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**THE NPL PORTABLE STANDARD FLAME: CHARACTERISATION OF
THE TEMPERATURE FIELD ABOVE THE BURNER USING
PRECISION RAYLEIGH SCATTERING THERMOMETRY**

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The NPL portable standard flame: characterisation of the temperature field above the burner using precision Rayleigh scattering thermometry

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ABSTRACT

Accurate measurement of combustion temperatures can significantly improve engine efficiency and reduce harmful emissions, particularly during the development phase. Non-perturbing optical diagnostic techniques are capable of measuring temperatures in such environments but are often technically complex and validation is challenging, with correspondingly large uncertainties (typically 5 % -10 % of temperature). This work aims to address this by developing a portable flame temperature standard, calibrated via the Rayleigh scattering thermometry technique, traceable to ITS-90. By taking in to account the variation in the differential Rayleigh scattering cross-section with both species and temperature, the post-flame temperature is determined with a relative uncertainty (1σ) of 0.5 %. The long-term reproducibility of the post-flame temperature was 0.2 % over a three month period, demonstrating the suitability of the system as a temperature standard. Additionally, the temperature field above the burner is fully characterised, indicating a uniform region where the temperature varies horizontally and vertically by at most ± 0.5 % and ± 1.0 % respectively for any point within 10 mm of the default measurement position. As the UK's National Measurement Institute, NPL provides calibration and consultation in all aspects of metrology and the standard flame facility should be a welcome addition to the facilities and services it provides.

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CONTENTS

EXECUTIVE SUMMARY

1	INTRODUCTION	5
2	RAYLEIGH SCATTERING THERMOMETRY	6
2.1	PRINCIPLE OF MEASUREMENT	6
2.2	ESSENTIAL THEORY	6
2.3	DETERMINATION OF δ	8
2.4	DETERMINATION OF T_2	14
3	THE BURNER SYSTEM	14
4	EXPERIMENTAL SETUP	15
5	MEASUREMENTS	17
5.1	AIR CALIBRATION	17
5.2	POST-FLAME MEASUREMENTS	17
6	RESULTS	18
6.1.1	Burner warm-up	18
6.1.2	Long-term temperature measurements	19
6.1.3	Temperature profiles	21
7	SUMMARY AND CONCLUSIONS	27
8	ACKNOWLEDGEMENTS	27
9	APPENDIX A: UNCERTAINTY BUDGET	28
10	APPENDIX B: TABULATED RESULTS	29
11	REFERENCES	37

1 INTRODUCTION

The accurate measurement and control of flame and combustion gas temperatures can lead to reductions in harmful pollutants, increases in combustion efficiency and reduced fuel consumption. Due to the high temperature and hostile nature of the combustion environment, optical diagnostics are increasingly becoming the preferred option. The current state-of-the-art favours active probing using laser-based techniques but passive techniques such as emission spectroscopy are also possible. These techniques have been applied to situations such as spark-ignition engines, burner optimisation, high-temperature gas furnaces and incinerators in an effort to improve process efficiencies and reduce pollutant levels [1-3]. However, due to both the complexity of the diagnostic techniques themselves and the harsh environments found in combustion, uncertainties are difficult to assess. Thus conservative estimates are often used and typically amount to between 5 % and 10 % of the process temperature [1-3] and currently, traceability of combustion thermometry to the International Temperature Scale of 1990 (ITS-90) does not exist.

The EMPRESS project 14IND04 – Enhancing process efficiency through improved temperature measurement [4], funded under the European Metrology Programme for Innovation and Research (EMPIR) aims to address this by developing and characterising a portable standard flame with known and reproducible temperature and species concentrations. Once characterised, the facility will be made available to three research groups working on different optical methods of flame thermometry. By encouraging this collaboration it is envisaged that the uncertainties associated with these optical methods will be quantified and reduced, with the longer term objective of improved measurements in industry.

We report here, the characterisation of the NPL portable standard flame using the Rayleigh scattering thermometry technique, traceable to ITS-90 [5] and determine the temperature field in the post-flame region with a relative uncertainty of 0.5 % - a factor of ten improvement on current techniques. The standard flame, based on a Hencken flat-flame diffusion burner (model #RD1.5X1.5 [6]) and associated gas flow control is presented, the temperature field above the burner is fully described and the long-term reproducibility is assessed. Factors affecting the flame temperature such as inlet gas temperature, atmospheric pressure and gas flow uncertainty are considered and an assessment of the overall uncertainty is given. The burner is shown in Figure 1 during operation, under interrogation with a 532 nm Nd-YAG CW laser.

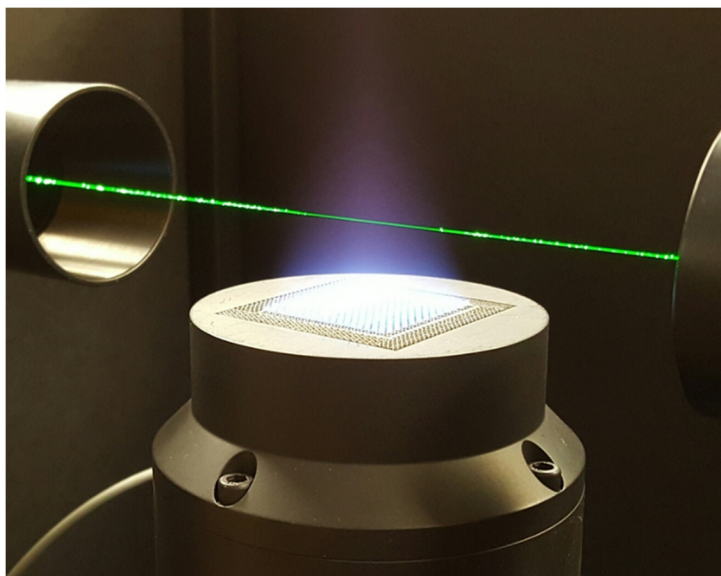


Figure 1 The Hencken flat-flame diffusion burner under laser interrogation.

2 RAYLEIGH SCATTERING THERMOMETRY

2.1 PRINCIPLE OF MEASUREMENT

Rayleigh scattering is the elastic scattering of light from atoms, molecules or very small particles - the wavelength of the scattered light is unchanged and the scattering amplitude is proportional to the weighted sum of the number of scatterers in the observation volume. In this work, two Rayleigh scattering signals are measured by observing a high intensity laser beam passing above the burner. Firstly, a calibration measurement is made by flowing dry air at ambient temperature T_1 through the burner giving a signal $S(T_1)$. Secondly, the signal for the post-flame species at an unknown flame temperature T_2 is measured giving $S(T_2)$. Assuming the solid angle of collection $\Delta\Omega$ is small and the entire width of the incident laser beam is included in the observation area, the signals so measured are given by:

$$S(T_1) = \eta I N_1 V \left(\frac{\partial \sigma}{\partial \Omega} \right)_{air} \Delta\Omega \quad S(T_2) = \eta I N_2 V \left(\frac{\partial \sigma}{\partial \Omega} \right)_f \Delta\Omega \quad (1)$$

where η is an instrument factor, I is the incident laser intensity and V is the observation volume. N_1 and N_2 are the number density of scatters present during the air calibration and post-flame measurements respectively. $(\partial \sigma / \partial \Omega)_{air}$ and $(\partial \sigma / \partial \Omega)_f$ are the Rayleigh differential scattering cross-sections of the air and post-flame gases respectively. For atmospheric pressure flames, the ideal gas law can be applied with negligible error, leading to:

$$N_1 k T_1 = N_2 k T_2 \Rightarrow T_2 = \frac{N_1}{N_2} T_1 \quad (2)$$

where k is the Boltzmann constant. By combining Equations 1 and 2 it is possible to write:

$$T_2 = \delta \frac{S(T_1)}{S(T_2)} T_1 \quad (3a)$$

$$\text{with} \quad \delta = \frac{(d\sigma/d\Omega)_f}{(d\sigma/d\Omega)_{air}} \quad (3b)$$

with T_1 measured with a calibrated PT100 sensor prior to the flame measurement. δ accounts for the difference in the scattering cross-section between the air calibration and flame measurements. To obtain T_2 , accurate knowledge of δ is required.

2.2 ESSENTIAL THEORY

The total Rayleigh scattering cross-section for a single species can be written as [7]:

$$\sigma(\lambda) = \frac{24\pi^3}{\lambda^4 N^2} \left(\frac{n(\lambda)^2 - 1}{n(\lambda)^2 + 2} \right)^2 \left(\frac{6 + 3p_0(\lambda)}{6 - 7p_0(\lambda)} \right) \quad (4)$$

where $n(\lambda)$ is the refractive index of the species at the laser wavelength λ , N is the number density of the species and $p_0(\lambda)$ is the depolarisation ratio, which is given by:

$$p_o = \frac{6\gamma^2}{45\alpha^2 + 7\gamma^2} \quad (5)$$

with γ and α the molecular anisotropy and mean molecular volume polarizability respectively. These are also wavelength dependent but for clarity we do not show this here. The depolarisation ratio $p_0(\lambda)$ is the ratio of horizontally to vertically polarised light scattered in to the perpendicular observation

direction for an unpolarised laser beam. The last term in brackets in Equation 4 is often called the *King Correction Factor*, F_K [7]. For this work, the probe laser propagates along the y-axis, is polarised along the z-axis and scattered light is collected at 90° along the x-axis. This is shown schematically in Figure 2.

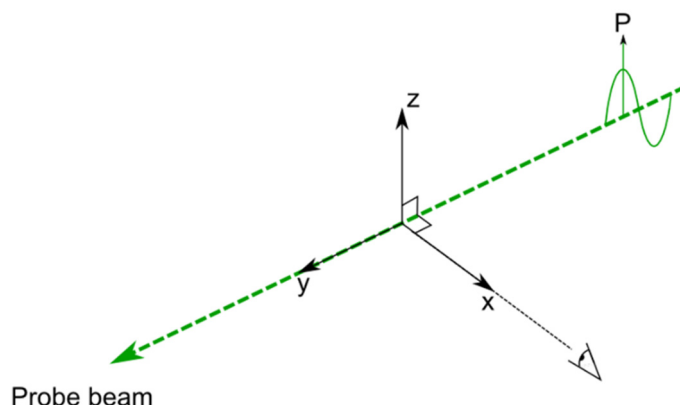


Figure 2 Rayleigh scattering measurement configuration used in this work.

Under these conditions the Rayleigh differential scattering cross-section – i.e. the scattering cross-section per unit solid angle is given by:

$$\frac{d\sigma(\lambda)}{d\Omega} = \frac{3\sigma(\lambda)}{8\pi} \left(\frac{2}{2 + p_0(\lambda)} \right) \quad (6)$$

Leading to

$$\frac{d\sigma(\lambda)}{d\Omega} = \frac{9\pi^2}{\lambda^4 N^2} \left(\frac{n(\lambda)^2 - 1}{n(\lambda)^2 + 2} \right)^2 \left(\frac{6}{6 - 7p_0(\lambda)} \right) \quad (7)$$

To simplify and remove the density dependence N , we make use of the molar refractivity $R_L(\lambda)$ [8] (with units of $\text{cm}^3 \cdot \text{mol}^{-1}$) given by:

$$R_L(\lambda) = \left(\frac{n(\lambda)^2 - 1}{n(\lambda)^2 + 2} \right) \frac{1}{\rho} \quad (8)$$

where ρ is the molar density N/N_A in units of $\text{mol} \cdot \text{cm}^{-3}$ at the pressure and temperature at which $n(\lambda)$ is measured. Substituting Equation 8 into Equation 7 gives a simplified working equation:

$$\frac{d\sigma(\lambda)}{d\Omega} = \frac{9\pi^2}{\lambda^4 N_A^2} R_L^2(\lambda) \left(\frac{6}{6 - 7p_0(\lambda)} \right) \quad (9)$$

It is important to re-iterate that $p_0(\lambda)$ is the depolarisation ratio for an unpolarised probe beam and that $d\sigma/d\Omega(\lambda)$ is the Rayleigh differential scattering cross-section for a linearly polarised probe beam measured perpendicular to both the probe beam's direction and polarisation.

In this work, we use a probe beam operating at $\lambda = 532 \text{ nm}$. Values for R_L and p_0 at this wavelength and STP conditions (0°C , 1 atm.) are tabulated in [8] and [9] respectively for all significant hydrocarbon/air combustion products. Together with the Rayleigh differential scattering cross-section, these are tabulated in Table 1.

Additionally, the OH radical has been omitted from this study, since at chemical equilibrium it amounts to approximately 0.3% by mole fraction, and calculations show that the net contribution to the Rayleigh scattering cross-section amounts to at most 0.1% (and significantly less either side of stoichiometric). It is worthy of note that other flame radicals such as NO and H have significantly smaller scattering cross sections than OH and tend to counterbalance the small rise in cross section from OH.

Table 1 Molar refractivity R_L , depolarisation ratio p_0 and Rayleigh differential scattering cross-section $(d\sigma/d\Omega)_i$ for significant hydrocarbon combustion products at STP conditions. All values are for a wavelength of $\lambda = 532$ nm.

Species	R_L / $\text{cm}^3 \cdot \text{mol}^{-1}$	$U_R(R_L)$ / %	p_0 $\times 10^2$	$(d\sigma/d\Omega)_i$ $\times 10^{-28} / \text{cm}^2$
N_2	4.4840	0.01	2.019	6.2965
O_2	4.0650	0.01	5.447	5.3831
CO_2	6.6900	0.01	7.526	15.003
H_2O	3.7820	0.02	0.060	4.3768
H_2	2.0860	0.11	1.784	1.3589
CO	5.0340	0.01	1.070	7.8469
Ar	4.2290	0.01	0.000	5.4688
<i>Air</i>	<i>4.3922</i>	<i>0.02</i>	<i>2.843</i>	<i>6.1014</i>

The molar refractivity for air is a special case and is needed to determine the difference in the scattering cross-section of the combustion gases relative to that of air, where an initial calibration measurement is made at known temperature. Table 2 shows how this is obtained from the mole-fraction weighted sum of R_L for the major species present in air.

