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A Review of Current Methods for Modelling Erosive Wear

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ABSTRACT

The wear process known as erosion occurs when discrete particles (e.g. solid particles or water droplets) strike a surface. The process causes surface damage, leading to material loss and mechanical degradation, potentially causing serious problems and economic loss. Material degradation is often unavoidable but the ability to model the erosion process allows engineers to improve component/system design through better material selection and the use of erosion reducing measures.

In this paper, the current methods available for modelling erosion will be reviewed. The simpler models are empirical models that use mass (or volume) loss equations to predict the erosion rate. Mechanistic models can account for fluid and particle properties and flow rates. The more complicated models include Monte Carlo based solid particle erosion, computational fluid dynamics based models and finite element based models.

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1 INTRODUCTION

Within many industrial sectors, coatings are commonly used to extend the in-service life of components within complex systems. The in-service conditions are often demanding and various factors can lead to wear of the coatings, potentially causing serious problems and economic loss. The wear process known as erosion occurs when discrete particles (e.g. solid particles or water droplets) strike a surface. The presence of water droplets in the flow of a steam turbine, for example, leads to erosion of the moving blade. Similarly, sand within oil/gas pipelines can lead to pipe thinning. Erosion is a complex surface damage process, strongly affected by mechanical and metallurgical factors such as shape, size, density, hardness and quantity of erosive particles, particle velocity and angle of impact and the properties of the substrate/coating.

The erosion process can cause damage to the coating, ultimately leading to material loss and mechanical degradation. The degree of degradation will generally depend on the system characteristics e.g. pressure, temperature and type of erosive particles, and will also be strongly dependent on the materials used in the system. The behaviour of the material being eroded will determine the erosion mechanism. For ductile metals material is removed through cutting and chip formation, while in brittle materials, material loss is due to crack formation and brittle fracture. Material degradation often cannot be avoided, but by selection of suitable materials and use of erosion reducing measures, a system fulfilling the requirements may be designed.

Many papers have been written on the experimental observations of erosive wear leading to a greater understanding of the mechanisms of erosion. Modelling of the erosion process naturally follows on from this experimental research. Modelling can be used to interpret existing experimental data, or to extend experimental data to conditions that cannot be tested with current facilities. Most significantly modelling erosion wear can assist design engineers both in the design of better experimental test facilities for material characterisation and in the design of improved industrial components/systems. The long-term industrial aim is to develop a system where the components are manufactured from appropriate materials, the materials degradation modes are identified and that models exist to predict material performance [1].

The aim of this paper is to review current methods of modelling erosion.

2 EXPERIMENTAL

There is a wide range of experimental tests that are used for studying erosion. Typical examples of these are direct impingement tests and tests performed in flow loops. Direct impingement tests involve shooting particles e.g. sand or water particles, at a sample [2, 3]. The sample can be tilted to investigate the effect of impact angle on erosion. Velocity effects can also be studied. Flow loops are used to investigate the effects of geometry, particle size and fluid properties on erosion. Flow loops can be single phase (e.g. gas only) or multiphase (e.g. gas/liquid/sand) allowing a range of conditions to be tested [3-6]. To successfully study erosion, laboratory facilities must be able to replicate the particle loadings, velocities and temperatures experienced in the industrial system. The information obtained from these types of experiments has been used to develop a range of erosive wear models.

3 MODELLING OF EROSIWE WEAR

Due to the complexity of erosive wear it is not surprising that there is no universally accepted predictive model. However, a number of erosive wear models have been developed over the years. These range from the simpler single mechanism models, to the more rigorous models involving complex geometries/interactions.

The simpler models are empirical models that use mass (or volume) loss equations to predict the erosion rate, and are often material specific. Mechanistic models can account for fluid and particle properties and flow rates. The more complicated models include Monte Carlo based solid particle erosion, computational fluid dynamics based models and finite element based models.

Although erosion can occur in a wide range of situations, many models are industry/system specific e.g. water droplet erosion of turbines or solid particle erosion in pipelines. Industry is understandably sensitive about the exact nature of the models they use. In many cases a fairly basic model will be used as a first pass assessment, to rank materials, followed by a more detailed model tailored to their requirements [7,8].

This section explores the main methods for modelling erosive wear documented in the literature.

3.1 EMPIRICAL MODELS

Some of the earliest models that were developed in order to understand and explain erosion phenomena were relatively simplistic, but were nevertheless capable of predicting some of the observed trends of erosion. The models correlated material properties with erosion rates, particularly for pure metals and simple alloys. An example of one of these early models is a model by Hutchings [9] for erosive wear by plastic deformation. Hutchings assumes that the erosive particle does not deform and that the problem can be analysed quasi-statically. By solving the equation of motion of the particle, Hutchings calculated the mass of the material removed as wear debris from impact of the particle at normal incidence. He summed this over many impacts to give an erosion rate (E) of

$$E = \frac{K\rho U^2}{2H} \quad (1)$$

where ρ is the density of the material being eroded, U is the initial particle velocity and H is the target surface hardness. K represents the fraction of material removed from the indentation as wear debris and is also known as the wear coefficient. The value of K can be thought of as a measure of the efficiency of the material removal process.

This model can be extended to include particles striking the surface at oblique angles, assuming that material is removed by individual particles in a cutting action, which occurs only over a narrow range of particle geometries and impact orientations. In this model the erosion rate is a function of the impact angle, θ :

$$E = \frac{K_1\rho U^n f_1(\theta)}{H} \quad (2)$$

where the velocity exponent, n , lies typically between 2.0 and 2.5 and is itself a function of θ .

This model is only valid for shallow angles of incidence as at higher angles the method of debris detachment changes. Hutchings compared the linear relationships predicted by equations (1) and (2) with experimental data for several pure metals and steels. Pure metals generally showed good correlation between hardness and resistance to erosion. While the above model is for erosive wear by ductile failure, Hutchings [9] also notes that models have been developed for erosion by brittle fracture. In this case material is assumed to be removed by the formation and intersection of cracks. These models estimate the contact force exerted on a particle during impact, either by assuming that the pressure resisting penetration is the quasi-static hardness of the surface, or by using a dynamic wave-propagation model to determine the impact pressure. The contact force is then used in a semi-empirical analysis to predict the extent and depth of the lateral cracks formed, and hence the volume of material removed. In this case, the most important material property for determining erosion rate is the fracture toughness.

Bingley and Flynn [10] compared the predictions of Hutchings model for erosion with plastic deformation with experimental data from a set of heat-treated steels. A reasonable agreement with experimental data was found for the dependence of erosion rate on hardness, if it was assumed that the wear coefficient, K , was also a function of hardness. A shortcoming of this model is the inability to account for particle shape. The shape of the indenting particle can strongly affect the deformation and hence the mechanism of material removed leading to differences between the model and experimental data. This limitation is similar to that of other simple models, hence few can be used to predict erosive wear in a general way, since most are only valid for the range of experiments for which they have been established.

For piping systems, DNV [11] have drawn up guidelines for the assessment of erosive wear in piping systems associated with production and transportation of oil and gas. The document provides procedures for calculation of erosion rates for typical components in a pipe. The procedure is intended to give conservative estimates for the erosion attack in order to avoid excessive erosion in the modelled system during operation.

The following equation is used as a basis for the estimation of erosion rates:

$$\dot{E}_L = \frac{\dot{m}_p K U_p^n F(\alpha)}{\rho_t A_t} \cdot C_{\text{unit}} \quad (3)$$

$\dot{E}_L$ is the erosion rate, referred to depth rather than mass of eroded material. This equation requires input of particle impact velocity, U_p , velocity exponent, n , particle impact angle, α , mass flow of sand impacting the area, $\dot{m}_p$, density of target material, ρ_t , area exposed to erosion A_t and the unit conversion parameter C_{unit} . The function $F(\alpha)$ characterises the ductility of the target material. Impact angles, impact velocities and the amount of sand hitting a surface are dependent on the flow characteristics, the grain size distribution and the component geometry. In the procedure these effects are accounted for by model/geometry factors. These factors also account for multiple impact of the sand particles, concentration of sand particles due to component geometry and model uncertainty. The document contains formulae for calculation of erosion rates in smooth straight pipes, welded joints, pipe bends, blunted tees, reducers, flexible pipes, intrusive erosion probes and headers. These models are based on extensive experimental investigations and are widely used in industry. They have been implemented in a commercially available computer programme called ERBEND and simulations show good agreement with pipe bend tests.

Another well-used empirical model for erosion in pipes is that of Salama [12] who uses a relationship similar to that given in equation (1) but with correction factors based on experimental and computational fluid dynamics (CFD) data. The model proposed by Salama is:

$$ER = \frac{1}{S_m} \frac{W V_m^2}{\rho_m} \frac{d}{D^2} \quad (4)$$

where ER is the erosion rate, W is the sand flow rate, V_m is the fluid mixture velocity, d is the sand size, D is the pipe internal diameter, ρ_m is the fluid mixture density and S_m is a geometry factor (e.g. 5.5 for pipe bends) which must be obtained experimentally. This model accounts for multiphase flow as it has terms for mixture density and mixture velocity. The accuracy of this model for pipe bends is illustrated in Figure 1, comparing model predictions with flowloop test data [12]

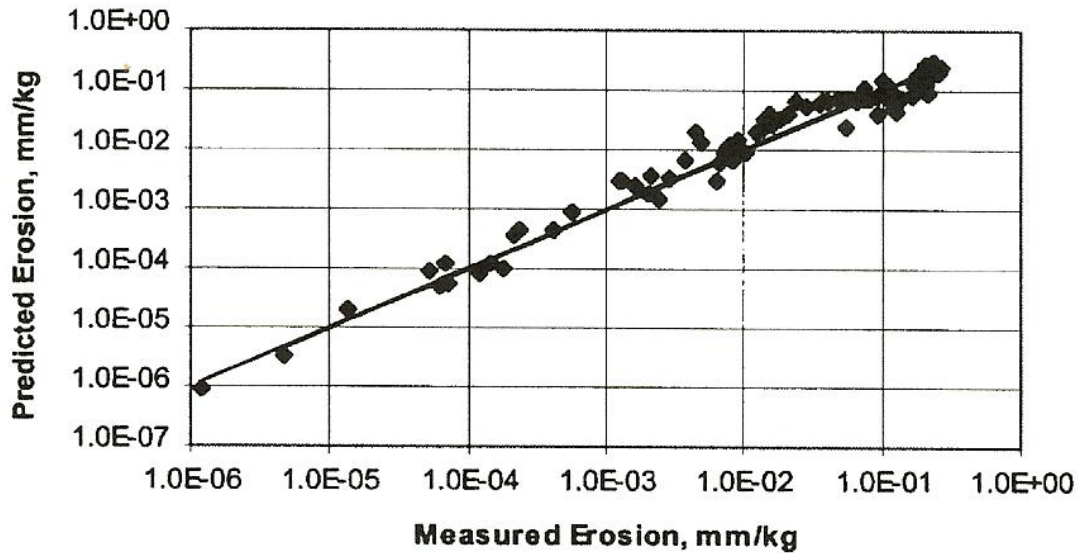


Figure 1. Correlation between empirical model predictions using equation (4) and measured sand erosion rate in pipe bends [12]

This empirical equation was found to be valid for other geometries such as tees, chokes and sand monitors, providing the value of the geometry factor, S_m , was calibrated using experimental data or CFD results.

