

A Strategy for Investigating the Influence of Processing Conditions on the Dimensional Stability of Injection Moulded Artifacts

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ABSTRACT

The influence of different processing conditions on shrinkage and warpage of a rectangular plate moulding has been assessed using the injection moulding simulation software package, Moldflow. Polypropylene and linear low density polyethylene were chosen as the feedstock materials. Six processing factors were considered over a range of values. A design of experiments approach has been used to limit the number of different combinations of processing conditions to twenty two.

The influence of different processing factors on the calculated shrinkage and warpage data has been analysed both qualitatively and quantitatively and at three different levels. These levels range in complexity from a very detailed analysis of how variations in the processing conditions affect local shrinkage and warpage to a more global treatment that suggests ways of enhancing the dimensional stability of the entire plate. The findings of this study are compared with those that have previously been published, and the potential problems that may be encountered in this type of study are discussed.

The analysis of both shrinkage and warpage data can only be used to reveal the quantitative links between dimensional instability and processing factors on a very detailed level i.e. shrinkage in a particular direction. This link is lost when shrinkage or warpage data are combined to give plate shrinkages or warpages, because of the difficulties of combining data spanning different ranges and units. The link can be restored if weighting factors are used, but it is not possible to generate these in an objective way.

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1. INTRODUCTION

All plastics shrink volumetrically by approximately 25% during cooling from a melt to a solid¹. However this can be reduced to levels of 2-3% for semicrystalline materials by cooling under pressure, as happens during the later stages of injection moulding. Even at these comparatively low levels, shrinkage can be a problem not only in the manufacture of precision injection mouldings but also for ensuring that snap fits, bottle caps or other mating parts function according to their original design^{2,3}. Excessive shrinkage can also result in ejection problems leading to damaged mouldings. Unbalanced shrinkage within a moulding causes the build up of residual stresses during cooling. These stresses are typically relieved by the part warping after it has been ejected; if this does not occur then they can ultimately lead to in-service failure of the moulding.

Allowances can be made for shrinkage by oversizing the mould. However, deciding by how much and where to make the allowance has traditionally relied on the experience of the toolmaker and mould designer. This situation is continually changing due to the increasing use of process simulation software packages such as Moldflow⁴ and C-Mold⁵. These packages are able to compute the dimensional instability of a moulding for different combinations of processing conditions for a given mould design and material. Changes in some of the processing conditions can be used to compensate for the variations in shrinkage and to a lesser extent warpage that exists between different grades of the same plastic. Generally part warpage can be reduced by extending the cooling time such that the moulding is effectively annealed within the mould cavity. However this practice limits productivity. Being able to control shrinkage by changing processing conditions is particularly useful when manufacturing items from differently coloured feedstocks in the same mould, as certain pigments can affect the shrinkage behaviour of plastics. Whilst it is acknowledged that changing the processing conditions can be used to limit dimensional instability, the question of which one or ones are the most effective to change and by how much remains. This is a complex issue as many of the processing variables cannot be independently altered. The problem has been addressed in a number of publications^{2, 6-14}. However most of these do not consider warpage or present a systematic methodology for exploring the relationships between different combinations of processing variables and dimensional stability.

This paper describes a strategy for investigating the relative influence of a range of processing conditions on the dimensional stability of a plate moulding simulated using Moldflow. The findings of this work will be used to define an experimental plan for assessing the role that different pigment types and reinforcing agents play in influencing the dimensional stability of injection moulded parts.

2. CAUSES OF DIMENSIONAL INSTABILITY

The factors that affect the dimensional stability of injection moulded parts have been described elsewhere^{11,15,16}, hence only a brief summary will be presented here. There are four main factors that affect shrinkage:

- changes in volume during cooling,
- crystallization,
- orientation,
- mould restraint.

As molten plastic is cooled it becomes progressively denser. The temperature at which it solidifies is marked by a discontinuity in a plot of specific volume versus temperature as shown in Figure 1. This transition is far more pronounced for semicrystalline polymers in comparison with amorphous materials due to the additional reduction in volume that occurs

through crystallization. The rate at which crystallization occurs depends on the type of polymer and its processing history. It can also be affected by the presence of additives such as pigments that accelerate crystallization by acting as nucleating agents. The degree and rate of crystallization can also be enhanced by both molecular and fibre orientation (where fibres are present) that results from melt shearing during mould filling. Typically high shear stresses are generated as the melt flows through narrow cross-section regions of the mould cavity. These shear fields distort the random coil equilibrium conformation of long chain polymers and are affected by the injection speed, the mould and melt temperatures and the geometry of both the cavity and the gate. As soon as the mould cavity has filled, the distorted polymer chain conformations begin to relax back towards a random coil configuration. The amount of molecular relaxation that occurs as the melt is cooled depends on the cooling rate which will depend on local conditions such as thickness and proximity to a gate. These effects tend to be more noticeable in semicrystalline polymers as molecular orientation enhances crystallization. Aligned fibres can significantly alter the shrinkage behaviour of the moulded part by restraining the movement of the matrix during the cooling phase. Features such as ribs and bosses may also limit shrinkage by pinning movement of the material within the mould cavity during cooling although this can lead to ejection problems and internal stress.

Unbalanced shrinkage can induce warpage if the residual stresses frozen into the moulding during cooling are sufficiently large to overcome the structural integrity of the part. These can be caused by frozen-in molecular orientation (flow induced residual stress) or by unbalanced cooling (thermally induced residual stress). Unbalanced cooling can occur if the cooling circuit is not well designed or if the moulding is a complex shape with marked variations in thickness. Thermally induced residual stress is present to some extent in most moulded parts. due to the outer layers solidifying first during cooling and restraining the contraction of the core region as it slowly solidifies. Usually the tensile compressive stresses in the surface layers are balanced by the tensile stresses that exist within the core, but unbalanced cooling can perturb this balance leading to warpage.

Part distortion can also be caused by applying excessive holding pressures during cooling that cause 'overpacking' of the material. Overpacking can be relieved by the part distorting after it has been ejected. Stresses are 'frozen-in' where this does not occur.

3. THE INFLUENCE OF PROCESSING CONDITIONS ON SHRINKAGE AND WARPAGE

Jou^{6,13} assessed the role that mould and melt temperature and cooling time has on the volumetric shrinkage of mouldings manufactured from polypropylene. The influence of different processing variables on volumetric shrinkage, ranked in order of importance, were found to be mould temperature (low is better) and melt temperature (low is better).

Yang and Liou⁹ investigated the influence of processing conditions on mouldings prepared by gas assisted injection moulding. The influence of melt temperature, injection rate and pressure and shot size on the shrinkage and warpage of a polystyrene moulding was investigated. Gas pressure and injection delay were also considered as variables. The melt temperature and shot size were found to be the most significant parameters.

The effects of changes in holding pressure, mould and melt temperatures on levels of crystallinity in two polypropylenes, one of which contained a nucleating agent, were assessed by Yang and Nunn⁸. Slow cooling enhances crystallization, as does a high melt temperature and the presence of nucleating agents. In the absence of nucleating agents all three variables were found to influence the degree of crystallinity of dumbbell mouldings. In order of decreasing significance these variables can be ranked as melt temperature, holding pressure

and mould temperature. The same ranking was obtained for the polypropylene that contained a nucleating agent except that mould temperature was no longer identified as being a significant variable. The regression results showed that increasing the value of each processing condition increased the degree of crystallinity. These observations can be explained by assuming that high melt temperatures increase the rate of crystal growth and extended cooling times permit more crystal growth. Increasing the holding pressure results in higher levels of crystallization occurring by raising the crystallization temperature (as shown in Figure 1).

Kumazawa⁷ investigated the role that cooling time, injection speed and melt and mould temperatures have on the shrinkage behaviour of rectangular plate mouldings of differing thicknesses. Using a blend of PC/ABS material it was found that shrinkage parallel and perpendicular to the flow are essentially independent of moulding conditions other than cooling time. Shrinkage through the thickness of the plates is particularly sensitive to cooling time.

Mamat et al¹⁴ agree with Walsh's¹⁷ decomposition of shrinkage into contributions from the volume dependence on pressure and temperature, crystallinity, mould restriction and material orientation effects. However they share Stitz's¹⁸ concern that it is difficult to use this approach for modelling due to the difficulties in obtaining good experimental data. As an alternative they suggest that statistical modelling (based on regression or kriging of measured data) will provide the most practical way forward.

Parallel and perpendicular shrinkages were measured in rectangular plaques moulded from filled (40% calcium carbonate) and unfilled polypropylene. The parameters varied were: holding pressure, holding time, mould and melt temperatures. Their results suggest that both parallel and perpendicular shrinkages increase with distance from the gate because of the decrease in holding pressure. The location of the cooling circuit was found to have a large effect on measured shrinkages causing higher shrinkages in those regions of the mould where relatively fast cooling occurred. These observations are valid for both filled and unfilled polypropylene. The authors conclude that holding pressure followed by holding time are the most significant parameters. Shrinkage decreases with increasing holding pressure, holding time and melt temperature, and increases with mould temperature and distance from the gate. Filled and unfilled polypropylene have similar parallel shrinkages, but the filled material has less perpendicular shrinkage. There is less tendency for plates manufactured from filled material to warp as shrinkage is more isotropic.

Salloum et al^{2,20} investigated the influence of processing conditions on the shrinkage of a centre-gated disc manufactured in four different thicknesses from high density polyethylene. Eight processing conditions (melt and mould temperature, injection and holding pressure, cooling and injection time, injection speed and holding pressure profile) were examined. The experimental protocol was to only vary one parameter at a time (between its upper and lower limits) with all other conditions maintained at their mean value. Their findings showed that, generally, the effect of the conditions can be ranked in decreasing order of importance as;

- holding pressure
- cooling time
- mould temperature
- maximum injection pressure
- melt temperature.

The other processing variables did not appear to have any influence over shrinkage. Interestingly, holding and injection times were found to be more significant for the thinner disc whilst cooling time assumed a more important role as the disc thickness increased.

4. THE APPLICATION OF DESIGN MATRICES TO THE INJECTION MOULDING PROCESS

Deciding which processing conditions to adjust and by how much to enhance dimensional stability whilst keeping productivity rates high and energy costs low is difficult due to the interactive nature of the processing parameters. This problem is made even more complex given the large number of different combinations that are possible. This difficulty can be overcome by adopting the statistical approach of factorial analysis. This method can be used to;

- identify the most significant processing variables,
- identify important interactions between them,
- determine how sensitive the moulding process is to variations in processing conditions.

Table 1 shows a comparison of the processing variables and number of different values of each factor considered by previous investigators. Clearly there is some disagreement over which processing conditions are considered to be most important. Based on the findings of these previous studies we have chosen to consider the following variables (with the number of values of each variable indicated in brackets);

- flow rate (4),
- melt temperature (3),
- mould temperature (3),
- holding pressure (2),
- holding time (2),
- cooling time (2).

Processing Variable	Moldflow ¹⁸	Salloum et al ¹²	Yang and Liou ⁹	Yang and Nunn ⁸	Mammat et al. ¹⁴	Jou ⁶	Girard et al ¹²⁰	Jones ¹⁰	This study
Melt temperature	✓(3)	✓	✓(3)	✓(2)	✓(3)	✓(2)	✓		✓(3)
Mould temperature	✓(2)	✓(2)		✓(2)	✓(3)	✓(2)	✓	✓(2)	✓(3)
Flow rate	✓(1)	✓	✓(3)*					✓(2)*	✓(4)
Ram travel	✓(1)		✓(3)						
Thickness	✓(4)								
Holding time	✓(3)				✓(11)			✓(2)	✓(2)
Cooling time	✓(4)	✓(4)					✓	✓(2)	✓(2)
Hold pressure	✓(1)	✓(3)		✓(2)	✓(3)	✓(2)	✓	✓(2)	✓(2)
Gas injection press. & delay			✓(3)						
Screw speed		✓						✓(2)	
Annealing time & temperature						✓(3)			
Injection Pressure & time		✓	✓(3)				✓		
Barrel Temperature								✓(2)	

Table : A comparison of processing variables that have been considered in the literature with those of this study (The levels of each variable are given in brackets).

* referred to as injection speed.

† Information on the number of levels is not available.

To consider all the possible combinations of each factor at each value in a full factorial array would require a very large number of experimental runs i.e. 289. To reduce the number of runs to a more manageable number statistical *Design of experiments*^{21,22} (DOE) software packages have been employed..

These packages enable the user to select fractional factorial arrays that represent subsets of the full factorial matrix. The DOE package *Doewin2*²³ was used to generate a selection of design matrices that made maximum use of the available information (as quantified by design performance indices) for the lowest cost (number of runs). The design matrix chosen (Table 2) caters for only single factor linear interactions in the 22 experimental runs. The matrix has the following characteristics; average variance

coefficient (AVC) of the estimated parameters =0.06, D-efficiency* =0.798 and a minimum detectable effect =1.495. Limiting the number of runs does limit the number of interactive terms that can be considered although some have been considered to constrain the D-efficiency and AVC values to within acceptable limits.

D-efficiency (determinant efficiency) is a scaled value of the determinant of the information matrix. It is used as a measure of D-optimality. A D-efficiency fraction of one is best. D-efficiency compares the design to a theoretical (but not always possible) orthogonally balanced design.

Orthogonality is a characteristic of an experimental design that ensures independence of estimates. If the underlying linear model is correct, an orthogonal design is associated with balanced data. More generally, orthogonality and balance are considered as separate design parameters. The degree of orthogonality is measured by the D-efficiency score. A balanced design is such that each column of a design array has the same number of values of that parameter. The data is balanced when it is distributed over the experimental region so that each point makes an equal contribution to the parameter estimates. $D\text{-efficiency} = \det(IM) / k$ i.e the determinant of the Information Matrix normalised by the number of rows.

5. DETAILS OF THE SIMULATION EXERCISE

Simulations of mould filling, cooling, packing and calculations of part shrinkage and warpage were carried out using Moldflow Version 9.4⁴. The materials properties of the polymers studied including their shrinkage coefficients were obtained from the Moldflow database. These data have been verified by Moldflow.

5.1 Materials selection

Details of the grades of linear low density polyethylene (LLDPE) and polypropylene (PP) used are given in Table 3. These material grades were chosen as they have a comparatively large window of injection times. This window was determined using a simple two dimensional strip representation of the mould cavity geometry as described below.

5.2 Details of the mould and cavity construction

The plate moulding chosen for this study is a model of a mould cavity designed by the NPL to ensure maximum anisotropy of properties such as Young's modulus in a square plate moulding²⁴.

The mould cavity and cooling circuit are contained within a block of P20 steel of dimensions 296 - 496 - 168 mm. Details of the cavity are shown in Figure 2 whilst the cooling circuit is shown in Figure 3. The baffle insert positioned on the underside of the plate ensures that heat is wicked away from the domed region of the cavity. A Reynolds number of 10000 was assumed for the water coolant. This is characteristic of turbulent flow and ensures efficient heat transfer from the melt to the cavity.

Run Number	Flow rate	Melt temperature	Mould temperature	Holding pressure	Holding time	Cooling time
1	min	min	max	min	min	min
2	max	min	min	max	max	max
3	max	max	min	min	min	max
4	0.66	max	opt	min	max	min
5	0.33	opt	min	min	max	min
6	0.66	max	max	min	max	min
7	max	opt	opt	max	max	min
8	max	max	max	min	min	max
9	min	max	opt	max	max	min
10	0.66	opt	min	max	min	max
11	max	min	max	max	max	max
12	0.33	max	opt	max	min	max
13	max	opt	max	max	max	min
14	min	opt	max	min	min	max
15	0.33	min	opt	min	max	max
16	0.66	max	min	min	max	min
17	0.33	max	max	max	min	max
18	0.33	min	min	min	max	max
19	min	max	min	max	max	min
20	min	opt	min	min	min	max
21	min	min	max	min	min	min
22	min	min	min	min	min	min

Table 2: The design matrix. The criteria used to select values for the parameters are described in section 5. The fractional values correspond to $\text{min} + (\text{max} - \text{min}) * 0.33$ or 0.66 .

	Linear low density polyethylene	Polypropylene
Supplier	Millenium Petrochemical Inc. (GA584-000)	Solvay (Eltex P RW211)
Melt Flow Index (g/10 min)	105.0	15.0
Ejection Temperature (°C)	84.5	105.0
No Flow Temperature (°C)	106	138.8
Melt Density (kg/m ³)	760	716.4
Thermal Conductivity (W/m°C)	0.24	0.13
Specific Heat (J/kg°C)	3177.7	2732.0

Table 3: Material properties.

5.3 Cavity performance during the injection moulding cycle

Figure 4 shows the progress of the flow front during mould filling. From this figure it is evident that the humped region near the gate is effective in producing a uniform flow front as the melt approaches the weir gate. This is achieved by locally extending the flow path between the sprue and the weir gate. It ensures that the melt proceeds to uniformly fill the plate region of the mould cavity and enhances anisotropy. Figure 5 shows the top and bottom surface temperatures at the end of filling. Warm areas inside the dome and close to the weir gate on the upper surface indicate that some improvements could be made to the design of the cooling circuit. The time taken for molten LLDPE to cool to the ejection temperature is shown in Figure 6. Regardless of the melt temperature most of the moulding has 'frozen' within 10 - 12s with the remaining time being used to cool the dome and sprue.

5.4 Selection of processing variables

Processing condition windows are normally chosen such that the mouldings produced will be commercially acceptable. Clearly it is not possible to follow this approach in a simulation exercise. The parameters used in this exercise are given in Tables 4 and 5, the reasons for choosing them are described below.

