

REPORT

ANAMET

The collected *ANALyse* Notes

Volume 2 - Numbers 11 to 20

N M Ridler

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ANAMET
The Collected ANAlyse Notes
Volume 2 - Numbers 11 to 20

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ABSTRACT

This report contains the second compilation of ten ANAlyse Technical Notes published originally by ANAMET - the automatic network analyser metrology club coordinated and administered by NPL. The Notes were first issued during the period April 1995 to November 1996. Each Note is reproduced here in its original form.

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Preface

This report contains the second compilation of ten *ANALyse* Technical Notes published by ANAMET - the Automatic Network Analyser (ANA) metrology (MET) club coordinated and administered by NPL. The club is an interest group of organisations and people involved in RF, microwave and millimetre-wave network analysis and measurements, and provides a forum for the rapid exchange of information, ideas and experience.

The club achieves these objectives through its publication series, biannual club meetings, and frequent measurement comparisons and investigations. ANAMET maintains three publication series: ANAMET News - the club's biannual newsletter; ANAMET Reports - which describe research investigations, etc, undertaken by the membership; *ANALyse* Notes - which provide technical information, often of a tutorial nature.

All three publication series are intended exclusively for the ANAMET membership. However, it was agreed at an early stage that "mature" *ANALyse* Notes could be made available to a wider audience as compilations issued as NPL Reports. The first ten *ANALyse* Notes were issued as *NPL Report DES 140* in October 1995. This compilation presents the second set of ten Notes. These Notes were issued originally to the ANAMET membership during the period April 1995 to November 1996. Each Note is reproduced here in its original form.

ANALyse Notes continue to be issued to the ANAMET membership, implying that further volumes of compiled Notes are to be expected in future. I would like to thank all the authors who have contributed to this compilation of Notes, the members of the ANAMET Steering Committee who reviewed and approved the contents of each Note and the National Measurement System Policy Unit of the Department of Trade and Industry for part-funding the ANAMET club.

It should be clearly understood that these *ANALyse* Notes were produced by, and for, the members of ANAMET. They were intended for fast dissemination of technical information for discussion purposes and do not necessarily represent an official viewpoint. No responsibility is accepted by the authors, ANAMET, or NPL for any use made of the information contained in these Notes.

NICK M RIDLER

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Additional publications

RIDLER, N.M. "ANAMET - Past, Present and Future", *26th Automated RF and Microwave Measurements Society (ARMMS) Conference Digest*, The Abbey Hotel, Malvern , April 1997.

RIDLER, N.M. and MEDLEY, J.C. "ANAMET comparison of reflection coefficient measurements at RF", *25th Automated RF and Microwave Measurements Society (ARMMS) Conference Digest*, The Limpley Stoke Hotel, Bath, November 1996.

RIDLER, N.M. and MEDLEY, J.C. "Comparison of ANA s-parameter measurements in X-band waveguide", *24th Automated RF and Microwave Measurements Society (ARMMS) Conference Digest*, Beech Hill Hotel Windermere, Cumbria, April 1996.

RIDLER, N.M. and MEDLEY, J.C. "A comparison of microwave reflection and transmission measurements in rectangular waveguide", in *Advances in Metrology and its Role in Quality Improvement and Global Trade* (Eds B S Mathur, V N Oja and P C Kothari), Narosa Publishing House, New Delhi, pp 364-368, February 1996.

RIDLER, N.M. (Ed), "ANAMET. The collected ANAlyse Notes Volume 1 - Numbers 1 to 10", *National Physical Laboratory Report DES 140*, October 1995.

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ANALyse

An occasional series of Notes produced by, and for, the Members of ANAMET

Number 11

April 1995

The Type N measurement comparison exercise –

Some additional results using different equipment.

Author: Doug Skinner, Head of Metrology, Marconi Instruments Ltd, Stevenage

Introduction

The first ANAMET comparison exercise involved measuring the VSWR of six terminations fitted with Type N, 50 ohm, coaxial connectors. There were three male and three female terminations with nominal VSWRs of 1.05:1, 1.2:1, and 1.5:1.

While the items were “resting” in the Standards Laboratory at Marconi Instruments the opportunity was taken to measure them on a number of different VSWR measuring instruments in the laboratory. In ANAMET News, Issue 3, the results using slotted lines and an autotester were reported, but interest has also been expressed in the results achieved by the other measuring instruments. These results were presented briefly at the ANAMET meeting held at Hewlett-Packard, Winnersh, in 1994. In this Note, the results have been plotted to show the VSWR difference from the ANAMET average of five readings which were used as a guide during the exercise.

Measuring instruments

The instruments available and used to make the measurement were:

An HP 8410 computer corrected network analyser with a full 12-term error correction programme. This network analyser is a modified HP 8410 which was modified at Marconi Instruments in 1975 by building a controller connected to a VAX computer. (This network analyser is located in the Standards Laboratory and is still in use today!)

A Marconi Instruments 6203 microwave test set with a 6210 six-port vector reflection analyser. (These were the measurements, made by Marconi Instruments, submitted to the ANAMET comparison exercise.)

A GR900LB precision slotted line fitted with a GR900/N male or female adaptor depending on the termination being measured. This slotted line was used from 1 GHz to 4 GHz.

An HP coaxial slotted line section type 809C/816A used from 5 GHz to 18 GHz.

An HP 8510A network analyser.

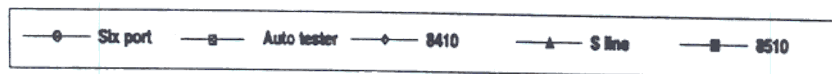
A Wiltron autotester type 560-97NF50-1. (This autotester was only used to measure the Type N Male terminations.)

Results

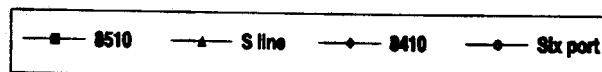
Before commencing the measurements, all the connectors were gauged to ensure compliance with the USA MIL Spec 39012.

The graphs show the VSWR difference from the ANAMET average of the five results submitted by the ANAMET members. The following identifiers are used to distinguish each instrument in the graphs.

(i) Male terminations



(ii) Female terminations



The results show good agreement between the different instruments and the only places where there are significant changes are where the HP 8410 is used to measure both male and female 1.5:1 terminations. This is mainly due to the fact that the HP 8410, as used in this configuration, is better used over a much narrower frequency band using a different calibration method. Unfortunately, time did not allow this to be done and where possible an attempt was made to use each equipment in a similar manner.

The slotted line measurements were clearly very time consuming to make and each slotted line was calibrated with the necessary adaptor connected to allow measurements to be made in each connector type. The slotted line values were all corrected for the end effect of the line including the adaptor.

Conclusions

The results do show some good agreement between the different devices at the industrial Standards Laboratory level and may be of help to those laboratories who do not have a network analyser available to make VSWR measurements.

ANALyse

An occasional series of Notes produced by, and for, the Members of ANAMET

Number 13

January 1996

New words for New Ideas - A look at some of the terminology recommended for expressing measurement uncertainty.

Author: Nick Ridler, NPL, Malvern

Background

Since the ISO published the "Guide to the expression of uncertainty in measurement"^[1] in 1993, much work is being done to interpret the guidance given on the subject of uncertainty in measurement.

One of the results of the publication of the Guide has been to introduce into the metrology vocabulary some new terms expressing the new ideas to be used when working with uncertainty in measurement. These new terms are given definitions in the ISO document but, like similar definitions relating to uncertainty found elsewhere in the literature^[2], the required precision of the language often obscures rather than clarifies the underlying idea. This is understandable since such definitions are constructed to withstand the close scrutiny of metrologists involved in a wide variety of disciplines.

This note discusses some of these new terms and the underlying ideas to show their application. The terms are; "standard uncertainty", "type A evaluation (of uncertainty)", "type B evaluation (of uncertainty)", "combined standard uncertainty", "expanded uncertainty" and "coverage factor".

Classifying uncertainty components

Over the years, considerable dialogue has taken place concerning *what* different types of uncertainty components there could be and *how* these should be evaluated. The different types were based on whether the uncertainty component was meant to represent a random or a systematic error. The often heated debate caused by this methodology and terminology was neatly side-stepped by the formulators of the ISO guidance document, who devised a technique and terminology linked directly to *how* components were evaluated and ignored what physical process governed their behaviour.

The ANALyse series of Technical Notes are produced by, and for, the members of ANAMET. They are intended for fast dissemination of technical information for discussion purposes and do not necessarily represent an official viewpoint. No responsibility is accepted by the author(s) or ANAMET for any use made of the information contained in this Note. *This note has been approved by the ANAMET Steering Committee.*

In short, if statistics are used to evaluate an uncertainty component (*e.g.*, from a repeated series of measurements) then it is termed a **"type A evaluation"**, and if other methods are used (*e.g.*, using manufacturers specifications, previous experience etc) then this is termed **"type B evaluation"**.

The term **"standard uncertainty"** refers to any uncertainty (either a component or an overall estimate of uncertainty) evaluated at the one standard deviation level. (The ISO document recommends evaluating all uncertainty at this level.)

The term **"combined standard uncertainty"** is used when a standard uncertainty is derived by combining the effects of contributing uncertainties. This is usually the final result.

Expressing uncertainty

The Guide recognised that "one standard deviation" wasn't always appropriate as an uncertainty statement to accompany a final result (*e.g.*, a result appearing on a certificate of calibration). Some customers for measurements require either an uncertainty interval approximating to a level of confidence (historically either 95 or 99%) or something representing a maximum likely error of measurement.

The term **"expanded uncertainty"** is used when the combined standard uncertainty is multiplied by the **"coverage factor"**. Typically, a coverage factor of two is used to convert the combined standard uncertainty into an interval with an approximate level of confidence of 95%. This means that the supplier of the measurement is 95% confident that the "true value" of the measurement lies within the expanded uncertainty of the value given.

Summary

Several new terms have appeared recently in connection with evaluating and expressing the uncertainty of a measurement. This Note has attempted to describe these terms so that the underlying ideas are shown rather than concentrating on using the precise definitions for these terms which can be found in the Guide^[1].

References

ISO, "Guide to the expression of uncertainty in measurement", First edition, 1993. (BSI equivalent publication: PD 6461: Part 3: 1995.)

ISO, "International vocabulary of basic and general terms in metrology", Second edition, 1993. (BSI equivalent publication: PD 6461: Part 1: 1995.)

ANALyse

An occasional series of Notes produced by, and for, the Members of ANAMET

Number 12

July 1995

Characteristic impedance of (X-band) waveguide - what is it?