These empirical models assume that the erosion rate is linearly proportional to sand production rates. This assumption is valid for most practical cases when sand concentration is low, but predictions will be conservative if sand concentration becomes high.

A common extension to the empirical erosion models is to include corrosion-related processes. Erosion-corrosion is the combined action of the mechanical process of solid particle erosion and the electrochemical process of corrosion. Erosion rate equations similar to those discussed above are used, but these equations have extra terms to account for the interaction with corrosion [13,14].

3.2 SEMI-EMPIRICAL MODELS

A good alternative to the empirical models is a semi-empirical model, which has a more mechanistic basis to it. This bridges the gap between the simpler models and the more complex CFD type models. A well-respected model has been developed by the University of Tulsa and used in its Sand Production Pipe Saver (SPPS) programme to predict erosion, accounting for several physical parameters and mechanisms. This model is more comprehensive than the empirical models and consists of flow modelling, particle tracking and more complex erosion equations.

This model is for simple geometries such as elbows and tees, and the relationship was developed based on extensive empirical information. The key point in this model is that a method was determined to find the characteristic impact velocity of the particles on the pipe wall, V_L . This characteristic impact velocity depends on many factors including pipe geometry and size, sand size and density, flow regime, velocity and fluid properties. To obtain the characteristic particle impact velocity the concept of stagnation length was introduced. This is the thickness of the fluid layer in contact with the pipe wall and is therefore the distance through which the particles must travel in order to strike the pipe wall. The equivalent stagnation lengths were obtained for elbow and tee geometries by erosion testing, flow modelling and particle tracking for sand in gas and liquid.

The model predicts the characteristic impact velocity of the particles, V_L , using a simplified particle tracking model based on two non-dimensional parameters Re and Φ . The Reynolds number of the particle, Re_0 , is defined as

$$Re_0 = \frac{\rho_m V_0 d_p}{\mu_m} \quad (5)$$

where V_0 is the equivalent flowstream velocity, ρ_m is the density of fluid (mixture) in the stagnation layer, μ_m is the viscosity of fluid (mixture) in the stagnation layer and d_p is the diameter of the particle.

Φ is a parameter proportional to the mass of fluid being displaced by the particle to the mass of the impinging particle:

$$\Phi = \frac{L \rho_m}{d_p \rho_p} \quad (6)$$

where L is the equivalent stagnation length and ρ_p is the particle density.

The density and viscosity of the fluid in the stagnation layer is computed based on the volume flow of gas and liquid at flowing conditions. The model assumes that the stagnation region is composed of a one-dimensional flow field that has a linear velocity in the direction of the particle motion and uses a simplified drag coefficient model. A comparison of the predicted impact velocities [4] with experimental data obtained by Lynn et al. [6] is shown in Figure 2. For single-phase flow the initial particle velocity is assumed to be the same as the flow stream velocity. For 2-phase flow, the sand particles in the liquid and gas phases are assumed to travel at velocities similar to their corresponding phase velocities. Studies of impact velocity found that particle impact velocity increases with particle diameter and impact velocity decreases with an increase in fluid viscosity

Once the characteristic impact velocities of the particles have been determined, an erosion equation can be used to calculate erosion and penetration rates for a specific geometry. For example, for elbows the maximum penetration rate is calculated using [4]

$$h = F_M F_S F_P F_{r/D} \frac{W V_L^{1.73}}{(D/D_0)^2} \quad (7)$$

where h is the penetration rate, F_M is an empirical constant accounting for material hardness, F_S is the empirical sand sharpness factor, F_P is the penetration factor for steel, $F_{r/D}$ is the penetration factor for elbow radius, W is the sand production rate, V_L is the characteristic particle impact velocity, D is pipe diameter and D_0 is the reference pipe diameter.

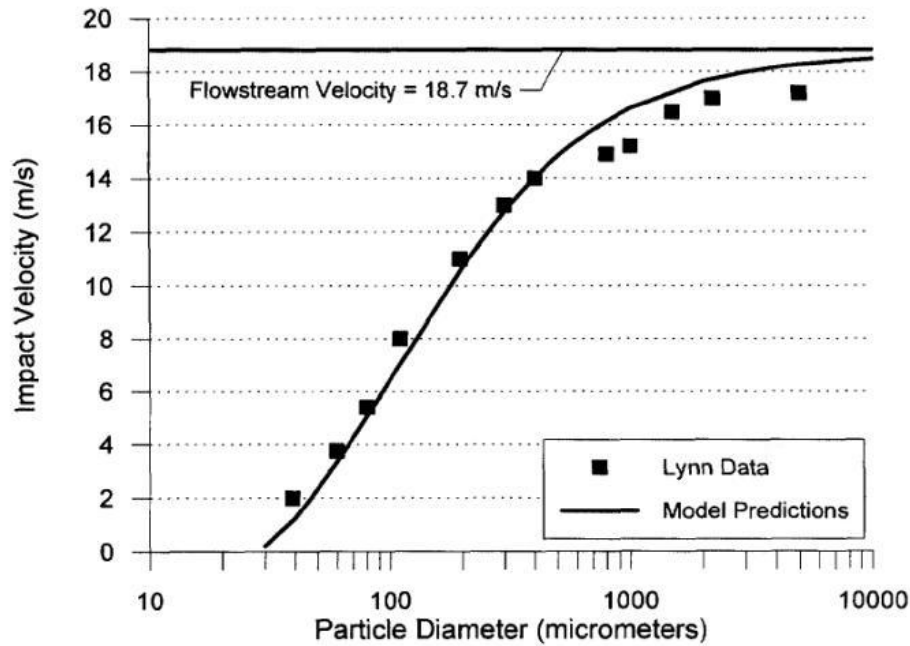


Figure 2. Impact velocity versus particle diameter for silicon carbide particles suspended in diesel oil [4]

This model has been further improved to allow for the complexities of multiphase flow, such as the four major flow patterns observed in vertical flow (annular, churn, slug and bubble flow) by taking into account local fluid velocities for each flow pattern [15]. Figure 3 shows comparisons of measured erosion with the mechanistic model predictions. There is good agreement especially for annular flow. Less experimental data was available for the other types of flow.

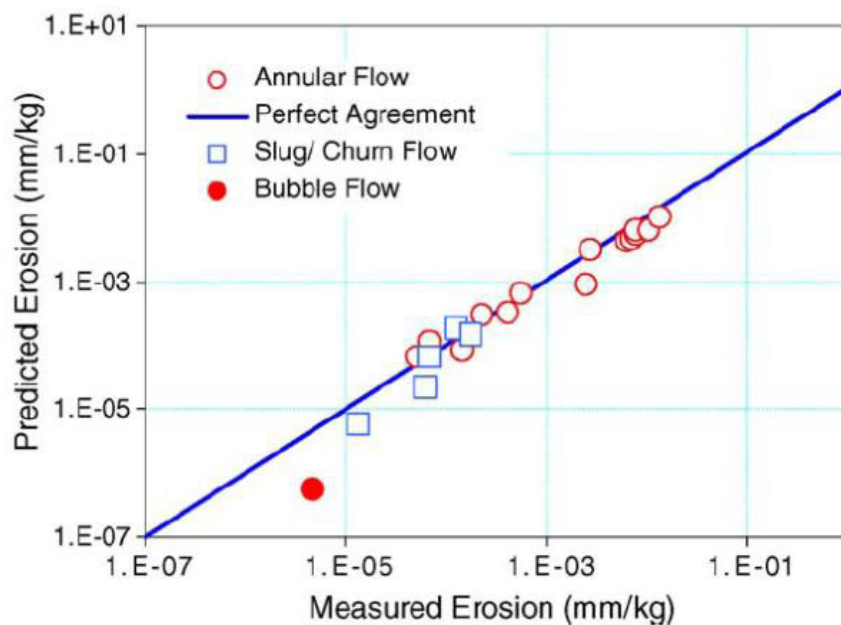


Figure 3. Comparison of measured erosion with mechanistic model predictions [15]

3.3 MONTE CARLO MODELLING

The definitive model for predicting solid particle erosion of thermal barrier coatings (TBC's) has been developed by Nicholls and co-workers [16-19]. The aim of their research has been to develop a computer model, based on Monte Carlo statistics, capable of predicting the erosion rate of electron beam (EB) physical vapour deposition (PVD) thermal barrier coatings (TBC's) under various different conditions relevant to gas turbine engine operation.

Monte Carlo simulation techniques were adopted in recognition of the stochastic nature of erosion processes. Particle size, properties, the material surface condition and the local dynamic impact condition can change continually, making predictions of erosion rates very difficult. Although many erosion models are available to describe brittle and ductile erosion, their use is restricted by the mode of material removal being modelled [16].

The advantage of using Monte Carlo techniques is that the most appropriate material removal model can be selected based on local impact conditions. Hence creating a tool capable of predicting erosion over a wide range of service conditions.

The authors' first consideration was the erosion mechanisms of EB PVD TBC's as well as their material properties. In TBC's material loss is via the initiation and propagation of cracks (brittle erosion mechanism). Although under certain conditions, e.g. above erodent softening temperatures, the EB PVD TBC's can exhibit a ductile type erosion response to impact angle. Brittle fracture still occurs but with enhanced loss of material at glancing impact angles.

EB PVD TBC's have a columnar microstructure, which improves erosion resistance. Erosion occurs via the accumulation of near surface (lateral) cracks in a number of adjacent columns, prior to the removal of the tops of the fractured columns [17]. The authors observed that cracks initiated in one column do not propagate to neighbouring columns, giving the coatings their strain tolerance and improving their erosion resistance. If an impacting particle interacts with more than one column it is possible for cracks to initiate in a cluster of adjacent columns.

The computer model has been set up with a number of input parameters i.e. material and system properties, and a single output, the calculated erosion rate. To model the EB PVD TBC's the columns were visualised as hexagons, each with 6 identical nearest neighbours. The model comprised 43 columns and 'wraps around' so that columns on the left edge are nearest neighbours to those on the right, and similarly those on top nearest neighbours to those on the bottom.

The model generates a look-up table of values for a random distribution of impacting particles, which includes a number of key values for a random selection of 1000 particles. The calculations for the particles include contact area, size of the plastic zone, maximum impact force and the critical velocity for that particular particle. When modelling erosion of brittle materials, the depth of the impact zone produced by the particle is very important. If the depth of the plastic zone can be calculated, the amount of material lost per impact can then be determined. The calculation of the depth of the plastic zone involves parameters such as Young's modulus, E , substrate hardness, H , and the radius of the indented volume. Equations are given for the size of the plastic zone for both spherical and angular particles. For the column that experiences the greatest impact, the crack depth is taken as the radius of the plastic zone. Due to the hemispherical shape of the plastic zone, neighbouring columns affected by the impact will have a plastic zone with maximum depth at the column boundary shared with the column experiencing greatest impact. This implies that cracking in the column in which the impact centred is most likely to occur within the bulk of the column and propagate towards the boundaries, whereas for neighbouring columns cracks are likely to initiate at the boundary and propagate into the column. Knowing the distance from the centre of impact to the column boundary and the radius of the plastic zone, the depth of the plastic zone in neighbouring columns can be calculated. The program

contains a look up table of distance from the centre of impact to column boundary for each neighbour as a function of radius.

