

Good Practice Guide No. 130

Co-ordinate measuring machine
task-specific measurement uncertainties

David Flack



Measurement Good Practice Guide No. 130

Co-ordinate measuring machine task-specific measurement uncertainties

David Flack
Engineering Measurement Division
Engineering Measurement Services Group

ABSTRACT

This guide covers methods to ascertain the measurement uncertainty associated with measurements made on a co-ordinate measuring machine. It describes the need to take in to account measurement uncertainty when making measurements on co-ordinate measuring machines. Methods for determining task-specific measurement uncertainties are covered with particular reference to ISO 15530.

© Crown Copyright 2013
Reproduced with the permission of the Controller of HMSO
and Queen's Printer for Scotland

September 2013

ISSN 1368-6550

National Physical Laboratory
Hampton Road, Teddington, Middlesex, TW11 0LW

Acknowledgements

The author would like to thank Dr. Alan Wilson for providing useful technical input. The work was funded by the UK National Measurement System Programme for Engineering & Flow Metrology.

Contents

Introduction.....	1
What this guide is about and what it is not?	1
Basic introduction to uncertainties	1
Expression of uncertainty in measurement	2
The contributions to the uncertainty budget.....	2
Expanded uncertainty	3
The statement of uncertainty	3
Confidence level.....	4
Type A and Type B contributions	5
Distributions	8
CMMs, traceability and uncertainties	10
CMM geometries/error maps and uncertainties	11
Decision rules and uncertainties	12
Summary of ISO 14253.....	15
CMM Measurement Uncertainties.....	18
Why is the MPE value not good enough?	19
Analytical approach for simple cases	19
Worked example using a substitution method	20
Worked example when substitution method not used.....	26
Worked example using a reversal method	28
Introduction to ISO 15530.....	30
Comparison technique – use of calibrated workpieces	30
Uncertainty evaluating software.....	31
Uncertainty evaluating software.....	33
Commercial implementations of the PTB model (Calypso/Quindos)	34
What if you are not a Calypso/Quindos user – PUNDITCMM?	36
Detailed description of PTB model based around Calypso implementation ...	38
Description of PTB model based around Quindos 7 implementation.....	41
Comparison of the techniques.....	42
What’s not covered in UES	43
Use of UES to design experiments.....	43
Advantages and disadvantages of the various methods	43
UES validity	43
UES missing uncertainty contributions.....	44
UES simulation time	44
Disadvantage of the comparison technique.....	44
Techniques to minimise measurement uncertainty	46

Minimising measurement uncertainty	47
Cleanliness	47
Form	47
Clamping	47
Stylus selection.....	48
Stylus qualification.....	48
Probing strategies	48
Temperatures/ thermal equilibrium.....	49
Redundancy	49
Comparison	49
Scanning.....	49
Probing force	50
Object	50
Interim checks of the CMM	50
Summary.....	52
Glossary of terms	54
Glossary of terms.....	55
Appendices.....	58
Appendix A Links to other useful sources of information	59
A.1 National and International Organisations.....	59
A.1.1 National Physical Laboratory	59
A.1.2 National Institute of Standards and Technology (NIST).....	60
A.1.3 EURAMET	60
A.2 Networks	61
A.2.1 Measurement Network – Engineering and Optical	61
A.2.2 Software Support for Metrology Programme (SSfM).....	61
A.3 National and International Standards.....	62
A.3.1 British Standards Institution (BSI)	62
A.3.2 International Organisation for Standardization (ISO)	62
A.4 Traceability	63
A.5 Training courses	64
A.5.1 Dimensional Measurement training: Level 1 – measurement User.....	64
A.5.2 Dimensional Measurement Training: Level 2 - Measurement Applier.....	65
A.5.3 Understanding and Evaluating measurement uncertainty	67
Appendix B CMM errors.....	68
Appendix C Monte Carlo – example code	69
Appendix D Further reading	71

List of Figures

Figure 1 A histogram of the 100 readings	5
Figure 2 Measured value with measurement uncertainty	8
Figure 3 Normal or bell shaped curve.....	8
Figure 4 An example of a rectangular distribution	9
Figure 5 Measurement on a CMM - flow chart (based on Figure B.1 of ISO 15530-4).	12
Figure 6 The two results and their associated uncertainties	13
Figure 7 Uncertainty of measurement: the uncertainty range reduces the conformance and non-conformance zones (Copyright BSI – extract from BS EN ISO 14253- 1:1999).....	14
Figure 8 Conformance or non-conformance.....	15
Figure 9 Output from the Calypso OVCM. Note the column marked measurement uncertainty.	35
Figure 10 Determining the residual error field of a CMM using a hole plate made from ZERODUR®, a low expansion glass ceramic. Notice that the stylus configuration allows measurement of the plate from both sides.....	36
Figure 11 The user interface of PUNDITCMM	37
Figure 12 The dialogue for extended stylus qualification in Calypso.	39
Figure 13 Input of the workpiece expansion coefficient uncertainty in Calypso. The example here is for a workpiece with an expansion coefficient of $11.5 \times 10^{-6} \text{ K}^{-1}$ with an uncertainty of $1 \times 10^{-6} \text{ K}^{-1}$	40
Figure 14 The output of the Matlab script to simulate rolling of a pair of dice.....	69
Figure 15 Output from the above Matlab script to demonstrate measurement of points on a circle	71

CMM measurement uncertainties

Preface

The author hopes that after reading this Good Practice Guide you will be able to better understand the specifications relating to co-ordinate measuring machines and the importance of measurement uncertainty to CMM measurements. The content is written at a simpler technical level than many of the published works on CMM uncertainty so that a wider audience can understand it. I am not trying to replace a whole raft of published papers, operator's manuals, specifications and standards, rather present an overview of good practice and techniques.

“Metrology is not just a process of measurement that is applied to an end product. It should also be one of the considerations taken into account at the design stage. According to the Geometrical Product Specification (GPS) model, tolerancing and uncertainty issues should be taken into account during all stages of design, manufacture and testing. The most compelling reason is that it is often considerably more expensive to re-engineer a product at a later stage when it is found that it is difficult to measure, compared to designing at the start with the needs of metrology in mind.” Professor Richard Leach 2003

GOOD MEASUREMENT PRACTICE

There are six guiding principles to good measurement practice that have been defined by NPL. They are:

The Right Measurements: *Measurements should only be made to satisfy agreed and well-specified requirements.*

The Right Tools: *Measurements should be made using equipment and methods that have been demonstrated to be fit for purpose.*

The Right People: *Measurement staff should be competent, properly qualified and well informed.*

Regular Review: *There should be both internal and independent assessment of the technical performance of all measurement facilities and procedures.*

Demonstratable Consistency: *Measurements made in one location should be consistent with those made elsewhere.*

The Right Procedures: *Well-defined procedures consistent with national or international standards should be in place for all measurements.*

Introduction

1

IN THIS CHAPTER

- What this guide is about and what it is not?
- Basic introduction to uncertainties
- CMMs, traceability and uncertainties
- CMM geometries/error maps and uncertainties
- Decision rules and uncertainties

This measurement good practice guide will provide an overview of methods used for determining the measurement uncertainty associated with a measurement made on a co-ordinate measuring machine. Many hundreds of thousands of measurements are made every year to check that components meet specification. Many of these measurements are made using co-ordinate measuring machines (CMMs). Decision rules in international standard ISO 14253 state that to demonstrate conformance with a specification, the measurement result should have an associated measurement uncertainty. If you are going to claim that your results are traceable, a requirement of ISO 9001, then an uncertainty needs to be associated with the result. But how do you calculate an uncertainty for measurements made on a CMM? This guide will indicate some of the methods available to the end user and give good practice in the application of these methods.

What this guide is about and what it is not?

It is intended that this guide should give enough information so that the metrologist can interpret the requirements of the international standards relating to CMM uncertainty. After reading this guide, the user should be aware of the various techniques available for estimating CMM uncertainty. This good practice guide is not intended to be an authoritative guide. The treatment of CMM uncertainties is a complex subject and this guide's aim is to give an overview of the subject, and point to more technical references on the subject.

Basic introduction to uncertainties

The formal definition of uncertainty is:

parameter, associated with the result of a measurement, that characterizes the dispersion of the values that could reasonably be attributed to the measurand

(PD 6461-4:2004 General metrology. Practical guide to measurement uncertainty).

Putting this statement another way, uncertainty is a parameter associated with a specific measurement result that expresses the range of values about that result within which we can confidently expect to find the true value of the measurand.

A complete statement of a measurement result from a CMM might look like this:

(50.005 ± 0.005) mm,

the range of values between which the true value of the measurand might lie is 50.000 mm to 50.010 mm.

NPL has produced a good practice guide on uncertainty of measurement, Bell S A 2001 *A beginner's guide to uncertainty in measurement* Measurement Good Practice Guide No. 11, Issue 2 (NPL). If the reader is unfamiliar with measurement uncertainty, it is advised they read this guide before reading the following section.

Expression of uncertainty in measurement

At its most basic, measurement uncertainty is a statement of how well someone thinks that they have measured something. Measurement uncertainty can, therefore, be considered as a guide to the quality of the measurement. There are a number of intimidating and highly mathematical explanations of how to estimate measurement uncertainty – the ISO *Guide to the Expression of Uncertainty in Measurement (GUM)* being the definitive article.

We shall present a somewhat simpler explanation here. There is no way to actually know the true value of a particular measurement because, whatever measurement process you use, there will always be some sort of error associated with that process. It follows from this that the result of any measurement is really only an estimate of the true value of that measurand, and so needs to be accompanied by an estimate of the uncertainty of the correctness of the stated result.

Systematic and random errors

Your measurement system can have two types of error. Systematic and random. Systematic errors are errors that are the same from one measurement to the next, for example, an error in the measurement scale. Random errors are those that are different from one measurement to the next.

Even after all the systematic effects have been corrected for, there remains an uncertainty due to both random effects and imperfect correction for the results of systematic effects. The overall uncertainty estimate gives a quantitative (or numerical) assessment of the reliability of the result, and allows us to compare results with one another in a meaningful manner. This can be helpful, as we shall see later.

The contributions to the uncertainty budget

Once we know our contributions to the uncertainty budget they need to be expressed and combined in some manner. If the measurand, Y , is a function of several input quantities, X_i , we can write

$$Y = f(X_1, X_2, \dots, X_n).$$

If there were only three input quantities we could write

$$Y = f(X_1, X_2, X_3).$$

For instance, if Y were the volume of a cube, then the three input quantities would be the length, breadth and height of the cube.

Very often, an estimate, y , of the measurand Y (in other words, y is the result of a measurement of Y) is determined from n other estimates of the input quantities ($x_1, x_2, \dots, x_n$) of input quantities ($X_1, X_2, \dots, X_n$) through some functional relationship f , rather than being measured directly, thus

$$y = f(x_1, x_2, \dots, x_n).$$

These input estimates ($x_1, x_2, \dots, x_n$) may either be quantities whose values and uncertainties are determined directly during the current measurement (for example, ambient temperature, humidity, *etc.*), or quantities whose values and uncertainties are brought into the process from external sources (for example, data from other certificates, handbooks, *etc.*)

Using our cube example again the input estimates are the measured values of length, breadth and height.

The combined standard uncertainty of the measurement result, written as $u_c(y)$, is determined from the estimated standard uncertainty of each input estimate x_i and written as $u(x_i)$. Each standard uncertainty associated with the input estimates is categorised according to the method in which its numerical value is obtained. Those uncertainties obtained by using statistical methods are termed Type A, whilst those obtained by methods other than statistical are termed Type B.

It should be noted that there is no direct correspondence between the Type A and Type B classifications and the old terms ‘random’ and ‘systematic’. Type A and B classifications refer to the manner in which the uncertainty was estimated, and not to the effect of the uncertainty on the measurement result (in other words an uncertainty previously referred to as ‘systematic’ might be obtained using statistical methods, a method previously thought of as reserved for ‘random’ uncertainties). Furthermore, an uncertainty that is a Type A at one level in the hierarchical national measurement system becomes a Type B at the next level down.

Expanded uncertainty

Once we have calculated all the Type A and Type B contributions, we have to combine them in to one number called the combined standard uncertainty. Assuming the contributions are not correlated (that is to say there are no inter-dependencies between contributions), the combined uncertainty is calculated by squaring each term, adding the squared terms together, and then taking the square root (a process known as quadrature addition).

The combined standard uncertainty is then multiplied by a coverage factor (k) to give the expanded uncertainty. The value of k for a confidence probability of 95 % is usually 1.96. This value is often rounded to 2 and the confidence probability expressed as approximately 95 %.

The statement of uncertainty

Assuming that we have a report with the results stated, for instance, in Table I and the uncertainties in Table II, the uncertainty of measurement will be stated in the following form.

The uncertainty of each mean value given in Table I has been calculated following the guidance given by ISO document ‘Guide to the expression of uncertainty in measurement’. The expanded uncertainties are given in Table II and each is based on a standard uncertainty multiplied by a coverage factor $k = 2$, providing a confidence probability of approximately 95 %.

The above statement is a good model if you need to quote an uncertainty. However, it will need to be modified if the k value is not 2, for instance when the number of degrees of freedom (see next box) is less than about thirty. An example statement in this case is given below.

The expanded uncertainty associated with the measurement of uniformity of diameter is based on a standard uncertainty multiplied by a coverage factor $k = 2.10$ which, for a t -distribution with $\nu_{\text{eff}} = 27$ degrees of freedom, provides a confidence probability of approximately 95 %.

What is a t -distribution?

The t -distribution is a probability distribution similar to the normal distribution. It differs from the normal distribution in that it has an additional parameter, called degrees of freedom, which changes its shape. A t -distribution with a high number of degrees of freedom (greater than 30) approaches a normal distribution.

Confidence level

We have just mentioned something called a confidence level. In the old days before *GUM* metrologists would make a number of measurements, calculate the mean, variance and standard deviation, assume a normal probability distribution and state the result as the mean plus or minus three times the standard deviation.

The old timers were describing the range within which the true value fell. Having made the assumption that the probability distribution is normal we are implying that no more than 3 in 1000 of the results would fall outside the bounds described by $\pm 3\sigma$. We will describe σ or sigma shortly but basically it is the standard deviation. The total range from -3σ to $+3\sigma$ is of course 6σ which may ring a bell for those readers with a background in statistical process control.

The expression ‘with 95 % confidence’ comes from the statistics and is actually trying to say that values fall within 2σ , or two standard deviations, of the mean. If we assume that the results are normally distributed then 5 % or 1 in 20 of the results in any given sample will fall outside those stated uncertainty limits.

If we look at the distribution of 100 readings on an artefact nominally 51 mm in size we might get a histogram plot very similar to that in Figure 1. The red line shows a normal distribution with the same mean and standard deviation. The mean of the distribution is 51.0000 mm with a standard deviation of 0.0012 mm (68 %). The 95 % confidence intervals are 50.9998 mm and 51.0003 mm.

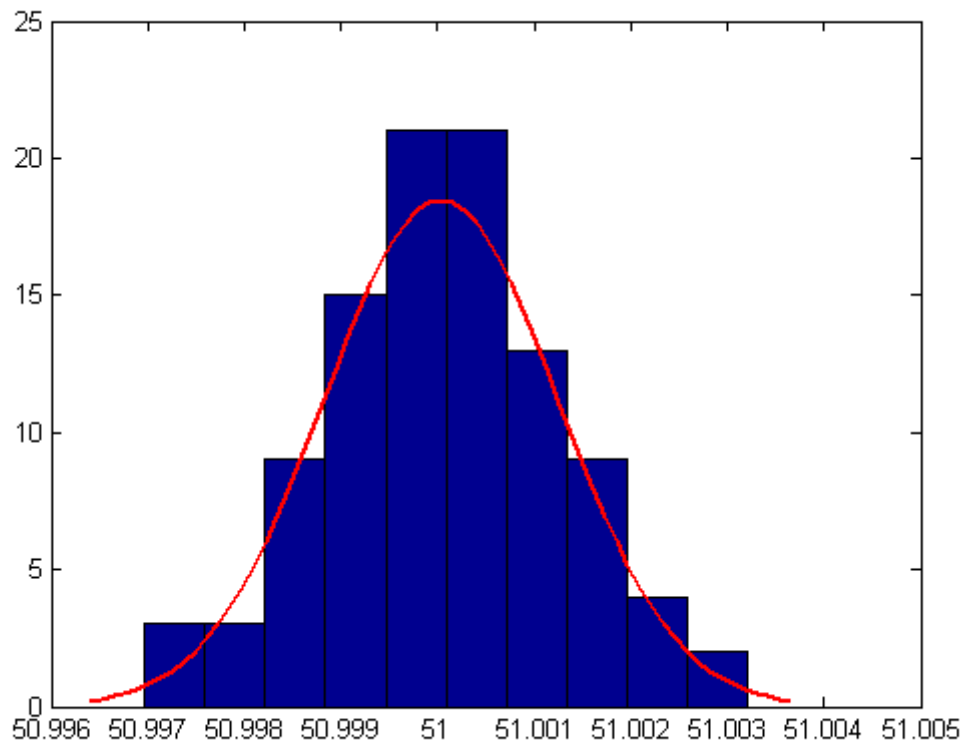


Figure 1 A histogram of the 100 readings

We can say that 68 % of readings are no more than 1σ away from the mean, 95 % are no more than 2σ away from the mean and 99.7 % no more than 3σ away from the mean. That is to say at 3σ only three readings in one thousand will lie outside the claimed boundaries.

