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Constitutive models and their
data requirements for use
in Finite Element Analysis
of adhesives under impact loading

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SUMMARY

Commercial Finite Element Analysis (FEA) software packages are widely used for stress analysis of adhesive joints. This report is a survey of constitutive models implemented in various FEA packages which are potentially suitable for adhesives under impact loading. Emphasis is given to the elastic - plastic models which take into account the pressure dependence of the plastic deformation. These models should be suitable for simulating behaviour of most structural adhesives or adhesives whose operating temperatures are lower than their glass transition temperature.

Under impact, the stress/strain distributions in a bonded joint are such that the strain rate in the adhesive is not uniform. Therefore, material models that handle the strain rate dependence of properties are needed. Various extensions of elastic - plastic models to include rate dependent behaviour are described.

The determination of parameters that define the yield behaviour of a material that is sensitive to hydrostatic stress requires tests under at least two different states of stress. In this report, it is shown how data from tensile and shear tests can be analysed to give these parameters. In the tensile tests, measurements of lateral and longitudinal strain are needed to generate plots of true stress against plastic strain which characterise the hardening behaviour. These measurements also allow a value for the flow parameter to be derived.

Finally, alternative constitutive models are considered which might be more suitable for simulating the behaviour of rubbery adhesives or for adhesives tested at temperatures just above their glass transition temperatures. These are the linear viscoelastic and the nonlinear elastic models. The latter are based on large strain constitutive theories developed for rubbers. An option exists where these two models can be used together to describe non-linear, elastic, strain rate dependent behaviour.

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Approved on behalf of Managing Director, NPL, by Dr. M K Hossain,
Director, Centre for Materials Measurement & Technology

LIST OF SYMBOLS

c	cohesion (ANSYS and LUSAS)
C_{ij}	material parameters for hyperelastic model
d	cohesion (ABAQUS)
D_i	material parameters for hyperelastic model
E	elastic modulus
E_h	hardening modulus from a yield stress vs. effective plastic strain plot
$\frac{1}{G_i}$	relative shear modulus of term i in Prony series
G	limiting shear modulus at $t \rightarrow \infty$
G_0	instantaneous shear modulus
$G_R(t)$	shear relaxation modulus
I_1	first invariant of the stress tensor
I'_1	first invariant of the plastic strain tensor
$\bar{I}_1$	first invariant of deviatoric strain tensor for hyperelastic model
$\bar{I}_2$	second invariant of deviatoric strain tensor for hyperelastic model
J	volume ratio
J_2	second invariant of the deviatoric stress tensor
J'_2	second invariant of the deviatoric plastic strain tensor
p	pressure
q	von Mises equivalent stress
W	strain energy density function of hyperelastic material
β	friction angle (ABAQUS)
γ_y	shear plastic strain
$\bar{\epsilon}_p$	equivalent plastic strain
$\dot{\bar{\epsilon}}_p$	equivalent plastic strain rate
$\dot{\epsilon}_{yt}$	plastic strain rate
$\dot{\epsilon}_e$	elastic strain rate
$\dot{\epsilon}$	strain rate
$\dot{\bar{\epsilon}}_{p0}$	static or reference equivalent plastic strain rate
ϵ_1	axial strain in uniaxial tension
ϵ_{1e}	elastic component of ϵ_1
ϵ_{1p}	plastic component of ϵ_1
ϵ_2	lateral strain in uniaxial tension
ϵ_{2e}	elastic component of ϵ_2
ϵ_{2p}	plastic component of ϵ_2
ϵ_i	engineering principal strains
ϵ_{ij}	component of the strain tensor
ϵ_{yc}	compressive plastic strain
ϵ_{yt}	tensile plastic strain (same as ϵ_{1p})

ζ	ratio of the yield stress in compression to the yield stress in tension corresponding to the same equivalent plastic strain
$\bar{\lambda}_i$	deviatoric principal extension ratios
λ_i	principal extension ratios
ν_e	elastic Poisson's ratio
ν_p	plastic Poisson's ratio
ν_t	Poisson's ratio calculated from total strains
ν_t	Poisson's ratio calculated from total engineering strains
σ_i	principal stresses
σ_{ij}	component of the stress tensor
σ_{in}	stress at first yield
σ_n	hydrostatic stress ($=I_1/3$)
σ_{y0}	static or reference yield stress
σ_{yc}	yield stress in compression
σ_{yt}	yield stress in tension
τ_i	relaxation time of term i in Prony series
τ_y	yield stress in shear
ϕ	angle of internal friction (ANSYS and LUSAS)
ϕ_f	dilation angle (ANSYS)
ψ	dilation angle (ABAQUS)

CONTENTS

1 INTRODUCTION.....	1
2 ELASTIC - PLASTIC MODELS	1
3 VISCOPLASTIC MODELS	5
4 DETERMINATION OF MATERIAL PARAMETERS.....	6
4.1 ELASTIC PROPERTIES.....	6
4.2 HARDENING BEHAVIOUR	7
4.3 CALCULATION OF THE PARAMETER ζ	7
4.4 CALCULATION OF THE DILATION ANGLE.....	8
4.5 DETERMINATION OF STRAIN RATE DEPENDENCE OF YIELD	9
5 ALTERNATIVE CONSTITUTIVE MODELS.....	10
5.1 LINEAR VISCOELASTICITY.....	10
5.2 HYPERELASTICITY	10
5.3 COMBINATION OF HYPERELASTIC AND LINEAR VISCOELASTIC MATERIAL MODELS.....	10
6 CONCLUSIONS	11
REFERENCES	11
APPENDIX 1 EVALUATION OF THE PARAMETER ζ FROM TENSILE AND SHEAR TESTS.....	13
APPENDIX 2 EVALUATION OF v_p WHEN THE LATTER VARIES WITH STRAIN.....	15
APPENDIX 3 EVALUATION OF PLASTIC STRAIN RATE FOR A CONSTANT STRAIN RATE TEST	16
APPENDIX 4 LINEAR VISCOELASTICITY.....	18
APPENDIX 5 HYPERELASTICITY	19
APPENDIX 6 COMBINATION OF HYPERELASTIC AND LINEAR VISCOELASTIC MODELS.....	22

1 INTRODUCTION

Finite Element Analysis (FEA) is used extensively in stress analyses of adhesive joints. This is due to the complex stress/strain distributions that exist in such joints and the inability of analytical methods to cope with such complexities.