In this model a velocity threshold was used to determine whether an impacting particle would initiate cracking. This critical velocity needed to initiate lateral cracking is given by:

$$V_{crit,l} = 105 \frac{E^{3/4} K_{IC}^3}{H^{13/4} \rho^{1/2} R^{3/2}} \quad (8)$$

where E is Young's modulus, K_{IC} is the critical stress intensity factor, H is hardness of the substrate, R and ρ are radius and density of the particle respectively which equates to a mass term. This is a qualitative estimate of the threshold, which is based on the crack length being equal to the size of the plastic zone. For each impacting particle the critical velocity needs to be calculated, then the model must determine whether the impacting particle is above or below this threshold. To account for a particle impacting a cluster of columns, a percentage of the particle mass is used in the calculations for each column based on the percentage of the contact area that is in contact with the column. The authors note that there is certain degree of error inherent in these calculations, which increases as the contact area decreases. But since the small contact areas are unlikely to cause cracking, this error should not be a major issue.

The last stages of the model are to calculate the volume of material removed per impact. The volume available for removal is calculated based on the size of the columns and the depth of the lateral crack. The model assumes that the central column and four nearest neighbours need to be cracked for material loss to occur. This was based on experimental observations of eroded surfaces.

The model has been validated against experimental data. Figure 4 shows the comparison between predicted and measured erosion rates for EBPVD TBC eroded with silica at 170 m/s at various impact angles. There is a good correlation between two sets of data with predicted results being within 20% of experimental data.

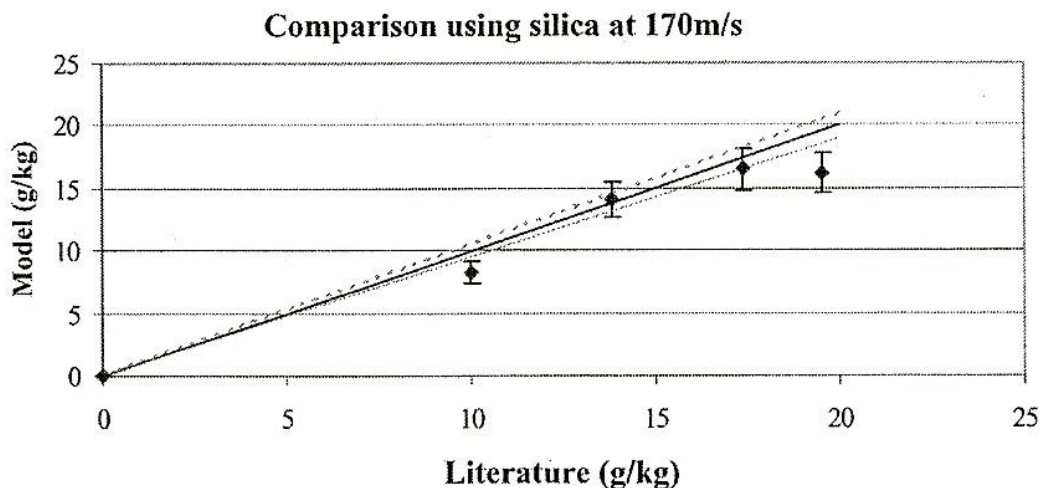


Figure 4. Graph showing the comparison between predicted and measured erosion rates (the superimposed lined is drawn for a direct comparison, the dashed lines represent 1 s.d) [18]

Figure 5 shows a comparison between measured and predicted erosion rates of various materials for erosion within a gas turbine under simulated coal fired combined cycle service conditions. Once again, good agreement was found between predicted and measured results.

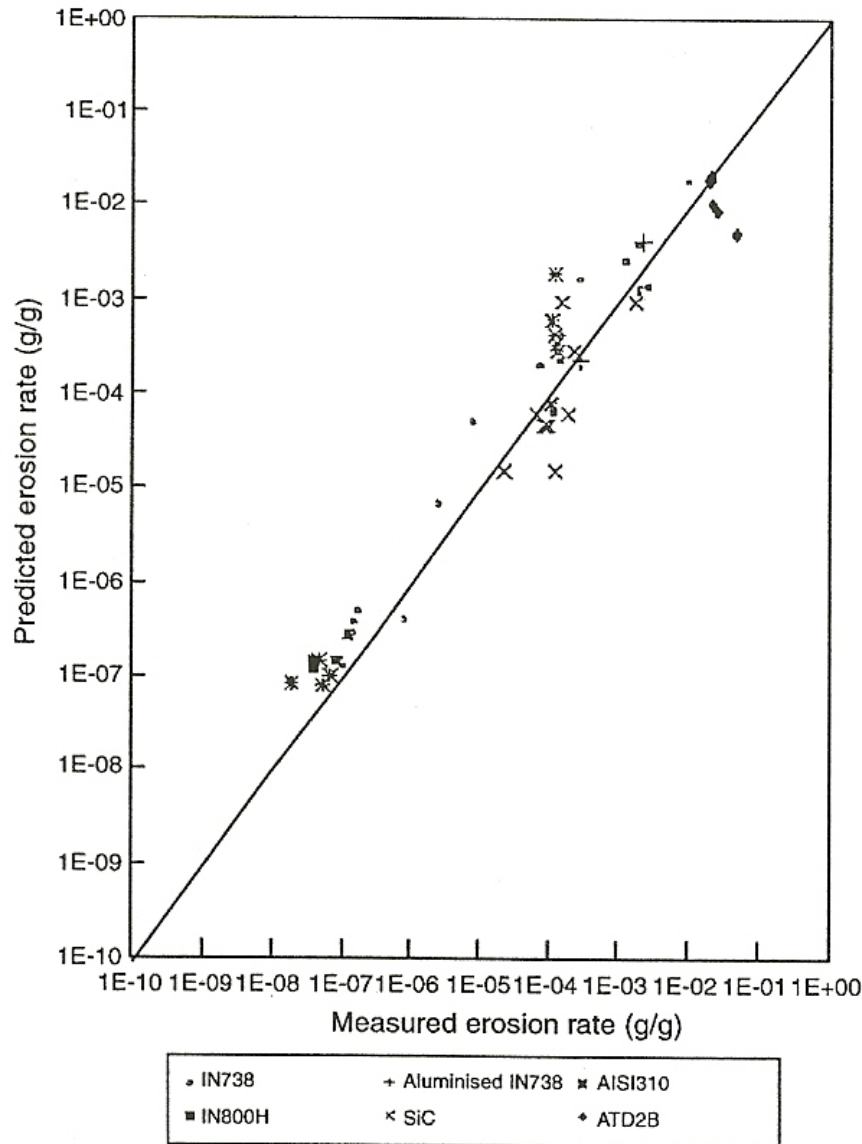


Figure 5. Graph of measured and predicted erosion rates for different materials [16]

This Monte Carlo method has also been used to study high temperature erosion-oxidation, by incorporating oxidation kinetics. This extended model allows incremental growth of an oxide on the surface as well as calculating material removal rates due to erosion [19].

3.4 COMPUTATIONAL FLUID DYNAMICS

An alternative approach to modelling erosion is the use of computational fluid dynamics (CFD). This method is particularly suited to predicting solid particle erosion inside pipe geometries. The disadvantage is that this technique is complicated and time consuming and as such is most appropriate for complex, non-standard geometries.

One of the difficulties in calculating erosion rates is predicting the proper percentage of particles impacting a specific location and the impact angle and impact velocity of each particle. It is well known that these factors affect the erosion process and are used as parameters in most erosion equations, for example, equations (2) and (3). For a simple geometry the particle impact angle and

velocity can be estimated quite accurately as the particles do not deviate much from their initial path. In the case of complex geometries, the particle trajectories can alter significantly making the modelling process much more challenging. In these situations CFD, particle tracking programmes and erosion equations can be used together as a tool to predict erosion damage. These techniques can be used to reflect the effects of particle size and concentration, fluid characteristics, flow rate and geometry. CFD erosion modelling has also been combined with corrosion models to study erosion-corrosion in pipes [20,21].

The DNV document for erosive wear in piping systems [11] suggests using CFD for complex geometries. Their recommended practice is to use CFD to calculate the flow field in a given geometry, from which the particle trajectories can be calculated. The impact velocities and impact angles can then be used to determine the resulting erosion. The erosion rate is calculated using an equation similar to that of equation (2) for individual particles and added to give an estimate of the overall erosion profile in the geometry.

An example of the use of CFD for erosion modelling is given by Salama [12]. He used commercially available CFD software to model turbulent flow and compute the average flow velocity assuming that the presence of solid particles does not change the flow field. In computing the flow field for a multiphase flow, the mixture velocity was used as the fluid entrance velocity and either the mixture or the gas properties could be used for the carrier fluid properties. After the flow solution was obtained from the CFD model, individual sand particles were introduced and tracked using a Lagrangian approach based around the Lagrangian equation of particle motion. The particle impingement velocity, angle and location were recorded and used to compute erosion rates. In Salama's analyses 100 to 1000 particles are generally tracked and all particles are assumed to be uniformly distributed at the entrance section unless there were data that suggested otherwise. A typical example of sand particle tracks predicted by CFD is shown in Figure 6.

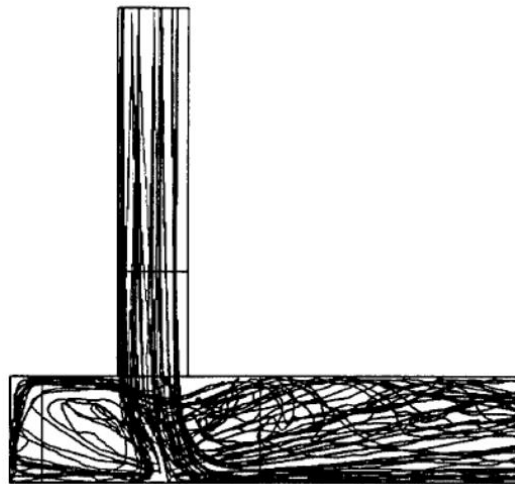


Figure 6. Predictions of sand particle tracks in a heater [12]

The predictions obtained from this detailed CFD analysis were compared with experimental data and also to predictions calculated by using an equation similar to equation (4) for erosion of elements in probes tested in a 3-inch flowloop, see Figure 7. The experimental results were obtained for probes installed in vertical and horizontal pipes in a three-phase (liquid, gas and sand) flow. Figure 7 shows that the predictions obtained compared reasonable well with the experimental data.