Type A and Type B contributions

This next section shows, in very basic terms, how Type A and Type B uncertainty contributions are determined.

Type A uncertainty evaluation

Type A evaluations are normally used to determine the repeatability or randomness of a measurement process. Should n independent observations of an input quantity be made under the same conditions of measurement, then the best possible estimate of the input quantity is the arithmetic mean or average of all the observations.

The mean can be written as

$$\bar{x} = \frac{1}{n} \sum_{k=1}^n x_k$$

where x_k are the individual measurements and n is the number of measurements. The sigma sign notation (Σ) is mathematical notation for ‘add up all the n readings’.

A measure of the dispersion of each individual observation around their mean is called the experimental standard deviation and is determined from the square root of the variance of the observations

$$s = \sqrt{\frac{1}{n} \sum_{k=1}^n (x_k - \bar{x})^2}$$

This standard deviation is for the particular set of n observations, and would be different to another set of observations of the same measurand. However, an estimate of the standard deviation for the entire population of possible values of the measurand can be made by replacing the n in the denominator of the equation above by $n - 1$

$$\sigma_x = \sqrt{\frac{1}{n-1} \sum_{k=1}^n (x_k - \bar{x})^2}$$

This standard deviation is the uncertainty associated with any one observation taken of the measurand. As would be expected, the variability of the mean value of several observations reduces as the number of observations increases and is given by the experimental standard error of the mean

$$\sigma_{\bar{x}} = \frac{\sigma_x}{\sqrt{n}}$$

The standard error of the mean is the standard uncertainty associated with the average of several observations. Even though the individual observations might not follow a normal distribution, the average values of sets of observations will generally have a normal probability distribution.

The degrees of freedom of the uncertainty estimation should be reported along with the results of the Type A evaluation. Broadly defined, the degrees of freedom, v_i are the number of terms used to obtain the estimation minus the number of constraints on those terms. For a Type A statistical analysis this results in

$$v_i = n - 1$$

In the case of a least squares fit evaluation, two constraints exist (both the slope of the fitted line and the intercept) and the number of degrees of freedom becomes

$$v_i = n - 2$$

Type B uncertainty evaluation

All uncertainties associated with input estimates that have not been obtained from repeated observations must be evaluated by scientific judgement. Using all possible sources of information, the metrologist needs to draw on his or her experience and general knowledge about the processes to make reliable decisions about Type B uncertainties.

It must be emphasised that Type B uncertainties are usually the most difficult to evaluate due to the rigorous scientific methodology required to substantiate a particular position about an uncertainty. If treated in a trivial manner, Type B components can produce meaningless results in uncertainty estimates. Possible Type B contributions include:

- ❑ previous measurement data on the same or similar system
- ❑ manufacturer's specifications
- ❑ values from calibration certificates
- ❑ uncertainties associated with reference data from handbooks
- ❑ previous experience with the behaviour of certain instrumentation

If the quoted uncertainty, taken from a handbook or calibration certificate, is stated to be a particular multiple of a standard deviation, then the standard uncertainty is obtained by dividing the expanded uncertainty by the multiplier. Similarly if a confidence level is given instead of a multiplier, then a normal distribution can be assumed and the standard uncertainty is obtained by dividing the expanded uncertainty by the appropriate factor for a normal distribution (*i.e.* 1.64 for 90 % confidence, 1.96 for 95 % confidence and 2.58 for 99 % confidence).

Note that generally the number of degrees of freedom associated with a Type B contribution is infinite. However, there are special cases where it may be necessary to calculate the number of degrees of freedom. In these cases it is necessary to refer to the GUM or chapter 2.

A short summary

We introduced the idea of the measurement uncertainty as a figure of merit for how good a measurement is considered to be - by the person who made the measurement. The implication of this statement is that the true value (which we may never exactly know) lies somewhere between the upper and lower bounds defined by the value of the stated uncertainty with a given confidence level. This is shown graphically in Figure 2. The dot shows the measurement results and the tee-ended line (error bar) shows the uncertainty of measurement.

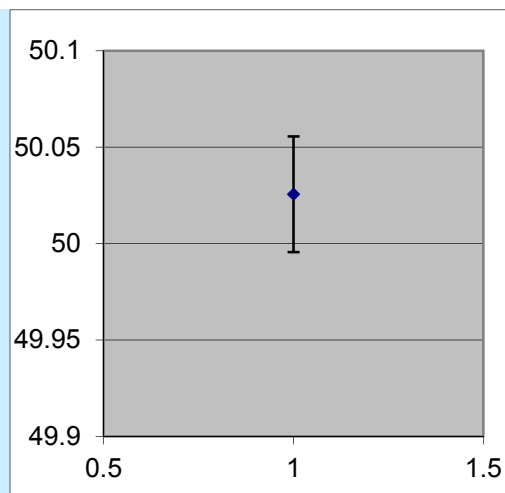


Figure 2 Measured value with measurement uncertainty

Distributions

There are various ways to specify a random variable. The most visual is the probability density function. The probability density function describes the shape of the characteristic curve derived from numerous repeated measurements. The probability density function represents how likely each value of the random variable is (as we saw in Figure 1).

The normal distribution, also called the Gaussian distribution, is an extremely important probability distribution. The distributions have the same general form, differing in their location and scale parameters: the mean (or average) and standard deviation (variability), respectively. A plot of the distribution is often called the bell curve because the graph of its probability density resembles a bell (Figure 3).

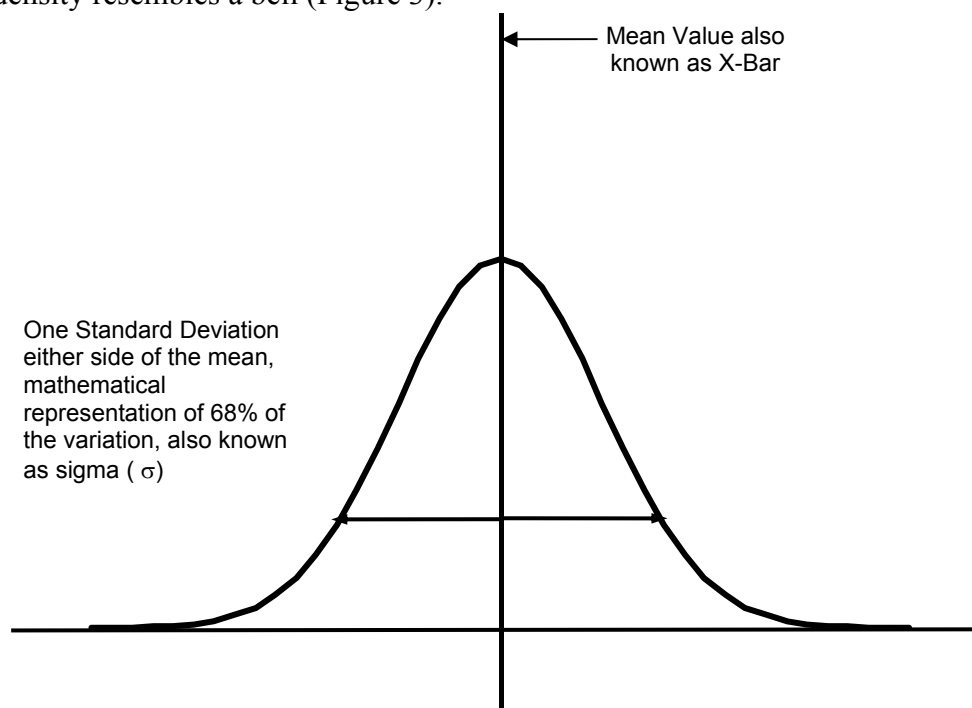


Figure 3 Normal or bell shaped curve

Some notable qualities of the normal distribution are:

The density function is symmetric about its mean value and as stated earlier:

68.27 % of the area under the curve is within one standard deviation of the mean,

95.45 % of the area is within two standard deviations,

99.73 % of the area is within three standard deviations.

The normal distribution has a symmetrical peaked form, while the rectangular distribution, as its name suggests is flat – having the same probability of a value occurring anywhere between two given values. Two examples of errors with rectangular distributions are instrument resolution and alignment errors.

Rectangular or normal distribution

If you throw a die hundreds of times and plot the results you will see that each number is equally likely. You have a rectangular distribution. If you throw two dice you will find that certain totals are more likely than others. You have a normal distribution.

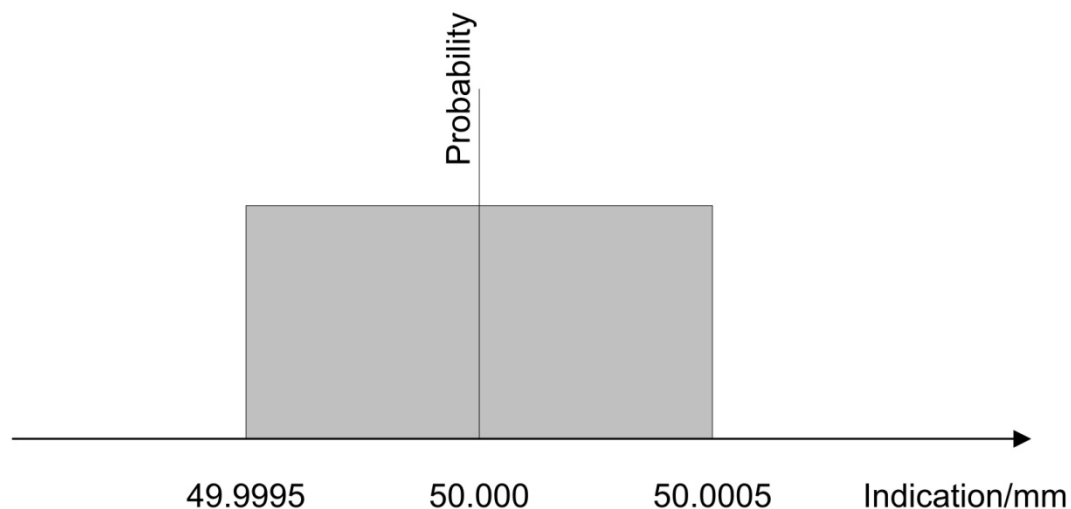


Figure 4 An example of a rectangular distribution

Figure 4 shows a typical rectangular distribution. In this case it is from an instrument with a resolution of 0.001 mm. The instrument has recorded 50.000 mm but we know the true value could be between 49.9995 and 50.0005 mm. The distribution is rectangular as each value is equally likely.

The contribution to the uncertainty, in this case, is found by halving the resolution and dividing by the square root of three (for a rectangular distribution), thus

$$\frac{\frac{0.001}{2}}{\sqrt{3}} = \frac{0.001}{\sqrt{12}}$$

There are even more unusual distributions such as the triangular and inverse triangular. The effect of these assorted distributions is to alter the weighting that is used in the calculation of an overall uncertainty for the total measurement system to which they contribute. More information can be found in the GUM.

CMMs, traceability and uncertainties

Traceability of CMMs is difficult to demonstrate. One of the problems is associating a measurement uncertainty with a result straight off the CMM.

For simple measuring devices, the uncertainty can be evaluated by an uncertainty budget according to the recommendations in the GUM. However, in the case of a CMM, the formulation of a classical uncertainty budget is impractical for the majority of the measurement tasks due to the complexity of the measuring process (see Figure 5).

It used to be the case that the only way to demonstrate traceability was to carry out tests recommended in ISO 10360. The disadvantage of this method was that it only demonstrated that the machine met its specification for measuring simple lengths, that is to say, it was not task-specific.

A better method to demonstrate traceability is to calculate a measurement uncertainty as described in ISO 15530-3, and will be covered later. This method makes use of calibrated artefacts to essentially use the CMM as a comparator. The uncertainty evaluation is based on a sequence of measurements on a calibrated workpiece or workpieces, performed in the same way and under the same conditions as the actual measurements. The differences between the results obtained from the measurement of the workpieces and the known calibration values of these calibrated workpieces are used to estimate the uncertainty of the measurements.

Alternative methods that are consistent with the GUM can be used to determine the task-specific uncertainty of coordinate measurements. One such method that evaluates the uncertainty by numerical simulation of the measuring process is described in ISO/TS 15530-4:2008.

To allow CMM users to easily create uncertainty statements, CMM suppliers and other third party companies have developed uncertainty evaluating software (UES), also known as virtual CMMs.

To quote ISO/TS 15530-4:2008

UES is based on a computer-aided mathematical model of the measuring process. In this model, the measuring process is represented from the measurand to the measurement result, taking important influence quantities into account.

In the simulation, these influences are varied within their possible or assumed range of values (described by probability distributions), and the measuring process is repeatedly

simulated, using possible combinations of the influence quantities. The uncertainty is determined from the variation of the final result.

CMM geometries/error maps and uncertainties

One of the reasons that it is difficult to carry out a classical uncertainty calculation on measurements made on a CMM is due to the number of error sources. Sources of errors in CMM measurements can be classified as spatial errors or computational errors. Spatial errors are errors in the measured position of a point on the surface of the workpiece and are determined by:

- the accuracy of the components of the CMM - the guideways, the scales, the probe system and the reference sphere used for probing system qualification;
- the environment in which the CMM operates - the ambient temperature, temperature gradients, humidity and vibration;
- the probing strategy used – the magnitude and direction of the probe force, the type of probe stylus used and the measuring speed of the probe; and
- the characteristics of the workpiece – elasticity, surface roughness, hardness and the mass of the component.

Computational errors are the errors in the estimated dimensions and form deviations of the workpiece and are determined by:

- the CMM software used to estimate the geometry of the workpiece;
- the numerical precision of the computer used on the CMM;
- the number and relative position of the measured points; and
- the extent to which the geometry departs from the ideal geometric form.

Geometric errors of a CMM are either measured directly using laser interferometers and specialist optics, such as those from a number of commercial suppliers or indirectly using sequential multi-lateration using, for instance, the Etalon LaserTRACER. Once measured these errors may be used to error correct the machine (computer-aided accuracy or CAA).

CMM geometric errors?

A CMM has twenty-one sources of kinematic error. Kinematic errors are errors in the machine components due to imperfect manufacturing or alignment during assembly. The straight-line motion of a moving component always involves six components of deviation from the nominal path:

- a) one positional deviation, in the direction of motion (linearity);
- b) two linear deviations orthogonal to the direction of motion (straightness); and
- c) three angular deviations (rigid body rotations - roll, pitch and yaw).

In addition there are the three squareness errors between the pairs of axes.

A comprehensive list of errors can be found in Appendix B.

Each of these geometric errors is measured and forms part of the CMM's error map. However, there are uncertainties in how well these geometric errors are measured. These uncertainties in turn mean that there is an uncertainty in the reported x , y , z ordinates of a contact point. A feature measured on a CMM uses many contact points and a series of calculations to report a value for that feature, for instance the diameter of a circle. The uncertainties in the co-ordinates of each point result in a subsequent uncertainty in the reported characteristic of the feature.

A model of a typical measurement on CMM and the quantities influencing the measurement process is shown in Figure 5.

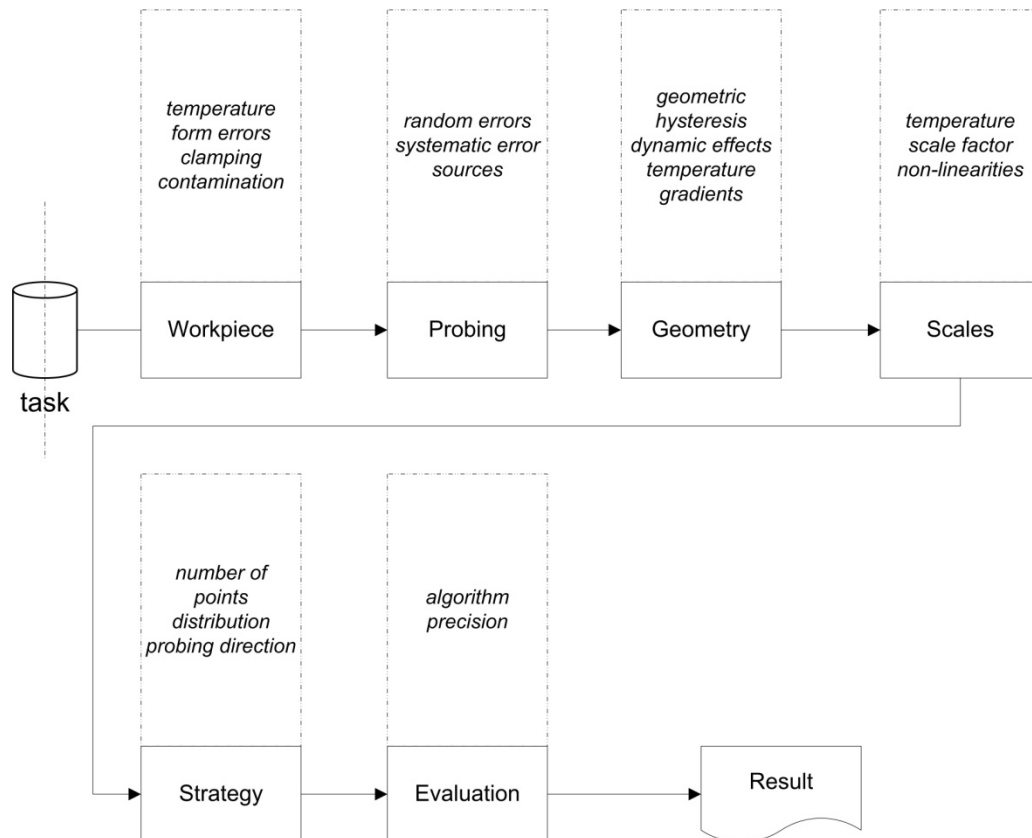


Figure 5 Measurement on a CMM - flow chart (based on Figure B.1 of ISO 15530-4).

