

Report

NPL REPORT MAT 22

Modelling the Long-term Deformation Behaviour of Polymers for Finite Element Analysis

**G. Dean, N. McCartney,
L. Crocker and R. Mera**

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JULY 2008

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Industry and Innovation Division

SUMMARY

Creep studies have been carried out under tension, compression and shear on two grades of poly(oxymethylene) supplied by Du Pont and Ticona. Measurements under tension have been modelled using a function with four material parameters. One of these parameters, related to a mean retardation time for the relaxation process responsible for creep, decreases with increasing stress and this gives rise to non-linear creep behaviour. Measurements of creep under compression and shear have enabled the model to be adapted to include the influence of the stress state.

In order for the model to be implemented in a finite element system, theoretical methods have been explored for using creep data to predict long-term deformation behaviour under situations where the stress varies with time. This is not straightforward when creep deformation is non-linear. An expression has been proposed for achieving this under a uniaxial tensile stress, and has been evaluated using experimental tests where the stress is increased or decreased in discrete steps. Solutions have also been obtained for changes in stress under a specified strain history from which good agreement has been obtained between the prediction of stress relaxation behaviour and experimental results.

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ISSN 1754-2979

National Physical Laboratory
Hampton Road, Teddington, Middlesex, TW11 0LW

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Approved on behalf of the Managing Director, NPL,
by Dr M G Cain, Knowledge Leader, Materials Team
authorised by Director, Industry and Innovation Division

CONTENTS

1	INTRODUCTION.....	1
2	MATERIALS.....	3
3	EXPERIMENTAL	4
4	A MODEL FOR CREEP IN POLYOXYMETHYLENE	5
4.1	MOLECULAR RELAXATION PROCESSES IN POLY(OXYMETHYLENE)	5
4.2	MODELLING NON-LINEAR CREEP UNDER UNIAXIAL TENSION.....	5
4.3	MODELLING CREEP UNDER UNIAXIAL COMPRESSION	9
4.4	EXTENSION OF THE MODEL TO MULTIAXIAL STRESS STATES.....	11
5	APPLICATION OF THE CREEP MODEL UNDER TIME-VARYING STRESS.....	14
5.1	EXPERIMENTAL.....	14
5.2	USE OF NON-LINEAR CREEP MODELS IN THE FINITE ELEMENT SYSTEM ABAQUS	16
5.3	PROPOSED APPLICATION OF THE CREEP MODEL FOR POLYMERS	19
5.4	GENERALISATION TO AN ARBITRARY STRESS OR STRAIN HISTORY ...	20
6	FURTHER WORK	23
7	CONCLUSIONS	24
	ACKNOWLEDGEMENT	25
	REFERENCES	25

1 INTRODUCTION

Polymeric materials are increasingly being used in the manufacture of components where a knowledge of deformation of the component under long-term loading is needed for confident design. Finite element analysis can be used to calculate stress and strain distributions within a component under service loads as well as overall changes in dimensions. The accuracy of results produced by a stress analysis depends on a number of factors including the validity of the materials model used to describe deformation behaviour. Polymeric materials are viscoelastic, so properties change with time under load as a result of relaxation processes in the molecular structure. Materials models for plastics in finite element systems generally represent the material as a series of elastic springs and viscous dashpots [1]. One problem with the application of such models is that a large number of spring and dashpot elements is needed to describe the change of properties with time over a long timescale since the relaxation time spectrum of molecular motions making up a relaxation process is usually very broad. A large number of parameters, therefore, need to be obtained in order to characterize material behaviour. Furthermore, without major adaptation, these models can only be used for low-stress applications where deformation behaviour is linear.

