

Digital Centring of Axicon Images

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ABSTRACT

The NPL designed monocentric axicon camera produces diffraction limited concentric ring patterns from point source targets. Using a very accurate plate measuring machine and a laser scanning device, 3D co-ordinates were obtained in a photogrammetric test experiment giving residuals of < 2000 nm in a volume of approximately one cubic metre corresponding to a photographic plate measurement uncertainty of < 100 nm, when compared with a Moore 5Z measuring machine. The current work is concerned with replacing the laser scanning device with computer image processing taking its input from a microscope image of the plate viewed with a commercial CCD camera.

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1 INTRODUCTION.

The aim of photogrammetric research at the National Physical Laboratory (NPL) has been to push the practical limits of close range photogrammetric survey for industrial applications. Two branches of the work developed, one based on increasing the accuracy of conventional photogrammetry, using commercial cameras and equipment, and the other exploiting the properties of the new monocentric axicon lens developed at NPL described in section (2). It is the latter approach which will be discussed in the present paper. This camera has been shown to be capable of producing surveys which agree with alternative measurements, using a three co-ordinate measuring machine, to the order of one part in a million of the overall size of the object. This was achieved using a purpose built laser scanning device.

There are obvious advantages in using less specialised hardware with, if necessary, specialised software to locate and measure the centres of the axicon images. This leads to a more robust system which may be used for a variety of measurements using different software drivers, online estimates of the uncertainties of the image co-ordinates and the possibility of fully automatic plate measurement.

2 THE NPL CAMERA

The NPL monocentric axicon camera was developed by J.M.Burch and C.Forno from earlier axicon imaging devices and illustrates a class of devices that do not form orthodox images. In this case the image generated by a point source in the object space is a diffraction pattern of concentric rings in the image space.

The camera consists of two concentric spheres of glass, the outer sphere being divided into two hemispheres separated by an opaque layer which excludes all rays which would not pass through both spheres. It turns out that this form of lens may be assembled with great precision so that the distortion is zero to the limits of present observation, over the entire field of view of $\pm 20^\circ$.

The original photogrammetric tests of this camera used a test object consisting of bearing balls kinematically mounted in conical depressions in aluminium columns which were in turn kinematically mounted on a steel base plate (see Burch and Forno 1984). In this experiment the photogrammetry agreed to 2 parts in 10^6 with a Moore 5Z measuring machine which was itself at the limits of its working accuracy.

In order to measure the 50mm square plates used by the NPL camera a purpose built laser scanning device was used in conjunction with a Zeiss ZKM measuring machine modified by the addition of plane mirror interferometers. This system was capable of locating the centres of target images with a repeatability of some 40nm.

The current work is aimed at producing a digital imaging system that will be more flexible than the purpose built laser system but equally accurate.

3 A MODEL OF THE AXICON IMAGE.

Burch and Forno (1982) have shown that the form of the intensity I near the centre of the image produced by the NPL monocentric axicon camera is of the form:

$$I(x,y) = A_0 (J_0(r/r_0))^2 \quad (1)$$

where

$$r = \sqrt{(x - x_0)^2 + (y - y_0)^2}$$

J_0 is the zeroth order Bessel function, x, y is the position in the plane of the image, x_0, y_0 is the image centre, A_0 is the image brightness and r_0 gives the physical size of the image in the plane of the detector. The form of this theoretical intensity is shown in figure (1).

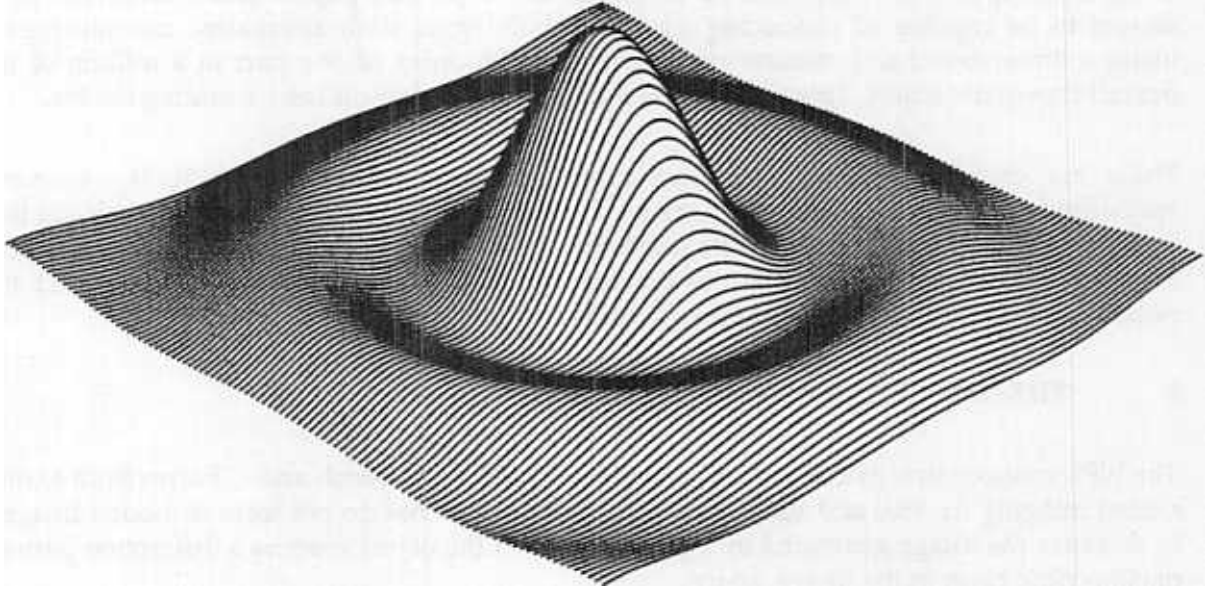


Figure 1 : Theoretical intensity plot for an axicon image.

In order to find the centre of such a pattern it is necessary to capture the intensity distribution on a CCD camera or similar device. Each effective pixel (in fact more correctly each address in the frame store) will contain a value obtained by convoluting the above intensity distribution with some filter function characteristic of the recording device and the control software. In the present treatment this pupil function will be assumed proportional to a delta function. The constant of proportionality G contains such things as the quantum efficiency of the camera and the electrical gain in its control circuit. Thus the value of the effective pixel at the i^{th} row and j^{th} column of the frame store is given by:

$$P_{ij} = GI_{ij} = f(r_{ij}) = GA_0 (J_0(r_{ij}/r_0))^2 \quad (2)$$

where

$$r_{ij} = \sqrt{(ai - (x_0 - x_{min}))^2 + (bj - (y_0 - y_{min}))^2}$$

and a and b are the "projected" pixel spacings i.e. they have been mapped out of the imaging device onto the plane of the aerial image being observed. In the case of an axicon image this is not as straightforward as it may seem because the aerial image is not focussed in a plane in the usual sense and it is this property that gives the system its excellent effective depth of focus. In fact in the experimental work the small axicon image was mapped onto the CCD array using a microscope objective and it was the very limited depth of field of this objective which defined the effective plane of the image. Thus in this model of the system it will be assumed that the depth of focus of the microscope objective is negligible and therefore the image plane is well defined. In the alternative case of interest where the image is located on a photogrammetric plate this is not a problem. If the four corners of the array are mapped onto the image plane at (x_{min}, y_{min}) , (x_{min}, y_{max}) , (x_{max}, y_{max}) and (x_{max}, y_{min}) then for an N by N array:

$$Na = x_{max} - x_{min} \quad \text{and} \quad Nb = y_{max} - y_{min}.$$

It is assumed that the array can be treated as though it was perfectly rectangular, equally

spaced and that the pixels all have the same pupil function. These conditions may be relaxed a good deal for a regime where the centre of the image is always brought back to the centre of the array but it is not intended to give a comparison of measurement strategies here.

