

# Determination of the point spread function of a coherence scanning interferometer

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## Abstract

Coherence scanning interferometry (CSI) combines the lateral resolution of a high power microscope with the axial resolution of an interferometer and provides a rapid and convenient means to measure surface topography. Although CSI measurements of features, such as step heights, have been demonstrated with nanometre accuracy, systematic errors have been reported in the measurement of more general features, such as sinusoids, where surface gradient can be significant.

Recently, methods of three-dimensional imaging, including digital holography, confocal microscopy, and CSI, have been analysed as linear shift invariant processes that are characterised by their point spread function (PSF). In this paper, it is shown that the linear theory leads to a straightforward model of fringe formation in CSI. The influence of the PSF on measurements of a sinusoidal surface profile and a method to measure the PSF of a CSI instrument are then discussed.

*Keywords:* Coherence scanning interferometry, Scanning white light interferometry, Surface measurement, Calibration.

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## 1. Introduction

Coherence scanning interferometry (CSI) provides a rapid and convenient means to measure surface topography and is an increasingly popular alternative to the contacting methods that form the basis of current international standards for surface measurement. Routine use of CSI for engineering metrology has, however, prompted comparison with the established methods. In 1990, Hillman was the first to report that optical measurements of a roughness standard artefact could differ significantly from those obtained from stylus profilometry [1]. Other observations include so-called “batwings” that are found at the edge of step discontinuities [2] and “fringe order errors” that occur at half-multiples of the wavelength [3]. It is found that spurious data is most likely to occur when the surface gradient is large such that a significant proportion of the scattered field falls outside the aperture of the objective. For example, figure 1 shows a CSI measurement of a surface with a sinusoidal profile with a pitch and amplitude of approximately 8  $\mu\text{m}$  and 233 nm respectively. In this case fringe order errors are responsible for the deviation from the ideal profile (shown dotted).

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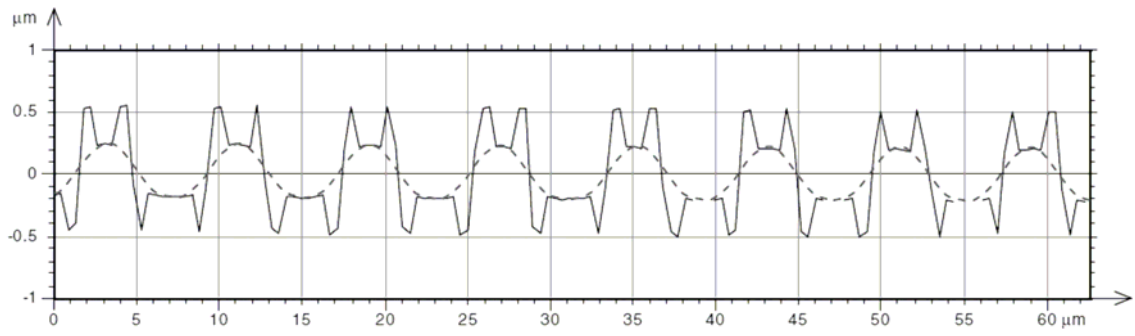


Fig. 1. CSI measurement of a sinusoidal grating

In a recent paper we have compared three-dimensional (3D) imaging systems, including holography, confocal microscopy, and CSI, as linear processes [4]. This analysis is strictly correct for the case of weakly scattering objects where the object causes a small perturbation in the illuminating field. With this assumption, each imaging system is characterized in the space domain by its point spread function (PSF) (its response to a point object) or equivalently in the frequency domain by its transfer function (TF), and the imaging process can be considered to be a 3D filtering operation. For CSI measurements of surface topography the assumption of weak scattering can be justified provided the field is returned to the instrument by a single scattering event and this assumption is usually valid for smooth surfaces with gradients that have a magnitude that is less than the numerical aperture of the objective.

In this paper we briefly review the linear theory of CSI imaging and show how the PSF is closely related to fringe order errors that are observed as the surface gradient increases. We describe how to calculate the PSF for ideal instruments and how the PSF can be measured in practice.