Alternative functions have been used to model the long-term deformation behaviour of polymers which can be traced to the parameters in spring and dashpot models but contain fewer parameters. A versatile function that is commonly used to model creep relaxation in semicrystalline polymers and secondary relaxations in glassy polymers expresses a creep compliance function in the form [2-4]

$$D(t) = D_0 + \Delta D \left(1 - \exp\left(- (t / t_0)^n \right) \right) \quad (1)$$

When the creep loading is carried out under uniaxial tension, $D(t)$ is then the tensile compliance function and is the ratio of the time varying tensile strain to the applied, constant tensile stress. The parameter D_0 is the tensile compliance in the limit of short times, ΔD is the magnitude of the creep relaxation process and is the difference between compliances in the limit of long and short times and t_0 and n are parameters that identify the time range spanned by the process. The parameter t_0 can be regarded as a mean retardation time for the process whilst n determines the distribution of retardation times

such that, the smaller is n , the broader is the process. Whilst the form of equation (1) is relatively simple and it captures the main features of a relaxation process, it cannot be expected to accurately describe compliance changes over the whole time frame spanned by the process. The results of experiments reported here, however, show that this is a convenient function to model creep behaviour of certain polymers over creep times $t \leq t_0$.

Under low levels of applied stress, compliances $D(t)$ do not vary with applied stress and deformation behaviour is therefore linear. With increasing stress, creep experiments on many polymers reveal a departure from linear behaviour. Non-linear creep is observed at moderate stress levels in tests of short duration, but the departure from linearity is evident at lower stress levels at longer times under load. The main factor contributing to this non-linear behaviour is a reduction in the retardation time parameter t_0 with increasing stress [5-9]. The effect of elevated stresses is therefore to increase molecular mobility so that the polymer creeps faster. Creep tests under uniaxial compression reveal that the dependence of t_0 on stress is different under tension from that under compression. This indicates that t_0 depends on stress state as well as the stress magnitude, and expressions for a relationship involving an effective stress have been proposed in previous work [8,9].

In this report, the above model for creep deformation has been applied to two grades of a poly(oxymethylene) material. Expressions have been derived that describe the observed non-linearity in creep behaviour and its dependence on stress state. This has allowed expressions to be proposed relating stress and strain components under a constant multiaxial stress. The main objective of this work is to incorporate this model into a finite element system to enable stress analyses to be carried out that take proper account of the time-dependent properties of polymers. Existing models for non-linear creep in finite element systems are based on time-dependent plasticity. It is shown here that these plasticity models are unable to describe accurately the creep behaviour of polymeric materials.

Whilst creep tests are a convenient way of determining material properties for materials where properties vary with time under load, a viscoelasticity theory is needed to reveal

how creep data can be used in applications where the stress is not simply constant with time but where some more general stress or strain history occurs. The theory for linear behaviour is well developed in terms of the Boltzmann superposition integral [1]. Here, changes in strain, resulting from incremental changes in stress that define the stress history, are obtained by adding creep compliance contributions from each increment that have been retarded by the time delay in the application of that increment. This approach cannot be used, without modification, when behaviour is non-linear since the response from each increment will depend on the stress level present when the increment is applied. In this report, the superposition integral is adapted to take account of non-linear behaviour.

In Section 4, the creep model is introduced, and relationships are derived for time-dependent strain components under a multiaxial creep stress. These equations are used with uniaxial tensile creep data to predict creep behaviour under uniaxial compression and shear and compared with experimental results. In Section 5, a theoretical foundation is proposed for using the creep model to predict the time-dependent strain under conditions where the stress is not constant but varies with time. The validity of the proposed developments is explored through comparisons of predicted deformations with results from experimental tests under different stress histories. In the first of these tests, the stress was increased in steps, the step duration being at least one day. Tests were also carried out under constant strain (stress relaxation tests) where the time-varying stress was measured. In both of these cases, the response to these stress histories is calculated using the creep model. These results are also used to demonstrate inaccuracies in the procedure used in ABAQUS when applied to polymers to determine the response to time-varying loads where creep behaviour is non-linear.

2 MATERIALS

The poly(oxymethylene) materials studied here were pharmaceutical grades supplied by Du Pont and Ticona. The Du Pont material was Delrin 1260 NC010 and was supplied as compression moulded plates 160 mm square and 3 mm thick. The Ticona material was Hostaform MT8U01 and was supplied as injection moulded plates 300 mm by 210 mm by 3mm thick.