Decision rules and uncertainties

An item is generally measured on a CMM to ascertain whether it conforms to a specification on a drawing. When making a measurement you may think that it is a simple matter of checking that the result falls within the tolerance band to prove conformance. This is not the case, as the following micrometre-based example shows.

The designer has specified that a hole should be $50 \text{ mm} \pm 0.005 \text{ mm}$ (top and bottom lines in Figure 6). The first operator measures the size with a traceable micrometer as 50.004 mm and states that the hole conforms to the drawing. However, the foreman, looking at this result, examines the uncertainty of the micrometer. The measurement uncertainty of the micrometer

is 0.003 mm and applying this uncertainty he realises that the actual size could lie between 50.001 mm and 50.007 mm. He gets the hole remeasured on a bore comparator that has a 0.001 mm uncertainty. The measurement comes out at 50.006 mm and conformance is not proven. As a general rule, the measurement uncertainty of the equipment should be no greater than ten percent of the tolerance band.

Note that, in this case, both measurement results agree to within their stated uncertainties. For operator 1, however, the measured value is less than the uncertainty away from the upper specification limit (USL)¹ and no real information has been obtained about whether the true value is inside or outside the specification limits.

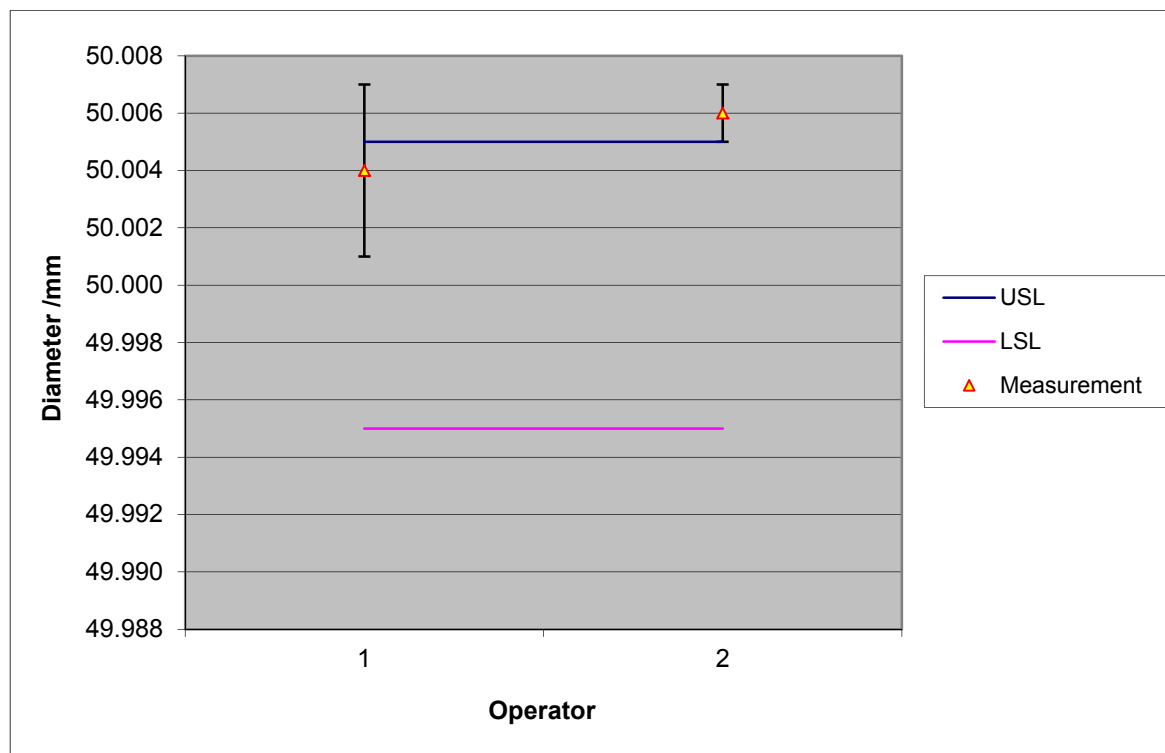


Figure 6 The two results and their associated uncertainties

ISO 14253 recommends that the following rules be applied for the most important specifications controlling the function of the workpiece (for example, a part measured on a CMM) or the measuring equipment (the CMM itself).

At the design stage the terms ‘in specification’ and ‘out of specification’ refer to the areas separated by the upper and lower tolerance (double sided) or either LSL or USL for a one sided specification (see Figure 7 areas 1 and 2, line C).

When dealing with the manufacturing or measurement stages of the process the LSL and USL are added to the measurement uncertainty. The conformance or non-conformance ranges are reduced by the uncertainty (see Figure 7, line D).

¹ Conversely LSL stands for lower specification limit.

These rules are to be applied when no other rules are in existence between supplier and customer. ISO 14253 allows for other rules to be agreed between customer and supplier. These other rules must be fully documented.

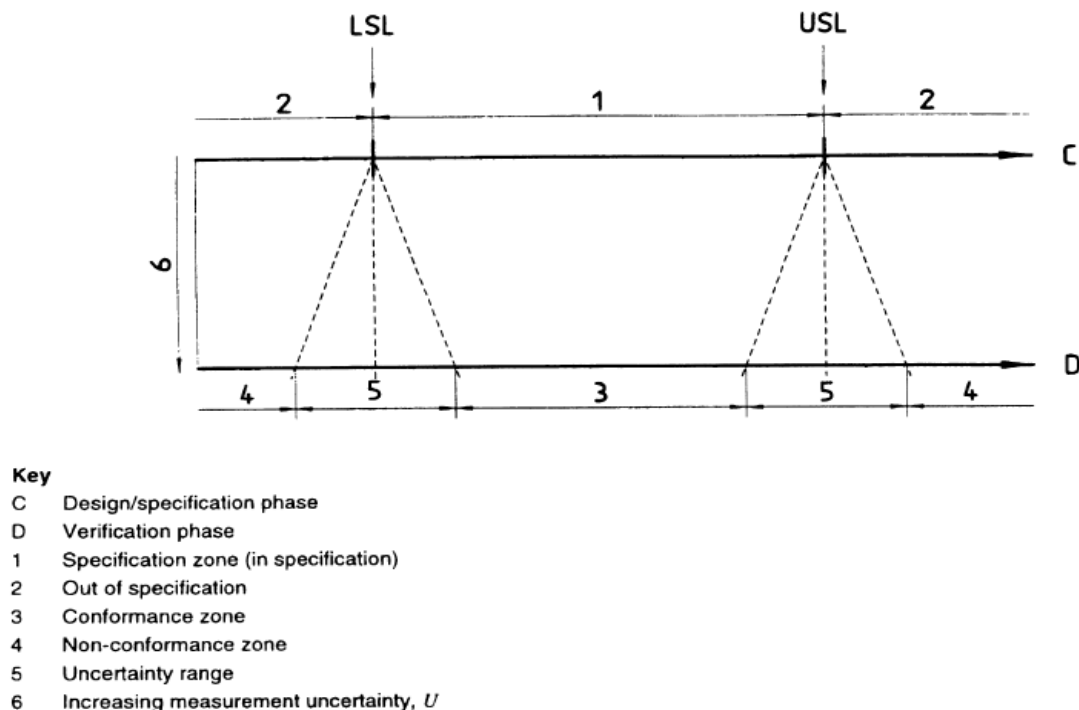


Figure 7 Uncertainty of measurement: the uncertainty range reduces the conformance and non-conformance zones (Copyright BSI – extract from BS EN ISO 14253-1:1999)

Conformance with a specification is proved when the result of measurement, complete statement,² falls within the tolerance zone or within the maximum permissible error of the specification for the measuring equipment (for example, the maximum permissible error of a CMM). Conformance is also proven when the measurement result falls within the tolerance zone reduced on either side by the expanded uncertainty. The conformance zone is linked to the LSL, USL and the actual expanded uncertainty.

Non-conformance with a specification is proved when the result of measurement, complete statement, falls outside the tolerance zone or outside the maximum permissible error of the specification for the measuring equipment. Non-conformance is also proven when the measurement result is outside the tolerance zone increased on either side by the expanded uncertainty. The non-conformance zone (4 in Figure 7) is linked to the USL, LSL and the expanded uncertainty.

Neither conformance nor non-conformance with a specification can be proven when the result of measurement, complete statement, includes one of the specification limits (for example, measurement 1 in Figure 6 and Figure 8).

² A complete statement of the result of a measurement includes information about the uncertainty of measurement.

It is important that the principle behind these rules is applied to a supplier/customer relationship where the uncertainty of measurement always counts against the party who is providing the proof of conformance or non-conformance, *i.e.* the party making the measurement. That is to say, the supplier will reduce the tolerance by their measurement uncertainty to prove conformance. The customer will increase the tolerance by their measurement uncertainty to prove non-conformance.

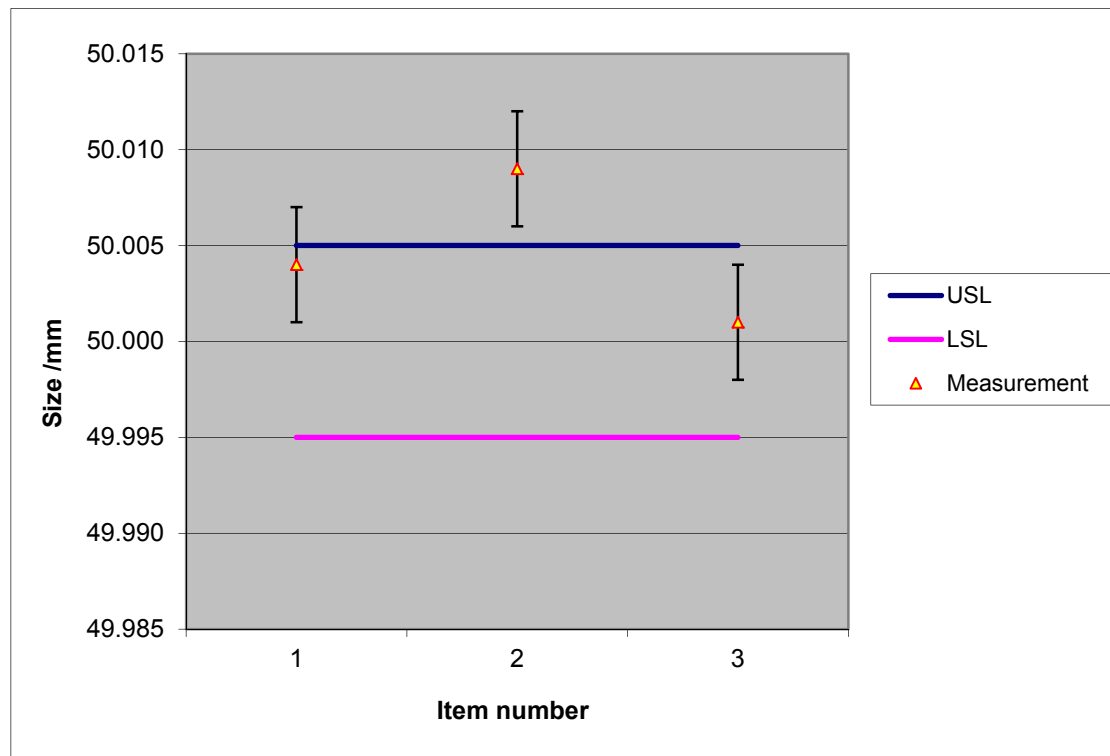


Figure 8 Conformance or non-conformance

Referring to Figure 8, three items have been measured. The purple line shows the LSL, the blue line the USL.

Measurement of item 1 - neither conformance nor non-conformance with a specification can be proven

Measurement of item 2 – non-conformance is proven

Measurement of item 3 – conformance is proven

In the case of item 1 the result of measurement, complete statement, straddles the USL and neither conformance nor non-conformance with a specification can be proven. In the case of item 2 the result of measurement, complete statement is above the USL and so non-conformance is proven. In the case of item 3 the result of measurement, complete statement is above the LSL and below the USL and so conformance is proven.

Summary of ISO 14253

ISO 14253 can be summarised in the following statements:

The supplier shall prove conformance in accordance with clause 5.2³ of BS EN ISO 14253 using their estimated uncertainty of measurement.

The customer shall prove non-conformance in accordance with clause 5.3⁴ of BS EN ISO 14253 using their estimated uncertainty of measurement.

When evaluating the measurement result the uncertainty is always at the disadvantage of the party with onus on proof.

In the past the measurement uncertainty has been ignored when ascertaining conformance with a specification as long as the uncertainty was one tenth of the specification width. This procedure is no longer acceptable.

It is, therefore, important when you are measuring items on a CMM to check against the specification the designer has put on the drawing that the uncertainty in the measurements is taken into account. In the next section the various methods of ascertaining CMM uncertainty will be discussed.

³ Rule for proving conformance with specification.

⁴ Rule for proving non-conformance with specifications.

CMM Measurement Uncertainties

2

IN THIS CHAPTER

- Why is the MPE value not good enough?
- Analytical approach for simple cases
- Introduction to ISO 15530
- Uncertainty evaluating software

The purpose of chapter 2 is to look at some simple cases that demonstrate an analytical approach to determining the measurement uncertainty of measurements made on a CMM. More rigorous techniques involving the use of uncertainty estimating software are then introduced.

Why is the MPE value not good enough?

NPL Good Practice Guide number 42 covers the verification of co-ordinate measuring machines. CMMs are specified in a way that gives an indication of how good they are at measuring features of size. Often in industry you will see this value quoted as an uncertainty associated with a CMM measurement.

One option that is widely used, but not necessarily recommended, is to use the CMM manufacturer's specified value for the maximum permissible error (MPE) of length measurement, $E_{L, MPE}$ as an uncertainty estimate. This error is usually expressed as $A + L/K$, where A and K are constants and L is the measured length. If no other information is available, use of $E_{L, MPE}$ is a start, but you have to bear in mind that the machine specification is not an uncertainty and that it only applies for a particular case, that is to say determining the length between two points with a particular stylus system. However, this method does not apply when determining inter-feature distances and angles, when reporting geometric form or when using long styli.

There are a number of reasons why the use of the MPE value is not good practice:

- the ISO 10360 test only samples the machine volume;
- only a single stylus is used for the verification test and that stylus may not represent the stylus used in every day measurements;
- the value quoted is only a specified value, an actual machine may easily meet its specification; and
- only tests the machines ability to measure the distance between two points. It says nothing of the machine's capability of performing other measurement tasks.

Analytical approach for simple cases

As stated earlier, a GUM-based analytical approach to CMM uncertainties is complex for all but the most trivial of examples. Examples where an analytical approach might be suitable is that of measuring the length of an artefact (between points on plane faces or sphere centres) or the diameter of a hole along an identified generator. Using a reversal technique to minimise CMM errors may also lend itself to this approach.

The basic principle in performing the uncertainty calculation is that the various contributions are first assessed. The Type A contribution can be assessed by calculating the standard deviation of the measurements.

Type B contributions may include the following:

- uncertainty in the measurement of temperature;
- uncertainties associated with stylus changes;

- uncertainty in the knowledge of the material expansion coefficient;
- uncertainties introduced by rounding of results;
- uncertainties in the calibrations of ancillary equipment; and
- uncertainties in the calibration of the CMM.

The Type A and Type B contributions are combined using the methods detailed in the GUM. The contributions should be tabulated as shown in Table 1.

Table 1 Blank table for uncertainty calculations

Component <i>i</i>	Value nm	Probability distribution	Divisor	u_i nm	v_i or v_{eff}
1					
2					
3					
4					
5					
6					
u_c		normal			
U		normal ($k = 2$)			

The number of degrees of freedom should be calculated using the Welch-Satterthwaite equation as given below.

$$v_{eff} = \frac{u_c^4}{\sum_{i=1}^N \frac{u_i^4(y)}{v_i}}$$

Worked example using a substitution method

This example is based around a component where there is a requirement to measure the length between two faces nominally 300 mm apart. The measurements are made by comparison with a 300 mm gauge block (the reference standard). Both items are made of steel. Measurements were made at a temperature that did not deviate from 20 °C by more than 0.25 °C.

The reference standard was mounted alongside and in the same axis as the component being measured. The component and the standard are then measured sequentially. The measured value of the component was adjusted by the amount of error identified in the measurement of the standard. The measurement strategy and the probe configuration were identical for the measurement of the reference artefact and the customer's component.

Both the component and the reference gauge block were measured five times along a machine axis and the following results obtained.

Table 2 Example measurement results for the reference standard and component

	Component	Reference standard
1	300.003 5 mm	300.000 7 mm
2	300.003 7 mm	300.000 8 mm
3	300.003 5 mm	300.000 6 mm
4	300.003 6 mm	300.000 8 mm
5	300.004 0 mm	300.000 9 mm
Mean	300.003 7 mm	300.000 8 mm
Standard Deviation	0.000 21 mm	0.000 12 mm

Notice that the mean and standard deviation of the results has been calculated.

