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Choosing Line Lengths for Calibrating Waveguide Vector Network Analysers at Millimetre and Sub-millimetre Wavelengths

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ABSTRACT

This report presents a strategy for determining suitable line lengths for calibrating waveguide Vector Network Analysers (VNAs) to enable precision measurements to be made at millimetre and sub-millimetre wavelengths. The strategy is based around the Thru-Reflect-Line calibration technique and establishes realisable line lengths for such calibrations. The method is used to obtain a set of lines suitable for use over the frequency range 90 GHz to 500 GHz but is also extendable to higher (and lower) frequencies. Some practical experiments in the frequency range 75 GHz to 110 GHz are provided to validate the method.

The report shows that, for each waveguide size, a single Line standard that provides a phase change with respect to the Thru standard of the order of a quarter wavelength ($\lambda/4$) is sufficient to enable TRL calibrations to be used at frequencies up to 110 GHz. However, at higher frequencies, such lines become too short (i.e. less than 1 mm) for practical use. It is therefore necessary to use two lines that give phase changes of the order of $3\lambda/4$ to enable TRL calibrations to be used to 325 GHz. At still higher frequencies (again, to avoid using lines that are too short for practical purposes), it becomes necessary to use three lines that give phase changes of the order of $5\lambda/4$ to enable TRL calibrations to be used to 500 GHz.

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Approved on behalf of the Managing Director, NPL,
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1. Introduction

In recent years, the use of Vector Network Analysers (VNAs) to provide complex-valued S -parameter measurements of microwave devices has been extended to frequencies above which traditional coaxial connectors are used [1], i.e. above 110 GHz. This is achieved using external frequency extension modules that are fitted with metallic rectangular waveguide test ports of the correct aperture size for the required measurement frequency range. These modules attach to a millimetre-wave controller unit used with the VNA [2]. This approach has been applied successfully over the entire millimetre-wave region (i.e. to 325 GHz [3]) and is now being extended for use at sub-millimetre wavelengths (i.e. one system has been reported recently that operates to 500 GHz [4]). However, there is, at present, no traceability to SI available from National Metrology Institutes (NMI) for S -parameter measurements at frequencies above 110 GHz. This has led the National Physical Laboratory (NPL), i.e. the UK's NMI, to begin a programme of research aimed at providing traceability in metallic rectangular waveguide sizes used above 110 GHz.

NPL already provides national waveguide standards at lower millimetre-wave frequencies (i.e. from 30 GHz to 110 GHz) [5, 6] to enable traceable S -parameter measurements to be made at these frequencies. These standards are high precision sections of waveguide with lengths chosen specifically to enable the Thru-Reflect-Line (TRL) calibration technique [7] to be implemented on the VNA. TRL is a so-called self-calibration technique [8] and possesses two important features that are attractive for precision metrology applications:

1. It requires that the waveguide Line standard only needs to be partly known. Specifically, it is assumed that the reflection coefficients, S_{11} and S_{22} , of the line are known but not the transmission coefficients, S_{21} and S_{12} ;
2. It is relatively straightforward to demonstrate traceability to SI through dimensional measurements of the apertures of the waveguides.

The other standards used during a TRL calibration are a Thru connection (where the VNA test port reference planes are joined together) and a Reflect connection (where a non-zero value of reflection is connected to both test ports). It is a requirement that all four S -parameters for the Thru connection are known. However, for the Reflect standard, the actual value of reflection does not need to be known – only that the same value of reflection is presented to both the VNA's test port reference planes.

The TRL calibration scheme achieves optimum performance when the Line standard produces a transmission coefficient phase change of 90° (i.e. a quarter wavelength, $\lambda/4$) with respect to the Thru standard. In practice, the TRL calibration scheme continues to work well when the phase change is far removed from this optimum condition. However, it fails completely when the phase change is either 0° , 180° ($\lambda/2$), or multiples thereof (i.e. $n\lambda/2$, where $n = 0, 1, 2, \dots$). Therefore, designs for waveguide TRL calibration schemes often choose the length of the Line standard such that a phase change of approximately 90° is achieved at or near the mid-band frequency for a given waveguide size.

A problem occurs when realising the above design criterion at higher millimetre-wave frequencies because the length of line needed to achieve a phase change of the order of 90° becomes very short (i.e. less than 1 mm). In practice, such short sections of line can

easily bend or become distorted during use, so this effectively produces an upper frequency limit for this $\lambda/4$ approach to realising the TRL calibration technique at millimetre and sub-millimetre wavelengths.

One method of avoiding using such short lengths of line is to utilise the Line-Reflect-Line (LRL) calibration technique [9]. Like TRL, the LRL calibration technique uses lines as the reference standards for calibrating the VNA. The LRL technique replaces the Thru standard (which can be considered as a Line standard of zero length) with a Line standard (which can be considered as a Thru standard of non-zero length). It is then the difference in length between the two Line standards that determines the effective bandwidth of the calibration. This means that both lines can be of significant length and so not be susceptible to damage during use. Some manufacturers use this approach to provide suitable line lengths as standards at millimetre-wave wavelengths. However, there are some significant drawbacks with using the LRL calibration technique for precision metrology applications. These are discussed in section 2 of this report.

This report presents a strategy whereby relatively long Line standards (i.e. greater than 1 mm in length) can be used with the TRL calibration technique for precision measurements at millimetre and sub-millimetre wavelengths. Section 3 of the report begins by giving the lengths of conventional Line standards used for TRL at millimetre wavelengths (i.e. to 325 GHz). This shows that it is inappropriate to use conventional TRL line lengths above 110 GHz. New lengths are then introduced to enable the TRL calibration technique to be applied to waveguide bands above 110 GHz in the millimetre-wave region (i.e. to 325 GHz). Section 4 extends this technique by showing how realisable lengths of line can be used with the TRL calibration technique at sub-millimetre wavelengths.

This report restricts the upper frequency of interest to 500 GHz, since this is the existing state-of-the-art for high frequency waveguide VNAs [4]. However, if necessary, the technique described in this report can easily be extended to include frequencies above 500 GHz.

2. COMPARING TRL AND LRL CALIBRATION SCHEMES

In principle, both TRL and LRL calibration techniques can be used for calibrating VNAs to enable high precision measurements to be achieved. Both calibration schemes use precision lines as the calibration standards. However, practical realisation of the LRL calibration technique exposes two drawbacks, when compared with the TRL calibration technique. These are described below:

1. LRL uses two lines whereas TRL uses only one line. The use of an additional line standard leads to the need for an extra flange connection during the calibration. This is illustrated in Figure 1, where Figure 1 (a) shows the flange connections that are needed during a TRL calibration and Figure 1 (b) shows the flange connections that are needed during an LRL calibration.