Moulding Parameter	Maximum	Minimum	Comments
Injection Time (s)	3.0	0.9	Range calculated using 2-D strip representation of dominant flow path
Flow Rate (cm ³ /s)	98.7	29.6	Flow rate through the sprue, determined from injection time.
Mould Temperature (°C)	60	20	Generic limits recommended by Moldflow
Melt Temperature (°C)	240	180	Generic limits recommended by Moldflow
Holding Pressure (MPa)	42	21	Based on computation of the maximum holding pressure for a 150 tonne machine.
Holding Time (s)	22	10	Minimum packing time depends on time taken for gate to freeze.
Cooling Time (s)	90	40	Based on the temperature distribution over the plate.

Table 4: Moulding parameters for LLDPE

5.4.1 Mould and melt temperatures

The range of melt and mould temperatures used are those suggested by the Moldflow database. It is assumed that the mean temperatures represent optimal values.

5.4.2 Injection time

The potential range of injection times for the plate moulding can be estimated using the Moldflow programme MABSTRIP. This programme models a melt flowing through a slice or 'strip' of the moulding that represents the dominant flow path through the cavity. The geometry of the cavity is represented by the strip as variations in thickness and length; changes in width and direction are not important. MABSTRIP computes the melt temperature and maximum shear stress for a range of injection times. The shortest injection times were discarded since at these high injection speeds shear heating of the melt causes its temperature to rise above that at which it is injected. At long injection times significant cooling of the melt occurs as it flows through the cavity that can cause filling problems. These limits define a temperature window ranging from the melt temperature at the point of injection, T_i to T_i -

20°C from which a span of injection times can be defined. It should be noted that this window of injection times may need to be reduced in size where the recommended shear stress for the material is exceeded.

Moulding Parameter	Maximum	Minimum
Injection Time (s)	5	1.6
Flow Rate (cm ³ /s)	53.22	17.8
Mould Temperature (°C)	60	20
Melt Temperature (°C)	240	200
Holding Pressure (MPa)	42	21
Holding Time (s)	22	12
Cooling Time (s)	90	50

Table 5 : Moulding parameters for PP. These were determined in an identical way to those for LLDPE.

The highest shear stresses were observed in the region of the weir gate. (It is possible to reduce these stresses by increasing the gate thickness with respect to that of the plate and thus extend the range of injection times that can be used).

Injection times for LLDPE ranged from 0.9 - 3s and 1.67 - 5s for PP, although for convenience these have been converted to flow rates.

5.4.3 Cooling time

Figure 6 shows the time taken for different regions of the plate mould to cool to the ejection temperature i.e. the temperature at which the plate can be ejected without distorting or being damaged by the ejector pins. The cooling time is defined as the time interval between the removal of the holding pressure and the point of ejection. The variation in cooling time reflects the changing thickness of the mould cavity with the weir gate freezing first followed by the plate and much later by the domed area preceding the gate.

During the cooling cycle the coolant temperature rose by a maximum of 0.3°C from its initial temperature of 40°C.

5.4.4 Holding pressure and profile

Calculations of shrinkage and warpage utilise the results from a packing analysis that simulates the part cooling under pressure after it has been filled. The packing analysis requires details of the holding pressure and the time over which it is applied, held and removed. In practice the maximum and minimum holding pressures are those used to generate mouldings without flash or sink marks. Flashing occurs when the holding pressure exceeds the clamping effect that the machine can apply to the mould. This maximum holding pressure is approximately given by:

$$P_{\max} \text{ (Pa)} = \frac{\text{Maximum.clamp.force.(N)}}{\text{Projected.area.of.moulding.(cm}^2\text{)}} * 100 * 0.5 \quad (1)$$

The factor 100 is for conversion of units whilst the factor 0.5 ensures that the maximum holding pressure is only about 50% of the machines clamp tonnage limit. For this exercise it was assumed that the maximum clamp tonnage would be 150t. (This corresponds to the Demag 150 moulding machine that was used in previous moulding trials²⁵). Setting a minimum holding pressure is difficult in the absence of actual mouldings if the criterion for the minimum is the absence of sink marks. The minimum holding pressure was therefore taken to be half that of the maximum. Clamp forces of 120t were required to contain the melt at the highest holding pressure of 42MPa.

Care was taken to ensure that the holding time was in excess of 10s to ensure that the melt in the plate cooled under pressure - noting that the rate limiting step is the time taken for the weir gate to freeze. Therefore only the area between the sprue and weir gate is influenced by changes in the holding time. It was assumed that the holding pressure would be applied and removed almost instantaneously in 0.2s as shown in Figure 7. The same holding profile was used for all combinations of processing conditions and for both materials. (It should be noted that the time dependence of the holding pressure profile can affect the shrinkage behaviour of mouldings^{1,26,27}).

Table 6 shows how the different combinations of processing conditions shown in Table 2 translate into flow rates, temperatures, pressures and times.

6. RESULTS AND DISCUSSION

Figures 8, 9, 10 and 11 show the calculated shrinkages parallel ($S_{\parallel}$) and perpendicular ($S_{\perp}$) to the flow direction for both PP and LLDPE plaques. The processing conditions for each material shown for each shrinkage direction represent the upper and lower limits of injection speed, melt and mould temperature (runs 8 and 22 of Table 2). Although constant holding pressures and times were used for the contour plots there were differences between the cooling times as indicated in Table 2. A low holding pressure was used for all of the plates shown in the figures to enhance shrinkage. These results show that LLDPE plates are more prone to shrinkage than those made from PP although it is possible that this reflects the differences in processing behaviour between the two materials. (Noting that it is difficult to directly compare the shrinkage behaviour of two different materials as dimensional instability depends on the processing conditions that are used.) The contour plots show that:

- more shrinkage occurs in the region between the point of injection and the weir gate than in the plate itself.
- shrinkage parallel to the flow direction is less than that calculated perpendicular to it for both materials.
- shrinkage parallel to the flow direction is more uniform over the surface of the plate when a high flow rate, melt and mould temperature and long cooling time are used.
- a high flow rate, melt and mould temperature and a long cooling time does not appear to affect the variation in shrinkage perpendicular to the flow direction over the plates surface but does seem to influence that which occurs between the injection point and the weir gate.

Figures 12 and 13 show the out-of-plane distortion of the mouldings (W_z) From these figures it can be seen that a high flow rate, melt and mould temperature and long cooling time affect the geometry of the deformed plate. The shape of the deformed plate appears to reflect the shrinkage behaviour of the region between the point of injection and the weir gate. Contraction of this area of the moulding can either be accommodated by the appearance of a highly stressed necked region or by the plate deforming yielding a barrel shape. Figure 12(A), for example, shows the plate curving suggesting that it follows the energetically more favourable option. Where the shape of the plate is less affected by shrinkage of the spreader region it forms a domed structure.

Run Number	Flow rate (cm ³ /s)	Melt temperature (C)	Mould temperature (C)	Holding pressure (MPa)	Holding time (s)	Cooling time (s)
1	29.62	180	60	21	10	40
2	98.75	180	20	42	22	90
3	98.75	240	20	21	10	90
4	75.71	240	40	21	22	40
5	52.66	210	20	21	22	40
6	75.71	240	60	21	22	40
7	98.75	210	40	42	22	40
8	98.75	240	60	21	10	90
9	29.62	240	40	42	22	40
10	75.71	210	20	42	10	90
11	98.75	180	60	42	22	90
12	52.66	240	40	42	10	90
13	98.75	210	60	42	22	40
14	29.62	210	60	21	10	90
15	52.66	180	40	21	22	90
16	75.71	240	20	21	22	40
17	52.66	240	60	42	10	90
18	52.66	180	20	21	22	90
19	29.62	240	20	42	22	40
20	29.62	210	20	21	10	90
21	29.62	180	60	21	10	40
22	29.62	180	20	21	10	40

Table 6: The twenty two combinations of processing conditions used to compute the shrinkage and warpage behaviour of a moulding manufactured from LLDPE. The corresponding table for PP can be constructed using the information given in Tables 2 and 5.

6.1 Links between dimensional instability and processing conditions

Looking for causal relationships between the shrinkage and warpage of plates and changes in particular processing conditions is not a trivial task. Unfortunately not only does shrinkage vary according to whether it is measured parallel or perpendicular to the direction in which the melt flows but also with *position* along each edge. Thus care must be taken to ensure that information is not lost by averaging for example the in-plane shrinkage measured parallel to the flow direction. Similar problems were encountered in analysing warpage data. Information relating to the geometry of the deformation is also lost if out-of-plane deformations are expressed only in terms of averages. To overcome these difficulties these data have been analysed in terms of:

- shrinkage between each pair of parallel edges as a function of distance along an edge,
- in-plane warpage along each edge in directions both parallel ($W_{\parallel}$) and perpendicular ($W_{\perp}$) to that of the flow,
- out-of-plane warpage,
- changes in the angle of the intersecting diagonals of a plate as characterized by an angle score - see annex 2 that enables in-plane distortion to be compared.

As there is an abundance of data with which to compare the dimensional instability of one run with another and to look for links between certain processing conditions and shrinkage or warpage it is necessary to summarize it by considering mean values, ranges and scores as described in Annex 2. (Recording the range λ (i.e. maximum-minimum) of shrinkage or warpage values to some extent compensates the loss of information that occurs when arithmetic means of the data are taken).

Three levels have been used to describe the dimensional instability of the plate moulding that vary in detail. These levels are defined as follows;

- Level 1 is used to describe local shrinkage and warpage i.e. along a particular edge.
- Level 2 is used to describe shrinkage *or* warpage of the whole plate.
- Level 3 describes the dimensional stability of the whole plate and takes into consideration both shrinkage *and* warpage.

6.1.1 Analysis of means (ANOM)

The principle of this analysis is to take the average value of, for example, the mean and range of the out-of-plane warpages that are obtained for each value of a processing factor. Taking melt temperature as an example, shrinkages parallel to the flow direction are divided into three groups representing melt temperatures of 180, 210 and 240 °C each containing a number of data sets. These data sets are then averaged to give a 'mean' of the means and a mean range for each group as shown in Figure 14. From this figure it is apparent that whilst the mean shrinkage, $S_{||}$ is very sensitive to both the type of material and holding pressure it is *insensitive* to variations in cooling or holding time. Correspondingly the mean *range* of shrinkage, $S_{||}$ is insensitive to holding pressure but sensitive to changes of material or holding time. The numbers on the vertical lines refer to the different levels of each factor where zero is the minimum. The position of these numbers on a given line shows the dependency of $S_{||}$ on the value of a factor. This can be illustrated by looking at the dependence of $S_{||}$ on flow rate. This shows a non linear dependence with the mean shrinkage progressively decreasing with increasing flow rate. Additional plots showing the dependencies of shrinkage and warpage and the angle score on different processing conditions can be found in Annex 3 (Figures A6 to A10). Plots of this nature can be used to define a set of processing conditions that can be used to control the 'quality' of mouldings. For example, the lowest shrinkage, $S_{||}$ can be obtained by manufacturing mouldings using the conditions that correspond to the lowest values of the mean warpage as indicated in Figure 14. However this does assume that interactions between the various factors *do not* influence shrinkage. Table 7 provides a summary of the sensitivity of different measurements to different processing factors. The results in this table show that each quantity is sensitive to a different ranking of processing conditions. This information can be used to control a particular aspect of dimensional instability, for example variations in flow rate can be used to limit out-of plane warpage but have very little affect on in-plane warpage in a direction parallel to that of the flow. Changing the holding pressure can be used to control the mean shrinkages both parallel and perpendicular to the flow direction but have little influence on the range of shrinkage values. Changing holding pressure will also affect in-plane warpage.

The analysis of means approach only provides qualitative information relating processing variables to changes in dimensional stability. Information concerning the interactions between the various processing factors and dimensional stability can be obtained by analysing the data using statistical methods.

6.1.2 Analysis of variance (ANOVA)

The analysis of variance is based on a linear regression model as defined implicitly in the design array. The analysis of variance interpretation of the dependence between the various

processing factors and shrinkage where it exceeds a 95% confidence limit is shown in Table 8. Relationships between the other processing factors and shrinkages either parallel or perpendicular to the flow direction were either undefined or significantly below 95%. The agreement between the ranked processing factors obtained using both the analysis of means and variance methods is generally very good. The analysis of means approach suggests causal relationships but isn't able to say whether or not the links are statistically significant. This is evident for the out-of-plane deformation results.

6.1.3 *Links between plate geometry and processing conditions*

Information relating the shape of the deformed plates to different processing conditions is lost following the approaches of both the analysis of means and variance. A route to correlate shapes with processing factors is to model the geometry using polynomials and then to look for trends in the coefficients. Seventh order polynomials were subsequently fitted to:

- the edge of the plate at the junction of the weir gate,
- a side edge parallel to the flow,
- the lines perpendicular and parallel to the flow that pass through the mid-point of the plate.

The polynomial coefficients a_0 to a_6 are plotted against different cooling times in Figure 15 to illustrate this type of analysis. Clearly there are trends in some of the coefficients i.e. a_4 for edge data and the mid-line that run parallel to the flow direction and this has been confirmed by a statistical analysis of all the polynomial coefficients. The results of this analysis suggests that only cooling time appears to be important in controlling the local geometry of the plate. This observation is of interest as cooling time was not identified as having a significant influence over the more global measures of shrinkage and warpage.

Correlating in-plane distortion of the plates with different processing factors can be assessed by using the angle scores. Tables 7 and 8 show that melt temperature has the most influence on in-plane distortion of the plates using both analysis of means and variance methods.

6.1.4 *Summary findings and comparisons with other studies*

We have so far concentrated on the links between different processing conditions and changes in local shrinkage or warpage. Whilst this information may be useful if one particular dimension of the part needs to be critically controlled it is often more important to reduce overall shrinkage or warpage. This can be achieved by amalgamating all the shrinkage data obtained in directions parallel and perpendicular to that of the flow for both materials and then analysing it with respect to different processing conditions. This analysis is summarised in Table 9 along with results from the literature. The information in this table can be used to reduce overall shrinkage. Table 9 shows a good agreement between the findings of the two analyses. The ranked processing factors are also comparable with the findings of other workers who have explored the links between processing and shrinkage. Some care needs to be taken in making direct comparisons with previously published work in that different authors may not have considered comparable ranges of processing variables or the effects of orientation. There is some anecdotal evidence to suggest that the influence a particular processing condition has on dimensional stability is affected by the range of values considered i.e. their dependence may be non-linear.

There is some concern that causal relationships can be exaggerated or masked if the levels of each factor are inappropriately chosen, i.e. too close together. The material factor although not shown is generally the most significant processing factor. However in terms of processing variables it is evident from Table 9 that shrinkage is most affected by holding pressure and the temperature of the melt.

In- and out-of-plane warpage data for both materials have been amalgamated and subsequently analysed with respect to different processing conditions. The results of this analysis are shown in Table 10. There have been few published studies linking warpage with processing factors. This may reflect the difficulty of measuring and interpreting the data. In comparison with the shrinkage results there is less agreement between the analysis of variance and means approaches. However it is apparent that holding pressure and both mould and melt temperatures are the most influential factors in controlling warpage.

It is apparent from the ANOVA and ANOM results shown in Tables 9 and 10 that the factors that can be used to control plate shrinkage or warpage have different rankings. Thus in practice adjusting the processing conditions to limit plate warpage may result in additional shrinkage or vice versa. To limit both shrinkage and warpage causal relationships between processing conditions and an amalgamation of all the shrinkage and warpage data are required. The results of this analysis (discussed in Annex 2, section 11.4) are shown in Table 10. From this table it is apparent that there are differences in the ranking of the processing conditions derived from the analysis of means and variances. It is apparent that holding time is important and also flow rate i.e. injection time. This finding is in agreement with the practice of extending the holding time to limit warpage that is used in injection moulding.

6.2 Controlling shrinkage and warpage by changing processing conditions

The work presented here shows that it is possible to control the dimensional stability of injection moulded parts on different levels ranging from a global reduction in shrinkage and/or warpage or control of the tolerance of a particular dimension. However to achieve the desired goal using the principles outlined in this report it is necessary to seek answers to the following questions.

- Which processing conditions will be considered?
- How many values of each factor will be considered?
- How will the range of values used for each factor be determined?
- What criteria will be used to select the DOE array?

How will shrinkage and warpage be measured?

- Will the shrinkage and warpage data be analysed qualitatively or quantitatively?
- At which level will the shrinkage and warpage data be analysed?

The factors affecting dimensional stability have been discussed, and comments have been made regarding the influences that each factor has upon shrinkage and warpage to specific dimensions of the moulding. Obtaining a meaningful and objective measure of the dependence of shrinkage and warpage of the entire plate upon each individual factor is more difficult, due to the complex interdependence of the processing conditions. At a detailed level some understanding can be gained of how changing the value of a particular factor will affect shrinkage or warpage from the ANOM analysis shown in figures 14 and A6 to A9. If these data are combined to give a plot of, for example, in-plane warpage of the whole plate then the first point to note is that the ordinate axes of the in-plane warpages parallel and perpendicular to the flow direction (Figures A7 and A8) are different. Directly combining the two sets of data will introduce a bias that will give undue weighting to the set of data that is of the greatest magnitude. This bias can be modified by introducing unequal weighting coefficients that are very difficult to determine objectively.