From the calibration certificate, the calibrated length of the 300 mm gauge block (reference standard) is 299.999 95 mm. The CMM has, therefore, an error over this length of $300.000\ 8 - 299.999\ 95 = 0.85\ \mu\text{m}$.

We can use this information to correct the measured size of the component. The corrected size of the component is, therefore, $300.003\ 7 - 0.000\ 85 = 300.002\ 85\ \text{mm}$.

We can now calculate the magnitude of all contributing uncertainties:

- (1) The component was measured five times and the standard deviation and standard error of the mean (SEOM) calculated, where n is the number of measurements. The contribution is,

$$\frac{\sigma}{\sqrt{n}} = \frac{0.00021}{\sqrt{5}} = 0.000\ 10\ \text{mm}.$$

The number of degrees of freedom associated with this contribution is 4 ($n - 1$).

- (2) The expansion coefficient of the component material, u_α , is estimated to be known to a part in 10^6 . Often this value is not known but an uncertainty of 1 ppm is a safe assumption for most common engineering materials. Assuming a systematic, rectangular *a priori* probability distribution, the contribution is,

$$\frac{u_\alpha L \Delta T}{\sqrt{3}};$$

and, substituting the values above, we have

$$\frac{1 \times 10^{-6} \times 300 \times 0.25}{\sqrt{3}} = 0.000\ 05\ \text{mm}.$$

where 0.25 °C is the deviation of the workpiece from 20 °C and 300 mm is the measured distance. Remember, for a rectangular PDF we divide by the square root of three. It is judged that this value of $u(x_i)$ is reliable to about 20%, therefore the number of degrees of freedom to assign is $\nu_i = (0.20)^{-2}/2 = 12.5$.

Note that the GUM gives the following expression for the number of degrees of freedom,

$$\nu_i = \frac{1}{2} \frac{u^2(x_i)}{\sigma^2[u(x_i)]} \approx \frac{1}{2} \left[\frac{\Delta u(x_i)}{u(x_i)} \right]^{-2}.$$

The quantity in large brackets is the relative uncertainty of $u(x_i)$; for a Type B evaluation of standard uncertainty it is a subjective quantity whose value is obtained by scientific judgement based on the pool of available information.

- (3) It has been determined that the CMM's workpiece thermometer has a possible error of 0.1 °C. Assuming a systematic, rectangular *a priori* probability distribution, the contribution is,

$$\frac{\frac{\alpha L u_T}{\sqrt{3}}}{\sqrt{3}} = \frac{11.7 \times 10^{-6} \times 300 \times 0.1}{\sqrt{3}} = 0.000\ 21\ \text{mm}.$$

It is judged that this value of $u(x_i)$ is reliable to about 20 %, therefore, the number of degrees of freedom to assign is $\nu_i = (0.20)^{-2}/2 = 12.5$.

- (4) The expansion coefficient of the CMM's scale material is estimated to be known to 1 ppm. Assuming a systematic, rectangular *a priori* probability distribution, the contribution is,

$$\frac{1 \times 10^{-6} \times 300 \times 0.25}{\sqrt{3}} = 0.000\ 05\ \text{mm}$$

where 0.25 °C is the deviation of the workpiece from 20 °C and 300 mm is the maximum measured distance. It is judged that this value of $u(x_i)$ is reliable to about 20 %, therefore, the number of degrees of freedom to assign is $\nu_i = (0.20)^{-2}/2 = 12.5$.

- (5) The CMM's scales have an expansion coefficient of 7.8 ppm. It is estimated that the CMM's scale thermometer has a possible error of 0.1 °C. Assuming a systematic, rectangular *a priori* probability distribution, the contribution is

$$\frac{7.8 \times 10^{-6} \times 300 \times 0.1}{\sqrt{3}} = 0.000\ 14\ \text{mm}.$$

It is judged that this value of $u(x_i)$ is reliable to about 20 %, therefore, the number of degrees of freedom to assign is $\nu_i = (0.20)^{-2}/2 = 12.5$.

- (6) The correction that is applied to the component measurements is based on measurements on a reference standard that has an associated uncertainty and a reference value that also has an associated uncertainty (from its calibration certificate).

- (a) The reference standard was measured five times and the standard deviation and SEOM calculated.

$$\frac{\sigma}{\sqrt{n}} = \frac{0.00012}{\sqrt{5}} = 0.000\ 06\ \text{mm}.$$

The number of degrees of freedom associated with this contribution is 4.

- (b) The reference standard has been calibrated at NPL with an uncertainty ($k = 2$) of 0.20 μm . Therefore the contribution is,

$$\frac{0.000\ 20}{2} = 0.000\ 10\ \text{mm}.$$

The expansion coefficient of the gauge block material is estimated to be known to 1 ppm (based on values in EN ISO 3650). Assuming a systematic, rectangular *a priori* probability distribution, the contribution is,

$$\frac{1 \times 10^{-6} \times 300 \times 0.25}{\sqrt{3}} = 0.000\ 05\ \text{mm}$$

where 0.25 °C is the deviation of the workpiece from 20 °C and 50 mm is the maximum measured distance. It is judged that this value of $u(x_i)$ is reliable to about 20 %, therefore, the number of degrees of freedom to assign is $\nu_i = (0.20)^{-2}/2 = 12.5$.

- (c) The CMM's workpiece thermometer has a possible uncorrected error of 0.1 °C. Assuming a systematic, rectangular *a priori* probability distribution, the contribution is,

$$\frac{11.7 \times 10^{-6} \times 300 \times 0.1}{\sqrt{3}} = 0.000\ 21\ \text{mm}.$$

It is judged that this value of $u(x_i)$ is reliable to about 20 %, therefore, the number of degrees of freedom to assign is $\nu_i = (0.20)^{-2}/2 = 12.5$.

- (d) The expansion coefficient of the scale material is estimated to be known to 1 ppm. Assuming a systematic, rectangular *a priori* probability distribution, the

contribution is,

$$\frac{1 \times 10^{-6} \times 300 \times 0.25}{\sqrt{3}} = 0.000\ 05\ \text{mm},$$

where 0.25 °C is the deviation of the workpiece from 20 °C and 50 mm is the maximum measured distance. It is judged that this value of $u(x_i)$ is reliable to about 20 %, therefore, the number of degrees of freedom to assign is $\nu_i = (0.20)^{-2}/2 = 12.5$.

- (e) The scale thermometer has a possible error of 0.1 °C. Assuming a systematic, rectangular *a priori* probability distribution, the contribution is,

$$\frac{7.8 \times 10^{-6} \times 300 \times 0.1}{\sqrt{3}} = 0.000\ 14\ \text{mm}.$$

It is judged that this value of $u(x_i)$ is reliable to about 20 %, therefore, the number of degrees of freedom to assign is $\nu_i = (0.20)^{-2}/2 = 12.5$.

The combined standard uncertainty of the correction is given by the expression:

$$u_c = \sqrt{\sum_{i=1}^5 u_i^2}$$

$$u_c = \sqrt{0.06^2 + 0.10 + 0.05^2 + 0.21^2 + 0.05^2 + 0.14^2} = 0.29\ \mu\text{m}.$$

- (7) The resolution of the CMM's output is 0.000 1 mm. This contributes to the uncertainty as the half width of a rectangular distribution thus,

$$\frac{0.0001}{\sqrt{12}} = 0.000\ 03\ \text{mm}.$$

Note that this contribution should be increased if the results are rounded for final use.

The combined standard uncertainty of the correction is given by the expression,

$$u_c = \sqrt{\sum_{i=1}^7 u_i^2}.$$

The expanded uncertainty is then found by multiplying the combined standard uncertainty by $k = 2$, based on the assumption that the number of degrees of freedom is sufficiently large. For this example the combined standard uncertainty is calculated as follows,

$$u_c = \sqrt{0.10^2 + 0.05^2 + 0.21^2 + 0.05^2 + 0.14^2 + 0.29^2 + 0.03^2} = 0.41 \mu\text{m}.$$

Thus the final expanded uncertainty ($k = 2$) based on the combined standard uncertainty calculated above is $0.82 \mu\text{m}$.

Table 3 Example (Substitution)

Component i	Value nm	Probability distribution	Divisor	u_i nm	ν_i or ν_{eff}
1	210	Normal	$\sqrt{5}$	100	4
2	87	Rectangular	$\sqrt{3}$	50	12.5
3	364	Rectangular	$\sqrt{3}$	210	12.5
4	87	Rectangular	$\sqrt{3}$	50	12.5
5	242	Rectangular	$\sqrt{3}$	140	12.5
6	290	Normal	1	290	∞
7	100	Rectangular	$\sqrt{12}$	30	∞
u_c		normal		410	133
U		normal ($k = 2$)		820	133

Using the Welch-Satterthwaite equation we can calculate the total number of degrees of freedom

$$\nu_{eff} = \frac{u_c^4}{\sum_{i=1}^N \frac{u_i^4(y)}{\nu_i}},$$

whence

$$\nu_{eff} = \frac{(0.410)^4}{\frac{0.100^4}{4} + \frac{0.050^4}{12.5} + \frac{0.210^4}{12.5} + \frac{0.050^4}{12.5} + \frac{0.140^4}{12.5}} = 133.$$

The corresponding k value is obtained from the t -distribution table in UKAS document M3003. This value of ν confirms the use of $k = 2$.

Worked example when substitution method not used

Sometimes, due to the number of features and range of sizes to be measured, the substitution method is not always practicable. It may also be that the tolerances required are large in comparison to the capability of the CMM. The following example is for an aluminium gearbox casing with a number of features and relatively large tolerances. The gear case was measured on a CMM using an 8.0 mm diameter ball-ended stylus and a measuring force of 0.10 N. To access the underside, the stylus system was replaced with a T-shaped stylus incorporating 3.0 mm diameter stylus tips.

Note that some test measurements were performed on a plain setting ring and on a known flat surface; however, this was not an application of the substitution method due to the disparity between the sizes of the features and the sizes of the reference artefacts.

- (1) All casings were measured at least three times and the standard deviation and SEOM calculated.

The contribution is $\frac{\sigma}{\sqrt{n}}$ where $n = 3$ or $n = 4$.

- (2) The expansion coefficient is estimated to be known to 1 ppm. Assuming a systematic, rectangular *a priori* probability distribution, the contribution is,

$$\frac{u_{\alpha} L \Delta T}{\sqrt{3}},$$

and substituting appropriately gives,

$$\frac{1 \times 10^{-6} \times 500 \times \Delta T}{\sqrt{3}},$$

where ΔT is the deviation of the workpiece from 20 °C and 500 mm is the maximum measured distance (465.75 mm on bores plus an allowance for flange measurement). It is judged that this value of $u(x_i)$ is reliable to about 20 %, therefore the number of degrees of freedom to assign is $\nu_i = (0.20)^{-2}/2 = 12.5$.

- (3) The thermometer has a possible error of 0.1 °C. Assuming a systematic, rectangular *a priori* probability distribution, the contribution is,

$$\frac{\alpha L u_T}{\sqrt{3}},$$

and substituting appropriately gives,

$$\frac{23 \times 10^{-6} \times 500 \times 0.1}{\sqrt{3}}.$$

It is judged that this value of $u(x_i)$ is reliable to about 20 %, therefore, the number of degrees of freedom to assign is $\nu_i = (0.20)^{-2}/2 = 12.5$.

- (4) The MPE for the machine is $1.3 + L/300 \mu\text{m}$. Assuming a systematic, rectangular *a priori* probability distribution, the contribution is,

$$\frac{1.3 + \frac{L}{300}}{\sqrt{3}} = 0.76 + \frac{L}{520},$$

where L is 500 mm. This value is known with a high number of degrees of freedom. Note that we are using the MPE as a contribution to the whole uncertainty budget. This is different to letting the MPE represent the whole uncertainty budget. Note that the MPE includes effects such as uncertainties in the thermometers as the ISO 10360 test is a black box test. The value here may not necessarily be the manufacturer's specified value but a value that represents the performance of the machine using the particular stylus combination used for the measurements.

- (5) Additional tests on a plain setting ring and a flat plate showed errors in measurement of up to 0.001 mm. Assuming a systematic, rectangular *a priori* probability distribution, the contribution is,

$$\frac{0.001}{\sqrt{3}}.$$

It is judged that this value of $u(x_i)$ is reliable to about 20 %, therefore the number of degrees of freedom to assign is $\nu_i = (0.20)^{-2}/2 = 12.5$.

- (6) The measurements necessitated a change of stylus towards the end of the measurement. It is estimated that measurements made on the same feature with the two stylus systems would not agree to better than 0.001 mm. This information is based on measurements of the co-ordinates of a test sphere with each stylus. Assuming a systematic, rectangular *a priori* probability distribution, the contribution is,

$$\frac{0.001}{\sqrt{3}}.$$

It is judged that this value of $u(x_i)$ is reliable to about 20 %, therefore, the number of degrees of freedom to assign is $\nu_i = (0.20)^{-2}/2 = 12.5$.

The combined standard uncertainty is given by the expression:

$$u_c = \sqrt{\sum_{i=1}^6 u_i^2}.$$

The expanded uncertainty is found by multiplying the combined standard uncertainty by $k = 2$.

Degrees of freedom

Using the Welch-Satterthwaite equation we can calculate the total number of degrees of freedom

$$v_{eff} = \frac{u_c^4}{\sum_{i=1}^N \frac{u_i^4(y)}{v_i}}$$

Worked example using a reversal method

In many cases certain errors associated with the CMM can be minimised using a reversal technique. In particular, errors of straightness can be removed using this technique. This example is for the measurement of a cylinder whose errors have been removed using a reversal technique. The reversal technique is covered in NPL Good Practice Guide No. 80.

The cylinder was measured on a CMM using an 8.0 mm diameter ball-ended stylus and a measuring force of 0.10 N. The cylinder was mounted horizontally in a precision vee block and the cylindrical surface contacted at a number of positions to define the cylinder axis. The cylinder axis was set to zero in the x and z directions. The end face of the cylinder was then contacted at a number of points; the end face being set to $y = 0$.

The cylinder was then contacted at 1000 points along generators A and B at $z = 0$ with the generators pointing in both the positive x and negative x directions of the CMM, to enable the straightness errors to be determined using a reversal technique of error separation. Measurements started at 20 mm from the datum face and finished at 20 mm from the opposite end.

The results were corrected to a temperature of 20 °C. A coefficient of thermal expansion of 11.7 ppm/°C was assumed.

At each orientation three runs are performed. The measured straightness values were of the order of 0.000 6 mm.

Type A

For the three runs at any given position there is a spread of readings due to:

- (a) how reproducible the machine is; and
- (b) possible thermal drift during the runs.

The maximum standard deviation of the three runs was 0.000 19 mm.

The contribution is $\frac{0.19}{\sqrt{3}} = 0.11 \mu\text{m}$ $\nu = 2$.

This is a slight overestimate as any drift will not affect straightness.

Type B

- (1) The accuracy of the CMM over 0.6 μm is estimated to be $\pm 0.1 \mu\text{m}$. Assuming a systematic, rectangular *a priori* probability distribution, the contribution is,

$$\frac{0.1}{\sqrt{3}} = 0.06 \mu\text{m}.$$

It is judged that this value of $u(x_i)$ is reliable to about 20 %, therefore the number of degrees of freedom to assign is $\nu_i = (0.20)^{-2}/2 = 12.5$.

- (2) The resolution of the CMM output is 0.1 μm . The contribution to the uncertainty is,

$$\frac{0.1}{\sqrt{12}} = 0.03 \mu\text{m}.$$

- (3) It was noted that the machine deviated from $z = 0$ by up to 0.1 mm. The radius of the cylinder was 50 mm.

$$50 \times \cos\left(\sin^{-1}\frac{0.01}{50}\right) = 49.999\ 99 \text{ mm}$$

This term can be ignored.

$$u_c = \sqrt{0.11^2 + 0.05^2 + 0.03^2} = 0.129 \mu\text{m},$$

$$\nu_{eff} = \frac{(0.129)^4}{\frac{0.11^4}{2} + \frac{0.060^4}{12.5} + \frac{0.030^4}{12.5}} = 3.73$$

The corresponding k value is obtained from the t -distribution table in UKAS document M3003. The appropriate value is $k = 2.96$.

The expanded uncertainty is $2.96 \times 0.129 \text{ } \mu\text{m} = 0.39 \text{ } \mu\text{m}$.

Introduction to ISO 15530

There are two methods outlined in international standards for determining the task-specific uncertainties associated with measurements made on a co-ordinate measuring machine. These methods are described in ISO 15530. Parts 1, 3 and 4 of the standard have been published. Part 2 – *Use of multiple measurement strategies in calibration artefacts* is in preparation.