When using a commercial FEA package, an important step in the analysis is to define the material's constitutive behaviour. The theme for this report is to identify constitutive models suitable for adhesives. Emphasis will be given to those models suitable for impact loading.

Deformation under impact will, in most cases, lead to large strains where relationships between stress and strain are non-linear. The most commonly used material models for describing non-linear behaviour are elastic-plastic models. These have been developed for metals and soils and assume that the non-linear behaviour is due to plastic deformation. Elastic - plastic models have been widely used for adhesives as well. Unlike metals, the yield in polymeric adhesives is usually dependent on the hydrostatic component of stress, or pressure.

The strain rate under impact loading will vary with time and position in an adhesive joint. As adhesives are visco-elastic materials, a difference in the stress - strain curve will be noted for changes in the applied strain rate. This effect is most pronounced when the adhesive is operating at temperatures close to its glass transition temperature; changes in the stress - strain curves are then observed for even small changes in strain rates. Therefore, the dependence of the adhesive's flow behaviour on strain rate will also need to be considered.

The determination of the parameters that define the yield behaviour of a material sensitive to pressure, requires tests under two different states of stress. Usually the models are expressed in terms of tensile and compressive yield stresses. In this report, it is shown how the yield parameters can be calculated from tensile and shear tests as well as from tensile and compressive tests. In the tensile tests, measurements of lateral and longitudinal strain are needed to generate plots of true stress against true plastic strain. Such plots characterise the hardening behaviour.

Sections 2 and 3 of this report deal with plastic and viscoplastic constitutive models, respectively. Section 4 describes the experiments required to determine the various material parameters that appear in the models of Sections 2 and 3. Section 5 describes constitutive theories not covered in Sections 2 and 3 and which might prove useful for modeling certain adhesives. Finally section 6 is a brief summary of the main conclusions and recommendations.

2 ELASTIC - PLASTIC MODELS

In this Section, it is assumed that elastic - plastic models can be used to model adhesives. In the low strain, linear region, the theory of linear elasticity using Hook's law is valid. In the non-linear region, plastic deformation is assumed to take place. The stress analysis when plastic deformation is occurring is described below.

Rate independent plasticity is based on three fundamental concepts:

- i) the yield criterion; determines the stress states necessary for plastic deformation to occur
- ii) the hardening rule; describes how the material's resistance to further yield changes with increasing strain

iii) the flow rule; defines the incremental plastic strains as a function of the stresses.

For metals, the classic von Mises yield criterion is usually assumed. This states that yielding is dependent only on the second invariant of the deviatoric stress tensor. Thus:

$$\sigma_y = \sqrt{\frac{1}{2}[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]} = \sqrt{3J_2} \quad (1)$$

where σ_i are the principal stresses, σ_y is the tensile yield stress and J_2 is the second invariant of the deviatoric stress tensor. In general, the tensile yield stress is not constant, i.e. the material will strain harden. Plastic deformation is assumed to take place under constant volume conditions, i.e. the 'plastic' Poisson's ratio, ν_p , is equal to 0.5.

The independence of yield on hydrostatic stress implies that the flow behaviour under tension is the same as that under compression. However, studies on polymers reveal that the chain structure generally leads to higher yield stresses in compression than in tension. Hence, a deviation from the classical von Mises yield criterion is observed.

Two main forms of pressure dependent criteria appear in the literature related to the yield of adhesives^{1,2}. They are usually referred to as the Drucker-Prager and Raghava criteria and are given by equations (2) and (3) respectively:

$$\sigma_y = \frac{\sqrt{3}(\zeta + 1)}{2\zeta} \sqrt{J_2} + \frac{(\zeta - 1)}{2\zeta} I_1 \quad (2)$$

$$\zeta \sigma_y^2 = 3J_2 + (\zeta - 1)\sigma_y I_1 \quad (3)$$

where I_1 is the first invariant of the stress tensor and ζ is the ratio of the yield stress in compression to the yield stress in tension corresponding to the same equivalent plastic strain, $\bar{\epsilon}_p$. The equivalent plastic strain as well as the equivalent stress are scalar quantities which allow the analysis of situations where multiaxial loading is present. For uniaxial tension they reduce to the tensile plastic strain and tensile stress, respectively. Equation (2) has originated from deformation theories of soils³ and has been subsequently used for polymers as well⁴. Equation (3) was proposed specifically for polymers⁵. Both equations revert to equation (1), for $\zeta=1$. Experimental methods for determining ζ are discussed in Section 4.

Both equations (2) and (3) can be found in various FEA packages. For example, ABAQUS⁶ has a general form of the Drucker - Prager criterion, called the 'linear extended Drucker Prager' model, written as:

$$d = t - p \tan \beta \quad (4)$$

The parameters β and d are called the friction angle and the cohesion of the material, respectively. These will be related to ζ and σ_y shortly (see equations 7 and 8). The remaining terms of equation 4 are the pressure, p , given by $-I_1/3$, and t which is defined as:

$$t = \frac{1}{2}q \left[1 + \frac{1}{K} - \left(1 - \frac{1}{K} \right) \left(\frac{r}{q} \right)^3 \right] \quad (5)$$

where r is the third invariant of the deviatoric stress tensor and q is the von Mises equivalent stress which from equation (1), is equal to $\sqrt{3J_2}$. For $K=1$, t is equal to q and yielding is independent of the third deviatoric stress invariant. Under such an assumption equation (4) can be written as:

$$d = \sqrt{3J_2} + \tan \beta \frac{I_1}{3} \quad (6)$$

If both sides of equation (6) are multiplied by $\left(\frac{\zeta+1}{2\zeta} \right)$ and the result compared with equation (2), the following identities are derived:

$$\tan \beta = 3 \left(\frac{\zeta-1}{\zeta+1} \right) \quad (7)$$

$$d = \left(\frac{2\zeta}{\zeta+1} \right) \sigma_y \quad (8)$$

The data that need to be determined for this model so far are β and σ_y . The options of perfect plasticity (σ_y is constant) or isotropic hardening (σ_y is a function of tensile plastic strain) are available.