## 2. Linear Theory

### 2.1. The point spread function.

We have recently published an analysis of 3D optical imaging methods, including CSI, in terms of linear systems theory [4]. It is shown that, if the object is described by its refractive index contrast,  $\Delta(\mathbf{r}) = 1 - n^2(\mathbf{r})$ , where,  $n(\mathbf{r})$  is the refractive index, then the fringe modulation,  $I(\mathbf{r})$ , of the interference pattern generated by a CSI instrument can be described by the linear, shift-invariant filtering operation,

$$I(\mathbf{r}) = \int_{-\infty}^{+\infty} H(\mathbf{r} - \mathbf{r}') \Delta(\mathbf{r}') d^3\mathbf{r}' \quad (1)$$

where  $d^3\mathbf{r}'$  represents the scalar  $dr'_x dr'_y dr'_z$ , and  $H(\mathbf{r})$  is the 3D PSF that defines the system response. In essence, the PSF of a CSI instrument is a function of the 3D coherent responses of the optical systems used to both illuminate and observe the light that is scattered from the object. Most CSI instruments use a Mirau objective for high resolution measurements and the numerical aperture of this lens usually limits the response of both the illumination and observation functions. In this case the PSF of an ideal aberration-free instrument system has been shown to be [4],

$$H_{\text{ideal}}(\mathbf{r}) = \text{Re} \left\{ \int_{-\infty}^{+\infty} k_0^2 S(k_0) G^2(\mathbf{r}, k_0) dk_0 \right\} \quad (2)$$

where,  $\text{Re}\{ \}$  denotes the real part,  $S(k_0)$  is the spectral density of the illumination and  $G(\mathbf{r}, k_0)$ , is the response of an ideal imaging system of numerical aperture,  $N_A$ , operating at a wavenumber  $k_0$ , given by,

$$G(\mathbf{r}, k_0) = \int_{-\infty}^{+\infty} \frac{\delta(|\mathbf{k}| - k_0)}{k_0^2} \text{step} \left( \mathbf{k} \cdot \hat{\mathbf{o}} - k_0 \sqrt{1 - N_A^2} \right) e^{i\mathbf{k} \cdot \mathbf{r}} d^3k \quad (3)$$

where,  $\hat{\mathbf{o}}$  is a unit vector in the direction of the optical axis and  $\delta( )$  and  $\text{step}( )$  represent a 3D Dirac delta function and a Heaviside step function, respectively.

In essence the PSF provides the interferogram that would be observed if a point-like object were measured by a CSI instrument. Figure 2 shows a slice through the PSF of an ideal instrument of  $N_A = 0.55$  with a Gaussian (at  $1/e^2$ ) distributed spectral density such that the central wavelength is 600 nm and the bandwidth approximately 135 nm.

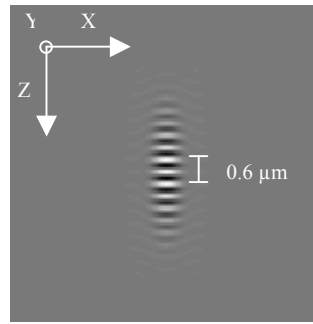


Fig 2. Ideal PSF ( $N_A = 0.55$ ; mean wavelength 0.6  $\mu\text{m}$ ; bandwidth 135 nm)

It can be seen that the PSF has a fairly compact form with fringes with a frequency of approximately half the mean wavelength extending in the axial direction. The lateral extent of the fringes (full width) is around 0.6  $\mu\text{m}$  and is approximately the mean wavelength for this objective. It is noted, however, that the PSF extends to a certain extent beyond this limit and this implies that the fringe pattern generated by a CSI instrument at a particular position in space depends in part on the topography of the surrounding area.

## 2.2. Foil model

Because we are concerned with surface scattering it is intuitive to consider only a thin ‘foil-like’ layer on the object surface. Indeed, for the case of a metal object, the illuminating field is rapidly attenuated and is only of significance within the skin depth [5]. The remainder of the object has no measurable effect. If the metal is a perfect conductor then the skin depth is infinitesimally small while the (complex) refractive index contrast is infinitely large and the scattered field will be exactly that which would be scattered from a thin foil with the same topography. In this case we can describe the object by an equivalent foil surface and can write,

$$I(\mathbf{r}) = \int_{-\infty}^{+\infty} H(\mathbf{r} - \mathbf{r}') \delta(r'_x) \delta(r'_y) \delta(r'_z - f(r'_x, r'_y)) d^3r' \quad (4)$$

where,  $f(r_x, r_y)$  describes the profile. We refer to this equation as the foil model of CSI.

As an example we calculate the response of the ideal system to a sinusoidal profile given by  $f(r_x, r_y) = A \sin(2\pi r_x / \lambda_g)$  where,  $A$  and  $\lambda_g$  are the amplitude and wavelength of the grating respectively. Figure 3

shows the foil model of the fringes generated by the ideal CSI system described above when measuring a sinusoidal profile with a pitch and amplitude of approximately 3.3  $\mu\text{m}$  and 233 nm respectively.