Author: Tim Hodgetts, NPL, Malvern

Waveguide, unlike coaxial-line, does not have a universal characteristic impedance, valid for every application, so there is no one answer. However, the question is usually asked by people who want to know what reflection or transmission coefficient they will see if they join together two lengths of waveguide with different cross-sections (or, sometimes, different dielectric fillings). There is an answer to this which is as meaningful as the corresponding answer when two different sections of coaxial-line are joined.

According to elementary transmission-line theory, when two transmission lines are joined together, the reflection coefficient Γ which is seen when looking from a line of characteristic impedance Z_{01} into a second line of characteristic impedance Z_{02} is given by

$$\Gamma = \frac{Z_{02} - Z_{01}}{Z_{02} + Z_{01}} \quad (1a) \quad \text{or} \quad \frac{1 + \Gamma}{1 - \Gamma} = \frac{Z_{02}}{Z_{01}} \quad (1b)$$

So the problem is equivalent to specifying the proper value of (Z_{02}/Z_{01}) .

For a coaxial-line of inner diameter a and outer diameter b , with a filling of material constants μ and ϵ , all the books on transmission lines give the characteristic impedance as

$$Z_0 = \frac{\sqrt{\mu/\epsilon}}{2\pi} \ln\left(\frac{b}{a}\right) \quad (2a) \quad \text{so} \quad \frac{Z_{02}}{Z_{01}} = \frac{\sqrt{\mu_2/\epsilon_2} \ln(b_2/a_2)}{\sqrt{\mu_1/\epsilon_1} \ln(b_1/a_1)} \quad (2b)$$

when two coaxial-lines are joined together.

The corresponding expression for rectangular waveguides at a frequency f , with the same material constants and with widths a_1 and a_2 and heights b_1 and b_2 is

$$\frac{Z_{02}}{Z_{01}} = \frac{\mu_2 b_2}{\mu_1 b_1} \frac{\sqrt{f^2 \mu_1 \epsilon_1 - 1/(2a_1)^2}}{\sqrt{f^2 \mu_2 \epsilon_2 - 1/(2a_2)^2}} = \frac{\mu_2 b_2 \lambda_{g1}}{\mu_1 b_1 \lambda_{g2}} \quad (3)$$

where λ_{g1} and λ_{g2} are the guide wavelengths. Z_{01} and Z_{02} are not separately defined, but this is no problem; only their ratio matters.

Equations (2b) and (3) only give accurate answers if the dimensions and material constants of the two lines are much the same (as when re-normalising, *i.e.* correcting from an actual piece of waveguide or coaxial-line to an ideal one of exact nominal size). When there are big differences between the lines, there is a reflection from the junction of the lines as well as from the second line. This effect may be important even if $(Z_{02}/Z_{01}) = 1$; a discussion of it for coaxial-lines in that case has been given in ANAlyse Note no. 3, in terms of an equivalent capacitance at the junction. A similar treatment is possible for two waveguides, but the equivalent junction capacitance (or, sometimes, inductance) is not defined absolutely but only relatively, like the characteristic impedance.

Both these treatments assume that the two lines or guides are joined as simply as possible, with a common axis of symmetry and (for the waveguide case) with corresponding sides of the guides parallel. Practical coaxial-connectors or waveguide flange joints must be of very good quality to satisfy these requirements, although first-order errors in symmetry often only lead to second-order errors in the predicted reflection coefficient, which are usually too small to matter. Also, the effects of some kinds of symmetry error can be calculated, at least if the two guides are much the same; a list of these, with tabulated results for various waveguide sizes, has been given in [1].

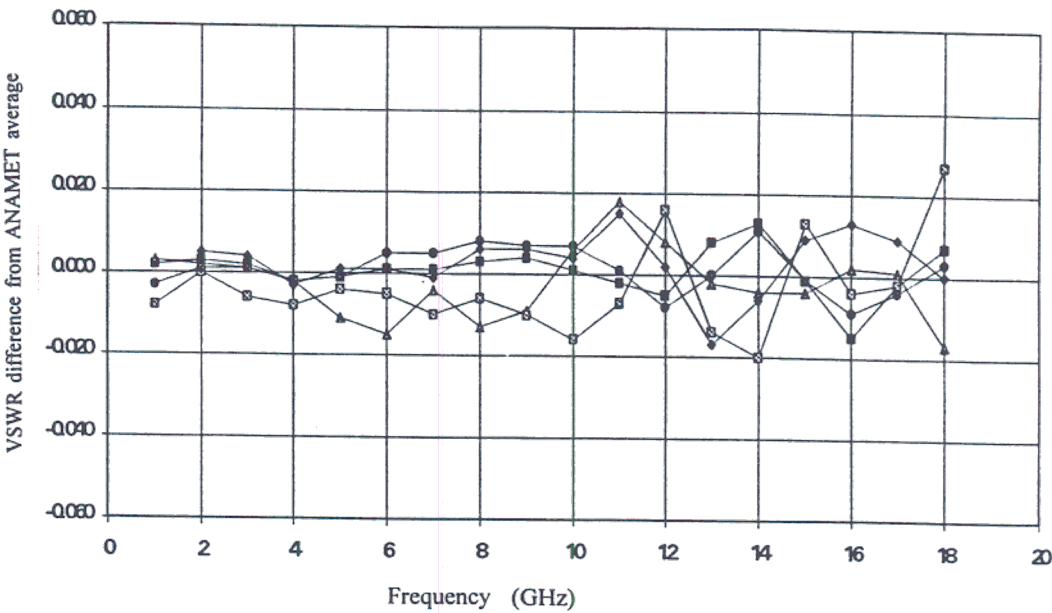
Predictions of transmission coefficients are even more complicated; they are subject to all the problems described above, and to some others of their own. If the joined lines are much the same, as above, essentially all the power arriving at the junction goes through it (the return loss is a second order effect); then the product of the two transmission coefficients s_{21} and s_{12} is theoretically 1, and this product ($s_{21}s_{12}$) is often more important than s_{21} or s_{12} alone. Unfortunately, this prediction ignores contact resistance in the joining connectors or flanges, which is a very tricky problem!

Reference

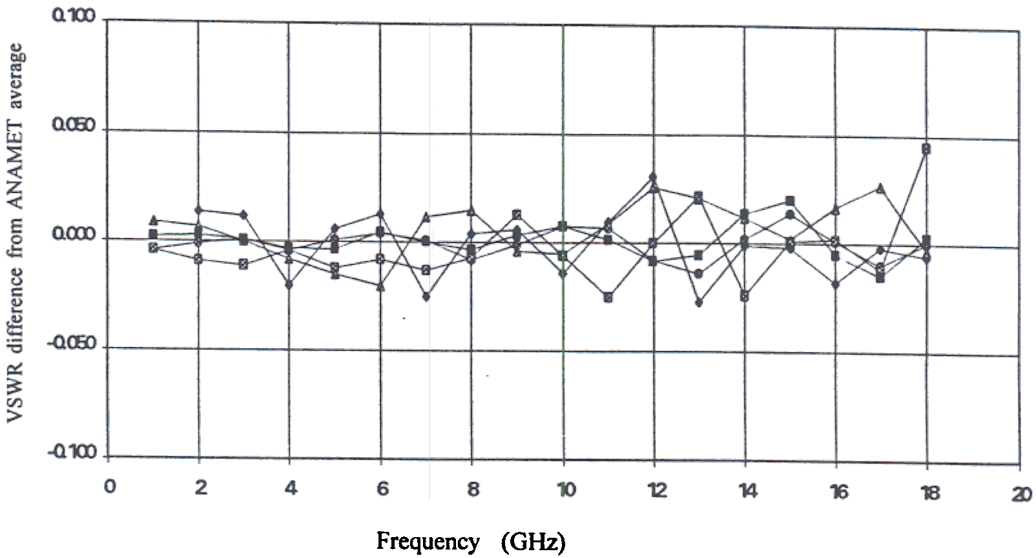
- [1] D J Bannister, E J Griffin and T E Hodgetts, "On the dimensional tolerances of rectangular waveguide for reflectometry at millimetric wavelengths", NPL Report DES 95, September 1989.