A further parameter that needs to be determined is the dilation angle, ψ . This is a parameter that appears in the flow rule and determines whether the flow rule is associated or non-associated. Associated flow implies that the plastic strain increments are normal to the yield surface and under such conditions $\psi = \beta$. The tangent of ψ is defined as:

$$\tan \psi = \frac{I_1'}{\frac{2}{\sqrt{3}} \sqrt{J_2'}} \quad (9)$$

where I_1' is the first invariant of the plastic strain tensor and J_2' is the second invariant of the deviatoric plastic strain tensor.

Under uniaxial tensile loading conditions, the invariants I_1' and J_2' are equal to:

$$I_1' = (1 - 2\nu_p) \epsilon_p, \quad J_2' = \frac{1}{3} (1 + \nu_p)^2 \epsilon_p^2 \quad (10)$$

where ϵ_{π} is the plastic strain in the loading direction and ν_p is the 'plastic' Poisson's ratio, i.e. the ratio of the transverse to the axial plastic strains. Substituting (10) in (9), the following relationship between ψ and ν_p is derived:

$$\tan\psi = \frac{3(1-2\nu_p)}{2(1+\nu_p)} \quad (11)$$

For $\nu_p=0.5$, i.e. yield at constant volume, $\psi=0$. When ψ is greater than zero, the material expands during plastic deformation and ν_p is smaller than 0.5. Section 4 describes an experimental method for determining ν_p and hence, ψ .

The Drucker - Prager yield criterion (equation 2) is implemented in other FEA packages in terms of different parameters. It is usually written as ^{7,8}:

$$\frac{6\sin(\phi)\sigma_n}{\sqrt{3}(3-\sin(\phi))} + \sqrt{J_2} - \frac{6c\cos(\phi)}{\sqrt{3}(3-\sin(\phi))} = 0 \quad (12)$$

where ϕ is again called the angle of internal friction, c is the cohesion and σ_n is the hydrostatic stress ($=I_1/3$). By rearranging and comparing equations (2) and (12), the following relationships between c , ϕ , σ_{π} and ζ are obtained:

$$\sin(\phi) = \frac{3(\zeta-1)}{3\zeta+1} \quad (13)$$

$$c = \frac{\zeta\sigma_{\pi}}{\sqrt{6\zeta-2}} \quad (14)$$

The Raghava yield criterion is implemented in LUSAS ⁷ in exactly the same form as equation (3). The parameters that need to be defined are the elastic properties and the initial values of the tensile and compressive yield stresses. Isotropic strain hardening is optional and the assumption of associated flow is made.

The Raghava criterion also exists as an option in ABAQUS. It is called the 'exponent Drucker Prager' model and it is written as:

$$aq^b - p = p_t \quad (15)$$

where a and b are material parameters and p_t is a hardening parameter. Assuming the special case of $b=2$, rearranging and comparing with equation (3), the following relationships are derived:

$$a = \frac{1}{3(\zeta-1)\sigma_{\pi}} \quad (16)$$

$$p_t = \frac{\zeta \sigma_y}{3(\zeta - 1)} \quad (17)$$

The data that need to be determined for the 'exponent' model are a , b , ψ and σ_y . The options of perfect plasticity (σ_y is constant) or isotropic hardening (σ_y is a function of ϵ_y) are available. The flow for this model is assumed to be non-associated.

A less commonly used pressure dependent yield criterion is the Mohr Coulomb. This has again originated from constitutive theories developed for soils³. It is available in LUSAS and it is given by:

$$(\sigma_1 - \sigma_3) + (\sigma_1 + \sigma_3) \sin \phi - 2c \cos \phi = 0 \quad (18)$$

where σ_1 and σ_3 are the maximum and minimum principal stresses respectively. In order to define c and ϕ in terms of ζ and σ_y , the loading conditions corresponding to tension ($\sigma_1 = \sigma_y$, $\sigma_2 = \sigma_3 = 0$) and compression ($\sigma_1 = \sigma_2 = 0$, $\sigma_3 = -\zeta \sigma_y$) are substituted in equation (18) to give two simultaneous equations that can be solved for c and ϕ . The following are obtained:

$$\sin \phi = \frac{\zeta - 1}{\zeta + 1}, \quad c = \frac{\sigma_y \sqrt{\zeta}}{2} \quad (19)$$

This model has similar data requirements to the Drucker Prager model of equation (12).

3 VISCOPLASTIC MODELS

The deformations in a typical joint loaded under impact are such that the strain rate is not uniform throughout the adhesive. Hence, if any of the constitutive theories described in Section 2 are to be used, options have to be available in the FEA software which will account for the extra variable, i.e. strain rate.

There are two ways of introducing a strain rate dependent yield. One is to separate the strain and strain rate effects and the other is to combine both dependencies into the material's parameters. Here, the material parameter that is assumed to vary with strain rate is σ_y .

The Cowper and Symonds model is a widely used strain rate dependent yield model which separates the strain rate from the strain effects^{6,9}. It assumes that the hardening curves corresponding to the various strain rates are 'similar' and hence can be defined by a single 'static' or 'reference' hardening curve and a multiplying factor. Thus:

$$\frac{\sigma_y}{\sigma_{y0}} = 1 + \left(\frac{\dot{\epsilon}}{\epsilon_p} \right)^{\frac{1}{p}} \quad (20)$$

where σ_{y0} is the 'static' or 'reference' tensile yield stress, σ_y and $\dot{\epsilon}_p$ are corresponding tensile yield stress and equivalent plastic strain rate, and C and P are material parameters that need to be determined.

Another example of separable strain and strain rate dependent yield model is the Eyring model. The strain rate dependence is accounted for by:

$$\frac{\sigma_y}{\sigma_{y0}} = 1 + b \ln \left(\frac{\dot{\epsilon}_p}{\dot{\epsilon}_{p0}} \right) \quad (21)$$

where $\dot{\epsilon}_{p0}$ is the equivalent plastic strain rate of the 'static test', i.e. when σ_y is equal to σ_{y0} and b is a material constant that needs to be determined.

An alternative to equations (20) and (21) is to define the strain rate dependence directly, in the form of numerical (σ_y/σ_{y0}) vs. $\dot{\epsilon}_p$ data.