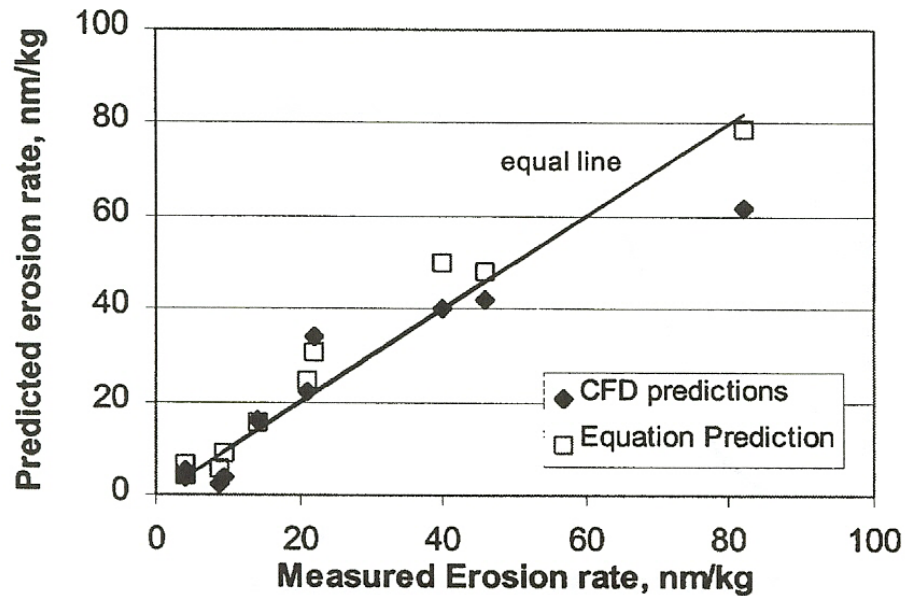


Figure 7. Comparison between measured and predicted erosion rate of sand probe elements [12]

Modelling of erosion using CFD has been studied extensively by The Erosion/Corrosion Research Centre at the University of Tulsa [22-25]. The research carried out within this centre has a strong focus on experimental research facilitating the comparison with modelling predictions.

For instance, Zhang et al. [24] have investigated the viability of using CFD to predict particle velocities. This was achieved by measuring the velocity of particles entrained in water approaching a target using a laser Doppler velocimeter (LDV). CFD simulations of the experiments were carried out and were found to match the data very well for fluid and particle velocities and impact speeds, demonstrating CFD is a viable method for predicting these parameters, see Figures 8-10.

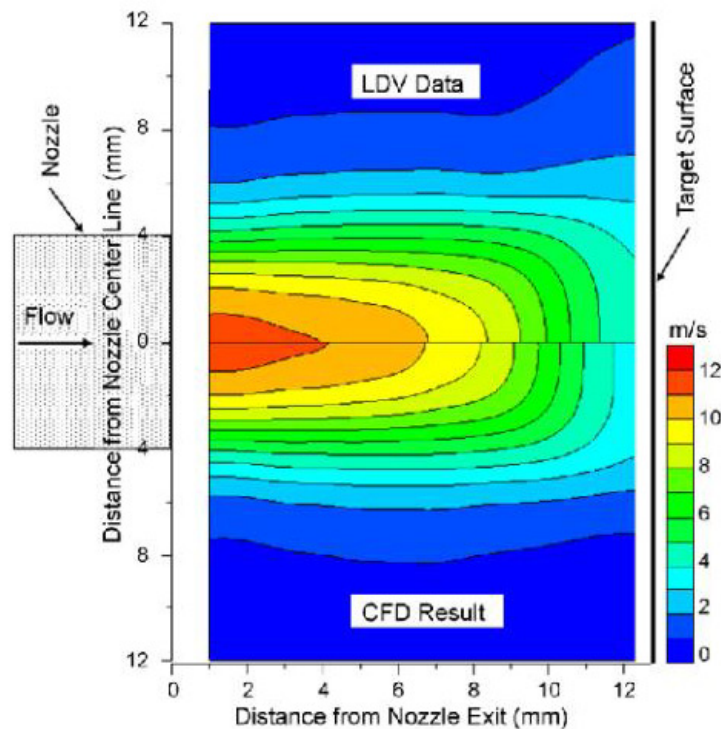


Figure 8. Axial fluid velocity, LDV data versus CFD result [24]

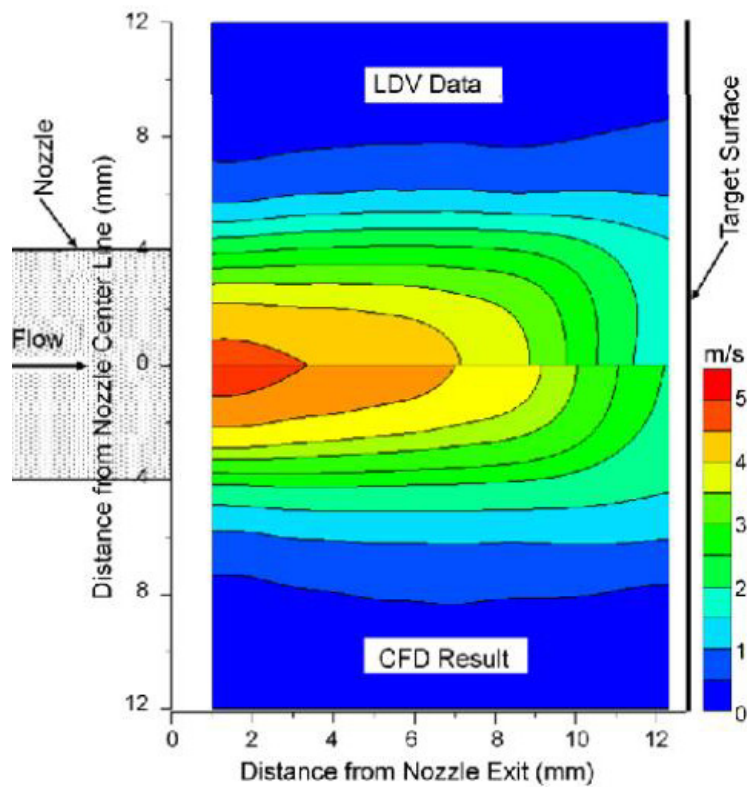


Figure 9. Axial particle velocity, LDV data versus CFD result [24]

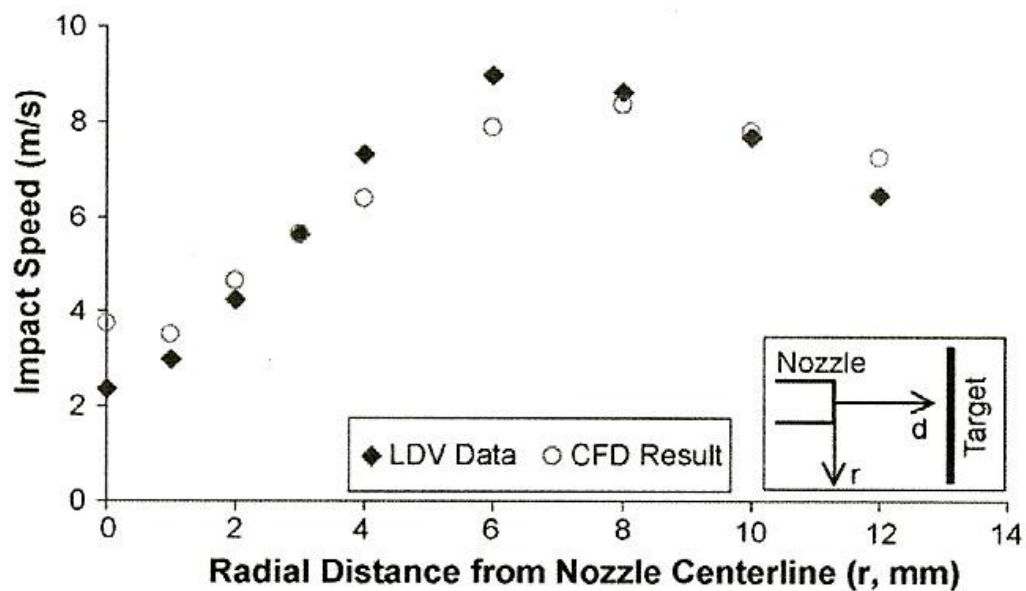


Figure 10. Particle impact speed, LDV data versus CFD result [24]

Within the Erosion/Corrosion Research Centre CFD modelling has also been used to investigate erosion in various complex pipe geometries. For example Chen et al. [22,23] have studied the relative erosion between plugged tees and elbows.

Plugged tees are often used in industry to replace elbows when erosion is expected, although it is unclear whether plugged tees reduce erosion. CFD modelling can be used to help understand the complex mechanisms taking place within these geometries. The erosion model used by Chen et al. was a three step process 1) continuous carrier fluid flow field simulation 2) particle tracking and 3) erosion calculations using information on particle impingements on the wall. A commercially available CFD code was used to solve the continuous carrier fluid flow equations. An important assumption of this model is that the particle-particle interactions are negligible, implying this model is only suitable for dilute systems. One-way coupling between the sand particles and carrier fluid was applied. It was assumed that particles did not affect the flow field.

After obtaining the flow field, the particle trajectories were calculated with the governing equation of particle motion in the fluid based on Newton's second law. Factors affecting particle motion include drag force, pressure gradient, buoyancy force and mass force.

Once a particle impinges on the wall, the accompanied energy lost must be accounted for to determine the reflected particle trajectory. The particle rebound dynamics can be treated in a statistical sense. Chen et al. used a stochastic particle rebound model. Impingement information such as impact speed and angle was gathered as particles hit the wall. From this information the erosion ratio, defined as the mass loss of the pipe wall due to erosion divided by the mass of particles impacting the wall, could be calculated as shown:

$$ER = AF_s V_0^n f(\theta) \quad (9)$$

ER is the erosion rate, V_0 is the particle impingement velocity, A is a material dependent coefficient, F_s is a particle shape coefficient, n is an empirical constant, 1.73, and $f(\theta)$ is a particle angle dependent function.

Analysis of CFD model predictions and experimental observations of the tee and elbow geometries highlighted that particle recirculation is an issue in plugged tees, as shown in Figure 11. This recirculation can potentially cause increased erosion at certain locations, although in practice the wall thickness can be increased in these areas to compensate.

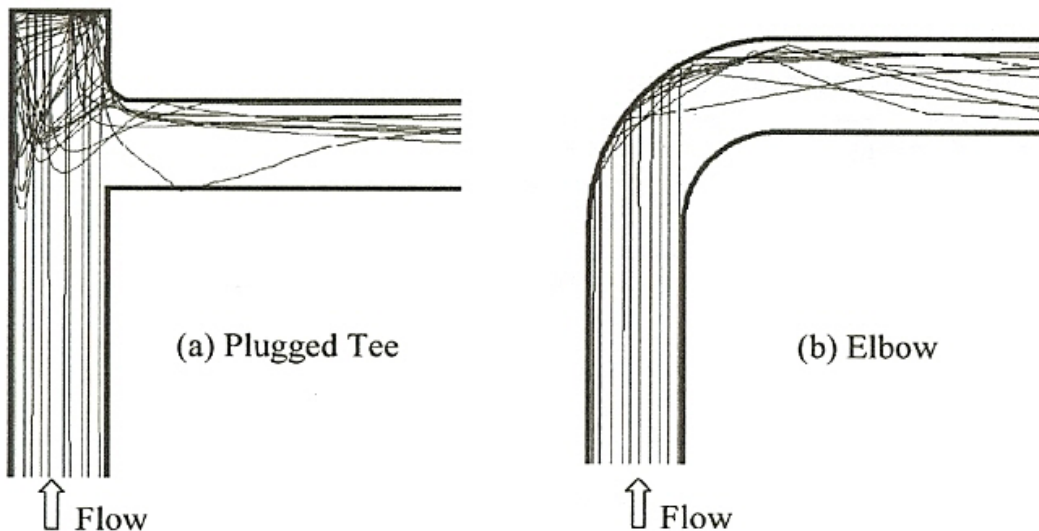


Figure 11. CFD predicted particle trajectories in the plugged tee and elbow geometries [23]