Male terminations

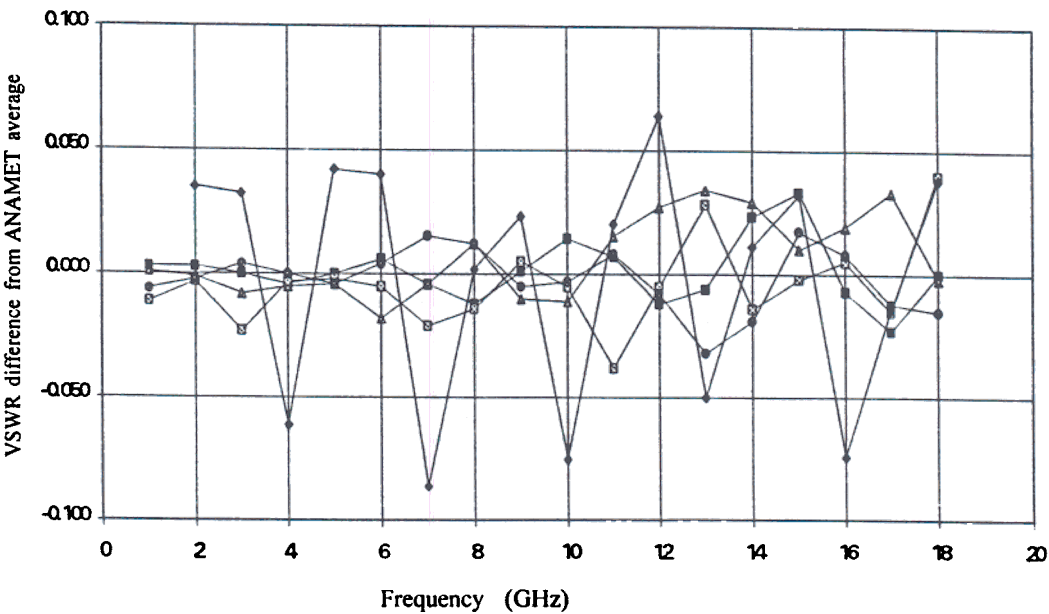
Nominal
VSWR = 1.05:1



Nominal
VSWR = 1.2:1

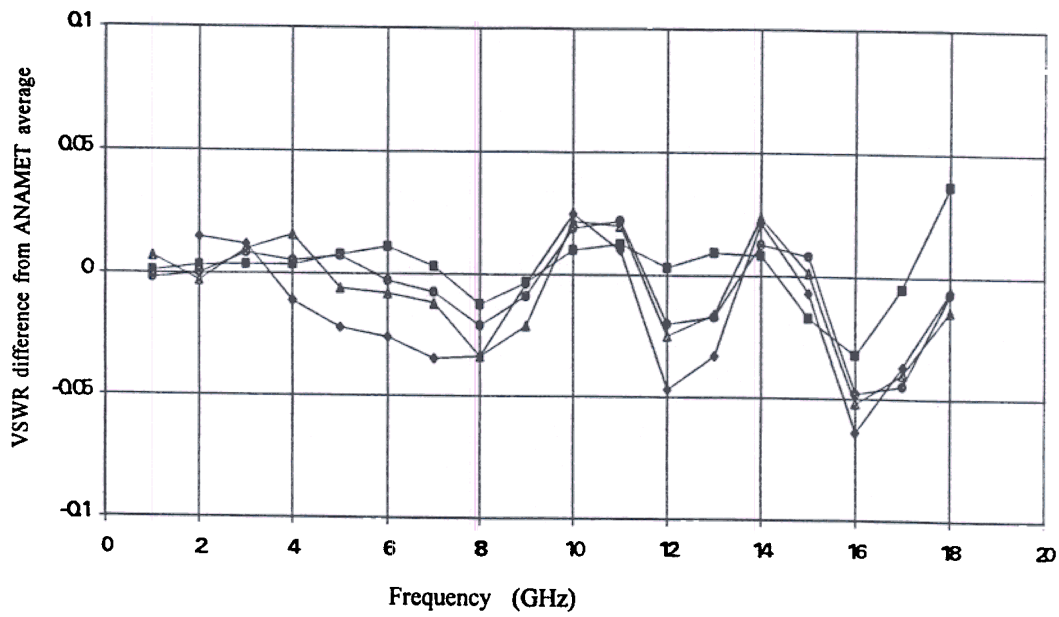


Nominal
VSWR = 1.5:1

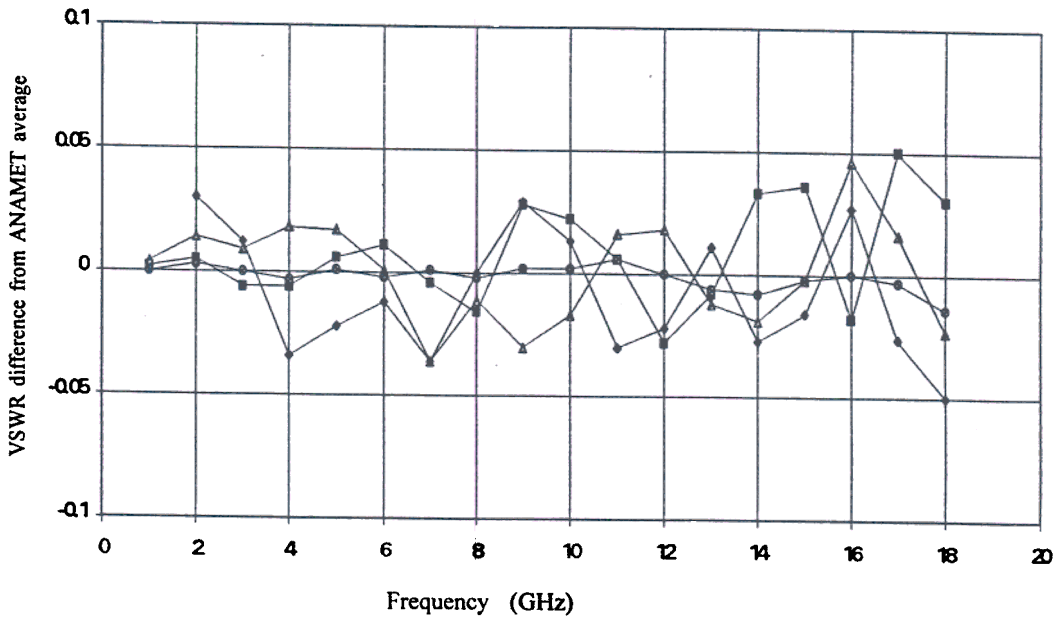


Female terminations

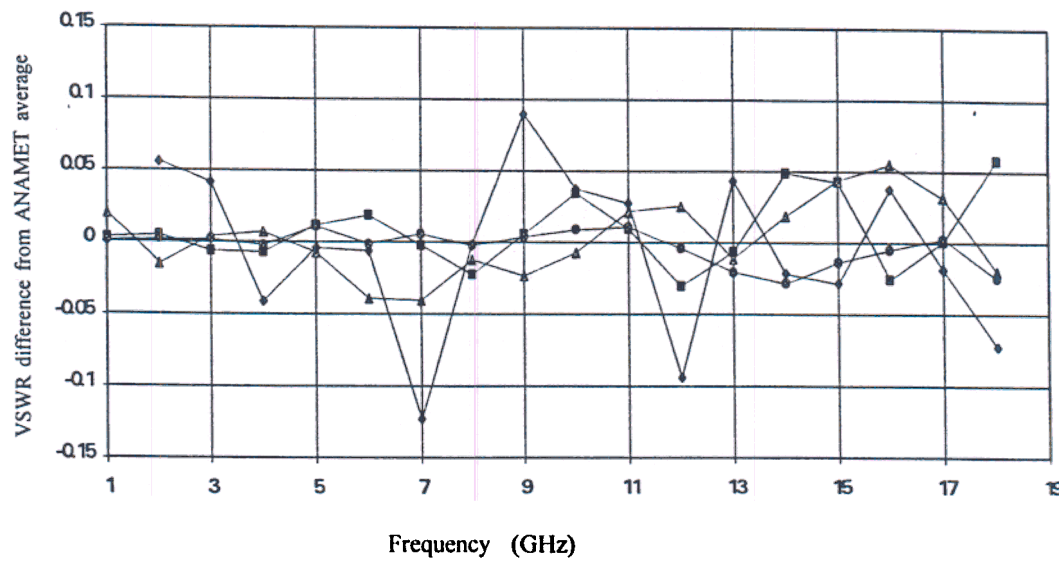
Nominal
VSWR = 1.05:1



Nominal
VSWR = 1.2:1



Nominal
VSWR = 1.5:1



ANALyse

An occasional series of Notes produced by, and for, the Members of ANAMET

Number 15

March 1996

Making a good estimate of a vector quantity

Authors: Nick Ridler and John Medley, NPL, Malvern

Introduction

A recent ANALyse Note (number 7) discussed how to obtain an estimate of a quantity being measured from a series of repeated determinations. This apparently trivial task was re-examined, placing emphasis on the problem of analysing data sets containing occasional unusual values. Such data sets can occur when a group of laboratories is involved in a measurement comparison exercise, or when a single laboratory is undertaking a statistical analysis of errors during an uncertainty assessment.

ANALyse 7 showed that, unlike the arithmetic mean (which is the conventional choice for an average value), the median remained unaffected by the inclusion of unusual data values. (In fact, the median produced a plausible average value for both the well-behaved data sets and data sets containing some unusual values.)

This note looks at extending the problem of obtaining an estimated value to the situation in which the measured values are vector quantities. This is particularly relevant to members of ANAMET who use vector network analysers. VNAs measure vector quantities, such as reflection and transmission coefficients, which have two components. They are typically quoted in either real and imaginary or magnitude and phase formats and can be represented as points in the complex plane (See Figure 1).

Vector measurement data

As with measurements in one-dimension (*i.e.*, scalar quantities), an average value is required which remains unaffected by an unusual value in the data set. Figures 1 and 2 show sets of measurement data in the first quadrant of the complex plane. The data set in Figure 1 contains eight measurements in a single, reasonably compact, group. Figure 2 shows the same data set, except that one of the data points has been moved away from the main grouping, illustrating a data set containing one measurement with a suspected large error.

One candidate for an average value for these data sets is the **spatial median**. It was used to provide an average value for the participants' results in the third ANAMET measurement comparison exercise.

The spatial median

The spatial median of a data set is the point for which the sum of the distances between it and the points of the data set is a minimum. In other words, for a data set $\{x_i ; i = 1, \dots, n\}$, the expression :

$$\sum_{i=1}^n |x_i - \mu|$$

is a minimum (over all possible choices of point μ) when μ is the spatial median.

The above definition makes no reference to the coordinate system used, requiring just a consistent definition of distance, (*i.e.*, distance means the same in every direction). An alternative would be to take medians of each component of the vector measurement, but the choice of coordinate system used to represent the data will affect the result. For example, the points defined by (median real, median imaginary) and (median magnitude, median phase) will in general be different. No such subjective choice is required to calculate the spatial median.

Further proposals for multi-dimensional medians can be found in the literature. However, the spatial median has the advantage that it is conceptually simple to understand (although a bit tricky to calculate, involving iterative calculations).

Discussion

The data sets from Figures 1 and 2 are shown again on a slightly expanded scale in Figures 3 and 4, together with their respective spatial medians (marked with a "×"). The position of the spatial median has not been changed greatly when the one data point is moved away from the main group, *i.e.*, it exhibits resilience to the effects of this one unusual value. In fact the unusual value could be moved away to infinity without unduly affecting the position of the spatial median. This is in contrast to the arithmetic mean, which goes to infinity as the unusual value goes to infinity.

The spatial median remains within the main grouping even if more than one data point is moved away from the group. (This is representative of the situation where there is more than one unusual value in a data set.) In fact, the spatial median (like its one-dimensional equivalent) remains with the majority of values until 50% of values have been removed from the group.

Conclusion

This note presents a version of the classical median statistic applicable to vector quantities. This version, called "the spatial median", has a number of desirable properties: (i) it is not affected by the method used to describe the data (*i.e.*, magnitude and phase, or real and imaginary components); (ii) it is very resilient to the presence of unusual values in the data set, and (iii) it is based on a simple concept – the position which minimises the sum of the distances to all the points in the data set.

ANALyse

An occasional series of Notes produced by, and for, the Members of ANAMET

Number 14

February 1996

How much variation should we expect from coaxial connector dial gauge measurements?

Author: Nick Ridler, NPL, Malvern

Introduction

To prevent damage to the measuring instrument caused by out-of-specification connectors, it is important to measure the position of the connector's inner conductor with respect to the outer conductor. This measurement is critical since it establishes whether the connectors of the artefact are outside specification and therefore unfit for measurement. The measurement is usually made using a dial gauge (often supplied with the network analyser calibration kit). However, there is little (if any) quantified information concerning the performance of these gauges.

The second ANAMET comparison exercise included comparing the participants' dial gauge measurements of the GPC-3.5 connectors with which the items were fitted. The overall results of the comparison have been reported in detail elsewhere [1] but this Note takes a closer look at the dial gauge measurements to see how much variation can be expected for this type of measurement. It uses a technique recommended [2] by ISO for assessing the accuracy of measurement methods and results, and establishes an upper limit, called the **Reproducibility Limit**, for the differences that might be expected between results obtained by different laboratories.

