

The effect of standing waves on the attenuation constant for a low-loss rectangular waveguide

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When standing waves are present in a rectangular waveguide, the resistive losses in the walls are not uniformly distributed. This affects the measurement of the attenuation constant. Here the phenomenon for the TE₁₀ mode in air-filled waveguide, where the deviation from the normal travelling wave attenuation constant is only about $\pm 1\%$ over the conventional bandwidth, is discussed. However, the paper also shows that nearer the cut-off frequency for the waveguide, the deviation could be as high as $\pm 100\%$.

Introduction: When standing waves are present in a rectangular waveguide, the current distribution in the walls is stationary, which is not the case for travelling waves. The attenuation constant for low-loss waveguide is usually defined explicitly in terms of the power travelling down the guide and the power lost per unit length [1] or by using perturbation methods [2, 3]. When a measurement is made, in the presence of standing waves, this definition is no longer valid, as the power lost is now a function of both the forward and reverse waves [4]. This paper describes this phenomenon for the TE₁₀ mode and compares an effective attenuation constant for standing waves, with that for normal travelling waves and its implications for impedance measurements. The paper also examines this phenomenon near the waveguide cut-off frequency.

Theory of standing wave attenuation constant: For a travelling TE₁₀ mode, there are three components of currents in the walls of the waveguide. The J_z currents, which are in the direction of propagation, are in phase with the E_y electric field and H_x magnetic field of the mode and they are described in [5, 6]. Whereas, the J_x and J_y currents are out of phase with the E_y field and have their peaks a quarter of a wavelength down the waveguide. The usual equations for the amplitudes of these currents linked to the amplitudes of the magnetic fields at the walls of the waveguide are given as:

$$|J_x| = |H_z| = \frac{\pi}{\omega\mu a} E_0 \cos\left(\frac{\pi x}{a}\right); |J_y| = |H_z| = \frac{\pi}{\omega\mu a} E_0, \\ |J_z| = |H_x| = \frac{\beta}{\omega\mu} E_0 \sin\left(\frac{\pi x}{a}\right), \quad (1)$$

where E_0 is the amplitude of the E_y field at the centre of the waveguide, a is the broad dimension of the waveguide aperture and β is the phase constant in the z direction. Also, ω is the angular frequency and μ is the permeability of the space inside the waveguide.

These three currents also varying in the z direction with the propagation factor $\exp(-\gamma z)$, where γ is the propagation constant. Now in the presence of standing waves, the reflected wave affects only the attenuation constant, α , so the propagation constant can be redefined as $\gamma = \alpha_D + j\beta$, where α_D is an effective attenuation constant for the length of guide, D . If the reflection coefficient of the termination is defined as $|\rho|\exp(j\theta)$, then adding the forward and reflected magnetic fields, at a distance z from the termination gives:

$$H_{x+} + H_{x-} = \frac{\beta}{\omega\mu} E_0 \sin\left(\frac{\pi x}{a}\right) (\exp(\gamma z) + |\rho| \exp(j\theta) \exp(-\gamma z)), \\ H_{z+} + H_{z-} = \frac{\pi}{\omega\mu a} E_0 \cos\left(\frac{\pi x}{a}\right) (\exp(\gamma z) + |\rho| \exp(j\theta) \exp(-\gamma z)), \quad (2)$$

where the $+$ or $-$ signs in the subscripts of expressions like H_{x+} and H_{x-} indicate the forward and reflected waves, respectively. The power lost in the walls of the waveguide, P_L , is given by the surface integral:

$$P_L = \frac{1}{2} R_s \iint (|J_x|^2 + |J_y|^2 + |J_z|^2) dS, \quad (3)$$

where R_s is the surface impedance of the waveguide walls.