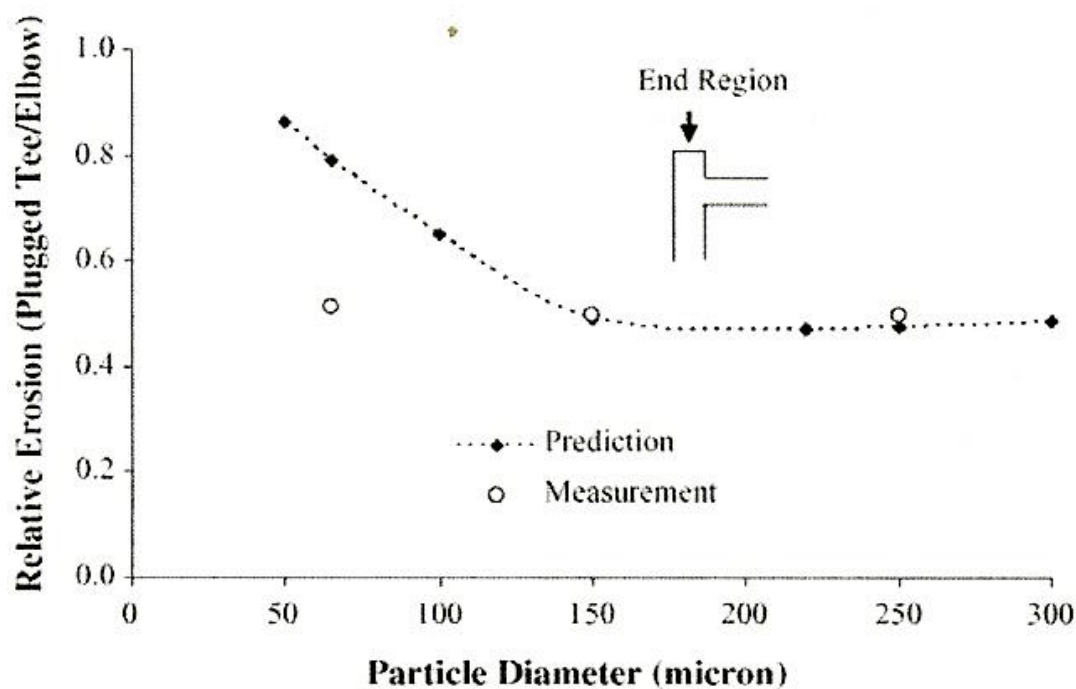


Figure 12. Predicted and measured relative erosion between the plugged tee and the elbow [23]

When comparing measured and predicted results, the CFD model was found to over-predict the erosion for smaller diameter particles, see Figure 12. Investigations found that for small particles, the model numerically exaggerated particle recirculation predictions. If the analysis was altered to only consider first impingement of particles, the correlation with experimental data was much improved.

In commercial CFD codes the particle impact velocities are calculated using a wall function to simulate the turbulent velocity field in the near wall region, and particles are assumed to have zero volume when particle impact information is being calculated. It has been shown [25] that if the CFD code is modified to account for both the turbulent velocity profile and particle size effects in the near-wall region, it helps to avoid nonphysical impacts and reduces the number of impacts by more than one order of magnitude for small particles. Hence this helps overcome the exaggerated particle recirculation predictions described earlier [23].

CFD has proven to be a very useful tool for erosion prediction, although because it is complicated and time consuming it is most suitable for predicting erosion in pipes of complex geometry, where simple models are not applicable.

3.5 FINITE ELEMENT MODELLING

Finite element (FE) modelling has become widely used and universally accepted in many industrial sectors, primarily for improving design processes and predicting performance. In recent years, advances in the capabilities of commercial FE packages have enabled this approach to be used more widely, especially in areas such as failure analysis. The availability of techniques such as element failure and element deletion have made it much more feasible to use finite element analysis (FEA) for erosion modelling. In terms of erosion modelling FEA has generally been used to model the single or multiple impact of solid particles onto a substrate [26-32], although FEA has also been used to simulate other complex wear processes such as the effect of wear generated debris on fretting wear [33]. Simulating the erosion process using FEA enables prediction of erosion behaviour of materials under different conditions. It may also enable identification of constants required for existing analytical models.

An advantage of FE modelling is its flexibility. The created models can be simple or complex, as required. The simpler FE models are generally 2D analysis. The benefits of these models are that they are quicker to set up and are less computationally expensive. They can be used to model a particular stage of the erosion process. This type of model is especially good for comparative work, e.g. comparing the behaviour of different materials. The disadvantage is that the model has to obey certain assumptions such as plane strain, plane stress or axisymmetry. With FE analyses there is a choice of solver enabling implicit or explicit integration. Explicit time integration uses a central difference rule to integrate the equations of time explicitly through time, using the kinematic conditions at one increment to calculate the kinematic conditions at the next increment. Explicit analyses require a small time increment size but the computational cost per increment is small. Explicit integration is particularly suited to analyses with complex contact interactions, high-speed dynamic events and for materials degradation and failure e.g. element removal. Consequently explicit time integration is the obvious choice for erosion modelling.

Bielawski and Beres [29] used FE modelling to study the surface stresses in erosion-resistant coatings. They used a 2D axisymmetric model of a single particle impacting a multilayer coating on a steel substrate. Explicit integration was used for the calculations. The single particle was modelled as a rigid sphere and contact conditions between the particle and substrate were set-up, with no friction applied. The relevant material properties were applied to the coating and substrate elements. The particle radius, mass and velocity were representative of the erosion conditions used in accelerated tests. The model was verified by comparing predictions from a case where the single particle impacts the substrate only (i.e. no coating) to those obtained from the Hertz contact model.

The FE model was used as a means to optimise the wear-resistant coating. It is known that multilayer coatings could perform better than their monolayer counterparts, although optimization of the coating architecture is difficult experimentally. The modelling approach was to use a tensile stress peak in the coating surface as an optimization parameter. Various coating configurations (both monolayer, bilayer and multilayer) were studied, looking at the effect of coating thicknesses and moduli, and the effect of bond layer. The aim of the work was to optimize the coating architecture by minimising the tensile stress in the coating, resulting from the single particle impact. The authors found that the best coating architecture, from the point of view of maximising tensile stress reduction, is to have a relatively low-modulus layer on top (optimum thickness exists) followed by a thick high-modulus bottom layer. This is demonstrated in Figure 13, which shows stress predictions for varying top layer thicknesses (bottom layer thickness is also adjusted to keep total coating thickness constant). The graph also shows the effect of modulus; the modulus for the top coating, E_1 , is kept constant while the bottom layer modulus, E_2 , is varied. The predicted stress is minimised when the top layer thickness is 0.75 microns, with a high-modulus bottom layer.

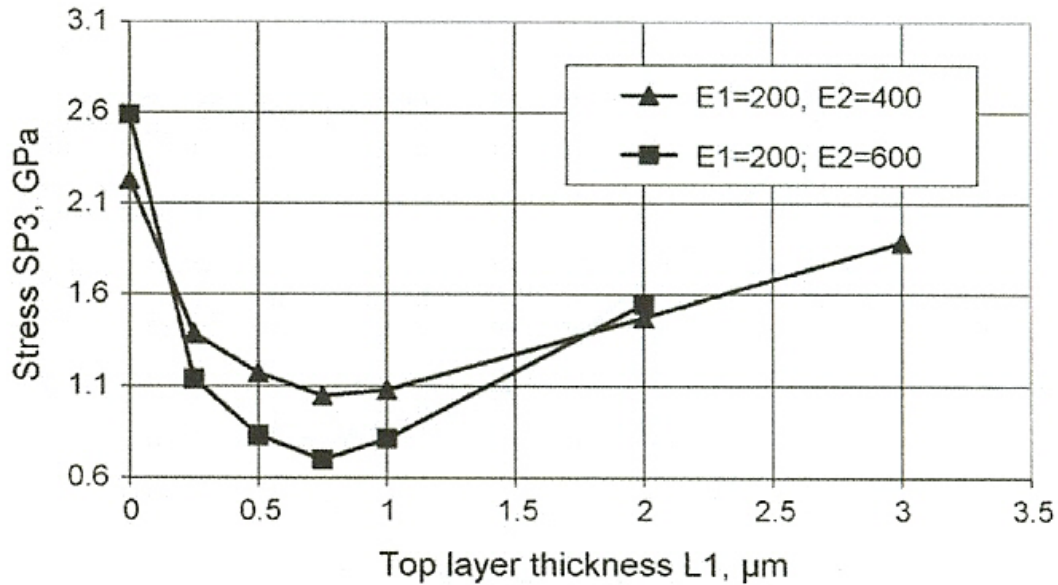


Figure 13. Magnitude of stress versus top layer thickness [29]

In recent years, more advance 3D FE models with element removal have been developed, allowing for simulation of actual material loss. Griffin et al. [30] created a 3D explicit 5-particle model with element deletion for representing solid particle erosion of an Alumina scale/MA956 substrate system. The scale and substrate were modelled using 3D solid elements, while the silica particles were analytical rigid surfaces. To prevent excessive element distortion, which could result from the impacts, adaptive meshing was used in the relevant impact sub-region, see Figure 14. This allows the mesh to adapt and move independently in a strain-free manner reducing element distortion. Contact is set up between the particles and Alumina, with the assumption of no friction between the two. All nodes within the scale sub-region are included in the contact formulation to enable contact to continue once element removal has occurred, exposing new surfaces in the process. The five particles are aligned to impact the surface at a specific angle, θ , which can be varied for different analyses. The particles are constrained such that their direction after impact is dependent on the topology of the eroded surface. Particle to particle contact is not considered giving a considerable computational time saving.

The authors used a tensile failure criterion to simulate the failure behaviour of the Alumina scale. This assumes that failure occurs once the pressure stress is equal to the tensile strength of the Alumina. Once an element has failed it is treated in one of two ways. If the face angle is greater than 60° , the element and its mass is removed from the model. If the face corner angle is less than 60° these elements are categorised as cracks, they are fundamentally intact and their mass is not removed from the model, see Figure 15.

For the ductile MA956 substrate a shear failure model was set up based on the value of the equivalent plastic strain at element integration points, although during this analysis no failure of substrate elements occurs.

Weight loss due to erosion was calculated as the number of failed elements after each impact multiplied by its mass. The maximum impact crater depth was calculated by measuring between nodes on the undamaged scale and nodes at the lowest part of the impact crater. The effect of impact angle was investigated. At the smallest impact angle (20°) there were signs of crack formation below the impact surface but no material loss. The maximum rate of erosion was found at the largest impact angle (90°). The trends observed were similar to experimental results found in literature, although no direct comparisons were made on a single graph due to differences in units of erosion (mg/mg compared to mm^3/kg).