A possible yield model where the strain and strain rate effects are not separable is obtained from a bi-linear representation⁹ of the stress - strain curve:

$$\sigma_y = \sigma_m \left(\frac{\dot{\epsilon}_p}{\dot{\epsilon}_p} \right) + E_h \left(\frac{\dot{\epsilon}_p}{\dot{\epsilon}_p} \right) \dot{\epsilon}_p \quad (22)$$

where σ_m is the stress at first yield and E_h is the hardening modulus from a yield stress vs. effective plastic strain plot. Both E_h and σ_m are functions of the effective plastic strain rate, $\dot{\epsilon}_p$. Therefore, there is no separation of the strain and strain rate effects. A more general model would be to include higher order terms of $\dot{\epsilon}_p$.

Another option for defining a non separable, strain and strain rate dependent yield, is to give complete hardening curves in the form of numerical stress - strain data for various strain rates. This option which is available in a number of FEA packages^{6,9} may lead to a large number of data that have to be entered in the input file.

4 DETERMINATION OF MATERIAL PARAMETERS

This Section describes experimental methods for determining the material parameters required in the constitutive models described in Sections 2 and 3.

4.1 ELASTIC PROPERTIES

The elastic parameters that are needed are the Young's modulus, E and the elastic Poisson's ratio, ν_e . The modulus may be determined from a uniaxial tensile test. The Poisson's ratio may be determined during the same test, by taking simultaneous measurements of the lateral

contraction of the specimen. An extensometer or any other device for measuring the displacement accurately is required for both measurements. Also note that if the shear modulus, G , is measured, ν_e can be calculated from $(E-2G)/2G$.

4.2 HARDENING BEHAVIOUR

The tensile stress at first yield and the complete tensile hardening curve are obtained from a tensile test. The hardening data are represented according to the limitations of the specific FEA software. For example, some packages allow only bi-linear representation, others allow for multi-point definitions. For many adhesives, the hardening curve will extend to strain values in excess of 10% and the associated reduction in the cross sectional area of the sample will be significant. In such cases the engineering stress, σ' , should be converted to true stress, σ , and the engineering strain, ϵ' , should be converted to true or logarithmic strain, ϵ , using:

$$\sigma = \frac{\sigma'}{(1 - \nu_e \epsilon')^2} \quad (23)$$

and

$$\epsilon = \ln(1 + \epsilon') \quad (24)$$

where ν_e is the ratio of the lateral to tensile total engineering strains obtained from a uniaxial tensile test.

4.3 CALCULATION OF THE PARAMETER ζ

The parameter ζ appears in all forms of pressure dependent criteria. It is defined as the ratio of the yield stress in compression, σ_{yc} , to the yield stress in tension, σ_{yt} , corresponding to the same equivalent plastic strain, $\bar{\epsilon}_p$. In the following discussion, it is assumed that the stress - strain curves in tension and compression are converted to true stress - true strain plots.

In cases where tensile and compressive data are available, the determination of ζ is as follows. The yield point in both tensile and compressive stress - strain curves is determined and plots of σ_{yc} vs. ϵ_{yc} (compressive plastic strain) and σ_{yt} vs. ϵ_{yt} (tensile plastic strain), are constructed. Note that the plastic strains are calculated by subtracting the elastic strains (equal to current stress divided by the elastic modulus) from the total strains. The parameter ζ is then calculated by dividing values of σ_{yc} and σ_{yt} which correspond to the same slope, i.e. $\sigma_{yc}/\epsilon_{yc} = \sigma_{yt}/\epsilon_{yt}$. This will ensure that the $\bar{\epsilon}_p$ corresponding to σ_{yc} and σ_{yt} are the same². Note that this procedure results in values of ζ which are generally functions of $\bar{\epsilon}_p$. If the dependence of ζ on $\bar{\epsilon}_p$ is not very strong, an average value, valid for the range of plastic strains of interest, can be used.

Compression tests can lead to inhomogeneous stress - strain states due to frictional effects between the sample and the loading platens. Evidence of such an occurrence is the 'barrelling' of cylindrical samples in compression, with the diameter half way through the length of the compressed sample being larger than the diameter at the top and bottom edges. This effect

might be reduced by the use of suitable lubricants but it is usually extremely difficult to eliminate it completely. The use of 'taller' specimens will also reduce the frictional effects, however these might lead to buckling. For this reason, instead of performing tensile and compressive tests, a more reliable alternative is to perform tensile and shear tests.

Plots of σ_y vs. ϵ_y and τ_y (shear yield stress) vs. γ_y (shear plastic strain) are now available. The shear plastic strains are calculated by subtracting the elastic strain components (equal to current stress divided by the shear modulus) from the total strains.

Using equations (2) and (3), the relationships between τ_y and σ_y corresponding to the same $\bar{\epsilon}_p$ are (see Appendix 1):

$$\frac{\sigma_y}{\tau_y} = \frac{\sqrt{3}(\zeta + 1)}{2\zeta} \quad (25)$$

$$\frac{\sigma_y}{\tau_y} = \sqrt{\frac{3}{\zeta}} \quad (26)$$

Equations (25) and (26) are for the Drucker - Prager and Raghava criteria, respectively. The procedure for calculating ζ is described below and supporting equations are given in Appendix 1.

- i) From corresponding σ_y and ϵ_y values calculate the 'tensile' slope, $S_t = \sigma_y/\epsilon_y$.
- ii) Calculate the corresponding 'shear' slope, S_s , from $S_s = S_t/2(1+\nu_p)$, where ν_p is the 'plastic' Poisson's ratio (see Section 4.4).
- iii) Determine the values of τ_y and γ_y such that $\tau_y/\gamma_y = S_s$. This ensures that the shear and tensile data correspond to the same $\bar{\epsilon}_p$ (see Appendix 1).
- iv) Calculate ζ by substituting the corresponding σ_y and τ_y in equation (25) or (26).

This procedure should be repeated for various points on the σ_y vs. ϵ_y curves. It is important to note that depending on the type of adhesive tested, ζ might be constant or vary with strain. If ζ is constant and the stress - strain data show stress maxima, the determination of ζ is greatly simplified. The shear and tensile plateau values are substituted in equations (25) or (26) to calculate ζ .

