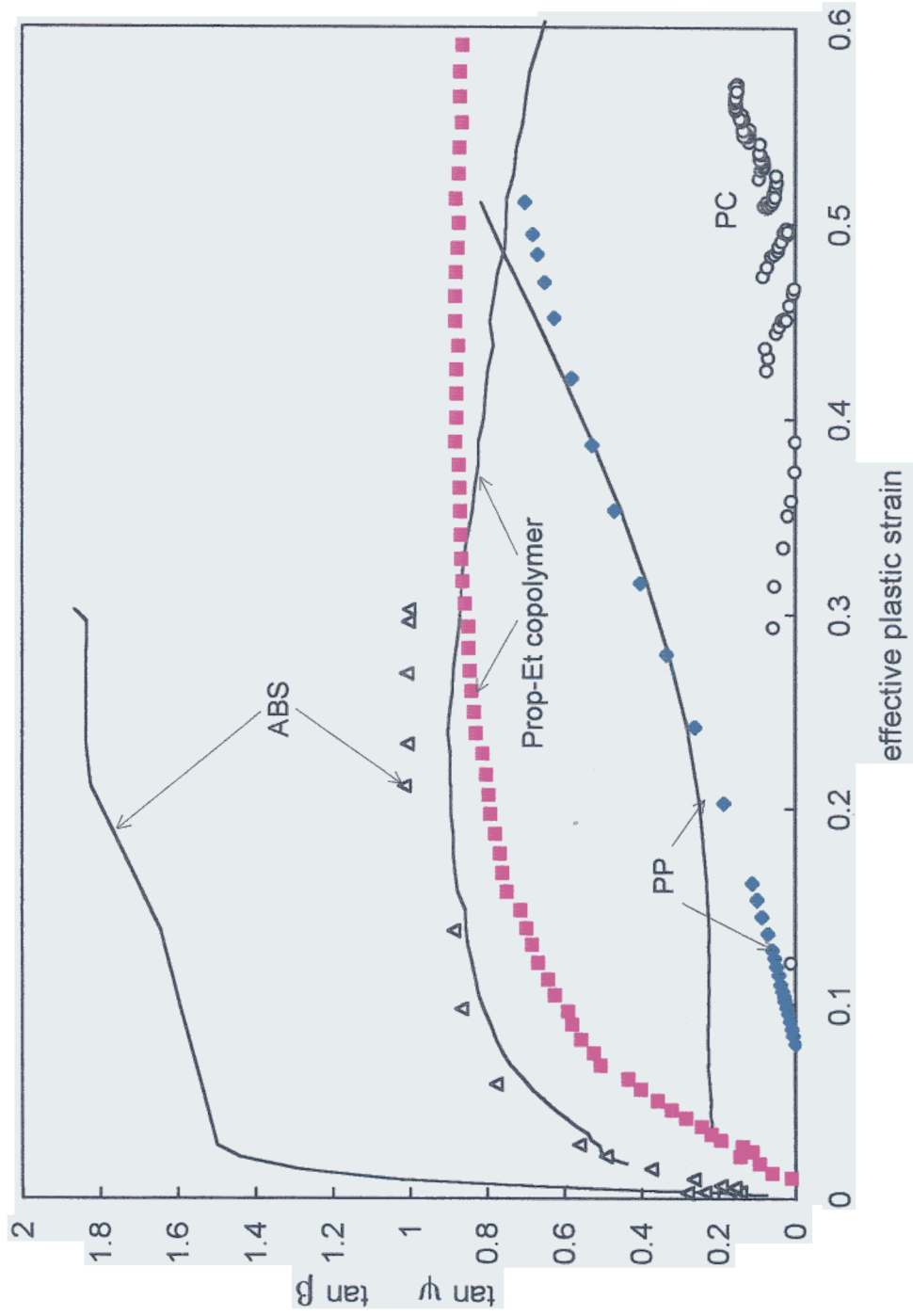


Fig. 13

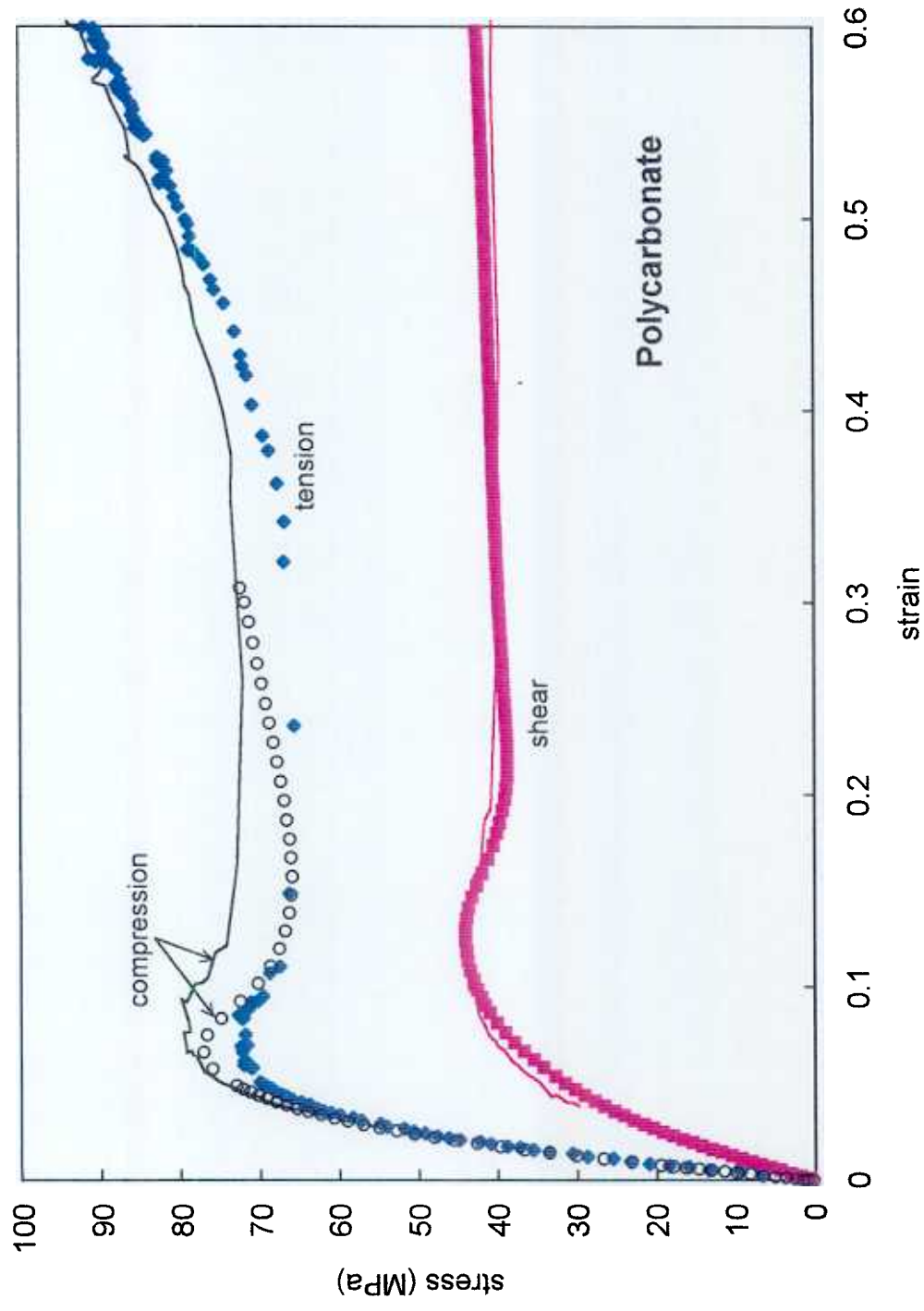


Fig. 14