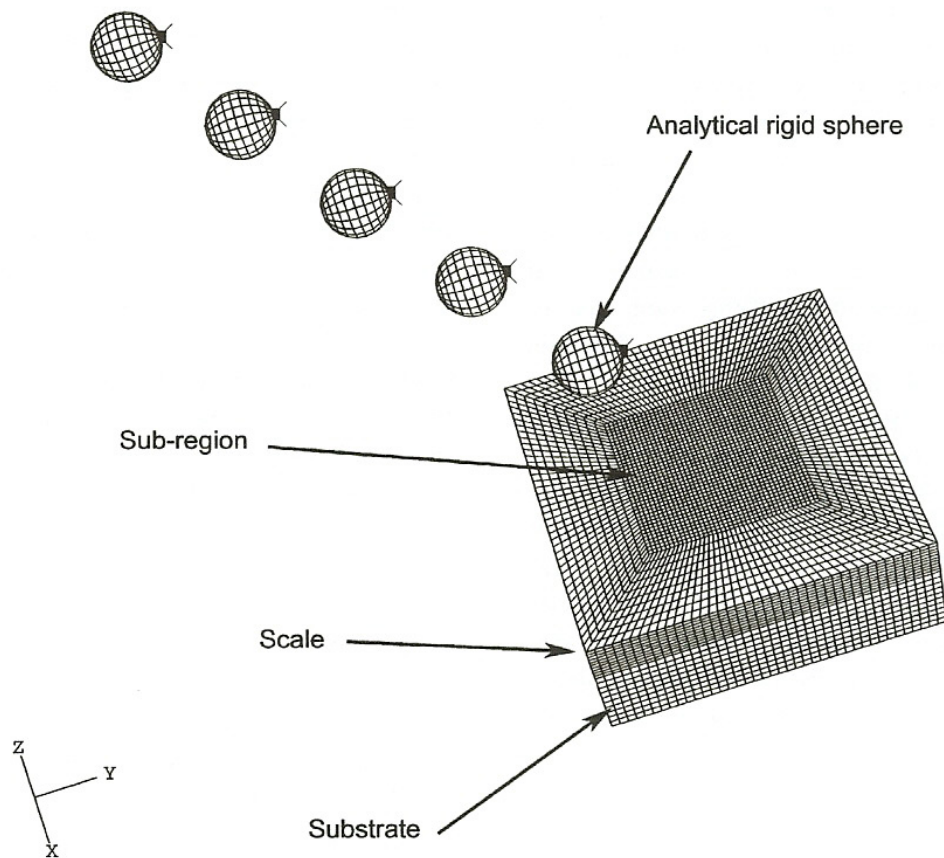


Figure 14. FE mesh of silica particles, scale, substrate and target sub-region [30]

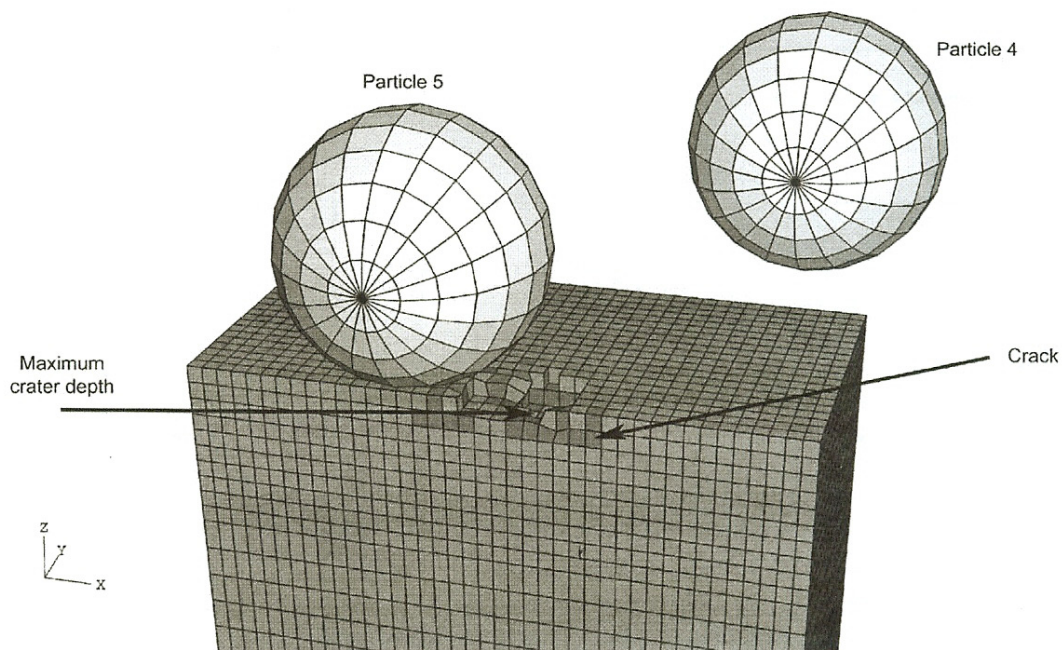


Figure 15. Cross-section of sub-region showing scale cracks and element deletion (erosion) after 5 impacts [30]

A similar approach was used by ElTobgy et al. [31] to analyse erosion of the ductile material Ti-6Al-4V. They chose a 3D explicit multiple impact model with 4 particles, after demonstrating that 3 or more particles were needed to simulate the erosion process.

When creating the model, ElTobgy et al used symmetry, allowing a half model with a more refined mesh to be used. A mesh sensitivity analysis was carried out to verify the refinement of the mesh. Contact was set up between the particles and substrate, and in this model friction was also included. The authors used the Johnson Cook material constitutive equations to represent the ductile substrate while the particles were meshed and defined as rigid bodies. A shear failure criterion, similar to that used by Griffin et al [30], was incorporated into the model to simulate material removal. The FE model was used to investigate the effects on the erosion rate of the number of particles, particle size, particle speed and impact angle. The trends predicted by the FE model showed reasonable agreement with experimental data, where available, and predictions from empirical models, which require a number of experimentally determined constants. Figure 16 shows comparison of data from the FE analysis and the empirical models of Bitter and Neilson and Gilchrist [34-36], showing the trend in erosion rate with impact angle. Although the FE simulation does not use any experimentally determined constants it was able to predict the velocity exponent within the same range proposed and found experimentally by other researchers.

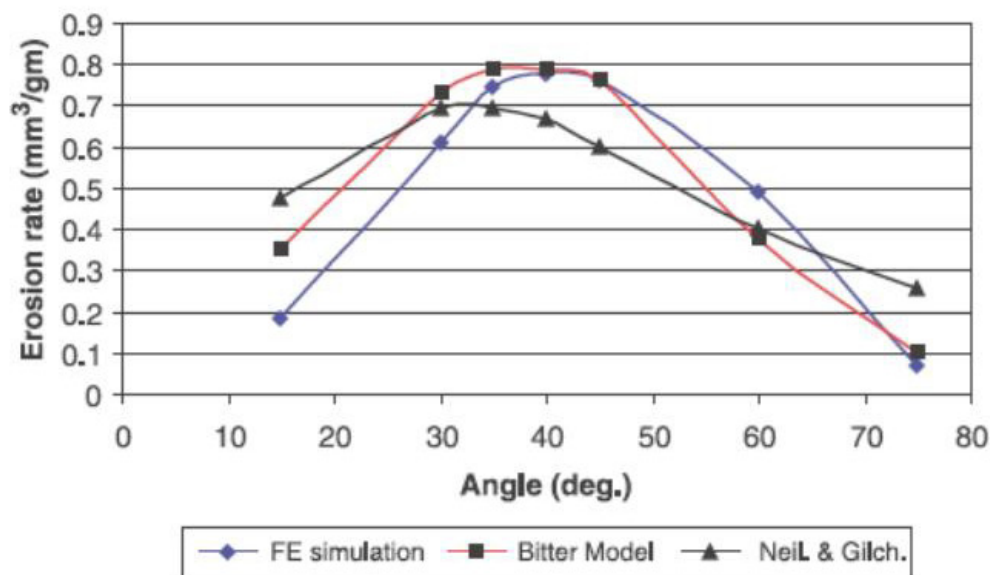


Figure 16. FE prediction of erosion rate compared with predictions from two empirical models [31]

One of the main advantages of using FE to simulate the erosion process is the ability to predict the residual stresses generated in the surface. The residual stress profile obtained from the ElTobgy et al. model is shown in Figure 17. This profile was observed to be comparable with residual stresses measured during shot peening where, although this is a different process as material is not removed, the effect of impacting particles on the residual stress is comparable to the one observed in the erosion process. FE can also be used to obtain difficult to measure data such as contact time and penetration depth.

Wang and Yang [32] have extended the 3D finite element erosion model further by including two exchangeable material sub-models to allow for ductile and brittle materials, using the Johnson-Cook and Johnson-Holmquist models respectively. They also moved from a multiple particle, single impact site analysis, to a non-coincident particle model. In the study 100 spherical particles were used,

divided into 10 groups of 10 particles. The particles in a single group hit the substrate simultaneously, at random locations. The Ti-6AL-4V alloy and silicon carbide ceramics were chosen as examples of ductile and brittle substrates. The study showed that for both ductile and brittle materials the model is able to predict trends such as change in erosion rate with impact angle, although the FE model slightly over predicted results in the ductile case and under predicted those in the brittle case, see Figures 17 and 18 respectively. Wang and Yang also highlighted the value of FE models in allowing the prediction of residual stresses and penetration depth, reporting similar predictions to those obtained by ElTobgy et al. [31].

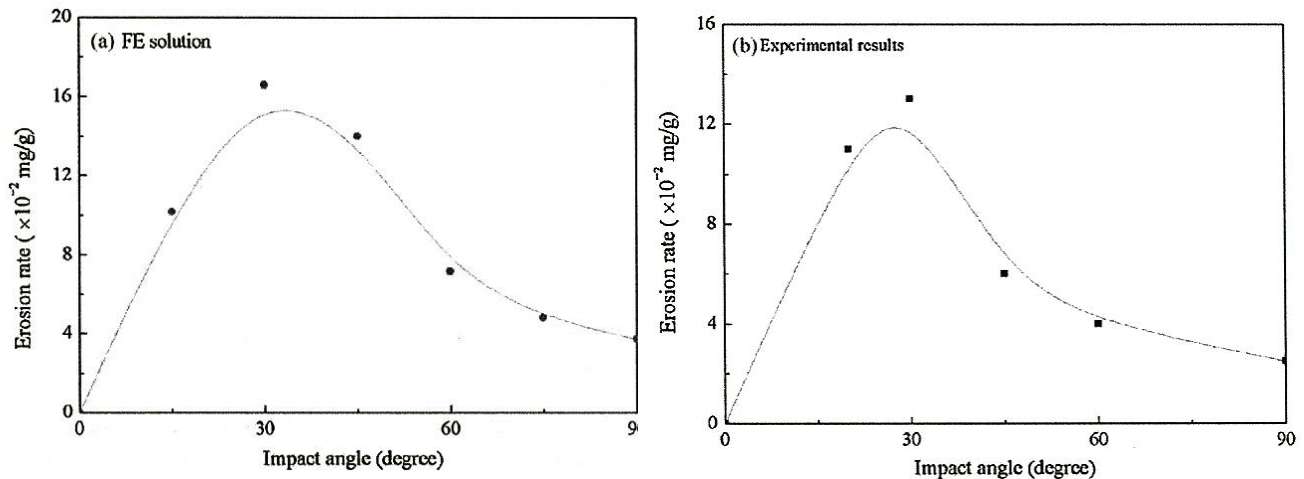


Figure 17. Variation of erosion rate with impact angle for ductile material (a) FE predictions (b) experimental results [32]