4.4 CALCULATION OF THE DILATION ANGLE

The last parameter that needs to be defined in the rate independent elastic - plastic models, is the dilation angle, ψ . This can be calculated using equation (11), if the Poisson's ratio, ν_p , in the plastic region, i.e. the ratio of the lateral to tensile plastic strains, is known. In the following discussion, it is assumed that the stresses and strains are converted to true stresses and true strains.

A procedure for determining ν_p when the latter is constant will be described below.

The data required are obtained from a uniaxial tensile test, during which both tensile extension and lateral contraction are measured. As a result, a plot of v_t vs. tensile strain can be constructed, where v_t is the 'total' Poisson's ratio, i.e. the ratio of the lateral to tensile total strains. The first part of such a plot, corresponding to the elastic region, will be a horizontal line, i.e. v_t will be constant and equal to the 'elastic' Poisson's ratio, v_e . Thereafter, depending on whether the v_p is higher, equal to or lower than the v_e , the 'total' Poisson's ratio, v_t , will rise, stay constant or fall, respectively, with strain. In all three cases, the value of v_t corresponding to large values of strain, will be equal to v_p , as the deformation will essentially be all plastic. In other words, the 'total' Poisson's ratio will tend to a value corresponding to the 'plastic' Poisson's ratio, for large strains.

The above can be summarised by the following relationships:

$$v_t = v_e \quad \text{for } \epsilon \leq \epsilon_{t0} \text{ where } \epsilon_{t0} \text{ is the total strain at first yield}$$

$$v_t < v_e \quad \text{for } \epsilon > \epsilon_{t0} \text{ and } v_p < v_e$$

$$v_t = v_e \quad \text{for } \epsilon > \epsilon_{t0} \text{ and } v_p = v_e$$

$$v_t > v_e \quad \text{for } \epsilon > \epsilon_{t0} \text{ and } v_p > v_e$$

$$v_t \approx v_p \quad \text{for large values of } \epsilon.$$

This procedure will not be suitable for the case where v_p varies with strain. Most FEA packages do not allow a definition of varying ψ or v_p . However, the procedure for calculating the plastic v_p in such cases, as a function of strain, is described in Appendix 2, for completeness.

Finally, note that the 'total' Poisson's ratio calculated from engineering strains, v_t' , is also needed to convert the engineering stresses to true stresses (see Section 4.2) and the 'plastic' Poisson's ratio, v_p , is needed in the calculation of ζ (see Section 4.3).

4.5 DETERMINATION OF STRAIN RATE DEPENDENCE OF YIELD

In order to determine the effect of the strain rate on the material's behaviour, additional tests, usually uniaxial tensile tests, need to be performed at various strain rates. For uniaxial tension, the equivalent plastic strain $\bar{\epsilon}_p$ and equivalent plastic strain rate $\dot{\bar{\epsilon}}_p$ are equal to the tensile plastic strain, ϵ_{yt} , and tensile plastic strain rate, $\dot{\epsilon}_{yt}$, respectively. As already mentioned in Section 3, the three most popular methods for defining the strain rate dependence of yield in FEA packages are:

- i) enter directly numerical data of σ_{yt} vs. ϵ_{yt} for various $\dot{\epsilon}_{yt}$,
- ii) enter corresponding data of multiplying factor $(\sigma_{yt}/\sigma_{y0})$ vs. $\dot{\epsilon}_{yt}$,
- iii) approximate the data with one of the models described in Section 3. For example, if the Cowper and Symonds law of equation (20) is to be used, plots of $\log(\sigma_{yt}/\sigma_{y0} - 1)$ vs. $\log \dot{\epsilon}_{yt}$ should be constructed. The material parameters P and C can then be calculated from the slope and the intercept of the 'line' which best fits the experimental data.

All of the above methods require the experimental data to be defined in terms of various constant values of $\dot{\epsilon}_y$. However, the tests are usually performed under constant strain rate, $\dot{\epsilon}$. Appendix 3 describes a method for determining $\dot{\epsilon}_y$ for a constant strain rate ($\dot{\epsilon}$) test.

5 ALTERNATIVE CONSTITUTIVE MODELS

The plasticity models described in Sections 2 and 3 are believed to be the most suitable for modeling impact of glassy, structural adhesives which are operating at temperatures below their glass transition temperature. For adhesives operating just above their transition temperature, the non-linear appearance of the stress - strain curve might not be predominantly due to plastic flow but due to dissipative losses caused by viscous effects or to non-linear elastic effects. In such cases, it might be more precise to use a viscoelastic or a non-linear elastic constitutive model.

5.1 LINEAR VISCOELASTICITY

Small strain, linear, viscoelastic models have been developed and are implemented in various FEA packages ^{6,8}. These can be used to model adhesives whose stress - strain relationship is linear and depends on rate of deformation as well as temperature. Linear viscoelasticity will generally lead to non-linear stress - strain curves, due to relaxation effects occurring during the loading phase of the test. The linear viscoelastic model is described in Appendix 4.

In most cases, linear relaxation effects are not enough to explain the amount of non-linearity present in the stress - strain curve and non - linear viscoelastic models should ideally be used. However, these are complex models where the Boltzmann superposition principle used in linear viscoelasticity is no longer valid, at least not in its original, unmodified form. This means that the assumption that, each increment in stress makes an independent contribution to the overall strain, is not correct. Because of their complexity, non-linear viscoelastic models have not yet been implemented in widely available FEA packages. However, some rubbery adhesives could be modeled using the option in ABAQUS which combines the linear large strain viscoelastic model together with a non-linear elastic or a hyperelastic model. The hyperelastic models will be described next, followed by a small Section on the combination of the viscoelastic and hyperelastic models, as used in ABAQUS.

5.2 HYPERELASTICITY

Hyperelastic models exist in various packages ^{6,8}. They are large strain, non-linear, elastic models based on constitutive theories of rubbers. They can be used to model non-linear materials which extend to large, recoverable strains and are almost incompressible, i.e. the Poisson's ratio is in the range of 0.45 to 0.5. When this model is used on its own, the assumption is made that the material is strain rate independent. The hyperelastic model is described in detail in Appendix 5.

5.3 COMBINATION OF HYPERELASTIC AND LINEAR VISCOELASTIC MATERIAL MODELS

In ABAQUS, it is possible to use the hyperelastic and the linear viscoelastic models together. The result of this combination would be a non-linear, strain rate dependent material which

could be useful for some rubbery adhesives (or adhesives tested just above their glass - transition temperature). As mentioned earlier, no plasticity exists in this model. The deformation is elastic and is recovered upon unloading with time.

