

Measurements of the quantized Hall resistance at low frequencies

by

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1. Introduction

From 1 January 1990 the primary standard of resistance has been based on the quantized Hall resistance (QHR) of a 2-dimensional electron gas established mainly in GaAs/AlGaAs heterostructures in high magnetic flux densities ($B > 5$ T) and low temperatures ($T < 1.5$ K). The Hall resistance is $R_H(i) = h/ie^2 = R_K/i$, where $R_K = 25812.807 \pm 0.005 \Omega$ is the von Klitzing constant and $i=1,2,3,4$, etc. is an integer defining the constant Hall resistance plateau. Until recently the Hall resistance has been measured with dc or very low (< 10 Hz) frequency ac. Using cryogenic current comparator bridge techniques the 1σ relative uncertainties achieved for the measurement of a room temperature resistor in terms of R_H approach 1 part in 10^9 . Since the first work published in 1993 [1], there has been an increasing effort [2,3] devoted to establishing R_H as an ac standard of resistance and phase angle for frequencies of a few kilohertz. The theory of the quantized Hall resistance predicts for these frequencies that its value should be unchanged from its dc value. However, the measured Hall resistance is modified by the impedances associated with the connecting cables and induced effects due to the presence of the large magnetic field. The purpose of the research described here is the investigation and elimination of these systematic errors.

2. Four terminal model

A simplified four terminal equivalent circuit of the device operated under fully quantized conditions (i.e. $\rho_{xx}=0$, implying zero dissipation in the direction of current flow) is shown in figure 1. The ac bridge has input connections I_H , V_H and I_L , V_L .