Mean $S_{\perp}$ Eqn. A2(4)	Range $S_{\perp}$ Eqn. A2(5)	Mean $S_{\parallel}$ Eqn. A2(4)	Range $S_{\parallel}$ Eqn. A2(5)	Angle Score	Mean $W_{\perp}$ Eqn. A2(8)	Range $W_{\perp}$ Eqn. A2(8)	Mean $W_{\parallel}$ Eqn. A2(9)	Range $W_{\parallel}$ Eqn. A2(9)	Range W_z Eqn. A2(10)
Material	Material	Hold Press	Material	Material	Material	Material	Hold press	Material	Material
Hold press.	Melt temp	Material	Hold time	Melt temp	Hold press	Flow rate	Material	Hold time	Flow rate
Mold temp.	Flow rate	Melt temp	Cool time	Flow rate	Mold temp.	Melt temp	Hold time	Cool time	Melt temp
Flow rate	Hold press.	Flow rate	Flow rate	Hold time	Flow rate	Hold time	Melt temp	Flow rate	Hold time
Hold time	Hold time	Mold temp.	Melt temp	Mold temp.	Hold time	Hold press	Mold temp.	Mold temp.	Mold temp.
Melt temp.	Mold temp.	Hold time	Mold temp.	Cool time	Melt temp	Mold temp.	Cool time	Melt temp	Cool time
Cool time	Cool time	Cool time	Hold Press	Hold press	Cool time	Cool time	Flow rate	Hold press	Hold press

Table 7: Analysis of mean results showing the sensitivity of different measurements to different processing factors. The factors are ranked in *decreasing* order of importance.

Most Important

----->

Least important

Mean $S_{\perp}$	Holding pressure	Mould temperature	Flow rate			
Range $S_{\perp}$	Holding pressure	Melt temperature	Holding time			
Mean $S_{\parallel}$	Holding pressure	Melt temperature	Flow rate	Holding time		
Range $S_{\parallel}$	Holding time	Mould temperature	Flow rate	Melt temperature		
Mean $W_{\perp}$	Holding pressure	Mould temperature	Holding time	Flow rate		
Range $W_{\perp}$	Melt temperature	Flow rate	Holding pressure	Holding time		
Mean $W_{\parallel}$	Holding time	Holding pressure	Melt temperature	Mould temperature	Flow rate	Cooling time
Range $W_{\parallel}$	Holding time	Flow rate	Cooling time	Mould temperature	Holding pressure	Melt temperature
Range W_z	Flow rate					
Angle score	Melt temperature	Holding time	Flow rate	Holding pressure	Mould temperature	

Table 8: Results of the analysis of variance (ANOVA) showing the sensitivity of different measurements to different processing factors. The factors are ranked in decreasing order of importance and are listed if their significance is in excess of $(1-\alpha)\%$ (where $\alpha=0.05$).

Yang and Nunn⁸	Girard et al²⁰	Salloum et al²	Yang and liou⁹	Mamma t et al¹⁴	Jou⁶	Jones¹⁰	ANOVA shrinkage	ANOM shrinkage
<i>Melt temp.</i>	<i>Holding pressure</i>	<i>Holding pressure</i>	<i>Melt temp.</i>	<i>Holding pressure</i>	<i>Mould Temp.</i>	<i>Melt Temp.</i>	<i>Holding pressure</i>	<i>Holding pressure</i>
<i>Holding pressure</i>	<i>Cooling time</i>	<i>Cooling time</i>	<i>Shot size</i>	<i>Holding time</i>	<i>Melt temp.</i>	<i>Mould temp.</i>	<i>Holding Time*</i>	<i>Holding time</i>
<i>Mould temp.</i>	<i>Injection Pressure</i>	<i>Injection Pressure</i>				<i>Cooling time</i>	<i>Melt temp.*</i>	<i>Flow Rate</i>
							<i>Flow Rate*</i>	<i>Melt Temp.</i>
							<i>Mould Temp.*</i>	<i>Mould Temp.</i>
								<i>Cooling Time</i>

Table 9 : A comparison of the influence of most significant processing factors (ranked in order of decreasing significance) on the amalgamated shrinkage that occurs in directions parallel and perpendicular to that of the flow direction. The asterisks shown for the ANOVA results indicate that there were no differences between these quantities. Comparisons with the literature are also presented.

ANOVA warpage	ANOM warpage	ANOVA angle scores	ANOM angle scores	ANOVA all data	ANOM all data
<i>Melt temp.</i>	<i>Holding pressure</i>	<i>Melt temp.</i>	<i>Melt temp.</i>	<i>Holding pressure</i>	<i>Flow rate</i>
<i>Holding pressure</i>	<i>Mould temp.</i>	<i>Holding time</i>	<i>Flow rate</i>	<i>Holding time</i>	<i>Holding time</i>
<i>Mould temp.</i>	<i>Melt temp.</i>	<i>Flow rate</i>	<i>Holding time</i>	<i>Flow rate</i>	<i>Melt temp.</i>
	<i>Holding time</i>	<i>Holding pressure</i>	<i>Mould temp.</i>	<i>Melt temp.</i>	<i>Holding pressure</i>
	<i>Cooling time</i>	<i>Mould temp.</i>	<i>Cooling time</i>	<i>Mould temp.</i>	<i>Mould temp.</i>
	<i>Flow rate</i>		<i>Holding pressure</i>	<i>Cooling time</i>	<i>Cooling time</i>

Table 10 : A comparison of the influence of different processing factors on amalgamated warpage, angle score and dimensional stability ranked in decreasing order of importance. All the shrinkage warpage and angle score data have been amalgamated in the two right hand columns to illustrate the effect of processing conditions on the dimensional stability of the plate as a whole.

7. CONCLUSIONS

The influence of different processing conditions on the shrinkage and warpage of a plate moulding simulated using Moldflow software has been both qualitatively and quantitatively assessed by using the analysis of means and variance respectively. Generally a good agreement was obtained using these two approaches although the former suggests causal relationships but is unable to identify whether the links are statistically significant or not.

The role that different processing conditions have on dimensional stability varies according to the level at which dimensional instability is considered i.e.

- the shrinkage or in-plane warpage in directions parallel or perpendicular to that of the flow,
- in-plane shrinkage or warpage and out-of-plane deformation,
- plate shrinkage or plate warpage,
- plate shrinkage and warpage combined.

The analysis of both shrinkage and warpage data can only be used to reveal the quantitative links between dimensional instability and processing factors on a very detailed level i.e. shrinkage in a particular direction. This link is lost when shrinkage or warpage data are combined to give plate shrinkages or warpages because of the difficulties of combining data spanning different ranges and units. The link can be restored if weighting factors are used but it is not possible to generate these in an objective way.

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10. ANNEX 1: SHRINKAGE AND WARPAGE CALCULATIONS

Shrinkage can be defined either in terms of a change in the volume of a part i.e. volumetric shrinkage (S_v) or as a change in one or more dimensions i.e. linear shrinkages (S_l). Whilst the linear shrinkage of an isotropic material should be given by

$$S_l = 1 - (1 - S_v)^{1/3} \quad A1(1)$$

in practice they are often greater or less than $S_v^{1/3}$. These differences reflect the influences of mould restraint or orientation effects on shrinkage behaviour.

10.1 Shrinkage calculations

Moldflow takes into consideration four factors in calculating the shrinkage parallel ($S_{||}$) to and perpendicular ($S_{\perp}$) to the flow direction^{17,19}, namely;

- mould restraint (M_r),
- molecular orientation (M_o),
- volumetric shrinkage (M_v)
- crystallinity (M_c).

Shrinkages parallel and perpendicular to the flow direction are calculated using the following functions

$$S_{||} = a_1 M_v + a_2 M_c + a_3 M_r + a_4 M_o'' + a_5$$

and

$$S_{\perp} = a_6 M_v + a_7 M_c + a_8 M_r + a_9 M_o^{\perp} + a_{10}.$$

With the exception of M_v (which relies on pressure-volume-temperature (pvT) data) all the other measures M_i are calculated. The measures are subsequently weighted by the coefficients a_1 to a_{10} . The coefficients are obtained by fitting equations 1 and 2 to shrinkage data derived from measurements of twenty eight rectangular tab mouldings. These rectangular mouldings are manufactured according to a range of processing conditions¹⁹ and allowed to anneal in a controlled environment for a period of time.

Shrinkage calculations are based on grid point results calculated over eleven evenly spaced points between the centre of the mould cavity thickness and the cavity wall. The cooling rate within each of the ten laminates is calculated at the instant at which the material freezes together with the shear stress and flow angle with respect to the elements local axis. The influence of the holding pressure on the volumetric shrinkage during cooling is also determined. Further details of the calculations may be found in reference 26.

10.2 Warpage calculations

In principle every region on a moulded part can warp either by a simple displacement in x, y or z or by rotating around one or more of these axes. Combinations of translations and rotations with respect to the orientation of the original design give rise to the distorted shapes observed in practice. Not all points within a moulded part have the freedom to move in all six directions either because of the presence of surrounding regions or because of the mould cavity design. Displacements are calculated with respect the coordinates of an undeformed plate as illustrated in Figure A1. All plots of displacements against position are with respect to this reference plate.

Warpage predictions are based on calculating the influence of shrinkage strains on the moulding using a structural analysis programme. It is necessary to determine whether or not part distortion can be interpreted using either linear or non-linear buckling analyses prior to

calculating warpage²⁶. Based on an interpretation of eigenvalues a linear buckling analysis was used for the plate moulding used in this paper. It should be noted that the wrong choice of model can affect the predicted shape of the distorted moulding.

11. ANNEX 2: THE DETERMINATION OF SHRINKAGE AND WARPAGE SCORES

Prior to determining the influence of different processing conditions on the dimensional stability of injection moulded artefacts, measures need to be defined that enable the amount of shrinkage and warpage to be quantified. These measures have to describe changes in the plates shape and location with respect to its original size and position. Although we have used the geometry of the plate these basic features are generic to any moulded object.

Figure A1 shows a comparison of an undistorted plate with one that has been deformed through a combination of shrinkage and warpage. The size and shape of the *deformed* plate shown in Figure A1 is determined by contributions from:

- shrinkage across the plate in directions both parallel ($S_{\parallel}$) and perpendicular ($S_{\perp}$) to that of the flow.
- out-of-plane shrinkage and warpage (S_z and W_z respectively).
- in-plane warpage both parallel to ($W_{\parallel}$) and perpendicular ($W_{\perp}$) to the flow direction.

Information relating to the geometry of the deformed plate can be obtained by using a variety of different measures i.e.

- changes in the curvature of the plate are reflected in the warpage derivatives $W_{\parallel}$, $W_{\perp}$ and W_z .
- in-plane distortion that can be assessed by comparing the differences between the angles, θ formed at the intersection of the in-plane diagonals of deformed plates with those of the undeformed plate.
- fitting empirical functions to a mesh that represents the deformed plate giving both shrinkage and warpage information in three dimensions.

11.1 Calculation of Level 1 shrinkage scores

The shrinkage of the plate along a line connecting for example edges 1 and 3 (as shown in Figure A1) is given by

$$s_{\perp,k} = \frac{|e_1(x, y_k, z) - e_3(x, y_k, z)| - |\tilde{e}_1(x, y_k, z) - \tilde{e}_3(x, y_k, z)|}{|e_1(x, y_k, z) - e_3(x, y_k, z)|} \quad A2(1)$$

where $e_1(x, y_k, z)$ is the k^{th} point on edge 1 of an undeformed plate and $\tilde{e}_1(x, y_k, z)$ is the corresponding point on the equivalent deformed edge. Through-thickness variations have been ignored since the thickness is small in comparison to the length of each side. Figure A2 shows that shrinkage varies with position and between pairs of opposite edges $\tilde{e}_1$ and $\tilde{e}_3$ and $\tilde{e}_2$ and $\tilde{e}_4$.

Distances between opposite edges of the deformed plate were determined from calculated arc lengths. The distance over the plates surface, a (arc length) can be computed at any point along an edge parallel or perpendicular to the direction of flow according to

$$a_{\parallel}(x_k, y, z) = \int_{e_2}^{e_4} \sqrt{1 + \left(\frac{\partial f(x_k, y, z)}{\partial y}\right)^2} dy \quad A2(2)$$

$$a_{\perp}(x, y_k, z) = \int_{e_1}^{e_3} \sqrt{1 + \left(\frac{\partial f(x, y_k, z)}{\partial x}\right)^2} dx \quad A2(3)$$

where $f(x,y,z)$ denotes the polynomial or spline function used to fit the z (out-of-plane) coordinates for edge i . The shrinkage at any point in x or y is then generated by substituting the arc length as, for example $a_{\perp}$ for $|\tilde{e}_1(x,y_k,z) - \tilde{e}_s(x,y_k,z)|$ in equation A2(1). Unfortunately this method of evaluating shrinkage cannot be applied to the regions closest to the plate edges. This is because of discontinuities that can arise in the path of the integrals in equations A2(2) and A2(3) if the outermost edges bow in towards the centre. The range over which equations A2(2) and A2(3) are applicable is shown schematically in Figures A3 and A4.

11.2 Calculation of Level 2 shrinkage scores

It is evident from Figure A2 that shrinkage varies with position. This information can be used to derive the mean and range of shrinkages for *each* edge of the plate i.e.

$$S_{\perp} = \text{mean}_k(S_{\perp,k}), \quad S_{\parallel} = \text{mean}_k(S_{\parallel,k}) \quad \text{A2(4)}$$

and

$$R_{\perp} = \text{range}_k(S_{\perp,k}), \quad R_{\parallel} = \text{range}_k(S_{\parallel,k}) \quad \text{A2(5)}$$

Where the shrinkage range, R is defined as the difference between the maximum and minimum shrinkage values.

11.3 Calculation of Level 3 shrinkage scores

The mean and range of shrinkages parallel and perpendicular to the flow direction can be combined to give a measure of the overall plate shrinkage according to

$$S = \sigma_1 S_{\parallel} + \sigma_2 S_{\perp} \quad \text{A2(6)}$$

and

$$R = \sigma_1 R_{\parallel} + \sigma_2 R_{\perp} \quad \text{A2(7)}$$

where $\sigma_1 + \sigma_2 = 1$ and $0 \leq \sigma_i \leq 1, i = 1, 2$. The parameter σ_i allows the contribution to the shrinkage score from the shrinkages parallel and perpendicular to the flow to be weighted if required. Values of $\sigma_i = 0.5$ were used in the current study.

11.4 Calculation of warpage scores

Plate warpage is normally thought of as some deformation of a planar surface occurring in the z direction, however in-plane distortion can also occur as shown in Figure A5. The overall plate warpage is then composed of the components $W_{\parallel}(x,y)$, $W_{\perp}(x,y)$ and $W_z(x,y)$ that describe the in-plane deflections parallel or perpendicular to the flow direction and the out-of-plane deformation in z . In-plane warpage data is (most easily) obtained from the edges where the deflections manifest themselves. For convenience dimensionless warpages were used in all three planes. These were obtained by normalising the data with respect to the length of the undeformed plate

11.5 Calculation of Level 1 warpage scores

In-plane warpages were determined for each of the plate edges following the procedure illustrated here for edges e_1 and e_2

$$W_{\perp}(e_1) = \text{mean}_k \cdot \frac{\tilde{e}_1(x,y_k,z)}{|e_1|}, \quad R(W_{\perp}(e_1)) = \text{range}_k \cdot \frac{\tilde{e}_1(x,y_k,z)}{|e_1|} \quad \text{A2(8)}$$

$$W_{\parallel}(e_2) = \text{mean}_k \cdot \frac{\tilde{e}_2(x_k, y, z)}{|e_2|}, \quad R(W_{\parallel}(e_2)) = \text{range}_k \cdot \frac{\tilde{e}_2(x_k, y, z)}{|e_{21}|} \quad \text{A2(9)}$$

where ‘ $\sim$ ’ denotes the deformed edge and $|e_i|$ the length of the undeformed edge e_i

The out-of-plane deformation in z was derived using the same empirically fitted surfaces as those used to generate the arc length data following equations A2(2) and A2(3). This was quantified by considering

$$R(W_z) = \text{range}_k \cdot f(x_k, y_k, z_k) \quad \text{A2(10)}$$

where $f(x_k, y_k, z_k)$ is the fitted surface at points x_k, y_k, z_k . The corresponding mean for out-of-plane warpage, W_z has not been analysed since Moldflow assigns an arbitrary Z value to the plate. This statistic would therefore yield little information of importance.

11.6 Calculation of Level 2 warpage scores

In-plane warpages along each edge can be combined to give measures of in-plane warpage that occur in directions both parallel and perpendicular to that of the flow directions according to

$$W_{\perp}^{(2)} = [W_{\perp}^{(1)}(e_1) + W_{\perp}^{(1)}(e_3)] / 2$$

$$R(W_{\perp}^{(2)}) = [W_{\perp}^{(1)}(e_1) + R(W_{\perp}^{(1)}(e_3))] / 2 \quad \text{A2(11)}$$

The superscripts indicate the level at which the warpage is being considered. Similar definitions apply to the warpage parallel to the direction of flow.

11.7 Calculation of Level 3 warpage scores

The in-plane warpages parallel and perpendicular to the flow direction can be amalgamated with the out-of-plane deformation to give an overall measure for warpage according to

$$W = \sigma_3 M(W_{\parallel}^{(2)}) + \sigma_4 M(W_{\perp}^{(2)}) + \sigma_5 M(W_z^{(1)}) \quad \text{A2(12)}$$

and

$$R = \sigma_3 R_{\parallel}^{(2)} + \sigma_4 R_{\perp}^{(2)} + \sigma_5 R_z^{(1)} \quad \text{A2(13)}$$

where $\sigma_3 + \sigma_4 + \sigma_5 = 1$ and $0 \leq \sigma_i \leq 1, i = 1, 2, 3$. The parameters σ_i allow the contribution to the shrinkage score from the mean warpage and range of warpages to be weighted if required although values of $\sigma_i = 0.5$ were used in the current study.