This is not a non-linear viscoelastic model. The viscoelasticity is still linear and its governing equations are similar to the ones described in Appendix 4, i.e. the time dependent part of the material's response, is not affected by strain. The difference now is that the relaxation coefficients are applied to the constants that define the hyperelastic behavior. When this combined model is used, it is only the behaviour at long times which is dependent on strain, i.e. hyperelastic. In other words, a separation of strain and time effects is made.

The proposed procedure for using this combined model is described in Appendix 6.

6 CONCLUSIONS

A finite element analysis of the response of adhesives to an impact loading requires the use of a materials model that is capable of describing the behaviour of the material under high strains where the relationship between stress and strain is non-linear. Since adhesives are viscoelastic, the dependence of properties on the rate of loading also needs to be included.

The most widely used models for impact analysis are elastic-plastic models that have been extended so that some of the parameters are rate dependent. These plasticity models have been developed for metals, and limitations in their applicability to adhesives are not clear.

The yield criteria in models that are relevant to adhesives need to include the sensitivity of yield behaviour to the hydrostatic component of stress. In general, the same material parameters need to be determined for these criteria.

The material parameters are:

$$E, \nu, \sigma_T(\epsilon_p), \nu_p, \lambda \text{ and } \psi$$

Stress/strain curves under two different states of stress are required to determine some of these material parameters. Tensile and shear tests have been recommended here. Measurements of lateral and longitudinal strains are needed throughout the tensile tests. If the specimen necks, these measurements must be made at local regions of the specimen.

An approximate analysis is obtained using $\lambda = \mu = 1$, $\nu_p = 0.5$ and $\psi = 0$. The significance of errors incurred by the use of these approximations for different loading situations is unknown.

Reliable methods for measuring tensile strain at strain rates above 1s^{-1} need to be developed.

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APPENDIX 1 EVALUATION OF THE PARAMETER ζ FROM TENSILE AND SHEAR TESTS

For pure shear loading conditions where a shear stress τ_y is applied, the stress invariants are equal to:

$$I_1 = 0, J_2 = \tau_y^2 \quad (1.1)$$

Substituting equation (1.1) in-equations (2) and (3), expressions relating the tensile yield stress to the shear yield stress corresponding to the same equivalent plastic strain, are obtained:

$$\frac{\sigma_y}{\tau_y} = \frac{\sqrt{3}(\zeta+1)}{2\zeta} \quad (1.2)$$

$$\frac{\sigma_y}{\tau_y} = \sqrt{\frac{3}{\zeta}} \quad (1.3)$$

Equations (1.2) and (1.3) are for the Drucker - Prager and Raghava criteria respectively. In order to proceed further, the equivalent plastic strain relationships need to be determined. In other words, the yield criteria need to be re-written in terms of strain invariants. Equations (1.4) and (1.5) correspond to the Drucker - Prager and Raghava criteria respectively:

$$\bar{\epsilon}_p = \frac{\sqrt{3}(\zeta+1)}{2\zeta} \frac{1}{(1+\nu_p)} \sqrt{J_2} + \frac{(\zeta-1)}{2\zeta} \frac{1}{(1-2\nu_p)} I_1' \quad (1.4)$$

$$\bar{\epsilon}_p = \frac{(\zeta-1)}{2\zeta} \frac{1}{(1-2\nu_p)} I_1' + \frac{1}{2\zeta} \sqrt{\frac{(\zeta-1)^2}{(1-2\nu_p)^2} (I_1')^2 + \frac{12\zeta}{(1+\nu_p)^2} J_2} \quad (1.5)$$

The expressions (1.4) and (1.5) are such that, for uniaxial tensile loading, $\bar{\epsilon}_p$ is equal to the tensile plastic strain, ϵ_y . For pure shear loading conditions, the strain invariants I_1' and J_2 are equal to:

$$I_1' = 0, J_2 = \frac{\gamma_y^2}{4} \quad (1.6)$$

When (1.6) is substituted in equations (1.4) and (1.5), the equivalent plastic strains in a shear test for Drucker - Prager and Raghava are given by equations (1.7) and (1.8) respectively:

$$\bar{\epsilon}_p = \frac{\sqrt{3}\gamma_y(\zeta+1)}{4(1+\nu_p)\zeta} = \epsilon_y \quad (1.7)$$

$$\bar{\epsilon}_p = \frac{\sqrt{3}\gamma_y}{2(1+\nu_p)\sqrt{\zeta}} = \epsilon_y \quad (1.8)$$

Equations (1.7) and (1.8) relate the tensile plastic strain to the shear plastic strain corresponding to the same equivalent stress.

Using equations (1.2) and (1.7) the following relationship is derived:

$$\frac{\sigma_{yt}}{\varepsilon_{yt}} = \frac{\tau_y}{\gamma_y} [2(1 + \nu_p)] \quad (1.9)$$

or

$$S_t = S_s [2(1 + \nu_p)] \quad (1.10)$$

Equation (1.10) was used in Section 4.3 in order to determine the parameter ζ .

APPENDIX 2 EVALUATION OF ν_p WHEN THE LATTER VARIES WITH STRAIN

Section 4 described a method which can be used to determine ν_p when the latter is constant. In cases where ν_p varies with strain or when it is desirable to examine the validity of the assumption that ν_p is constant, the following procedure can be used. The experimental data that are needed are again uniaxial tensile data. These should be converted to true stress - true strain for large strains.

i) For any value of axial strain, ϵ_1 , calculate the elastic, ϵ_{1e} , and plastic, ϵ_{1p} , components, using:

$$\epsilon_{1e} = \frac{\sigma}{E} \quad (2.1)$$

$$\epsilon_{1p} = \epsilon_1 - \epsilon_{1e} \quad (2.2)$$

where E is the elastic modulus and σ is the stress corresponding to the strain ϵ_1 .

ii) Calculate the elastic, ϵ_{2e} , and plastic, ϵ_{2p} components of the lateral strain, ϵ_2 , which corresponds to ϵ_1 , from:

$$\epsilon_{2e} = \nu_e \epsilon_{1e} \quad (2.3)$$

$$\epsilon_{2p} = \epsilon_2 - \epsilon_{2e} \quad (2.4)$$

where ν_e is the elastic Poisson's ratio.

iii) Calculate the plastic Poisson's ratio, ν_p , from:

$$\nu_p = \frac{\epsilon_{2p}}{\epsilon_{1p}} \quad (2.5)$$

The above procedure should be repeated for a range of ϵ_1 values in order to determine the variation of ν_p with ϵ_1 . In many cases an average value might have to be defined as FEA packages do not allow for a varying ν_p .