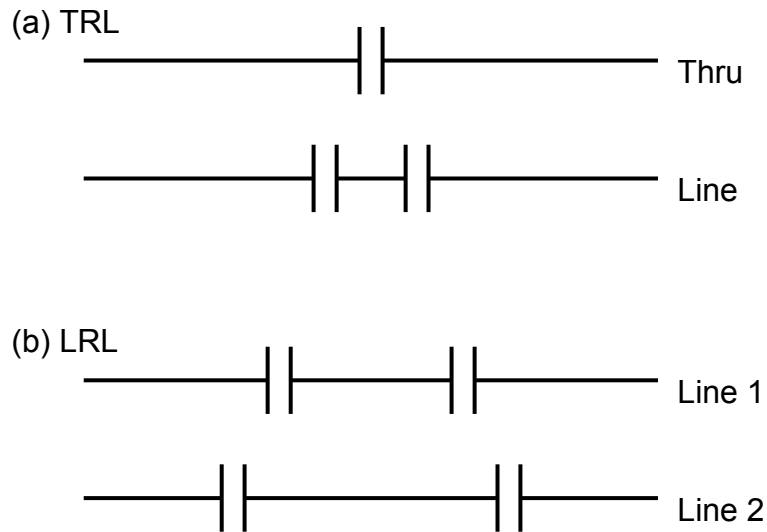


Figure 1: showing the number of flange connections used when connecting standards in: (a) TRL calibration; and, (b) LRL calibration. The connection of the Reflect standard is not shown, since this is the same for both calibration schemes

In both cases, the connection of the Reflect standard (usually a short-circuit) to both test ports is not shown, as it is the same for both TRL and LRL. Figure 1 shows that TRL requires making three pairs of flange connections whereas LRL requires making four pairs of flange connections. A flange connection at millimetre and sub-millimetre wavelengths is likely to be a significant source of reflection (and hence error) due to the inevitable mechanical tolerances and misalignments of both the waveguide apertures and the flange dimensions (including dowel pins, etc). This suggests favouring the calibration technique that uses the smaller number of flange connections, i.e. TRL.

2. In both TRL and LRL calibration, the first standard (i.e. the Thru in TRL and the first Line connection in LRL) is required to be fully known - i.e. all four S -parameters need to be known, including the phase of the transmission coefficients, S_{21} and S_{12} . In the case of a TRL calibration, it is reasonable to assume that these phases will have a value that is effectively 0° . However, for an LRL calibration, it is necessary to determine these phases based on the physical length of the line used as the standard. This length is determined by a dimensional measurement and therefore will have an uncertainty associated with it to take account of the inevitable measurement error. This uncertainty will be due to both the uncertainty in the dimensional measuring instrument and the uncertainty due to the length of the line itself.¹ This uncertainty impacts directly on the accuracy of a VNA calibrated using the line to achieve an LRL calibration and will be significant at millimetre and sub-millimetre wavelengths.

¹ This will include a contribution to the uncertainty due to the temperature at which the line was measured dimensionally and then subsequently used electrically. The dimensional and electrical measurements are usually made at different temperatures in different measurement laboratories.

For the above two reasons, LRL is not the preferred calibration technique for precision VNA measurements at millimetre and sub-millimetre wavelengths. Instead, TRL is the calibration method of choice for these frequencies.

3. CALIBRATION AT MILLIMETRE WAVELENGTHS

3.1 TRL using one $\lambda/4$ line

For a TRL calibration in a given waveguide band, the phase change due to the Line standard with respect to the Thru standard will vary from a minimum value, $\phi_{\min}$ (at the lowest recommended frequency in the band), to a maximum value, $\phi_{\max}$ (at the highest recommended frequency in the waveguide band). (Recommended frequencies for waveguide bands at millimetre wavelengths can be found in existing document standards [10, 11].)

In order that the TRL algorithm performs equally well at the waveguide band edges, and for both $\phi_{\min}$ and $\phi_{\max}$ having values lying between 0° and 180° , we set:

$$\phi_{\max} = 180 - \phi_{\min} \quad (1)$$

where:

$$0^\circ < \phi_{\min} < 180^\circ$$

and

$$0^\circ < \phi_{\max} < 180^\circ.$$

At a given frequency, f , within a waveguide band, the guide wavelength, λ_g , is given by:

$$\lambda_g = \frac{\lambda}{\sqrt{1 - \left(\frac{\lambda}{\lambda_0}\right)^2}} \quad (2)$$

where

$$\lambda_0 = 2a$$

$$\lambda = \frac{v}{f}$$

and

$$v = \frac{c}{\sqrt{\epsilon_r}}$$

where

λ_0 = waveguide cut-off wavelength (m)

- a = waveguide broad wall² dimension (m)
- λ = free space wavelength (m)
- f = frequency (Hz)
- v = speed of electromagnetic waves in air (m.s⁻¹)
- c = speed of electromagnetic waves in vacuum (defined as 299,792,458 m.s⁻¹ exactly)
- ϵ_r = relative permittivity. ($\epsilon_r = 1.000649$ for ‘standard’ air at a temperature of 23 °C, 50 % relative humidity, and 1013.25 hPa atmospheric pressure [12].)

The lowest recommended frequency in a waveguide band corresponds to the maximum guide wavelength, $\lambda_{g(\max)}$, and the maximum recommended frequency in a waveguide band corresponds to the minimum guide wavelength, $\lambda_{g(\min)}$.

For a given line length, l , at the minimum recommended frequency, we have:

$$\phi_{\min} = \frac{360}{\lambda_{g(\max)}} \times l \quad (3)$$

and, at the maximum recommended frequency, we have:

$$\phi_{\max} = \frac{360}{\lambda_{g(\min)}} \times l \quad (4)$$

Substituting equations (3) and (4) into equation (1), we obtain:

$$\frac{360}{\lambda_{g(\min)}} \times l = 180 - \frac{360}{\lambda_{g(\max)}} \times l \quad (5)$$

Re-arranging equation (5) gives:

$$l = 0.5 \times \left(\frac{1}{\lambda_{g(\min)}} + \frac{1}{\lambda_{g(\max)}} \right)^{-1} \quad (6)$$

Equation (6) is used to determine the length of the TRL Line standard (the so-called “ $\lambda/4$ line”), in each of the millimetre-wave waveguide bands [10], as shown in Table 1.

² Values for the broad wall dimension of all waveguide sizes considered in the report are given in Appendix A.