Table 2 Calculation of the Molar refractivity of air at STP conditions. All values are for a wavelength of $\lambda = 532$ nm.

Species	X_i	R_L / $\text{cm}^3 \cdot \text{mol}^{-1}$	$R_L(532 \text{ nm, Air})$ / $\text{cm}^3 \cdot \text{mol}^{-1}$
N_2	0.78084	4.4840	$R_L = \sum_i^n X_i R_L(532 \text{ nm}, i) = 4.3922$
O_2	0.20946	4.0650	
Ar	0.00934	4.2290	
Tot:	0.99964		

By making use of Equation 9 and Table 1, the Rayleigh differential scattering cross-section can be calculated for air and any relevant combustion species at STP conditions.

2.3 DETERMINATION OF δ

Definition: Equivalence ratio ϕ

For any combustion reaction, the equivalence ratio is defined as the ratio of the volume of fuel to the volume of air (oxidiser) divided by the same ratio for stoichiometric conditions

$$\phi = \frac{(V_{fuel}/V_{air})}{(V_{fuel}/V_{air})_{ST}} \quad (10)$$

Stoichiometric conditions occur when the ratio of fuel to air is such that there is no excess oxygen after combustion, so a value of $\phi = 1$ corresponds to complete balanced combustion and generally produces close to the maximum post flame temperature. For the propane fuel used in this work, when $\phi = 1$, $V_{fuel}/V_{air} = 0.0419$, i.e. the volume of fuel burned is 4.19 % the volume of the air consumed.

In order to calculate the flame temperature T_2 , the value of δ must be determined and although the value of $(d\sigma/d\Omega)_{air}$ is known (and can be taken from Table 1), the value of $(d\sigma/d\Omega)_f$ depends strongly on the equivalence ratio ϕ and weakly on T_2 . This leads to a difficulty in solving Equation 3a/b. A strategy developed to overcome this difficulty follows.

Since the Rayleigh differential scattering cross-section of the post-flame gas is given by the mole-fraction weighted sum of the individual scattering cross-sections of each species present, i.e. $(d\sigma/d\Omega)_f = \sum_i X_i(d\sigma/d\Omega)_i$, we need to determine the temperature dependence of each species separately and then combine the contributions. We can recast Equation 9 by making use of Equation 5 and the relationship between α and R_L given by:

$$\alpha = \frac{3R_L}{4\pi N_A} \quad (11)$$

This leads to an expression that defines the differential Rayleigh scattering cross-section of species i , in terms of α_i and γ_i only:

$$\left(\frac{d\sigma}{d\Omega}\right)_i = \frac{16\pi^4}{\lambda^4} \left(\alpha_i^2 + \frac{7}{45} \gamma_i^2 \right) \quad (12)$$

So, the temperature dependence of $(d\sigma/d\Omega)_i$ can be determined completely by finding the temperature dependence of α_i and γ_i . For small fractional increases in α_i and γ_i , given by $(\Delta\alpha_i/\alpha_i)$ and $(\Delta\gamma_i/\gamma_i)$, the absolute change in $(d\sigma/d\Omega)_i$ can be written as:

$$\Delta \left(\frac{d\sigma}{d\Omega} \right)_i = \frac{\partial}{\partial \alpha} \left(\frac{d\sigma}{d\Omega} \right)_i \alpha_i \left(\frac{\Delta\alpha_i}{\alpha_i} \right) + \frac{\partial}{\partial \gamma} \left(\frac{d\sigma}{d\Omega} \right)_i \gamma_i \left(\frac{\Delta\gamma_i}{\gamma_i} \right) \quad (13)$$

By substituting in the partial derivatives, we arrive at the result:

$$\Delta \left(\frac{d\sigma}{d\Omega} \right)_i = \frac{32\pi^4}{\lambda^4} \left(\alpha_i^2 \left(\frac{\Delta\alpha_i}{\alpha_i} \right) + \frac{7}{45} \gamma_i^2 \left(\frac{\Delta\gamma_i}{\gamma_i} \right) \right) \quad (14)$$

The temperature dependence of α for typical combustion species can be determined from *Hohm et al* [10] and varies between 0.2 % and 1.4 % for a temperature rise of 1000 K, depending on species.

The temperature-dependence of the molecular anisotropy, γ has not been established experimentally. However, it is possible to use theoretical arguments to assess the appropriate trend. *Ab initio* calculations of the polarizability of H_2 reveal a larger relative increase in the anisotropy than in the polarizability with increasing inter-nuclear distance [9, 11]. For typical combustion species the increase is estimated to be of the order of 2 % per 1000 K rise in temperature [9]. This seems reasonable since we see from Equation 12 that the contribution to the differential scattering cross-section of the anisotropy term is weighted by a factor 7/45. An increase in the anisotropy of 2 % / 1000 K is thus assumed for this work. The uncertainty in the flame temperature due to this approximation is less than ± 0.15 % [1].

Table 3 shows a summary of the calculated parameters needed to determine the Rayleigh differential scattering cross-section at 532 nm for all significant combustion species at any temperature. The first six rows show the parameters used in the cross-section calculations, and the final four rows show results of these calculations. Row 7 shows the baseline scattering cross-sections for each species at 300 K and Rows 8, 9 and 10 respectively show the fractional changes in α_i , γ_i and Rayleigh differential scattering cross-section per 1000 K temperature rise.

Table 3 Summary of the calculated parameters needed to determine the Rayleigh differential scattering cross-section at 532 nm for all significant combustion species at any temperature.

Row	Parameter	N_2	O_2	CO_2	H_2O	H_2	CO	Ar	Air
1	R_L [cm ³ .mol ⁻¹]	4.4840	4.0650	6.6900	3.7820	2.0860	5.0340	4.2290	4.3922
2	$\alpha_i \times 10^{24}$ [cm ³]	1.7776	1.6115	2.6521	1.4993	0.82696	1.9956	1.6765	1.7412
3	$P_0 \times 10^2$	2.0194	5.2580	7.5257	0.0600	1.7839	1.0702	0.0000	2.8430
4	$\gamma_i \times 10^{25}$ [cm ³]	7.0009	10.4453	20.8621	1.0060	3.0568	5.6896	0.0000	8.1771
5	α_i/γ_i	2.539	1.543	1.271	14.904	2.705	3.508	-	2.129
6	F_K	1.0345	1.0934	1.1375	1.0010	1.0304	1.0181	1.0000	1.0490
7	$(d\sigma/d\Omega)_i \times 10^{28}$ [cm ²]	6.2965	5.3831	15.003	4.3768	1.3589	7.8469	5.4688	6.1014
8	$\Delta\alpha_i/\alpha_i \times 100$ [1000 K rise]	0.2	0.7	1.1	0.5	1.4	0.4	0.0	-
9	$\Delta\gamma_i/\gamma_i \times 100$ [1000 K rise]	2	2	2	2	2	2	0	-
10	$\frac{\Delta(d\sigma/d\Omega)_i}{(d\sigma/d\Omega)_i} \times 100$ [1000 K rise]	0.5	1.6	2.4	1.0	2.8	0.8	0.0	-

The procedure to calculate $\delta(\phi, T_2)$ is now described:

First the Rayleigh differential scattering cross-section for all major combustion species over the possible range of flame temperatures is calculated. The temperature range of interest for propane/air combustion is $\{1600 \text{ K} < T_2 < 2400 \text{ K}\}$. This is achieved by making use of Rows 7 and 10 in Table 3 as follows:

$$\left(\frac{d\sigma}{d\Omega}\right)_i (@T_2) = \left(\frac{d\sigma}{d\Omega}\right)_i (@STP) \left(1 + \Delta \left(\frac{d\sigma}{d\Omega}\right)_i / \left(\frac{d\sigma}{d\Omega}\right)_i \left(\frac{T_2 - 300}{1000}\right)\right) \quad (15)$$

Table 4 shows a summary of these calculations.

Table 4 The Rayleigh differential scattering cross-section of major combustion species versus flame temperature at $\lambda = 532 \text{ nm}$.

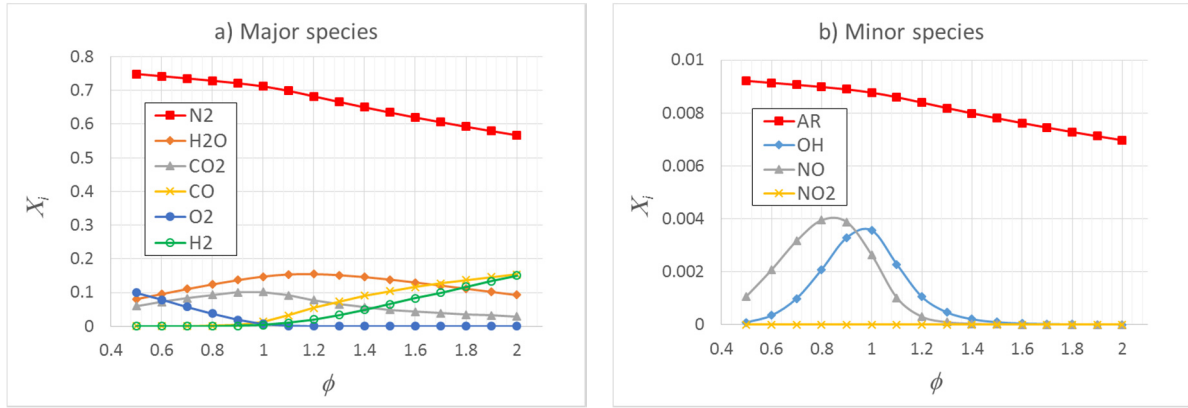
T_2 / K	$(d\sigma/d\Omega)_f \times 10^{28} @ T_2 / \text{cm}^2$						
	N_2	O_2	CO_2	H_2O	H_2	CO	Ar
1600	6.3362	5.4922	15.4629	4.4339	1.4088	7.9326	5.4688
1700	6.3392	5.5006	15.4982	4.4383	1.4126	7.9392	5.4688
1800	6.3423	5.5090	15.5336	4.4426	1.4164	7.9458	5.4688
1900	6.3453	5.5174	15.5690	4.4470	1.4203	7.9523	5.4688
2000	6.3484	5.5258	15.6044	4.4514	1.4241	7.9589	5.4688
2100	6.3515	5.5342	15.6397	4.4558	1.4280	7.9655	5.4688
2200	6.3545	5.5426	15.6751	4.4602	1.4318	7.9721	5.4688
2300	6.3576	5.5510	15.7105	4.4646	1.4356	7.9787	5.4688
2400	6.3606	5.5593	15.7459	4.4690	1.4395	7.9853	5.4688

Secondly, the equilibrium species concentrations for propane/air combustion over the full range of equivalence ratios and possible flame temperatures is calculated using chemical the equilibrium software *Gaseq* [12]. The range over which calculations have been made are: $\{0.5 < \phi < 2.0\}$ and $\{1600 \text{ K} < T_2 < 2400 \text{ K}\}$. The software provides an option to calculate the equilibrium mole-fraction species concentrations at a defined pressure and temperature and this is used here.

For the Hencken burner used in this work, a flame with a temperature of up to 10% below the adiabatic flame temperature T_{ad} can be stabilised for $\{0.7 < \phi < 1.4\}$. So the ranges over which the species concentrations have been calculated in both T_2 and ϕ are more than adequate. The adiabatic flame temperature is the maximum possible flame temperature achievable if there are no heat losses from the flame i.e. to the burner surface or radiated to its surroundings. This is never achieved in practice and is why we need to measure the flame temperature and not rely on calculations. Table 5 shows an example of these calculations for $\phi = 1.0$ and Figure 3 shows the major and minor species concentrations versus ϕ for $T_2 = T_{ad}$. The inlet gas temperature T_0 is assumed to be 64°C which was the mean burner surface temperature and it is assumed that the gases acquire this temperature before they leave the burner.

Table 5 Equilibrium species concentrations X_i versus T_2 for $\phi = 1.0$ for major combustion species.

T_2 / K	$X_i(@ T_2)$						
	N_2	O_2	CO_2	H_2O	H_2	CO	Ar
1600	0.720381	0.000119	0.115753	0.154505	0.000093	0.000209	0.008856
1700	0.720209	0.000262	0.115457	0.154351	0.000190	0.000483	0.008855
1800	0.719880	0.000530	0.114883	0.154067	0.000358	0.001012	0.008851
1900	0.719302	0.000991	0.113863	0.153579	0.000631	0.001954	0.008845
2000	0.718345	0.001734	0.112176	0.152783	0.001052	0.003511	0.008835
2100	0.716853	0.002855	0.109566	0.151547	0.001672	0.005919	0.008820
2200	0.714642	0.004448	0.105766	0.149704	0.002549	0.009417	0.008797
2300	0.711514	0.006581	0.100550	0.147062	0.003747	0.014205	0.008764
2400	0.707282	0.009267	0.093797	0.143403	0.005334	0.020376	0.008720

**Figure 3** Propane/air equilibrium combustion products at T_{ad} versus equivalence ratio: a) major species and b) selected minor species. In both cases $T_0 = 64^\circ\text{C}$. $P = 1 \text{ atm}$.