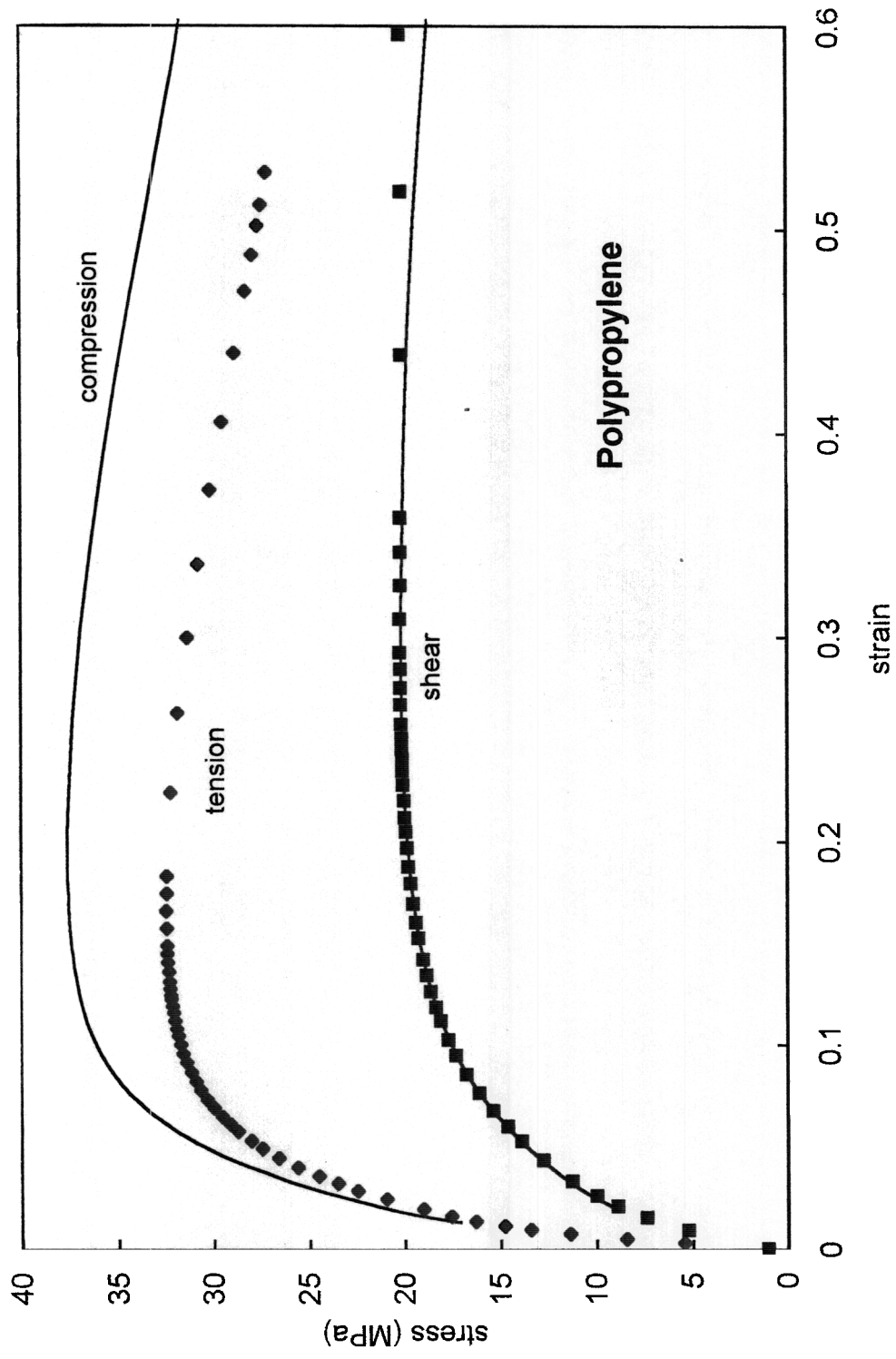


Fig. 15

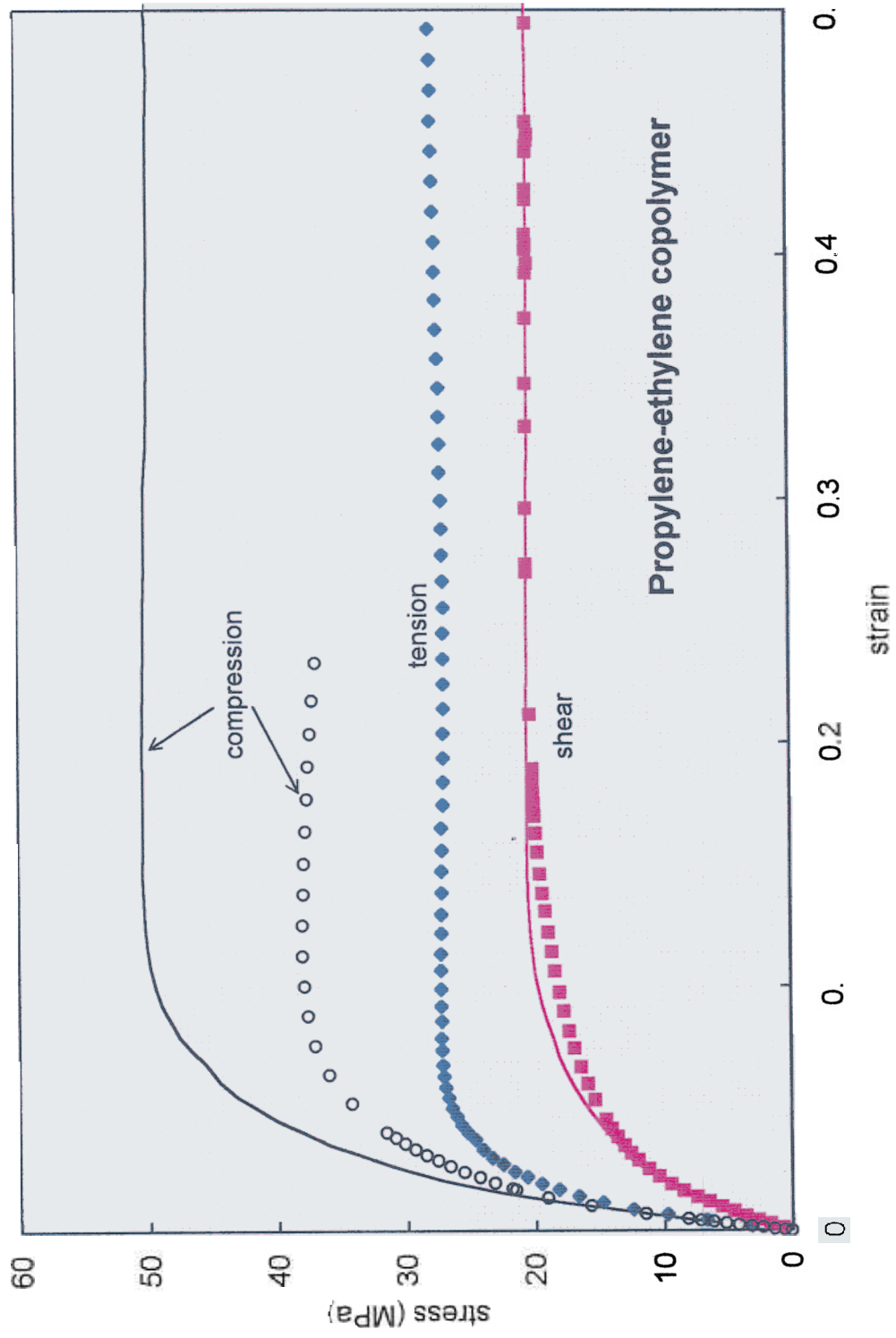


Fig 6