TABLE 1: TRL line lengths, l , for millimetre-wave waveguide sizes

Waveguide size	Frequency range (GHz)	$\lambda_{g(\max)}$ (mm)	$\lambda_{g(\min)}$ (mm)	l (mm)	$\Phi_{\min}$	$\Phi_{\max}$
WR-28	26.5 – 40	18.647	8.814	2.993	58	122
WR-22	33 – 50	15.071	7.051	2.402	57	123
WR-19	40 – 60	12.083	5.860	1.973	59	121
WR-15	50 – 75	9.930	4.717	1.599	58	122
WR-12	60 – 90	8.436	3.948	1.345	57	123
WR-10	75 – 110	6.472	3.228	1.077	60	120
WR-08	90 – 140	5.809	2.518	0.878	54	126
WR-06	110 – 170	4.822	2.085	0.728	54	126
WR-05	140 – 220	3.803	1.602	0.564	53	127
WR-04	170 – 260	2.987	1.357	0.467	56	124
WR-03	220 – 325	2.214	1.090	0.365	59	121

The lines listed in Table 1 become increasingly shorter as the waveguide size gets smaller (i.e. as the frequency increases). This leads to lengths that are too short for most practical purposes. This is because a short line is likely to become deformed (i.e. bent or twisted) during use. Therefore, as a rule of thumb, a line of less than 1 mm in length is considered here to be too short for practical purposes. This means that only waveguide sizes up to WR-10 (75 GHz to 110 GHz) are suitable for the conventional “ $\lambda/4$ line” TRL technique.

3.2 TRL using one $3\lambda/4$ line

An alternative method, described in this report, is to continue to use the TRL calibration scheme, but to use line lengths that give phase changes of around $3\lambda/4$ (instead of $\lambda/4$) at the mid-band frequency for each waveguide band. In effect, instead of using phase changes in the region between 0° and 180° (centred on 90°), longer lengths of line are used to enable phase changes to vary in the region between 180° and 360° .

It is well known [13] that TRL calibration schemes work optimally when the difference in phase between the through connection and the line connection is

$$(2n + 1) \lambda/4 \quad \text{where } n = 0, 1, 2, \dots \quad (7)$$

And similarly, TRL calibration schemes fail completely when this phase difference is

$$(2n) \lambda/4 \quad \text{where } n = 0, 1, 2, \dots \quad (8)$$

Therefore, if we set $n = 1$ in equation (7), optimum calibration performance occurs at $3\lambda/4$ (i.e. 270°). The associated calibration failure will occur at $\lambda/2$ (180°), setting $n = 1$ in equation (8), and λ (360°), setting $n = 2$ in equation (8). We therefore set a minimum phase criterion such that the phase can vary by 30° away from the two calibration failure points of 180° and 360° .³ In other words, the phase can vary from $(180 + 30 =) 210^\circ$ to $(360 - 30 =) 330^\circ$.

³ The minimum phase criterion of 30° has been chosen somewhat arbitrarily. Published values for this criterion are typically as low as 16° [13]. So, a value of 30° is considered to be a more than adequate minimum phase separation criterion.

We can now follow a similar procedure as given in sub-section 3.1 to determine what length of line is needed to achieve satisfactory calibration across each waveguide band in the sizes smaller than WR-10. However, since the phase now changes from 180° to 360° , equation (1) is modified as follows:

$$\varphi_{\max} = (360 + 180) - \varphi_{\min} = 540 - \varphi_{\min}$$

where:

$$180^\circ < \varphi_{\min} < 360^\circ$$

and

$$180^\circ < \varphi_{\max} < 360^\circ.$$

Equation (6) is therefore modified to become:

$$l = 1.5 \times \left(\frac{1}{\lambda_{g(\min)}} + \frac{1}{\lambda_{g(\max)}} \right)^{-1} \quad (9)$$

Equation (9) is used to determine the TRL line length, and equations (3) and (4) are used to determine the maximum and minimum phase change produced by this line length. This information is shown in Table 2.

TABLE 2: TRL line lengths, l , for phase changes centred around $3\lambda/4$

Waveguide size	Frequency range (GHz)	$\lambda_{g(\max)}$ (mm)	$\lambda_{g(\min)}$ (mm)	l (mm)	$\varphi_{\min}$	$\varphi_{\max}$
WR-08	90 – 140	5.809	2.518	2.635	163	377
WR-06	110 – 170	4.822	2.085	2.183	163	377
WR-05	140 – 220	3.803	1.602	1.691	160	380
WR-04	170 – 260	2.987	1.357	1.400	169	371
WR-03	220 – 325	2.214	1.090	1.096	178	362

Table 2 clearly shows that, for all these waveguide sizes,

$$\varphi_{\min} < 210^\circ$$

and

$$\varphi_{\max} > 330^\circ.$$

It is also clear that the range of phase values produced by each line, when used across the full bandwidth of each waveguide size, includes both 180° and 360° and so the TRL calibration will fail completely at these frequencies.

In summary, although using a longer line enables phase changes centred around $3\lambda/4$ to be used, the longer length of line produces excessive phase changes. This means that the minimum and maximum acceptable phase limits ($\varphi_{\min} = 210^\circ$ and $\varphi_{\max} = 330^\circ$) are exceeded and also the frequency range includes frequencies where complete calibration failure occurs (i.e. when $\varphi = 180^\circ$ and $\varphi = 360^\circ$).

3.3 TRL using two $3\lambda/4$ lines

An alternative method for using phase changes ranging between 210° and 330° , centred on $3\lambda/4$, is to use two lines, i.e. each line covers only a part of the full frequency range of a given waveguide size but, when used together, both lines cover the full waveguide band.