Thirdly, the mole-fraction weighted Rayleigh differential scattering cross-section of the combustion gas at T_2 is calculated and $\delta(\phi, T_2)$ determined using the following equations:

$$\left(\frac{d\sigma}{d\Omega}\right)_f (@T_2) = \sum_i X_i(@T_2) \left(\frac{d\sigma}{d\Omega}\right)_i (@T_2) \quad (16)$$

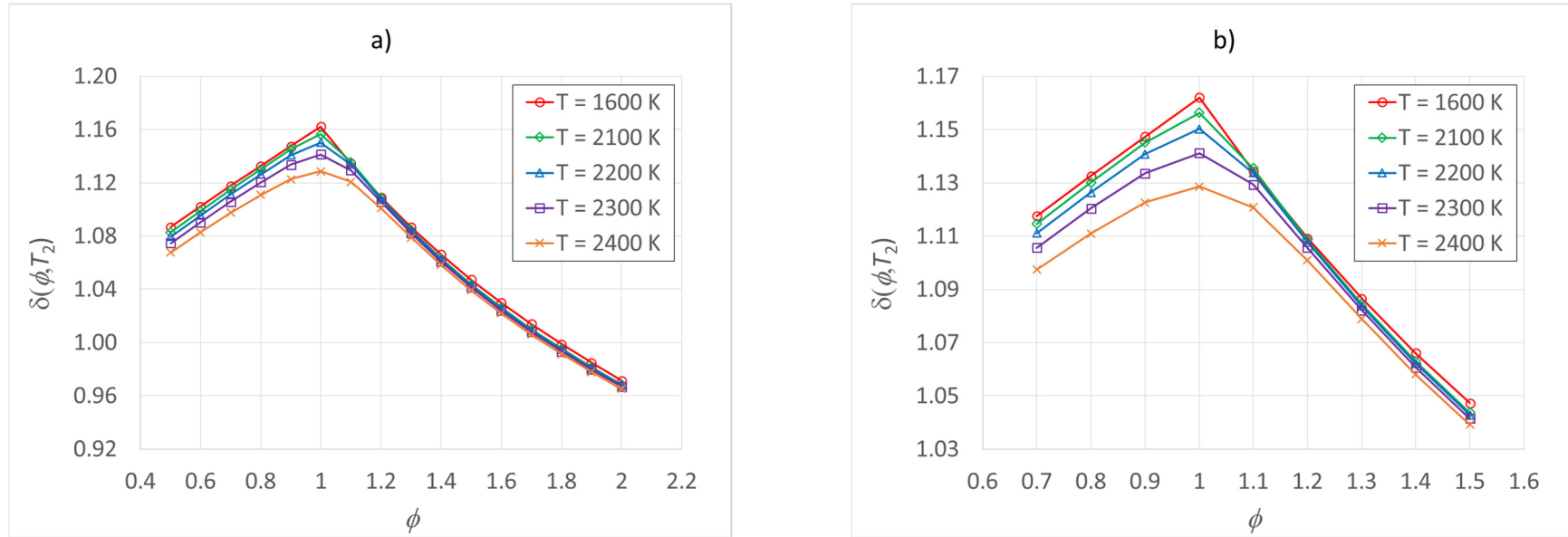
$$\delta(\phi, T_2) = \left(\frac{d\sigma}{d\Omega}\right)_f (@T_2) / \left(\frac{d\sigma}{d\Omega}\right)_{air} (@STP) \quad (17)$$

Table 6 and Figure 4 show the value of $\delta(\phi, T_2)$ for the range of interest in this work and we see that it varies by up to 20 % with ϕ (ranging from 0.97 when $\phi = 2.0$, to 1.17 when $\phi = 1.0$) and up to 3 % with T_2 . If we had assumed the scattering cross-section remained unchanged between the air calibration and flame measurements, the temperature could be in error by up to $\approx 17\%$.

Finally, Since T_2 is close to T_{ad} , we make an initial estimate of $\delta(\phi, T_2)$ by assuming $T_2 = T_{ad}$, where T_{ad} is calculated using the chemical equilibrium software *Gaseq* [12].

Table 6 The value of $\delta(\phi, T_2)$ for propane/air combustion over the range of interest in this work.

T_2 /K	$\delta(\phi, T_2)$															
	$\phi = 0.5$	0.6	0.7	0.8	0.9	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
1600	1.08691	1.10235	1.11756	1.13257	1.14740	1.16205	1.13439	1.10927	1.08665	1.06611	1.04730	1.02996	1.01387	0.99885	0.98477	0.97151
1700	1.08702	1.10257	1.11791	1.13305	1.14805	1.16266	1.13475	1.10905	1.08601	1.06521	1.04626	1.02886	1.01277	0.99780	0.98380	0.97064
1800	1.08680	1.10248	1.11795	1.13325	1.14842	1.16276	1.13517	1.10897	1.08557	1.06453	1.04545	1.02801	1.01193	0.99700	0.98306	0.96999
1900	1.08615	1.10195	1.11757	1.13301	1.14832	1.16204	1.13558	1.10897	1.08525	1.06401	1.04482	1.02733	1.01125	0.99637	0.98249	0.96949
2000	1.08490	1.10081	1.11654	1.13209	1.14741	1.16008	1.13581	1.10893	1.08496	1.06354	1.04426	1.02673	1.01066	0.99581	0.98199	0.96905
2100	1.08282	1.09879	1.11456	1.13010	1.14517	1.15635	1.13547	1.10866	1.08455	1.06300	1.04365	1.02610	1.01004	0.99522	0.98144	0.96856
2200	1.07959	1.09550	1.11116	1.12644	1.14085	1.15025	1.13376	1.10778	1.08375	1.06219	1.04282	1.02527	1.00923	0.99445	0.98072	0.96788
2300	1.07475	1.09040	1.10569	1.12037	1.13360	1.14118	1.12935	1.10559	1.08213	1.06076	1.04148	1.02400	1.00802	0.99329	0.97961	0.96682
2400	1.06773	1.08283	1.09739	1.11102	1.12263	1.12859	1.12075	1.10090	1.07891	1.05817	1.03921	1.02192	1.00607	0.99144	0.97784	0.96513

**Figure 4.** $\delta(\phi, T_2)$ for the range of interest in this work: a) full range of calculation, b) reduced range showing variation of δ with T_2 in detail.

2.4 DETERMINATION OF T_2

To determine the post-flame temperature T_2 we employ an iterative procedure:

1. An initial estimate of the post-flame temperature is made using equation 3a:

$$\langle T_2(0) \rangle = \delta(\phi, T_{ad}) \frac{S(T_1)}{S(T_2)} T_1 \quad (18)$$

Where $S(T_1)$ and $S(T_2)$ are the Rayleigh scattering signals measured in air (calibration point) and the post-flame regions respectively and T_1 is measured via a calibrated PT100 sensor.

2. A new value of $\delta(\phi, \langle T_2(0) \rangle)$ is calculated.
3. A new estimate of T_2 is then calculated:

$$\langle T_2(n+1) \rangle = \delta(\phi, \langle T_2(n) \rangle) \frac{S(T_1)}{S(T_2)} T_1 \quad (19)$$

4. Steps 2 and 3 are repeated until convergence is achieved.

For a given ϕ , δ is a monotonic decreasing function of T and only changes by at most 4% over the full range of possible flame temperatures. For this reason, the iterative procedure described above converges to the true flame temperature quickly - within three iterations. The difference between $\langle T_2(0) \rangle$ and $\langle T_2(3) \rangle$ is at most 1%, so it is necessary to iterate to obtain the required 0.5% uncertainty but the change in T_2 is not large.

3 THE BURNER SYSTEM

Figure 5 shows a schematic of the gas metering and burner control system. The Hencken burner is provided with dry air and propane gas of 95 % purity via Bronkhorst [13] mass flow controllers (MFCs), calibrated with an uncertainty of less than 1 %. A UV-TRON [14] flame detection sensor observes the burner at all times and triggers a fuel cut-out switch on the propane line ten seconds after the flame out condition is detected – this safety feature gives sufficient time to ignite the burner when propane flows but does not allow significant release of flammable gas into the laboratory environment. Additionally, the atmospheric pressure and inlet gas temperatures are monitored and the burner's position can be adjusted via a motorised XYZ translation stage. Labview [15] software is used to monitor and control the burner system including the facility to programmatically adjust or ramp the airflow rate from 1 to 30 SLPM and the flame equivalence ratio (ϕ) from 0.5 to 2.5.

The burner produces an array of small diffusion flames which combine to form a flat sheet and highly uniform post-flame region. By not premixing the fuel and air, the flame is stabilised higher above the burner surface than for a premixed flame [1] reducing heat loss and producing a flame temperature close (within 3 %) to the theoretical maximum adiabatic flame temperature.

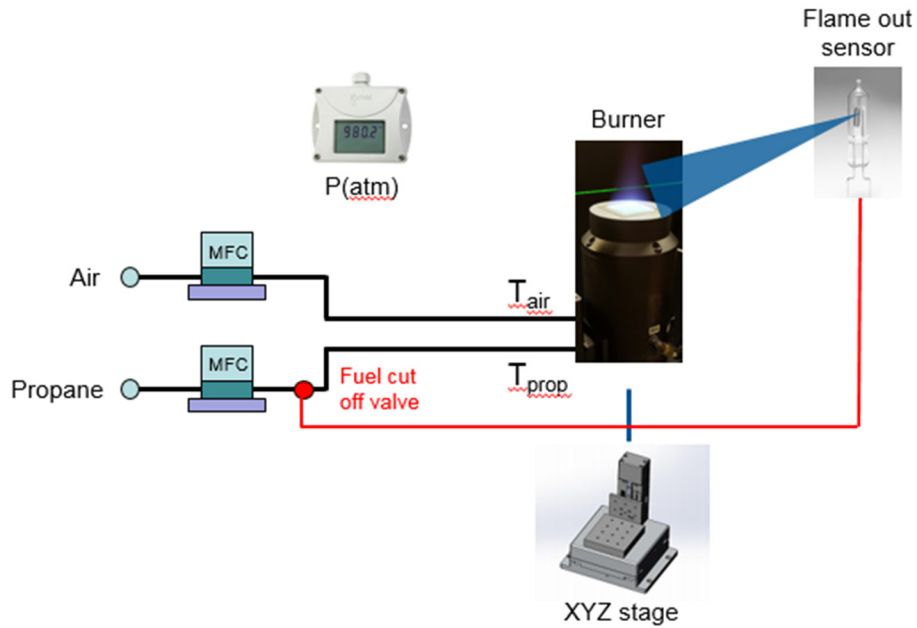


Figure 5 The STD flame gas metering and burner control system.

4 EXPERIMENTAL SETUP

The experimental setup of the precision laser Rayleigh scattering thermometry system is shown in Figure 6. A probe laser beam passes above the burner and the Rayleigh scattered light is collected perpendicular to it.

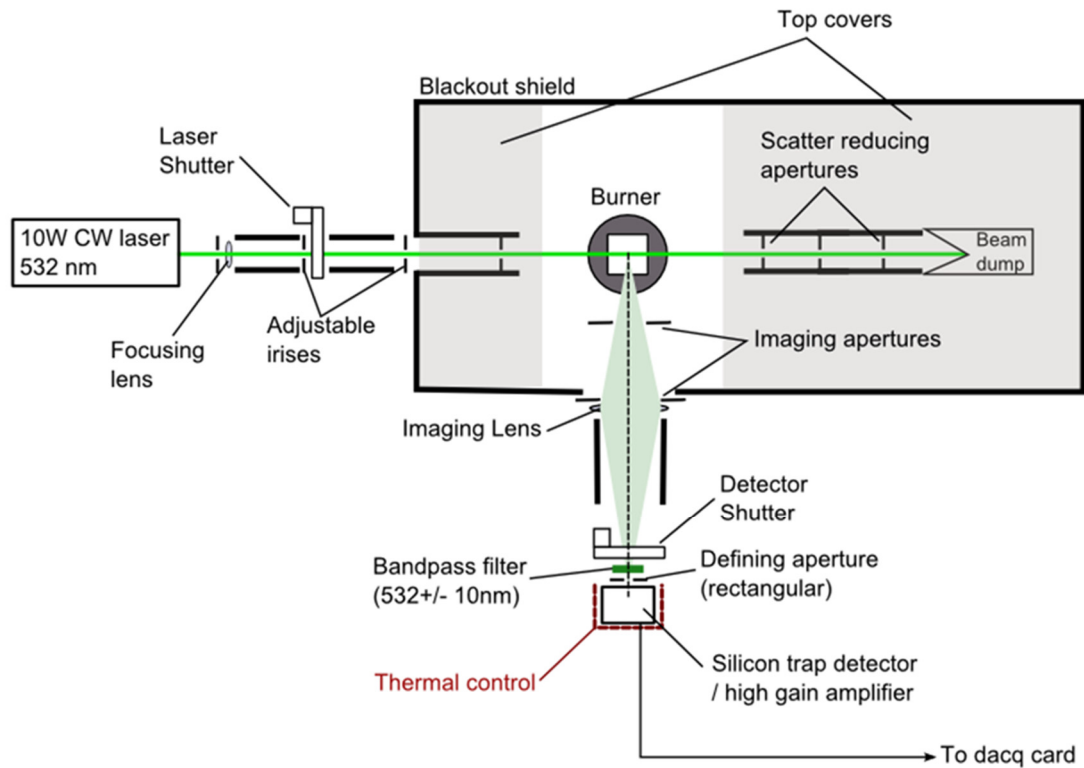


Figure 6 Experimental setup: the laser Rayleigh scattering thermometry system.