The first method is to use the technique described in ISO 15530 part 3 and to use the method of substitution. This technique works well, but relies on suitable reference standards or calibrated workpieces being available and meeting the similarity conditions listed in clause 5.2. To quote the standard: *The aim of this part of ISO 15530 is to provide an experimental technique for simplifying the uncertainty evaluation of CMM measurements. In this experimental approach, measurements are carried out in the same way as actual measurements, but with calibrated workpieces or measurement standards of similar dimension and geometry instead of the unknown objects to be measured.*

The preferred technique to establish the uncertainty of measurement is to make use of uncertainty evaluating software (UES) that makes use of Monte Carlo methods. Monte Carlo methods are a class of computational algorithms that rely on repeated random sampling to compute their results. In affect, you are running the CMM program many times on a *Virtual CMM*. There are several commercial implementations of UES including those that are optional extras for Zeiss Calypso and Leitz Quindos CMM operating systems, and the stand-alone package Pundit from Metrosage. The technique for calculating task-specific measurement uncertainties using simulation is outlined in ISO 15530 part 4. At NPL, scientists have access to the Leitz and Calypso implementations of UES, both of which are based around previous work by PTB and NPL, and Pundit.

Comparison technique – use of calibrated workpieces

BS EN ISO 15530-3:2011 *Geometrical Product Specifications (GPS) -- Coordinate measuring machines (CMM): Technique for determining the uncertainty of measurement -- Part 3: Use of calibrated workpieces or measurement standards* specifies the evaluation of measurement uncertainty for results of measurements obtained by a CMM when using calibrated workpieces (a comparator/substitution technique). This standard provides an experimental technique for simplifying the uncertainty evaluation of CMM measurements, whose approach (substitution measurements) leads to measurements being carried out in the same way as actual measurements, but with calibrated workpieces of similar dimension and geometry instead of the unknown workpieces to be measured. The standard also covers non-substitution measurements and the uncertainty evaluation procedure.

A non-substitution measurement is a measurement where the uncorrected indication of the CMM is used as a result, whereas a substitution measurement is where both a workpiece and a check standard are measured in order to provide additional corrections for systematic errors of the CMM.

The substitution method makes use of a master part (calibrated workpiece) or check standard with defined similarity requirements.

Using the CMM as a comparator (substitution method) significantly enhances the accuracy of the final measurement result on any artefact. The technique is particularly useful when assessing the effect of using long or unusual stylus configurations. This technique requires the use of good quality standards such as end bars, ring and plug gauges, *etc.*

The method requires similarity of the following:

- the dimension and geometry of the workpiece and standard and the calibrated workpiece or standard (within 10 % for dimensions and 5° for angles);
- the evaluation procedure (*e.g.* handling, clamping, elapsed time, measuring force, speed); and
- the environmental conditions (*e.g.* temperature, stabilisation time *etc.*).

The check standard should be matched to the measuring task, that is to say, a ring gauge for an internal diameter measurement, or an end bar for the measurement of the distance between two planes. The reference standard should be mounted alongside and in the same axis as the workpiece being measured. The workpiece and the reference standard are then measured sequentially. The measured value of the workpiece is then adjusted by the amount of error identified in the measurement of the standard. The measurement strategy and the probe configuration shall be identical for the measurement of the reference standard and the workpiece.

Similarity conditions are covered in more detail in clause 5.2 of ISO 15530.

A sequence of measurements is then performed in the same way and under the same conditions as the actual measurements. The only difference is that, instead of measuring the workpiece, one or more calibrated workpieces are measured. The differences between the results obtained from the measurements and the known calibration values of the calibrated workpieces are used to estimate the uncertainty of the measurements. The full procedure is covered in clause 7 of the standard.

ISO 15530-3 contains examples of application of this technique in Annex A. Two examples are given; the first example covers the measurement of pump housings and the second the calibration of a ring gauge on a laboratory CMM.

Uncertainty evaluating software

Another way of calculating CMM measurement uncertainty makes use of uncertainty evaluating software (UES) also known as a virtual CMM. ISO/TS 15530-4:2008 *Geometrical Product Specifications (GPS) -- Coordinate measuring machines (CMM): Technique for determining the uncertainty of measurement -- Part 4: Evaluating task-specific measurement uncertainty using simulation* specifies requirements (for the manufacturer and the user) for the application of (simulation-based) uncertainty evaluating software to measurements made with CMMs, and gives informative descriptions of simulation techniques used for evaluating task-specific measurement uncertainty. Furthermore, it describes testing methods for such

simulation software, along with advantages and disadvantages of various testing methods. Finally, it describes various testing procedures for the evaluation of task-specific uncertainty determination by simulation for specific measurement tasks carried out on CMMs, taking into account the measuring device, the environment, the measurement strategy and the object. This document describes the general procedures without restricting the possibilities of the technical realisation. Guidelines for verification and evaluation of the simulation package are included.

Uncertainty evaluating software simulates measuring a component hundreds of times (Monte Carlo simulation) each time calculating new features and characteristics based on a changing set of input points. The input points each individually vary based on uncertainties present in the knowledge of the machine's geometry and its effect on the error map.

Two types of virtual CMM are defined: the on-line and the off-line. If the evaluation is done as similar as possible to the evaluation of the real data, that is to say using the same CMM software and the same evaluation sequence as the actual measurement (including identical stylus system qualification and object orientation measurements), then the virtual CMM is deemed to be on-line. However, if separate software is used, that is to say, not used in the actual measurements and an evaluation sequence is used that is only similar to the actual measurement, then the virtual CMM is deemed to be off-line.

A good explanation of the concept of the virtual CMM appears in section 3 of the PTB report *Traceability of Coordinate Measurements According to the Virtual Measuring Machine*. To summarise the key points of this section of the report:

The virtual CMM technique makes use of a point-by-point simulation of the individual measurements with an as-exact-as-possible modelling of the elementary uncertainty contributions which disturb each of the probing points.

There are three basic contributions, known systematics (but not always known by the user), unknown systematics (reproducible only over short time periods) and random contributions.

The evaluation of the point coordinates during the simulation is as similar as possible to the evaluation of real data.

The entire simulation is repeated a statistically significant number of times.

All known systematic contributions remain constant, all unknown systematic contributions are varied from simulation run to simulation run throughout their ranges and throughout the spectrum of their possible function shapes, taking into account the assumed probability distributions. All truly random contributions are also varied throughout their ranges taking into account assumed probability distributions.

After each simulation run, the differences between the actual result and the simulated result are calculated. After all the simulation runs have been completed a statistical evaluation of these differences is made. Finally, the expanded uncertainty U at ($k = 2$) is reported.

Monte Carlo simulation, the basis of UES, is covered in a supplement to the GUM concerned with the propagation of probability distributions through a mathematical model of measurement as a basis for the evaluation of uncertainty of measurement, and its implementation by a Monte Carlo method. The treatment applies to a model having any number of input quantities, and a single output quantity. The described Monte Carlo method is a practical alternative to the GUM uncertainty framework. The supplement is *Evaluation of measurement data – Supplement 1 to the "Guide to the expression of uncertainty in measurement" – Propagation of distributions using a Monte Carlo method*.

Monte Carlo simulation?

Monte Carlo simulation is a method whereby you can simulate performing an experiment many, many times. For instance, if you wanted to know the distribution and mean of throwing a pair of dice ten thousand times you could perform the experiment. Such an experiment could be quite tedious. On the other hand you could program a computer to produce two numbers between 1 and 6 where each number was equally likely (rectangular or uniform distribution). This simulation could be repeated ten thousand times in seconds and the results plotted as a histogram. Note that in this case the probabilities of each outcome (2 to 12) are easily determined analytically, but this is not always the case.

If you like computer programming why not give this example a try. How does the outcome vary with the number of simulations? How does the simulation compare with predicted? You may need to look up an algorithm for producing random numbers that are uniformly distributed.

Note that a throw of 2 can only be produced one way (1+1) whereas a throw of seven can be produced in the following six ways (1+6, 6+1, 5+2, 2+5, 3+4, 4+3). The total number of outcomes is 36 (6×6).

For the more adventurous, calculate a number of points that describe a circle based around the origin 50 mm in diameter. Now perform a Monte Carlo simulation of 100 simulations allowing each point to vary by 0.005 mm based on a Gaussian distribution. Calculate the 100 resultant circles and see how the calculated diameter varies.

Example answers are given in Appendix C.

For those who are really keen, NPL has developed software for measurement uncertainty analysis that can be downloaded from the NPL website.

Website: www.npl.co.uk/mathematics-scientific-computing/mathematics-and-modelling-for-metrology/software-for-measurement-uncertainty-evaluation

Uncertainty evaluating software

So how is a virtual CMM implemented in practice on a typical CMM? The first thing that has to be done is to measure the residual error field; this measurement is usually performed by the

manufacturer. This task is performed using an artefact such as a hole plate, ideally made from a thermally invariant material. The hole plate is measured in at least six orientations (Figure 10). Typical orientations are parallel to the XY-plane (two heights), the XZ-plane (measured from both sides) and the YZ-plane (measured from both sides).

Next, temperature gradients are measured around the machine. Finally, the kinematic chain of the machine has to be described, that is to say the sequence of the moving axes. This information describes the real CMM to the simulation software. In use, further input parameters are needed. The first input parameter takes the form of a special probe qualification routine. Other input parameters are required relating to the uncertainty in the thermal expansion of the component and the component's surface texture. Once all these input parameters are known, the software repeatedly simulates collecting individual points with a Gaussian spread and propagates these values through all the calculations necessary to produce the desired result. You effectively run the CMM program repeatedly (100 to 200 times) on a virtual CMM varying the input parameters, describing the machine, its environment *etc.*, slightly each time. The output is all the characteristics requested, each printed out with an associated measurement uncertainty.

The beauty of this method is that, as long as the conditions do not change, the Monte Carlo simulation only has to be performed once for a particular component. A further advantage of the technique is that it allows 'what-if' calculations to be performed. That is to say, before receiving a component you can see what effect varying the number of points and stylus configurations will have on the quality of your measurements. You can, in some cases, even allow the software to help you choose which CMM you are going to use for a particular component. The virtual CMM goes part of the way to achieve traceability in co-ordinate metrology and work is on-going to include scanning and rotary tables.

Commercial implementations of the PTB model (Calypso/Quindos)

The PTB virtual CMM has been incorporated as an option in CMM software from Zeiss and Leitz. NPL's Zeiss UPMC 550 has installed on it the Calypso OVCMM (O standing for on-line) for calculation of uncertainties of geometric elements from measurements made using single point probing. This package will be used to describe a practical implementation of UES. The Calypso OVCMM takes in to account

- Systematic residual deviations of the geometry of the CMM. These deviations are assessed by measurements of ball or hole plates and subsequent parametric error analysis.
- Fluctuations of the geometry deviations due to temporal and spatial temperature gradients.
- Direction-dependent systematic residual deviations of the stylus system
- Random deviations of the probing system (assessed by repeated probings of the same points, *e.g.* on the reference sphere; hysteresis is included in this contribution.
- Uncertainty caused by linear workpiece expansion.
- Uncertainty in qualification of the stylus tip diameter.
- Uncertainty in qualification of the stylus tips of a stylus system.
- Surface texture of the workpiece surface.

- Uncertainty of the standards used for connection to the international SI units.

In order to determine the uncertainties, the Calypso OVCMM uses the method of the virtual CMM to calculate an uncertainty range for each individual measurement. In the case of complex measured values (for example a construction or theoretical element), this range consists of the uncertainties from the entire measurement chain. The virtual CMM takes the errors and deviations of all previous measuring tasks involved in the measurement into account, *i.e.*, including the alignment and the reference features.

The virtual CMM determines the measurement uncertainty in compliance with GUM, ISO 14253 and ISO 15530. To this end, the CNC run is simulated for all characteristics of a measurement plan. The result of this simulation corresponds to the dispersion range of a measurement of 200 workpieces with a random variation of the known influencing factors. In this way, a range that is representative for the measuring results is calculated. Finally, the resulting variance is used to ascertain the uncertainty for each characteristic and to output it in the measurement report.

	Actual	Nominal	Meas. uncertainty	Upper Tol.	Lower Tol.	Deviation
	Dia_Cyl_Datum_Y					-
	35.0197	35.0000	0.0017	0.0500	0.0000	0.0197
Diameter of LSBF cylinder Datum Y						
	Cylind_Cyl_Datum_Y					-
	0.0025	0.0000	0.0009	0.0050		0.0025
Cylindricity of LSBF cylinder Datum Y						

Figure 9 Output from the Calypso OVCMM. Note the column marked measurement uncertainty.

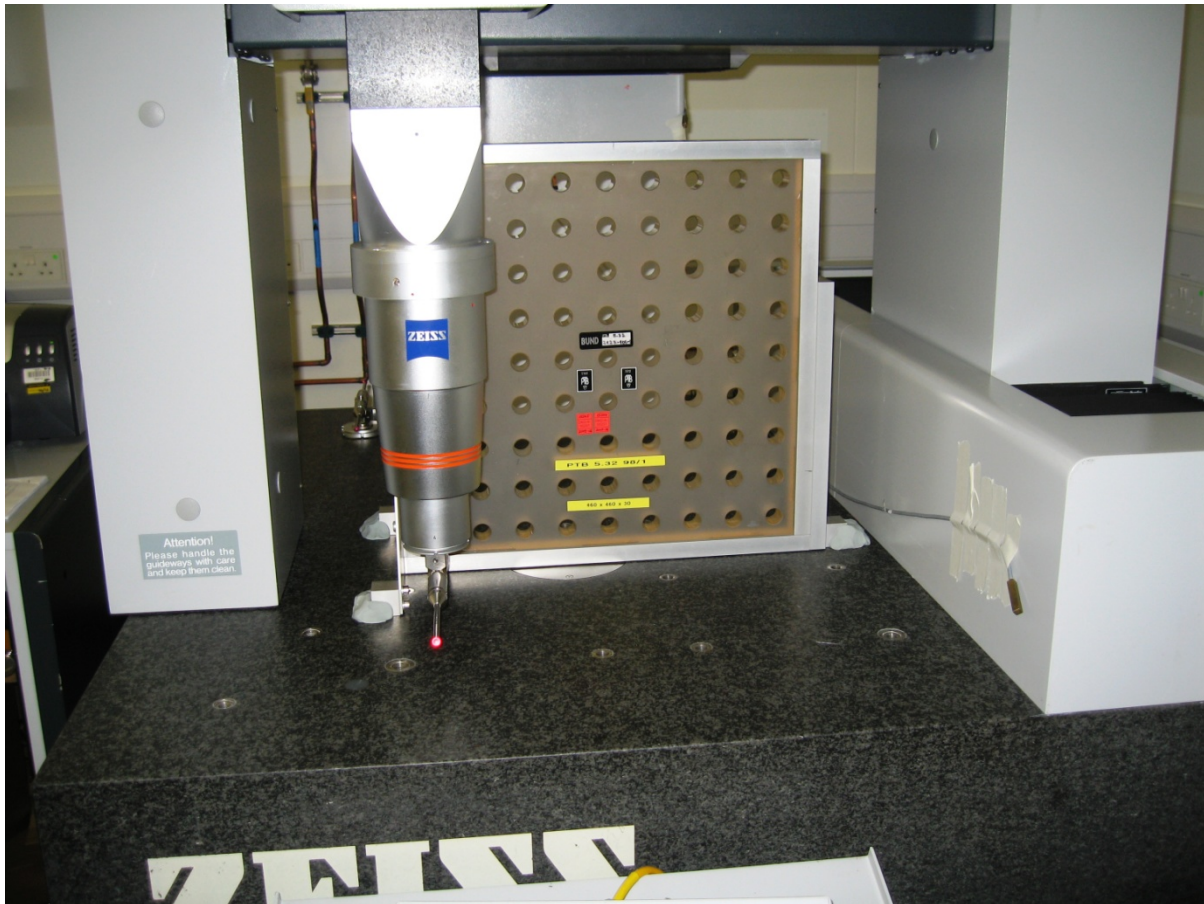


Figure 10 Determining the residual error field of a CMM using a hole plate made from ZERODUR®, a low expansion glass ceramic. Notice that the stylus configuration allows measurement of the plate from both sides.

NPL's PMM-C Infinity CMM also has an installation of the PTB VCMM. This is the Calypso 7 implementation and is based around PTB's VCMM2. VCMM2 builds on the original VCMM and incorporates effects due to form error.

What if you are not a Calypso/Quindos user – PUNDITCMM?

If you do not have access to a CMM with software from Leitz or Zeiss, there is a stand-alone CMM uncertainty package on the market from Metrosage⁵. This commercially available software package called PUNDITCMM (Figure 11) enables evaluation of task-specific measurement uncertainties from knowledge of the machines MPE. Essentially the software simulates measurement of the part on a number of CMMs, with the appropriate geometry, that would have the stated MPE. This method is also known as off-line virtual CMM. The method used is called simulation by constraints.

The software allows the user to define CMM geometry and workspace dimensions, and to enter CMM performance data. These data could include ISO 10360 or B89 test results, or just the CMM manufacturer's specifications, or perhaps some other available information. The more specific the data provided by the user, the greater the precision of the uncertainty

⁵ Volcano, CA, USA: Metrosage LLC. <www.metrosage.com/punditcmm.html>

estimates. An entire suite of CMMs can be stored, allowing comparison of their respective abilities to adequately perform one or more specified measurement tasks.

The software simulates part inspections, gathering statistical data on the variability of each measurement result. These simulations are presented in a graphical format. The user can see how a specified inspection protocol will perform in providing the level of certainty required. If it is found that the estimated uncertainty is too large, alternative strategies, such as taking additional measurement points or using a different probe system, can be quickly tried, evaluated and compared. More details of PUNDITCMM can be found at www.metrostage.com.