The following procedure is used to determine the lengths of the two TRL lines:

1. Determine the length, l_1 , for the first line that gives the minimum phase change, $\varphi_{\min}$ ($= 210^\circ$), at the minimum recommended frequency for the given waveguide size, using equation (3) re-arranged, as;

$$l_1 = \frac{\lambda_{g(\max)}}{360} \times \varphi_{\min} = \frac{\lambda_{g(\max)}}{360} \times 210 \approx 0.583\lambda_{g(\max)}$$

2. Determine the useable upper frequency limit for this line, i.e. the frequency at which the maximum phase change, $\varphi_{\max}$ ($= 330^\circ$), occurs. This is done by first calculating the guide wavelength corresponding to this maximum phase change, using equation (4) re-arranged, as:

$$\lambda_g = \frac{360 \times l_1}{\varphi_{\max}} = \frac{360}{330} \times l_1 \approx 1.091l_1$$

then, calculating the frequency corresponding to λ_g , using equation (2) re-arranged, as:

$$f = \frac{v \sqrt{1 + \left(\frac{\lambda_g}{\lambda_0} \right)^2}}{\lambda_g} \quad (10)$$

3. Determine the length, l_2 , for the second line that gives the maximum phase change, $\varphi_{\max}$ ($= 330^\circ$), at the maximum recommended frequency for the given waveguide size, using equation (4) re-arranged, as;

$$l_2 = \frac{\lambda_{g(\min)}}{360} \times \varphi_{\max} = \frac{\lambda_{g(\min)}}{360} \times 330 \approx 0.917\lambda_{g(\min)}$$

4. Determine the useable lower frequency limit for this line, i.e. the frequency at which the minimum phase change, $\varphi_{\min}$ ($= 210^\circ$), occurs. This is done by first calculating the guide wavelength corresponding to this minimum phase change, using equation (3) re-arranged, as:

$$\lambda_g = \frac{360 \times l_2}{\varphi_{\min}} = \frac{360}{210} \times l_2 \approx 1.714l_2$$

then calculating the frequency corresponding to λ_g using equation (10).

This procedure results in the line choices and frequency ranges shown in Table 3. Notice that, for each waveguide size, the bandwidth of each line is such that, when used together, the lines provide continuous frequency coverage across the full bandwidth of each of the waveguide sizes. In fact the bandwidth of each line overlaps, to some extent, the bandwidth of the other line. Notice also that all line lengths are greater than or equal to 1 mm (this being the length referred to earlier as a minimum realistic acceptable length for a line standard). We can therefore say that the use of two TRL lines, giving phase changes of the order of $3\lambda/4$, produces satisfactory calibration coverage to at least 325 GHz.

TABLE 3: TRL calibration using two lines of lengths l_1 and l_2

Waveguide Size	Frequency range (GHz)	l_1 (mm)	l_1 frequency range (GHz)	l_2 (mm)	l_2 frequency range (GHz)
WR-08	90 – 140	3.387	90 – 109	2.309	106 – 140
WR-06	110 – 170	2.811	110 – 133	1.912	129 – 170
WR-05	140 – 220	2.217	140 – 169	1.469	166 – 220
WR-04	170 – 260	1.741	170 – 209	1.244	197 – 260
WR-03	220 – 325	1.291	220 – 274	1.000	246 – 325

4. CALIBRATION AT SUB-MILLIMETRE WAVELENGTHS

At the present time, waveguide aperture sizes for use at sub-millimetre wavelengths have not been standardised. However, proposals have been published [14, 15] for defining waveguide sizes at these frequencies. We will follow here the proposal given in [14] since this method provides a waveguide band that corresponds to the band used for the commercially available measurement system reported in [4].

4.1 TRL using two $3\lambda/4$ lines

If we try using the TRL method using two lines, as described in sub-section 3.3, then the required line lengths are shown in Table 5. These lengths include values that are less than the practical minimum length of 1 mm. Note also that there is a ‘gap’ (from 318 GHz to 320 GHz) in the frequency coverage for the WR-2.8 line size.⁴

TABLE 5: TRL calibration using two lines of lengths l_1 and l_2

Waveguide Size	Frequency range (GHz)	l_1 (mm)	l_1 frequency range (GHz)	l_2 (mm)	l_2 frequency range (GHz)
WR-2.8	260 – 400	1.147	260 – 318	0.809	320 – 400
WR-2.2	325 – 500	0.951	325 – 394	0.651	380 – 500

4.2 TRL using three $5\lambda/4$ lines

To cater for these higher frequency waveguide bands, we need to extend the approach described in sub-section 3.3 by using line lengths that give phase changes of around $5\lambda/4$ (i.e. setting $n = 2$ in equation (7)) at the mid-band frequency for each waveguide

⁴ We follow here the waveguide naming convention given in [14], i.e. WR-2.8 and WR-2.2 for these two bands.

band. This will enable longer line lengths to be used to produce phase changes that vary in the region from 360° to 540° .

We therefore set a minimum phase criterion such that the phase can vary from 25° away from the two calibration failure phases of 360° and 540° (i.e. setting $n = 2$ and $n = 3$ in equation (8)), so that the minimum and maximum phases are⁵:

$$\varphi_{\min} = (360 + 25)^\circ = 385^\circ$$

and

$$\varphi_{\max} = (540 - 25)^\circ = 515^\circ.$$

We then follow a similar procedure as described in sub-section 3.3:

1. Determine the length, l_1 , for the first line that gives the minimum phase change, $\varphi_{\min}$, ($= 385^\circ$) at the minimum recommended frequency for the given waveguide size, using equation (3) re-arranged, as:

$$l_1 = \frac{\lambda_{g(\max)}}{360} \times \varphi_{\min} = \frac{\lambda_{g(\max)}}{360} \times 385 \approx 1.069 \lambda_{g(\max)}$$

2. Determine the useable upper frequency limit for this line, i.e. the frequency at which the maximum phase change, $\varphi_{\max}$, ($= 515^\circ$) occurs. This is done by first calculating the guide wavelength corresponding to this maximum phase change, using equation (4) re-arranged, as:

$$\lambda_g = \frac{360 \times l_1}{\varphi_{\max}} = \frac{360}{515} \times l_1 \approx 0.699 l_1$$

then, calculating the frequency corresponding to λ_g , using equation (10).

3. Determine the length, l_2 , for the second line that gives the maximum phase change, $\varphi_{\max}$ ($= 515$ degrees), at the maximum recommended frequency for the given waveguide size, using equation (4) re-arranged, as:

$$l_2 = \frac{\lambda_{g(\min)}}{360} \times \varphi_{\max} = \frac{\lambda_{g(\min)}}{360} \times 515 \approx 1.431 \lambda_{g(\min)}$$

4. Determine the useable lower frequency limit for this line, i.e. the frequency at which the minimum phase change, $\varphi_{\min}$ ($= 385$ degrees), occurs. This is done by first calculating the guide wavelength corresponding to this minimum phase change, using equation (3) re-arranged, as:

⁵ This minimum phase criterion of 25° is less than the value of 30° used for phase changes in the 180° to 360° region. This is to allow more of a given line's bandwidth to be used during the calibration (recognizing that this bandwidth is being reduced by using longer lengths of line). However, a phase criterion of 25° is still considerably more than other published values (e.g. 16° , as used in [13]) and so it is still considered a conservative choice for this value.

$$\lambda_g = \frac{360 \times l_2}{\phi_{\min}} = \frac{360}{385} \times l_2 \approx 0.935 l_2$$

then calculating the frequency corresponding to λ_g , using equation (10).