The calculations

The scatter of the participants' dial gauge readings was found using the inter-quartile range (IQR - See ANALyse 8 for more details) for each set of connector measurements. The largest IQR value for the connectors was $5.8 \mu\text{m}$ and this figure was used (See Appendix) to determine a value for the **Reproducibility Limit** of $12 \mu\text{m}$.

Since most dial gauges have graduated scales with minor divisions of $0.0001''$, this reproducibility limit is approximately equal to five minor divisions on such scales.

Summary and conclusions

The results reported by the participants in the second ANAMET comparison exercise have been used to establish a reproducibility limit for dial gauge measurements of GPC-3.5 connectors. This tells us that, in general, the difference between measurements reported by any two laboratories for this type of connector can be expected to be less than $12\ \mu\text{m}$ (for gauges in good working order). *I.e.*, in the absence of any traceability, this is a typical degree of variation for this type of measurement.

Although these conclusions have been made from measurements of GPC-3.5 connectors, dial gauges for other connector types have much in common with these gauges, leading us to expect variations in other connector measurements to be not dissimilar to those observed above. However, further work is needed on other connector types to substantiate this prediction. Indeed, the fourth ANAMET measurement comparison is including type-N connector dial gauge measurements as part of the exercise as a useful start on the data gathering.

References

- [1] RIDLER, N.M. and MEDLEY, J.C., A comparison of complex scattering coefficient measurements in 50 ohm coaxial line to 26.5 GHz. *NPL Report DES 138*, June 1995.
- [2] ISO 5725, *Accuracy (trueness and precision) of measurement methods and results*. (Six parts.) First edition, 1994.

Appendix: Determining the Reproducibility Limit from the IQR

The IQR can be used to provide an estimate of the standard deviation, s , using:

$$s \approx 0.74 \times \text{IQR}$$

The largest IQR value for the connectors was $5.8\ \mu\text{m}$, so the associated standard deviation is estimated as:

$$s = 0.74 \times 5.8 = 4.3\ \mu\text{m}$$

This value can be taken as equivalent to the reproducibility standard deviation, s_R , since from experience the repeatability of any one gauge is considerably better than the reproducibility between different gauges.

This reproducibility standard deviation can be used to estimate the reproducibility limit, R , using:

$$R \approx 2.8 \times s_R$$

Therefore, from our measurements, we have:

$$R \approx 2.8 \times 4.3 \approx 12\ \mu\text{m}$$

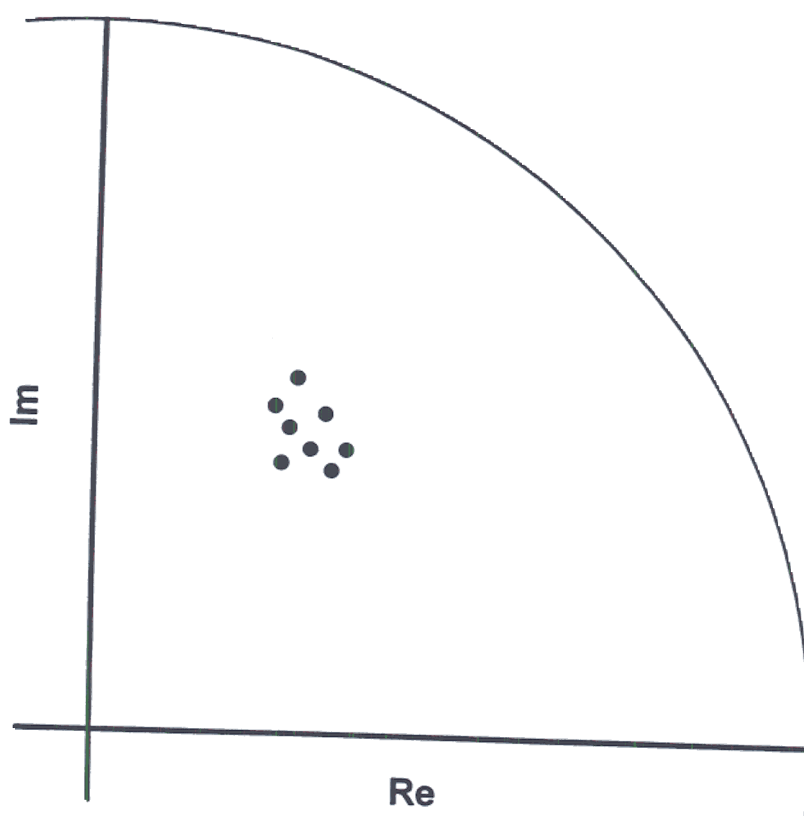


Figure 1: A set of vector measurements in the first quadrant of the complex plane. These measurements are relatively well-behaved.

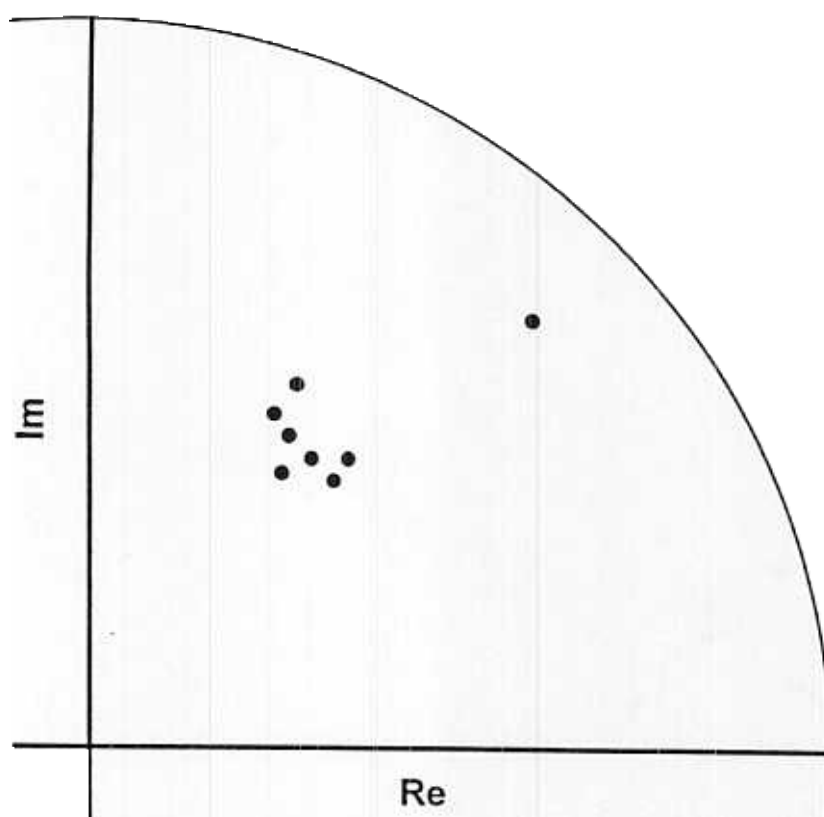


Figure 2: The same data set as figure 1 except one point has been moved away from the main grouping. This value is unusual with respect to the main grouping.

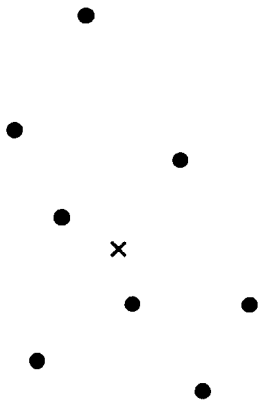


Figure 3: A close-up view of the data set in figure 1. The position of the spatial median is marked with a "x".



Figure 4: A close-up view of the data set in figure 2. The position of the spatial median is marked with a "x".

ANALyse

An occasional series of Notes produced by, and for, the Members of ANAMET

Number 16

April 1996

Assessing the quality of an estimated value - Part 2: the Median Absolute Deviation

Authors: Nick Ridler and John Medley, NPL, Malvern

Introduction

This note presents a development of the techniques discussed in ANALyse 8, which can be used to produce a figure for the reliability of an estimated value derived from a series of repeated determinations. For example, such techniques can be used when evaluating random error contributions to a single measurement system or for summarising the results from measurement comparison exercises, such as those coordinated by ANAMET.

ANALyse 8 looked at using the standard deviation and the inter-quartile range as estimators of the dispersion in a series of scattered values. This note introduces the Median Absolute Deviation (MAD) as an alternative to these estimators.

The MAD definition

For a series of values, the MAD is defined here as the median of the absolute differences between each value and the median of the series of values. In other words, finding the MAD is done in three steps:

Step one – Find the median for the original series of observations (by following the techniques given in ANALyse 7);

Step two – Calculate the absolute difference between each of the original values and the median calculated in step one, giving us a new series of "absolute difference" values;

Step three – Find the median for our series of absolute difference values.

Expressed mathematically, the MAD for a series of n values $x_1, \dots, x_n$, is given by;

$$MAD = median \{ |x_i - \bar{x}| ; i = 1, \dots, n \}$$

where $\bar{x}$ is the median for the series of values.

Properties

Like the inter-quartile range, examined in *ANALyse* 8, the MAD estimate exhibits resilience to unusual, or outlying, values present in a series of observations. In fact, it is even more resilient than the inter-quartile range, providing an interval the size of which remains relatively unaffected even when up to 50% of the values are well away from the main group of values.

The interval $\pm \text{MAD}$ is symmetric about the median and contains half the total number of values in a given set of values.

A useful relationship between MAD and the more common standard deviation, $s(x)$, for large samples drawn from a normal distribution, is:

$$s(x) \approx 1.483 \times \text{MAD}$$

This can be used to compare intervals produced by each statistic.

Scalar examples

Example 1: Find the MAD for the following series of seven (well-behaved) observations;

7.58, 7.69, 7.75, 7.72, 7.62, 7.64

Step 1: the median of these values is 7.65.

Step 2: the absolute difference between each of the original values and the median calculated in step one gives us the following new series of "absolute difference" values;

0.07, 0.04, 0.10, 0.07, 0.03, 0.01

Step 3: the median for the series of absolute difference values is 0.04.

The MAD is therefore 0.04 for the series of observations.

By comparison, the standard deviation for these observations is 0.06. This agrees well with the relationship given above between MAD and the standard deviation.

Example 2: Find the MAD for the following series of seven observations (containing the unusual value of 7.95);

7.58, 7.69, 7.95, 7.72, 7.62, 7.64

Step 1: the median of these values is still 7.65.

Step 2: the series of absolute difference values is now;

0.00, 0.07, 0.04, 0.30, 0.07, 0.03, 0.01

Step 3: the median for the series of absolute difference values is still 0.04.

The MAD is therefore still 0.04 for the modified series of observations (containing the unusual value).

By comparison, the standard deviation for the modified series of observations is 0.12. This value is approximately twice the standard deviation figure arrived at from the relationship between MAD and the standard deviation.