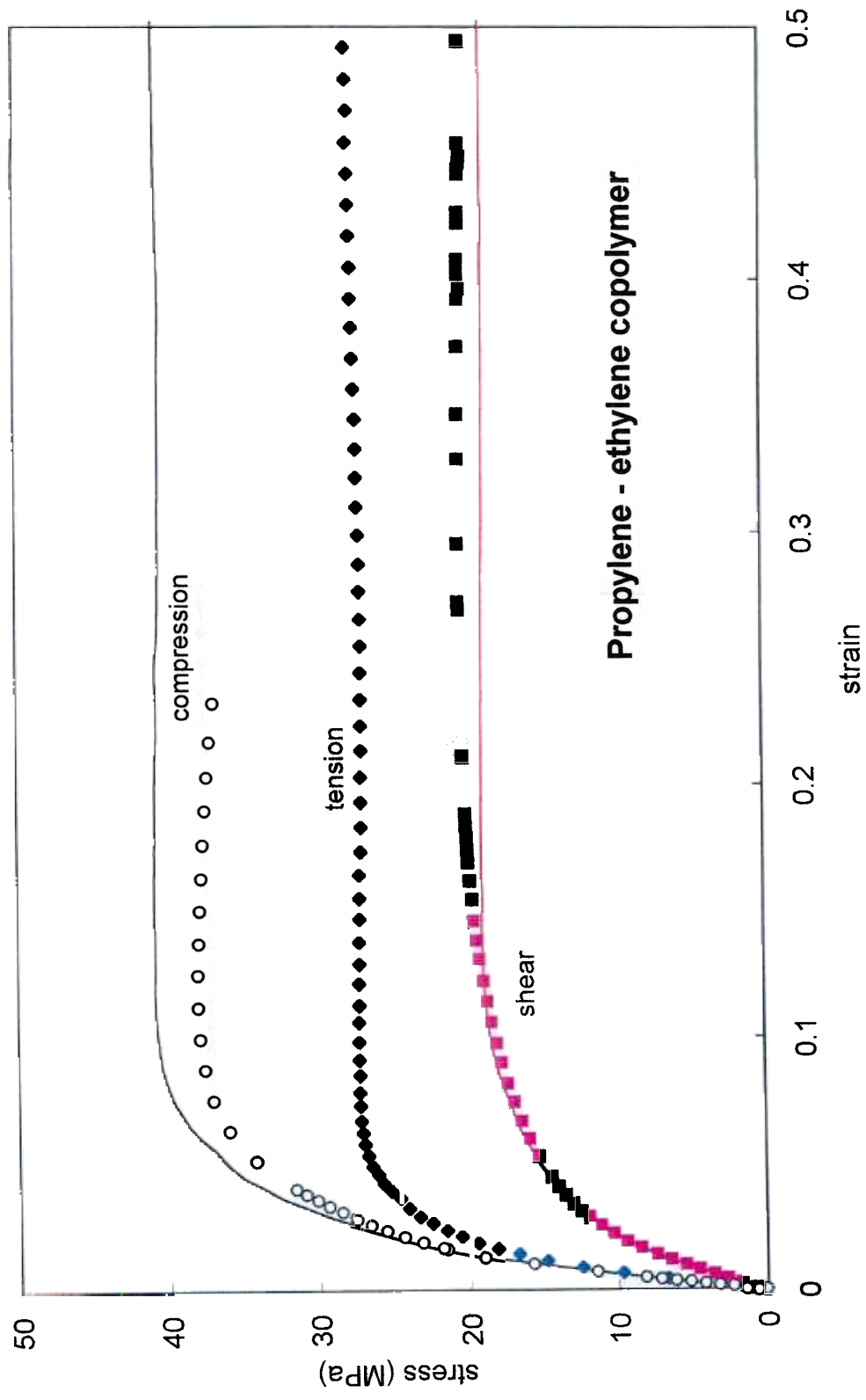


Fig. 17

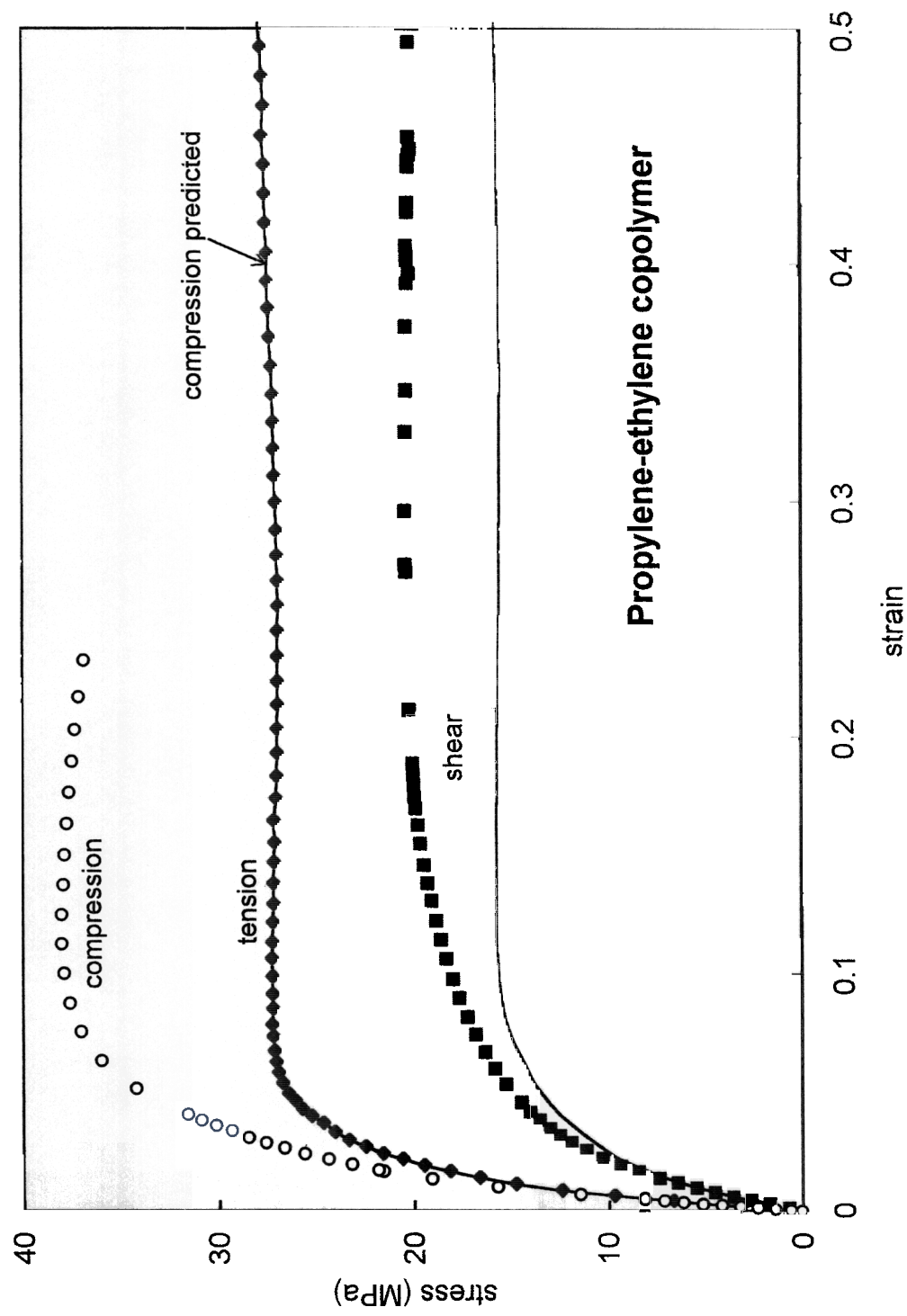


Fig. 18

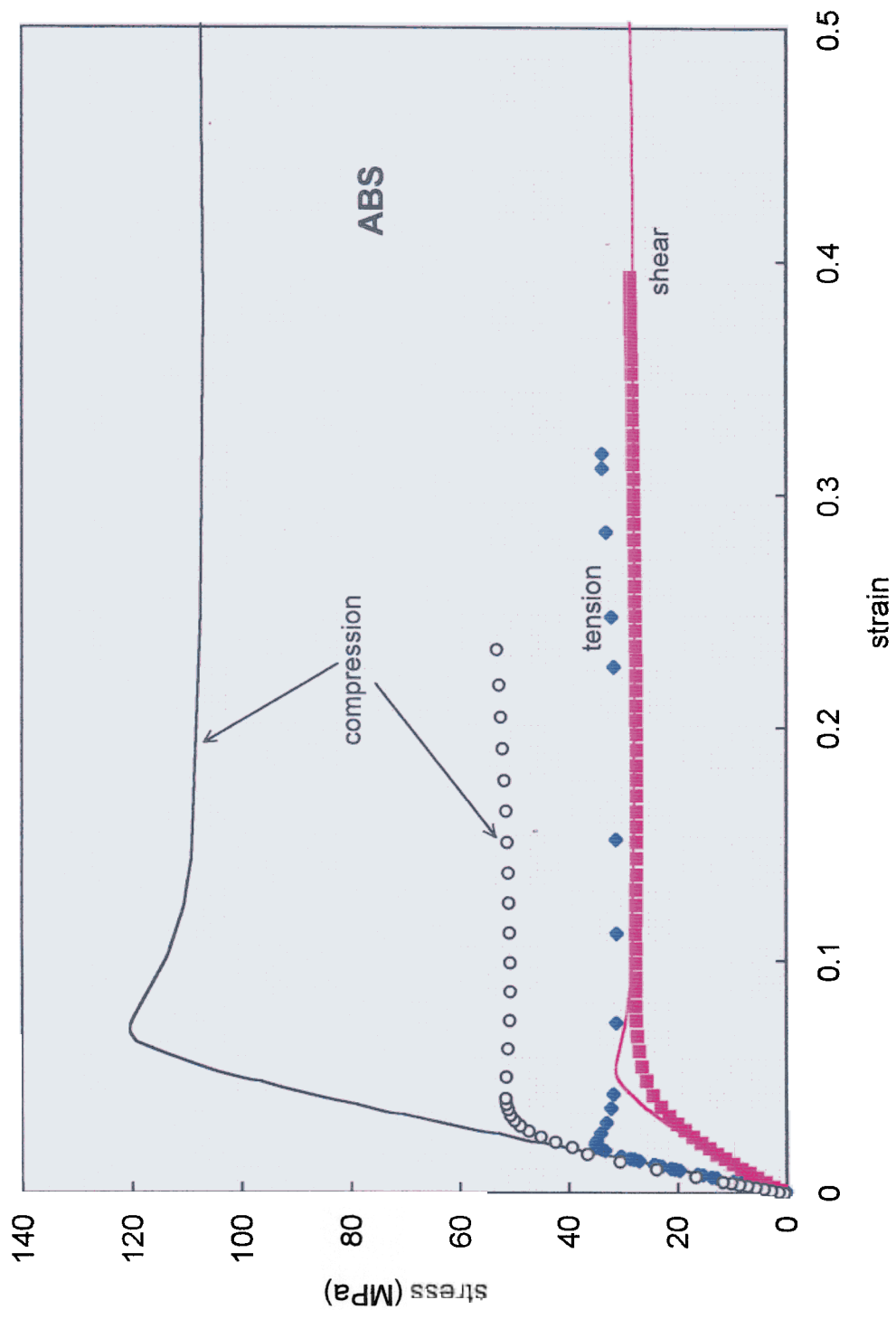