This procedure results in the choices for the lengths l_1 and l_2 , as shown in Table 6.

Notice that, for both waveguide sizes, the calibration bandwidths achieved using l_1 and l_2 are such that, when used together, there is a gap in the frequency coverage across the complete waveguide band. For example, for WR-2.8, the gap is from 293 GHz to 331 GHz and for WR-2.2 the gap is from 364 GHz to 413 GHz. In order to fill this gap, a third line of length l_3 is chosen such that its useable bandwidth will cover this gap region.

The length, l_3 , is chosen so that the TRL algorithm performs equally well at the lower and upper frequencies of the gap region. To do this, we follow a similar procedure as given in sub-section 3.1, except ϕ_{lower} and ϕ_{upper} are used (where ϕ_{lower} and ϕ_{upper} occur at the lower and upper frequencies of the gap region, respectively).

However, since the phase now varies in the region between 360° and 540° , equation (1) is modified as follows:

$$\phi_{\text{upper}} = (360 + 360 + 180) - \phi_{\text{lower}} = 900 - \phi_{\text{lower}}$$

where:

$$360^\circ < \phi_{\text{lower}} < 540^\circ$$

and

$$360^\circ < \phi_{\text{upper}} < 540^\circ.$$

Equation (6) is therefore modified to become:

$$l_3 = 2.5 \times \left(\frac{1}{\lambda_{g(\text{lower})}} + \frac{1}{\lambda_{g(\text{upper})}} \right)^{-1} \quad (11)$$

Equation (11) is used to determine the TRL line length, l_3 , where the values $\lambda_{g(\text{lower})}$ and $\lambda_{g(\text{upper})}$ are the guide wavelengths at the lower and upper frequencies of the gap region and are calculated using equation (2).

The upper and lower useable frequencies for l_3 are determined as follows:

1. Calculate the maximum guide wavelength, $\lambda_{g(\text{max})}$, corresponding to the minimum allowable phase change, $\phi_{\min}$, using equation (3) re-arranged as:

$$\lambda_{g(\text{max})} = \frac{360}{\phi_{\min}} \times l_3 = \frac{360}{385} \times l_3 \approx 0.935 l_3$$

2. Calculate the useable lower frequency limit for l_3 using $\lambda_{g(\text{max})}$ and equation (10).

3. Calculate the minimum guide wavelength, $\lambda_{g(\min)}$, corresponding to the maximum allowable phase change, $\phi_{\max}$, using equation (4) re-arranged as:

$$\lambda_{g(\min)} = \frac{360}{\phi_{\max}} \times l_3 = \frac{360}{515} \times l_3 \approx 0.699l_3$$

4. Calculate the useable upper frequency limit for l_3 using $\lambda_{g(\min)}$ and equation (10).

The length values and associated frequency ranges for l_3 are given in columns 7 and 8 of Table 6.

We can now see that the calibration bandwidths achieved by l_1 , l_2 and l_3 are such that, when used together, provide continuous frequency coverage across the full waveguide band. In fact the bandwidth of each line overlaps, to some extent, the bandwidth of the neighbouring line. Notice also that all line lengths are greater than 1 mm (this being the length referred to earlier as a realistic minimum acceptable length for a line standard). We can therefore say that the use of three TRL lines, giving phase changes of the order of $5\lambda/4$, produces satisfactory calibration coverage to 500 GHz.

TABLE 6: TRL calibration using three lines of lengths l_1 , l_2 and l_3

Size	Frequency range (GHz)	l_1 (mm)	l_1 frequency range (GHz)	l_2 (mm)	l_2 frequency range (GHz)	l_3 (mm)	l_3 frequency range (GHz)
WR-2.8	260 – 400	2.104	260 – 293	1.262	331 – 400	1.633	288 – 337
WR-2.2	325 – 500	1.744	325 – 364	1.016	413 – 500	1.337	360 – 418

5. PRACTICAL EXPERIMENTS

Until recently, NPL did not own any high precision lines in waveguide sizes smaller than WR-10. Therefore, in order to test the calibration strategy described in this report, two different calibrations were undertaken using lines in WR-10 already belonging to NPL, in particular, a 1.08 mm line and a 3.07 mm line. Two devices were then measured - a near-matched load and a mismatched load (with voltage reflection coefficient varying between 0.05 and 0.35).

The first experiment involved performing a conventional TRL calibration using a $\lambda/4$ line (i.e. the 1.08 mm line). The second experiment involved performing a TRL calibration using the 3.07 mm line. This line provides a phase change of approximately 171° at 75 GHz and 343° at 110 GHz. This means that the line can be used to provide calibrations where the phase varies within the range from 180° to 360° . However, the range of phases achieved by this line includes 180° and so the TRL calibration will fail completely at this frequency. We can use equations (3) and (4), re-arranged, along with equation (10), to determine:

- The frequency at which the calibration will fail completely (when $\phi = 180^\circ$);
- The recommended minimum and maximum frequencies (when $\phi_{\min} = 210^\circ$ and $\phi_{\max} = 330^\circ$, respectively) for using this line as a TRL calibration standard.

This results in the following frequencies:

- i) Calibration failure frequency: 76.5 GHz;
- ii) Recommended minimum frequency: 82 GHz;
Recommended maximum frequency: 107 GHz.

5.1 Near-matched load – results

5.1.1 Reflection coefficient magnitude

Figure 2 shows the measurements of the near-matched load obtained using the conventional TRL calibration (using the 1.08 mm line) along with the measurements obtained by the TRL calibration using the 3.07 mm line. For the 3.07 mm line, the calibration failure at 76.5 GHz is clearly visible. For the remainder of the frequency range, the agreement between the two calibration schemes looks promising.