Vector applications

The definition of MAD given in this note can be extended easily to provide a dispersion indicator for vector data in two dimensions. Instead of finding the median for the original set of observations (step one, above), we find the Spatial Median for the set of *vector* observations (as discussed in ANAlyse 15). Step two of the MAD calculation is very similar to the scalar application except that the absolute difference is found between each of the original vector values and the spatial median (calculated in step one). *I.e.*, we calculate the *distance*, in the complex plane, between each vector value and the spatial median value. This produces a new series of values which are all *scalar* quantities. Finally, we find the median for our series of absolute difference values (as before). This is the MAD for our series of vector values.

We can interpret the MAD (scalar) value, for a series of vector values represented as points in the complex plane, as the radius of the circle centred on the spatial median that contains half of the data points.

Vector examples

To illustrate the performance of the MAD as a dispersion indicator for series of complex, vector, quantities, we will look at two data scatter plots. (These are similar to the plots used in ANAlyse 15 to illustrate the resilience of the position of the spatial median to unusual, outlying, values.)

Figure 1 shows eight relatively well-behaved data points (represented by the black dots in the complex plane). The position of the spatial median (as in ANAlyse 15) is marked with a "×". The circle is the data dispersion indicator, centred on the spatial median, with radius MAD.

Figure 2 shows the same data set, except one of the data points has been moved away from the main grouping (representing the inclusion of an unusual value). As before, the spatial median is marked with a "×" and the dispersion in the data is represented by a circle, centred on the spatial median, with radius MAD.

The radius of the circle (*i.e.*, the value for MAD) has increased only slightly with the inclusion of the unusual value in the set of vector data. This illustrates the MAD's relative resilience to outlying values in sets of vector measurements.

Conclusions

This note has demonstrated methods of calculating the MAD for both scalar and vector measurement data. It has shown that, for both types of data, the MAD exhibits a high degree of resilience to unusual values in measurement data sets. This is a highly desirable property when analysing data of this type.

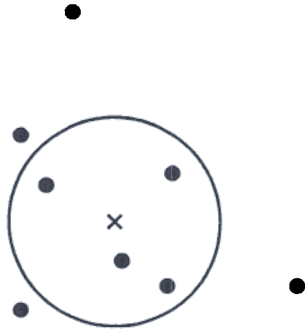


Figure 1: A data scatter plot for "well-behaved" data – The "x" locates the spatial median and the circle has radius MAD.



Figure 2: The same data set as figure 1 except one data point has been moved away from the main grouping. Spatial median and circle of radius MAD, as before.

ANALyse

An occasional series of Notes produced for, and by, the Members of ANAMET

Number 17

May 1996

“Two into one *can* go”

NPL’s expression of uncertainty for scattering coefficient measurements

Author: J P Ide, NPL, Malvern

Introduction

The Impedance section of NPL’s RF and Microwave Standards Division at Malvern issue calibration certificates for the scattering coefficients of a device, referred to a defined impedance, usually 50Ω for coaxial devices. In these certificates the default method is to supply the real and imaginary parts of the scattering coefficient, and a *single* figure for the uncertainty, which is applicable to both those parts. This note sets out to explain the reasoning behind the use of a single figure rather than having separate figures for each part.

Why Real and Imaginary rather than another format?

An ANA can produce results in a variety of formats, each of which is suitable for some particular use and none of which are universally used or accepted. Real and Imaginary parts are used throughout the process of generating the results because part of the process involves making multiple measurements and performing statistical operations on the data. This process does not necessarily produce valid results for other formats. Once the data processing is complete the results can be transformed into a variety of formats as can be seen later.

Typical presentation of results

A typical result for the voltage reflection coefficient (VRC) of a 50Ω Termination at 2.0 GHz might look like this.

Frequency (GHz)	VRC 50Ω Termination		
	Real	Imaginary	Uncertainty
2.0	0.0022	-0.0003	0.0017

This means that the most likely position of the VRC is at $0.0022 - j 0.0003$ and that NPL is more than 95% confident that the true (unknown and unknowable) value lies within a circle of radius 0.0017, centered on that point. This is shown in Figure 1.

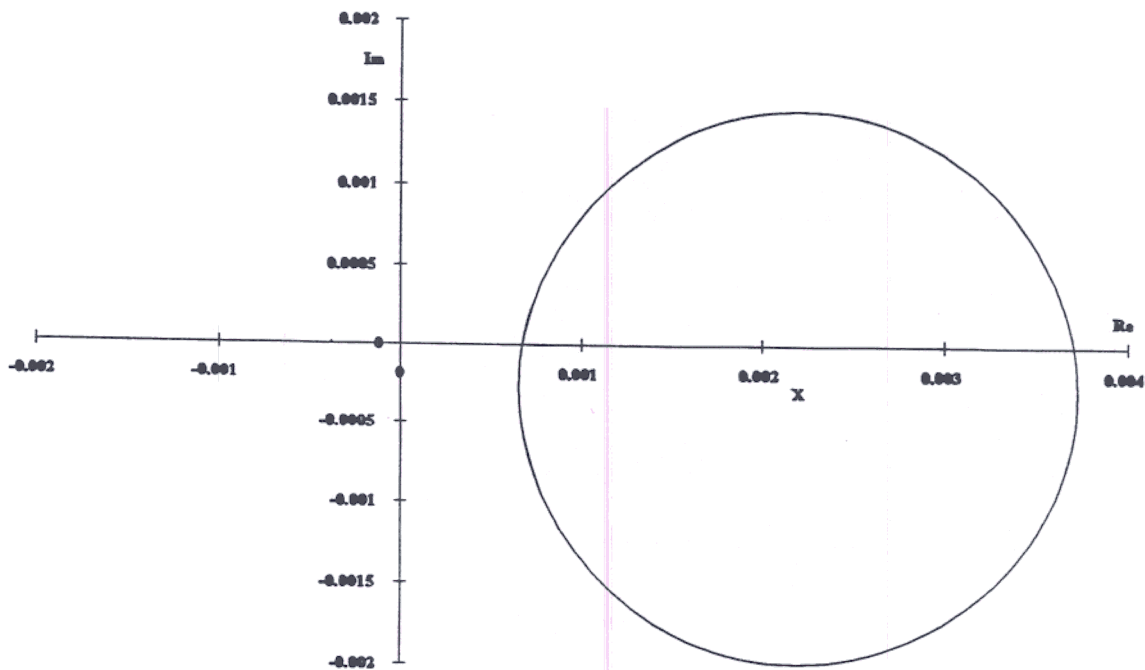


Figure 1

Conversion to other formats

The transformation from real and imaginary to magnitude and phase does not change the reference plane and so the same circle of uncertainty applies as shown in Figure 2. This means that the uncertainty ($\delta\Gamma$) in the magnitude (Γ) is given by the radius of the circle while the uncertainty ($\delta\theta$) in the phase (θ) is calculated from the arcsin ($\delta\Gamma/\Gamma$). For a well-matched device, the uncertainty in the magnitude is often greater than the magnitude value itself, causing the arcsin expression for the phase uncertainty to become meaningless. Under these circumstances the origin is within the circle of uncertainty and the phase is indeterminate which is represented on the calibration certificate by a phase uncertainty of $\pm 180^\circ$.

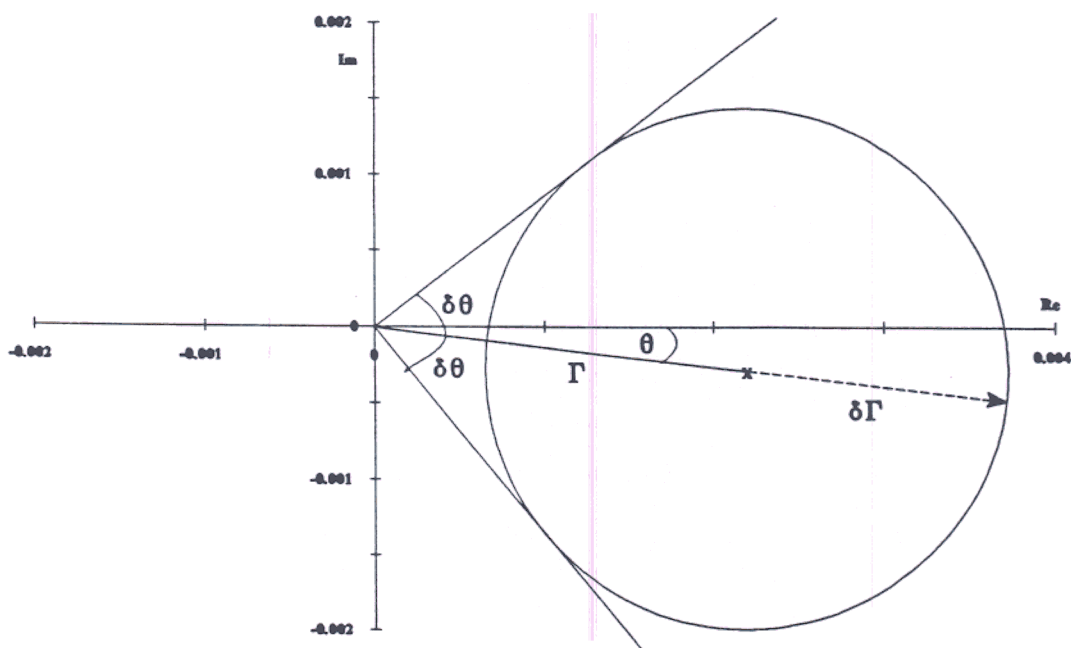


Figure 2

ANALyse

An occasional series of Notes produced by, and for, the Members of ANAMET

Number 18

June 1996

Hidden problems when employing partial differentiation in uncertainty assessments

Author: B Williams, DRA Aquila, SESC

Introduction

In analysing a measurement equation that is a function of two or more variables, $y = f(x, w, v \dots)$, it is fairly common practice and quite convenient to employ Newton's "Total Derivative". This will yield the overall error in y for *small finite errors* ($\delta x, \delta w, \delta v \dots$, in x, w and v respectively) when y is differentiated with respect to each of these variables separately:-

$$\delta y = \left[\frac{\partial y}{\partial x} \right] \cdot \delta x + \left[\frac{\partial y}{\partial w} \right] \cdot \delta w + \left[\frac{\partial y}{\partial v} \right] \cdot \delta v \dots \dots \dots (1)$$

The Problem

Using this approach in Impedance measurements, where phase information is involved, has led to some suspect results where the derived expression *appears* to work admirably over most of the range but then yields odd answers in particular regions.

The following is a good example of the case in point. It was taken from a mathematics textbook as a worked example for "A" level:-

$$a^2 = b^2 + c^2 - 2bc \cos A \quad (2)$$

and the instruction is to evaluate the change in "a" for small changes in b, c and angle A . (This is analogous to the situation arising from $\frac{1}{4}\lambda$ technique where b and c are reflection coefficients and A is the angle between them).