To achieve the lowest measurement uncertainty, four critical aspects need to be addressed: the laser, the detector, the imaging system, and the background scatter:

The laser

Since the magnitude of the Rayleigh signal depends on the intensity and polarisation of the probe laser beam, appropriate laser selection is critical, if measurement uncertainties are to be minimised. After careful consideration, a Coherent (Verdi G10) [16] operating at 532 ± 2 nm was selected. The G10 is an Optically Pumped Semi-conductor Laser (OPSL), producing a vertically polarised beam ($>99\%$) with a maximum output power of 10 W (CW). After a 20 minute warm-up period, the laser power was stable to 0.2% over an eight hour period and the polarisation was not observed to change.

The detector

Rayleigh scattering signals are small – for these experiments, they amounted to approximately 4 nW and 0.6 nW for measurements in ambient air and flames respectively. It is therefore important to use a detector with high gain and stability, and low noise. We chose to use a trap detector, consisting of three Hamamatsu 1331 [17] ($10 \text{ mm} \times 10 \text{ mm}$) silicon photodiodes. The photocurrent is amplified by a transimpedance amplifier utilising an OPA128LM integrated circuit with a gain of 10^9 V A^{-1} . The detector-amplifier combination is thermally insulated and temperature stabilised to within ± 25 mK at around 30 K. Temperature stabilisation minimises gain changes in the amplifier and offset drifts.

The imaging system

The probe laser beam was focussed to a beam waist of 0.1 mm, 20 mm above the centre of the burner and then captured by a high-efficiency beam dump. The Rayleigh scattered light was collected perpendicular to the probe beam, over a 0.1 sr solid angle (defined by two knife-edged imaging apertures) via a 1:1 imaging system with a 75 mm diameter lens doublet. It was then focused on to a laser-line filter ($\lambda = 532 \pm 2$ nm, 10 ± 2 nm FWHM) and on to a $3 \text{ mm} \times 3 \text{ mm}$ precision square defining aperture on the front of the of the trap detector.

Although the trap detector is designed to collect as much of the incident light as possible via multiple reflections, the incoming light is diverging. This results in a variation of the measured signal with vertical position of the detector aperture, as different amounts of light are collected by the trap. To determine the optimum position for the detector and minimise this variation, its position was adjusted using a small translation stage and the signal monitored. The results of this exercise are shown in Figure 7, yielding an optimum Y-position of 5.6 mm. When the detector is in this position, small vertical variations in the position of the probe beam, such as those caused by flame fluctuations, do not significantly change the measure Rayleigh signal. This position was used throughout the measurements.

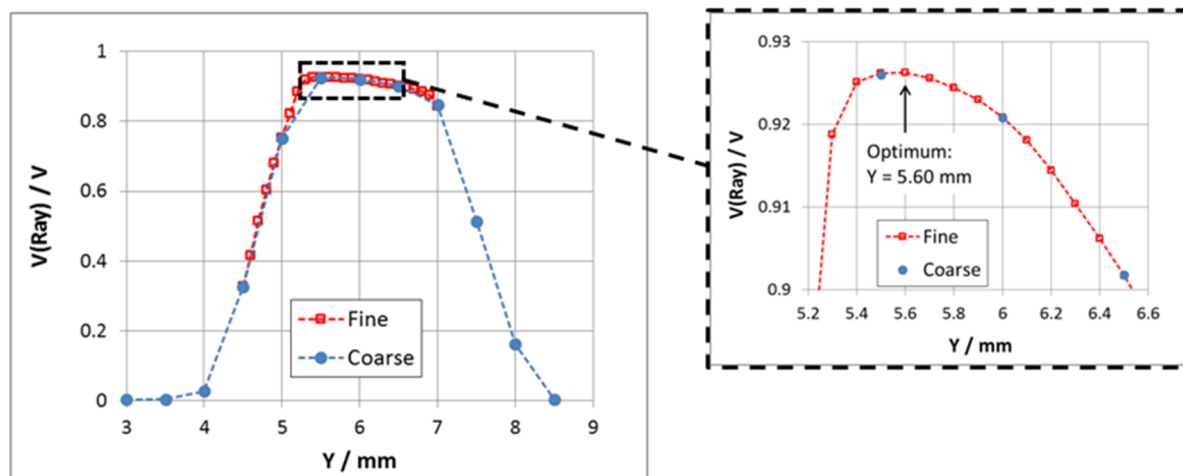


Figure 7 Line-up: Measured signal with the Rayleigh beam imaged onto the trap detector versus vertical position of detector.

Background scatter

With laser probe beam powers of up to 10 W and typical Rayleigh signals of several nanowatts, it is critical that the background (non-Rayleigh) scattered laser light is minimised. We achieved this using a blackout shield around the burner and blackened lens tubes containing a number of apertures along the probe beam path. An iterative process followed, in which three adjustable irises (before the burner) and three scatter reducing apertures (after the burner) were adjusted in size and position until the signal was minimised. However, without an absolute baseline figure, it is difficult to assess the remaining level of scatter. To address this issue, after each adjustment, air, nitrogen (99.99% purity) and helium (99.999% purity) were consecutively flowed through the burner and the Rayleigh signals measured. By assuming that the signals contain a constant background component and a component proportional to the Rayleigh scattering cross-section of each gas, it was possible to determine the background scatter level relative to one of the three gases. After optimisation, the final level of background scatter was 0.022 % of the measured Rayleigh signal for air. This is equivalent to 0.15 % of the typical flame Rayleigh signal, which is bordering on being a significant systematic error – for this reason, the background scatter was re-determined regularly and subtracted from the measured signals prior to further processing.

5 MEASUREMENTS

Software written in Labview 2015 [15] was used to control the laser power, detector and laser shutters, and the data acquisition process. For each data point, the Rayleigh, ambient background and detector zero signals were measured and when required (i.e. during air calibration), the temperature of a calibrated PT100 was also measured. Through a network connection to the burner control system, the air flow-rate, propane flow-rate, equivalence ratio, gas inlet temperatures and the position of the XYZ-stage were also captured. Following each measurement, all data was saved to file for later processing.

The raw detector voltages were captured by a 16 bit National Instruments [15] PCIe-6321 data acquisition card. A single capture consisted of measuring 100 k samples at 100 kHz sample rate (1 second acquisition time), removing any spikes and then averaging all remaining points. The measurement sequence consisted of: 1) detector and laser shutters closed – measure the detector zero, 2) detector shutter open, laser shutter closed – measure the ambient background, 3) detector and laser shutters open – measure the Rayleigh + ambient background signal and 4) determine the absolute Rayleigh signal by subtracting the ambient background signal. In all cases, the detector zero offset was small due to good thermal control of the trap detector, but any such errors were removed by this process. Post-processing of the captured data was performed with Labview 2015 to determine the post-flame temperature T_2 according to the method described in section 2.

5.1 AIR CALIBRATION

At the start of a day's measurement campaign, prior to any flame measurements, the Rayleigh signal for air was determined (see $S(T_1)$, Equation 1). The laser was first switched on and set to 9 W indicated power and the burner control software started with an air-flow rate of 23 SLPM. The calibrated thin film PT100 sensor was placed adjacent to the surface of the burner centre and the Rayleigh scattering signal 20 mm above this point was monitored. Once the measured signal had stabilised – typically with a standard deviation of less than 0.02 %, and the temperature of the air exiting the burner was stable, a definitive measurement of the air calibration point was made $[T_1, P_{atm}, S(T_1)]$, and the PT100 sensor moved away from the burner. Atmospheric pressure was recorded for the calibration point and subsequently monitored during flame measurements and a small correction applied for any changes.

5.2 POST-FLAME MEASUREMENTS

For all post-flame measurements an airflow rate of 23 SLPM was used. This was found to give both a superior uniform flat flame and minimum heat loss to the burner surface for all equivalence ratios (ϕ 's) used in this work. This was verified by measuring the burner surface temperature with a thermal imager

immediately after extinguishing the flame, which yielded a mean surface temperature over the central 30 mm, for $\phi = \{0.8 - 1.4\}$ of 64 °C. In characterising the temperature field above the burner, three distinct measurement sets were performed:

Burner warm-up: to establish the time taken for the post-flame temperature to reach equilibrium and identify any correlations with the inlet reactants temperature, a small PT100 sensor was attached to the upper body of the burner (with no direct view of the flame), the flame ignited ($\phi = 1.4$, 23 SLPM airflow) and the Rayleigh signal monitored 20 mm above the burner centre.

Long-term temperature measurements: to establish the long-term stability and reproducibility of the post-flame temperature, ten measurements were performed for $\phi = \{0.8 - 1.4\}$ in 0.1 steps, over a three month period. All measurements were performed in the region 20 mm above the burner centre.

Temperature profiles: to establish the spatial variation in the temperature field above the burner, a series of profiles in the x and y directions were measured at heights of 10 mm, 20 mm and 30 mm above the burner surface. The temperature profiles were measured from $R_x = -20$ mm to $R_x = +20$ mm and from $R_y = -20$ mm to $R_y = +20$ mm, i.e. R_x is the position in the x direction relative to the burner centre. These measurements were then repeated with the addition of a nitrogen (N_2) gas co-flow. The co-flow was delivered through a mesh surrounding the central flame/air tubes. During the measurements, the N_2 output regulator was held at a gauge pressure of 0.5 bar and no other flow control was used. Figure 7 shows the burner, the scan orientations and the N_2 co-flow region.

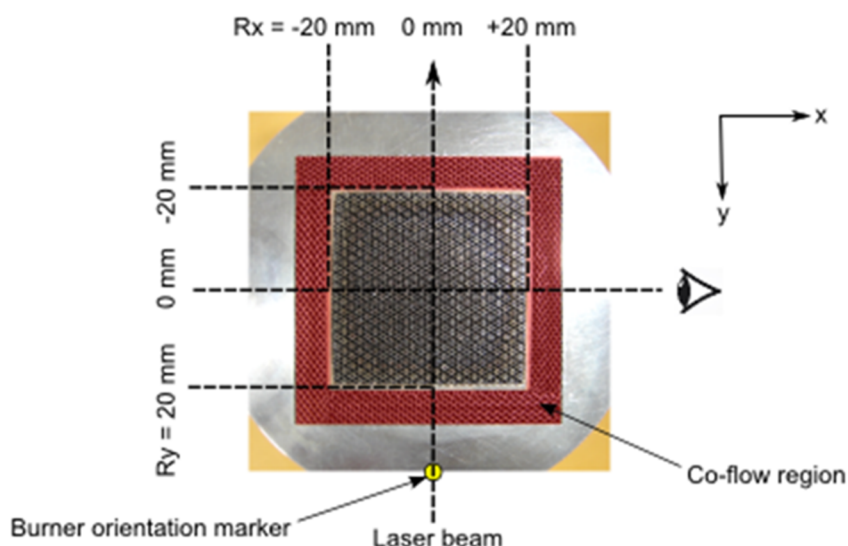


Figure 8 The standard flame (viewed from above): R_x and R_y scan directions are shown. The yellow spot in the figure (also present on the burner itself) indicates the measurement system geometry.

6 RESULTS

6.1.1 Burner warm-up

Figure 9 shows a comparison of the post-flame temperature T_2 and burner upper body temperature T_b immediately after burner ignition for 23 SLPM airflow and $\phi = 1.4$. There is clear evidence of a correlation between the two, with a relative increase in T_2 from the time of ignition to stability of approximately 1 % of temperature. Fitting an exponential to either data set reveals a time constant of 35 minutes. For all subsequent tests, measurements were not commenced until approximately

100 minutes (three time constants) after ignition. This guaranteed a post-flame temperature reproducible to 0.2 %. Similar warm-up characteristics were seen for all ϕ in the interval $\{0.8 < \phi < 1.4\}$.

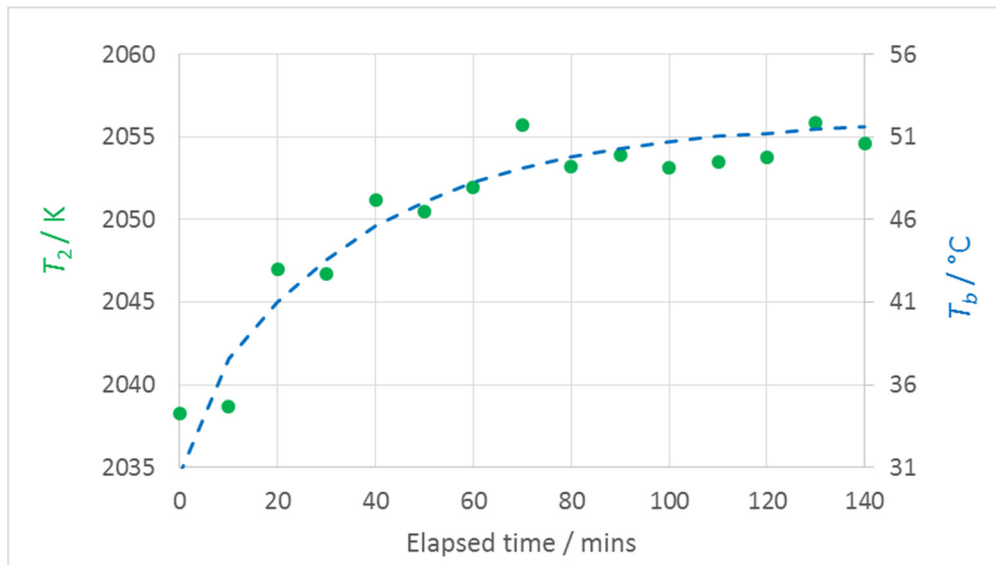


Figure 9 Burner warm-up test. $\phi = 1.4$, 23 SLPM airflow. The increase in post-flame temperature T_2 is proportional to the increase in burner upper body temperature T_b .