4 POTENTIAL ACCURACY OF LOCATION OF THE LIVE IMAGE

In order to get algebraic estimates of x_0 and y_0 from a linear least squares fitting technique it is necessary to assume that initial estimates $\bar{A}_0$, $\bar{r}_0$, $\bar{a}$ and $\bar{b}$ of A_0 , r_0 , a and b are known and linearise equation (2) to give:

$$P_{ij} = f(\bar{r}_{ij}) + \left(\frac{\partial f}{\partial x_0}\right) \Delta x_0 + \left(\frac{\partial f}{\partial y_0}\right) \Delta y_0 \quad (3)$$

where

$$\begin{aligned} \bar{r}_{ij} &= \sqrt{(ai - (\bar{x}_0 - x_{min}))^2 + (bj - (\bar{y}_0 - y_{min}))^2} \\ x_0 &= \bar{x}_0 + \Delta x_0 \quad \text{and} \quad y_0 = \bar{y}_0 + \Delta y_0 \end{aligned} \quad (4)$$

The sum of squares of the differences between the pixel values predicted by equation (3) and the measured values may be minimised with respect to Δx_0 and Δy_0 to give the best fit values of $\langle \Delta x_0 \rangle$ and $\langle \Delta y_0 \rangle$ as follows:

$$\langle \Delta x_0 \rangle = \frac{\sum f_y^2 \sum (f_{ij}(P_{ij} - f)) - \sum (f_x f_y) \sum (f_y (P_{ij} - f))}{\sum f_x^2 \sum f_y^2 - (\sum (f_x f_y))^2} \quad (5)$$

$$\langle \Delta y_0 \rangle = \frac{\sum f_x^2 \sum (f_{ij}(P_{ij} - f)) - \sum (f_x f_y) \sum (f_x (P_{ij} - f))}{\sum f_x^2 \sum f_y^2 - (\sum (f_x f_y))^2} \quad (6)$$

$$\text{where} \quad f_x = \left(\frac{\partial f}{\partial x_0}\right), \quad f_y = \left(\frac{\partial f}{\partial y_0}\right) \quad \text{and} \quad f = f(\bar{r}_{ij})$$

Similarly if it is assumed that the pixel values all have the same variance σ_p^2 the expected variances of the estimates of $\langle \Delta x_0 \rangle$ and $\langle \Delta y_0 \rangle$ may be written as:

$$\sigma^2(\langle \Delta x_0 \rangle) = \sigma_p^2 \frac{\sum f_y^2}{\sum f_x^2 \sum f_y^2 - (\sum (f_x f_y))^2} \quad (7)$$

$$\sigma^2(\langle \Delta y_0 \rangle) = \sigma_p^2 \frac{\sum f_x^2}{\sum f_x^2 \sum f_y^2 - (\sum (f_x f_y))^2} \quad (8)$$

From equations (4) defining x_0 in terms of the constant $\bar{x}_0$ and the correction Δx_0 the variance $\sigma^2(x_0)$ is equal to the variance of $\langle \Delta x_0 \rangle$ and similarly for y_0 . Thus if the image is circularly symmetric and it has been brought to the centre of the pixel array the mixed sum $\sum f_x f_y$ vanishes and (7) and (8) simplify to:

$$\sigma^2(\langle \Delta x_0 \rangle) = \frac{\sigma_p^2}{\sum f_x^2} \quad (9)$$

$$\sigma^2(\langle \Delta y_0 \rangle) = \frac{\sigma_p^2}{\sum f_y^2} \quad (10)$$

These equations show that the standard error, σ , of the estimates of the image co-ordinates are inversely proportional to the root mean square gradient of the image function f . A proof of this result is given in appendix (1) which follows Ackermann (1984) where examples of its use in air survey work are discussed.

In the particular case of the axicon image:

$$\begin{aligned}
 f_x &= \left(\frac{\partial A_0 (J_0(r_{ij}/r_0))^2}{\partial x_0} \right) \\
 &= \frac{2A_0 J_0 J_1}{r_0 r_{ij}} (a_i - x_0 + x_{min})
 \end{aligned} \tag{11}$$

In the light of equations (9) and (10) it is interesting to plot f_x^2 as a function of position in the image plane, as shown in figure (2), to highlight the portions of the image which contribute to the accuracy of $\langle x_0 \rangle$.

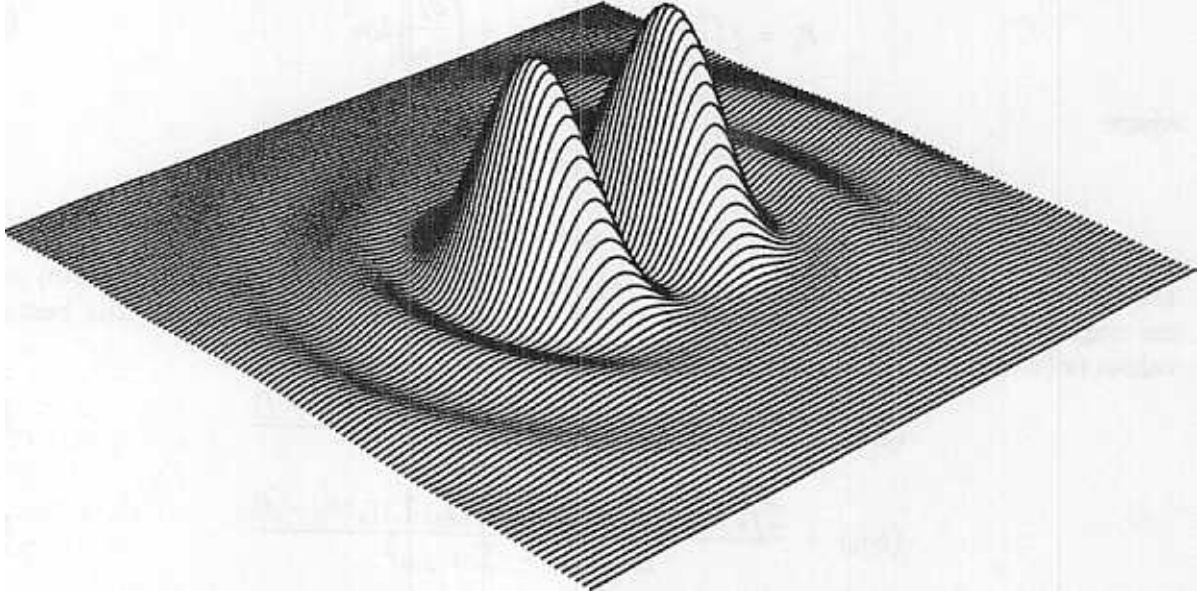


Figure 2 : A plot of f_x^2 as a function of position in the image plane.