Fig. 19

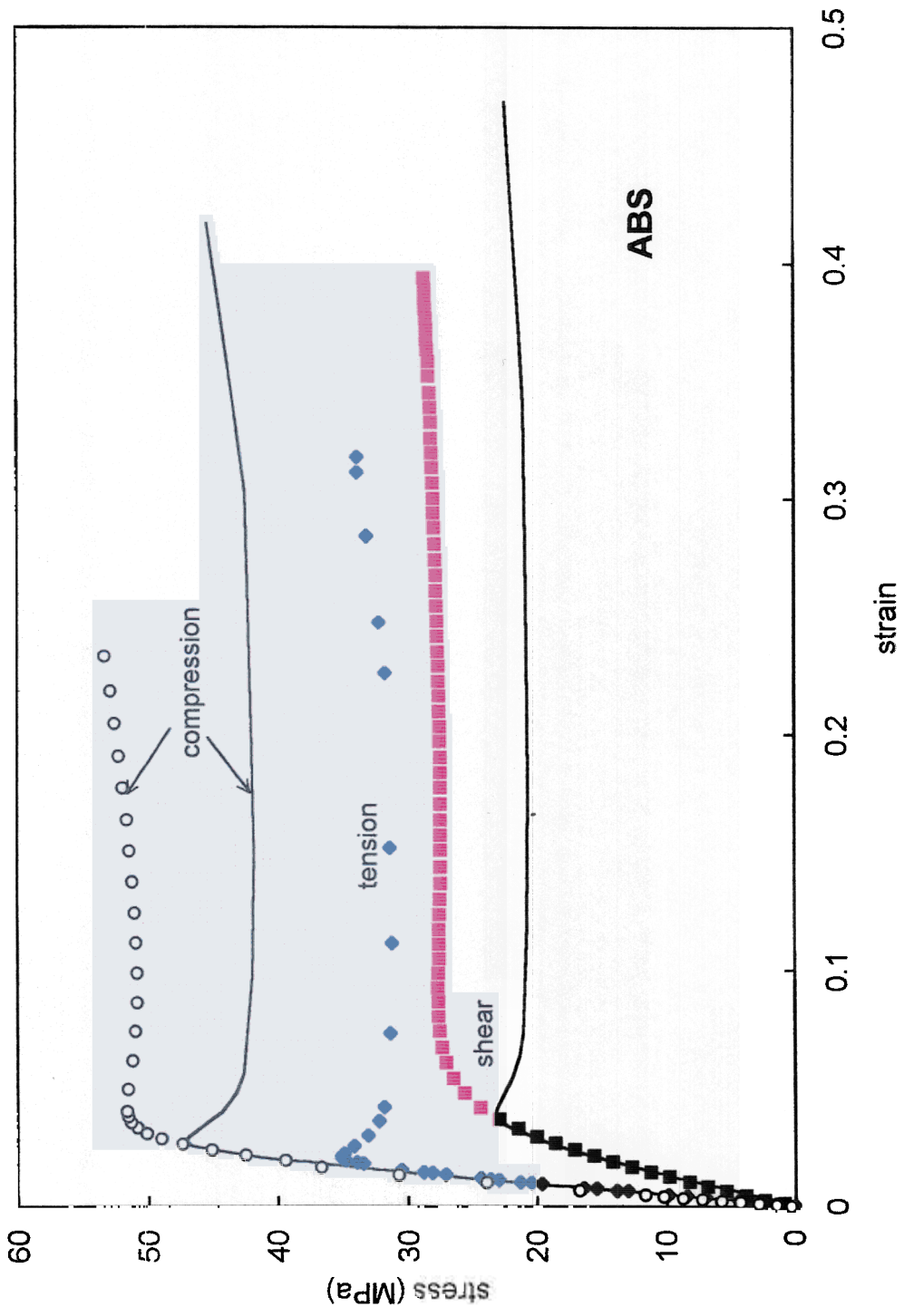
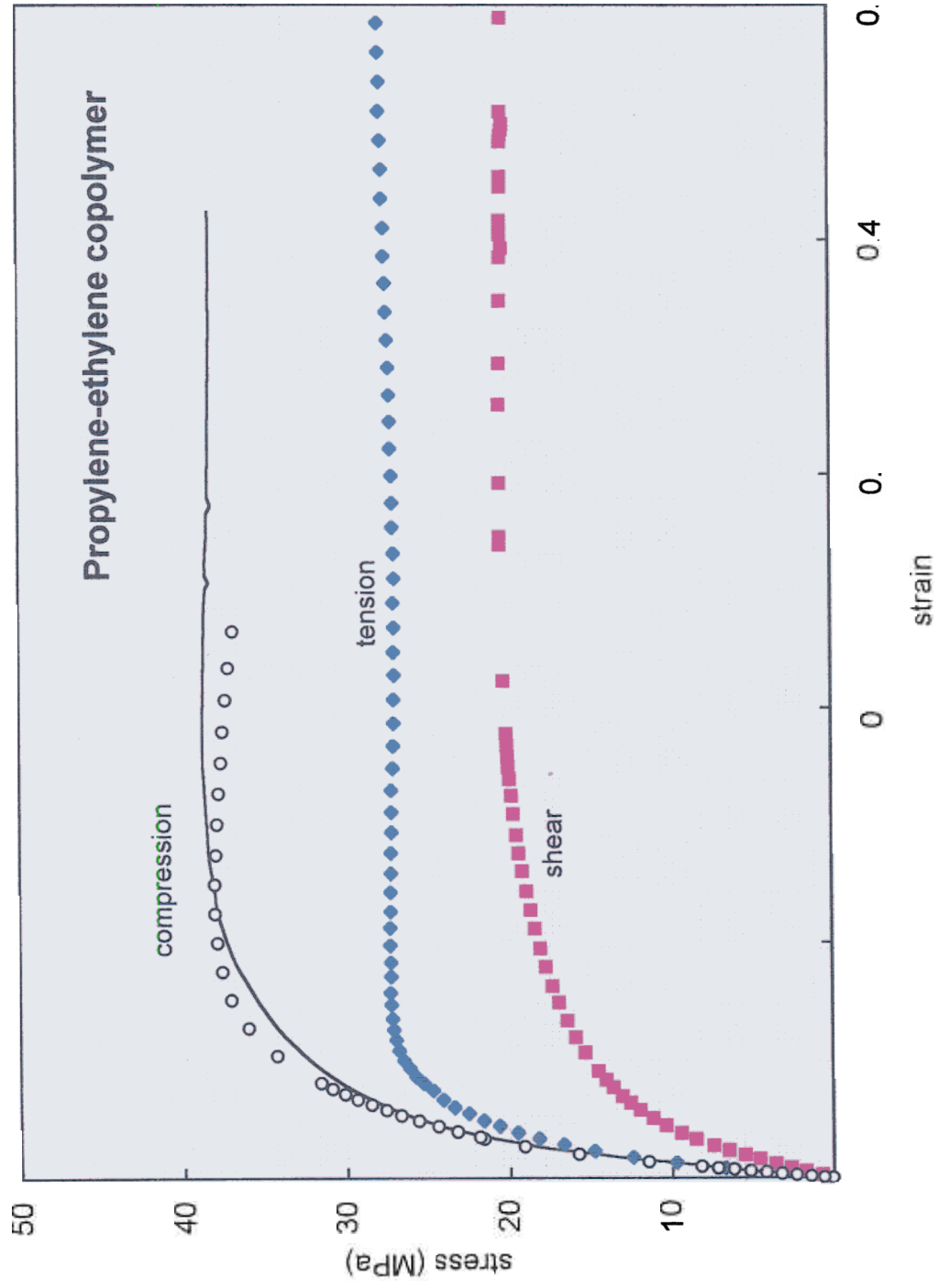