6.1.2 Long-term temperature measurements

Figure 10 a) shows the mean temperature T_2 measured 20 mm above the burner centre for $\{0.8 < \phi < 1.4\}$ over a three month period (10 days measurements). The adiabatic flame temperature T_{ad} is also shown and represents the theoretical maximum temperature the flame can attain if there are no heat losses. This is never achieved in practice since there are always heat losses to the burner surface and the external environment. For all ϕ , the temperature is lower than T_{ad} by between 1 % and 2 %. Additionally, the measurement data sits symmetrically below the T_{ad} curve, indicating that the mass flow controllers used to meter the air and propane are operating within their calibration uncertainty. If this had not been the case, a systematic offset would be clearly evident.

The standard deviation of the temperature measurements is shown in Figure 10 b). Depending on ϕ , it ranges from 2.4 K to 4.6 K (0.12 % to 0.2 % of T_2). This is a remarkable level of reproducibility and demonstrates the suitability of the Hencken burner and associated gas flow and control system as a standard flame. The overall uncertainty in each temperature measurement is estimated to be $U_r(T) = 0.5 \%$. An uncertainty budget for determining the post-flame temperature can be found in Appendix A and Tabulated results for the mean and standard deviation of the measurements can be found in Appendix B Table B1.

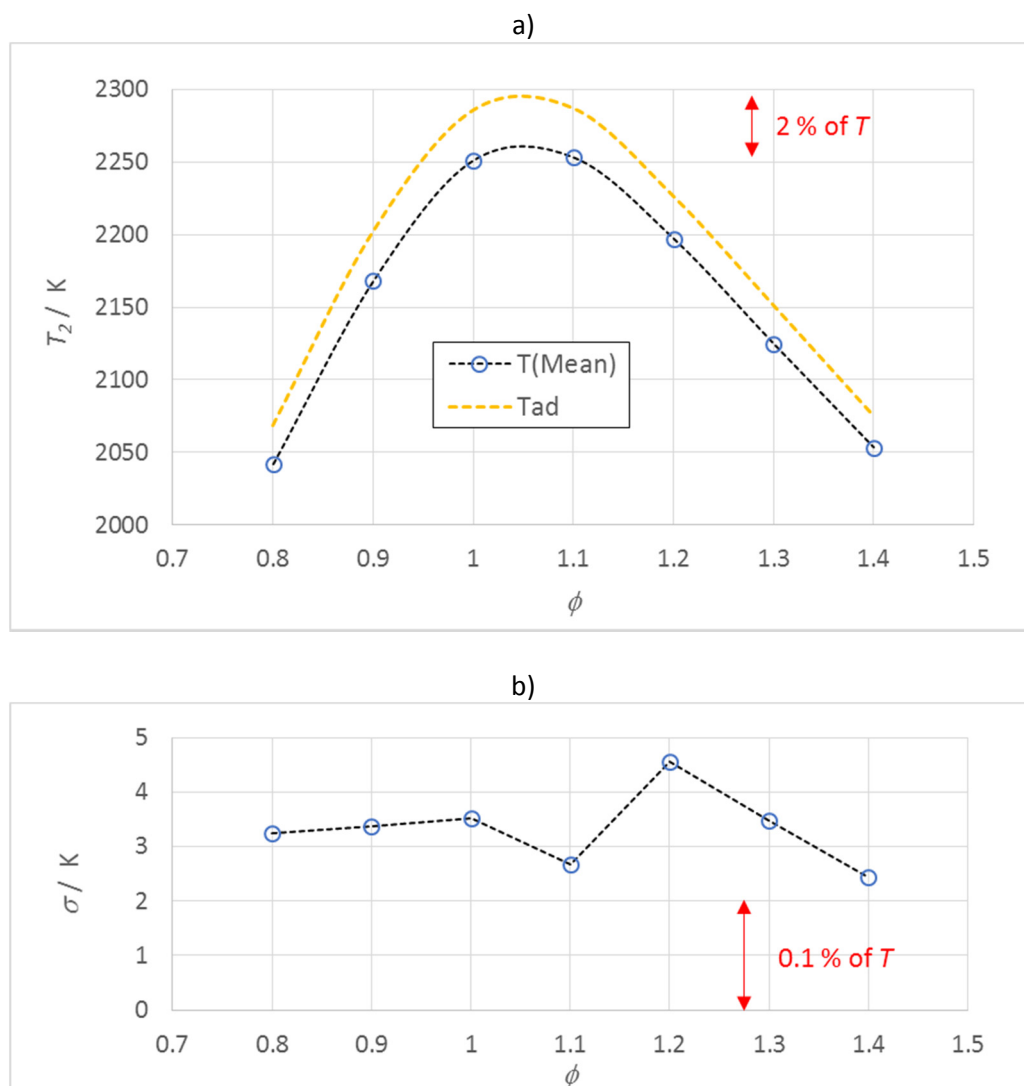


Figure 10 Long-term temperature measurements for $\phi = \{0.8 - 1.4\}$ taken over a three month period: a) mean and b) standard deviation, of 10 measurement sets. All measurements were made 20 mm above the burner centre.

Figure 11 a) shows the change in temperature T_2 relative to the mean, measured 20 mm above the burner centre for $\{0.8 < \phi < 1.4\}$ over a three month period (10 days measurements) and the environmental parameters during each test are given in Figure 11 b), with T_1 , $T_{in}(\text{air})$ and $T_{in}(\text{fuel})$ the air calibration, inlet air and inlet propane temperatures respectively and $P(\text{atm})$ the atmospheric pressure during the measurement. The stability and reproducibility of the STD flame is clearly evident. Additionally there is no apparent correlation between the post-flame temperature and the environmental parameters.

Tables B2 and B3 in Appendix B, provide a tabulated version of the data given in Figure 11 a). Additionally, Tables B10, B11 and B12 give tabulated data for the mole fractions of the reactants, major and minor species respectively, calculated at the measured mean flame temperature for each ϕ .

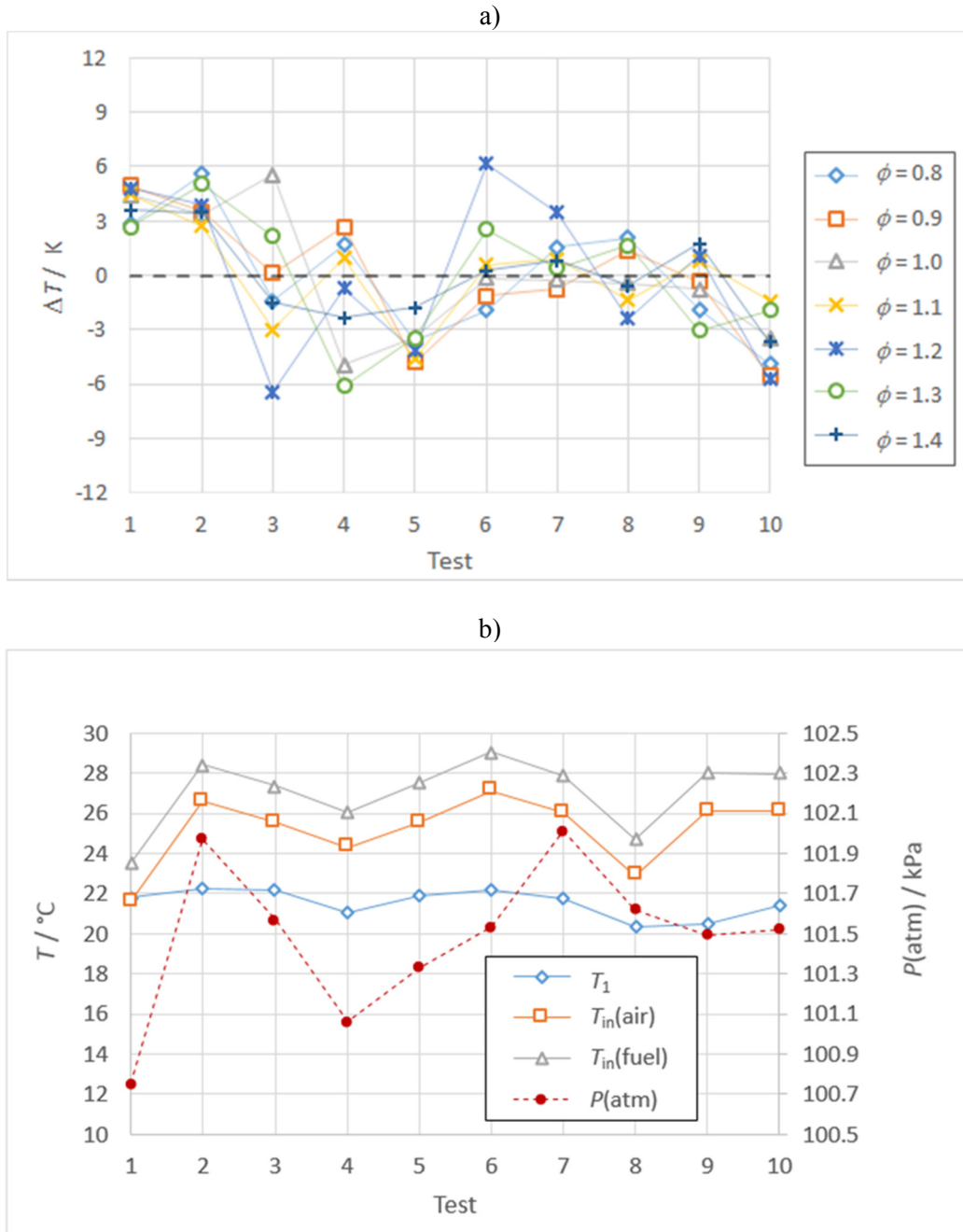


Figure 11. Long-term measurements: a) change in post-flame temperature 20 mm above the burner centre relative to the mean of the 10 days measurements, b) change in the environmental parameters during tests.

6.1.3 Temperature profiles

Figures 12, 13 and 14 show the horizontal temperature profiles for $\phi = 1.0$, $\phi = 1.4$ and $\phi = 0.8$ respectively, based on the mean of ten measurements. (R_x, R_y, R_z) is the measurement location relative to a point 20 mm above the burner centre (see Figure 8), and negative R_z are further away from the burner. For each figure, the profiles are given for a) no co-flow and b) N_2 co-flow, with c) and d) giving the corresponding standard deviations based on ten measurements.

In the central post-flame region, the standard deviations for the N_2 co-flow measurements are significantly less than those without co-flow for $\phi = 1.0$ and $\phi = 1.4$, but similar for $\phi = 0.8$. This

suggests that the secondary combustion (outer blue flame), present for $\phi > 1.0$, is a significant source of the *signal* noise – the co-flow is observed to ‘lift off’ this secondary flame. This reduction in standard deviation with N₂ co-flow is most probably due to a combination of two factors: 1) an improvement in the stability of the measurement region (temperature/species) and 2) the removal of the outer flame background. In the case of the latter, the Rayleigh thermometry system images the probe laser beam through a narrow laser line filter – this should remove a majority of the background. However, earlier spectroscopic measurements of the flame show that there is emission from the C₂ swan bands over the optical bandwidth of the filter: we take account of this by measuring the flame with the laser shutter closed and open and calculate the difference. The difficulty with this method is that the background often contains a semi-periodic component (flame flicker), and the two signals cannot be measured simultaneously – this results in increased noise in the recovered Rayleigh signal once the background is subtracted.

For the outer regions of the post flame where there is mixing with ambient air or N₂ co-flow gas (estimated for $R_y > 12$ mm), the validity of the Rayleigh thermometry is in doubt. This is because, in calculating the Rayleigh temperature, assumptions are made about the gas composition, which are needed to determine the Rayleigh scattering cross-sections, which in turn are used to calculate the temperature. If the composition is far from the expected equilibrium, temperature errors will result. The standard deviations of the temperature measurements for $R_y > 12$ mm are seen to be large and this probably indicates both large fluctuations in temperature and species concentrations. Although genuine temperature differences are expected as R_x or R_y increase – we expect T to fall, the change in species concentrations leads to difficulty in quantifying errors. By noting that the total Rayleigh scattering cross-section for typical post-flame gases differs from that for pure air or N₂ by at most 14 %, we may, with a reasonable degree of certainty, say that the temperature error due to poor knowledge of the species will not be larger than this in the post-flame/ambient air/N₂ region.

The temperature profiles exhibit a number of features. Most significantly, in all cases, the N₂ co-flow makes the horizontal temperature profile more uniform and radially symmetric. Additionally, the width of the uniform temperature region reduces with height above the burner. This is expected as observations of the flame show the same narrowing of the luminous post flame gases.

For $\phi = 1.0$, Figure 12 shows that temperatures are similar with or without N₂ co-flow and generally fall slightly with vertical distance from the burner. Without co-flow, the apparent high temperature lobes seen for $R_z = +10$ mm (10 mm above the burner) for $R_x = R_y \approx \pm 12$ mm are reproducible and seen in earlier measurements – this may be real or an artefact of poor mixing and poor validity of the Rayleigh temperature interpretation. For $\phi = 1.4$, Figure 13 shows that, in the central region, temperatures are depressed by approximately 35 K (1.5 %) for all heights above the burner when the N₂ co-flow is active compared to the no co-flow measurements. Additionally, the temperature profiles are more uniform in the central region with a narrower horizontal extent when the co-flow is active. Again, the temperature falls with vertical height above the burner. For $\phi = 0.8$, Figure 14 shows that temperatures are higher in the centre when the co-flow is active. The co-flow also has the effect of producing a symmetric but downward (as $|R_x|$ or $|R_y|$ increase) curved horizontal temperature profile for all R_z . Again, the temperature falls with vertical height above the burner. When $R_z = +10$ mm (10 mm above the burner), the measurements made with no co-flow exhibit a periodic and reasonably radially symmetric temperature profile. Visually the flame does not look flat for $\phi = 0.8$ which may explain the periodic structure.