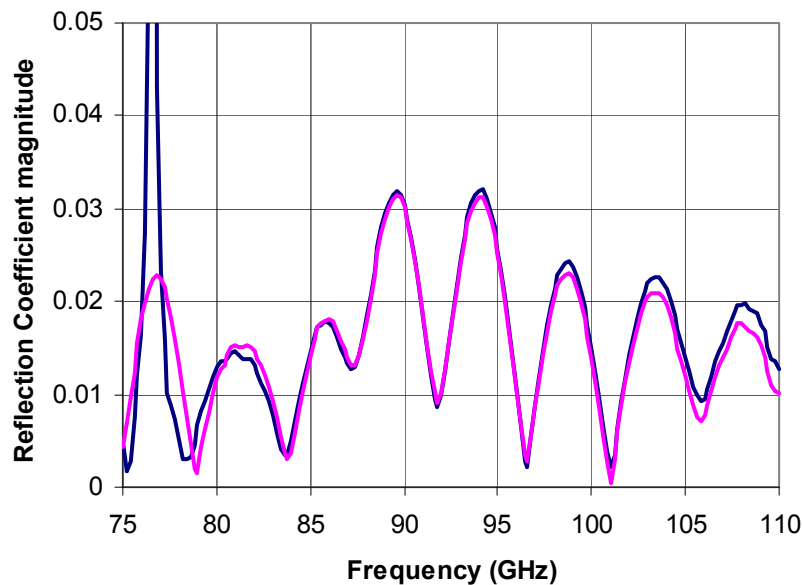


Figure 2: measurements of the reflection coefficient magnitude of a near-matched load using 1.08 and 3.07 mm TRL lines

To further investigate the level of agreement between the two calibration schemes, a calibration repeatability exercise was performed using the conventional $\lambda/4$ line (i.e. the 1.08 mm line). A repeatability exercise establishes the minimum level of variability that can be achieved for a given measurement process under prescribed repeatability conditions [16]. In this case, the repeatability conditions consisted of using the same VNA, the same calibration and measurement procedure, the same operator and the same operating conditions. Three repeated calibrations/measurements were made under these conditions and the differences between each pair of results were compared with the differences between the results obtained using the two calibration schemes, as shown in Figure 2. The three sets of repeatability differences (i.e. between: repeats 1 and 2; repeats 1 and 3; and, repeats 2 and 3) are shown in Figure 3, along with the difference between measurements made using the two calibration schemes. In each case, the

difference, Γ_{diff} , between two reflection coefficient magnitude values, $|\Gamma_1|$ and $|\Gamma_2|$, has been calculated at each frequency, using:

$$\Gamma_{\text{diff}} = |\Gamma_1| - |\Gamma_2|$$

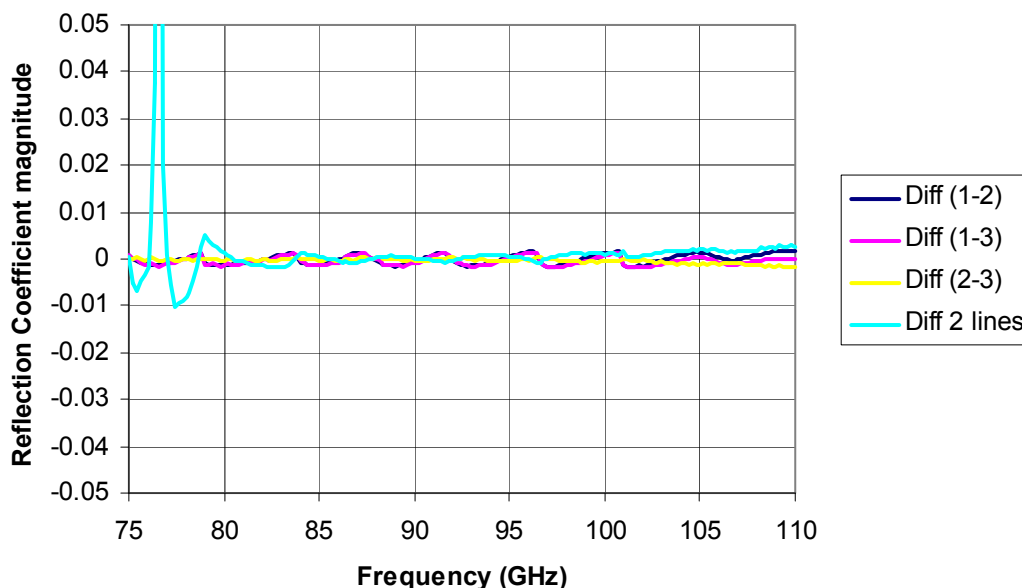


Figure 3: difference between the measurements in Figure 2, and the repeatability differences for the measurements using the 1.08 mm TRL line

Figure 3 shows that the differences between the results obtained using the two different calibration schemes above approximately 80 GHz is comparable to the difference observed with the repeatability exercise. This shows that the differences in the results achieved using the two different calibration schemes are not due to any significant systematic difference in the measurements i.e. the results from both calibration schemes are deemed equivalent above approximately 80 GHz. This agrees well with the predicted frequency range of 82 GHz to 107 GHz for the calibration using the 3.07 mm line.

5.1.2 Reflection coefficient phase

Since the reflection coefficient phase of a well-matched load (i.e. with a reflection coefficient magnitude close to zero) is difficult to determine experimentally, this is not a useful parameter to use to compare sets of results obtained from different measurement set-ups. Therefore, a comparison of the reflection coefficient phase of the near-matched load is not included in this report.

5.2 Mismatched load – results

5.2.1 Reflection coefficient magnitude

Figure 4 shows the measurements of the mismatched load obtained using the two different calibration schemes. Again, for the calibration using the 3.07 mm line, the expected calibration failure at 76.5 GHz is clearly visible. As before, the difference between these two measurements is compared with differences observed during the repeatability exercise. This is shown in Figure 5.

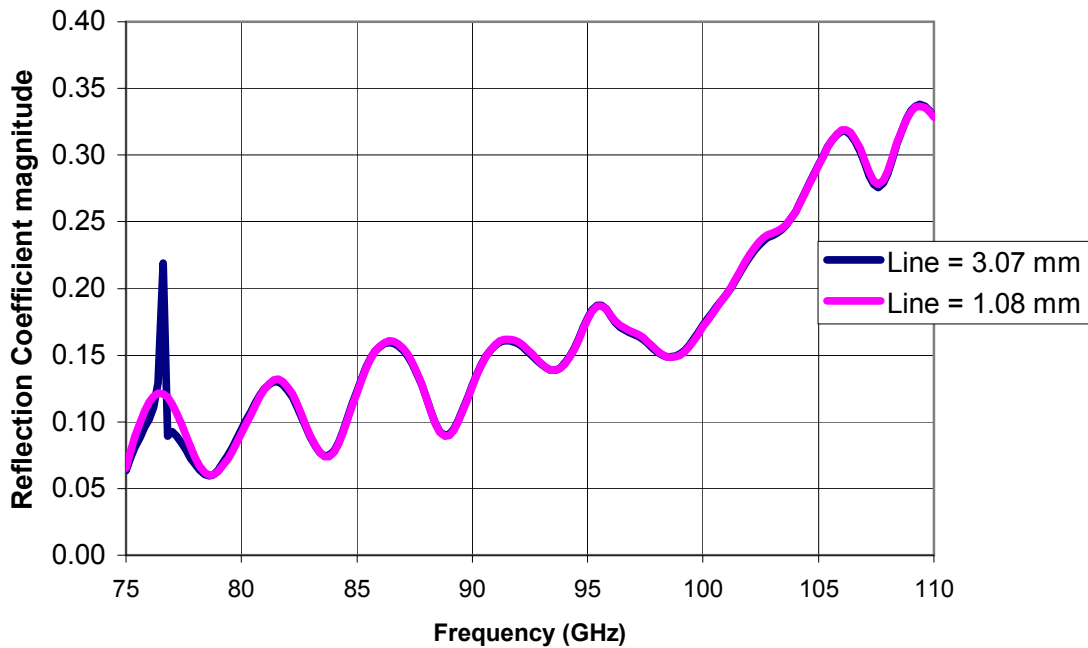


Figure 4: measurements of the reflection coefficient magnitude of a mismatched load using 1.08 and 3.07 mm TRL lines