Fig. 20



Fig

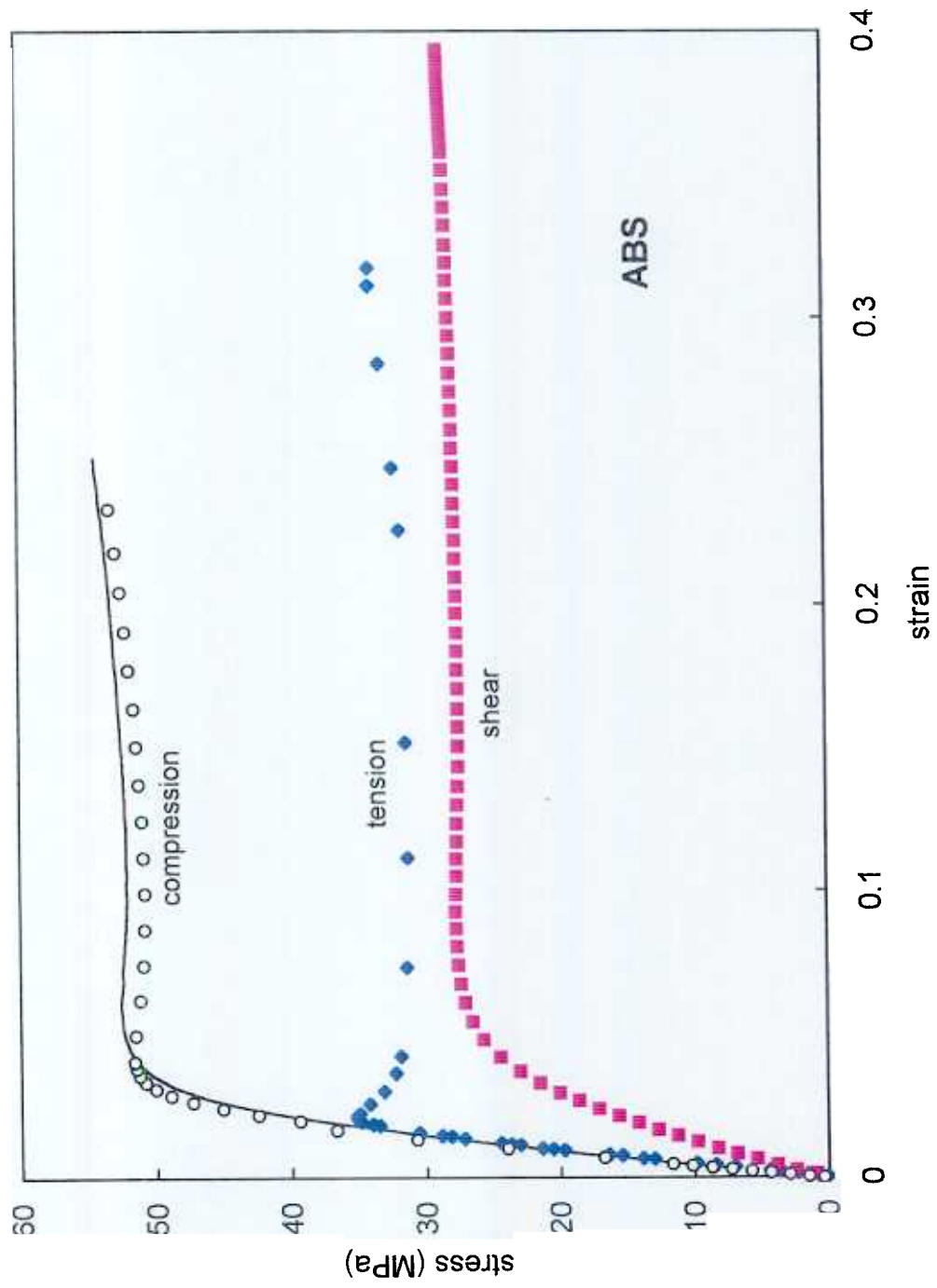


Fig. 22

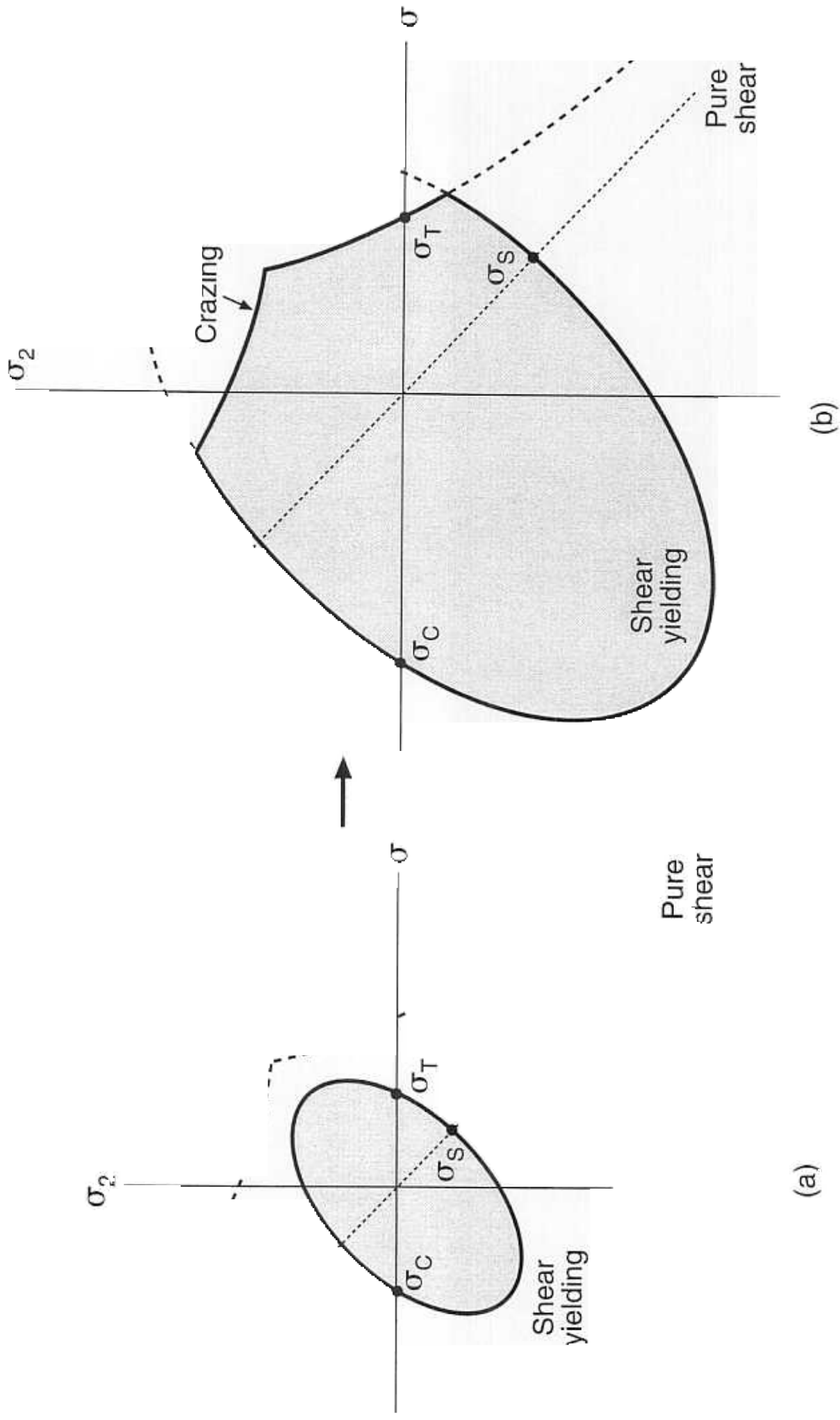