Figure (2) shows it is only necessary to window the main peak of the aerial image to get most of the information that contributes to the accurate determination of $\langle x_0 \rangle$.

In some practical cases A_0 , r_0 , and b/a may not be known but they can be fitted from the array of pixel values. As x_{min} and y_{min} just offset the origin of co-ordinates they will be set to zero in the following analysis.

If $A = GA_0$, $R_0 = r_0/a$, $X_0 = x_0/a$, $Y_0 = y_0/b$, $\lambda = b/a$ and some constant, C , is used to represent the effects of background illumination; equation (2) becomes:

$$P_{ij} = GI_{ij} = f(R_{ij}) = A(J_0(R_{ij}/R_0))^2 + C \tag{12}$$

where $R_{ij} = \sqrt{(i - X_0)^2 + \lambda^2(j - Y_0)^2}$

and equation (4) becomes:

$$\begin{aligned}
 P_{ij} &= f(\overline{R_{ij}}) + \left(\frac{\partial f}{\partial X_0} \right) \Delta X_0 + \left(\frac{\partial f}{\partial Y_0} \right) \Delta Y_0 + \left(\frac{\partial f}{\partial A} \right) \Delta A \\
 &\quad + \left(\frac{\partial f}{\partial R_0} \right) \Delta R_0 + \left(\frac{\partial f}{\partial \lambda} \right) \Delta \lambda + \Delta C
 \end{aligned}$$

where

$$\left(\frac{\partial f}{\partial X_0} \right) = 2AJ_0J_1 \frac{(i - X_0)}{R_{ij}R_0}$$

$$\left(\frac{\partial f}{\partial Y_0}\right) = 2AJ_0J_1 \frac{(j - Y_0)}{R_{ij}R_0}$$

$$\left(\frac{\partial f}{\partial A}\right) = J_0^2$$

$$\left(\frac{\partial f}{\partial R_0}\right) = 2AJ_0J_1 \frac{R_{ij}}{R_0^2}$$

$$\left(\frac{\partial f}{\partial \lambda}\right) = -2AJ_0J_1\lambda \frac{(j - Y_0)}{R_{ij}R_0}$$

This six parameter fit should be capable of describing an aerial axicon image of arbitrary intensity A and arbitrary scale R_0 imaged on a CCD array of arbitrary aspect ratio λ . Unfortunately these equations have only been solved numerically, therefore closed formulae like (7) and (8) have not been derived for the expected variances of the estimated parameters.

5 ESTIMATION OF THE CENTRE OF A LIVE AXICON IMAGE.

In order to investigate the properties of the aerial image two examples were recorded using a microscope objective ($\times 60$) plus an extension tube fitted directly onto a Pulnix CCD camera. This camera was linked via a pixel clocked 512×512 frame store with 256 grey levels, supplied by Frame Store Technology Ltd., to an Acorn Archimedes computer. The coherent point source of illumination used for the experiments was the demagnified image of an unexpanded HeNe laser beam reflected by a 10mm diameter bearing ball. A typical image is shown in figure (3).

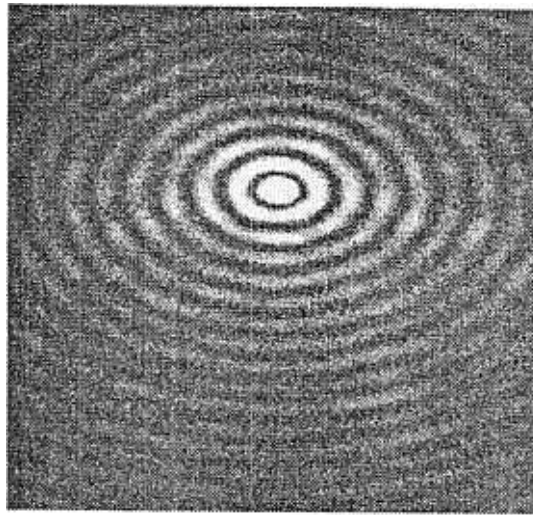


Figure 3 : A live image from the NPL camera as captured on the frame store. It may be noted that the grainy appearance of the image and the high lights in some of the rings are coherence effects due to the use of laser light.

The first problem in any nonlinear least squares fitting is to find good initial values for the nonlinear variables. With the aerial image from the NPL camera this is not very difficult as the image is dominated by the central peak as can be seen from its theoretical form in figure (1). A simple and accurate way of finding the centre of this peak, which is significantly more intense than the background noise, is to compute the co-ordinates of its

centre of gravity x_{CG} and y_{CG} given by:

$$x_{CG} = x_{min} + \frac{\sum iaP_{ij}}{\sum P_{ij}} \quad \text{and} \quad y_{CG} = y_{min} + \frac{\sum jbP_{ij}}{\sum P_{ij}} \quad (19)$$

Unfortunately if these formulae are used directly they tend to be dominated by the background in regions of the frame remote from the peak, for this reason it is usual to exclude any pixel from the sum which has a value below some fraction α of the maximum pixel value in the frame. Table 1 gives the results for the computed centres of gravity of the image shown in figure (3) for different values of α .

It is seen from the table that if the image is thresholded with α is greater than 0.25 only the central region remains and the centre of gravity of the image after thresholding gives a good estimate of the image centre.

α	x_{CG}/a	y_{CG}/b
0.1		
0.2		
0.3		
0.4		
0.5		
0.6		
0.7		
0.8		
0.9		

Table 1 : Computed centres of gravity of axicon images.

The positions of the centres of gravity computed for α between 0.3 and 0.7 are all visually indistinguishable from the apparent image centre.

The centre of gravity results for $\alpha = 0.8$ and $\alpha = 0.9$ were used as starting values for a nonlinear least squares fitting based on Householders algorithm for the solution of the linearised problem. In both cases, for this particular captured frame, the results were:

$$x_{LS} = 254.1122 a \quad \text{and} \quad y_{LS} = 323.1178 b.$$

This repeatability from somewhat different starting values merely illustrates that the least squares program was working and is not intended to imply that the true image centres could be found to a tenthousandth of a pixel! The true accuracy of the estimates depends on the variance of the individual pixel values and the accuracy with which they are recorded in the frame store.

With a pixel clocked frame store like the one used, Raynor and Seitz (1990) find that the centre of an image may usefully be estimated to one hundredth of a pixel using the information on a single digitized frame. It is seen from table (1) that at the tenth-of-a-pixel level the fast centre of gravity algorithm is very good for $\alpha \approx 0.5$.