In some cases, measurements close to the burner surface ($R_z = 10$ mm, i.e. 10 mm above the burner) appear to be higher than the adiabatic flame temperature. This is probably due to the species not having reached local thermodynamic equilibrium close to the flame surface. Simple 1D chemical kinetic models have shown that equilibrium is reached around this height or slightly higher. The consequence of this is that the predicted species used to determine the Rayleigh scattering cross-section will be in error – this will lead to an error in the inferred temperature.

Tabulated data for the post-flame temperature profiles and their standard deviations are given in appendix B. Tables B4, B5 and B6 give the results of scanning in the R_x direction, for $R_z = 0$ mm, $R_z = +10$ mm and $R_z = -10$ mm respectively (corresponding to heights of 20 mm, 10 mm and 30 mm above the burner). Tables B7, B8 and B9 give the results of scanning in the R_y direction, for $R_z = 0$ mm, $R_z = +10$ mm and $R_z = -10$ mm respectively.

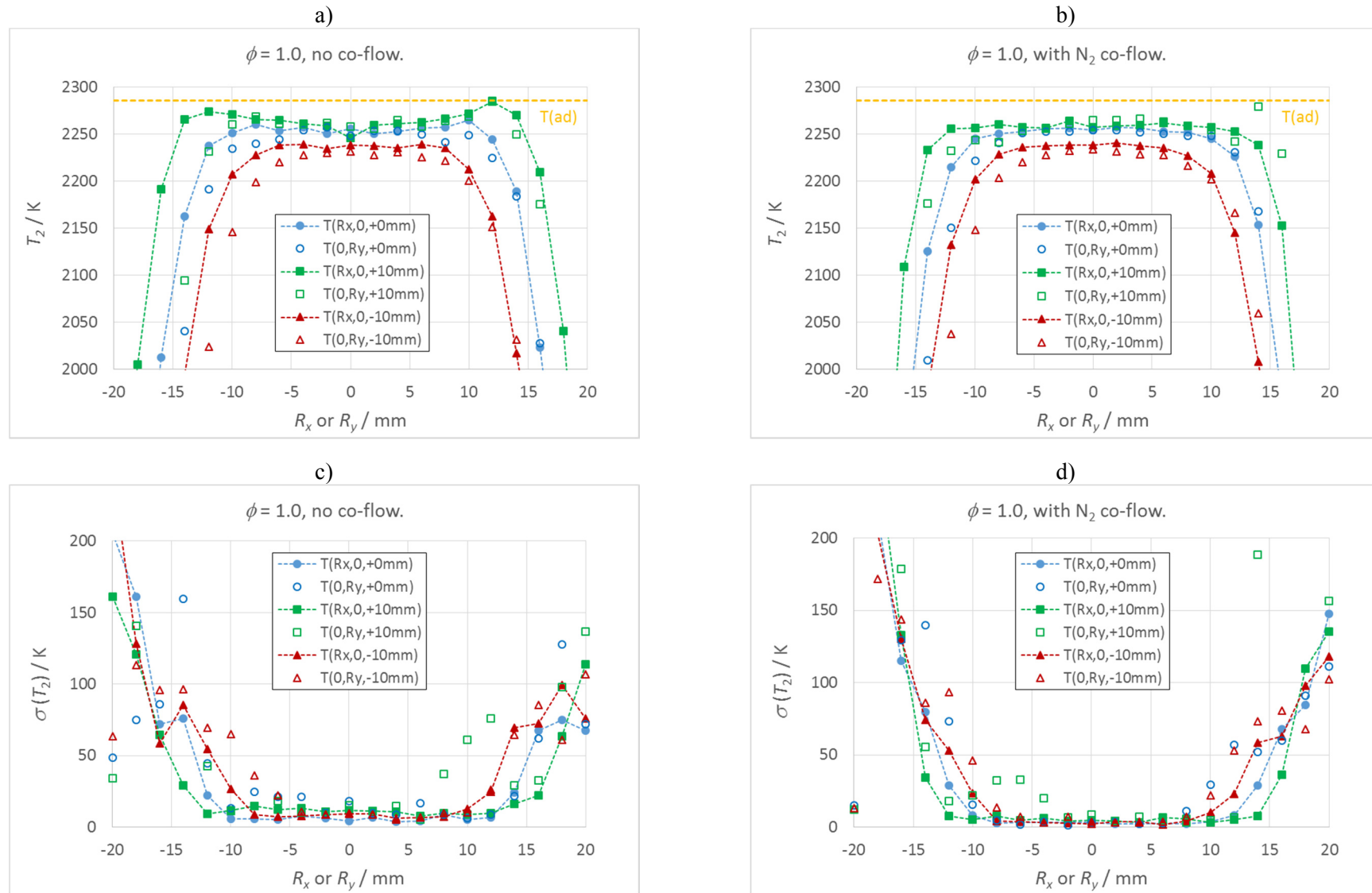


Figure 12 Temperature profiles for $\phi = 1.0$, with a) no co-flow, b) N_2 co-flow, and the measurement standard deviation with c) no co-flow and d) N_2 co-flow. (R_x, R_y, R_z) is the measurement location relative to a point 20 mm above the burner centre (see Figure 8), and negative R_z are further away from the burner.

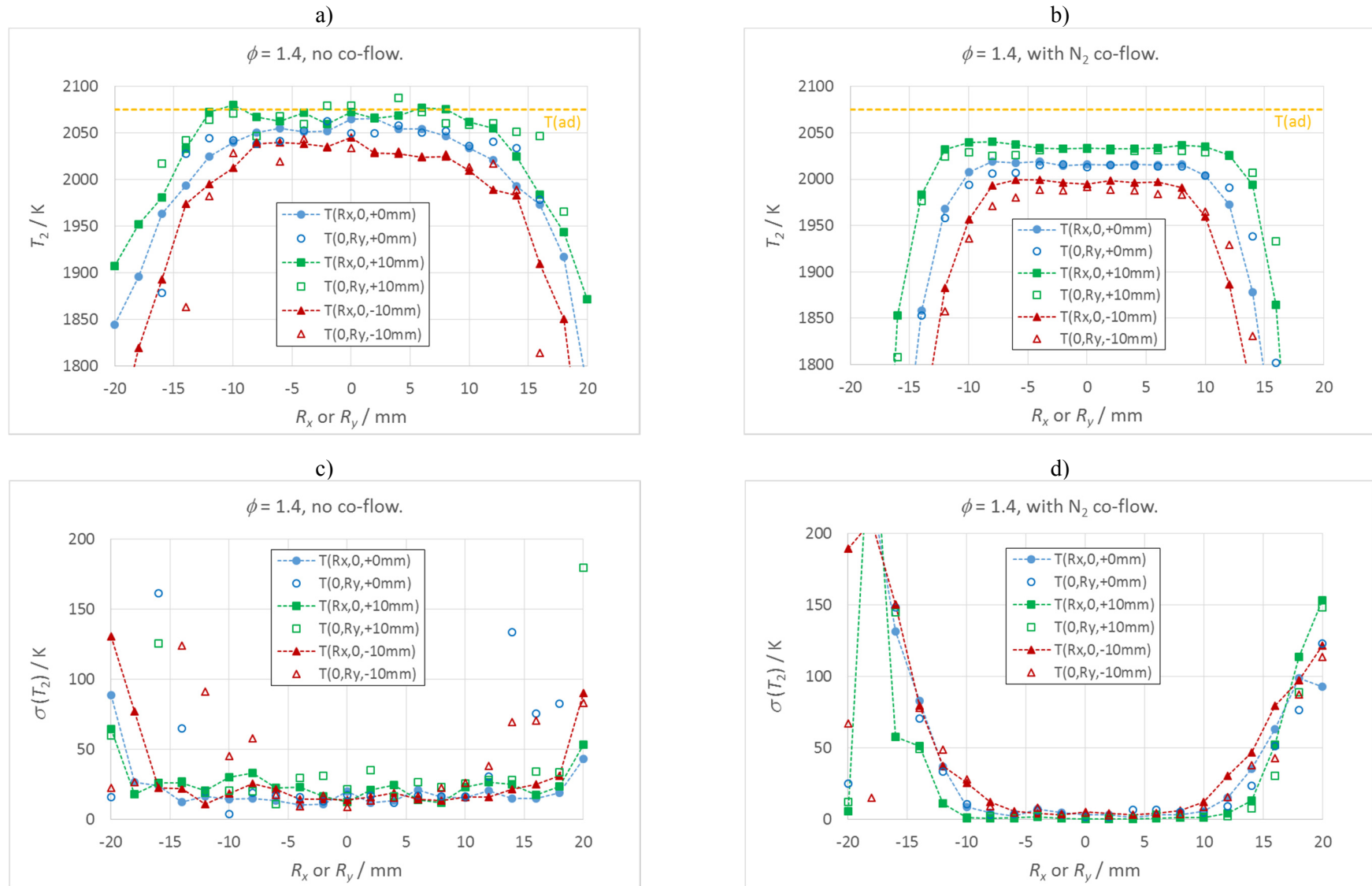


Figure 13 Temperature profiles for $\phi = 1.4$, with a) no co-flow, b) N_2 co-flow, and the measurement standard deviation with c) no co-flow and d) N_2 co-flow. (R_x, R_y, R_z) is the measurement location relative to a point 20 mm above the burner centre (see Figure 8), and negative R_z are further away from the burner.

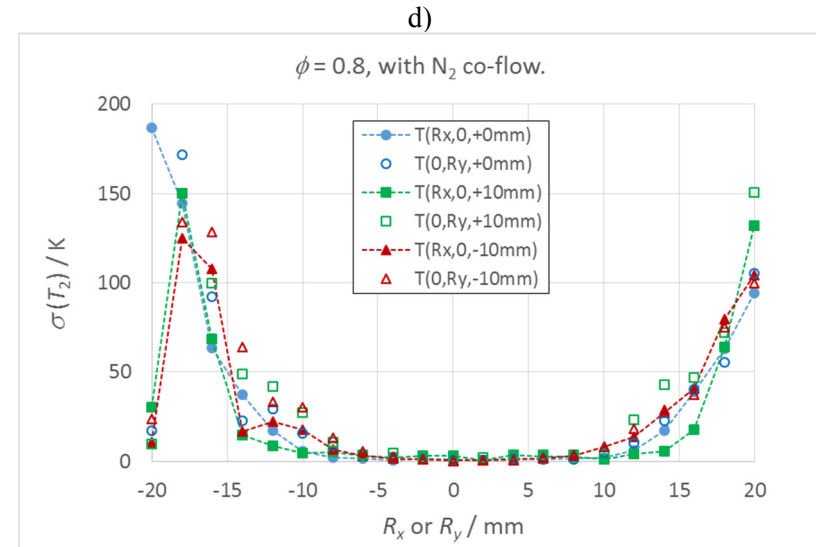
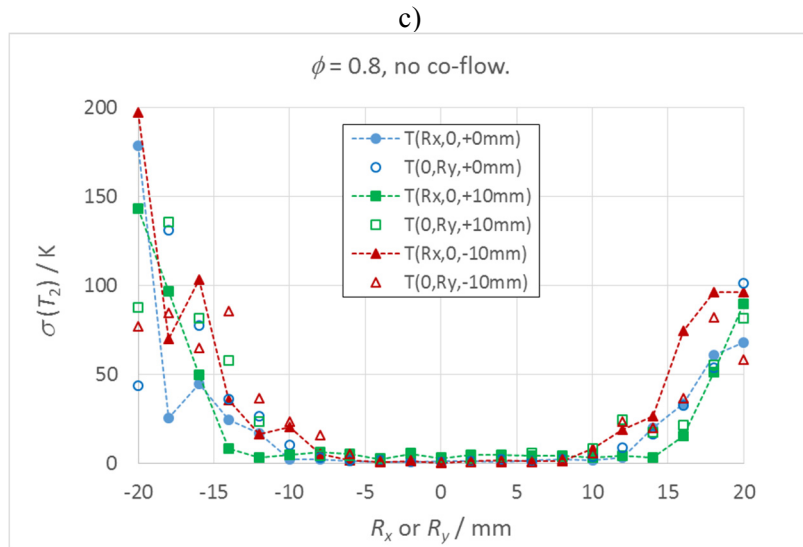
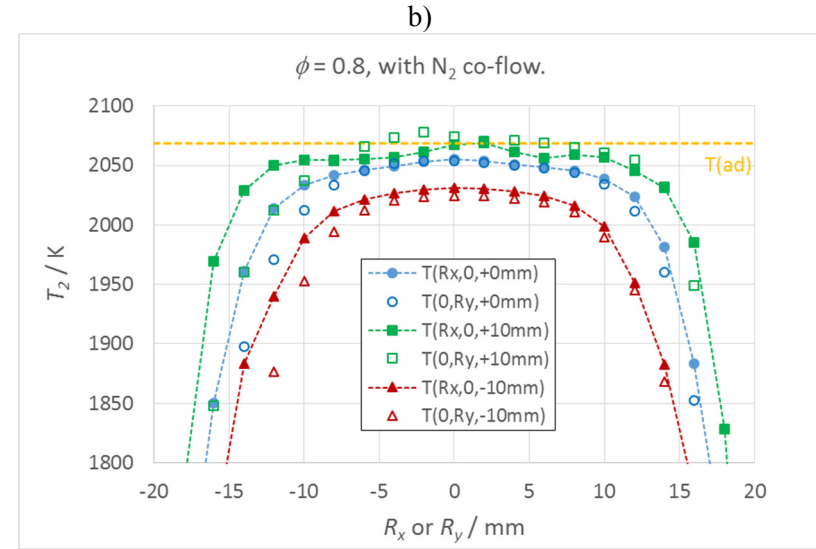
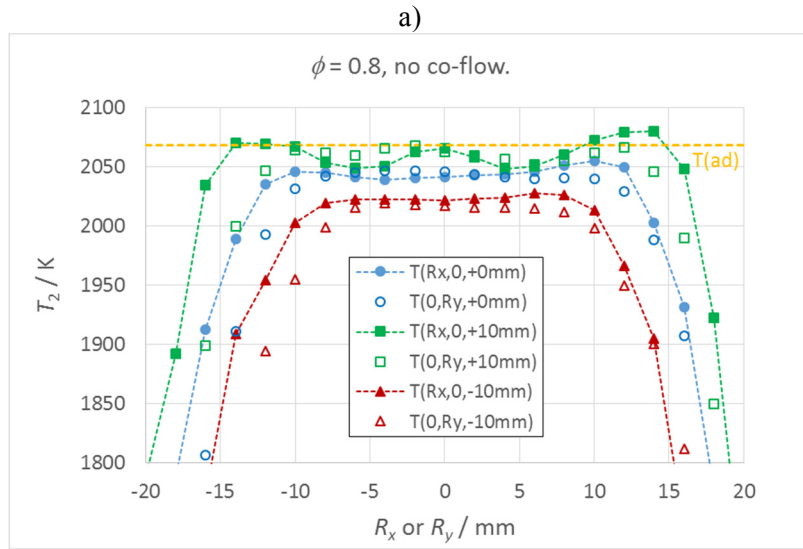