APPENDIX 3 EVALUATION OF PLASTIC STRAIN RATE FOR A CONSTANT STRAIN RATE TEST

The tensile yield stress, σ_y , is a function of the plastic strain, ϵ_y :

$$\sigma_y = f(\epsilon_y) \quad (3.1)$$

where f is determined experimentally from a tensile test at constant strain rate, $\dot{\epsilon}$. From the usual assumption regarding the additive decomposition of the total strain, the following is obtained:

$$\dot{\epsilon} = \dot{\epsilon}_e + \dot{\epsilon}_y \quad (3.2)$$

where $\dot{\epsilon}_e$ and $\dot{\epsilon}_y$ are the elastic and plastic strain rates respectively. The elastic strain rate can be written as:

$$\dot{\epsilon}_e = \frac{\dot{\sigma}_y}{E} \quad (3.3)$$

where E is the elastic modulus. Combining equations (3.1), (3.2) and (3.3):

$$\dot{\epsilon} = \frac{1}{E} f'(\epsilon_y) \dot{\epsilon}_y + \dot{\epsilon}_y = \frac{1}{E} \frac{df}{d\epsilon_y} \dot{\epsilon}_y + \dot{\epsilon}_y \quad (3.4)$$

or, solving for $\dot{\epsilon}_y$:

$$\dot{\epsilon}_y = \frac{\dot{\epsilon}}{1 + \frac{E_h}{E}} \quad (3.5)$$

where E_h is $df/d\epsilon_y$, i.e. the slope of the tensile yield stress against tensile plastic strain curve.

For a linear elastic - perfectly plastic material, E_h is zero and from equation (3.5), $\dot{\epsilon}_y$ is constant and equal to $\dot{\epsilon}$. For the case of linear hardening (bi-linear stress - strain curve), E_h is constant and $\dot{\epsilon}_y$ is again constant but smaller than $\dot{\epsilon}$. For the general case where the function f in equation (3.1) is neither a constant nor linear, $\dot{\epsilon}_y$ will vary throughout the test. Since the ratio E_h/E will generally decrease with increasing strain, $\dot{\epsilon}_y$ will increase during the test and will approach a value equal to $\dot{\epsilon}$ at large strains.

The above observations need to be taken into account when defining the strain rate dependence of yield. The representation of the hardening curve will have consequences on the validity of constant $\dot{\epsilon}_y$ tests that most FEA packages assume.

APPENDIX 4 LINEAR VISCOELASTICITY

The time dependent material behaviour is approximated with the Prony series for the shear relaxation modulus, $G_R(t)$, ⁶:

$$\frac{G_R(t)}{G_0} = 1 - \sum_{i=1}^N \bar{g}_i^{-p} (1 - e^{-t/\tau_i}) \quad (4.1)$$

where N (number of terms in Prony series), $\bar{g}_i^{-p}$ (relative modulus of term i) and τ_i (relaxation time of term i) are material parameters that need to be defined. G_0 is the instantaneous shear modulus and it is related to $\bar{g}_i^{-p}$ through:

$$G_0 = G + \sum_{i=1}^N G_0 \bar{g}_i^{-p} \quad (4.2)$$

where G is the limiting shear modulus at $t \rightarrow \infty$. The instantaneous shear modulus G_0 has to 'correspond' to the Young's modulus and the Poisson's ratio entered in the elastic definition of the material, i.e. E and ν should be obtained from a high rate test. An independent relaxation function for the bulk modulus, similar in form to equation (4.1), can be used as well.

Knowing $G_R(t)$ from equation (4.1), allows calculations to be made of $\sigma(t)$, as a function of strain history, or the stress - strain curves at specified strain rates. For example for a shear test, in which a time varying strain $\gamma(t)$ is applied, the response $\tau(t)$ is given by:

$$\tau(t) = \int_0^t G_R(t-s) \frac{d\gamma(s)}{ds} ds \quad (4.3)$$

Equation (4.1) is based on the Boltzmann superposition principle for evaluating the response for a number of simple loading programmes.

The constants $\bar{g}_i^{-p}$ and τ_i appearing in equation (4.1), may be entered directly or calculated by the FEA package ⁶ from experimental shear relaxation or creep curves. In the latter case, the experimental data have to be entered in the form of 'normalized shear modulus ($G_R(t)/G_0$) or compliance ($J_R(t)/J_0$) vs. time' data, G_0 and J_0 being the instantaneous shear modulus and compliance respectively.

An option exists to define the shift function describing the effects of temperature on the viscoelastic material behaviour by the reduced or pseudo time concept.

APPENDIX 5 HYPERELASTICITY

Hyperelastic models are used to describe the behaviour of materials that exhibit elastic response up to large strains. These materials are described in terms of an elastic potential scalar function W (or strain energy density function) whose derivative with respect to a strain component, ϵ_{ij} , determines the corresponding stress component, σ_{ij} . This can be expressed by:

$$\sigma_{ij} = \frac{\partial W}{\partial \epsilon_{ij}} \quad (5.1)$$

The strain energy density is given by:

$$W = \sum_{i+j=1}^N C_{ij} (\bar{I}_1 - 3)^i (\bar{I}_2 - 3)^j + \sum_{i=1}^N \frac{1}{D_i} (J_{cl} - 1)^{2i} \quad (5.2)$$

where N is a material parameter which defines the number of terms in the strain energy function and C_{ij} and D_i are temperature dependent material parameters. The maximum value that N can take is 6. $\bar{I}_1$ and $\bar{I}_2$ are the deviatoric strain invariants and they are equal to:

$$\bar{I}_1 = \bar{\lambda}_1^2 + \bar{\lambda}_2^2 + \bar{\lambda}_3^2 \quad (5.3)$$