Fig. 23

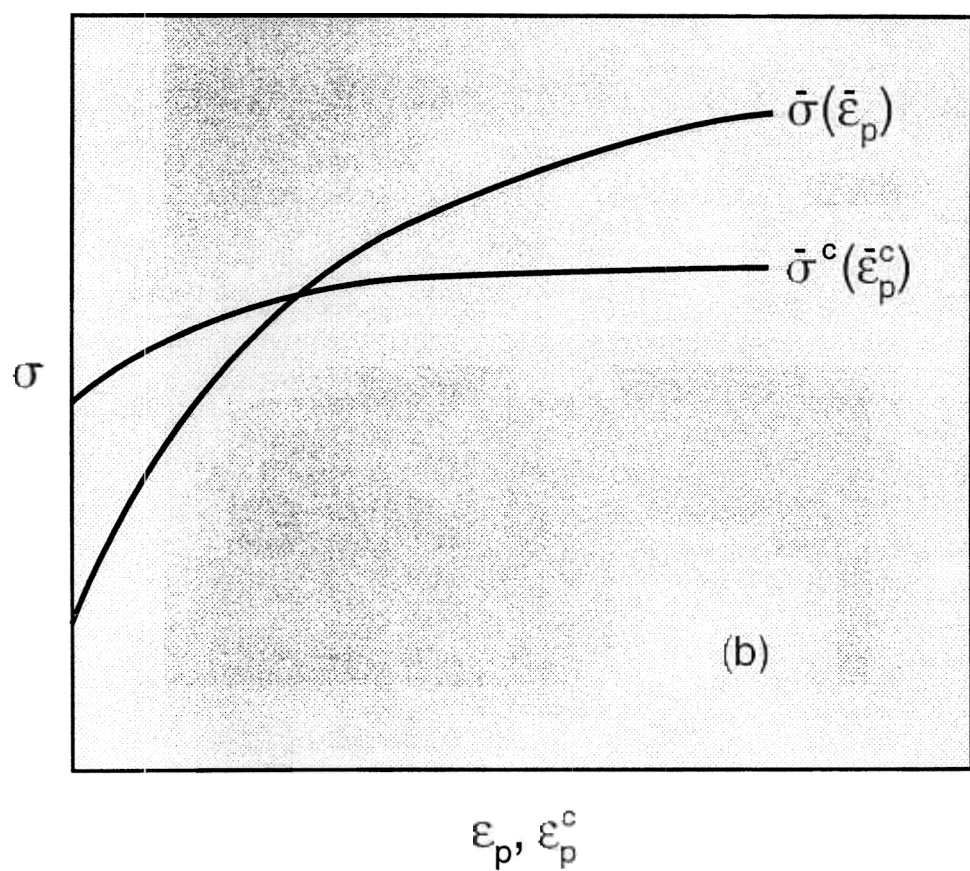
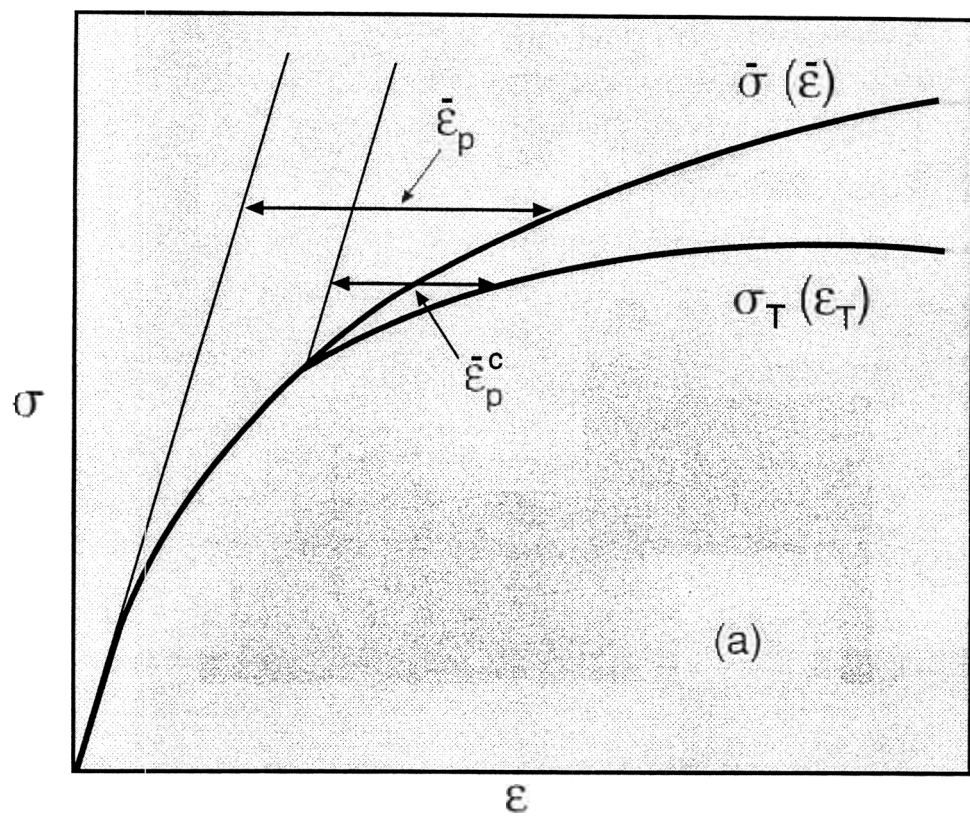