11.8 Determination of angular scores

The angle formed at the intersection of the diagonals of an undeformed square plate is given by $\theta(L_{\perp}, L_{\parallel})$. Since $L_{\perp} = L_{\parallel}$ then $\theta = \pi/2$ and so $\cos(\theta) = 0$. The mean x and y values for the four edges of the plate are given by $M(W_{\parallel})$ and $M(W_{\perp})$. Taking the dot product of the vectors formed by these vertices and normalizing yields the cosine of the angle θ of the deformed plate as illustrated in Figure A1. The angle θ is then taken as the angle score.

11.9 Determination of overall plate dimensional instability

Due to the difficulty of choosing suitable weightings for the parameter σ level 3 shrinkage and warpage scores were not used in determining the sensitivity of the plates to changes in the processing conditions. Instead the results from the ANOVA and ANOM analysis of level 2 data were ranked according to their positions in Tables 7 and 8 i.e. holding pressure that

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had a consistently high score in Table 8 also had the highest score in Table 10, hence it has a high rank.

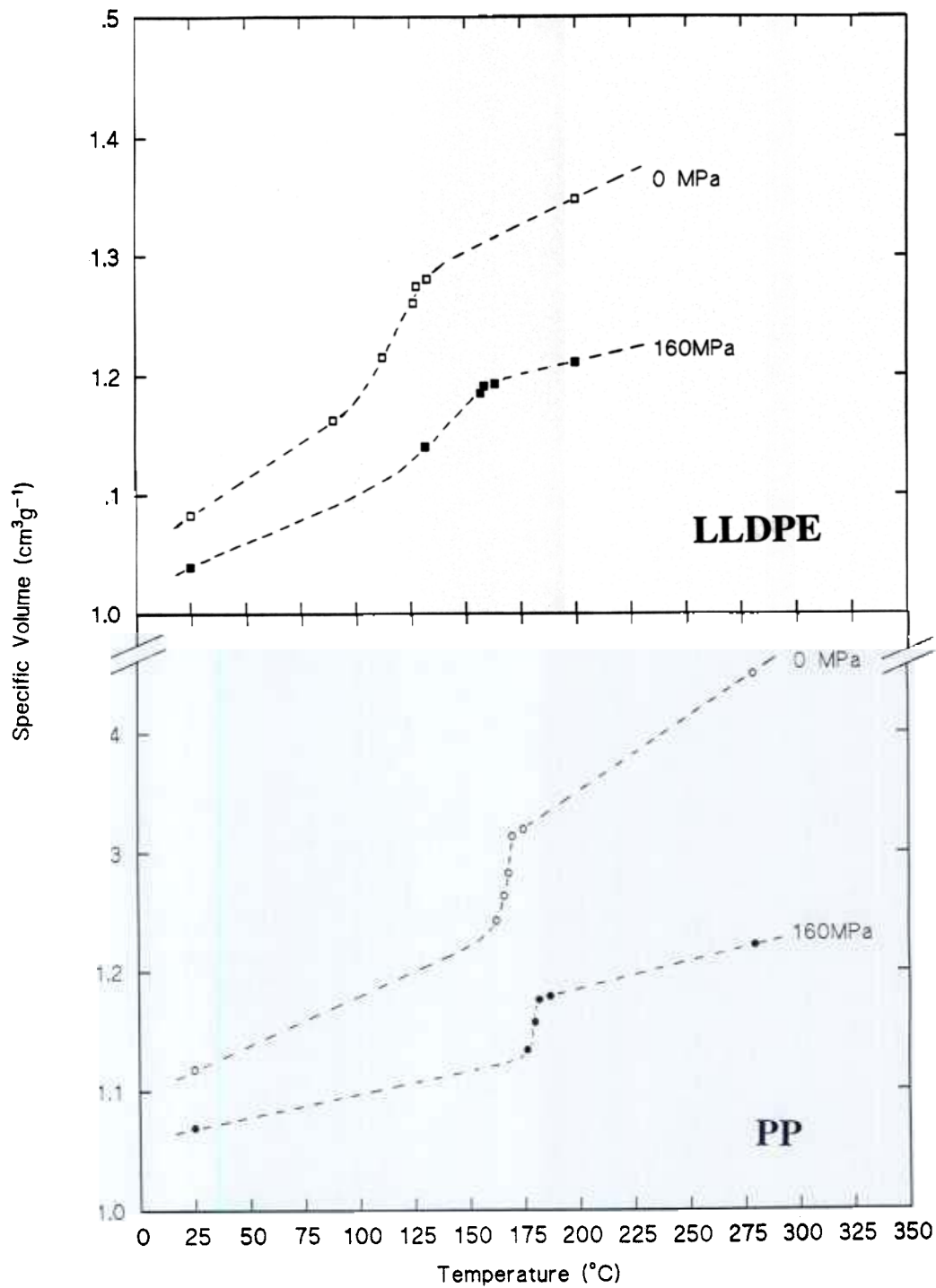


Figure 1: The temperature dependence of specific volume for the grades of LLDPE and PP described in Table 1 at the pressures indicated (taken from the Moldflow database). Notice that in comparison with PP the transition from melt to solid occurs over a much broader range of temperatures in LLDPE.

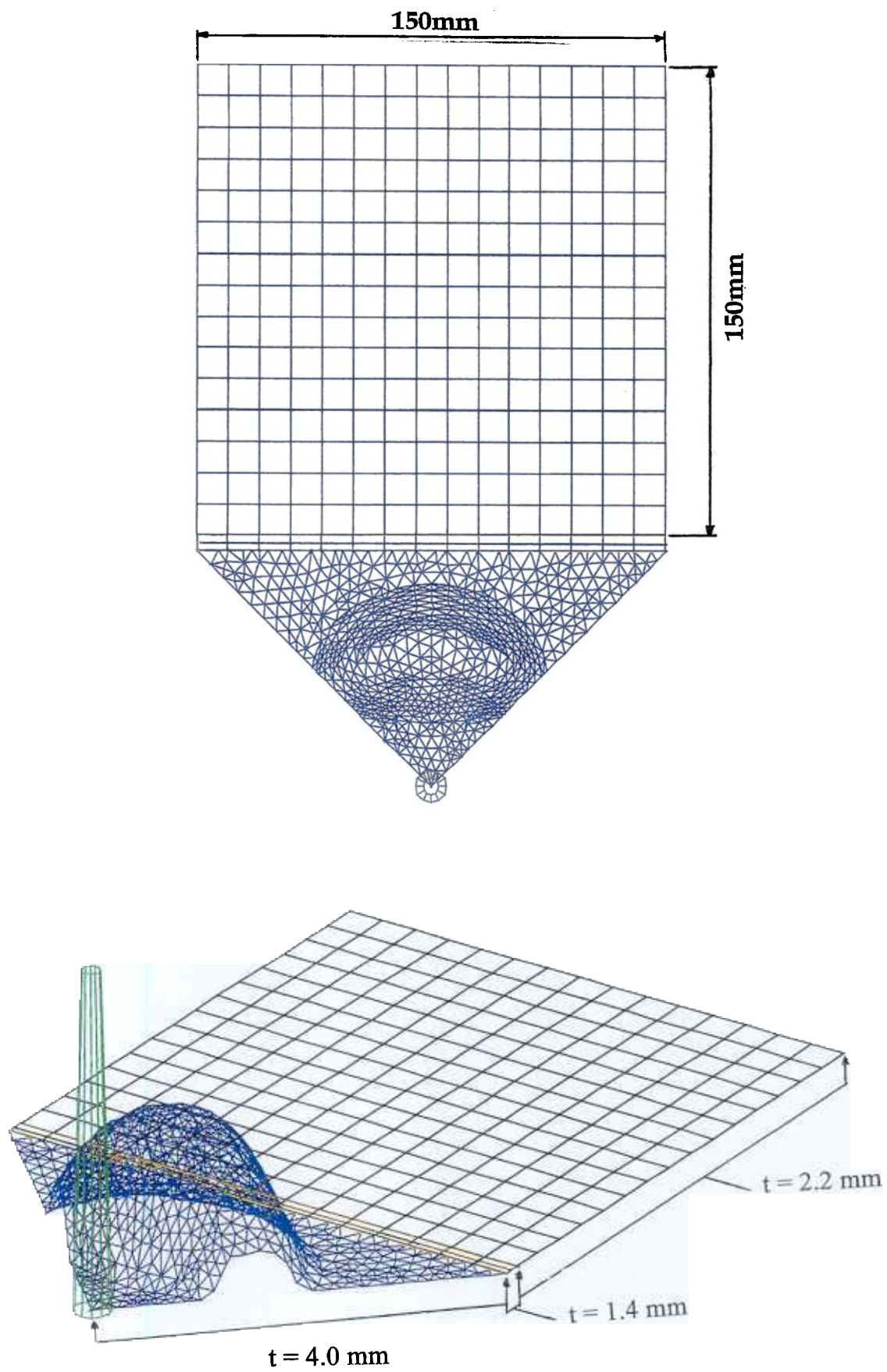


Figure 2: Cavity dimensions where t denotes thickness.

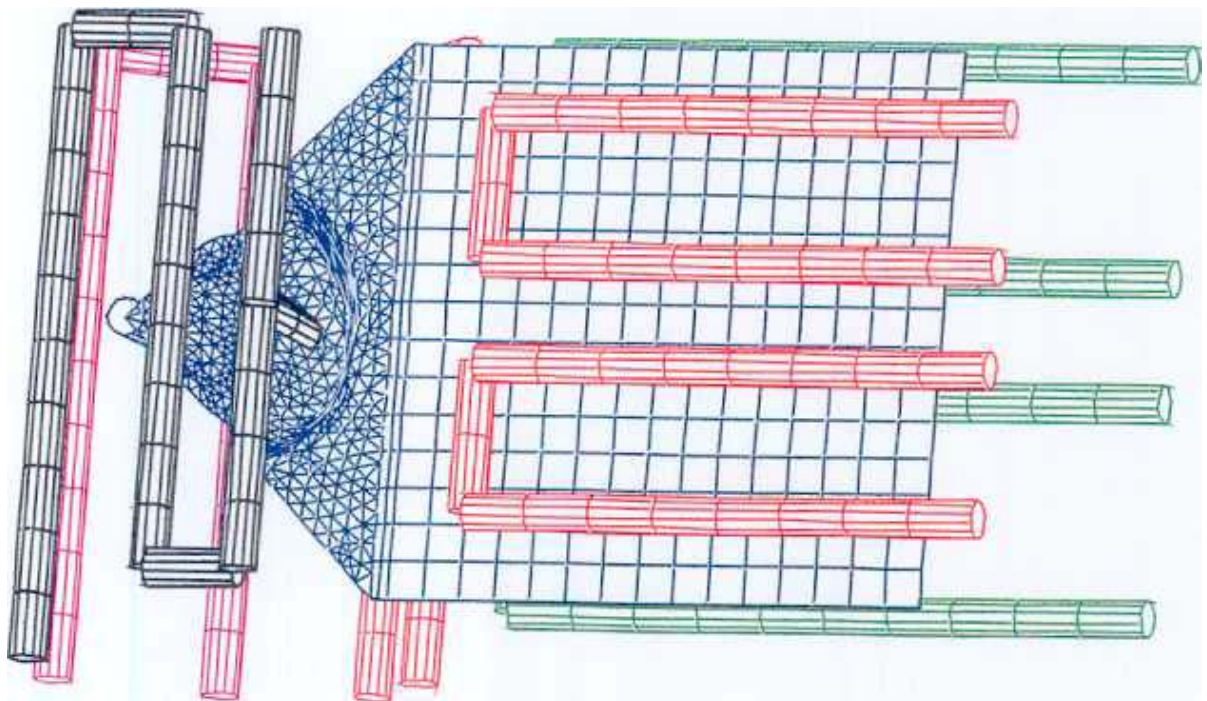
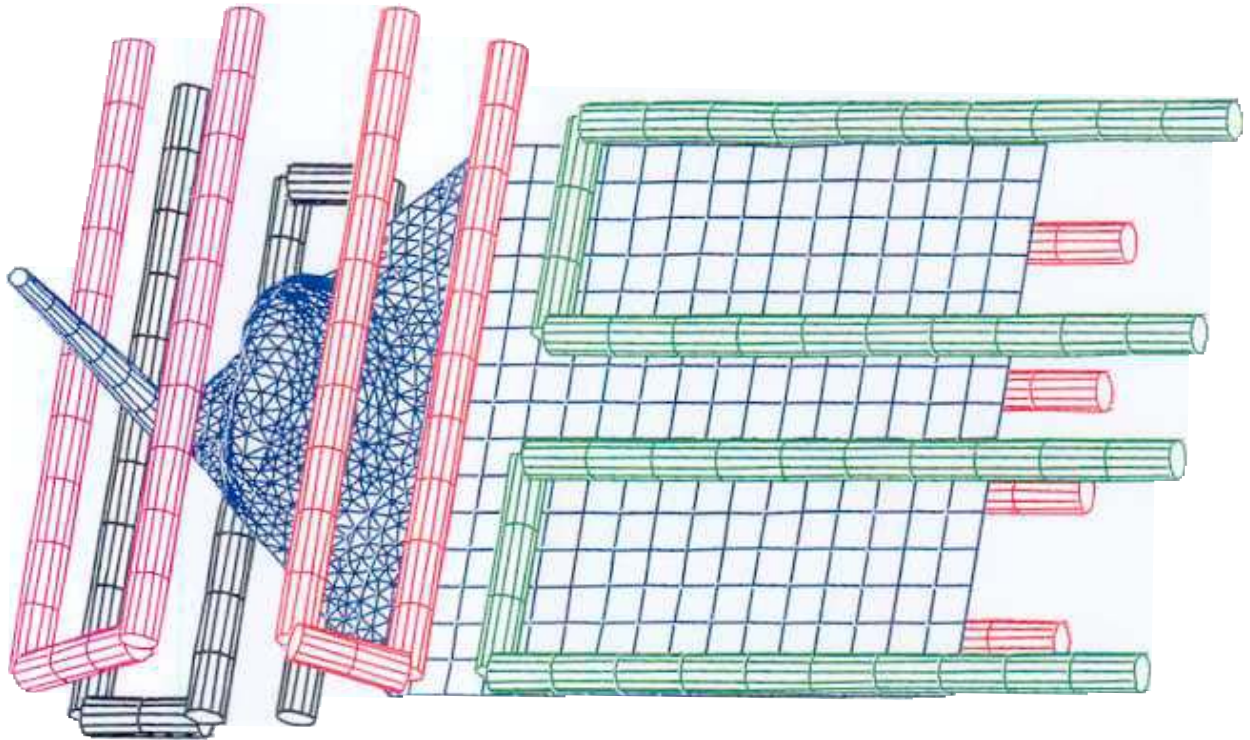


Figure 3: Details of the cooling circuit and its position with respect to the mould cavity.
a) View showing the upper surface of the moulding
b) View from the underside of the cavity showing the position of the baffle beneath the dome.

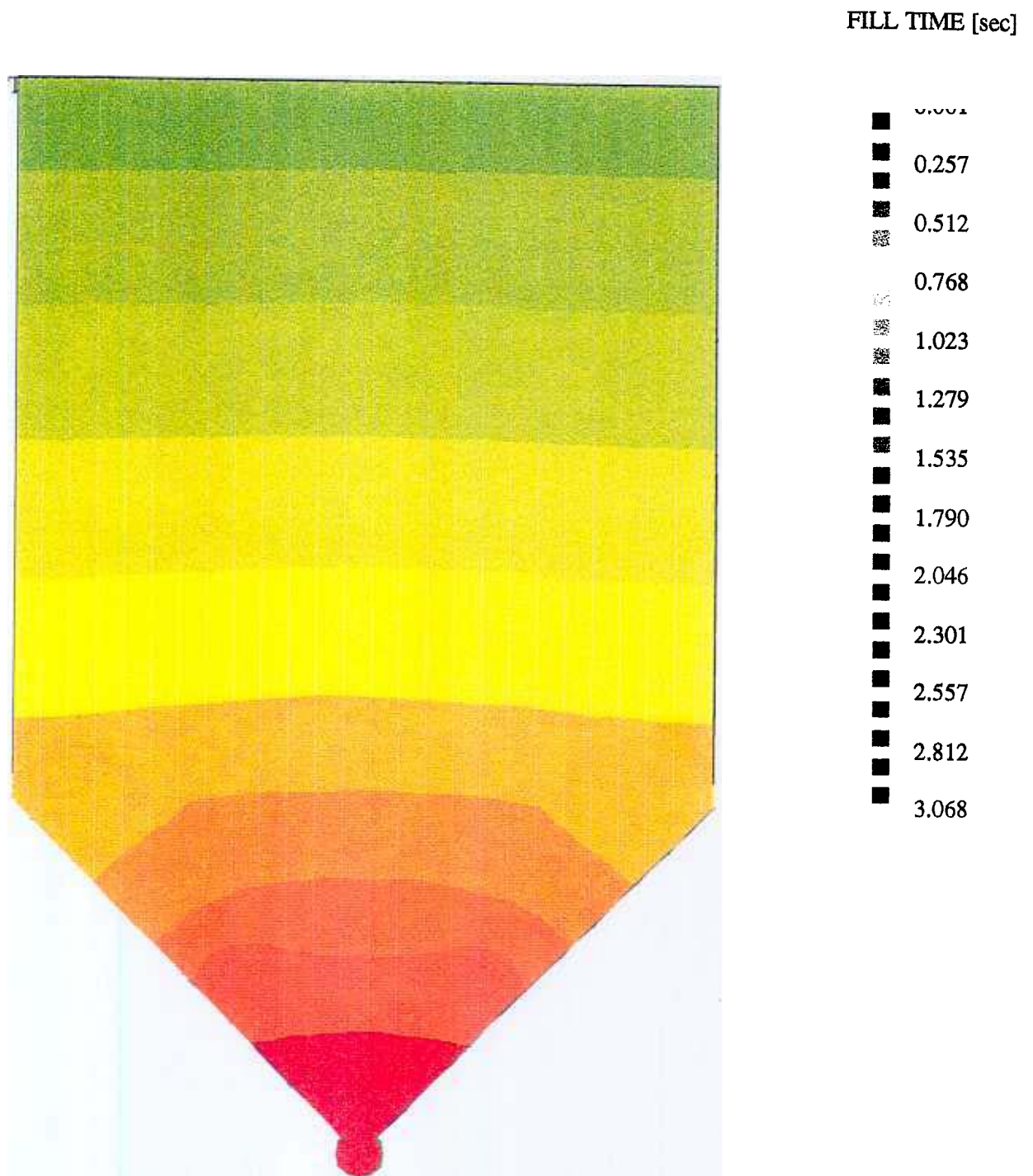


Figure 4: Contour plot showing the flow of melt through the cavity as a function of time for LLDPE. The time for the cavity to fill is 1.2s (melt temp. = 180 °C, mould temp. = 40 °C).