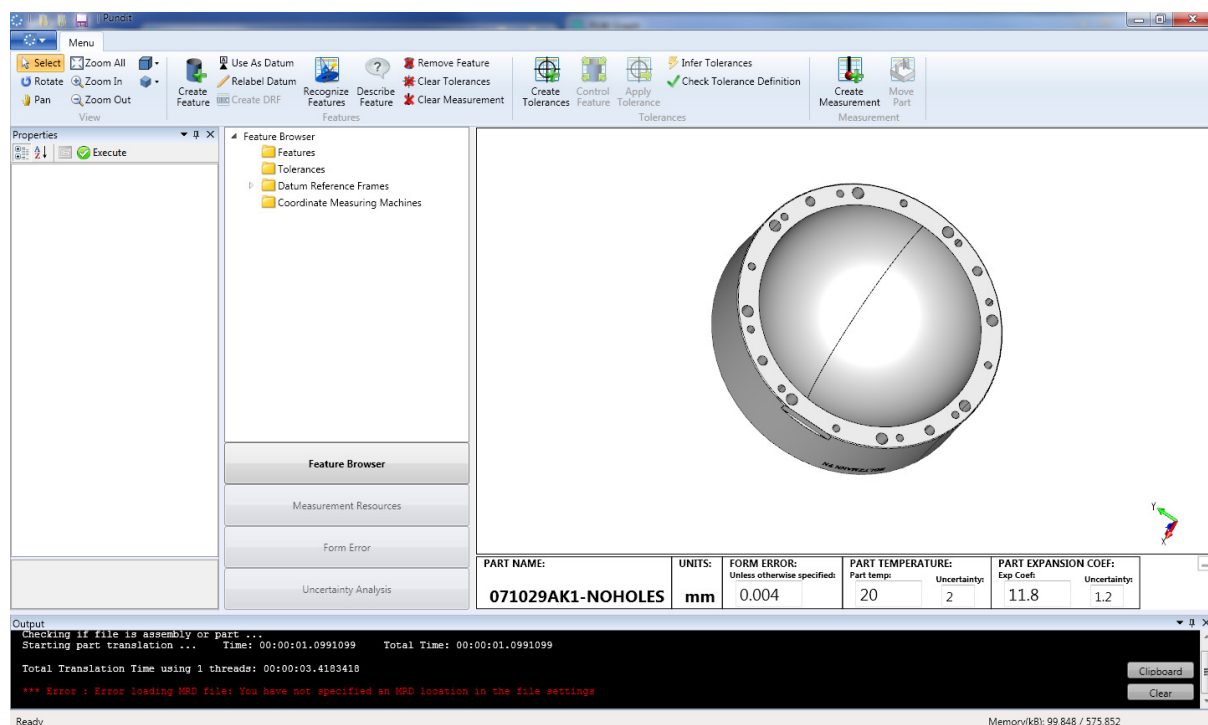


Figure 11 The user interface of PUNDITCMM

Simulation by constraints?

Phillips *et al.* at NIST named methods that use complete parametric error specification of the CMM (like the virtual CMM) full parametric simulation (FPS). They then suggested a generalisation of FPS in order to make it easier to implement and more robust. They call their approach simulation by constraints (SBC). When using SBC, you start by building a very large set of virtual CMMs, each having a different (random) error state. Every virtual CMM is characterized by 21 parametric errors.

A further virtual CMM based on the Expert CMM project is reported by Balsamo *et al.* (Balsamo A, Di Ciommo M, Mugno R, Rebaglia B, Ricci E, Grella R. Evaluation of CMM uncertainty through Monte Carlo simulations. Annals of the CIRP, 1999; 48: 425-428).

Detailed description of PTB model based around Calypso implementation

Measurement of the uncertainty contributions

The determination of the input parameters of the virtual CMM is carried out by service engineers from Zeiss as part of the software installation process.

The contributions to the virtual CMM are determined using a technique that involves measuring a ball plate in a number of orientations in the CMM volume. In the case of Cartesian CMMs, the required object positions are three pairs of two each parallel to the same two coordinate axes but with two different offsets from the slideways. This allows the calculation of the six errors of each motion axis (Appendix B). That is to say, three rotational, one position and two deviations from straightness. The artefact is generally made from a nominally thermally invariant material in order to assess the errors of the CMM in isolation from the effect of temperature variations on the reference object (see Figure 10). This data is stored in the folder named mac (see Table 4).

Typical temperatures and gradients are measured as part of the installation process. Temperatures and temperature gradients that prevail during the full error assessment describe the reference state. The gradients are used to predict uncertainties due to distortions in the CMM structural components and hence the uncertainty contributions to roll, pitch, yaw and squareness errors.

Drift is assessed by repeated measurements of a sphere located in a part of the volume where measurements normally occur. A typical stylus configuration is used. The CMM axes are kept continually moving between sphere measurements as in normal use. The procedure is repeated several times on different days. Drift is defined as the displacement during an individual drift measurement divided by the hours elapsed during that measurement.

Other characteristics that the software needs to know about include the kinematic characteristics of the CMM, that is to say the position of the scales given in universal coordinates (essentially the sequence of the moving axes). This information is stored in the kch folder.

The data from the experimental test are stored in the exchange folder along with files that describe the characteristics of the machine. This folder structure is consistent with that defined by PTB (see Table 4).

Table 4 A description of the various sub-folders in the exchange folder

Folder	description
env	A file which contains the uncertainty parameters to describe different environmental conditions
kch	A file which describes the kinematic characteristics of the CMM
mac	A file which describes the errors of the slideways (component errors and their uncertainties)
pro	A file which describes the uncertainties of the probing process.
pro_base	A file which describes the base characteristic (behaviour) of the specific probe head

pro_cal	A file which describes the diameter of the reference sphere, the uncertainty of the sphere's calibration and the probe offset.
wkp	A file which describes the surface texture as a R_z value of the simulated part surface

Use of the Calypso virtual CMM

When using the OVCMM, the user writes the program exactly as they would normally. There are no special requirements. However, the user has to carry out an extended stylus qualification (see Figure 12). The extended qualification allows determination of the uncertainty influence from the particular stylus used. Figure 12 also shows the measuring points used and indicates where data from multiple probings are recorded. In the case below, twenty-four points are collected at the equator and seven points on a polar circle. The '5' refers to the number of repeat points in each of four probing directions (directions of each axis plus one oblique direction). This test has to be carried out with each new stylus used. The resultant parameters calculated are U_{dir} and $U_{repetitive}$. Data for the extended qualification is stored in the folder pro (see Table 4).

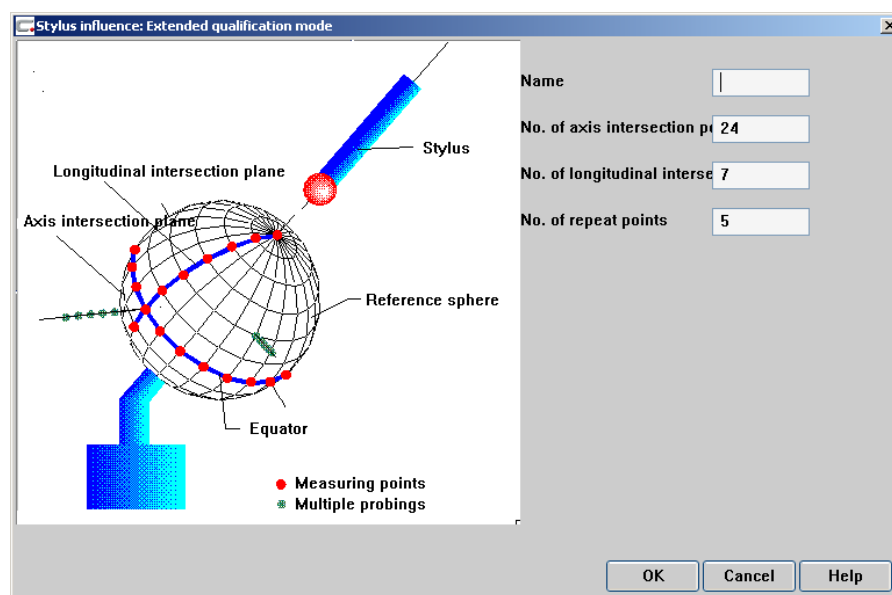


Figure 12 The dialogue for extended stylus qualification in Calypso.

The user also has to specify the contributions attributable to the workpiece. These include terms for the uncertainty in the knowledge of the workpiece expansion coefficient (see Figure 13), stored in the folder env and the value of the workpiece surface texture (R_z) stored in the folder wkp. The surface texture file can be changed by the user to represent a typical measured R_z for the part.

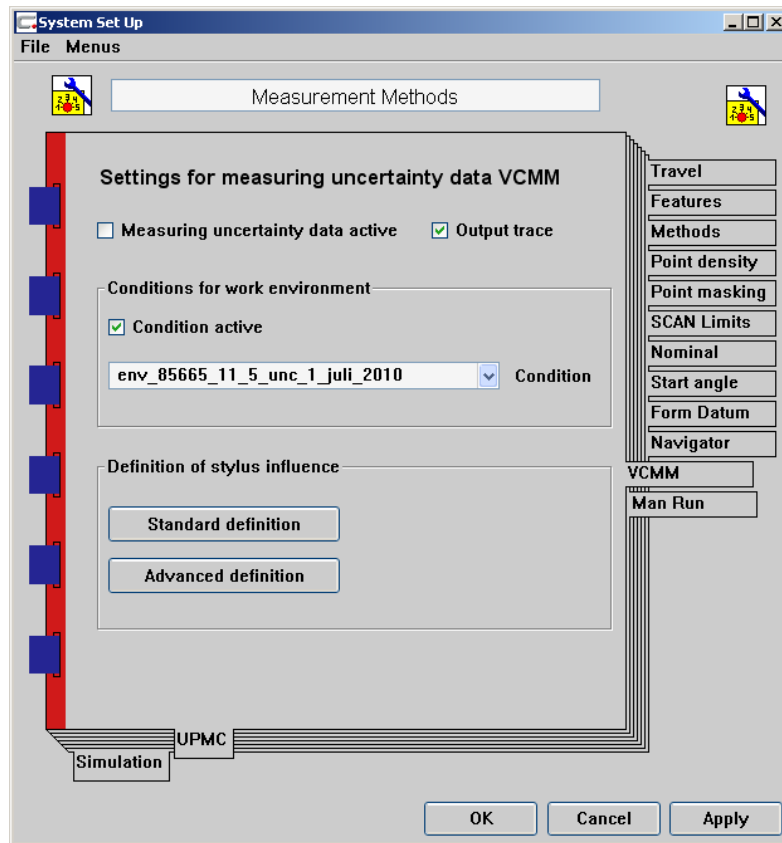


Figure 13 Input of the workpiece expansion coefficient uncertainty in Calypso. The example here is for a workpiece with an expansion coefficient of $11.5 \times 10^{-6} \text{ K}^{-1}$ with an uncertainty of $1 \times 10^{-6} \text{ K}^{-1}$.

Care must be taken when placing the component in the volume of the CMM. It is often only practicable to assess the residual error map over part of the whole volume of the CMM and the component needs to be placed in this restricted volume for the virtual CMM to be able to perform its calculations.

It should also be noted that the extended qualification has to use a 30 mm diameter reference sphere, so qualifying small diameter stylus types may be problematical if not impossible.

The program is then run as normal and the measurement data collected. On program completion an additional run is performed to recompute the data with the virtual CMM turned on. Depending on the complexity of the program this additional run can take some time to complete. This is a one-off process for measurements made with this CNC program and this stylus configuration.

For each measurement characteristic the actual value is output with an associated task-specific measurement uncertainty (see Figure 9).

Interim checks

The CMM needs to be checked regularly for possible changes in its measurement uncertainty. As previously described, the CMM's uncertainty is fully assessed when the virtual CMM

software is initially installed and reassessed over longer time intervals. In order to build confidence in the validity of the results of these full uncertainty assessments it is good practice to carry out simpler interim checks at shorter intervals. A suitable method would be to measure a calibrated hole plate and check that the results and estimated uncertainties are commensurate with the calibrated value and associated uncertainty.

A useful document covering the PTB virtual CMM and its installation is Trapet, E., Franke, M., Härtig, F., Schwenke, H., Wäldele, F., Cox, M., Forbes, A., Delbressine, ., Schellekens, P., Trenk, M., Meyer, H., Moritz, G., Guth, Th., Wanner, N.: Traceability of Coordinate Measurements According to the Method of the Virtual Measuring Machine, Final Project Report MAT1-CT94-0076, PTB-report F-35, Part 1 and 2, ISBN 3-89701-330-4, 1999

Description of PTB model based around Quindos 7 implementation

In principle, the Quindos implementation is very similar to the Calypso implementation; however, there are subtle differences in its use. The Quindos7 implementation is based around the PTB VCMM2, a later incarnation of the VCMM used in Calypso.

Using the Quindos implementation, the program must be run once and the complete run, including all of the measured points saved, to a file. This file is then simulated in a separate Quindos7 process and the calculated uncertainty for each feature is stored to the appropriate element.

All the data for use by the CMM is stored in a similar exchange folder structure but the files are XML-based rather than being text-based.

As Quindos is a text-based language, additional commands are implemented:

VCmm2PrbChngCheck to determine the probe changing uncertainty;

VCmm2QualCheck to determine the probe specific uncertainties; and

VCmm2SetScenario to set roughness, shape, expansion coefficient and temperature scenarios.

From the additional commands it can be seen that the Quindos 7 implementation accounts for automatic probe changes and the artefact shape.

Comparison of the techniques

3

IN THIS CHAPTER

- What's not covered in UES
- Use of UES to design experiments
- Advantages and disadvantages of the various methods

This chapter aims to give the reader a comparison of the techniques defined in ISO 15530. Whilst comparing the techniques, it's worth remembering that in the past users often either had too much belief in the correctness of the measurement results or conversely mistrusted the measuring results to the extent they asked for unjustifiably high measurement effort or too expensive measurement equipment. Either of these techniques is better than doing nothing at all.

What's not covered in UES

The virtual CMM goes part of the way to achieve traceability in co-ordinate metrology and work is on-going to include scanning and rotary tables.

As will be seen later, uncertainty contributions due to the object are not included. Chapter 4 gives details of these terms and how they can be combined with the output from the OVCMM.

Use of UES to design experiments

A further advantage of the UES technique is that it allows 'what-if' calculations to be performed. That is to say, before receiving a component, you can see what effect varying the number of points and stylus configurations will have on the quality of your measurements. You can, in some cases, even allow the software to help you choose which CMM you are going to use for a particular component. In this case, the predicted uncertainty can be compared with the tolerance that needs to be achieved. A UES can also allow investigations of changing some aspect of the measurement (e.g. the thermal environment, the stylus configuration or the point sampling strategy) and monitoring how the measurement uncertainty varies.

Advantages and disadvantages of the various methods

A number of methods have been discussed for ascertaining the measurement uncertainty associated with a measurement performed on a CMM. Each has its advantages and disadvantages.

UES validity

One disadvantage of the UES based around the PTB model is that the calculated uncertainties are only valid whilst the CMM is in the same state as when it was initially assessed. That is to say that the CMM is in the same environment and that the geometric errors have not changed. It is, therefore, necessary to carry out interim checks to make sure that the uncertainties produced by the UES are still valid. By using a comparison technique, you are essentially assessing the CMM on-the-day.

UES missing uncertainty contributions

It is impossible to incorporate all uncertainty contributors in UES. Therefore, it is important to document which uncertainty contributors are included and which are not, as emphasised in ISO 15530-4. Current implementations of UES focus mainly on hardware uncertainties. One important uncertainty contributor, for discrete tactile measurements, that is missing in current UES is feature form deviations although this omission is being addressed by work at PTB on OVCMM2.

UES simulation time

Both commercial UES (OVCMM and PUNDITCMM) are offline software packages and uncertainties are calculated before or after the actual CMM measurement. This results in extra workload and is undesirable for most users. Monte Carlo simulations can take a lot of time to run, which can be an issue for online operation.

Disadvantage of the comparison technique

The comparison technique can be time consuming and to an extent reduces the versatility of the CMM. The additional measurement of reference artefacts can also be time consuming and expensive.

However, the comparison technique does mean that you are evaluating your task-specific measurement uncertainty directly against a traceably calibrated artefact. However, traceable artefacts may not be available for all the characteristics you need to evaluate as part of the measurement program.

Techniques to minimise measurement uncertainty

4

IN THIS CHAPTER

- Minimising measurement uncertainty

There are a number of ways of minimising the measurement uncertainty associated with measurements made on a CMM. The following chapter will discuss some of these methods and indicate ways in which the magnitude of these uncertainties can be estimated.

Minimising measurement uncertainty

Cleanliness

To minimise uncertainty, it is important that rules are in place relating to the cleaning of the part and the stylus system. Particles of dust on either the reference sphere used for stylus system qualification or the part can increase the measurement uncertainty. It is good practice to have a documented cleaning process in place for the part, the stylus tips and the reference sphere.

Form

The form of an object is something that cannot be changed, but it is instructive to ascertain the effect of form on the measurements.