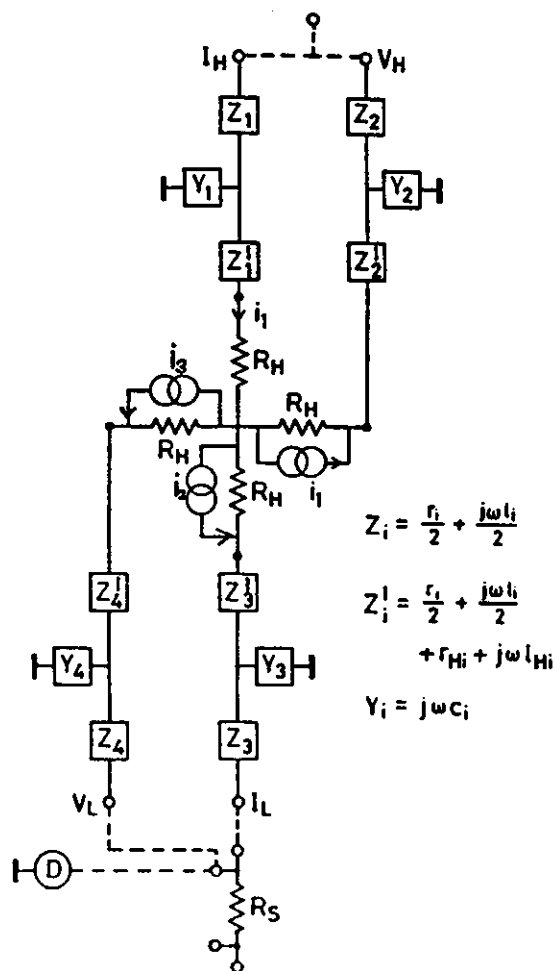


Figure 1

which are connected to the device by long (≈ 5 m) coaxial cables. The device itself is represented by four equal resistances R_H with three parallel current sources i_1 , i_2 and i_3 , where i_1 is equal to the current flowing in R_H in the counter-clockwise adjacent arm (or clockwise if B is in the opposite direction), and similarly for i_2 and i_3 . The impedances and admittances of the coaxial cables are respectively, for $i=1-4$, $z_i=r_i/2+j\omega l_i/2$ and $y_i=j\omega c_i$, where r_i , l_i and c_i are their resistance, inductance and capacitance (it is assumed that both G_i , the cable leakage conductance, and the capacitance between device contacts can be neglected). The total impedances are $z'_i=z_i+r_{Hi}+j\omega l_{Hi}$, where r_{Hi} and l_{Hi} are contact resistances and inductances respectively, associated with the Hall device. In a good device $r_H \approx 0$ but in a GaAs heterostructure (PTB/1) mounted in a standard EUROMET TO8 header l_H had typical values of 40 μH at $i=4$ and 160 μH at $i=2$, and also exhibited much larger quasi-inductances caused by a number of mechanical resonances in the frequency range 0-6 kHz. However, a device (BIPH 14/3) mounted in a specially designed printed-circuit header with well-guarded connections and very short (≈ 1 mm) bonding wires, exhibited no mechanical resonances and had maximum values of l_H of 80 μH for $i=2$. The inductance l_H appears to be proportional to B^2 (or R_H^2) and has no obvious dependence on frequency but its origins are not understood. Since the cable inductance is only ≈ 2 μH , l_H dominates the usual cable corrections derived from the cable parameters. The formula for the measured value of R_H becomes $R'_H=R_H[1+\omega^2(l+l_H)c-j\omega rc]$ (if $l_i=l$, $c_i=c$, $r_i=r$, $l_{Hi}=l_H$ and $r_{Hi}=0$) [2].

Because of the special properties of the quantized Hall device [3] it is possible to mitigate the effects of l_H and r_H (if it exists) by connecting the bridge to the Hall device configured as a two terminal device (see dashed lines in figure 1). In these circumstances it can be shown that:

$$R'_H = R_H \left[1 + \frac{(z_1+z'_1)(z_2+z'_2)}{R_H^2} + \frac{(z_3+z'_3)(z_4+z'_4)}{R_H^2} + z_2 y_2 + z_3 y_3 \right] \quad (1)$$

The effect of terms $\omega^2 l_{Hi} l_H / R_H^2$ in equation (1) is relatively small ($\approx 1 \times 10^{-8}$), and the dominant terms are those containing the cable correction, $\sum z_i y_i$. The latter have the same magnitude as for the normal four terminal configuration but are of the opposite sign.

3. AC Bridge

The ac bridge used for the measurements is shown in figure 2. It is of conventional design except the Hall device is connected to the bridge by the two terminal configuration referred to in 2. The in-phase balance is determined with the phase-sensitive detector D by adjusting the tap of an 8-decade inductive voltage divider (IVD) while the quadrature balance is similarly determined using a tapped 6-decade IVD connected to a 1 pF capacitor. B is adjusted to be at the centre of the $R_H(i)$ plateau as determined from dc measurements with $\rho_{xx}=0$. This condition may not be quite correct for ac measurements and it is probably necessary to investigate the plateau region more carefully by measuring $\rho_{xx}(B)$ as a function of current and frequency. A bridge is being constructed for this purpose. The standard resistor $R_s=R_q(i)$, where $R_q(2)=12906.4 \Omega$ and $R_q(4)=6453.2 \Omega$ are quadrifilar, Gibbing's type resistors with calculable frequency dependencies [4]. The in-phase ratio $R_s/R_H(i)$ has been measured as a function of frequency from 0.4-6.4 kHz, with a type A uncertainty of 3×10^{-8} when the device current was restricted to $\approx 25 \mu\text{A}$. A measurement with dc was made using an automated CCC bridge.