The solution is given as:-

$$\delta a = \left[\frac{b - c \cos A}{a} \right] \cdot \delta b + \left[\frac{c - b \cos A}{a} \right] \cdot \delta c + \left[\frac{bc \sin A}{a} \right] \cdot \delta A \quad \dots \dots (3)$$

An alternative method is to use a software spreadsheet in which it is easy to vary Equation (2) with respect to any one of the variables, holding the others constant, and then to compare that result with the equivalent component of equation (3). (Remembering, of course, the square root implied and to work in radians).

This, in fact, works very well in the case of varying b and c , i.e. the spreadsheet agrees with the first two terms of equation (3). However, when the same technique is applied to the third term of equation (3) a discrepancy arises, although this is not immediately obvious.

In the textbook example the nominal value of angle A is given as 60° . Graph 1 shows the results for δa plotted against $\delta A = 0^\circ$ to 10° , for both the spreadsheet and the third component of equation (3). A small difference between the two "curves" is just discernible at $\delta A = 10^\circ$. If we change the nominal value of angle A to 100° , Graph 2, the difference is clearer.

Continuing this process, Graph 3 shows the differences when angle A has a nominal value of 150° and Graph 4 illustrates the situation when angle A is nominally 170° ! Now the discrepancy cannot be overlooked!

What therefore is wrong? Is it a misunderstanding of "small print", i.e. what constitutes a small finite change, for even on Graph 4 (the worst case thus far), there is reasonable agreement provided δA is limited to $\pm 2^\circ$. But consider the case where the nominal value of angle A tends to 180° . $\sin A$ will now tend to zero and the third component of equation (3) will indicate that any finite value of δA will have no effect - which is clearly nonsense!

ANALyse Note No 6 recommends always working in rectangular coordinates and, indeed, the author has subscribed to that philosophy ever since the early nineteen seventies when Professor Don Woods demonstrated that computers can have difficulties in distinguishing between the vectors $-a-jb$ and $+a+jb$ when executing a polar conversion. However in this particular instant the measuring system, quite naturally, yields its data in polar form and the subsequent process of separating a residual reflection coefficient, from that of the DUT, is conveniently solved by the cosine rule (Equation 2). This also applies to many other problems in vector analysis.

The problem here is not so much the coordinates but rather the application of partial differentiation. Comparing equations (2) and (3); varying angle A , linearly, by $\pm \delta A$ in equation (2) has a *trigonometric* effect on the value of a , whereas the same variation in equation (3) has a *linear* effect since the differential coefficient is fixed at the nominal value of A . Thus, despite the textbook, equation (3) is **not valid**!

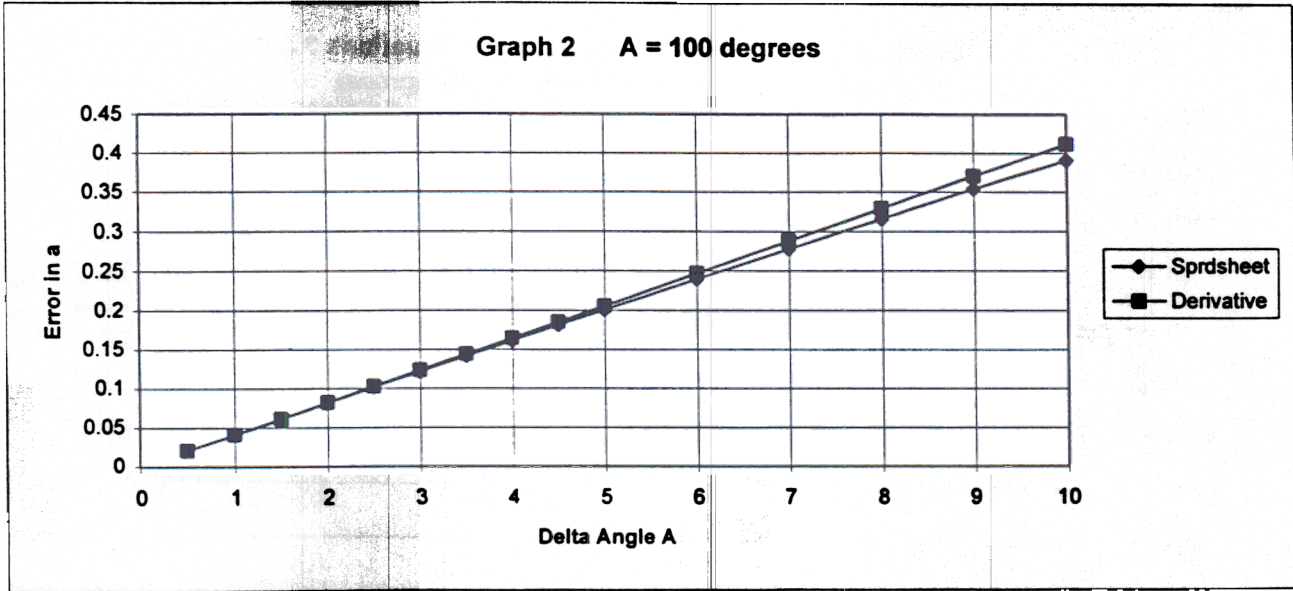
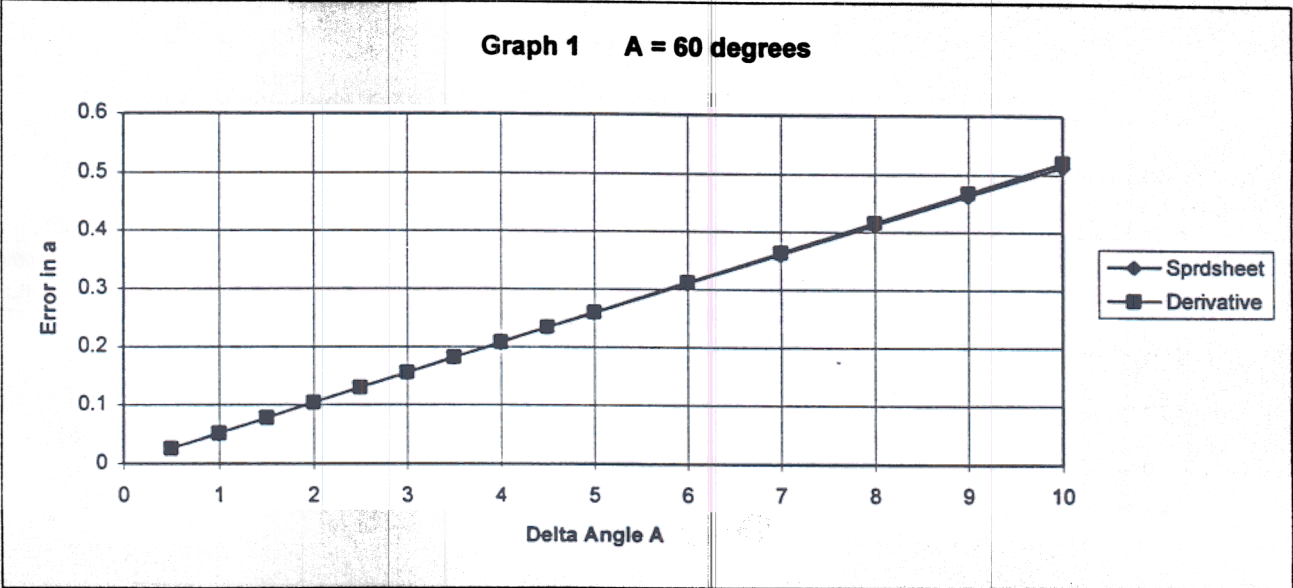
Recommendation