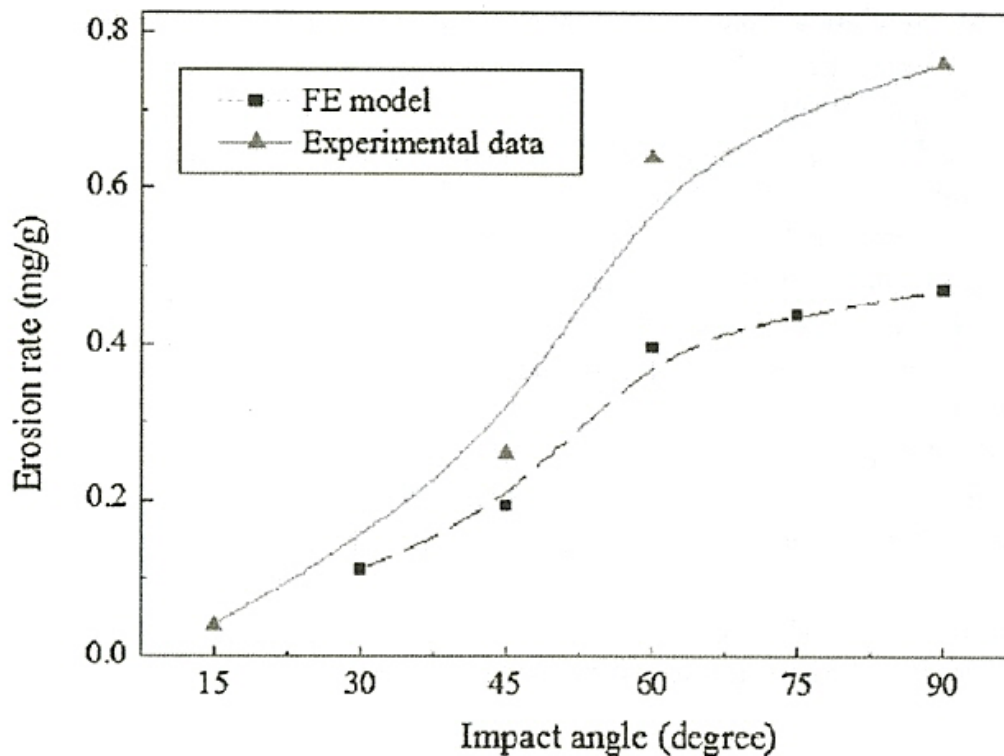


Figure 18. Variation of erosion rate with impact angle for brittle materials, showing FE predictions and experimental data [32]

3.6 COUPLED FE AND MESH FREE METHODS

A limitation of 3D FE modelling is the degree of mesh refinement achievable. Finer meshes give the more accurate predictions, but are often not suitable in 3D explicit models due to the computational cost (in explicit analyses increment size is directly linked to the smallest element dimension within the mesh). Another issue is the possible mesh distortion caused by the impact of a high-energy particle, although this can be alleviated to some degree by using adaptive meshing techniques [30]. An alternative to using a meshed FE model is to use the mesh free technique [37]. Mesh free methods have been developed over recent years. In these methods the area of interest is discretised by a scattered set of particles rather than an element grid, avoiding mesh distortion problems. However mesh free methods are usually more expensive in computational time. Thus coupled FEA and mesh free methods have been investigated to draw on the advantages of each method.

Wang and Yang [37] have proposed such a method for analysing erosive wear. The authors used a 3D explicit dynamic analysis, employing the Johnson-Cook model to represent the ductile Ti-6Al-4V material. The half model with symmetry was set up so that the area of impact was mesh free, while the section with less deformation uses FE discretisation, see Figure 19. A cumulative damage model with the coupling algorithm of smoothed particle hydrodynamics (SPH) was used for the mesh free area.

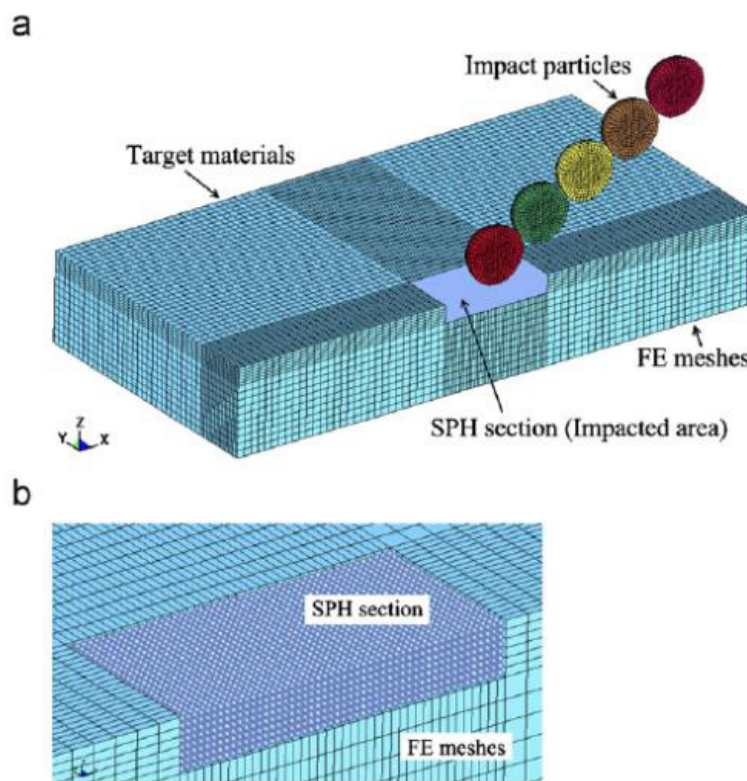


Figure 19. Erosion model showing (a) FE mesh and (b) mesh free region in the area of impact [37]

A total of 1000 SPH particles were used to represent the area of impact. The model was used to predict the effect of impact angle and impact velocity on erosive wear, with results that agreed well with published data [34,35], see Figure 20. The model was also able to predict residual stresses, crater depth and energy transformation.

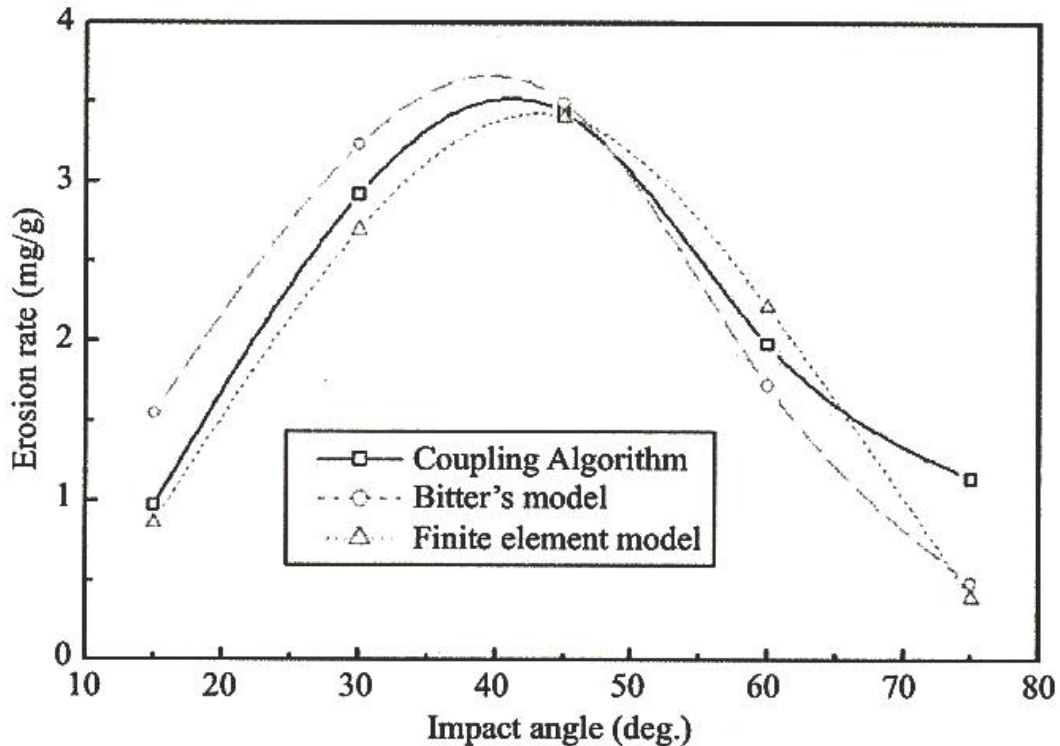


Figure 20. Comparison of coupled FE and mesh free analysis predictions versus FE predictions and results from an empirical model [37]

3.7 MICROSCALE DYNAMIC MODEL

Following on from FE/mesh free techniques the next level, in terms of modelling complexity, would be models based on fundamental physics laws, such as molecular dynamics (MD) simulation. In the MD method, Newton's equation of motion is used to predict trajectories of atoms or molecules in a system. Due to computational constraints, this approach is often used to simulate a process with only approximately 10^4 atoms, making it difficult to apply this method to a realistic erosion process.

In order to bridge the gap between MD models and macroscale models, a micro-scale dynamic model (MSDM) was developed [38]. Chen and Li [39,40] have applied this MSDM model to the solid particle erosion process, simplifying it to a 2D configuration. The substrate and impacting particle are discretised and mapped onto a discrete lattice. During erosion the deformation of a lattice site can be influenced by the impact of the particle or by the interaction with neighbouring sites. The site-site interaction is a function of the materials mechanical properties such as Young's modulus, yield strength, work hardening and fracture strain. Bonds between adjacent sites are broken when the strain exceeds the fracture strain. If all bonds connecting a lattice site to adjacent sites are broken, that volume of material is removed, see Figure 21. By changing the strength of the bond between lattice sites, the effect of interfacial bond strength can be investigated.

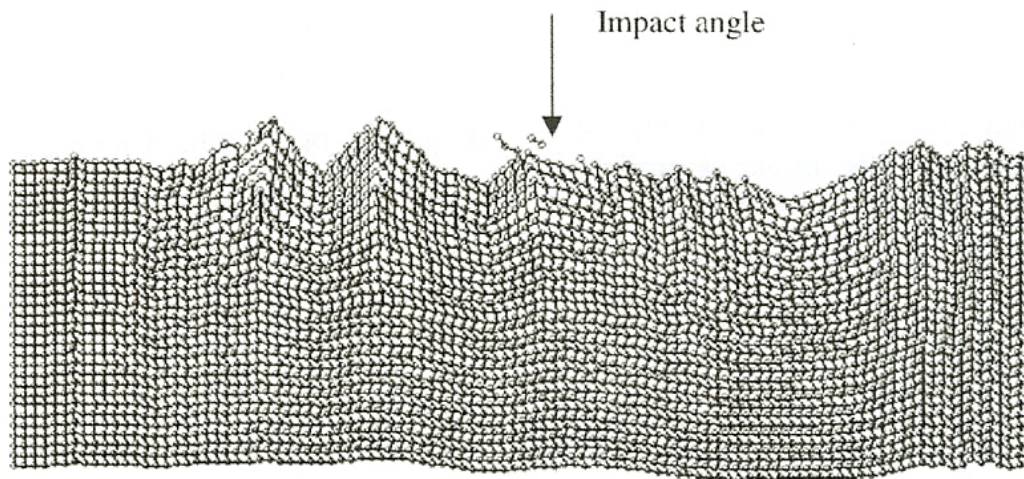


Figure 21. The surface morphology of the MSDM model after impact of 30 particles showing removal of material [39]

Chen and Li have used this model to study particles impacting both ductile (copper) and brittle (SiC) materials [39] as well as a composite of these two (copper as matrix, SiC as reinforcing phase) [40]. The erodent particle was assumed to have a square shape although the influence of particle shape on crack propagation in brittle materials was investigated. In this study the initiation and propagation of radial and lateral cracks was observed. The authors considered the effect of particle velocity and impact angle on erosion of ductile and brittle materials. The predicted trends using the MSDM model are similar to both experimental and analytical trends reported in the literature, e.g. Figure 22. The magnitude of erosion rates cannot be compared though as the erosion loss from the MSDM model is represented by the number of lattice sites removed from the surface rather than an absolute erosion rate.