3 EXPERIMENTAL

Creep tests have been carried out under uniaxial tension using specimens consisting of rectangular bars of length 160 mm. These were clamped in creep machines such that one clamp is fixed and the other is attached to a lever arm, which supports an applied load at the other end. A pair of extensometers was attached to opposite faces of the specimen, and the gauge length of each extensometer was 25 mm. The tensile strain was determined from the average of the extensometer readings to eliminate the influence of specimen bending on the strain measurement. Engineering values of stress and strain were used in the derivation of creep compliances because strain levels rarely exceeded 0.03.

Creep tests under uniaxial compression were carried out on specimens 12 mm long $\times$ 10 mm wide and approximately 4 mm thickness. Specimens were loaded along the length direction between parallel platens in a universal test machine. An alignment jig was attached to one of the platens to enable the faces to be set parallel using slip gauges. The surfaces of the platens were lubricated with light oil, and the top and bottom faces of the specimens were machined smooth and accurately parallel. Extensometers were used to measure changes in the platen separation on loading, and nominal strain values were derived from the original specimen length. Corrections were made to these strain values of around 0.005 arising from initial displacements in the regions of the specimen/platen interface. The corrections were derived from the departure from linearity near the origin in a plot of load against platen displacement.

Shear tests were carried out using a version of a test reported by Arcan [10]. In this test, a plate specimen with V-shaped notches machined into opposite edges is loaded in simple shear. The notches define a geometry such that a predominantly pure shear stress is established in the central region of the specimen. The notch tip separation in the specimen was 12 mm and the radius of the notches was 1.5 mm. Shear strains were measured using a purpose-built extensometer which measured the relative displacement of points separated by a distance of 3 mm at the centre of the specimen. In the calculation of shear stresses and strains, corrections were applied for the non-uniform shear stress between the notches and for a contribution from bending to measured displacements.

4 A MODEL FOR CREEP IN POLYOXYMETHYLENE

4.1 MOLECULAR RELAXATION PROCESSES IN POLY(OXYMETHYLENE)

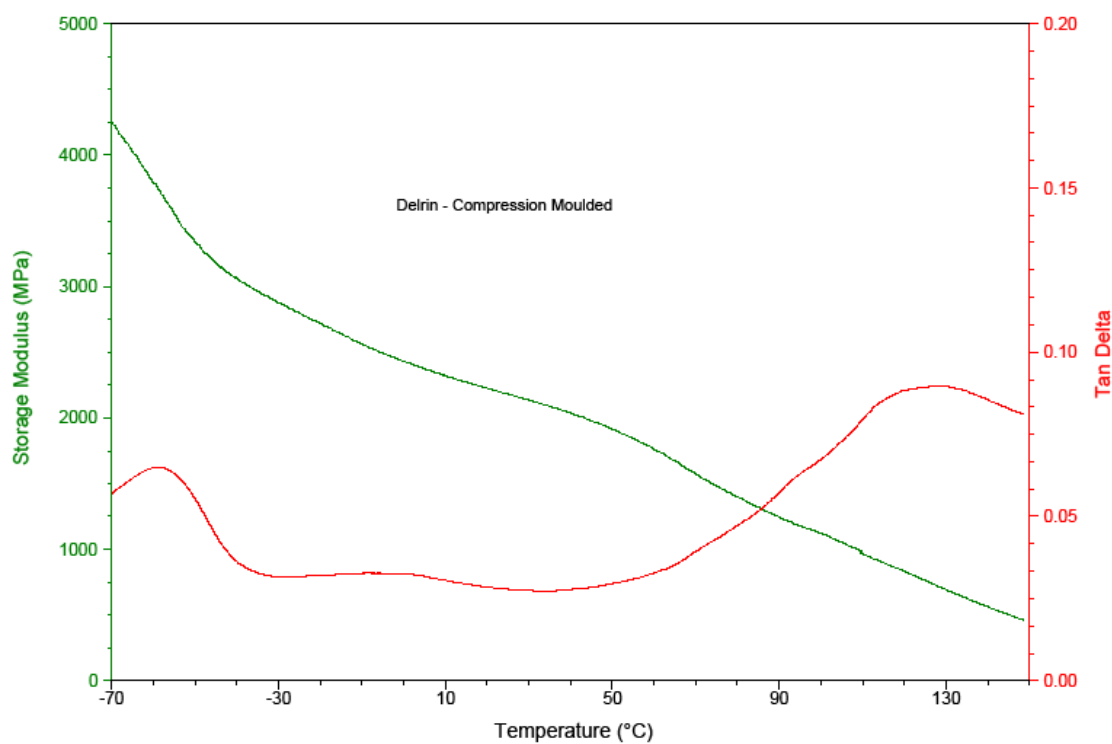
Dynamic mechanical measurements over a wide temperature range can be used to reveal the presence of the various relaxation processes in a polymer that are responsible for viscoelastic behaviour. Dynamic property curves for each of the grades studied here were measured at a frequency of 3Hz and are shown in figures 1a and 1b. Three relaxation regions are evident centred at temperatures around -70 °C, -5 °C and 120 °C. The high temperature process is believed to be associated with the crystalline phase [11] and is responsible for long-term creep at ambient temperatures measured in the work reported here. There is some evidence that the relaxation process centred at -5 °C is not fully relaxed at a temperature of 23 °C, and there is possibly some overlap with the low-temperature region of the high temperature process. This overlap would be evident in creep tests at very short times, but will be much less and maybe negligible at times greater than 10 s.