Fig 24

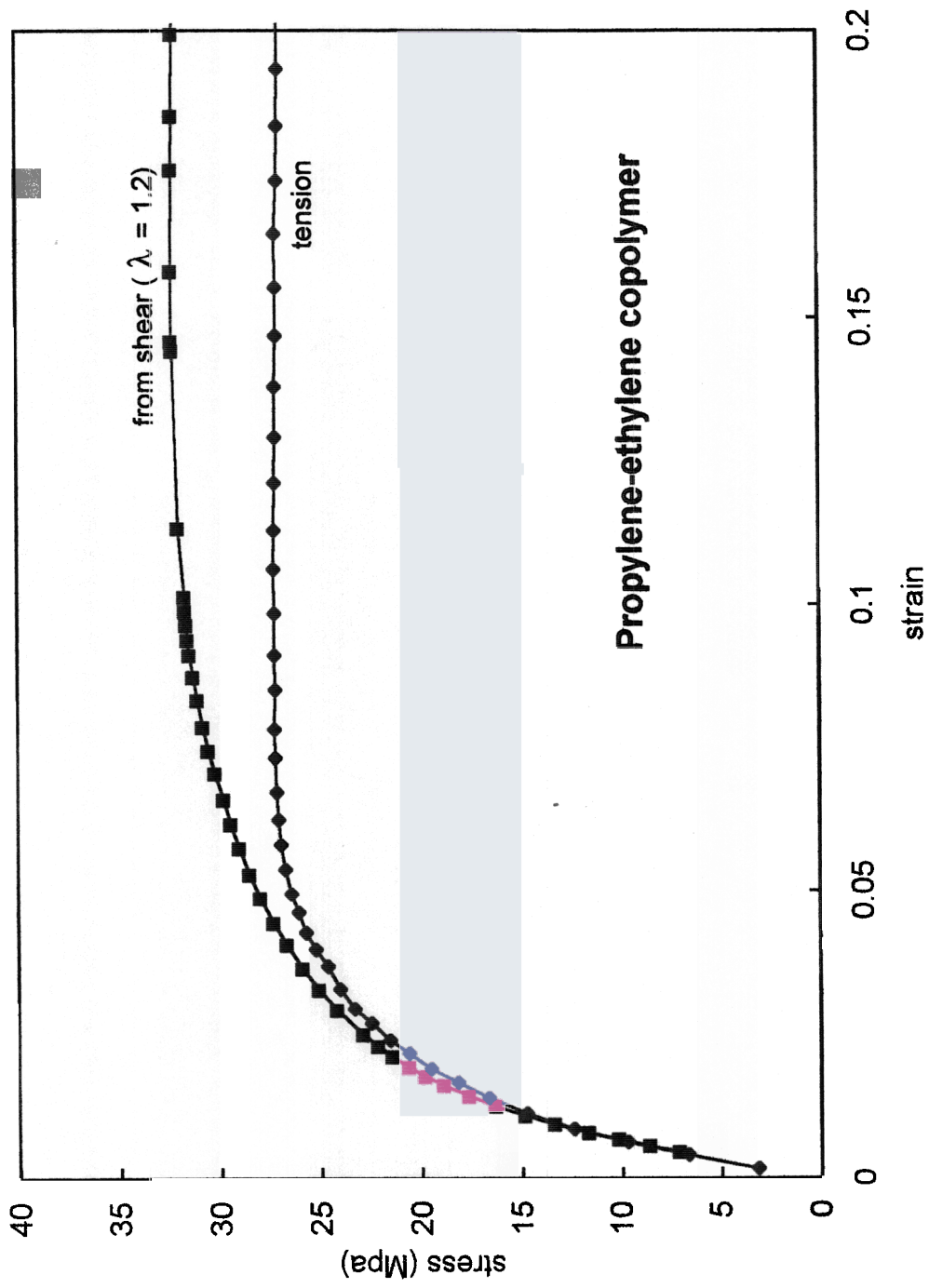


Fig. 25

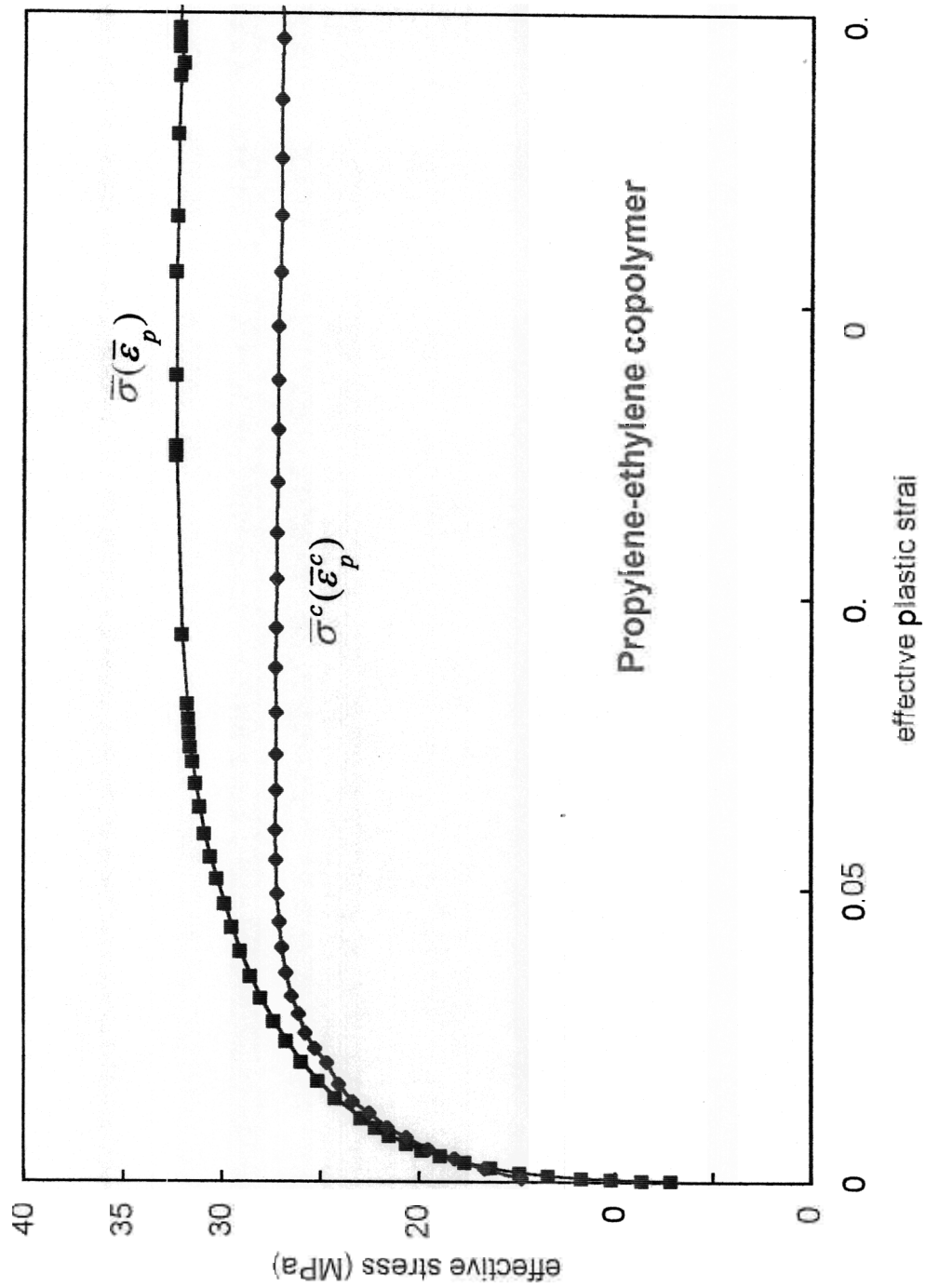


Fig 26

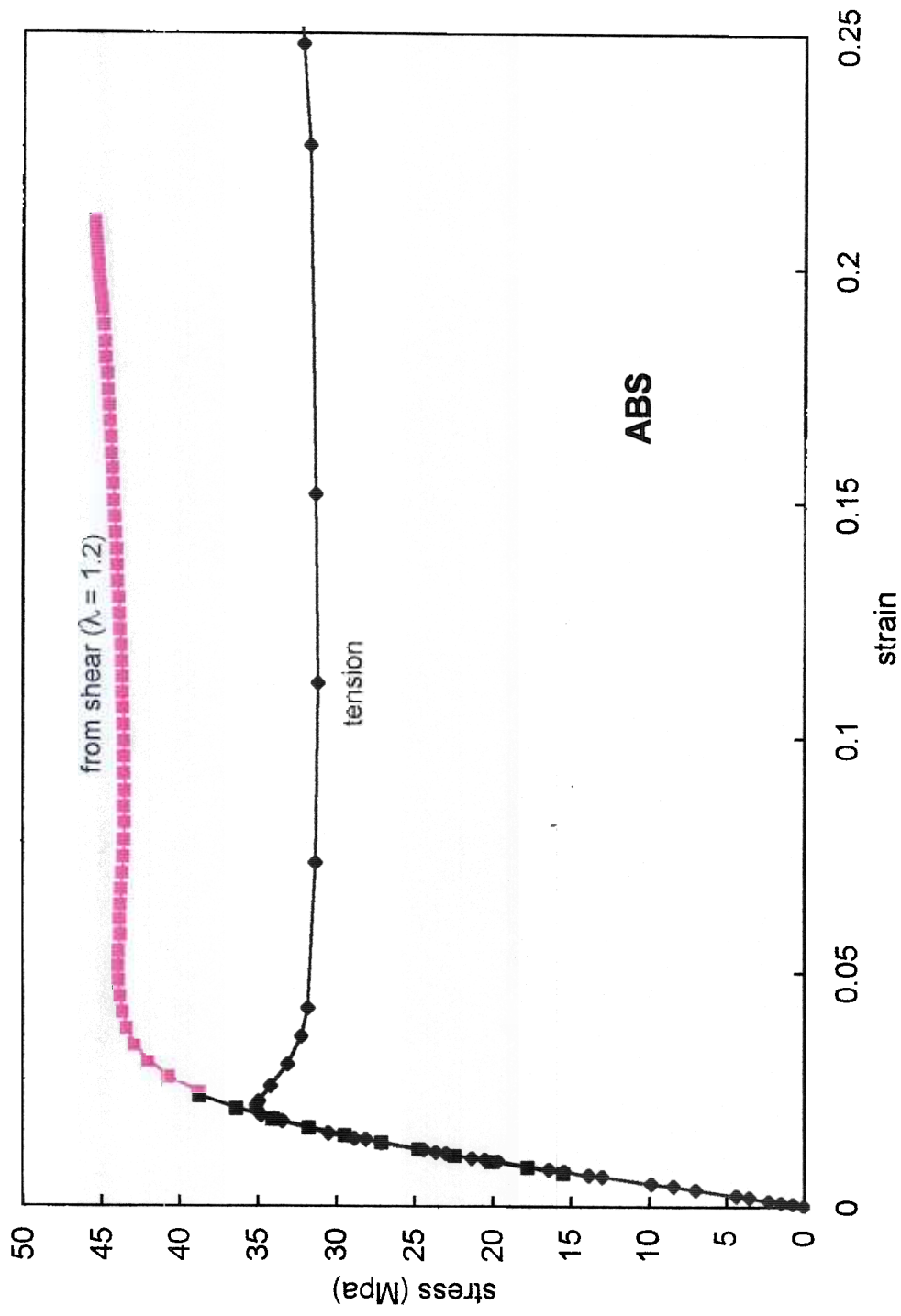