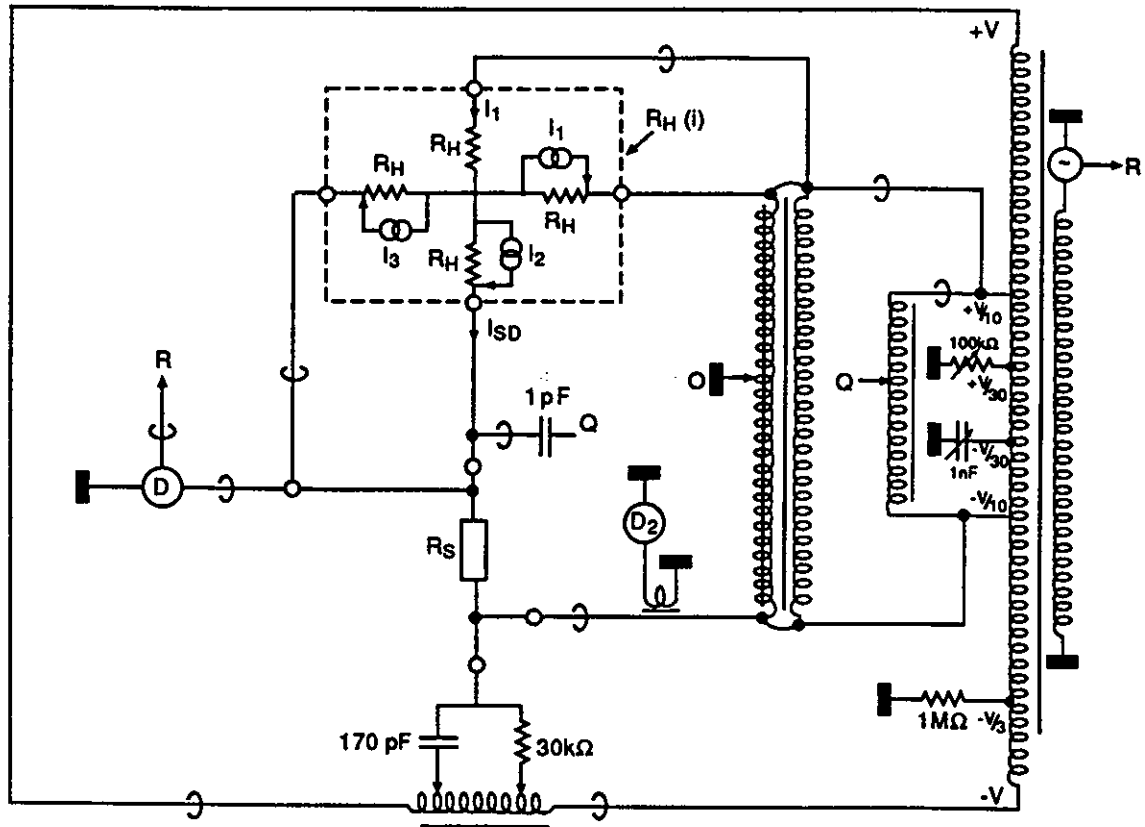


Figure 2

4. Measurements

Measurements have been made on four devices with various values of current, frequency, B and temperatures of 0.4 K and 1.25 K for each of the plateau indices $i=2$ and 4. For illustrative purposes figure 3 shows a fairly typical measurement of $[R_q(2)/R_H(2)-1]$ for device PTB/1 plotted as a function of frequency f . The mechanical resonances observed in this device were carefully avoided. The data have been corrected for the calculated frequency dependence of $R_q(2)$ and the cable and I_H effects according to equation (1). The residual value of $[R_q(2)/R_H(2)-1]$ (excluding the dc value) has been arbitrarily fitted to a linear frequency dependence with slope $\alpha(2)=-0.239 \times 10^{-6}/\text{kHz}$. If the frequency dependence of $R_H(2)$ is zero as predicted theoretically and that of $R_q(2)$ is correctly calculated then the residual value of $[R_q(2)/R_H(2)-1]$ should be constant and equal to the dc value.

A similar measurement procedure for $i=4$ can also be fitted to a linear frequency dependence in the range 0-2 kHz but with a much smaller slope $\alpha(4)=-0.037 \times 10^{-6}/\text{kHz}$. Within an uncertainty of 3×10^{-8} the correct dc value is predicted when the fitted line is extrapolated to $f=0$. Results for other devices exhibit similar behaviour but with somewhat different values of $\alpha(i)$.