To estimate the effect of form, measure the component using the intended point distribution and then a further nine times randomly shifting points within the point sampling interval. The n individual results X_i and their average X_{av} are calculated. This calculation is performed for several representative parts and may need to be repeated if the manufacturing process changes. The form contribution is calculated as:

$$U_{form} = 2 \sqrt{\frac{1}{n-1} \sum (X_i - X_{av})^2}$$

Clamping

Many parts will change shape when inappropriately clamped. Make sure that the clamping method does not distort the part. Aim to clamp the part in a manner similar to its eventual use.

The uncertainty due to placement of the workpiece and clamping can be estimated as follows. Carry out at least ten measurements repeatedly placing the workpiece on the CMM with the fixture in different locations while changing the object orientation each time. Place and clamp the object in the fixture anew each time. The n individual results X_i and their average X_{av} are calculated. This calculation is performed for each measured feature. The clamping contribution is calculated as:

$$U_{clamp} = 2 \sqrt{\frac{1}{n-1} \sum (X_i - X_{av})^2}$$

Stylus selection

Stylus tips make contact with the workpiece. They generally consist of synthetic monocrystalline, highly spherical industrial rubies (Al_2O_3) mounted on a shaft. Ruby is a hard ceramic material ensuring minimum wear of the stylus ball. There are numerous types and sizes available for a variety of applications; the shafts on which the styli are mounted are usually non-magnetic stainless steel, ceramic, carbon fibre or tungsten carbide.

Ruby ball styli are suitable for the majority of probing applications. To maximise the accuracy of measurements taken, the user should follow the following rules:

- keep the stylus system simple;
- use a short, stiff stylus;
- use the largest ball tip possible;
- check that the stylus tip has not come loose from the stem;
- keep it clean;
- use the most appropriate stylus type; and
- let the stylus thermally stabilise after handling.

The user should be aware of the extra uncertainty introduced by, for example, changing a stylus configuration during a measurement run and by the indexing of a probe head.

Further good practice on stylus selection can be found in *Measurement Good Practice Guide No. 43, CMM Probing*.

Stylus qualification

You can minimise uncertainty introduced by the qualification process by making sure the stylus system is qualified regularly. Requalification is particularly important if the temperature is not stable and you are using tee- or star-shaped stylus systems.

Probing strategies

Probing strategies can have an influence on the measurement uncertainty, both from the point of view of the number of points, but also in their distribution. This influence can be particularly important when measuring partial features or when evaluating the form of a feature.

Obviously the fewer points you have on a feature, the less information you have about that feature but equally important is the distribution of points. It does not make sense to collect hundreds of points around the circumference of a cylindrical feature if you then only collect data in two planes. The aim should be to evenly distribute points over the feature.

Further good practice on number and distribution of probing points can be found in *Measurement Good Practice Guide No. 41, CMM Measurement Strategies*.

Temperatures/ thermal equilibrium

To minimise measurement uncertainty, it is important that the temperature is as close to 20 °C as can be practically achieved and that both spatial and temporal temperature gradients are kept to a minimum. Any spatial gradients will potentially distort the CMM. Any temporal gradients mean that you are essentially using a different CMM to measure the component from day-to-day.

In addition, if the temperature of the part is changing with time, perhaps because it has just been handled or machined, then that can increase uncertainties.

Therefore, where available, use should be made of any temperature correction routines in the CMM software. Items should be left long enough to thermally stabilise before measurement ideally in the environment in which they are to be measured. Thermal stabilisation times should be determined either by monitoring the part temperature or by making repeated measurements of the part until no change in size is seen.

The main source of uncertainty will then be any uncertainty in the knowledge of the thermal expansion coefficient of the material.

Redundancy

Measurement redundancy can help you to gain confidence in the measurements and may enable estimation of certain uncertainty contributions.

The easiest way to introduce redundancy, especially during long runs, is to remeasure a feature that was measured earlier in the run. Remeasurement potentially allows identification of drift or errors such as part movement due to poor clamping.

The next form of redundancy is more involved and requires multiple measurement of the part, either in the same part of the machine volume, in a different part of the machine volume or in a different part of the machine volume with a rotation of the part. Such a test will give an idea of the relationship between location and results. Such test may identify a 'sweet spot' for the machine where errors are minimised.

Comparison

We have already mentioned comparison techniques and how they can be used to evaluate sources of uncertainty and correct for some error sources. It is good practice to apply the comparison principle when the aim is to minimise sources of uncertainty.

Scanning

It is difficult to assign an uncertainty when measurements are made on the CMM in scanning mode. One method is to measure the item at least five times at the normal scanning speed and then at least five times at a quasi-static scanning speed (very low scanning speed).

Calculate the average result for the low speed (X_{low}) and the average for the intended scanning speed (X_{high}). The scanning related uncertainty is calculated as

$$U_{dyn} = \frac{2}{\sqrt{3}} |X_{high} - X_{low}|.$$

Probing force

In a similar manner the contribution to the uncertainty due to probing force can be estimated. Carry out the measurements five times at the intended probing force and then five times with double the probing force. Again calculate the averages X_{high} and X_{low} . The probing force contribution is calculated as:

$$U_{force} = \frac{2}{\sqrt{3}} |X_{high} - X_{low}|.$$

Object

As can be seen from the above, the component (object) being measured introduces a number of sources of uncertainty. We can assign an overall uncertainty due to the object as follows when using UES

$$U_{object} = \sqrt{U_{form}^2 + U_{force}^2 + U_{clamp}^2 + U_{dyn}^2} \quad (k=2)$$

Interim checks of the CMM

Interim checks of the CMM using calibrated artefacts allow routine evaluation of the CMM performance and may help to identify problems that could contribute to an increased measurement uncertainty.

An interim check can be as simple as a partial ISO 10360 test or may involve a more complex assessment using an artefact such as a ball plate. Interim checks should be carried out for two reasons:

- (1) to check that the machine still meets specification; and
- (2) to establish that the outputs from any UES in use is still giving valid results.

Summary

5

IN THIS CHAPTER

- Summary

This measurement good practice guide has provided an overview of the Methods that can be used to ascertain the measurement uncertainty of CMM-based measurements. The key good practice guidelines for carrying out CMM performance verification can be summarised as follows:

- don't rely on MPE;
 - the MPE only samples the machine volume;
 - may not adequately reflect how you use the CMM
- be aware of the various international specification standards relating to uncertainties in general and more specifically CMM measurement uncertainties;
 - ISO 15530 substitution based techniques;
 - ISO 15530 simulation based techniques;
 - ISO 14253 understand the importance of uncertainty
- try and make use of uncertainty evaluating software (Online Virtual co-ordinate Measuring Machine (OVCMM)), the software uses
 - the points measured on the object
 - the knowledge of all the uncertainty contributions
 - from a qualified calibration laboratory
 - from user
 - uncertainty calculation based on repeated simulations. Points varied through their expected ranges and with their expected probability distributions
 - needs regular monitoring by user
 - interim check artefact
- be aware of the difference between on-line and off-line virtual CMMs
- be aware of the advantages and disadvantages of the two methods
- don't ignore measurement uncertainty for measurements made on a CMM

When using the virtual CMM, follow the CMM operator checklist below

- Is the machine calibration still valid?
- Have interim checks been performed and are they still valid?
- Have all the styluses been calibrated after qualification?
- Are the proper names attributed to the probes?
- Have the appropriate environmental conditions been chosen?
- Is the temperature correction active?
- Are the temperature sensors attached to the workpiece?
- Has thermal equilibrium been reached?
- Are the styluses clean?

Glossary of terms

6

IN THIS CHAPTER

- Glossary of terms

Glossary of terms

Terms defined below are based on the VIM (*International Vocabulary of Basic and General Terms in Metrology*) and various ISO standards.

Ball plate	A mechanical artefact comprising a number of spheres mounted in fixed positions on the plate, usually in a single plane.
CMM	A measuring system with the means to move a probing system and capability to determine spatial coordinates on a workpiece surface.
Combined standard uncertainty	A standard measurement uncertainty that is obtained using the individual standard measurement uncertainties associated with the input quantities in a measurement model.
Complete measurement result	measurement result including the expanded measurement uncertainty
Coverage factor	number larger than one by which a combined standard measurement uncertainty is multiplied to obtain an expanded measurement uncertainty
Expanded measurement uncertainty	product of a combined standard measurement uncertainty and a factor larger than the number one
Geometric errors	The departures from the ideal geometry caused by a lack of mechanical perfection in the moving elements of the CMM. Note: The most commonly encountered geometrical errors include: roll, pitch, yaw, straightness in both the horizontal and vertical orientations and positioning error, in each axis; there are also squareness errors between pairs of axes.

Hole plate	A mechanical artefact that comprises a flat plane pierced with holes whose axes are normal to one of the surfaces.
Task-related calibration	The set of operations which establish, under specified conditions, the relationship between values indicated by a CMM and the corresponding known values of a limited family of precisely defined measurands which constitute a subset of the measurement capabilities of a CMM.
Traceability	
UES	Uncertainty evaluating software
Uncertainty of measurement	non-negative parameter characterising the dispersion of the quantity values being attributed to a measurand, based on the information used

Appendices

7

IN THIS CHAPTER

- Appendix A Links to other useful sources of information
- Appendix B CMM errors
- Appendix C Monte Carlo – example
- Appendix D Further reading

Appendix A Links to other useful sources of information

A.1 National and International Organisations

A.1.1 National Physical Laboratory

"When you can measure what you are speaking about and express it in numbers you know something about it; but when you can not express it in numbers your knowledge is of a meagre and unsatisfactory kind."

Lord Kelvin, British Scientist (1824 – 1907)



The National Physical Laboratory (NPL) is the UK's national measurement institute and is a world-leading centre of excellence in developing and applying the most accurate measurement standards, science and technology available. For more than a century NPL has developed and maintained the nation's primary measurement standards. These standards underpin an infrastructure of traceability throughout the UK and the world that ensures accuracy and consistency of measurement.

NPL ensures that cutting edge measurement science and technology have a positive impact in the real world. NPL delivers world-leading measurement solutions that are critical to commercial research and development, and support business success across the UK and the globe.

Good measurement improves productivity and quality; it underpins consumer confidence and trade and is vital to innovation. We undertake research and share our expertise with government, business and society to help enhance economic performance and the quality of life.

NPL's measurements help to save lives, protect the environment, enable citizens to feel safe and secure, as well as supporting international trade and companies to

innovation. Support in areas such as the development of advanced medical treatments and environmental monitoring helps secure a better quality of life for all.

NPL employs over 500 scientists, based in South West London, in a laboratory, which is amongst the world's most extensive and sophisticated measurement science buildings.

The National Physical Laboratory is operated on behalf of the National Measurement Office by NPL Management Limited, a wholly owned subsidiary of Serco Group plc. For further information: Switchboard 020 8977 3222 | www.npl.co.uk/contact

A.1.2 National Institute of Standards and Technology (NIST)

NIST is the equivalent of NPL in the United States of America. Founded in 1901, NIST is a non-regulatory federal agency within the U.S. Department of Commerce. NIST's mission is to promote U.S. innovation and industrial competitiveness by advancing measurement science, standards, and technology in ways that enhance economic security and improve our quality of life.

The NIST web site at www.nist.gov often contains documents relevant to this guide in pdf format. In particular the document *The validation of CMM task specific measurement uncertainty software* can be downloaded from the NIST website.

A.1.3 EURAMET

The European Association of National Metrology Institutes (EURAMET) is a Regional Metrology Organisation (RMO) of Europe. It coordinates the cooperation of National Metrology Institutes (NMI) of Europe in fields like research in metrology, traceability of measurements to the SI units, international recognition of national measurement standards and related Calibration and Measurement Capabilities (CMC) of its members. Through Knowledge Transfer and cooperation among its members EURAMET facilitates the development of the national metrology infrastructures.

EURAMET serves the promotion of science and research and European co-operation in the field of metrology.

This is realised by the following measures in particular:

- development and support of European-wide research co-operation in the field of metrology and measurement standards;
- development, regular updating and implementation of a European Metrology Research Programme (EMRP);
- support of members and associates when applying for research funds for the purpose of European cooperative projects;
- co-ordination of joint use of special facilities;
- improvement of the efficiency of use of available resources to better meet metrological needs and to assure the traceability of national standards;

- technical co-operation with metrology institutes beyond EURAMET and with other regional and international metrology organisations;
- performing the tasks of a Regional Metrology Organisation (RMO) with the objective of worldwide mutual recognition of national measurement standards and of calibration and measurement certificates;
- promotion and co-ordination of scientific knowledge transfer and experience in the field of metrology;
- representing metrology at the European level and promoting best practice to policy and political decision makers with regard to the metrological infrastructure and European co-operation;
- co-operation with European and international organisations responsible for quality infrastructure, in particular by participation in the preparation of harmonized technical documents.

For more information visit the EURAMET web site at: www.euramet.org

A.2 Networks

A.2.1 Measurement Network – Engineering and Optical

This special interest group reflects a range of interests from a number of sectors, including advanced manufacturing and engineering, transport and energy. It aims to ensure that the needs of members with an interest in dimensional, mass, temperature and optical measurement are reflected in the range of events held under the Measurement Network. These events provide a forum which enable members to exchange views and information.

For further information visit the website at: www.npl.co.uk/measurement-network/engineering-and-optical-resources/

A.2.2 Software Support for Metrology Programme (SSfM)

SSfM is an programme that underpins the NMS, focussing on the use of mathematics and computing in metrology. It aims to achieve a balance between research and development, whilst also extending the range of techniques and applications available to meet the continually changing needs of metrology. The overall aim of the SSfM Programme is to tackle a wide range of generic issues, some of which are problems in metrology that require the application of established software engineering practices, whilst others require advances in mathematics, software engineering or theoretical physics. The programme, thus, includes work in metrology, mathematics, software and theoretical physics, with strong links between the various disciplines.

The SSfM Club is aimed at users and suppliers of metrology software, giving them a say in the direction of the Programme. It is the focal point for the distribution of general information arising from the Programme.

Further details can be found at: www.npl.co.uk/mathematics-scientific-computing/software-support-for-metrology/

A.3 National and International Standards

A.3.1 British Standards Institution (BSI)

BSI started in 1901 as a committee of engineers determined to standardise the number and type of steel sections in order to make British manufacturers more efficient and competitive. The BSI Group is now the oldest and arguably the most prestigious national standards body in the world and is among the world's leading commodity and product testing organisations. Website www.bsi-group.com.

A.3.2 International Organisation for Standardization (ISO)

The International Organization for Standardization (ISO) is a worldwide federation of national standards bodies from some 140 countries.

The mission of ISO is to promote the development of standardisation and related activities in the world with a view to facilitating the international exchange of goods and services, and to developing cooperation in the spheres of intellectual, scientific, technological and economic activity.

ISO's work results in international agreements that are published as International Standards.

Further information on ISO can be found at: www.iso.ch

The following BS and ISO specifications are relevant to this guide.

Evaluation of measurement data – Guide to the expression of uncertainty in measurement JCGM 100:2008 (GUM 1995 with minor corrections)

DD ISO/TS 23165: 2006 Geometrical product specifications (GPS). Guidelines for the evaluation of coordinate measuring machine (CMM) test uncertainty

BS 7172:1989 British Standard Guide to Assessment of position, size and departure from nominal form of geometric features.

Document VDI/VDE 2617 *Accuracy of co-ordinate measuring machines* may also be of interest to users of this guide.

Evaluation of measurement data – Supplement 1 to the "Guide to the expression of uncertainty in measurement" – Propagation of distributions using a Monte Carlo method JCGM 101:2008

ISO/TS 15530-1:2013 Geometrical product specifications (GPS) -- Coordinate measuring machines (CMM): Technique for determining the uncertainty of measurement -- Part 1: Overview and metrological characteristics

ISO 15530-3:2011 Geometrical product specifications (GPS) -- Coordinate measuring machines (CMM): Technique for determining the uncertainty of measurement -- Part 3: Use of calibrated workpieces or measurement standards

ISO/TS 15530-4:2008 Geometrical Product Specifications (GPS) -- Coordinate measuring machines (CMM): Technique for determining the uncertainty of measurement -- Part 4: Evaluating task-specific measurement uncertainty using simulation

A.4 Traceability

Traceability in measurement is the concept of establishing a valid calibration of a measuring instrument or measurement standard, by a step-by-step comparison with better standards up to an accepted or specified standard. In general, the concept of traceability implies eventual reference to an appropriate national or international standard.

The National Physical Laboratory is the United Kingdom's national standards laboratory. It operates at the heart of the National Measurement System (NMS) which is the infrastructure designed to ensure accuracy and consistency in every physical measurement made in the UK. Chains of traceability link UK companies' measurements directly to national standards held at NPL.

For the majority of industrial applications, companies can establish a link to national measurement standards through the calibration and testing services offered by United Kingdom Accreditation Service (UKAS) accredited laboratories, which are in turn traceable to NPL. However, for challenging or novel measurements to the highest standards of accuracy, which are not catered for by UKAS-accredited laboratories, NPL can often provide a traceable measurement solution directly to industry.

The United Kingdom Accreditation Service is the sole national accreditation body recognised by government to assess, against internationally agreed standards, organisations that provide certification, testing, inspection and calibration services.