4.2 MODELLING NON-LINEAR CREEP UNDER UNIAXIAL TENSION.

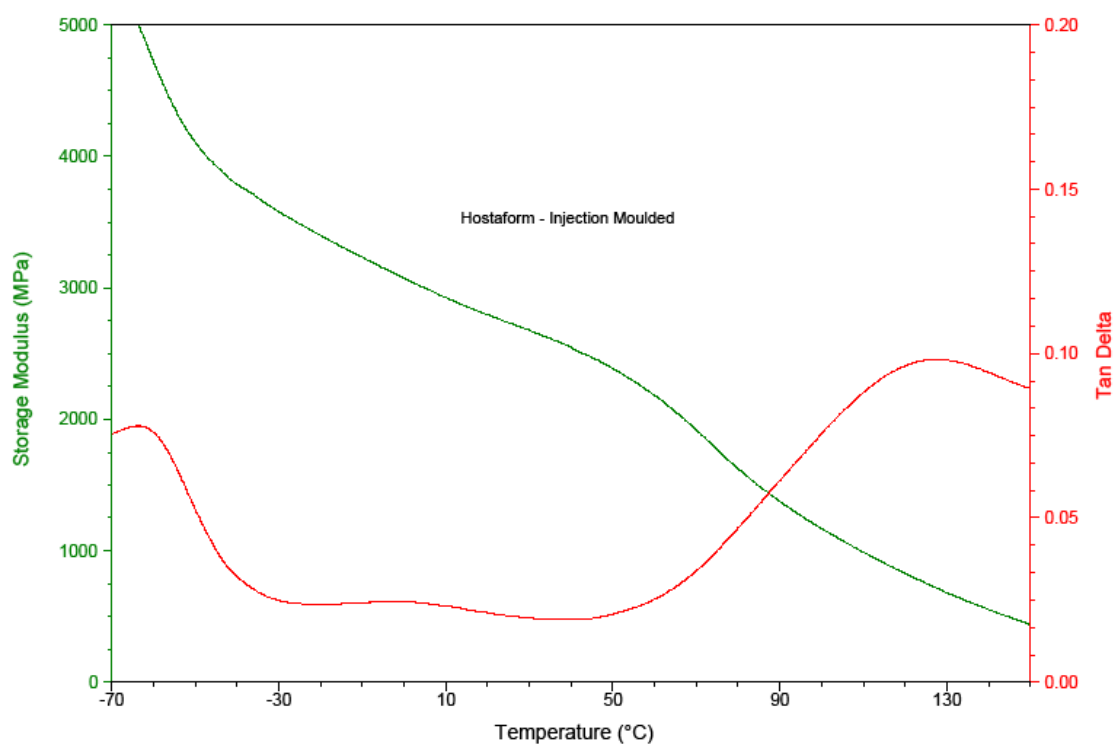
The development of the creep model for poly(oxymethylene) has been carried out using results obtained on the compression moulded material supplied by Du Pont. Figure 2 shows changes in the tensile creep compliance with log time measured under different tensile loads. Here the time-dependent compliance $D(t)$ is defined by the ratio of the time-dependent strain $\varepsilon(t)$ to the applied constant tensile stress σ_T thus

$$D(t) = \varepsilon(t) / \sigma_T \quad (2)$$

The continuous lines with each set of experimental data have been obtained using equation (1) with the parameter values listed in Table 1.