Using Equations (1) and (2) gives:

$$|J_{x+} + J_{x-}|^2 = A (\exp(2\alpha_D z) + |\rho|^2 \exp(-2\alpha_D z) + 2|\rho| \cos(2\beta z - \theta)), \\ |J_{y+} + J_{y-}|^2 = B (\exp(2\alpha_D z) + |\rho|^2 \exp(-2\alpha_D z) + 2|\rho| \cos(2\beta z - \theta)), \\ |J_{z+} + J_{z-}|^2 = C (\exp(2\alpha_D z) + |\rho|^2 \exp(-2\alpha_D z) - 2|\rho| \cos(2\beta z - \theta)),$$

where:

$$A = \left(\frac{\pi}{\omega\mu a} E_0 \cos\left(\frac{\pi x}{a}\right)\right)^2; B = \left(\frac{\pi}{\omega\mu a} E_0\right)^2; C = \left(\frac{\beta}{\omega\mu} E_0 \sin\left(\frac{\pi x}{a}\right)\right)^2. \quad (4)$$

Now for most waveguides, even those used above 500 GHz, if D is of the order of λ_C , then $\alpha_D D \ll 1$, within the normal waveguide bandwidth, where λ_C is the free-space wavelength at the cut-off frequency for the waveguide. Assuming $\exp(2\alpha_D D) \cong 1 + 2\alpha_D D$, by using Equations (3) and (4) the power lost in the walls over a distance, D , from the termination is:

$$P_L = \frac{1}{2} R_s \left(\frac{E_0}{\omega\mu}\right)^2 \left(\left(\left(\frac{\pi}{a}\right)^2 (a + 2b) + \beta^2 a \right) (1 + |\rho|^2) D \right. \\ \left. + \left(\left(\frac{\pi}{a}\right)^2 (a + 2b) - \beta^2 a \right) \frac{2|\rho|}{k_z} \sin(\beta D) \cos(\beta D - \theta) \right), \quad (5)$$

where b is the shorter dimension of the waveguide aperture.

The power travelling down the waveguide using Poynting's vector is given by:

$$P = \frac{\beta a b E_0^2}{4\omega\mu}. \quad (6)$$

If P is the power arriving at the termination, then using the effective attenuation constant, α_D , the power lost by the forward and reverse waves in the distance, D , is given by:

$$\text{Power lost} = P (\exp(2\alpha_D D) - 1 + |\rho|^2 (1 - \exp(-2\alpha_D D))) \cong 2\alpha_D D P (1 + |\rho|^2). \quad (7)$$

Now Equations (5) and (7) both relate to the losses in the waveguide. So, combining these two equations, along with Equation (6) gives an expression for α_D as,

$$\alpha_D = \alpha + \frac{R_s}{\omega\mu a b k_z} \left(\left(\frac{\pi}{a}\right)^2 (a + 2b) - \beta^2 a \right) \\ \frac{2|\rho|}{(1 + |\rho|^2)\beta D} \sin(\beta D) \cos(\beta D - \theta), \quad (8)$$

where α is the travelling wave attenuation constant and is given by the usual equation [5, 6]:

$$\alpha = \frac{R_s}{\omega\mu a b \beta} \left(\left(\frac{\pi}{a}\right)^2 (a + 2b) + \beta^2 a \right) \\ = \frac{R_s}{b\eta \left(1 - \left(\frac{f_c}{f}\right)^2\right)^{\frac{1}{2}}} \left(1 + \frac{2b}{a} \left(\frac{f_c}{f}\right)^2 \right), \quad (9)$$

where η is the characteristic impedance of free space. This equation is equivalent to the equations given in recent international standards [7, 8].

Rewriting Equation (8), using Equation (9) gives:

$$\frac{\alpha_D}{\alpha} = 1 + \frac{\left(2 \left(1 + \frac{b}{a}\right) \left(\frac{f_c}{f}\right)^2 - 1 \right)}{\left(1 + \frac{2b}{a} \left(\frac{f_c}{f}\right)^2 \right)} \\ \times \frac{2|\rho|}{(1 + |\rho|^2)\beta D} \sin(\beta D) \cos(\beta D - \theta). \quad (10)$$

Equation (10) is illustrated in Figure 1 where the ratio of the standing wave attenuation, α_D , to the travelling wave attenuation, α , over the