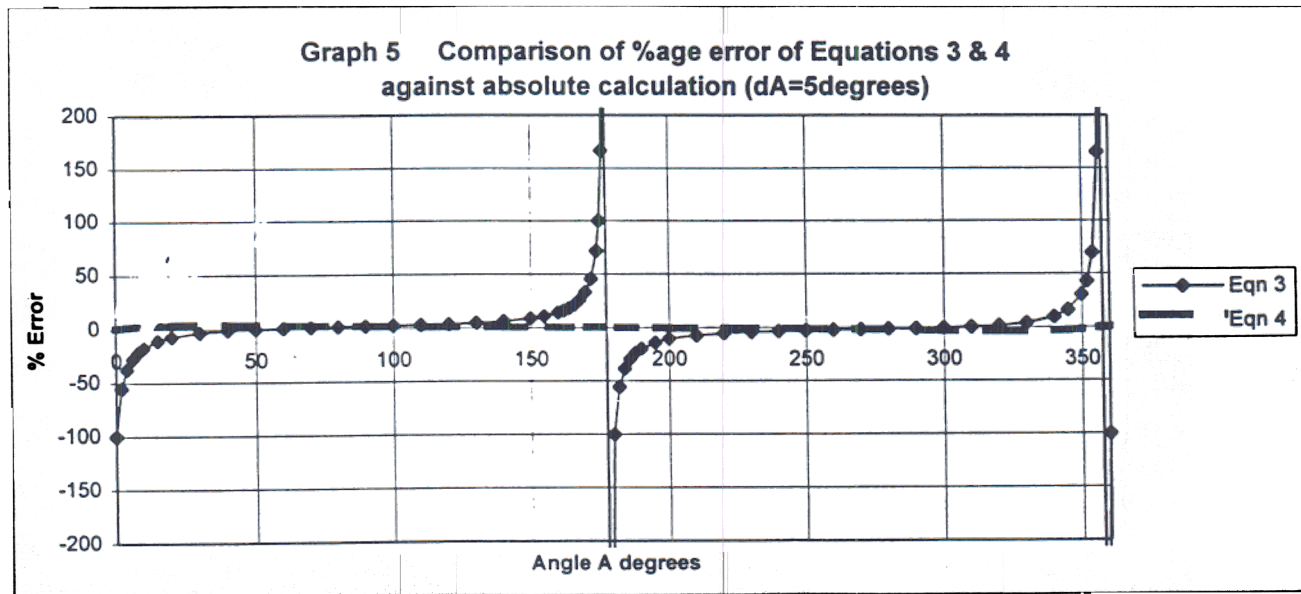
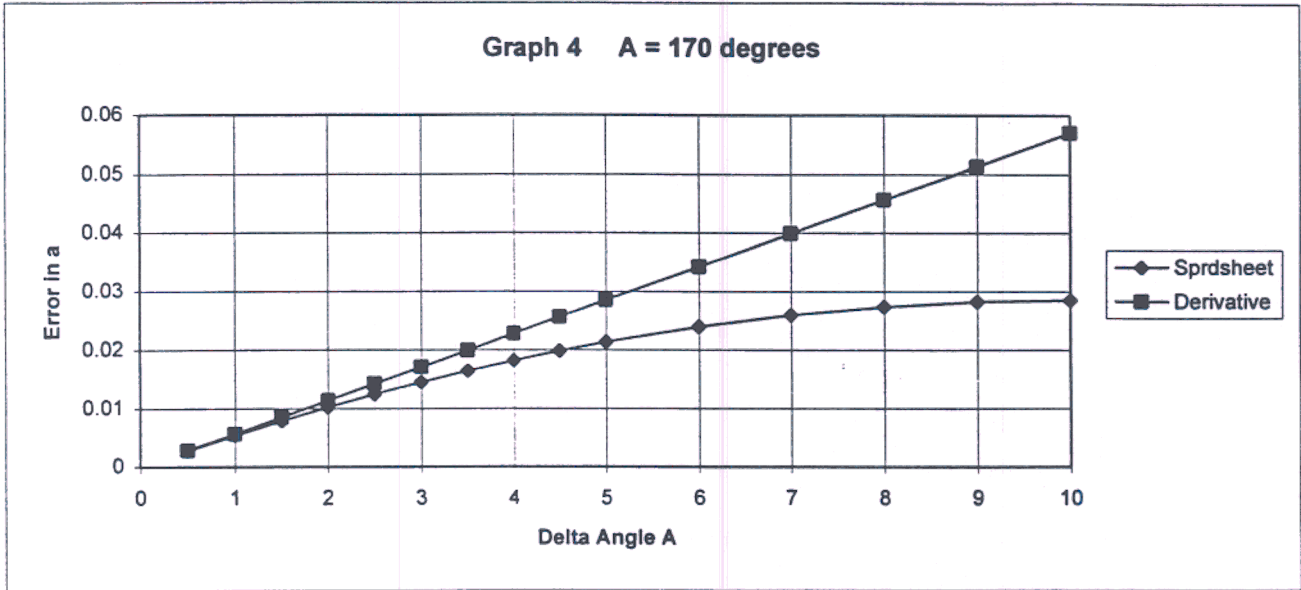
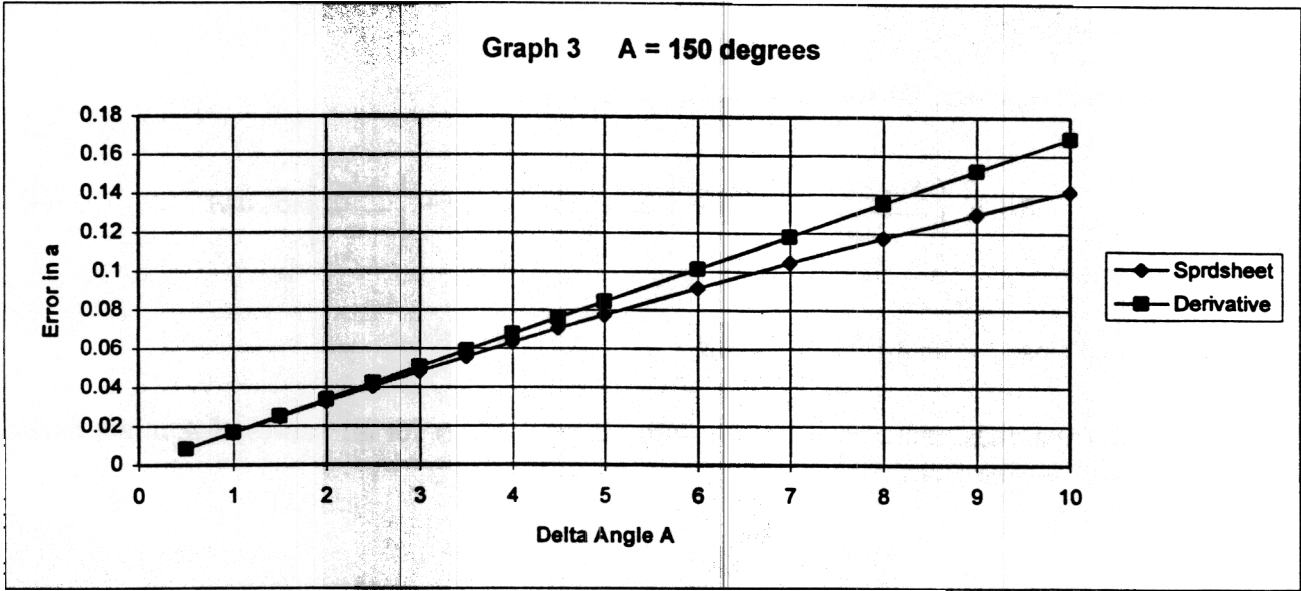
Differentiate equation (2) with respect to b,c and CosA :-

$$\delta a = \left[\frac{b - c \cos A}{a} \right] . \delta b + \left[\frac{c - b \cos A}{a} \right] . \delta c + \left[\frac{-bc}{a} \right] . \delta \cos A \dots\dots (4)$$

where: $\delta \cos A = \cos(A \pm \delta A) - \cos A$

Equation (4) agrees, more consistently, with the spreadsheet for all values of A in the range 0° to 360° – See Graph 5 !





ANALyse

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Number 19

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Partial differentiation in uncertainty assessments - Part 2: Simplifying the problem

Author: Manuel Rodríguez, INTA, Spain

Introduction

As a continuation to Mr Williams' contribution to the problem of partial derivatives in uncertainty calculations (ANALyse 18), let us simplify the problem to a minimum: by reducing the expression of the dependent variable to a straightforward function of a unique independent variable x , we will be able to represent the problem graphically and see things in the light of primary concepts of differential calculus.

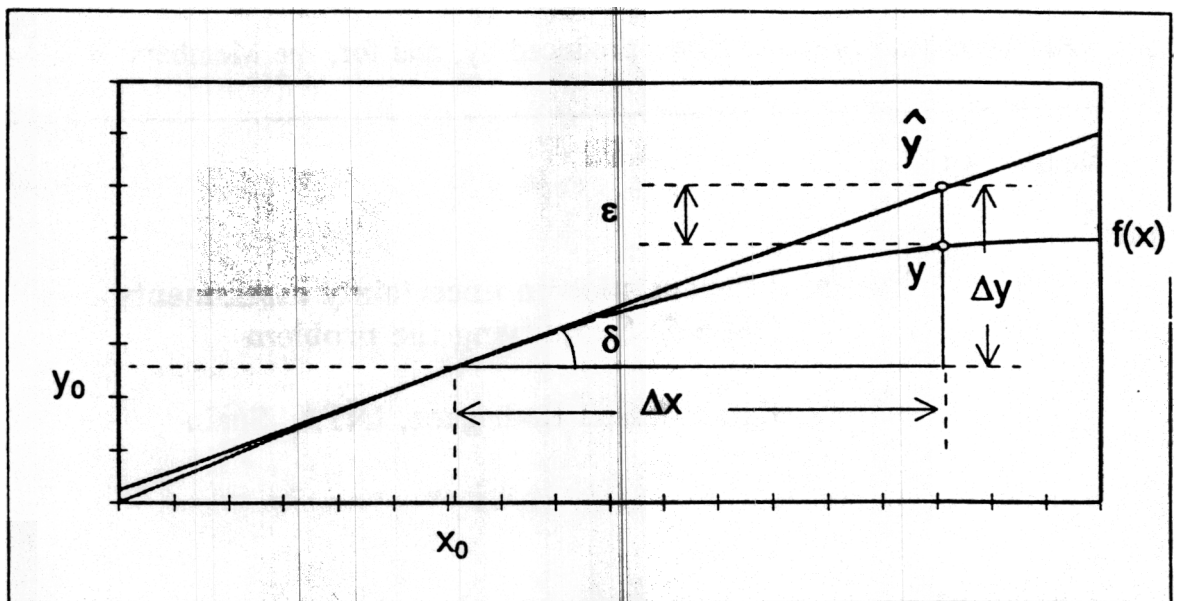
In a generic sense, one could state the aim of partial differentiation (and its application to uncertainty estimation) as: *trying to predict the behaviour of a certain function from our knowledge of it at certain given points.*

The problem

We have chosen a generic function, $y = f(x)$, in order to illustrate what differentiation really means. The first derivative of y is given by:

$$dy = f'(x).dx$$

And its meaning is: when we move around a given point (x_0, y_0) , the incremental change in y , Δy , is given approximately by the incremental change in x , Δx , times the "slope" of the curve at point (x_0, y_0) . This is illustrated on page 2.



we have:

$$\Delta y = \tan \delta \cdot \Delta x$$

$$\tan \delta = f'(x) |_{x_0}$$

Of course, an estimated value $\hat{y}$ ($= y_0 + \Delta y$) will never exactly coincide with the value of the function at the point x_0 , i.e., $y = f(x_0 + \Delta x)$, unless the derivative, or the "slope", remains constant within a certain range of values.

And here is where some confusion may arise, as to what extent the expression of the derivative is exact. In my opinion, this method for calculating incremental changes is always exact, provided that:

- 1) The incremental changes are small enough to assure that we are in the vicinity of (x_0, y_0) , the point for which we do know the derivative.
- 2) The rate of change of the first derivative is small enough (the second derivative comes to help us estimate how fast the "slope" of the curve is changing).
- 3) One admits the existence of a residual, unavoidable error, whose estimation is the task of the metrologist. This error, ϵ , is dependent on the above parameters:

$$\hat{y} = y_0 + \Delta y = f(x_0 + \Delta x) + \epsilon(\Delta x, f'(x) |_{x_0}, f''(x) |_{x_0})$$

Let us consider three examples relating the simple, well-known cosine function, $y = -\cos(x)$. The dependence of y with respect to the independent variable is similar to the example given in ANAlyse 18. We also use the same incremental changes in x , from 0° to 10° .

$$y = -\cos(x)$$
$$dy = \sin(x).dx$$

Δx (°)	Δy	$y = -\cos(x_0 + \Delta x)$	$\hat{y} = y_0 + \Delta y$	ϵ
0	0.0000	0.0000	0.0000	0.0000
1	0.0175	0.0175	0.0175	0.0000
2	0.0349	0.0349	0.0349	0.0000
3	0.0524	0.0523	0.0524	0.0000
4	0.0698	0.0698	0.0698	0.0001
5	0.0873	0.0872	0.0873	0.0001
6	0.1047	0.1045	0.1047	0.0002
7	0.1222	0.1219	0.1222	0.0003
8	0.1396	0.1392	0.1396	0.0005
9	0.1571	0.1564	0.1571	0.0006
10	0.1745	0.1736	0.1745	0.0009

Example 2 $x_0 = 170^\circ;$ $y_0 = 0.9848;$
 $f'(x_0) = 0.1736;$ $f''(x_0) = -0.9848.$

$\Delta x(^{\circ})$	Δy	$y = -\cos(x_0 + \Delta x)$	$\hat{y} = y_0 + \Delta y$	ϵ
0	0.0000	0.9848	0.9848	0.0000
1	0.0030	0.9877	0.9878	0.0002
2	0.0061	0.9903	0.9909	0.0006
3	0.0091	0.9925	0.9939	0.0014
4	0.0121	0.9945	0.9969	0.0024
5	0.0152	0.9962	1.0000	0.0038
6	0.0182	0.9976	1.0030	0.0054
7	0.0212	0.9986	1.0060	0.0074
8	0.0242	0.9994	1.0091	0.0097
9	0.0273	0.9998	1.0121	0.0122
10	0.0303	1.0000	1.0151	0.0151

Clearly, errors are larger than in the previous example. This is due to the fact that the slope is changing rapidly around 170° . An incremental change of 10° can no longer be considered "small", since for $x = 170^\circ \pm 10^\circ$, the first derivative changes from 0.3420 to 0.