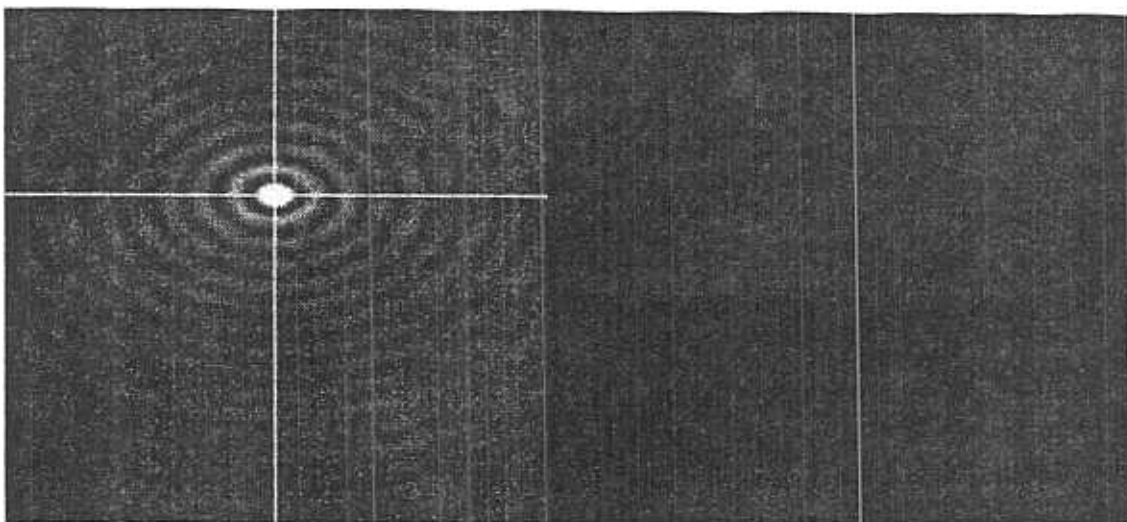


Figure 4 : The left hand frame shows the same image as in figure (3) but printed with about half the exposure and having superimposed on it a cross indicating the position of the centre of gravity corresponding to $\alpha = 0.9$. The right hand frame, which is very nearly black, shows the modulus of the difference between the captured image and the least squares fit using equation (12). The significance of this result will be seen later by comparison with figures (8) and (9).

It is interesting to note that the excellent fit between the captured image and equation (12) is only possible if the exposure is sufficiently low that the elements of the CCD array are behaving linearly. The small residuals almost invisible in the dark half of figure (4) are due to speckle generated by the laser illumination.

It is seen that the effect of the photography is to emphasise the variation in regions where PS happens to fall on the linear part of the probability curve in figure (4) and reduce the variation to almost zero for very high and very low intensities. The main effect of this is to make the ring structure of the image much more visible and it might be hoped that this emphasis on the information in the outer rings might more than compensate for the spurious noise necessarily introduced by the photographic process. This assertion may be investigated in a more systematic way by noting that in the photographic case equations (9) and (10) become:

$$\sigma^2(\langle \Delta x_0 \rangle) = \frac{\sigma_p^2}{\sum F_x^2} \quad (29)$$

$$\sigma^2(\langle \Delta y_0 \rangle) = \frac{\sigma_p^2}{\sum F_y^2} \quad (30)$$

Thus the regions contributing to an accurate centring of the target image may be illuminated by plotting F_x^2 as a function of position in the image plane.

6 A MODEL OF THE PHOTOGRAPHIC IMAGE.

Unfortunately most of the images to be investigated are photographic images and the simple Bessel function form of equation (1) has to be modified to allow for the response of the photographic emulsion.

There are several empirical descriptions of the photographic process all of which describe the low sensitivity at low exposures and the saturation of the emulsion at very high exposures with a relatively linear region in between. A simple model of this behaviour is given by assuming that at least n photons have to fall on a given grain of the emulsion before it is bound to be blackened when the plate is developed.

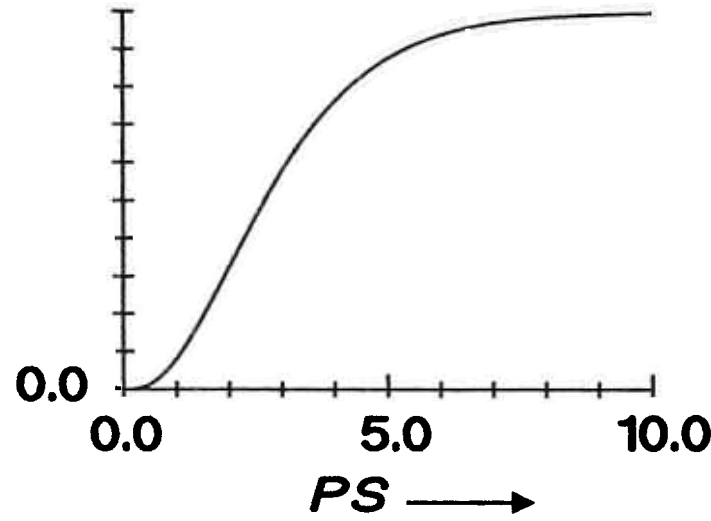


Figure 4 : A plot of $P_{n > 3}$ as a function of PS .

In the present work the observed image seems well modelled by taking a value of $n = 3$. If the area of a grain is S and the probability per unit area of a photon falling on the emulsion is P then the probability $P_{n > 3}$ of at least 3 photons falling on a grain is shown in appendix (2) given by:

$$P_{n > 3} = 1 - e^{-PS} \left(1 + PS + \frac{(PS)^2}{2} \right) \quad (20)$$

where P is the product of the intensity I and the exposure time T giving

$$P = IT.$$

If the grains are so small that they are not resolved by a linear video input device then the effective darkening of the plate will be proportional to $P_{n > 3}$. Let us say that the coefficient of proportionality is G then the optical density ρ is given by:

$$\rho = G \left[1 - e^{-IST} \left(1 + IST + \frac{(IST)^2}{2} \right) \right] \quad (21)$$

where from (12)

$$I = A_0 (J_0(R/R_0)^2 + C/G) \quad (22)$$

This is a fairly crude model of the photographic process, for a detailed discussion see Mees (1954) , but it has many of the qualitative features of more detailed models and its simple analytic form make it particularly useful for the present purpose. The equations may be slightly simplified by grouping together constants which always occur together. Let $STA_0 = g$ and $STC / G = c$ then (21) and (22) become:

$$\rho = G \left[1 - e^{-J} \left(1 + J + \frac{J^2}{2} \right) \right]$$

where

$$J = g (J_0 (R / R_0)^2 + c$$

The video camera sees the light transmitted through the photographic plate hence the pixel values P_{ij} will be given by an equation of the form:

$$P_{ij} = L (1 - \rho_{ij})$$

where ρ_{ij} is the value of ρ corresponding to R_{ij} . Therefore from (23), (24) and (25) P_{ij} may be written as:

$$P_{ij} = L - m \left[1 - e^{-J_{ij}} \left(1 + J_{ij} + \frac{J_{ij}^2}{2} \right) \right]$$

where $m = LG$ and

$$J_{ij} = g (J_0 (R_{ij} / R_0)^2 + c$$

or in the simplest possible form

$$P_{ij} = F (R_{ij})$$

then the results (5), (6), (7), (8), (9) and (10) may be carried over to this case of the photographic image by substituting $F (R_{ij})$ for $f (R_{ij})$. The form $F (R_{ij})$ of this video image of the photographic image is shown in figure (5).