Figure 14 Temperature profiles for $\phi = 0.8$, with a) no co-flow, b) N_2 co-flow, and the measurement standard deviation with c) no co-flow and d) N_2 co-flow. (R_x, R_y, R_z) is the measurement location relative to a point 20 mm above the burner centre (see Figure 8), and negative R_z are further away from the burner.

7 SUMMARY AND CONCLUSIONS

In conclusion, a portable flame temperature standard has been developed and calibrated using the Rayleigh scattering thermometry technique, traceable to ITS-90. By taking in to account the variation in the differential Rayleigh scattering cross-section with both species and temperature, it has been possible to measure the post-flame temperature (for the default measurement position – 20 mm above the burner centre) with a relative uncertainty (1σ) of 0.5 %. Additionally, for the same measurement position, the long-term temperature reproducibility (over a three month period) was found to be 0.2 % of T , further demonstrating the suitability of the system as a temperature standard. By careful control of the equivalence ratio ϕ , the STD flame system is also capable of realising a number of fixed post-flame temperatures in the range $T \propto \{2042 \text{ K} - 2254 \text{ K}\}$ with known species concentrations. Tabulated results for temperature and species are given in Tables B1 and B12 respectively. By providing this data, it should be possible to greatly improve validation of other optical thermometry techniques in which data reduction can be complex.

Post-flame temperature profiles indicate a uniform region where the temperature varies horizontally and vertically by at most $\pm 0.5 \%$ and $\pm 1.0 \%$ respectively for any point within 10 mm of the default measurement position. Additionally, tabulated results are provided in the Appendix, which may be useful when calibrating optical techniques that rely on line-of-sight signal collection such as emission or absorption spectroscopy.

Recently, the STD flame has returned from field trials [4]. Recalibration of the flame following these trials indicated that the flame temperature has not changed - i.e. it is within the $\pm 0.2 \%$ reproducibility reported earlier. Further trials employing emission/absorption spectroscopy and Laser Induced Grating Scattering (LIGS) are currently underway and will also be reported in future publications.

The long-term reproducibility and stability of the standard flame makes it an ideal candidate for the calibration of optical thermometry techniques found in both universities and combustion research institutions. As the UK's National Measurement Institute, NPL provides calibration and consultation in all aspects of metrology and the standard flame facility should be a welcome addition to the facilities and services it provides.

8 ACKNOWLEDGEMENTS

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9 APPENDIX A: UNCERTAINTY BUDGET

Table A1 shows the uncertainty budget for the temperature measured 20 mm above the centre of the burner ($R_z = 0$ mm) using Rayleigh scattering Thermometry. For regions closer to the burner surface, where thermal and chemical equilibrium may not have been reached, and beyond ± 10 mm horizontally from the central position, where the gas composition is uncertain, the uncertainties will be significantly larger.

In accordance with the *Guide to the expression of uncertainty in measurement* [18], the *Multiplier* (column 5) is used to convert a *rectangular* uncertainty contribution (Type-B) in to the equivalent *normal* uncertainty contribution, and is equal to $1/\sqrt{3}$. The *Sensitivity coefficient* (column 6) describes how the output estimate (in this case flame temperature) varies with changes in the input estimate (i.e. it is the partial derivative of the output estimate w.r.t. the input estimate).

For example, considering the *Flow-meter uncertainty*⁽⁺⁾ – this has a rectangular uncertainty distribution of $\pm 1\%$, indicating that the indicated flow could be expected to be anywhere within $\pm 1\%$ of the true value. We convert this to the equivalent normal distribution by multiplying it by $1/\sqrt{3}$ giving $\pm 0.58\%$ and then by the sensitivity factor of 0.4 (determined by offsetting the propane flow by $+1\%$ and determining the new flame temperature) yielding an overall uncertainty of 0.23% in flame temperature.

Table A1 Uncertainty budget for the temperature 20 mm above the centre of the NPL STD flame.

Source	Type	Distr.	Size / $\pm\%$	Multiplier	Sensitivity Coefficient	Size (1σ) / $\pm\%$
Molar refractivity data	B	Rect	0.20	0.58	1.00	0.12
Temperature dependence of polarisability	B	Rect	0.20	0.58	1.00	0.12
Flow-meter uncertainty ⁽⁺⁾	B	Rect	1.00	0.58	0.40	0.23
Chemical equilibrium assumption	B	Rect	0.30	0.58	1.00	0.17
Air calibration PRT	A	Norm	0.05	1.00	1.00	0.05
Background scattered signal	A	Norm	0.10	1.00	0.50	0.05
Laser stability	A	Norm	0.20	1.00	1.00	0.20
Inlet air temperature (15-25 °C)	B	Rect	3.00	0.58	0.10	0.17
Atmospheric pressure	B	Rect	5.00	0.58	0.05	0.15
Gas purity	B	Rect	2.50	0.58	0.05	0.07
Flame temperature reproducibility	A	Norm	0.20	1.00	1.00	0.20
Total uncertainty (combined in quadrature)						0.50

10 APPENDIX B: TABULATED RESULTS

Temperature

Table B1 Mean post-flame temperature 20 mm above the burner centre for 10 measurements made over a three month period.

ϕ	$T_f(\phi) / \text{K}$	$\sigma(T) / \text{K}$	$\sigma(T) / \%$	T_{ad} / K
0.8	2041.8	3.3	0.16	2068.3
0.9	2168.0	3.4	0.16	2201.8
1	2251.1	3.5	0.16	2285.5
1.1	2253.3	2.7	0.12	2287.1
1.2	2197.3	4.6	0.21	2226.6
1.3	2124.9	3.5	0.16	2151.4
1.4	2053.4	2.4	0.12	2075.2

Table B2 Absolute change in post-flame temperature 20 mm above the burner centre for 10 measurements made over a three month period.

$\Delta T_f(\phi)$ (re: mean of 10 tests) / K										
ϕ	1	2	3	4	5	6	7	8	9	10
0.8	2.8	5.6	-1.4	1.7	-3.6	-1.9	1.6	2.1	-1.9	-4.9
0.9	4.9	3.5	0.1	2.7	-4.8	-1.2	-0.8	1.4	-0.3	-5.6
1	4.5	3.4	5.6	-5.0	-3.4	-0.1	-0.2	-0.4	-0.7	-3.4
1.1	4.5	2.7	-3.0	0.9	-4.6	0.6	0.9	-1.4	0.8	-1.4
1.2	4.8	3.9	-6.4	-0.7	-4.2	6.1	3.5	-2.4	1.1	-5.7
1.3	2.7	5.1	2.2	-6.1	-3.4	2.6	0.4	1.6	-3.0	-1.9
1.4	3.6	3.5	-1.5	-2.3	-1.8	0.3	0.8	-0.6	1.7	-3.7

Table B3 Percentage change in post-flame temperature 20 mm above the burner centre for 10 measurements made over a three month period.

$\Delta T_f(\phi)$ (re: mean of 10 tests) / %										
ϕ	1	2	3	4	5	6	7	8	9	10
0.8	0.14	0.27	-0.07	0.08	-0.18	-0.09	0.08	0.10	-0.09	-0.24
0.9	0.23	0.16	0.01	0.12	-0.22	-0.05	-0.04	0.06	-0.02	-0.26
1	0.20	0.15	0.25	-0.22	-0.15	-0.01	-0.01	-0.02	-0.03	-0.15
1.1	0.20	0.12	-0.13	0.04	-0.21	0.03	0.04	-0.06	0.04	-0.06
1.2	0.22	0.18	-0.29	-0.03	-0.19	0.28	0.16	-0.11	0.05	-0.26
1.3	0.13	0.24	0.10	-0.29	-0.16	0.12	0.02	0.08	-0.14	-0.09
1.4	0.17	0.17	-0.07	-0.11	-0.09	0.01	0.04	-0.03	0.08	-0.18

Table B4 Horizontal temperature profile in the R_x direction for $R_z = 0$ mm, 20 mm above the burner: a) Temperatures, b) Standard deviation of the measurements. Note: the SD's do not represent the uncertainty of the temperature measurement but can be used as an indicator of the flame stability.

R_x / mm	a) $T_f(R_x, 0, 0)$ / K						b) $\sigma(T_f(R_x, 0, 0))$ / K					
	No co-flow			With N ₂ co-flow			No co-flow			With N ₂ co-flow		
	$\phi = 1.0$	$\phi = 1.4$	$\phi = 0.8$	$\phi = 1.0$	$\phi = 1.4$	$\phi = 0.8$	$\phi = 1.0$	$\phi = 1.4$	$\phi = 0.8$	$\phi = 1.0$	$\phi = 1.4$	$\phi = 0.8$
-20	1623	1845	1592	1311	980	1459	205	88	179	243	222	187
-18	1772	1896	1788	1609	1285	1665	161	27	25	215	229	144
-16	2012	1964	1912	1916	1633	1850	72	24	45	115	131	64
-14	2162	1994	1989	2125	1858	1961	76	12	24	79	83	38
-12	2237	2025	2035	2215	1968	2014	22	17	17	29	37	17
-10	2252	2040	2046	2245	2008	2033	6	14	2	8	9	6
-8	2260	2051	2045	2250	2019	2042	6	15	2	3	5	2
-6	2253	2055	2041	2253	2017	2046	5	13	1	3	2	2
-4	2257	2051	2039	2256	2019	2049	8	10	1	3	7	1
-2	2251	2052	2041	2257	2015	2053	6	11	1	3	5	2
0	2256	2065	2042	2254	2016	2055	4	20	1	3	3	1
2	2250	2066	2043	2257	2016	2054	7	12	1	2	3	1
4	2253	2054	2044	2257	2016	2051	4	13	1	2	1	2
6	2257	2054	2046	2253	2015	2049	4	21	2	3	3	1
8	2258	2047	2051	2252	2016	2045	9	16	2	2	3	1
10	2265	2034	2055	2245	2004	2039	5	15	2	4	5	2
12	2245	2021	2050	2226	1973	2024	7	21	3	8	15	6
14	2190	1993	2003	2154	1878	1982	24	15	20	29	35	17
16	2023	1973	1932	1971	1704	1883	67	15	33	68	63	39
18	1826	1917	1778	1702	1441	1721	75	19	61	84	99	63
20	1528	1778	1467	1287	1007	1409	68	43	68	148	93	94

Table B5 Horizontal temperature profile in the R_x direction for $R_z = +10$ mm, 10 mm above the burner: a) Temperatures, b) Standard deviation of the measurements. Note: the SD's do not represent the uncertainty of the temperature measurement but can be used as an indicator of the flame stability.

R_x / mm	a) $T_f(R_x, 0, +10 \text{ mm})$ / K						b) $\sigma(T_f(R_x, 0, +10 \text{ mm}))$ / K					
	No co-flow			With N ₂ co-flow			No co-flow			With N ₂ co-flow		
	$\phi = 1.0$	$\phi = 1.4$	$\phi = 0.8$	$\phi = 1.0$	$\phi = 1.4$	$\phi = 0.8$	$\phi = 1.0$	$\phi = 1.4$	$\phi = 0.8$	$\phi = 1.0$	$\phi = 1.4$	$\phi = 0.8$
-20	1791	1907	1790	1392	950	1482	161	65	143	253	6	30
-18	2005	1952	1892	1740	1415	1781	121	18	97	256	300	150
-16	2191	1981	2034	2109	1853	1969	64	26	50	133	58	69
-14	2266	2033	2070	2233	1983	2029	29	26	9	34	51	15
-12	2274	2073	2070	2256	2032	2050	9	20	3	8	11	9
-10	2271	2081	2067	2257	2040	2055	12	30	5	5	1	5
-8	2266	2068	2054	2261	2040	2055	15	33	6	7	1	5
-6	2265	2062	2049	2257	2037	2055	12	23	5	5	1	3
-4	2261	2072	2051	2256	2034	2057	13	23	2	6	1	2
-2	2259	2060	2063	2264	2033	2061	11	16	5	4	1	3
0	2246	2073	2066	2257	2033	2067	12	13	3	5	0	3
2	2260	2066	2058	2259	2033	2069	11	21	5	4	0	1
4	2261	2069	2049	2260	2033	2062	11	25	5	3	0	4
6	2263	2077	2050	2262	2034	2056	8	14	4	7	1	3
8	2266	2076	2061	2259	2037	2059	10	12	4	6	1	3
10	2272	2062	2072	2258	2035	2057	9	23	3	3	1	1
12	2285	2055	2079	2253	2026	2046	10	27	4	5	4	4
14	2271	2025	2080	2239	1994	2031	16	25	3	8	13	6
16	2210	1984	2048	2152	1864	1985	22	18	15	36	52	18
18	2040	1943	1922	1851	1532	1828	63	24	51	110	113	64
20	1740	1872	1691	1324	951	1500	114	54	90	135	153	132

Table B6 Horizontal temperature profile in the R_x direction for $R_z = -10$ mm, 30 mm above the burner: a) Temperatures, b) Standard deviation of the measurements. Note: the SD's do not represent the uncertainty of the temperature measurement but can be used as an indicator of the flame stability.