As before, the difference between the results obtained using the two different calibration schemes above approximately 80 GHz is comparable to the differences observed with the repeatability exercise. This shows that the results obtained using the two calibration schemes are equivalent over this frequency range.

The traces in Figure 5 also show a sinusoidal ‘ripple’, where the frequency difference between adjacent maxima is approximately 5 GHz. This is likely to be due to interactions between residual errors remaining in the measurement system after calibration. (No VNA calibration procedure is able to perfectly correct for, or remove, all systematic errors in the measurement process. The errors remaining after calibration, i.e. after correction, are called residual errors [17].)

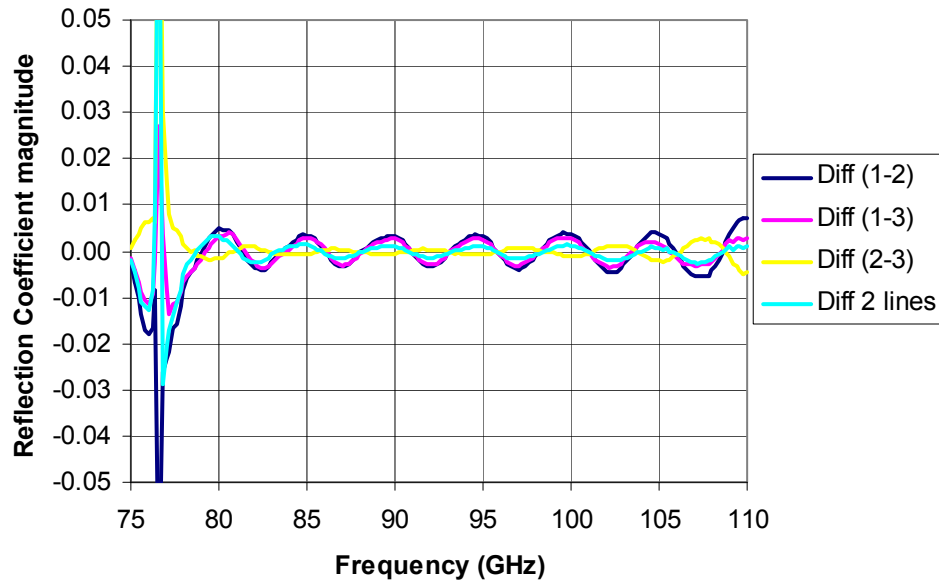


Figure 5: difference between the measurements in Figure 4 and the repeatability differences for the measurements using the 1.08 mm TRL line

5.2.2 Reflection coefficient phase

For the mismatched load (where the magnitude of the reflection coefficient is significantly greater than zero), a comparison can be made between the measured reflection coefficient phases using both calibration schemes. This is shown in Figure 6, with the associated difference and repeatability data shown in Figure 7. In each case, the difference, φ_{diff} , between two reflection coefficient phase values, $|\varphi_1|$ and $|\varphi_2|$, has been calculated at each frequency, using:

$$\varphi_{\text{diff}} = |\varphi_1| - |\varphi_2|$$

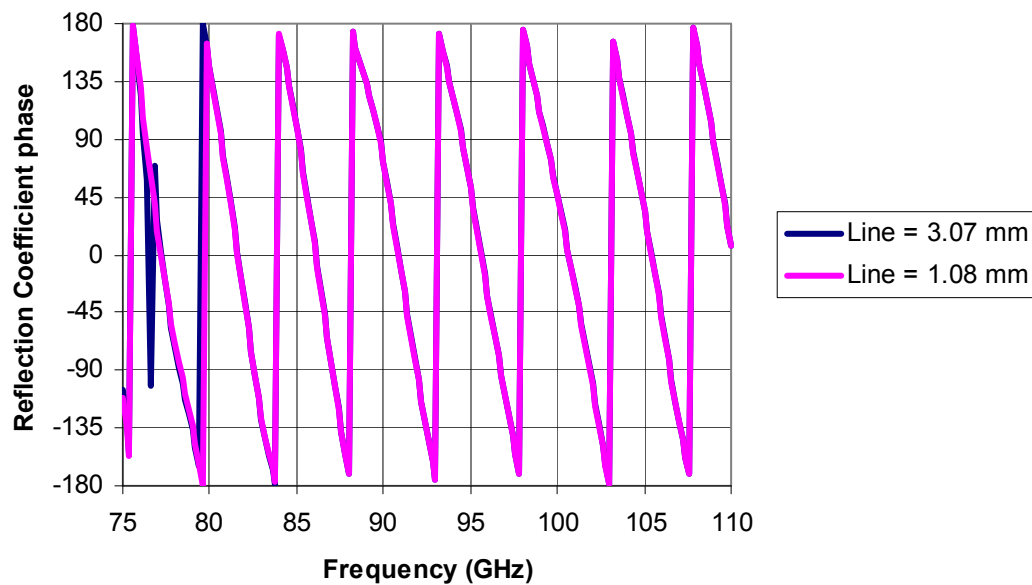


Figure 6: measurements of the reflection coefficient phase of a mismatched load using 1.08 and 3.07 mm TRL lines

As before, the differences between the results obtained using the two different calibration schemes above 80 GHz are comparable to the differences observed with the repeatability exercise (Figure 7). This further demonstrates that the results obtained using the two calibration schemes are equivalent at these frequencies. Also note that the traces in Figures 7 show a characteristic ripple, similar to that shown in Figure 5, which is likely to be due to residual errors remaining in the measurement system after calibration.