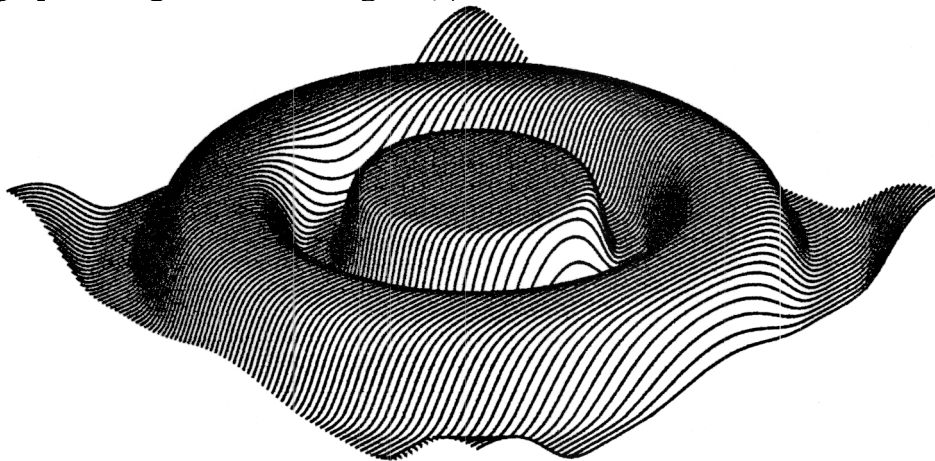


Figure 5 : the *inverse* video image of a photograph of an axicon image i.e. the actual video image is dark at the centre. In fact the inverse image is often used for target location as it lends itself to the standard centre of gravity algorithm.

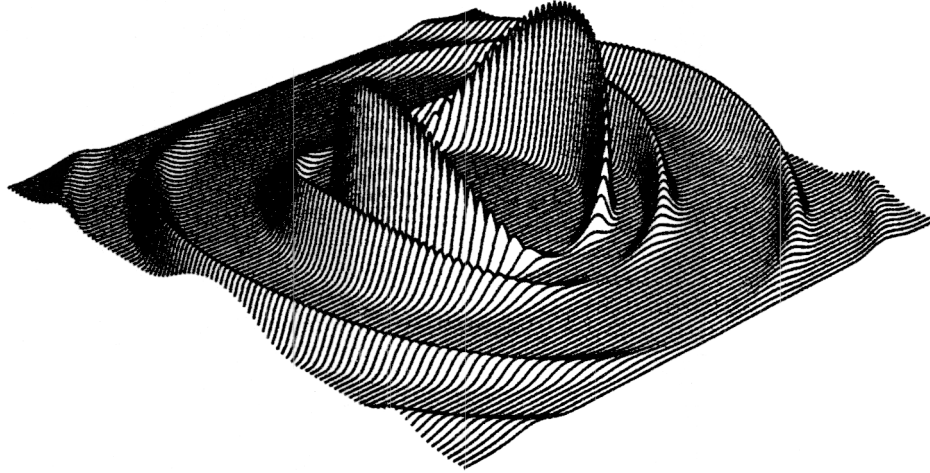


Figure 6 : F_x^2 as a function of position in the image plane.

It is interesting to note that, using this model of the photographic process, although the outer rings of the image are enhanced the gradients of the sides of the central peak are also enhanced and the major contribution to the accuracy of the centre is still from the region adjacent to the central peak. As the contrast of the photographic stage is increased so the contributions to F_x^2 become very large but from very small bands of the image plane.

In the limit of a binary image with no intermediate grey levels between black and white the contributions to F_x^2 become delta functions from the lines of transition between black and white. In this case the image has forgotten the relative slopes of the transitions and all edges contribute equally. An improved model which goes over more smoothly to this behaviour is suggested in the section 8.

In fact the negative image defined by $N_{ij} = 255 - P_{ij}$ was used in the least squares program. From (26) N_{ij} is given by:

$$N_{ij} = 255 - P_{ij} = m \left[1 - e^{-J_{ij}} \left(1 + J_{ij} + \frac{J_{ij}^2}{2} \right) \right] + k \quad (31)$$

where $k = 255 - L$.

7 ESTIMATION OF THE CENTRE OF A PHOTOGRAPHIC AXICON IMAGE.

In order to capture an experimental example of a photographic axicon image on video, apparatus was set up as described for the live axicon image in section 5 but with the laser source, the bearing ball and the NPL centrax camera replaced by a photographic plate, taken by the camera for a photogrammetric survey, and a diffuse illuminator. In theory these images taken and viewed in white light should, for the first few rings at least, be less noisy than the corresponding speckled images captured live with laser light, after that the spectral spread of the source will obliterate the rings. In fact the first three or four rings could be seen in the best images but they were very noisy. Due perhaps to antiquity of the original plates, or the partial coherence of the illumination, the plates had many large defects in the form of dark spots and scratches which in many cases were more visible than target images. One of the better images is shown in figure (7).

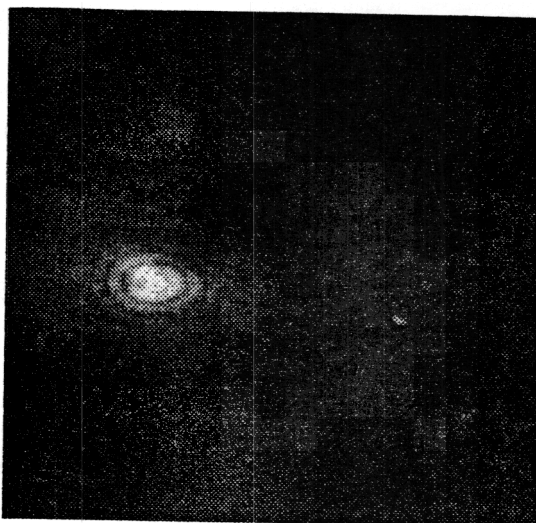


Figure 7 : The inverse video image of one of the better photographs of an axicon image from a photogrammetric plate. It should be noted that to get this illustration the raw pixel values have been subtracted from 225 and the first 90 grey levels of the resulting image set to black.

It is seen from figure (7) that the photographic images can indeed be made clearly visible. There are two major problems with this, (i) the settings necessary to make the target image clearly visible vary from image to image on the plate and (ii) the target images must be distinguished from other high contrast imperfections in the plate at an early stage to avoid wasting computing power on irrelevant objects. These two requirements put the general automatic image finding task beyond the capability of the resources allocated to this project using conventional computing techniques.