1(a)



1(b)

Figure 1 Temperature dependence of the dynamic mechanical properties under tension of two grades of poly(oxymethylene) studied.

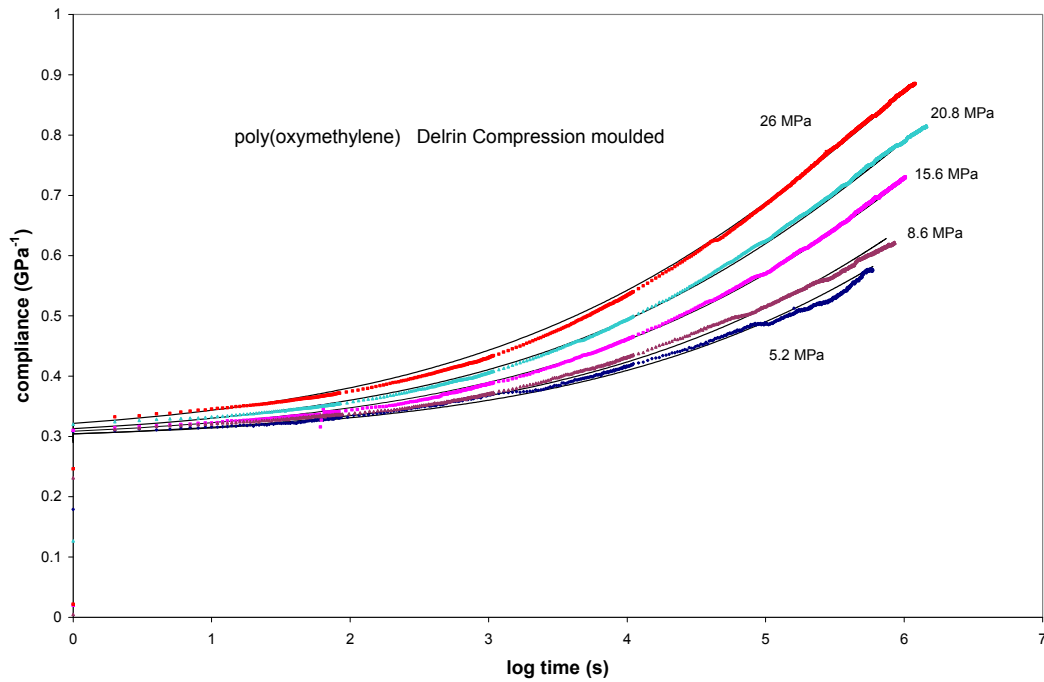


Figure 2 Tensile creep compliance curves for Delrin at different levels of stress. Curve fits are obtained using equation (1) and the parameters in table 1.

Table 1 Values for the parameters in equation (1) used to obtain the fits to data in Figure 2.

σ_T (MPa)	D_0 (GPa ⁻¹)	ΔD (GPa ⁻¹)	n	t_0 (s)
5.2	0.29	0.9	0.25	2.6×10^7
8.6	0.29	0.9	0.25	1.5×10^7
15.6	0.29	0.9	0.25	5.2×10^6
20.8	0.29	0.9	0.25	2.3×10^6
26	0.29	0.9	0.25	9.0×10^5

An optimisation procedure was used initially to obtain the parameter values giving the best fit. These results showed no systematic variation in the values of the parameters D_0 , ΔD or n with stress. Constant values for these parameters were therefore chosen for all stress levels and the values for t_0 were then determined. These values are recorded in

the table. This approach to the selection of parameter values assumes that the only effect of increasing stress is to reduce the retardation time parameter t_0 , and this is responsible for a shift in curves to shorter time with no change in shape. Whilst the possibility of small changes in the values of the other parameters with stress cannot be discounted, the accuracy and validity of the form of the creep function used to describe the time-dependent compliance, equation (1), is insufficient to establish the extent of any change. Any dependence of these parameters on stress is unlikely to significantly reduce the accuracy of predictions for poly(oxyethylene) compared with when these parameters are assumed constant.