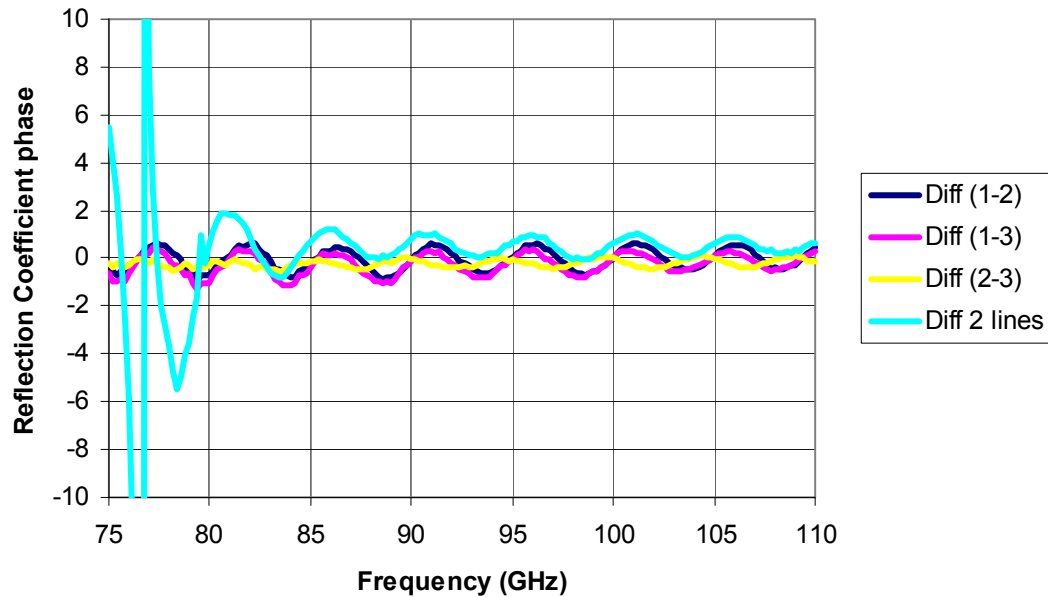


Figure 7: difference between the measurements in Figure 6 and the repeatability differences for the measurements using the 1.08 mm TRL line

6. RECOMMENDATIONS

Based on the work presented in this report, the following recommendations can be made regarding line choices for implementing TRL calibrations to cover full waveguide bands:

- i) For waveguide sizes used up to 110 GHz (i.e. WR-10 and larger sizes), a single $\lambda/4$ line can be used: see Table 7.⁶ These line lengths show good agreement with lengths that are used currently for the existing national waveguide standards [18], as shown in Appendix B.
- ii) For waveguide sizes WR-08 to WR-03, two $3\lambda/4$ lines are used: see Table 8.
- iii) For waveguide sizes WR-2.8 and WR-2.2, three $5\lambda/4$ lines are used: see Table 9.

⁶ In this table, and the tables that follow, line lengths are specified to only two places of decimal (i.e. to the nearest 0.01 mm). This is because this level of precision is considered adequate for specifying the length of each line. (Recall that, for TRL calibrations, the length of the line standard does not need to be known exactly [8].)

In addition, where more than one line is used to cover a complete waveguide band (as in Tables 8 and 9) the frequency where the change between two lines occurs is chosen to be close to the middle frequency for the overlap region for the two lines.

TABLE 7: TRL using one $\lambda/4$ line

Waveguide size	Frequency range (GHz)	Line length (mm)
WR-28	26.5 – 40	2.99
WR-22	33 – 50	2.40
WR-19	40 – 60	1.97
WR-15	50 – 75	1.60
WR-12	60 – 90	1.35
WR-10	75 – 110	1.08

TABLE 8: TRL using two $3\lambda/4$ lines

Waveguide size	Frequency range (GHz)	Line 1 length (mm)	Line 2 length (mm)	Line changeover frequency (GHz)
WR-08	90 – 140	3.39	2.31	108
WR-06	110 – 170	2.81	1.91	131
WR-05	140 – 220	2.22	1.47	168
WR-04	170 – 260	1.74	1.24	203
WR-03	220 – 325	1.29	1.00	260

TABLE 9: TRL using three $5\lambda/4$ lines

Waveguide size	Frequency range (GHz)	Line 1 length (mm)	Line 2 length (mm)	Line 3 length (mm)	Line 1 to 2: changeover frequency (GHz)	Line 2 to 3: changeover frequency (GHz)
WR-2.8	260 – 400	2.10	1.61	1.26	291	334
WR-2.2	325 – 500	1.74	1.33	1.02	362	416

7. SUMMARY

This report has presented a method whereby TRL calibrations can be implemented in waveguide used at millimetre and sub-millimetre wavelengths. The method enables calibrations to be achieved across the full bandwidth of each given waveguide size using lines of reasonable lengths (i.e. lengths that are greater than 1 mm).

Line choices have been given to enable this technique to be implemented to 500 GHz, this being the current upper frequency limit of commercially available VNAs. Undoubtedly, the technique could be extended to higher frequencies, should the need arise in the future.

Realisations of some of the calibration schemes described in this report will be implemented by NPL in the coming years to enable precession measurement facilities to be put in place at millimetre and sub-millimetre wavelengths.

8. ACKNOWLEDGEMENTS

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APPENDIX A: Broad wall dimensions of waveguides

Table A1 gives the broad wall dimension, a , in both mm and mil (thousandths of an inch), of the waveguide sizes considered in this report. The source of information for these dimensional values is [10], for all waveguide sizes up to WR-03, and [14], for the WR-2.8 and WR-2.2 waveguide sizes. The table also gives the equivalent WG waveguide name, where appropriate.

TABLE A1: Broad wall dimension for waveguide sizes included in this report

WR	28	22	19	15	12	10	08	06	05	04	03	2.8	2.2
a (mm)	7.112	5.690	4.775	3.759	3.099	2.540	2.032	1.651	1.295	1.092	0.864	0.711	0.559
a (mil)	280	224	188	148	122	100	80	65	51	43	34	28	22
WG	22	23	24	25	26	27	28	29	30	31	32	-	-

APPENDIX B: TRL lines used by NPL at lower millimetre-wave frequencies

Table B1 shows the lines that are currently used by NPL to perform TRL calibrations at millimetre-wave frequencies up to 110 GHz. These lines are used as $\lambda/4$ lines and so provide phase changes within the region from 0° to 180° .

TABLE B1: NPL's existing TRL lines

Waveguide size	Frequency range (GHz)	Length (mm)
WR-28	26.5 – 40	2.97
WR-22	33 – 50	2.42
WR-15	50 – 75	1.61
WR-10	75 – 110	1.08