There are two solutions of this difficulty; either a novel and highly parallel computing technique could be developed or we could settle for a hybrid technique using the computer to assist a human operator. In the current situation it was necessary to follow the second course of action. An alternative approach is discussed in section (8).

In the light of the above discussion it is assumed that a human operator makes the critical decision to accept any candidate object as a useful target image. Once a target image has been selected the centre of gravity may be found as before using only pixels with values above 90% of the maximum pixel value in the frame. This centre was not so close to the apparent centre of the image as in the case of the aerial image.

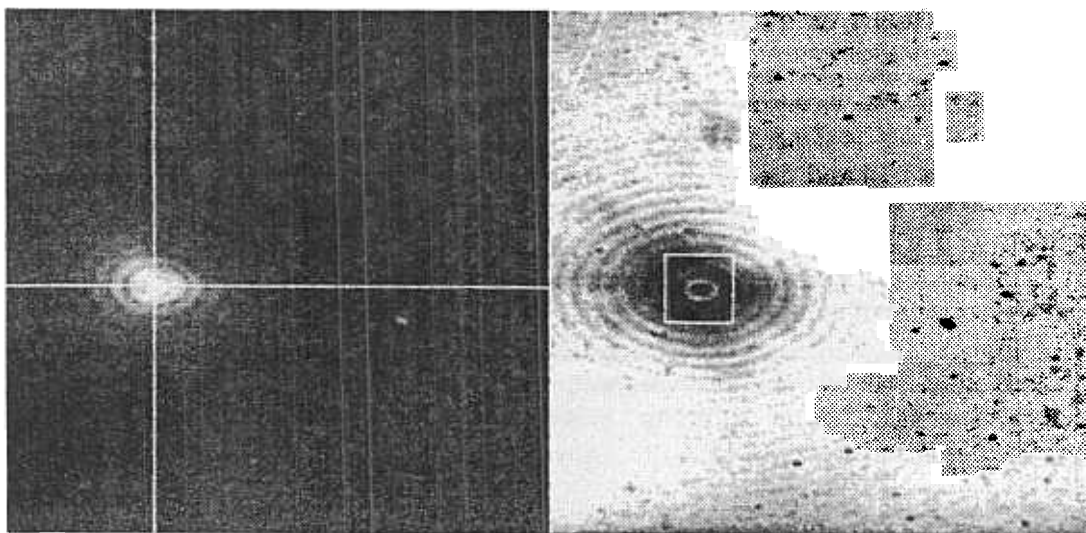


Figure 8 : Centre of gravity and least squares fit to the image shown in figure (7). The left hand frame shows the centre of gravity of the pixels with values greater than 90% of the maximum pixel value which is seen to be shifted to the left by bright glints near the centre of the image. The right hand frame shows the magnitudes of the residuals at each pixel position. Only the pixels within the square were used in the least squares calculation.

As with the previous work on the aerial image a window was set up bounded by the lines $x_0 + 31$, $x_0 - 32$, $y_{CG} + 31$ and $y_{CG} - 32$, as shown in figure (8). This was necessary to reduce the number of equations to $64 \times 64 = 4096$ which was the limit that could be easily handled on the Archimedes. It is seen from the right hand side of figure (8) that the residuals are quite low (dark area) within the window near the centre of gravity but become much larger (white area) when the fit is extrapolated beyond the window to the rest of the frame. This is in marked contrast to the results on the aerial image where the fit within an identical window gave a very good fit over the entire frame verifying that the assumed form of the image was indeed correct.

The most obvious strategy to try to overcome this problem is to increase the area of the frame used in the least squares fit. The result of this strategy is shown in figure (9), where the area considered has been increased by a factor of four but only a quarter of the pixels are used in order to keep the number of equations down to 4096. In this case the outer part of the observed ring structure is better fitted at the expense of the central region and the generation of large numbers of spurious outer rings.

In the fitting k was very nearly confounded with m and k was given a fixed value to avoid a spurious singularity in the least squares equations. This confounding was so nearly exact that large variations in the value taken for k had no visible effect on the fit. In fact a separate series of numerical experiments was done using exact data rather than real recorded images to establish that m and k were independent variables. Remarkably the key results for x_0 , y_0 and r_0 were virtually independent of the change in size of the region of pixels used in the fit and the value taken for k as illustrated in table (2).

It is interesting to note that although the least squares fit is unable to account for the picture outside the fitted window, indicating that it is not using a true description of the form of the image, it does give a centre much closer to the observed centre of symmetry than the centre of gravity of the pixels greater than 90% of the maximum pixel value. This is because the least squares result for x_0 and y_0 is much less prone to errors due to noise than the centre of gravity.

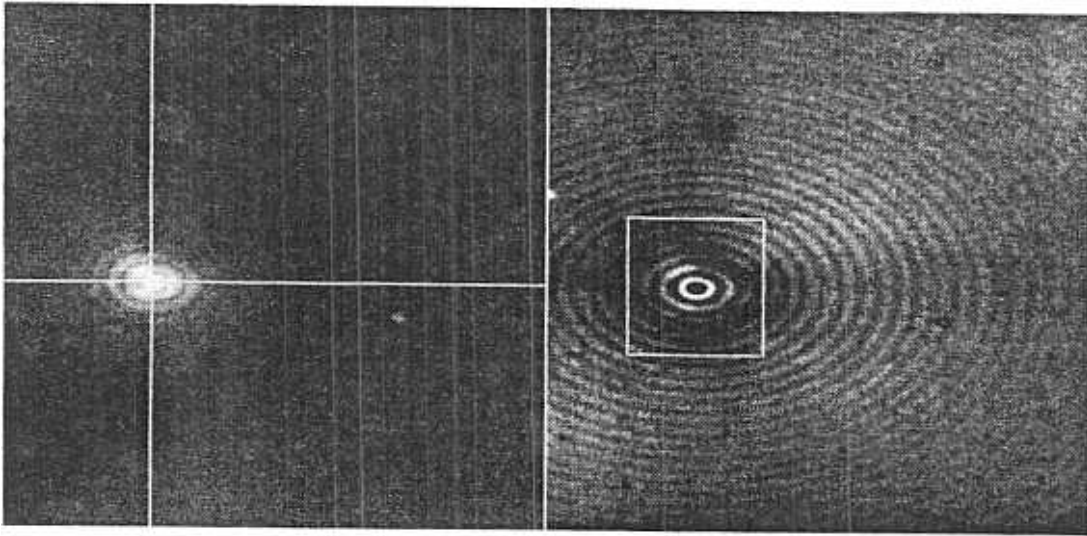


Figure 9 : The left hand frame of this figure is identical to the left hand frame of figure (8). The right hand frame shows the magnitudes of the residuals at each pixel position. Only the pixels within the square were used in the least squares calculation.

	x_0	y_0	r_0	g	c	m	k	$k+m$
small pixel area	140.976	242.327	5.588	9.353	4.467	171.401	0	171.401
	140.973	242.327	5.587	9.830	3.922	121.310	50	171.310
	140.972	242.326	5.587	10.639	3.243	81.163	90	171.163
large pixel area	140.731	242.206	5.599	9.350	3.497	179.656	0	179.656
	140.730	242.210	5.598	10.272	2.911	129.205	50	179.205
	140.729	242.222	5.597	12.195	2.141	88.365	90	178.365

Table 2 : results of the least squares fitting to the photographic images. k is kept fixed during each fit but its value as it is very nearly confounded with m . k values greater than about 125 cause the routine to fail. In all cases the centre of gravity was $x_{CG} = 138.643$ and $y_{CG} = 243.153$.