The dependence of the parameter t_0 upon stress has been found to follow the empirical relationship

$$t_0 = A \exp(-\alpha \sigma_T) \quad (3)$$

where A and α are additional material parameters. This is demonstrated by the accurately linear variation of $\log_e t_0$ with stress shown in figure 3 for the parameter values given in table 1. Values for the parameters A and α have been obtained from figure 3 and are recorded in Table 2. Equation (3) implies a dependence of t_0 upon stress at all value of stress and, therefore, no transition from linear to non-linear behaviour. A smooth transition from linear to non-linear behaviour would be expected in practice which implies that equation (3) should become flatter at stresses below around 5 MPa and should cut the $\log_e t_0$ axis horizontally so that there is a smooth transition from tensile to compressive behaviour. Use of equation (3) will probably not have a serious effect on the predictive accuracy of the model, but maybe this should be investigated in further work. Creep under compression is considered in the next section.

**Table 2 Values for some of the creep model parameters
measured on compression moulded Delrin**

A (s)	α (MPa ⁻¹)	μ	ν
5.9×10^7	0.16	0.865	0.4

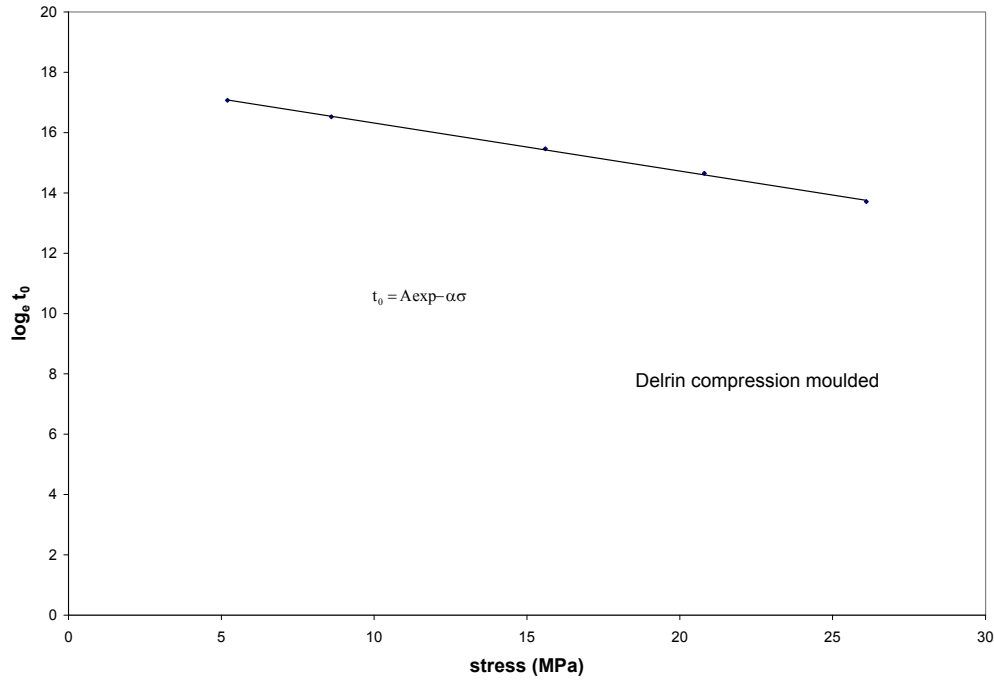


Figure 3 Plot of $\log_e t_0$ against stress. Values for t_0 were obtained from the curve fits to the data in figure 2.