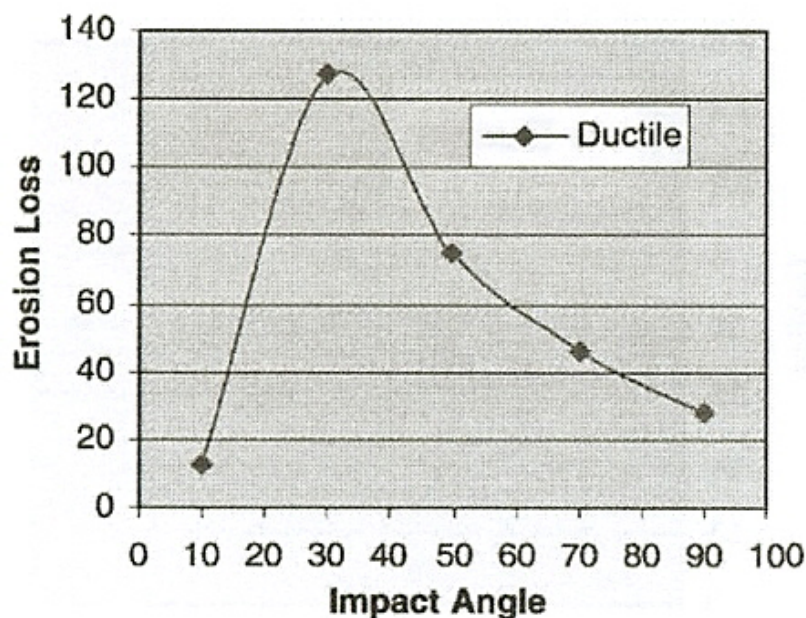


Figure 22. The relationship between impact angle and erosion loss for a ductile material [39]

4 DISCUSSION

Each of the different types of model discussed in this report has their own advantages and disadvantages. The empirical models are quick and simple to use and can identify trends such as the effect of impact angle or particle size/velocity on erosion rate. Due to the simplistic nature of these models they are commonly used as a tool for material ranking. In calculation of the erosion rate the models require input of some material properties such as hardness, and information about the impacting particle such as initial velocity. The main feature of these models is that they all include a coefficient or factor, which is obtained from experimental data. Disadvantages are that these types of models are normally valid for only one mechanism (e.g. a particular mode of material removal or a specific pipe geometry), also the empirical models do not generally account for particle shape. These limitations mean that the models cannot predict erosive wear in a general way since most are only valid for the range of experiments for which they have been established.

Within the field of pipe erosion, an empirical model has been extended to improve the predictive capability. This was achieved by accounting for several physical parameters and mechanisms, such as the characteristic impact velocity of multiple particles on the pipe walls. The model is more comprehensive than empirical models, it includes flow modelling and particle tracking as well as using a more complex erosion equation to calculate erosion rate. A disadvantage is that due to the increased complexity more parameters are required and the model is still based on extensive empirical information. The model was shown to calculate impact velocities accurately and was able to account for differing flow patterns observed in multiphase flow. This model is used in the Sand Production Pipe Saver program, which is available to member companies of the Erosion Corrosion Research Centre at the University of Tulsa. This model is only applicable to the more straightforward pipe geometries where particle impact angle and velocity can be estimated quite accurately. For complex, non-standard pipe geometries where particle trajectories can alter significantly, the use of CFD is recommended.

With CFD the computer programme performs the calculations required to simulate the interactions of liquids and gases with surfaces. This is used to predict the flow field from which particle trajectories and hence impact velocities and impact angles can be calculated. With this form of modelling there is still a need to use an erosion equation to calculate the erosion rate. This equation will often include an empirical constant. This method generally gives good correlation with experimental data for calculation of particle velocities and also for erosion rates. One issue was that for smaller particles numerically exaggerated particle recirculation predictions were observed.

For solid particle erosion of TBC's, the definitive model is the Monte Carlo model developed by Nicholls and co-workers. The model is based closely on the observed erosion mechanisms of EB PVD TBC's and incorporates the columnar microstructure of these TBC's in the form of hexagonal columns. The advantage of the Monte Carlo technique is that the most appropriate material removal model can be selected based on local impact conditions, creating a tool capable of predicting erosion in a wide range of service conditions. In contrast to the empirical models, this erosion model is able to handle a range of different impacting particles. A random distribution of particles is chosen from a selection of 1000 particles, each with a look-up table of key variables calculated by the model. The input parameters required by the model are the TBC material properties such as Young's modulus, hardness and critical stress intensity factor. A velocity threshold was used to establish whether an impacting particle would initiate cracking and calculations of plastic zone size are used to determine material loss. There is a degree of error inherent in these calculations, which increases as contact area decreases, but as small contact areas are unlikely to cause cracking, this error is less of an issue. By its very nature, this model is material specific, but it has wide industrial application as these coatings are often used in highly demanding service environments such as on turbine blades in high temperature environments. The model has been validated against experimental data and predicted erosion rates showed good agreement, being within 20% of experimental data.

In terms of a more general approach for modelling solid particle erosion of a substrate, FEA is an appropriate choice. An advantage of FEA modelling is its flexibility. Models can be simple 2D analyses, useful for comparative work, or complex 3D dynamic analyses when greater predictive accuracy is required. The substrate material type can be changed easily, although care needs to be taken over the material model used to represent the substrate. In setting up the model, no experimentally determined erosion constants are required, only the material properties needed for the material model are used. FE models have the ability to predict material loss (erosion) and cracking through the use of advanced tools such as element failure and deletion. The method can model multiple impacts, though the number of particles is usually small e.g. 5-10, due to computational constraints. The main issue with FEA modelling is the balance between mesh density and computational cost. A finer mesh yields greater accuracy, but the degree of mesh refinement is limited due to the use of the explicit algorithm and the corresponding increase in computational cost for smaller elements. Due to the high impact nature of the erosion process, FE meshes can be prone to excessive element distortion. There are meshing techniques such as adaptive meshing that can be used to help alleviate this problem. The erosion rates predicted by FE models show reasonable agreement with experimental data. One of the main advantages to using FEA to simulate the erosion process is the ability to obtain difficult to measure data such as residual stress, contact time and penetration depth.

Mesh free modelling methods have been developed in recent years as an alternative to FEA. In these methods the geometry is discretised by a scattered set of particles rather than an element grid. Because this is still a relatively new modelling approach it has yet to become as widely known as FEA. The mesh free approach is more computationally expensive than FEA, but mesh distortion problems are avoided. An FE mesh, in which the region of impact has been replaced with a mesh free region, has been developed. The predictions obtained using this model correlate well with the standard FE predictions and empirical models. The model was also able to predict residual stresses, crater depth and energy transformation. The ability to combine FEA and mesh free methods enables the advantages of both methods to be exploited.

The most complex form of modelling is molecular dynamic simulation based on fundamental physics laws. This method is currently not suitable for use in predicting realistic erosion processes due to model size/computational constraints. Instead a micro-scale dynamic model has been developed to bridge the gap between the macroscale models and the much more complex molecular dynamics models. In using this MSDM model, the solid particle erosion process was simplified to a 2D configuration. Multiple particle impacts were studied although unlike most other models where spherical particles have been used as the erodent, a square particle shape was selected. The model is based on the interaction between lattice sites and material mechanical properties are required to calculate whether a bond between two lattice sites remains intact or is broken. Material loss occurs when all bonds connecting a lattice site are broken. The model can predict cracking as well as erosion. Although the predicted trends appear similar to experimental data, the calculation of erosion loss as the number of lattice sites removed from the surface means direct comparisons with experimental data cannot be made.

5 CONCLUSIONS

Erosion is known to be a complex process that can be detrimental to industrial components with demanding in-service conditions. Experimental research carried out over many years has led to greater understanding of the erosion mechanisms. In recent years this knowledge has been applied to the development of models with the aim of predicting erosion wear.

Due to the complexity of the erosion process, no universally accepted predictive model exists. Instead a number of models have been developed, ranging from the simpler empirical models to the more recent computational models. This report has presented an overview of the types of erosion models

available, concentrating on two main scenarios: particulate erosion of material (e.g. water droplet erosion of turbine blades) and solid particle erosion within pipelines (e.g. sand within oil/gas pipelines).

Empirical models exist for predicting solid particle erosion in both the above-mentioned scenarios. By their very nature, these models are heavily dependent on experimental data in the form of erosion coefficients. These models are suitable for predicting erosion rates in well-defined situations where a wealth of experimental data already exists. They are useful in comparative work e.g. material ranking. If modelling of a more general erosion process, or a more complex geometry is required, the empirical models are no longer applicable. For erosion within pipes of complex geometry, CFD has proven a useful tool, although this method is still reliant on erosion equations and in some cases, experimentally determined erosion coefficients. As well as dealing with complex geometries this method can also account for the effects of different types of flow.

The Monte Carlo method for predicting erosion has been developed especially for BD PVD TBC's. It is a comprehensive model that takes into account the specific erosion mechanisms for this material, and has been validated against experimental data.

The 'computer based' methods, such as FEA etc, all approach erosion in a more general sense. These methods do not rely on experimentally determined erosion coefficients, using only material parameters and therefore reducing the number of experiments required. These types of models are useful for investigating the mechanisms responsible for erosion. They have the ability to predict measureable data such as cracking and material loss and also difficult to measure data such as residual stress. Another feature of this type of modelling is the fact that predictions can be presented visually enhancing the understanding of the mechanisms taking place.

When modelling erosion, the type of modelling approach selected needs to be considered carefully. Choice of model will mainly depend on the complexity of the scenario being modelled, and what the required outputs are. A key factor will also be the availability of relevant experimental erosion data. For systems where a wealth of data already exists, it is likely that industry will continue to use the simpler models to obtain a first pass assessment of the erosion rate. But when new materials are being considered, it could become more cost effective to move away from these types of models towards more computer based models such as CFD and FEA, reducing the requirement for extensive testing. In terms of research areas it is envisaged that these two main areas will continue to be developed creating more advanced, realistic models. The molecular dynamics type modelling is not yet suitable for erosion modelling as the model size required for representing erosion exceeds the current computational capability. The FEA plus mesh free methods approach is interesting. Mesh free methods are not as widely known as FEA. But the combination of both brings many benefits and is likely to be an area that is developed in the future.

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