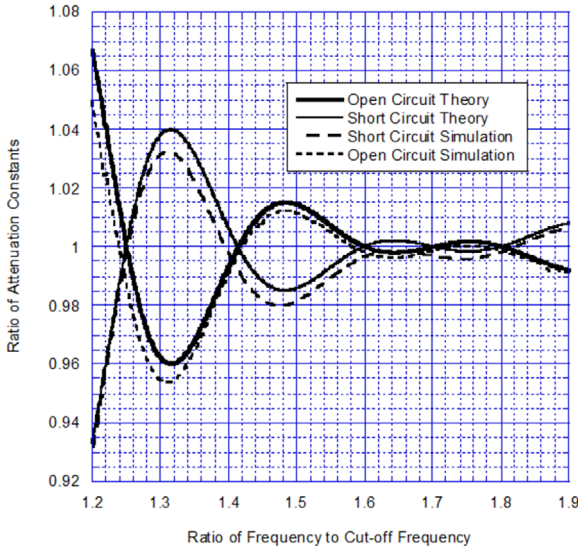


Fig. 1 The ratio of the standing wave attenuation constant to the travelling wave attenuation constant for both an open and a short circuit termination. The length of the waveguide is equal to the cut-off wavelength. The curves show the theory from Equation (10) compared to the simulation from Equation (11)

normal bandwidth of a waveguide is shown. The frequency has been normalised to the cut-off frequency so that the graph applies to any size of waveguide. The terminations have been chosen to be either an open or a short circuit. So $|\rho| = 1$, and $\theta = 0$ or π . The length of the waveguide, D , has been taken as λ_c . Also, in Figure 1 are the results of an electromagnetic simulation using CST Microwave Studio. The simulation used CST's frequency domain solver with perfect E- and H-planes for the short and open circuit terminations, respectively. (The walls of the waveguide had a relatively low value of conductivity, 50,000 S/m, to exaggerate the deviation between α_D from α in Equation (8).) The simulation gave the magnitude of the scattering parameter S_{11} , which was then used to produce a value for the ratio of attenuation constants, as follows:

$$|S_{11}| = \exp(-2\alpha_D D), \quad (11)$$

and

$$\frac{\alpha_D}{\alpha} = -\frac{\ln(|S_{11}|)}{2\alpha D}.$$

By examining Figure 1, it can be seen that there are two effects taking place. The first relates to the numerator of the first term in Equation (10), which goes to zero as the frequency increases and then after that goes negative. The term is:

$$2 \left(1 + \frac{b}{a} \right) \left(\frac{f_c}{f} \right)^2 - 1. \quad (12)$$

The zero occurs when $f = 1.699f_c$ for $a = 2.25b$, and this is visible in Figure 1. (The situation where $a = 2.25b$ occurs for WR-90 (X band) waveguide.) The second effect concerns the second term in Equation (10) where for an open or a short circuit termination the term becomes:

$$\pm \frac{\sin(2\beta D)}{2\beta D}. \quad (13)$$

This particular shape can be clearly seen with the zero values given by:

$$2\beta D = n\pi, \text{ or } f = f_c \left(1 + \left(\frac{n}{4} \right)^2 \right)^{\frac{1}{2}} \quad (14)$$

for $n = 1, 2, 3 \dots$

The values of f/f_c for $\alpha_D = \alpha$, are 1.25, 1.41, 1.6, 1.8 for $n = 3, 4, 5$ and 6, respectively. The combination of these two effects is that at these particular frequencies there is no difference between the two attenuation constants. Also, the first effect greatly reduces the size of any differences so that, except at the lower frequencies within the bandwidth,

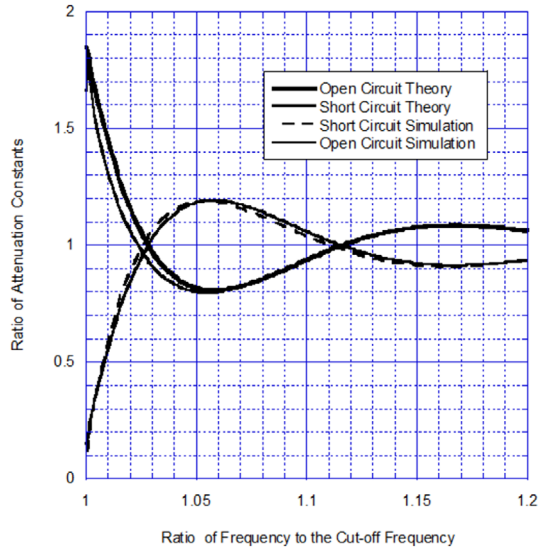


Fig. 2 The ratio of the standing wave attenuation constant to the travelling wave attenuation constant for both open and short circuit terminations. In view of the more complex theory, this graph has been plotted for a cut-off frequency of 6.57 GHz, however it will not change much for higher frequencies

the differences are no more than $\pm 1\%$. For values of $D > \lambda_c$, these differences would be further reduced.