4.3 MODELLING CREEP UNDER UNIAXIAL COMPRESSION

Creep tests under uniaxial compression and shear have been carried out to explore the nature of relationships between strain and stress for multiaxial creep stresses. Measurements of creep compliance under a compressive stress of 25 MPa are shown in figure 4. A tensile creep curve at a stress of 26 MPa is shown for comparison. The compression curve has the same shape as the tensile curve but is shifted to longer times. Assuming that the creep behaviour under tension and compression is the same at low stresses where behaviour is linear, this result demonstrates that the reduction in t_0 with stress is less under compression than under tension. The difference between uniaxial tensile and compressive stresses of the same magnitude is the sign of the hydrostatic component. This suggests that in the relationship between t_0 and stress given by equation (3), the stress should be replaced by an effective stress $\bar{\sigma}$ that is a combination of shear and hydrostatic parts. Thus equation (3) under a multiaxial stress will become

$$t_0 = A \exp(-\alpha \bar{\sigma}) \quad (4)$$

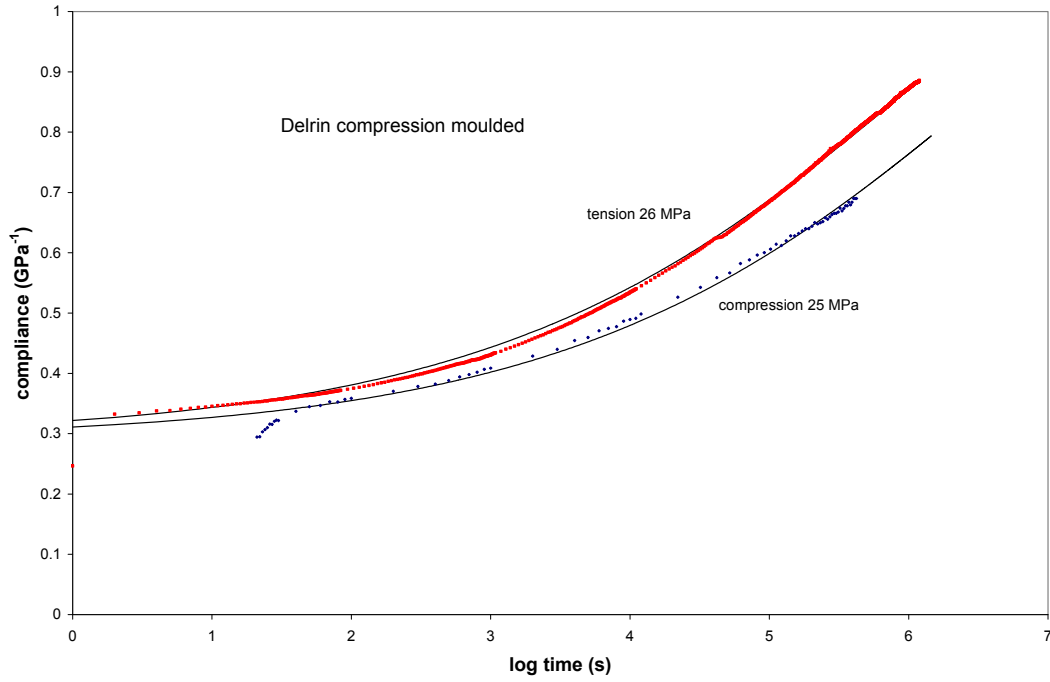


Figure 4 Comparison of creep compliance curves for Delrin obtained under uniaxial tension and compression at similar levels of stress.

Assuming the effective stress is a simple linear combination of shear and hydrostatic components, $\bar{\sigma}$ will take the form

$$\bar{\sigma} = \mu\sigma_e + \lambda\sigma_m \quad (5)$$

where μ and λ are parameters that determine the sensitivity of the effective stress to the shear and hydrostatic components of stress σ_e and σ_m , respectively. These components of stress are given in terms of principal components of a multiaxial stress σ_1 , σ_2 and σ_3 by

$$\sigma_e = \frac{1}{\sqrt{2}} [(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]^{1/2} \quad (6)$$

and

$$\sigma_m = \frac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3) \quad (7)$$

It is possible to define the effective stress $\bar{\sigma}$ as being numerically equal to the uniaxial tensile stress σ_T . The parameters μ and λ are then related, and equation (4) becomes

$$\bar{\sigma} = \mu\sigma_e + 3(1-\mu)\sigma_m \quad (8)$$

Therefore, under a uniaxial compressive stress $-\sigma_C$ so that σ_C is positive

$$\bar{\sigma} = (2\mu-1) \sigma_C \quad (9)$$

The continuous line alongside the compression data in figure 4 has been obtained using equation (1) with the same parameters obtained for tensile data, shown in table 1, but with a value of t_0 of 3.2×10^6 s. Substituting this value for t_0 together with equation (9) into equation (4) enables a value of $\mu = 0.865$ to be determined.