Example 3 $x_0 = 180^\circ$;
 $f'(x_0) = 0$;

$$y_0 = 1;$$
$$f''(x_0) = -1.$$

$\Delta x(^{\circ})$	Δy	$y = -\cos(x_0 + \Delta x)$	$\hat{y} = y_0 + \Delta y$	ϵ
0	0.0000	1.0000	1.0000	0.0000
1	0.0000	0.9998	1.0000	0.0002
2	0.0000	0.9994	1.0000	0.0006
3	0.0000	0.9986	1.0000	0.0014
4	0.0000	0.9976	1.0000	0.0024
5	0.0000	0.9962	1.0000	0.0038
6	0.0000	0.9945	1.0000	0.0055
7	0.0000	0.9925	1.0000	0.0075
8	0.0000	0.9903	1.0000	0.0097
9	0.0000	0.9877	1.0000	0.0123
10	0.0000	0.9848	1.0000	0.0152

In this case, the incremental changes are evaluated as zero because the first derivative goes to zero at $x_0 = 180^\circ$. The absolute errors ϵ almost coincide with those calculated for $x_0 = 170^\circ$, which seems reasonable since both the first and the second derivatives are quite similar.

In my opinion, it does not seem sensible to define relative errors, since they would be dependent upon the actual value of y . One method of normalising the absolute errors could be with respect to the maximum range of variation of the dependent variable within the area of interest:

$$\epsilon_{norm} = \frac{\epsilon}{y_{max} - y_{min}}$$

In our example, $y_{max} - y_{min} = 2$. For $x_0 = 180^\circ$, the normalized error so obtained ranges from 0.01% for $\Delta x = 1^\circ$ to 0.75% for $\Delta x = 10^\circ$.

ANALyse

An occasional series of Notes produced by, and for, the Members of ANAMET

Number 20

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The uncertainty in the median

Author: Nick Ridler, NPL Malvern

Introduction

At the present time, the median has been used within ANAMET to analyse the results produced by its measurement comparison exercises. It has been seen to provide a useful summary value for the comparison data especially when the data contains unusual, or outlying, values. The reason behind this usefulness is that, unlike the sample mean which is the conventional choice of summary value, the median is relatively unaffected by outlying values in a data set. (See, for example, ANALyse 7.)

This Note considers one of the implications of using the median as a summary value for analysing repeat measurements produced by a single measurement system. In particular, having found the median for a series of repeat determinations of a quantity, what is the *uncertainty* in the median? To help answer this question, we will assume that our measurement results conform, in the main, to the assumption of normality except there are occasion outlying values "contaminating" the sets of results.

Relating MAD, standard deviation and the uncertainty in the median

We will use the spread indicator MAD (the median absolute deviation - see ANALyse 16), as the starting point for an estimate of the uncertainty in the median. A useful relationship relating MAD to the standard deviation, $\sigma(x)$, is given by;

$$\sigma(x) \approx 1.483 \times \text{MAD} \quad (1)$$

(Strictly speaking, this is only true for large sample sizes drawn from a normal distribution.)

Also, the standard deviation for the median, (*i.e.*, the standard uncertainty in the median, $s(\bar{x})$) derived from n repeat observations, is given by;

$$s(\bar{x}) = \sqrt{\frac{\pi}{2n}} \sigma(x) \quad (2)$$

Substituting equation (1) into (2), we get;

$$s(\bar{x}) = \sqrt{\frac{\pi}{2n}} 1.483 \times \text{MAD}$$

or

$$s(\bar{x}) = \frac{1.86}{\sqrt{n}} \times \text{MAD} \quad (3)$$

Therefore, for a series of n repeat measurements, we can use equation (3) to calculate the standard uncertainty in the median from the MAD.

Median versus mean

When analysing repeat measurements, the median, along with its standard uncertainty, can be used in place of the mean and its standard uncertainty. In fact, for data which does not conform rigidly to the normal distribution assumption, the median is likely to provide a "better" estimate, and this is demonstrated by its uncertainty interval being smaller than that for the mean.

Example 1 - Ten measurements

The following ten measurements of the capacitance of a 10 pF capacitor at 400 MHz were made using an ANA [1] (they have been placed in ascending order of size):

10.721 pF; 10.802 pF; 10.818 pF; 10.822 pF; 10.822 pF;
10.824 pF; 10.829 pF; 10.832 pF; 10.833 pF; 10.833 pF.

This gives a median for the ten results of;

$$\bar{x} = \frac{1}{2} (10.822 + 10.824) = 10.823 \text{ pF}$$

To find the uncertainty in the median, first we find the median of the absolute deviations from the above median value – the MAD calculation. The absolute deviations from the above median are given below:

0.102 pF; 0.021 pF; 0.005 pF; 0.001 pF; 0.001 pF;
0.001 pF; 0.006 pF; 0.009 pF; 0.010 pF; 0.010 pF.

The median of these absolute deviations is:

$$\text{MAD} = \frac{1}{2} (0.006 + 0.009) = 0.0075 \text{ pF}$$

We now use equation (3) to convert the MAD into the standard uncertainty in the median:

$$s(\bar{x}) = \frac{1.86}{\sqrt{10}} \times 0.0075 = 0.004 \text{ pF}$$

Our result is therefore:

$$\text{Capacitance} = (10.823 \pm 0.004) \text{ pF}$$

We can compare this uncertainty estimate with the more conventional method of calculating the standard uncertainty in the mean [2,3]. The mean, $\bar{x}$, and experimental standard deviation, $s(x_i)$, for the above ten results is:

$$\bar{x} = 10.8136 \text{ pF}$$

$$s(x_i) = 0.0338 \text{ pF}$$

This leads to a standard uncertainty in the mean of:

$$s(\bar{x}) = \frac{s(x_i)}{\sqrt{n}} = 0.011 \text{ pF}$$

Our result is therefore:

$$\text{Capacitance} = (10.814 \pm 0.011) \text{ pF}$$

This is considerably larger than the standard uncertainty in the median – How can this be?

Closer examination of the ten capacitance measurements reveals that the lowest value is far removed from the other nine values. (The outlier syndrome strikes again!) If we reject the suspected outlier and re-do our calculations, we can see what effect the suspected outlier has on the calculations. (Strictly speaking, it is metrologically unethical to reject results purely on subjective grounds, such as; "I don't like the look of this one . . . let's get rid of it!". But how many of us are guilty of such practises?!)

Example 2 - Ten measurements reduced to nine

Having rejected the lowest capacitance value we can re-calculate the median for the remaining nine. (It will now be the fifth value in the ordered results.):

$$\bar{x} = 10.824 \text{ pF}$$

The absolute deviations from this median are now:

$$\begin{array}{ccccccccc} & 0.022 \text{ pF;} & 0.006 \text{ pF;} & 0.002 \text{ pF;} & 0.002 \text{ pF;} & & & & \\ 0 \text{ pF;} & 0.005 \text{ pF;} & 0.008 \text{ pF;} & 0.009 \text{ pF;} & 0.009 \text{ pF;} & & & & \end{array}$$

The median of these absolute deviations is:

$$\text{MAD} = 0.006 \text{ pF}$$

Again, using equation (3) to convert the MAD into the standard uncertainty in the median, we have:

$$s(\bar{x}) = \frac{1.86}{\sqrt{9}} \times 0.006 = 0.004 \text{ pF}$$

Our result is therefore:

$$\text{Capacitance} = (10.824 \pm 0.004) \text{ pF}$$

To complete the calculation exercise, we re-calculate the mean and its standard uncertainty for the nine results.

$$\bar{x} = 10.8239 \text{ pF}$$

$$s(x_i) = 0.0099 \text{ pF}$$

This leads to a standard uncertainty in the mean of:

$$s(\bar{x}) = \frac{0.0099}{\sqrt{9}} = 0.003 \text{ pF}$$

Our result is therefore:

$$\text{Capacitance} = (10.824 \pm 0.003) \text{ pF}$$

Discussion

The Table below summarises the calculations in the previous examples.

	Calculations based on ten capacitance measurements	Calculations based on nine capacitance measurements
The median and its standard uncertainty	$(10.823 \pm 0.004) \text{ pF}$	$(10.824 \pm 0.004) \text{ pF}$
The mean and its standard uncertainty	$(10.814 \pm 0.011) \text{ pF}$	$(10.824 \pm 0.003) \text{ pF}$

Quite clearly, the tenth value in our set of capacitance results has upset the calculation of the mean and its standard uncertainty. The mean value is skewed from the majority of values (which lie around 10.824 pF) and the standard uncertainty provides an unrealistically large interval when viewed in the context of the grouping of the other nine values. A seasoned metrologist may well examine the data closely and reject the tenth value leaving him (at least) with a guilty conscience. However with modern automated measuring instruments, such as ANAs, producing vast amounts of data, such close scrutiny of results is not always practicable.

The above scheme of using the median and its associated standard uncertainty is an alternative method of estimating realistic uncertainty intervals in data sets where unusual values may or may not occur, without the need to reject values. Their use has been proposed elsewhere [4] for the treatment of measurement comparison data.

Conclusions

The median and its associated standard uncertainty can be useful estimators for evaluating the size of an uncertainty component using statistical techniques (the so-called type A method [3]). Typical uncertainty components which could be treated in this way are electrical noise in an ANA and the connection repeatability of an item to the ANA test port.

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- [2] NAMAS, "The expression of uncertainty and confidence in measurement for calibration", *United Kingdom Accreditation Service*, NIS 3003, Edition 8, May 1995.
- [3] ISO, "Guide to the expression of uncertainty in measurement", *International Organization for Standardization*, Genève, Switzerland. First edition, 1993.
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