$$\bar{I}_2 = \bar{\lambda}_1^{-2} + \bar{\lambda}_2^{-2} + \bar{\lambda}_3^{-2} \quad (5.4)$$

where $\bar{\lambda}_i$ are the deviatoric principal extension ratios ($\bar{\lambda}_i = J^{-1/3} \lambda_i$), J is the volume ratio (equal to $\lambda_1 \lambda_2 \lambda_3$) and λ_i are the principal extension ratios, i.e. the ratio of the deformed length to the original length. The relationship between the extension ratio, λ_i and the engineering strain, ϵ_i , is:

$$\lambda_i = 1 + \epsilon_i \quad (5.5)$$

The last term of equation (5.2) represents the hydrostatic (volumetric) work. J_{cl} is defined as:

$$J_{cl} = \frac{J}{J_{th}} = \frac{\lambda_1 \lambda_2 \lambda_3}{(\lambda_1 \lambda_2 \lambda_3)_{th}} \quad (5.6)$$

where $(\lambda_1 \lambda_2 \lambda_3)_{th}$ is the volume change due to temperature changes.

For incompressible behaviour, $\lambda_1 \lambda_2 \lambda_3 = 1$, D_i are set to zero and equations (5.3) and (5.4) can be written as:

$$\bar{I}_1 = \lambda_1^2 + \lambda_2^2 + \lambda_3^2 \quad (5.7)$$

$$\bar{I}_2 = \lambda_1^2 \lambda_2^2 + \lambda_1^2 \lambda_3^2 + \lambda_2^2 \lambda_3^2 \quad (5.8)$$

Assuming incompressible behaviour and $N=1$, W becomes:

$$W = C_{10}(\bar{I}_1 - 3) + C_{01}(\bar{I}_2 - 3) \quad (5.9)$$

Equation (5.9) is the classical Mooney - Rivlin law. For a uniaxial tension test, $\lambda_1 = \lambda$, $\lambda_2 = \lambda_3 = \lambda^{-1/2}$. Substituting these relations in equation (5.9) and differentiating with respect to λ gives the following relationship for the uniaxial engineering stress:

$$\sigma = 2 \left(\lambda - \frac{1}{\lambda^2} \right) \left(C_{10} + C_{01} \frac{1}{\lambda} \right) \quad (5.10)$$

For $N=2$, there are five terms in W , for $N=3$, nine terms etc.

The material parameters for up to $N=2$ can be determined by the FEA package ⁶ from a least squares fit on experimental data. It is recommended that three experiments are performed which will result in a good characterization of a material. These are uniaxial, equibiaxial and planar (pure shear) tests.

A uniaxial tension test is usually preferred over a uniaxial compression test as the latter usually suffers from an inhomogeneous stress - strain state due to frictional effects between the sample and the loading platens. The tension test is performed by pulling a dumbbell shaped sample. Note that if both compression and tension test data are used, difficulties might arise in the least squares fit, leading to inaccurate calculations of the model's parameters.

Tension tests are also usually preferred in biaxial experiments over the compression tests due to practical difficulties with the experimental set-up. In addition, it can be shown that for a fully incompressible material, the compressive equibiaxial test is equivalent to the tensile uniaxial test which is far easier to perform. The equibiaxial tension test is performed on square sheets with gripping tabs in the shape of a cruciform, in a biaxial testing machine.

Planar (or pure shear) tests are performed on thin, short and wide rectangular samples. Wide loading grips are used to grip the sample along its wide edges and they are moved apart either in tension or compression. The planar tension and compression tests can be shown to be equivalent for an incompressible material.

When it is necessary to allow some compressibility in the material model, the coefficients D_i must also be given. For a general case, these can be calculated by the FEA package if volumetric test data in the form of pressure vs. volume ratio are defined.

In the case where volumetric test data are not available but the Poisson's ratio, ν , is known from uniaxial test data, the following procedure is suggested:

i) Choose $N=1$. This should be a sufficiently accurate model for strains less than 100%.

ii) Enter the uniaxial, biaxial and planar test data in a preliminary run and allow the FEA package to calculate the constants C_{10} and C_{01} , i.e. assume incompressibility.

iii) Calculate the initial Young's modulus from:

$$E_0 = 6(C_{10} + C_{01}) \quad (5.11)$$

This should agree with the slope at the origin of the uniaxial nominal stress - strain curve.

iv) From E_0 and ν calculate the initial bulk modulus, K_0 :

$$K_0 = \frac{E_0}{3(1-2\nu)} \quad (5.12)$$

v) Calculate D_1 from:

$$D_1 = \frac{2}{K_0} \quad (5.13)$$

vi) Define the material's hyperelastic behaviour by entering directly C_{10} , C_{01} and D_1 and continue with the analysis.

Non zero values of D_1 affect the uniaxial, equibiaxial and planar stress results. However, since the material is assumed to be only slightly compressible ($\nu = 0.45$ to 0.5), the techniques described for obtaining the C_{ij} should give sufficiently accurate values even though they assume that the material is fully incompressible.

APPENDIX 6 COMBINATION OF HYPERELASTIC AND LINEAR VISCOELASTIC MODELS

This procedure is only valid for the case where the hyperelastic behaviour is the classical Mooney - Rivlin formulation with $\nu=0.5$. Similar procedures can be followed for more general cases.

- i) Define C_{10} and C_{01} . This could be done directly or through providing appropriate experimental data (see Appendix 5). The strain rate for these tests should be sufficiently high to ensure that no relaxation effects are allowed to take place. This is due to the fact that the instantaneous response of the material will be calculated from these parameters.
- ii) From the above definitions calculate the 'instantaneous' response i.e. the initial shear modulus from:

$$G_0 = 2(C_{10} + C_{01}) = \mu_1 + \mu_2 \quad (6.1)$$

- iii) Define the time dependent behaviour through $\bar{g}_i^p$ and τ_i . The shear moduli $\bar{g}_i^p$ are normalized with respect to the 'instantaneous' modulus calculated from equation (6.1). The other alternative is to provide experimental data, e.g. shear normalized moduli vs. time (see Section 4). The value of the normalized shear modulus at $t \rightarrow \infty$, should also be defined. ABAQUS will then use the least squares fit method to calculate the viscoelastic parameters.