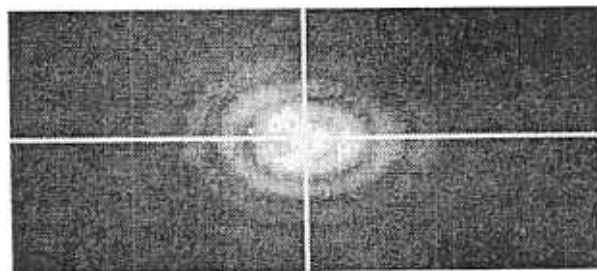


Figure 10 : the central portion of the image showing the asymmetry of the centre of gravity.

8 CONCLUSIONS.

The excellent fit for the aerial image shows that the CCD is reasonably linear and the assumed form of the image is correct. Indeed such an image could be located very accurately by this technique. Also with a live image the various sources of deviation from the ideal image might well be randomised or calibrated out.

Unfortunately in the case of the photographic image, which was the main target of the work, none of these useful properties is present and the images are difficult to distinguish using the computer program alone without human intervention.

The best solution to the problem of recognising target images in the presence of high contrast nonrandom noise is to use a spacial light modulator to perform a very fast, highly parallel, optical correlation with a numerically generated ideal image. It would appear that the 'signature' of a true target image is quite unique, no scratch or dust particle seen on a real plate could be confused with a target image with its characteristic ring structure. This suggests that excellent results should be obtained from this technique. A system of this kind has been demonstrated for another purpose by Smectic Technology 1993.

It is obvious from this study that the model of the process given by equation (31) is not sufficient to represent the photographic image as observed on the photogrammetric plates. There are several possible reasons for this:

- a) The photogrammetric images were taken in white light.
- b) A crude local model for the photographic process was used.
- c) The finite numerical aperture of the microscope + camera system was ignored

It was noticed during the work that spurious features on the plate, such as scratches and dust particles, were imaged with apparently high acuity. This suggests that b) and c) do not seriously degrade the image at this magnification. It appears therefore that a) is the main cause of the differences between the model and the photographic image.

To get a more accurate model of the white light image it would be necessary to start by convoluting the intensity given by equation (1), scaled for each wavelength, with the presumed spectral distribution of the white light source before applying any corrections for the photographic process itself. Even if this spectral distribution were known it is unlikely that the resulting formula would be sufficiently simple in form to be useful in a least squares fitting routine. It would also be interesting to retake the photogrammetric photographs using monochromatic light to test the modelling of the photographic process but this has been left to a future study.

It was noted in the discussion of figure (6) that as the contrast of the photographic process increased the main contribution to $\sum F_x^2$ and hence to the centring of the target image, is concentrated near the first ring of that image where the gradient of the pixel values is greatest. If the numerical aperture of the camera plus microscope system, which transfers the photographic image onto the CCD array, is taken into account it limits the maximum height of the peaks in figure (6). Once this maximum height is reached further increase in the contrast shifts the main contribution to $\sum F_x^2$ out to higher and higher order rings. Taking a more realistic model of the photographic process which gives it a finite spacial resolution

has a similar effect. This slightly more detailed model goes over relatively smoothly to the pure black and white case of infinite contrast where only the line length of the black to white transition contributes to the accuracy of the estimated values of the co-ordinates of the target image centre.

It is tempting to compute the uncertainty in the estimated values of x_0 and y_0 using formulae like equations (29) and (30) which assume that the actual pixel values differ from the ideal values only by some uncorrelated noise. Observation of the images shows that this is not true, as there are many patterns in areas remote from the target images on a scale of five or more pixels. Thus adjacent pixels are, in fact, highly correlated. This spacially correlated noise is different for each target image on the plate. It is very likely that using the formulae for uncorrelated noise underestimates the uncertainty.

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10 APPENDIX (1) : a proof of the mean square gradient theorem.

Let us consider a case where the pixel values P_{ij} are given by some function f of the co-ordinates relative to some image centre at x_0 and y_0 then P_{ij} may be written in the form:

$$P_{ij} = f(x_i - x_0, y_j - y_0) \quad A1$$

Expanding A1 in a Taylor series we obtain to first order:

$$P_{ij} = f(x_i - \bar{x}_0, y_j - \bar{y}_0) + f_x(x_i - \bar{x}_0, y_j - \bar{y}_0) \Delta x_0 + f_y(x_i - \bar{x}_0, y_j - \bar{y}_0) \Delta y_0 \quad A2$$

$$\text{where } x_0 = \bar{x}_0 + \Delta x_0, y_0 = \bar{y}_0 + \Delta y_0, f_x = \left(\frac{\partial f}{\partial x_0} \right) \text{ and } f_y = \left(\frac{\partial f}{\partial y_0} \right).$$

In the rest of this appendix the arguments of f etc. will be reduced to the form f_{ij} , so A2 becomes:

$$P_{ij} - f_{ij} = f_{xij} \Delta x_0 + f_{yij} \Delta y_0 \quad A3$$

The set of equations, corresponding to different values of i and j , represented by A3 may be solved by a least squares method to give estimates of Δx and Δy . These values of Δx_0 and Δy_0 are then used to improve the estimates $\bar{x}_0$ and $\bar{y}_0$ of x_0 and y_0 and the process is repeated until the correction is found to be negligible.

The simplest way to write down the least squares solution to A3 is to use the standard form:

$$y = Ax \quad \text{and} \quad D(y) = \sigma^2 I \quad A4$$

Where the first part of A4 represents the linearised equations to be solved and the second part represents the statistical properties of the measured values, in particular A4 asserts that the P_{ij} are statistically independent and of equal accuracy. $D(y)$ is defined as a matrix with the variances of the elements of y on its diagonal and the corresponding covariances elsewhere.