Evaluation of the contributions derived from the quadrature balance indicate that the QHR is an excellent standard of phase with a time constant which does not exceed 10^{-10} s.

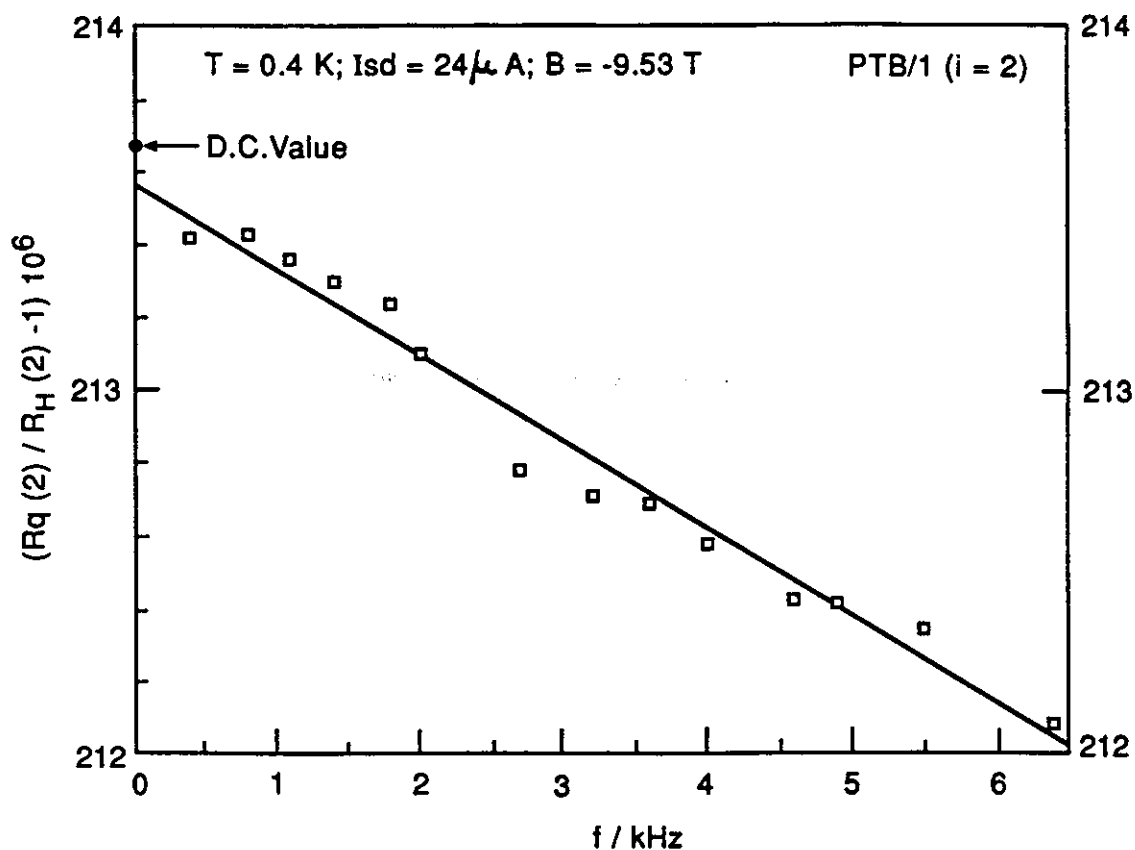


Figure 3

5. Conclusions

It has been shown that it is possible to make precise measurements of the quantised Hall resistance at low frequencies. The results indicate that $R_H(i) = R_s[1 + \alpha(i)\omega]$, where $\alpha(2) \approx 0.25 \times 10^{-6} / \text{kHz}$ and $\alpha(4) \approx 0.04 \times 10^{-6} / \text{kHz}$. More measurements are planned to further investigate and if possible eliminate this linear dependence on frequency. If this can be done the resulting "perfect" standard of ac resistance will allow better measurements to be made in terms of the calculable capacitor and consequently will lead to a better determination of R_K in SI units.

References

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