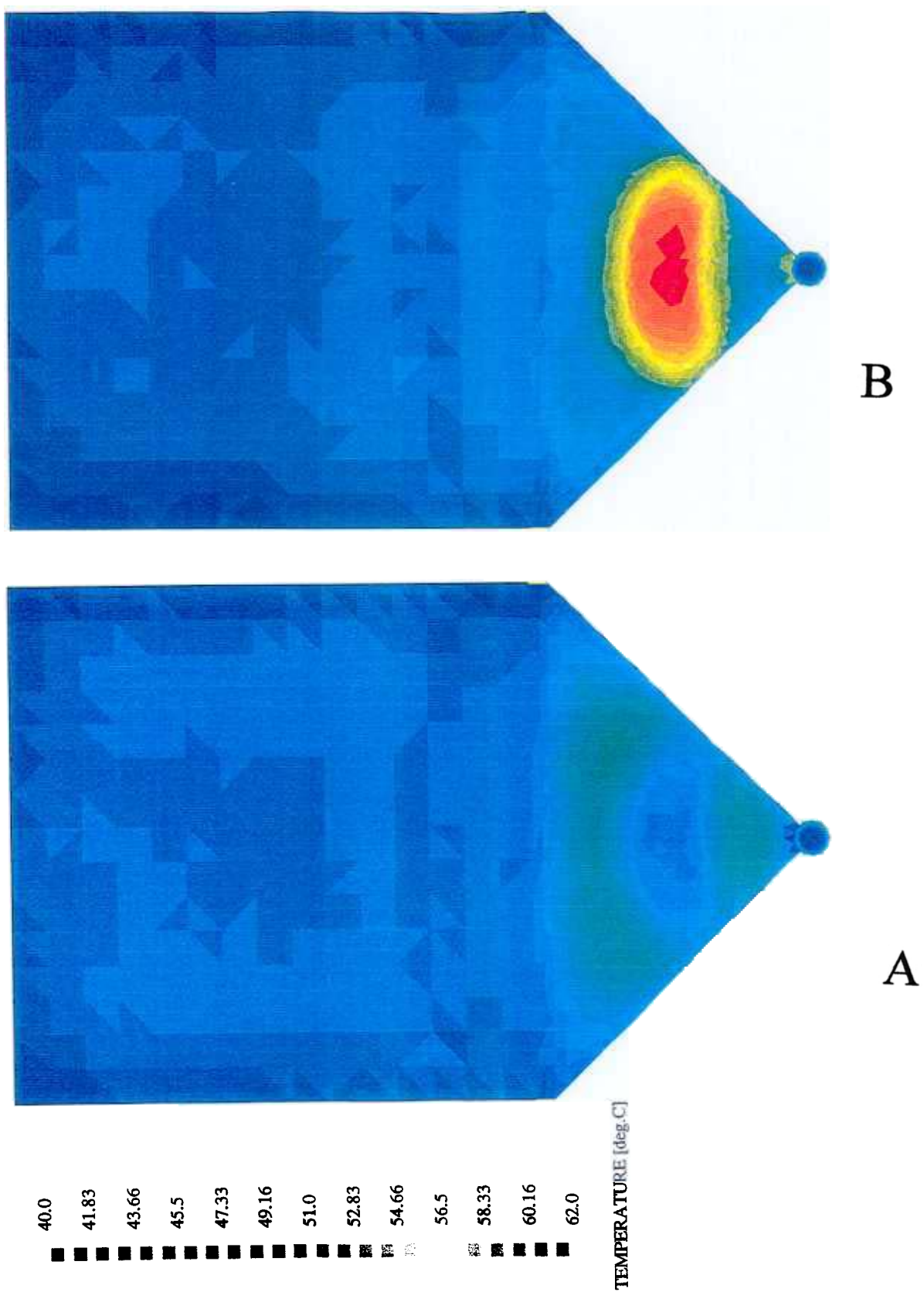


Figure 5: The top (A) and bottom (B) temperatures of a plate of LLDPE. These contour plots show that the baffle on the underside of the cone region of the spreader could be more efficient at removing heat during mould cooling. The upper surface temperature plot shows some unevenness in cooling on either side of the cone. The conditions used to generate this plot are given in run 5 of Table 6.

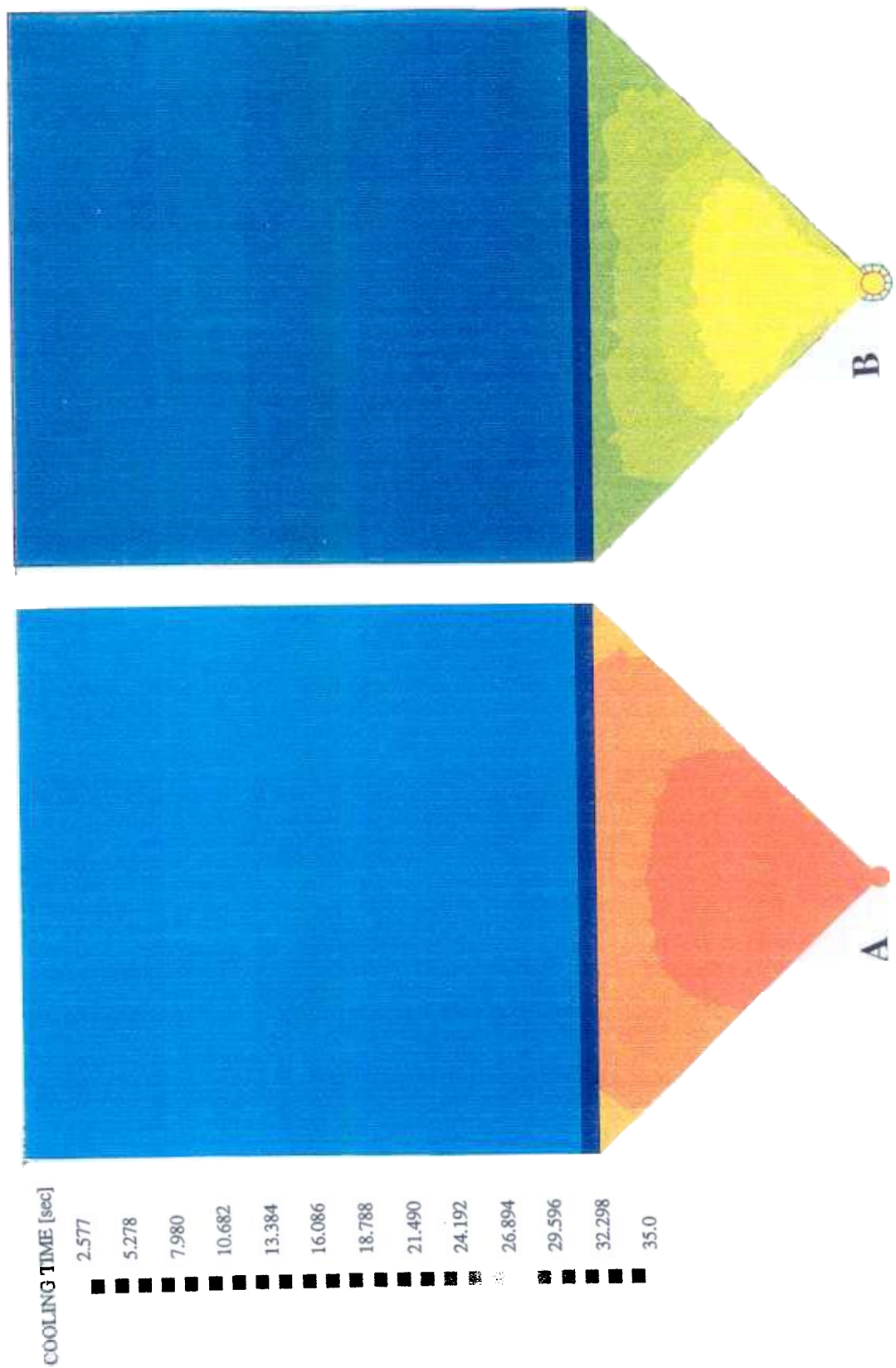


Figure 6: The time taken for the molten LLDPE to cool to the ejection temperature is a reflection of the cavity geometry. The processing conditions used to generate plots A and B correspond to runs 8 and 22 respectively in Table 6.

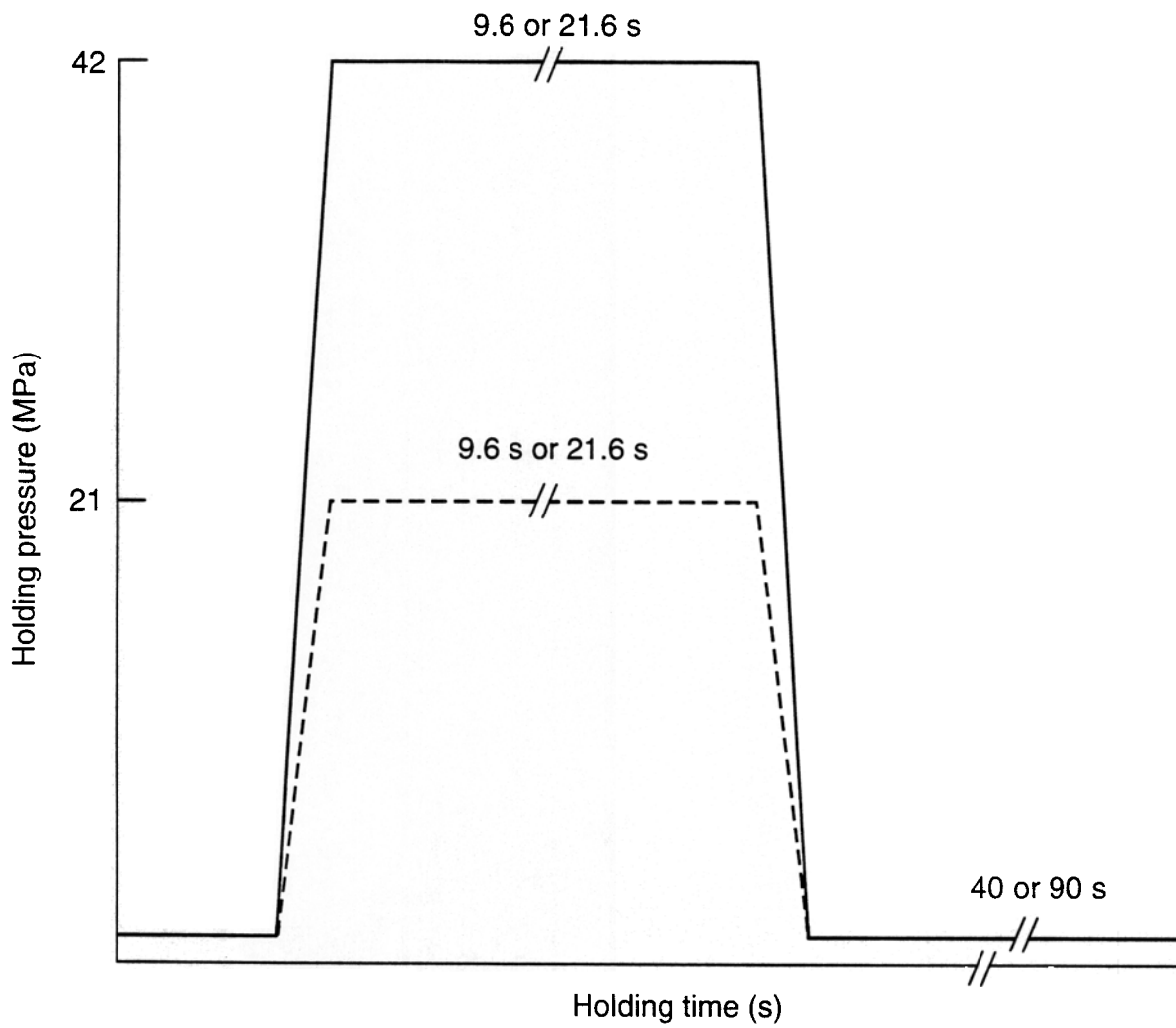


Figure 7: Details of the holding pressure profile for LLDPE. Holding pressures of either 21 or 42MPa are applied and removed over a period of 0.2s. The cooling time after removal of the holding pressure is either 40 or 90s.

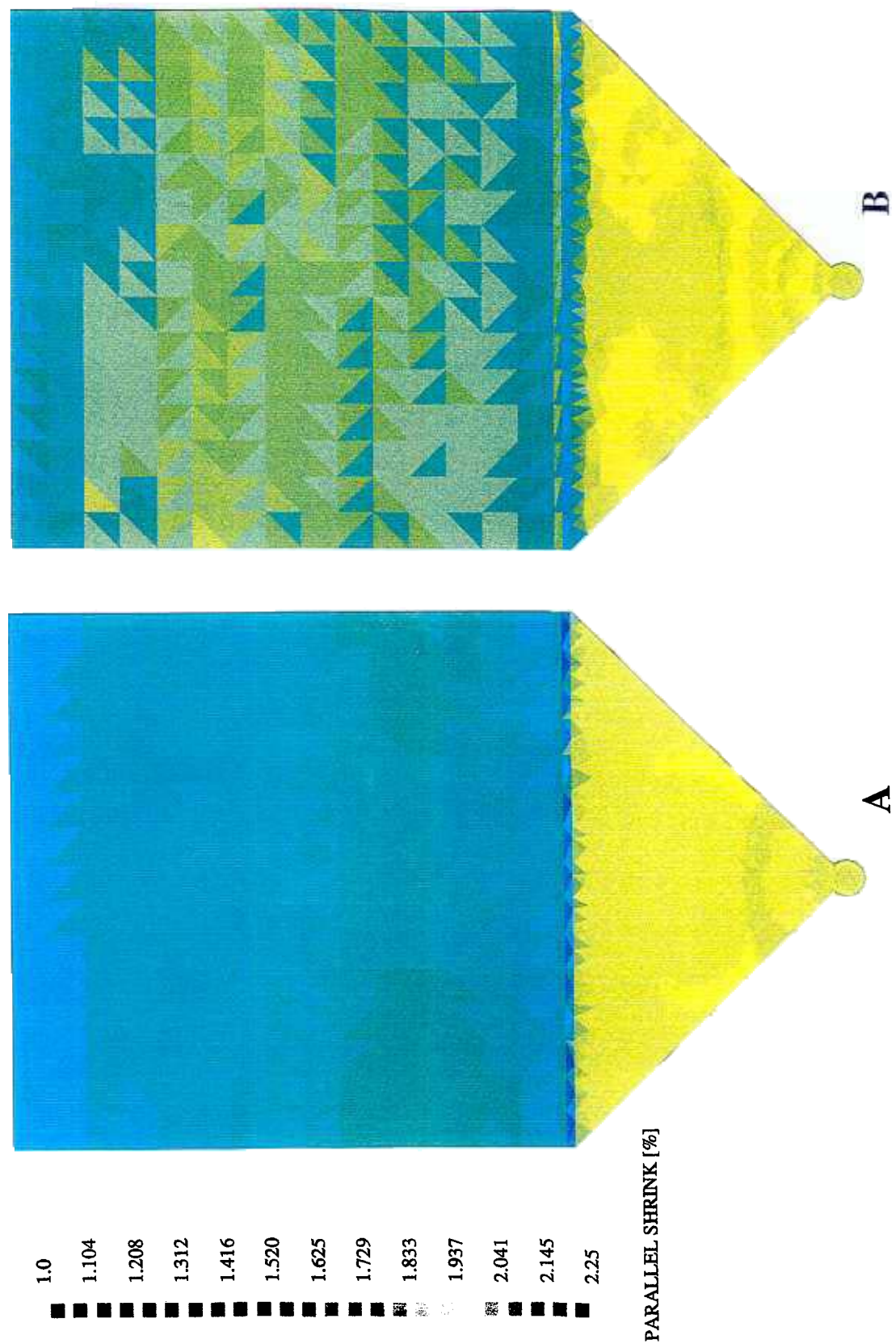


Figure 8: A contour plot of the shrinkage of a PP plate in a direction parallel to that of the flow. The processing conditions used to generate plots A and B correspond to runs 8 and 22 respectively in Table 6.