The formal least squares solution to A4 is:

$$\langle x \rangle = (A^T A)^{-1} A^T y \quad A5$$

and

$$D(\langle x \rangle) = (A^T A)^{-1} \sigma^2 \quad A6$$

where from A3 and A4 if i and j run from 1 to n :

$$\mathbf{x} = \begin{pmatrix} \Delta x \\ \Delta y \end{pmatrix}, \quad \mathbf{y} = \begin{pmatrix} P_{11} - f_{11} \\ \vdots \\ P_{nn} - f_{nn} \end{pmatrix} \quad \text{and} \quad \mathbf{A} = \begin{pmatrix} f_{x11} & f_{y11} \\ \vdots & \vdots \\ f_{xnn} & f_{ynn} \end{pmatrix} \quad A7$$

Using these associations we have:

$$(\mathbf{A}^T \mathbf{A}) = \begin{pmatrix} \sum f_x^2 & \sum (f_x f_y) \\ \sum (f_x f_y) & \sum f_y^2 \end{pmatrix}$$

hence

$$(\mathbf{A}^T \mathbf{A})^{-1} = \frac{1}{\sum f_x^2 \sum f_y^2 - (\sum (f_x f_y))^2} \begin{pmatrix} \sum f_y^2 & -\sum (f_x f_y) \\ -\sum (f_x f_y) & \sum f_x^2 \end{pmatrix}$$

Armed with this result the explicit solution follows directly from A5 and A6:

$$\langle \Delta x_0 \rangle = \frac{\sum f_y^2 \sum (f_{ij} (P_{ij} - f)) - \sum (f_x f_y) \sum (f_y (P_{ij} - f))}{\sum f_x^2 \sum f_y^2 - (\sum (f_x f_y))^2} \quad A8$$

$$\langle \Delta y_0 \rangle = \frac{\sum f_x^2 \sum (f_{ij} (P_{ij} - f)) - \sum (f_x f_y) \sum (f_x (P_{ij} - f))}{\sum f_x^2 \sum f_y^2 - (\sum (f_x f_y))^2} \quad A9$$

$$\sigma^2(\langle \Delta x_0 \rangle) = \sigma^2 \frac{\sum f_y^2}{\sum f_x^2 \sum f_y^2 - (\sum (f_x f_y))^2} \quad A10$$

$$\sigma^2(\langle \Delta y_0 \rangle) = \sigma^2 \frac{\sum f_x^2}{\sum f_x^2 \sum f_y^2 - (\sum (f_x f_y))^2}$$

As $\bar{x}_0$ and $\bar{y}_0$ are constant during any given step of the calculation, including the last step, A10 and A11 may be rewritten as:

$$\sigma^2(\langle x_0 \rangle) = \sigma^2 \frac{\sum f_y^2}{\sum f_x^2 \sum f_y^2 - (\sum (f_x f_y))^2} \quad A12$$

$$\sigma^2(\langle y_0 \rangle) = \sigma^2 \frac{\sum f_x^2}{\sum f_x^2 \sum f_y^2 - (\sum (f_x f_y))^2}$$

Further if f is symmetrical is either x or y and the effects of the edges of the image window may be neglected or cancel by symmetry then the mixed sum $\sum (f_x f_y)$ is approximately zero. If the centre of symmetry of the image was also symmetrically placed with respect to the projected pixel positions then the zero would be exact but this is unlikely for captured images. In short if $\sum (f_x f_y)$ may be neglected A12 and A13 reduce to the mean square gradient theorem:

$$\sigma^2(\langle x_0 \rangle) = \frac{\sigma^2}{\sum f_x^2} \quad \text{and} \quad \sigma^2(\langle y_0 \rangle) = \frac{\sigma^2}{\sum f_y^2}$$

11 APPENDIX (2) : the probability of at least three.

The simple theory of the photographic process used here assumes that the light sensitive grains form a regular pavement on the emulsion surface, they are all of the same area S and they are perfectly opaque. It is also assumed that at least three photons must strike a grain before it is bound to blacken on development.

Let the probability per unit area of a photon striking a grain be P and let each grain be subdivided into N subgrains of area δS where δS is so small that the probability that more than one photon falls on it may be neglected. Then the probability of a photon falling on a subgrain is just $P\delta S$ and the probability that no photons fall on a subgrain is $(1 - P\delta S)$.

Hence the probability P_0 that no photons fall on an entire grain is given by:

$$\begin{aligned} P_0 &= (1 - P\delta S)^N \quad \text{or} \quad \ln P_0 = N \ln (1 - P\delta S) \\ &\quad \text{but} \quad \lim_{x \rightarrow 0} \{\ln(1 - x)\} = -x \\ \therefore \text{as } \delta S &\rightarrow 0 \text{ and } N \rightarrow \infty \text{ in such a way that } S = N\delta S \\ \ln P_0 &\rightarrow -PN\delta S \quad \text{or} \quad P_0 \rightarrow e^{-PS} \end{aligned}$$

Let P_1 be the probability of one photon striking a grain then:

$$\begin{aligned} P_1 &= NP\delta S (1 - P\delta S)^{N-1} \quad \text{or} \quad \ln P_1 = \ln PS + (N - 1) \ln (1 - P\delta S) \\ \therefore \text{as } \delta S &\rightarrow 0 \text{ and } N \rightarrow \infty \text{ in such a way that } S = N\delta S \\ \ln P_1 &\rightarrow \ln PS - PN\delta S \quad \text{or} \quad P_1 \rightarrow PS e^{-PS} \end{aligned}$$

Let P_2 be the probability of two photons striking a grain then:

$$\begin{aligned} P_2 &= \frac{N(N-1)}{2!} (P\delta S)^2 (1 - P\delta S)^{N-2} \\ \text{or} \quad \ln P_2 &= \ln \{N(N-1)(P\delta S)^2\} + (N-2) \ln (1 - P\delta S) - \ln 2 \\ \therefore \text{as } \delta S &\rightarrow 0 \text{ and } N \rightarrow \infty \text{ in such a way that } S = N\delta S \\ \ln P_2 &\rightarrow 2 \ln PS - PS - \ln 2 \quad \text{or} \quad P_2 = \frac{(PS)^2}{2} e^{-PS} \end{aligned}$$

Therefore the probability of at least three photons striking a grain $P_n > 3$ which is equal to $1 - P_0 - P_1 - P_2$ is given by:

$$P_{n > 3} = 1 - e^{-PS} \left\{ 1 - PS - \frac{(PS)^2}{2} \right\}$$

the first step, in the case of a continuous variable, is to transform the variable into a binary variable. This is done by choosing a threshold value, and then classifying the observations as "above" or "below" the threshold. In the case of a categorical variable, the first step is to assign numerical values to the categories. This is done by choosing a reference category, and then assigning values to the other categories relative to the reference category.

The second step is to calculate the mean of the transformed variable. This is done by summing the values of the transformed variable and dividing by the number of observations. The third step is to calculate the standard deviation of the transformed variable. This is done by summing the squared deviations from the mean and dividing by the number of observations. The fourth step is to calculate the correlation coefficient between the transformed variable and the original variable. This is done by dividing the covariance between the transformed variable and the original variable by the product of the standard deviations of the two variables.

The fifth step is to calculate the coefficient of determination, which is the square of the correlation coefficient. This coefficient represents the proportion of the variance in the original variable that is explained by the transformed variable.

$$R^2 = \frac{SSR}{SST} = \frac{SSR}{SSE + SSR} = \frac{SSR}{SST}$$

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The sixth step is to calculate the coefficient of determination, which is the square of the correlation coefficient. This coefficient represents the proportion of the variance in the original variable that is explained by the transformed variable.

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