R_x / mm	a) $T_f(R_x, 0, -10 \text{ mm})$ / K						b) $\sigma(T_f(R_x, 0, -10 \text{ mm}))$ / K					
	No co-flow			With N ₂ co-flow			No co-flow			With N ₂ co-flow		
	$\phi = 1.0$	$\phi = 1.4$	$\phi = 0.8$	$\phi = 1.0$	$\phi = 1.4$	$\phi = 0.8$	$\phi = 1.0$	$\phi = 1.4$	$\phi = 0.8$	$\phi = 1.0$	$\phi = 1.4$	$\phi = 0.8$
-20	1462	1710	1486	1090	833	1263	247	131	197	215	189	11
-18	1575	1819	1579	1427	1162	1502	128	77	70	203	209	125
-16	1794	1893	1773	1775	1510	1737	59	22	103	130	150	108
-14	1994	1974	1909	1976	1729	1883	85	22	36	74	80	17
-12	2149	1996	1955	2132	1883	1940	55	11	16	53	37	22
-10	2208	2013	2003	2202	1956	1989	27	18	21	24	26	18
-8	2228	2039	2019	2228	1993	2012	9	25	5	4	12	7
-6	2238	2040	2022	2236	1999	2021	7	22	2	4	6	3
-4	2239	2038	2023	2237	1999	2027	8	14	1	3	4	2
-2	2234	2035	2022	2239	1997	2030	9	14	1	3	3	1
0	2238	2045	2022	2238	1995	2031	9	14	0	2	5	1
2	2238	2028	2023	2241	1999	2030	9	16	1	4	4	1
4	2236	2028	2024	2237	1996	2028	6	19	2	4	3	1
6	2240	2024	2027	2235	1997	2024	7	14	1	2	4	2
8	2236	2025	2027	2227	1991	2016	7	13	1	4	6	3
10	2213	2010	2013	2208	1959	1999	13	16	8	10	12	8
12	2163	1990	1967	2145	1887	1951	25	16	19	23	30	14
14	2017	1983	1905	2008	1768	1883	69	22	27	59	47	27
16	1864	1910	1746	1826	1603	1775	73	25	75	63	79	41
18	1589	1850	1549	1542	1301	1551	99	31	96	98	97	80
20	1349	1653	1338	1378	1004	1238	76	90	96	118	122	104

Table B7 Horizontal temperature profile in the R_y direction for $R_z = 0$ mm, 20 mm above the burner: a) Temperatures, b) Standard deviation of the measurements. Note: the SD's do not represent the uncertainty of the temperature measurement but can be used as an indicator of the flame stability.

R_x / mm	a) $T_f(0, R_y, 0 \text{ mm}) / \text{K}$						b) $\sigma(T_f(0, R_y, 0 \text{ mm})) / \text{K}$					
	No co-flow			With N ₂ co-flow			No co-flow			With N ₂ co-flow		
	$\phi = 1.0$	$\phi = 1.4$	$\phi = 0.8$	$\phi = 1.0$	$\phi = 1.4$	$\phi = 0.8$	$\phi = 1.0$	$\phi = 1.4$	$\phi = 0.8$	$\phi = 1.0$	$\phi = 1.4$	$\phi = 0.8$
-20	1369	1061	1335	1126	953	1191	48	16	44	15	25	17
-18	1608	1392	1568	1478	1317	1500	75	211	131	201	205	172
-16	1805	1879	1806	1790	1645	1746	86	162	78	129	148	92
-14	2040	2028	1911	2009	1853	1898	159	65	36	140	70	23
-12	2191	2045	1993	2151	1958	1971	45	19	27	73	34	29
-10	2234	2042	2032	2222	1994	2012	13	4	10	15	11	16
-8	2240	2038	2042	2241	2006	2034	25	19	3	5	3	9
-6	2245	2041	2045	2251	2007	2045	21	17	2	2	4	3
-4	2254	2053	2046	2253	2015	2051	21	16	2	4	6	2
-2	2257	2063	2047	2253	2016	2054	10	13	1	1	3	1
0	2248	2050	2046	2255	2013	2054	18	17	1	3	2	1
2	2253	2050	2044	2254	2015	2052	10	17	1	4	2	1
4	2254	2058	2041	2252	2014	2050	9	12	2	2	6	2
6	2250	2051	2040	2251	2014	2048	17	17	1	3	7	1
8	2242	2052	2040	2248	2014	2044	9	16	2	11	5	1
10	2249	2036	2040	2248	2004	2034	6	16	4	29	5	5
12	2225	2041	2030	2231	1991	2012	9	30	9	57	9	10
14	2184	2034	1988	2168	1938	1960	22	134	16	52	24	23
16	2028	1979	1907	1994	1802	1852	62	75	32	60	51	40
18	1786	1785	1778	1738	1590	1700	128	83	54	91	77	55
20	1555	1359	1551	1397	1207	1385	72	277	101	111	123	105

Table B8 Horizontal temperature profile in the R_y direction for $R_z = +10$ mm, 10 mm above the burner: a) Temperatures, b) Standard deviation of the measurements. Note: the SD's do not represent the uncertainty of the temperature measurement but can be used as an indicator of the flame stability.

R_x / mm	a) $T_f(0, R_y, +10 \text{ mm})$ / K						b) $\sigma(T_f(0, R_y, +10 \text{ mm}))$ / K					
	No co-flow			With N ₂ co-flow			No co-flow			With N ₂ co-flow		
	$\phi = 1.0$	$\phi = 1.4$	$\phi = 0.8$	$\phi = 1.0$	$\phi = 1.4$	$\phi = 0.8$	$\phi = 1.0$	$\phi = 1.4$	$\phi = 0.8$	$\phi = 1.0$	$\phi = 1.4$	$\phi = 0.8$
-20	1386	1186	1399	1005	916	1081	34	60	88	12	12	10
-18	1654	1713	1684	1513	1393	1536	141	300	136	330	324	293
-16	1924	2017	1899	1993	1808	1847	321	126	82	179	145	100
-14	2094	2042	2000	2176	1977	1960	287	27	58	55	49	49
-12	2232	2064	2047	2233	2024	2013	42	19	23	18	11	42
-10	2261	2071	2064	2244	2029	2037	11	20	5	22	1	28
-8	2269	2047	2062	2242	2025	2054	15	20	4	32	2	11
-6	2261	2068	2060	2254	2026	2066	18	11	4	33	1	4
-4	2260	2060	2066	2256	2031	2073	10	30	3	20	2	5
-2	2262	2079	2068	2261	2033	2078	11	31	6	7	1	3
0	2258	2080	2063	2265	2033	2075	15	21	3	9	0.4	3
2	2256	2067	2060	2265	2032	2071	11	35	3	4	1	2
4	2265	2088	2056	2267	2030	2071	15	16	4	7	1	4
6	2258	2073	2052	2263	2031	2069	5	26	6	5	3	4
8	2265	2061	2055	2259	2031	2065	37	23	4	3	1	4
10	2269	2059	2062	2255	2029	2061	61	26	8	5	2	4
12	2286	2060	2066	2242	2025	2054	76	28	25	7	2	23
14	2250	2051	2046	2279	2007	2032	29	28	17	189	7	43
16	2176	2046	1990	2230	1933	1949	33	34	21	324	31	47
18	1950	1966	1850	1928	1693	1740	98	34	56	271	89	72
20	1587	1494	1620	1308	1195	1236	137	179	82	156	148	151

Table B9 Horizontal temperature profile in the R_y direction for $R_z = -10$ mm, 30 mm above the burner: a) Temperatures, b) Standard deviation of the measurements. Note: the SD's do not represent the uncertainty of the temperature measurement but can be used as an indicator of the flame stability.

R_x / mm	a) $T_f(0, R_y, -10 \text{ mm}) / \text{K}$						b) $\sigma(T_f(0, R_y, -10 \text{ mm})) / \text{K}$					
	No co-flow			With N ₂ co-flow			No co-flow			With N ₂ co-flow		
	$\phi = 1.0$	$\phi = 1.4$	$\phi = 0.8$	$\phi = 1.0$	$\phi = 1.4$	$\phi = 0.8$	$\phi = 1.0$	$\phi = 1.4$	$\phi = 0.8$	$\phi = 1.0$	$\phi = 1.4$	$\phi = 0.8$
-20	1277	1090	1250	1028	868	1055	63	22	77	13	67	24
-18	1475	1095	1424	1315	1213	1307	113	26	85	172	15	134
-16	1608	1459	1611	1633	1499	1598	96	206	65	144	150	128
-14	1791	1863	1767	1851	1723	1779	96	124	85	86	78	64
-12	2024	1982	1895	2038	1857	1876	69	91	36	93	49	33
-10	2146	2029	1955	2148	1936	1952	65	45	24	46	28	30
-8	2199	2040	1999	2204	1971	1994	36	58	16	14	9	13
-6	2220	2019	2016	2220	1980	2012	22	18	6	7	4	6
-4	2228	2044	2020	2228	1989	2020	11	9	1	3	8	2
-2	2230	2035	2018	2232	1988	2024	9	17	2	7	4	1
0	2231	2034	2017	2234	1991	2025	10	9	1	4	5	0.3
2	2228	2030	2016	2232	1989	2024	9	13	1	3	3	1
4	2231	2030	2016	2229	1988	2022	5	15	1	3	3	1
6	2226	2025	2015	2228	1984	2019	7	17	1	4	4	2
8	2221	2027	2012	2216	1983	2010	10	22	2	7	3	4
10	2201	2013	1998	2202	1965	1990	10	26	6	22	9	8
12	2152	2017	1950	2166	1929	1946	26	38	23	53	16	18
14	2031	1989	1901	2059	1831	1868	65	70	20	73	38	29
16	1791	1814	1811	1799	1709	1767	85	71	36	81	43	38
18	1619	1352	1622	1609	1469	1565	61	228	82	68	87	75
20	1424	1137	1431	1299	1098	1265	107	83	58	102	113	100

Species

The equilibrium species concentrations for the reactants versus ϕ are given in Table B10. The equilibrium concentrations for the major and minor species at $T_2(\phi)$, for an inlet temperature of 64 °C at $P = 1$ atm. are given in Tables B11 and B12 respectively. The inlet temperature of 64 °C was the mean burner surface temperature (over all ϕ) measured with a calibrated thermal imager and it is assumed that the reactants are at this temperature when they exit the burner.

Table B10 Mole fractions for reactants: Propane/air.

Mole fraction, inlet (reactants)				
ϕ	N_2	O_2	Ar	C_3H_8
0.8	0.755573	0.202705	0.009289	0.032433
0.9	0.752522	0.201887	0.009251	0.036340
1	0.749496	0.201075	0.009214	0.040215
1.1	0.746494	0.200270	0.009177	0.044059
1.2	0.743516	0.199471	0.009140	0.047873
1.3	0.740562	0.198678	0.009104	0.051656
1.4	0.737631	0.197892	0.009068	0.055410

Table B11 Mole fractions for post-flame species (major).

Mole fraction, post-flame (Major species)								
ϕ	$T_f(\phi)/K$	Ar	N_2	H_2O	CO_2	CO	O_2	H_2
0.8	2041.8	0.008987	0.729156	0.124322	0.093250	0.000885	0.037401	0.000251
0.9	2168.0	0.008898	0.722041	0.137392	0.101370	0.003492	0.019009	0.000923
1	2251.1	0.008781	0.713170	0.148468	0.103289	0.011694	0.005468	0.003117
1.1	2253.3	0.008603	0.699433	0.154460	0.093085	0.030830	0.000658	0.009475
1.2	2197.3	0.008394	0.682707	0.154816	0.078413	0.053482	0.000073	0.020225
1.3	2124.9	0.008188	0.666001	0.151511	0.066013	0.073361	0.000010	0.033801
1.4	2053.4	0.007990	0.649900	0.145719	0.056339	0.090122	0.000002	0.049222

Table B12 Mole fractions for post-flame species (minor).

Mole fraction, post-flame (minor)					
ϕ	$T_f(\phi)/K$	OH	H	O	NO
0.8	2041.8	0.001845	0.000034	0.000176	0.003692
0.9	2168.0	0.002858	0.000142	0.000301	0.003572
1	2251.1	0.003034	0.000415	0.000272	0.002291
1.1	2253.3	0.001838	0.000732	0.000096	0.000791
1.2	2197.3	0.000852	0.000785	0.000023	0.000230
1.3	2124.9	0.000377	0.000664	0.000005	0.000070
1.4	2053.4	0.000170	0.000512	0.000001	0.000023

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