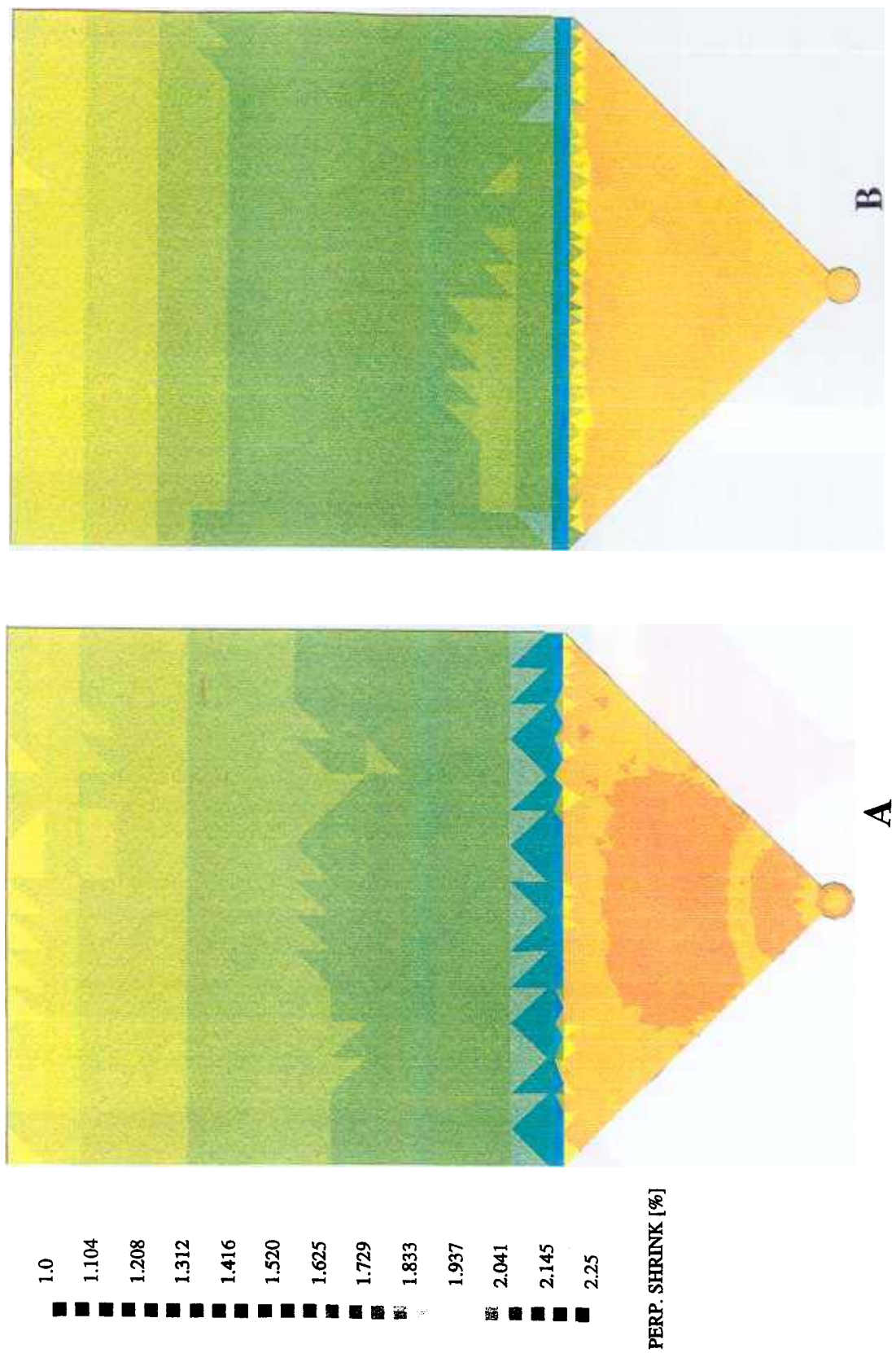


Figure 9: A contour plot of the shrinkage of a PP plate in a direction perpendicular to that of the flow. The processing conditions used to generate plots A and B correspond to runs 8 and 22 respectively in Table 6.

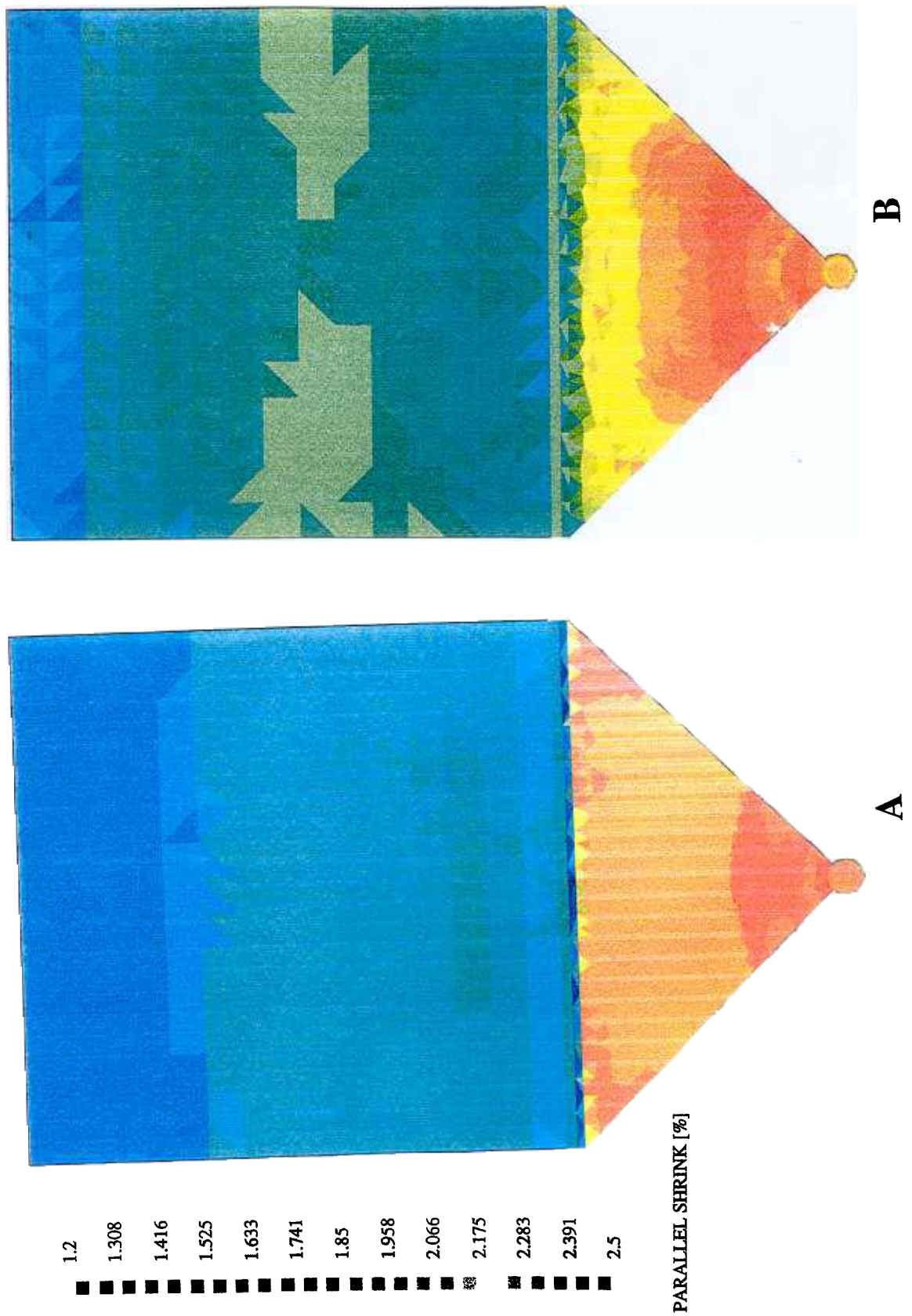


Figure 10: A contour plot of the shrinkage of an LLDPE plate in a direction parallel to that of the flow. The processing conditions used to generate plots A and B may be obtained by substituting the conditions given in Table 4 into runs 8 and 22 respectively in Table 2 .

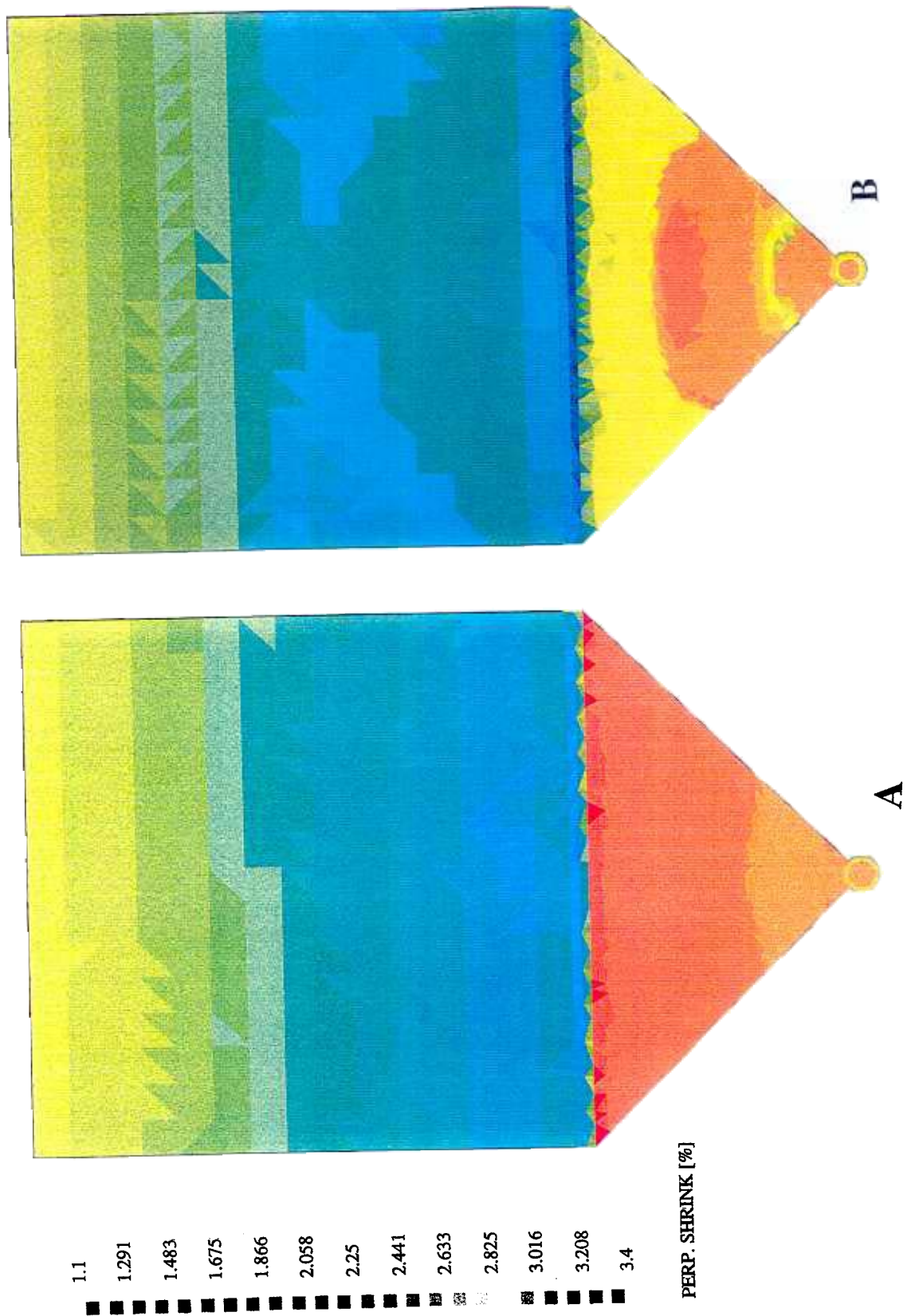


Figure 11: A contour plot of the shrinkage of an LLDPE plate in a direction perpendicular to that of the flow. The processing conditions used to generate plots A and B may be obtained by substituting the conditions given in Table 4 into runs 8 and 22 respectively in Table 2 .

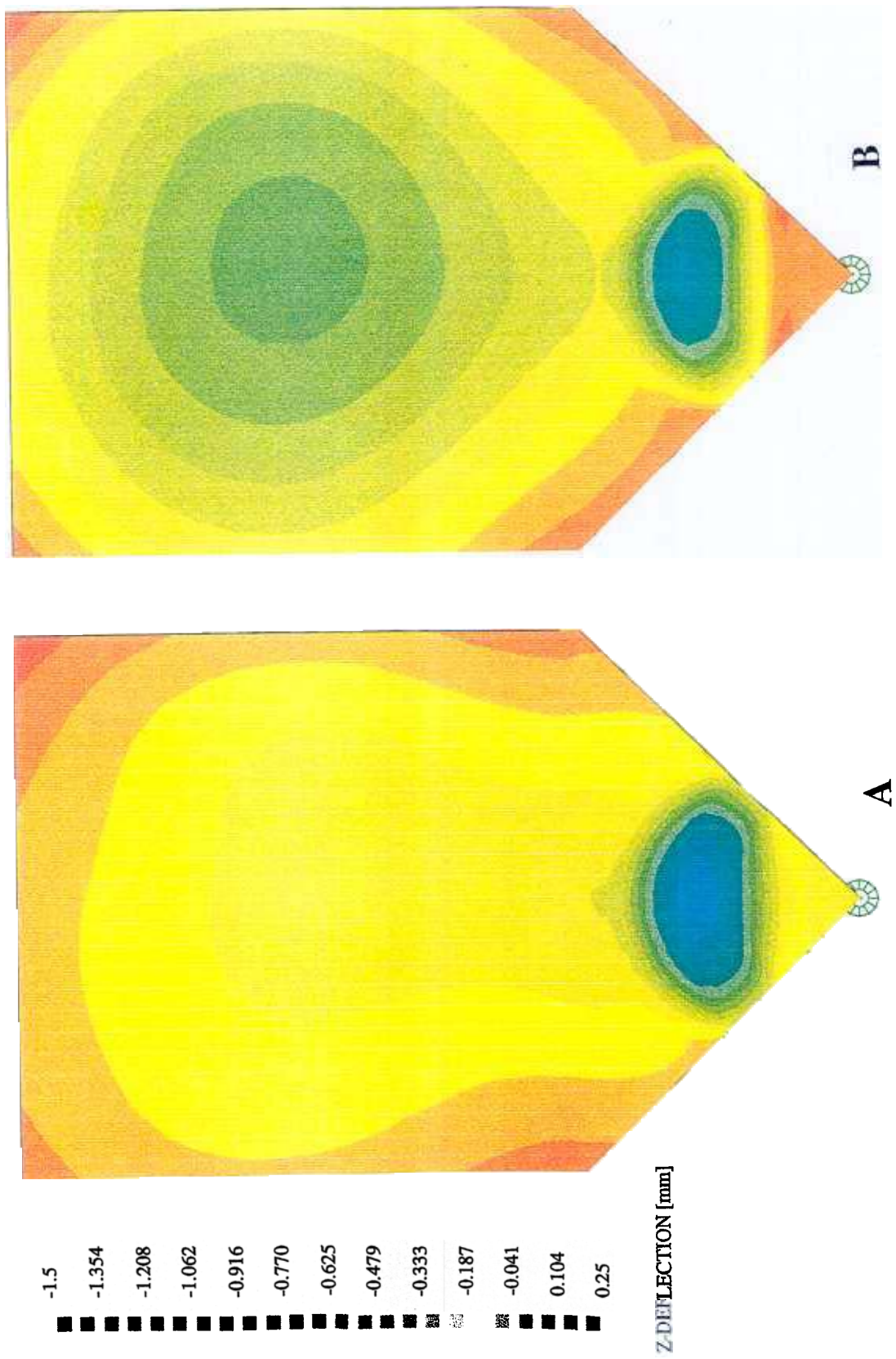


Figure 12: Out-of-plane distortion of PP mouldings. The processing conditions used to generate the plots A and B correspond to runs 8 and 22 respectively in Table 6.