The region near the cut-off frequency: The region of frequencies from the cut-off frequency up to the lowest frequency of the normal bandwidth is not usually used in measurements, due to rapid changes in the propagation constant. However, to complete the theory presented here, this will now be briefly considered. At the cut-off frequency, Equation (9) predicts an infinite value for the attenuation constant. This is not correct as the waveguide losses modify the propagation constant and a complete theory is given in [1, 9].

A modified propagation constant, γ_M , is needed to predict correctly the behaviour near the cut-off frequency. This is given by solving for α_M and β_M , the components of the modified propagation constant:

$$\gamma_M^2 + \beta^2 + (1 - j)G = 0, \text{ where } \gamma_M = \alpha_M + j\beta_M \text{ and } G$$

$$= 2 \left(\frac{\omega \epsilon_0 R_s}{b} \right) \left(1 + \left(\frac{2b}{a} \right) \left(\frac{f_c}{f} \right)^2 \right) = 2\alpha_M \beta_M = 2\alpha\beta, \quad (15)$$

where ϵ_0 is the permittivity of free space.

At the cut-off frequency, $\alpha \rightarrow \infty$, and $\beta \rightarrow 0$, but by using Equation (15), α_M does not become infinite nor does β_M go to zero. The cut-off frequency can then be redefined as the point where α_M and β_M have the same numerical value. This occurs at a frequency just below the lossless waveguide cut-off frequency [9]. Since $\alpha \cong \alpha_M$ and $\beta \cong \beta_M$ at frequencies in the normal waveguide bandwidth, these new modified propagation constants only need to be used as the frequency approaches the lossless waveguide cut-off frequency. The low-loss condition, $\alpha_D D \ll 1$, used in the main theory, is still valid at these frequencies. The results are shown in Figure 2. Using Equation (14), the values of f/f_c for $\alpha_D = \alpha$, are 1.031 and 1.118 for $n = 1$ and 2. The presence of the surface resistance, R_s , means that the theory is more complex than that described in the previous section. To construct Figure 2, the cut-off frequency was chosen as 6.57 GHz, corresponding to WR-90 (X Band) waveguide.

Conclusion: For most waveguides, the attenuation is not a limit to measurements as the attenuation constant for travelling waves, given in Equation (9), is approximately 0.1 dB/m at 10 GHz and about 250 to 650 times that value (depending on the waveguide size) at around 500 GHz, where usually only short lengths of waveguide are used. The process of measuring the impedance of a termination may well involve standing waves but this paper has shown that these standing waves will only cause a $\pm 1\%$ variation in the attenuation constant. This will result in approx-

imately a similar variation in the amplitude of the reflection coefficient of the termination. So only measurements requiring very low uncertainty will need any correction.

One of the most sensitive methods for measuring the attenuation constant is by using a resonant cavity. In this case, the resonant frequencies are given by Equation (13) and so there will be no difference between the measured value and the travelling wave value of the attenuation constant at these frequencies.

Both Figures 1 and 2 show a close agreement between the theory given in this paper and the simulations, which has the advantage of mutual verification. Practical measurements of this effect have proved difficult at centimetre wavelengths (e.g. X band), as it requires measuring the attenuation constant to an accuracy of less than $\pm 1\%$. It may be possible with practical measurements at higher frequencies or in waveguides with higher losses, assuming that additional losses due to any flange misalignment can be mitigated.

Finally, Figure 2 shows the unexpected result that the attenuation constant goes to zero for the short circuit termination near the cut-off frequency, whereas the attenuation constant is double the traveling wave value for the open circuit termination.

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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