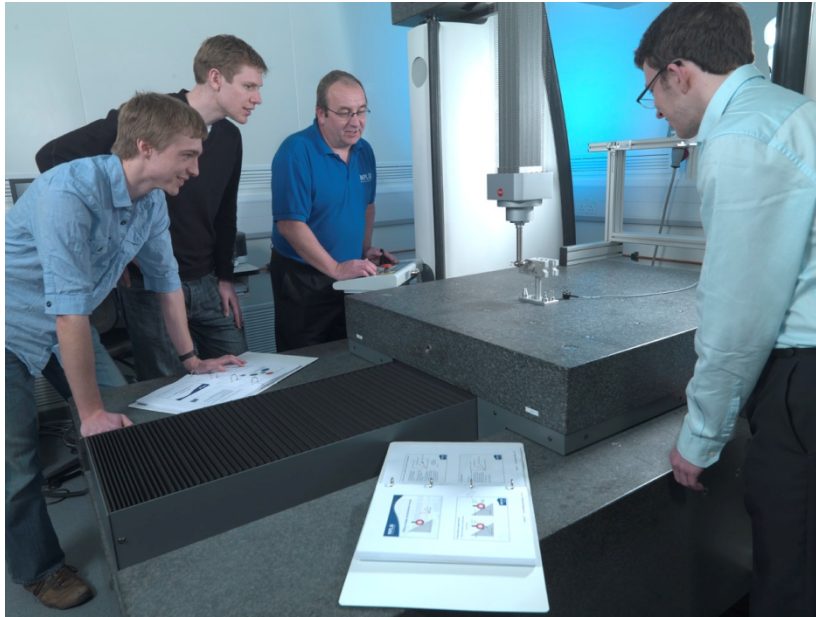
Accreditation by UKAS demonstrates the competence, impartiality and performance capability of these evaluators.

UKAS is a non-profit-distributing private company, limited by guarantee. UKAS is independent of Government but is appointed as the national accreditation body by the Accreditation Regulations 2009 (SI No 3155/2009) and operates under a Memorandum of Understanding with the Government through the Secretary of State for Business, Innovation and Skills.

UKAS accreditation demonstrates the integrity and competence of organisations providing calibration, testing, inspection and certification services.

Further information on UKAS can be found at: www.ukas.com.

A.5 Training courses



A.5.1 Dimensional Measurement training: Level 1 – measurement User

A three day training course introducing measurement knowledge focusing upon dimensional techniques.

Aims & Objectives

To provide:

- the underpinning knowledge and expertise for anyone who uses measurement tools or requires an appreciation of the importance of measurement,
- the principle knowledge and practical training for people who are required to use dimensional measurement techniques to complete their daily tasks; and
- the tools to instil and encourage questioning culture.

Enabling:

- An understanding of the fundamentals of standards, traceability, calibration, uncertainty, repeatability, drawing symbols and geometrical tolerances, the importance of the relationship between tolerances and measuring equipment and be able to question the measurement.

Level 1 is applicable to all industrial sectors as a stand-alone qualification or as a building block for further NPL Dimensional Measurement Training levels – 2 & 3.

Course Content**Day 1 - Geometric Product Specification (GPS) A**

Including what is GPS, drawing practice and geometrical tolerances.

Day 2 - Measurement Principles and Methods A

Including successful measurements, standards, traceability, calibration, uncertainty, units, relationship between tolerances and measuring equipment using micrometers and callipers, repeatability and reproducibility of measurements.

Day 3 - Measurement Principles and Methods B

Including the relationship between tolerances and measuring equipment by the use of height gauges, dial test indicators, dial gauges, plug gauges, gap gauges and temperature effects.

NB: Fundamental Measurement Calculation is incorporated into all 3 days including powers, scientific notation and triangles. This is achieved by understanding the relationship of these calculations when applied to tolerance zones and practical measuring tasks.

A workbook of evidence must be completed successfully during the training course and, where required, post assessment tasks can be set for each individual to be completed in the workplace.

A.5.2 Dimensional Measurement Training: Level 2 - Measurement Applier

A four day training course for those who have a good basic understanding of measurement principles gained through the Level 1 training course.

Aims & Objectives

To provide:

- the underpinning knowledge and expertise for anyone who uses measurement tools or requires an appreciation of the importance of measurement,
- the principle knowledge and practical training for people who are required to use co-ordinate measurement techniques to complete their daily tasks; and
- the tools to instil and encourage questioning and planning culture

Enabling:

- a visible return on investment for a manufacturing organisation in the form of various production cost savings and an upskilled workforce,
- a reduction in re-work time and waste on the production line - faults and problems will be detected earlier in the production process; and
- An in-depth appreciation of *why* measurement is carried out and not simply *how*

Level 2 is applicable to all industrial sectors as a stand-alone qualification or as a building block for further NPL Dimensional Measurement Training levels – 3 & 4.

A workbook of evidence must be completed successfully during the training course and, where required, post assessment tasks can be set for each individual to be completed in the workplace.

Course Content

Geometric Product Specification (GPS) B

Content covered:

GPS standards; Envelope tolerance; Size Principles; ISO Limits and Fits Projected tolerance; Free state condition; Virtual condition; Maximum Material Condition principles; Geometrical tolerancing measurements using first principle measuring equipment; Surface texture principles.

Measurement Principles and Methods C

Content covered:

Calibration; Uncertainties; Traceability; Procedures; First Principle Measurement; Angle plate; Gauge blocks; Surface plate; Height micrometer; Sine bar or sine table.

Process Control A

Content covered:

Statistical Process Control theory; Variation – common, special causes; Prevention versus detection; Collecting and calculating data when using measuring tools; Callipers; micrometers; Basic charts – Tally chart/Frequency Table, Histogram, Control Chart; Reacting to variation; Benefits of process control; Standard deviation; Capability indices; Fundamentals of Gauge R&R.

Measurement Principles and Methods D

Content covered:

Taper calculations; Angles; Diameters; Searching for triangles; Chords; Radians; Manipulation of formula.

Co-ordinate Principles A

Content covered:

Application of equipment: First principles; Co-ordinate Measuring Machine; Optical and vision machines; Articulating arm; Laser tracker; Projector; Microscopes; Height gauge with processor; Contour measurement equipment.

Machine performance: Calibration standards; Self-verification/artefacts; Measurement volume.

Alignment Techniques: 321/point system alignment; Flat face alignment; Axes alignment; Car line/engine centre line.

Machine appreciation: Ownership; Care; Respect; Cost; Contribution to the business.

Work Holding: Fixturing; Rotary table; Clamping; How to hold the

part; Influence of component weight, size, shape; Free state; Restrained state.

Co-ordinate geometry: Points; Plane; Line; Circle; Cylinder; Cone; Sphere; Ellipse.

Sensor Types: Probing Strategies; Relevant standards; Environment.

Measurement Strategies: Number of points; Partial arc; Contact/non contact.

Co-ordinate methods A (OEM Training - equipment specific)

Content covered:

First principles; Co-ordinate Measuring Machine; Optical and vision machines; Articulating arm; Laser tracker; Projector; Microscopes; Height gauge with processor; Contour measurement equipment.

A.5.3 Understanding and Evaluating measurement uncertainty

Aims & Objectives

- Give improved understanding of measurement uncertainty, methods for measurement uncertainty evaluation and the circumstances in which those methods apply
- Support the provision of scientifically defensible results
- Emphasise model-based uncertainty evaluation
- Help make better use of the GUM
- Help tackle more challenging uncertainty evaluation problems than those covered directly by the GUM

Module details

The course is based on the considerable experience gained by the presenters of the needs of practitioners in the area of measurement uncertainty evaluation. The course covers:

- Underpinning statistical concepts
- Developing and using a measurement model
- Formulating a problem of measurement uncertainty evaluation
- The conventional approach to uncertainty evaluation of the GUM
- Other approaches to uncertainty evaluation, including a Monte Carlo method
- Extensions to problems with more than one measurand
- Examples and case studies
- Current international work on the GUM including the GUM Supplements

Delegates are encouraged to bring their own problems of uncertainty evaluation, for which there will be ample discussion time.

Appendix B CMM errors

Geometry errors of a three Cartesian axes CMM

xtx(x) position error in x	ytx(y) straightness error in y	ztx(z) straightness error in z
xty(x) straightness error in x	yty(y) position error in y	zty(z) straightness error in z
xtz(x) straightness error in x	ytz(y) straightness error in y	ztz(z) position error in z
xrx(x) roll error in x	yry(y) pitch error in y	zrx(z) pitch error in x
xry(x) pitch error in x	yry(y) roll error in y	zry(z) yaw error in x
xrz(x) yaw error in x	yrz(y) yaw error in y	zrz(z) roll error in x
ywx squareness error in yx	zwx squareness error in zx	zwy squareness error in zy

Appendix C Monte Carlo – example code

The following example is written in Matlab and shows how the roll of two dice can be simulated. See what happens if you vary the number of simulations.

```
%
% matlab code to demonstrate simulating the roll of a pair of dice
%
a=1;% lowest possible roll of 1 dice
b=6;% maximum possible roll of 1 dice
simulations=10000; % number of simulations
rng('shuffle'); % seed random number generator
roll1 = randi([a,b],1,simulations); % results for first die
roll2 = randi([a,b],1,simulations); % results for second die

total=roll1+roll2; % total throw
hist(total,11) % plot histogram with 11 bins (2 to 12)
```

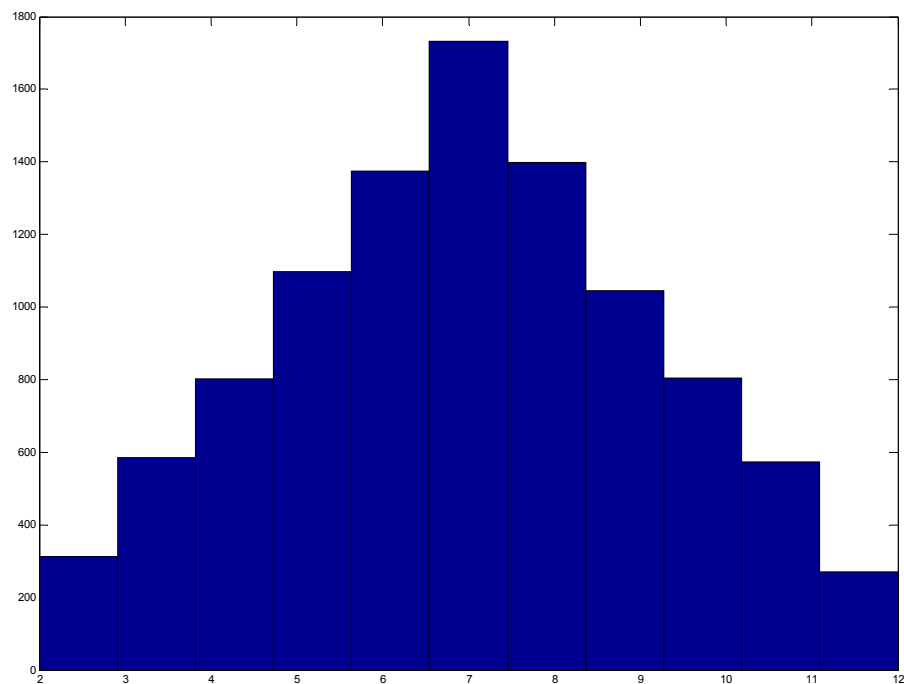


Figure 14 The output of the Matlab script to simulate rolling of a pair of dice

The next example simulates adding random variations to measurement points that define a circle and shows the effect on calculated radius.

```
% Matlab code to simulate measuring a circle

nomradius=50;%nominal radius of circle
points=66; % number of measured points
simulations=1000; % number of simulations
```

```

% preallocate arrays for speed
x=zeros(points,1);
y=zeros(points,1);
x1=zeros(1,points);
y1=zeros(1,points);
radius=zeros(simulations,1);

s=size(x1);
% =====
% Define parameters for circle fitting
% =====
x0=[0,0]'; % Estimate of the circle centre.
r=nomradius; % Estimate of the circle radius.
tolp=0.001; %Tolerance for test on step length.
tolg=0.001; %Tolerance for test on gradient.
w=ones(s(2),1); % Weighting = 1

% calculate a circle of nominal points
for R = 1:points
    angle=R*(2*pi/points);
    x(R)=nomradius.*sin(angle);
    y(R)=nomradius.*cos(angle);
end

% now vary the points and calculate radius for each simulation
for j=1:simulations
    % the following could be done in fewer commands but this method
    is used
    % for clarity (r and r2 could be arrays added to x and y).
    for i=1:points
        a=0; % lower limit of variation
        b=0.005; %upper limit of variation
        r = a + (b-a).*randn(1,1); % variation for x
        r2 = a + (b-a).*randn(1,1); % variation for y
        x1(i)=x(i)+r; % new x
        y1(i)=y(i)+r2; % new y
    end

    % =====
    % fit circle to X and display results
    % =====
    X=[x1;y1]';

    [x0n, rn, d0, sigmah, conv, Vx0n, urn, GNlog, a, R] = ...
        ls2dcircle6(X, x0, r, tolp, tolg, w);

    radius(j)=rn;
end
hist(radius,20)

```

⁶ This routine is available by contacting NPL

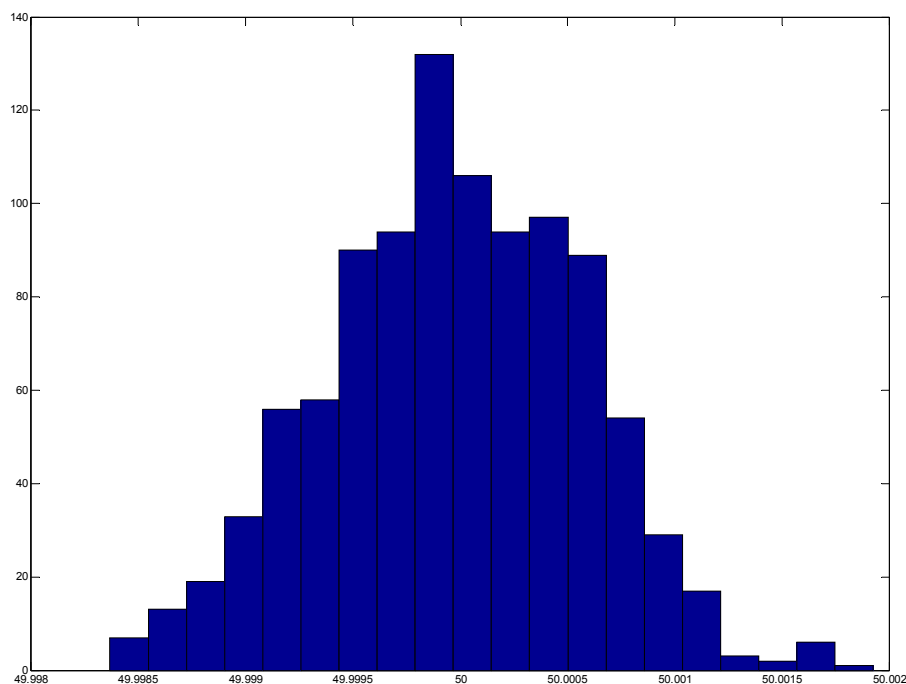


Figure 15 Output from the above Matlab script to demonstrate measurement of points on a circle

Appendix D Further reading

ISO 14253-1:1998, Geometrical Product Specifications (GPS) – Inspection by measurement of workpieces and measuring equipment – Part 1: Decision rules for proving conformance or non-conformance with specifications, ISO, 1998.

ISO/TS 15530-4:2008, Geometrical Product Specifications (GPS) – Coordinate measuring machines (CMM): Technique for determining the uncertainty of measurement – Part 4: Evaluating task-specific measurement uncertainty using simulation, ISO, 2008.

R. Wilhelm, R. Hocken, H. Schwenke, Task specific uncertainty in coordinate measurement, *CIRP Annals-Manufacturing Technology* 50 (2) (2001) 553–563.

H. Schwenke, B. Siebert, F. Wäldele, H. Kunzmann, Assessment of uncertainties in dimensional metrology by Monte Carlo simulation: proposal of a modular and visual software, *CIRP Annals-Manufacturing Technology* 49 (1) (2000) 395–398.

ISO/IEC Guide 98-3:2008, Uncertainty of measurement – Part 3: Guide to the expression of uncertainty in measurement (GUM:1995), ISO/IEC, 2008.

Trapet, E., Franke, M., Härtig, F., Schwenke, H., Wäldele, F., Cox, M., Forbes, A., Delbressine, ., Schellekens, P., Trenk, M., Meyer, H., Moritz, G., Guth, Th., Wanner, N.: Traceability of Coordinate Measurements According to the Method of the Virtual

Measuring Machine, Final Project Report MAT1-CT94-0076, PTB-report F-35, Part 1 and 2, ISBN 3-89701-330-4, 1999

Baldwin J, Summerhays K, Campbell D, Henke R. Application of Simulation Software to Coordinate Measurement Uncertainty Evaluations. *Measure* 2007; 2:40–52

Balsamo A, Di Ciommo M, Mugno R, Rebaglia B, Ricci E, Grella R. Evaluation of CMM uncertainty through Monte Carlo simulations. *Annals of the CIRP*, 1999; 48: 425-428.

United Kingdom Accreditation Service, M3003, The Expression of Uncertainty and Confidence in Measurement, Edition 3, November 2012.

Coordinate Measuring machines and Systems, Robert J. Hocken, Paulo H. Pereira, Taylor and Francis, 2012