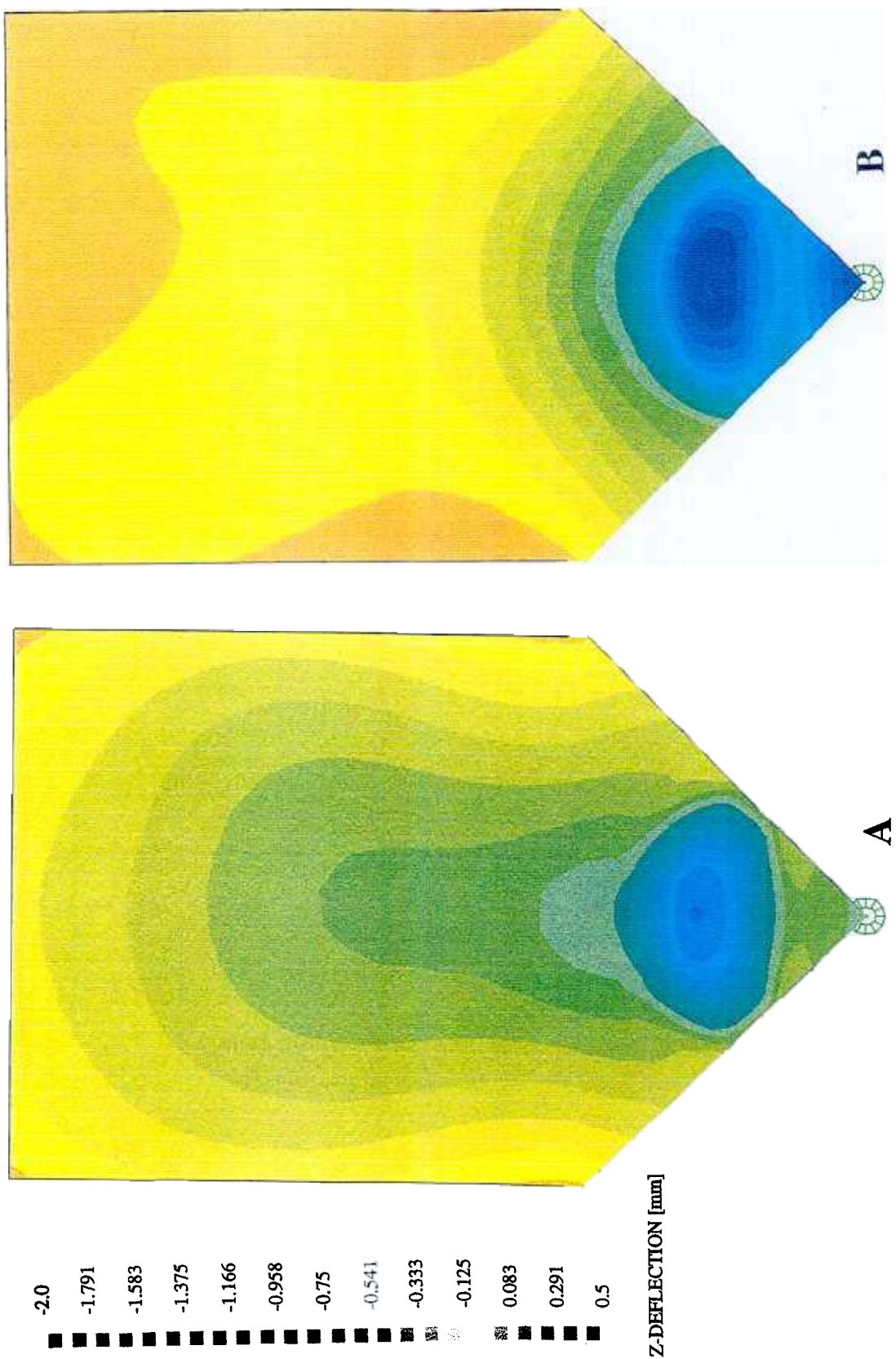


Figure 13: Out-of-plane distortion of LLDPE mouldings. The processing conditions used to generate plots A and B may be obtained by substituting the conditions given in Table 4 into runs 8 and 22 respectively in Table 2 .

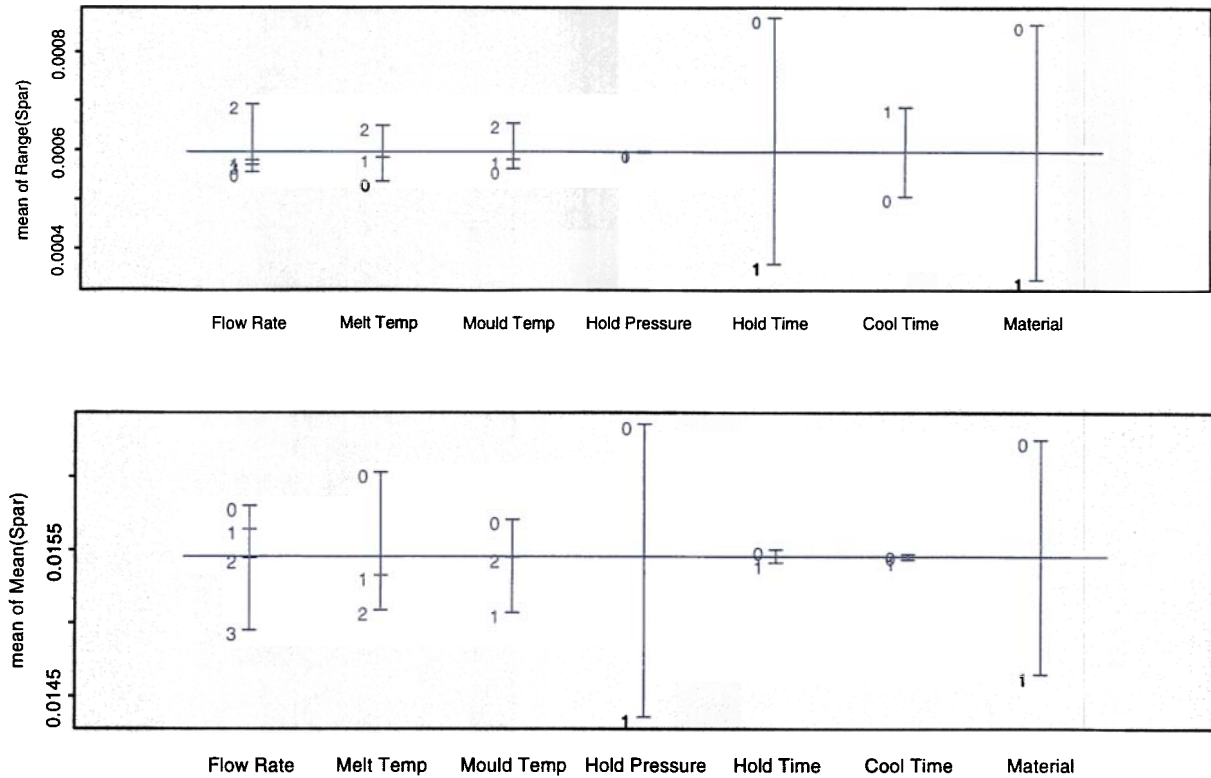
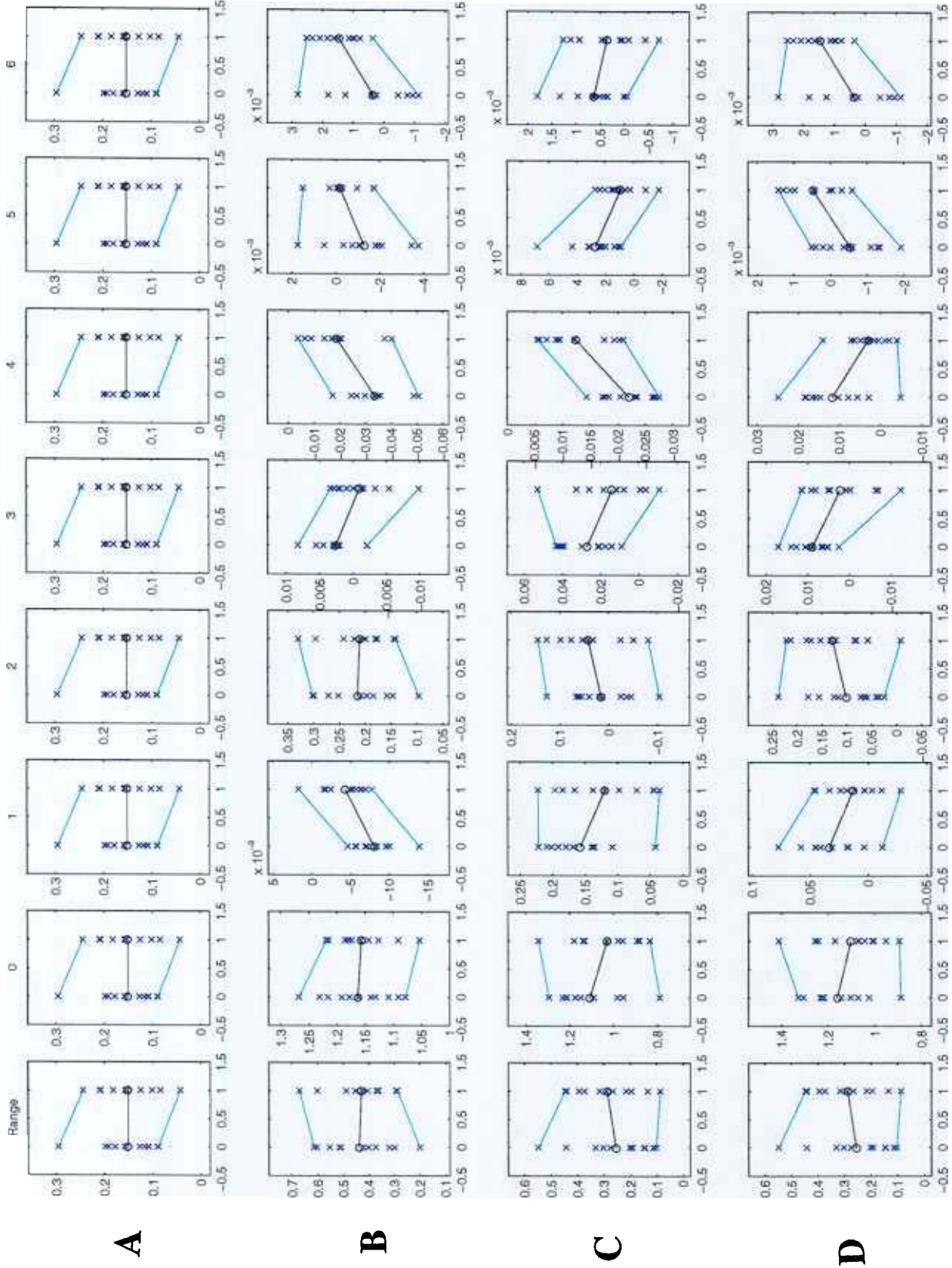


Figure 14: The dependence of the shrinkage that occurs in a direction parallel to that of the flow on different processing conditions. The numbers on each vertical line represent different values of a particular factor. The horizontal line is the grand mean of the averaged data. The trends observed with each processing condition indicate the direction in which a factor could be altered in order to gain a further reduction in shrinkage. However this can be counterproductive, for example, increasing the melt temperature reduces the overall shrinkage of the plate between the two edges running perpendicular to the gate but *increases* the rate at which these edges converge.

Figure 15: The dependence of warpage on cooling time. The plots are the range and values of the polynomial coefficients a_0 to a_6 . The numbers on the ordinates refer to the magnitude of the coefficients whilst those on the abscissae represent different cooling times i.e 0 = 40s and 1 = 90s. A refers to edge data obtained parallel to the flow direction whilst B refers to edge data obtained perpendicular to it. C and D represent the mid-lines parallel and perpendicular to the flow direction respectively.



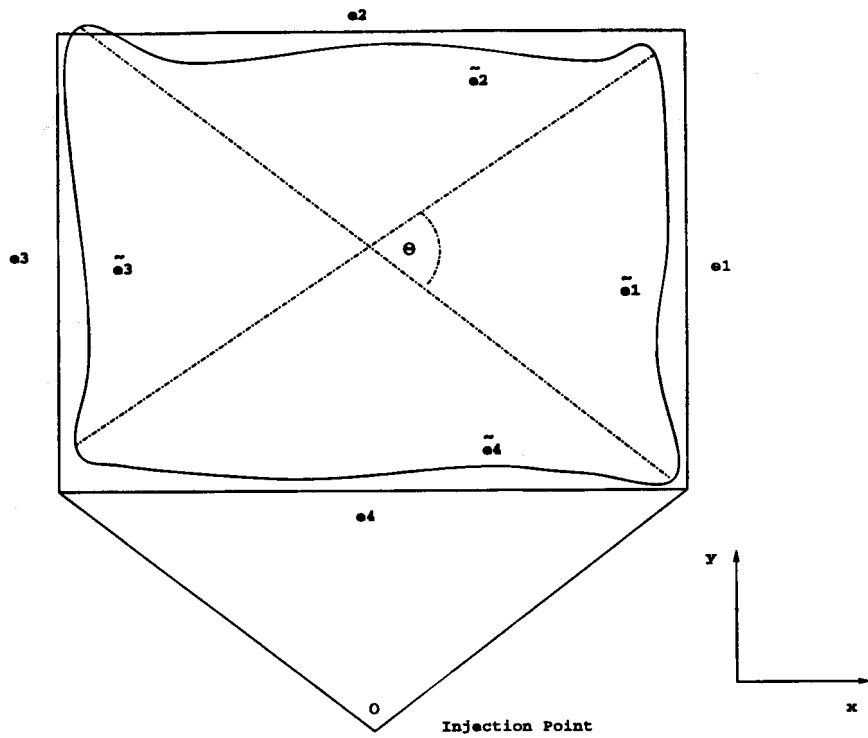


Figure A1: A comparison of the projections of the undeformed plate with edges e_1 , e_2 , e_3 and e_4 and a deformed plate with edges $\tilde{e}_1$, $\tilde{e}_2$, $\tilde{e}_3$ and $\tilde{e}_4$ on to the xy plane. The angle, θ formed by the intersections of the diagonals of the deformed plate is also shown.

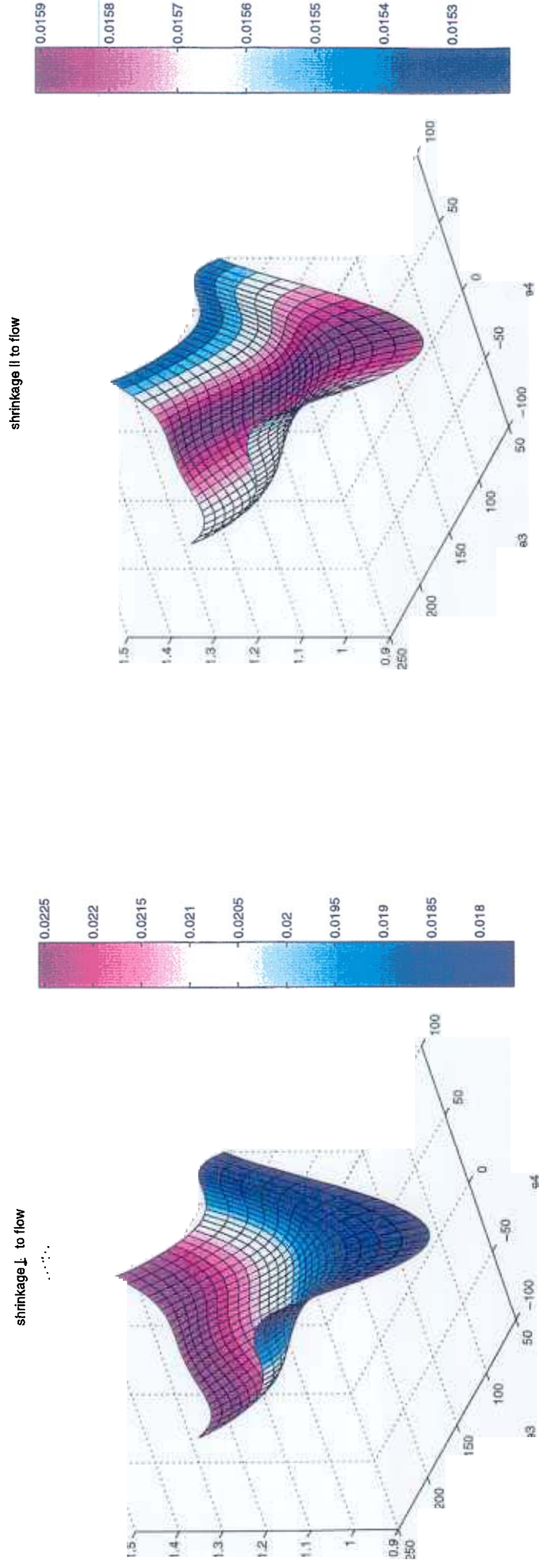


Figure A2: Shrinkage of the plate parallel to the flow direction (left) and perpendicular to it (right) as projected on to the xy plane. The origin is located at the injection point and the coordinates refer to the undeformed plate dimensions including the gate region.

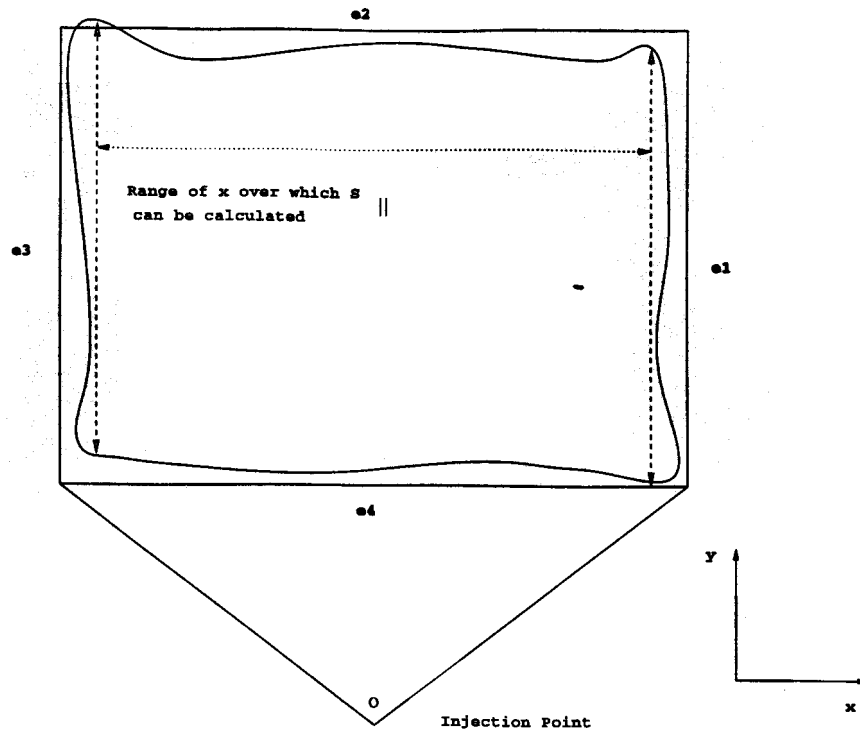


Figure 1: Injection Velocity

Figure A3: Equation A(2) can be used to compute the shrinkage parallel to the direction of flow between the dashed line boundaries.