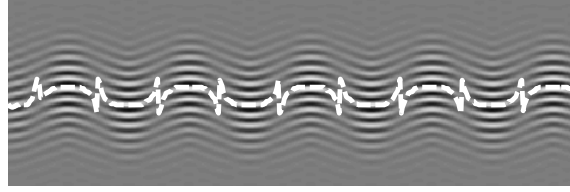


Fig 3. Interferogram generated by the foil model for sinusoidal grating 3.3  $\mu\text{m}$ .

The dotted white lines show the measured profile as defined by the peak fringe modulation. Fringe order errors similar to those in the experimental results presented in figure 1 can be observed, however, it is noted that the pitch is smaller (and the gradient correspondingly larger) in this ideal case.

Although the foil model was introduced with reference to a perfect conductor, it can be used to model surface scattering from most homogeneous engineering materials. We have discussed the validity of the foil model and have demonstrated the effect of aberrations on the PSF and measured topology elsewhere [6]. In this paper, however, we discuss the measurement of the PSF of a CSI instrument in practice.

### 3. Measurement of the Optical Point Spread Function

If an object of known shape is measured it is possible to calculate the PSF of the measuring instrument as a de-convolution and this can be accomplished most easily in the spatial frequency domain (k-space). Taking the Fourier transform of equation ?? and re-arranging, the transfer function,  $\tilde{H}(\mathbf{k})$ , is given by,

$$\tilde{H}(\mathbf{k}) = \tilde{I}(\mathbf{k}) / \tilde{\Delta}(\mathbf{k}) \quad (5)$$

where tilde denotes transformed quantities. It is clear from equation (5) that it is important to choose an appropriate object that contains all spatial frequencies within the pass-band of the instrument. For this reason a precision ruby sphere with a nominal diameter of 125  $\mu\text{m}$  was chosen as a calibration object. Figure 4 shows a section through the resulting interferogram.

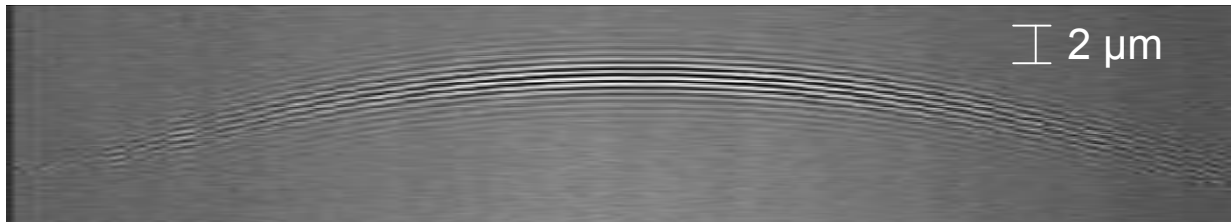


Fig 4. Interferogram; ruby sphere of 125  $\mu\text{m}$  diameter.

It is noted that the fringes are of higher contrast toward the top of the ball and the fringes become less clear as the gradient increases. Taking the (3D) Fourier transform of the interferogram, dividing by the transform of a foil model of the spherical surface, and returning to the space domain, gives the PSF shown in figure 5.

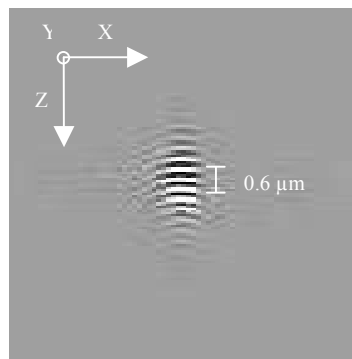


Fig 5. Measured PSF

Comparison of figure 5 and figure 2 shows that the lateral extent of the PSF has increased and this is responsible for the low contrast fringes toward the edges of figure 4.

#### 4. Discussion and Conclusions

In this paper we have considered a linear theory of imaging that accurately describes CSI instrumentation provided that multiple scattering effects are negligible. In this case, the system response can be characterised in the space domain by its 3D point spread function (PSF) or equivalently in the frequency domain by its transfer function (TF). Additionally the surface of a homogenous material can be replaced by a foil-like object of the same surface topography. This foil model of CSI illustrates that the 3D PSF plays a pivotal role in the characterization of surface topography. The PSF can be thought of as a filter which mixes the response from parts of the object within its bounds. If the surface gradient is small the mixing has little effect and CSI would be expected to work even in the presence of severe aberration. As the surface gradient becomes steeper, however, this mixing has a greater effect on the measurements.

It is clear from this work that the optical performance of a CSI instrument can be summarised by its PSF and we have shown that it is possible to measure the PSF by using a suitable artefact. Since the PSF is a function of system parameters including numerical aperture, source spectrum and optical aberrations it can be compared to that of an ideal system and can be used as a diagnostic tool and in certain cases a correction can be applied. Further work is underway to investigate the implementation of these ideas in practice.

#### Acknowledgements

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