Fig. 27

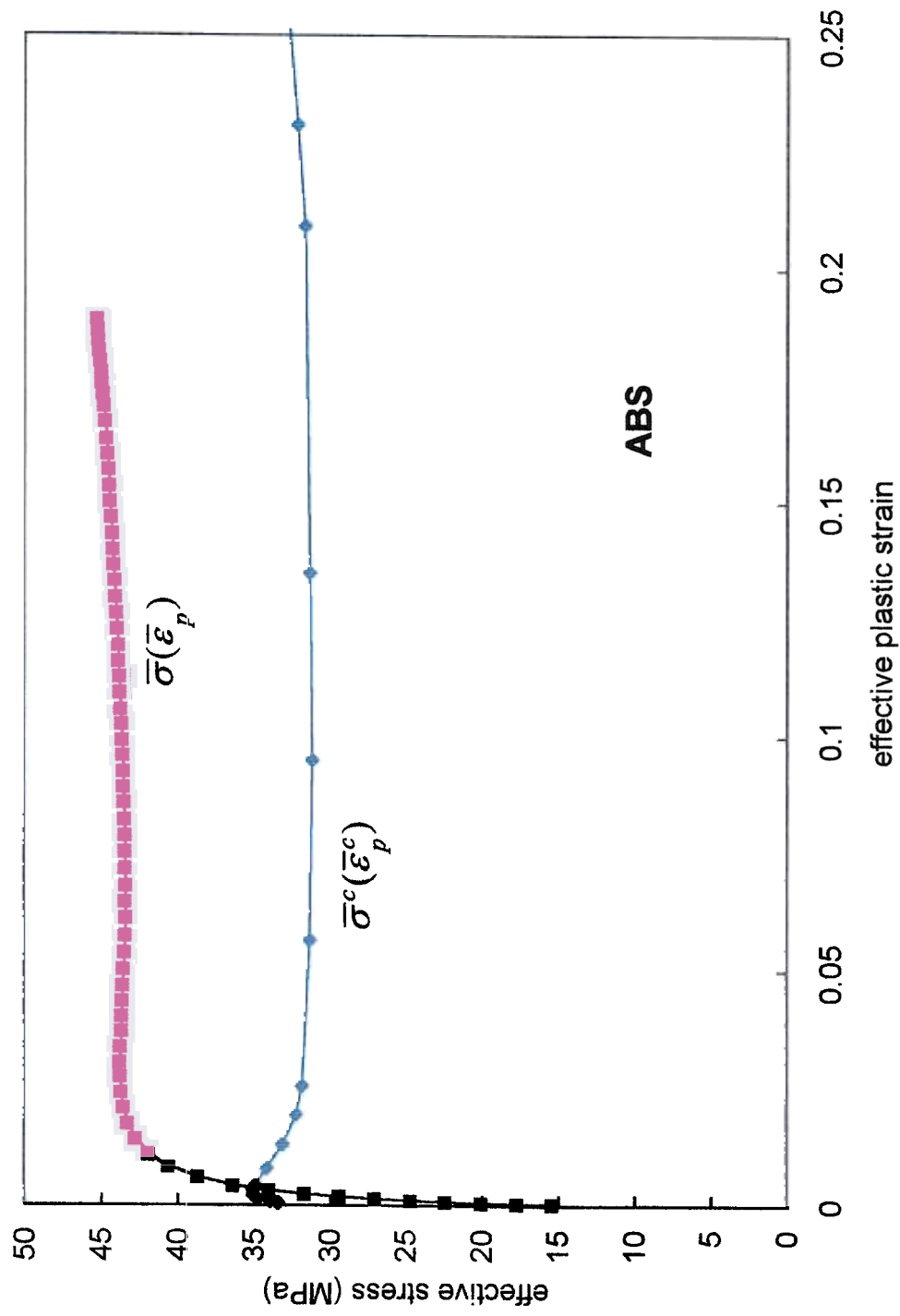


Fig. 28

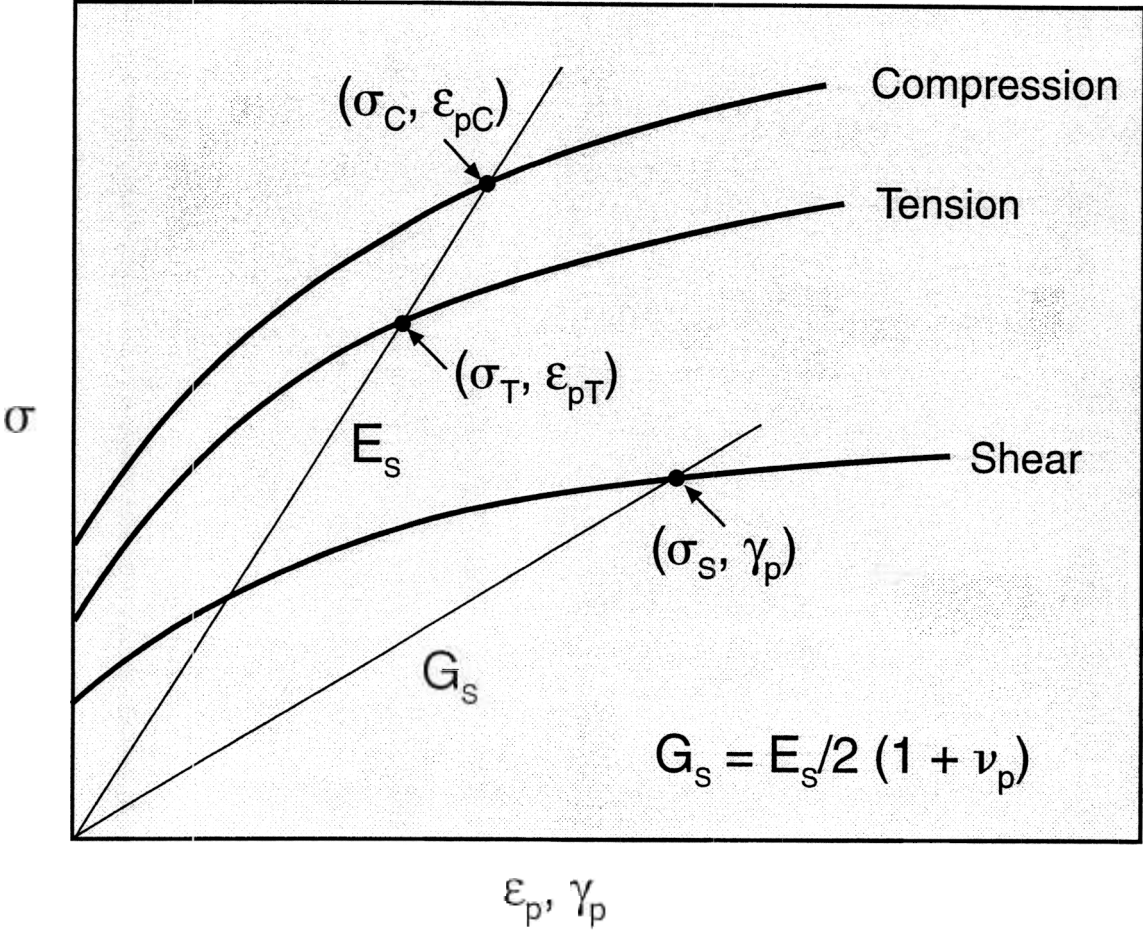


Fig. A