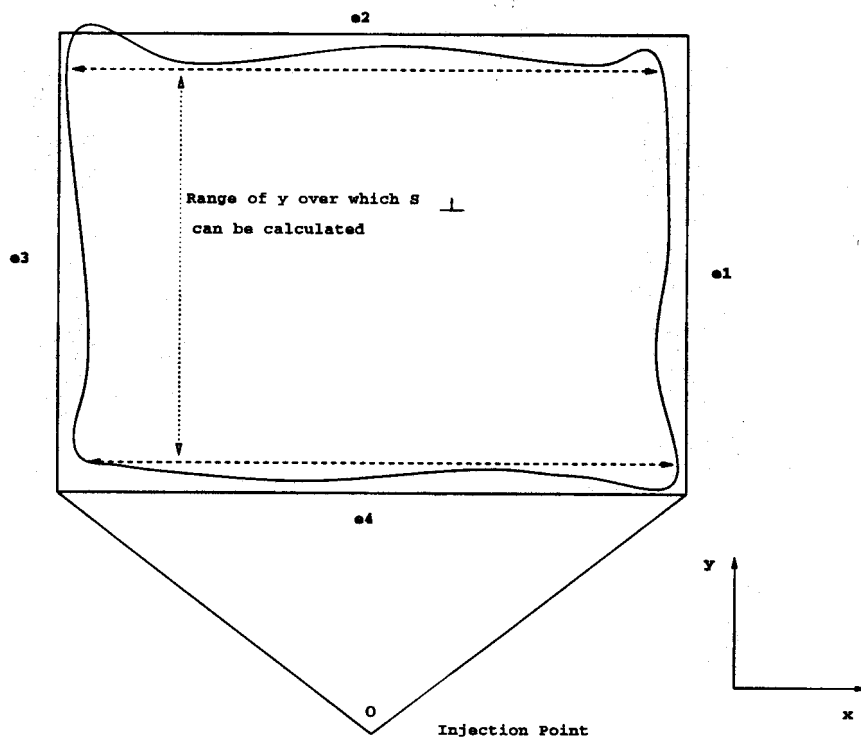


Figure A4: Equation A(3) can be used to compute the shrinkage perpendicular to the direction of flow between the dashed line boundaries.

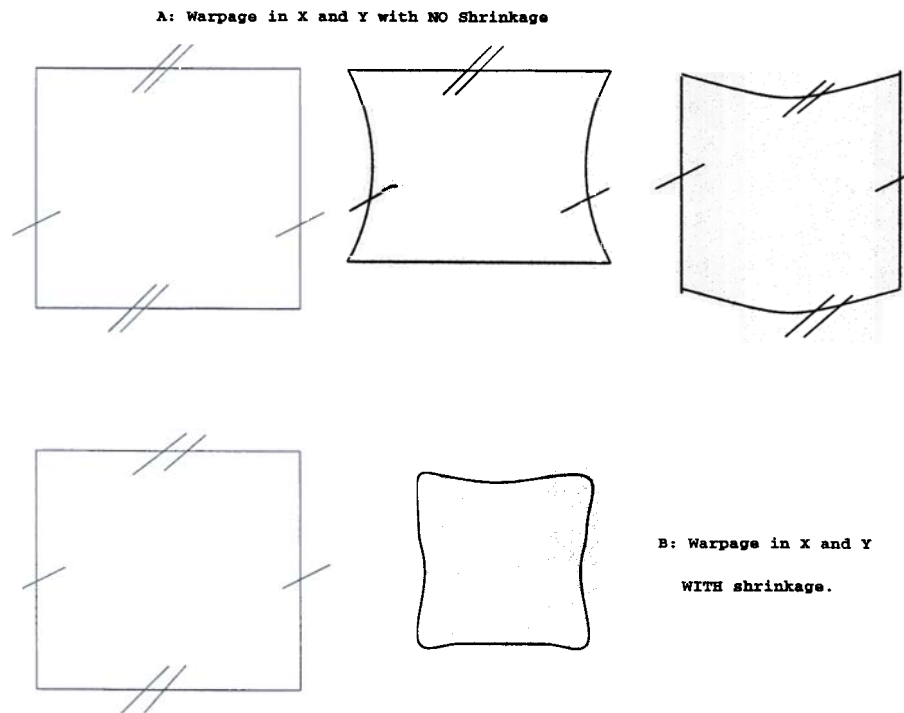


Figure A5: In-plane warpage without shrinkage can occur as shown in A. The single and double diagonal lines represent edges of the same length. In-plane warpage combined with shrinkage is shown in B.

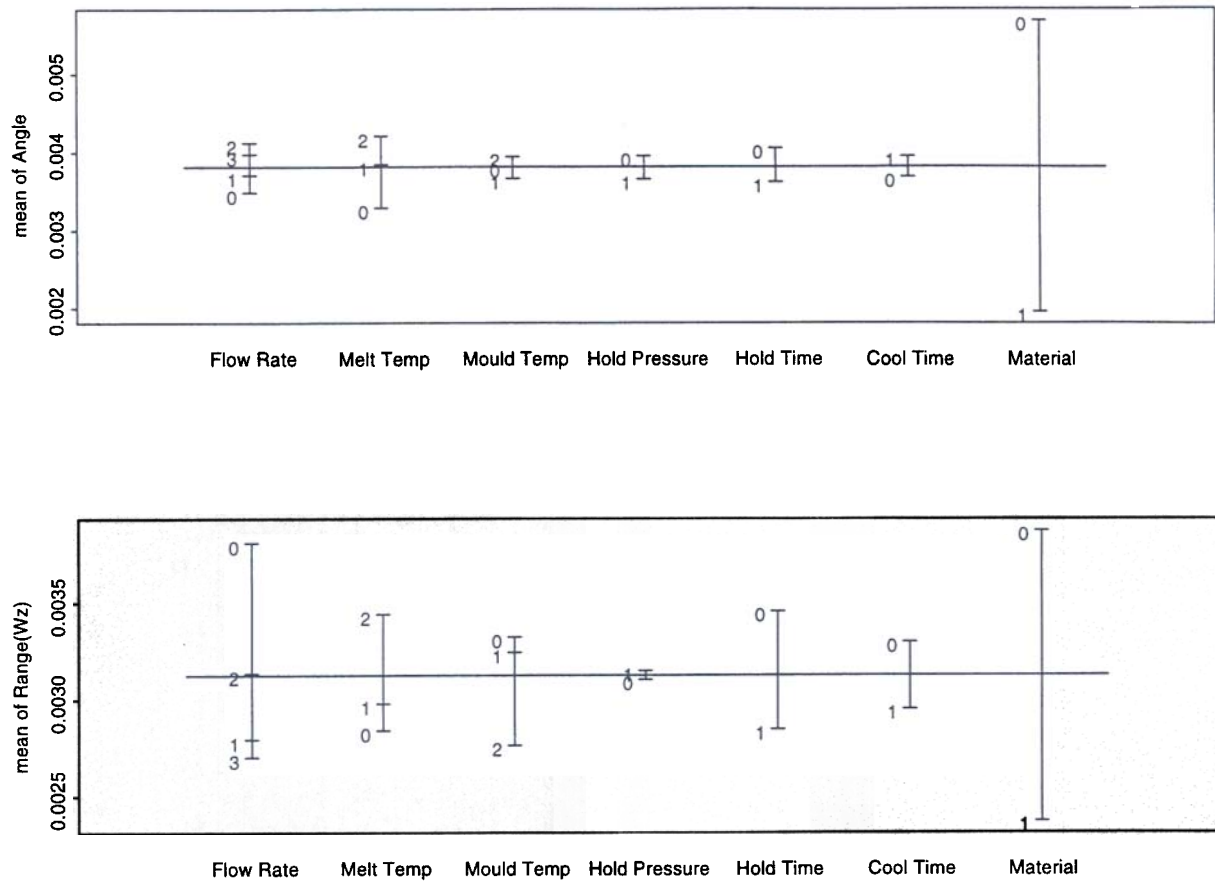


Figure A6: The dependence of the angle of the intersecting diagonals of deformed plates on different processing conditions. The numbers on each vertical line correspond to different levels of each factor with 0 being the minimum. The lower figure shows how the variation in the out-of-plane warpage depends on different processing conditions.

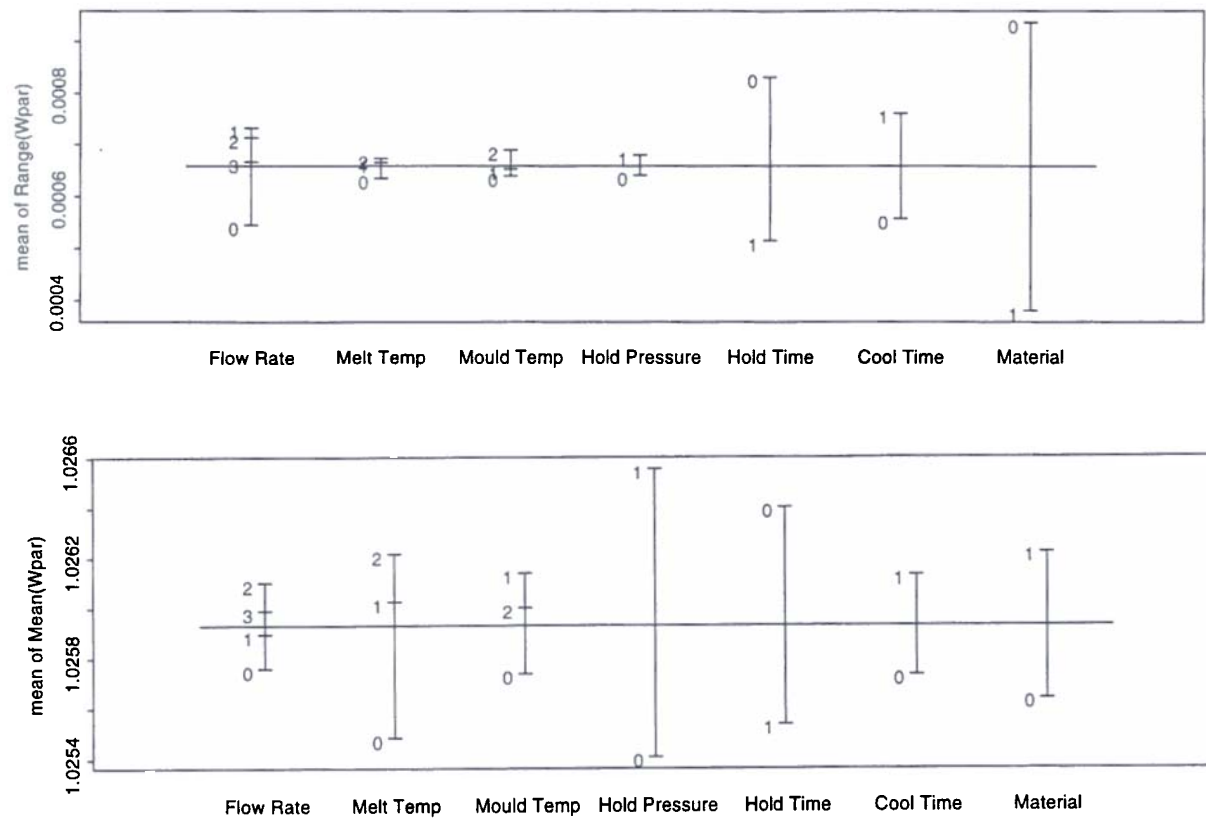


Figure A7: The dependence of the in-plane mean warpage and range of warpages that occur in a direction parallel to that of the flow on different processing conditions.

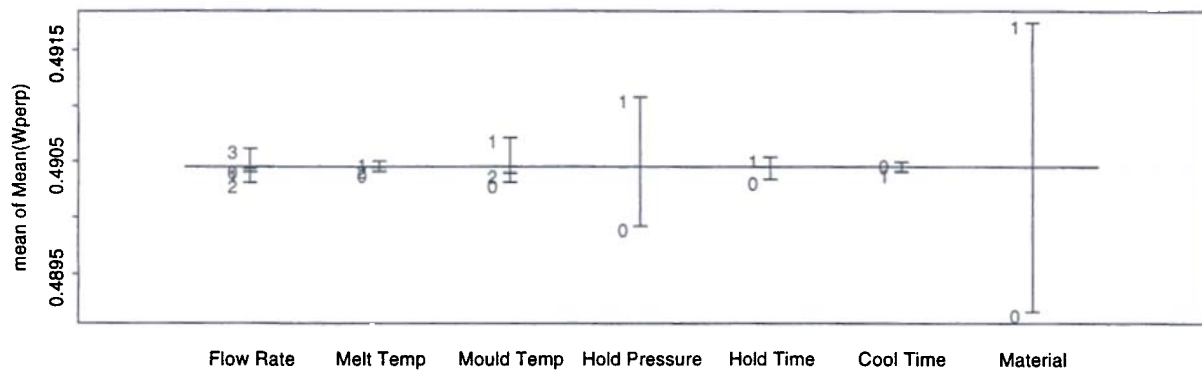
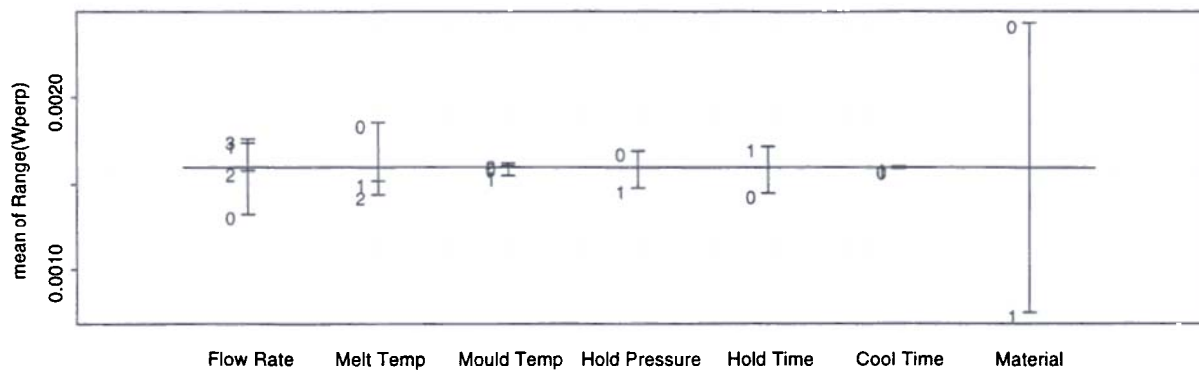


Figure A8: The dependence of the in-plane mean warpage and range of warpages that occur in a direction perpendicular to that of the flow on different processing conditions.

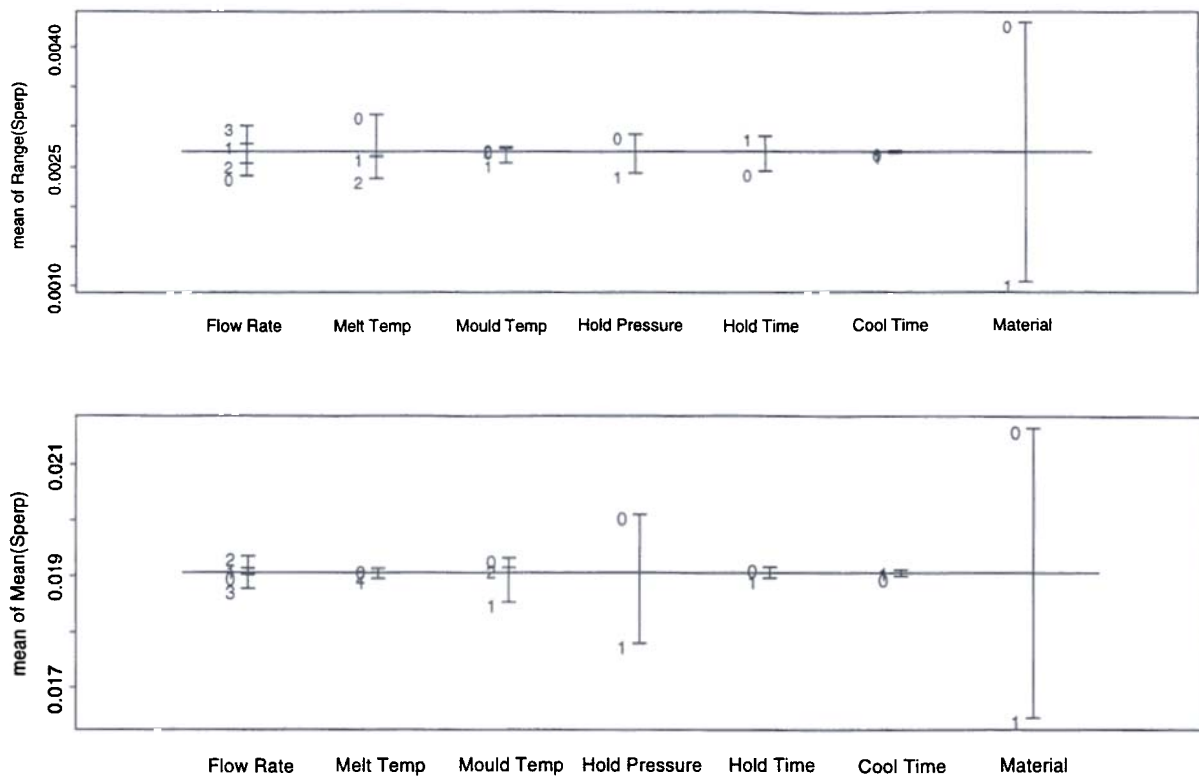


Figure A9: The dependence of the mean shrinkage and range of shrinkage that occurs in a direction perpendicular to that of the flow on different processing conditions.