4.4 EXTENSION OF THE MODEL TO MULTIAXIAL STRESS STATES

In a Cartesian coordinate system x_i , $i = 1, 2$ or 3 , multiaxial stress and strain fields are specified by tensor components σ_{ij} and ε_{ij} , respectively. Under a constant multiaxial creep stress, multiaxial creep strains $\varepsilon_{ij}(t)$ are given by the sum of creep strains arising from the shear and the hydrostatic components of the stress. Creep strains are then given by

$$\varepsilon_{ij}(t) = \frac{J(t)}{2}(\sigma_{ij} - \sigma_m \delta_{ij}) + \frac{B(t)}{3}\sigma_m \delta_{ij} \quad (10)$$

where $J(t)$ and $B(t)$ are creep compliance functions under pure shear and hydrostatic stresses, respectively, and $\delta_{ij} = 1$ when $i = j$ and 0 when $i \neq j$. The determination of creep strains from a known creep stress therefore requires the measurement of two, time-dependent material properties $J(t)$ and $B(t)$. It is very difficult to measure either of these properties, but, as with isotropic, elastic behaviour, it is possible to relate these functions to the creep compliance measured under tension $D(t)$ as follows

$$\begin{aligned} B(t) &= 3D(t)(1-2\nu) \\ \text{and} \quad J(t) &= 2D(t)(1+\nu) \end{aligned} \quad (11)$$

Here ν is Poisson's ratio and is assumed, for the moment, to be independent of time. Combining equations (10) and (11) therefore gives a relationship for multiaxial creep strains in terms of measurable quantities. It is proposed that in this relationship, $D(t)$ is given by equation (1) with t_0 given by equation (4) so that the retardation time for both the shear and the hydrostatic strain components is the same and determined by the value for the effective stress producing the multiaxial creep. Thus $D(t)$ should be replaced by $D(t, \bar{\sigma})$

In order to explore some of the assumptions behind the generalisation of the model to multiaxial behaviour, creep experiments have been carried out under a shear stress for comparison with predictions. Under a pure shear stress σ_s , the creep shear strain ε_s is given, using equation (10) with equation (11), by

$$\varepsilon_s(t) = D(t, \bar{\sigma})(1 + \nu)\sigma_s \quad (12)$$

from which the shear creep compliance $J(t) = \varepsilon_s(t)/\sigma_s(t)$ can be calculated.

From equation (1),

$$D(t, \bar{\sigma}) = D_0 + \Delta D \left(1 - \exp \left(- \left(\frac{t}{t_0(\bar{\sigma})} \right)^n \right) \right) \quad (13)$$

and from equation (8), since under a pure shear stress $\sigma_m = 0$, $\bar{\sigma}$ is given by

$$\bar{\sigma} = \sqrt{3}\mu\sigma_s \quad (14)$$

So, from equation (4)

$$t_0(\bar{\sigma}) = A \exp - \sqrt{3}\alpha\mu\sigma_s \quad (15)$$

Measurements have been made of creep deformation under shear on specimens of the cast Delrin. A shear creep compliance curve under a stress of 14.8 MPa is shown in figure 5. The continuous line is the calculated compliance obtained from equation (12) with equations (13) and (15) using values for the parameters in these equations recorded in tables 1 and 2. The value for Poisson's ratio is recorded in table 2 and was determined from a conventional tensile test at a speed of 1 mm/min. The observed agreement between measured and predicted compliances in figure 5 is very good. Despite this, a number of assumptions have been made in generalising the creep model to multiaxial stress states, and these should be verified in further work. The assumption that Poisson's ratio is constant was made to allow shear and bulk compliances to be derived from measured tensile data alone. An alternative approximation, that the bulk modulus has negligible relaxation and is therefore constant with time, has been explored, and the calculated shear compliance curve based on this is shown in figure 5. It can be seen from the results on the compression moulded Delrin material that the assumption of a constant Poisson's ratio is better.

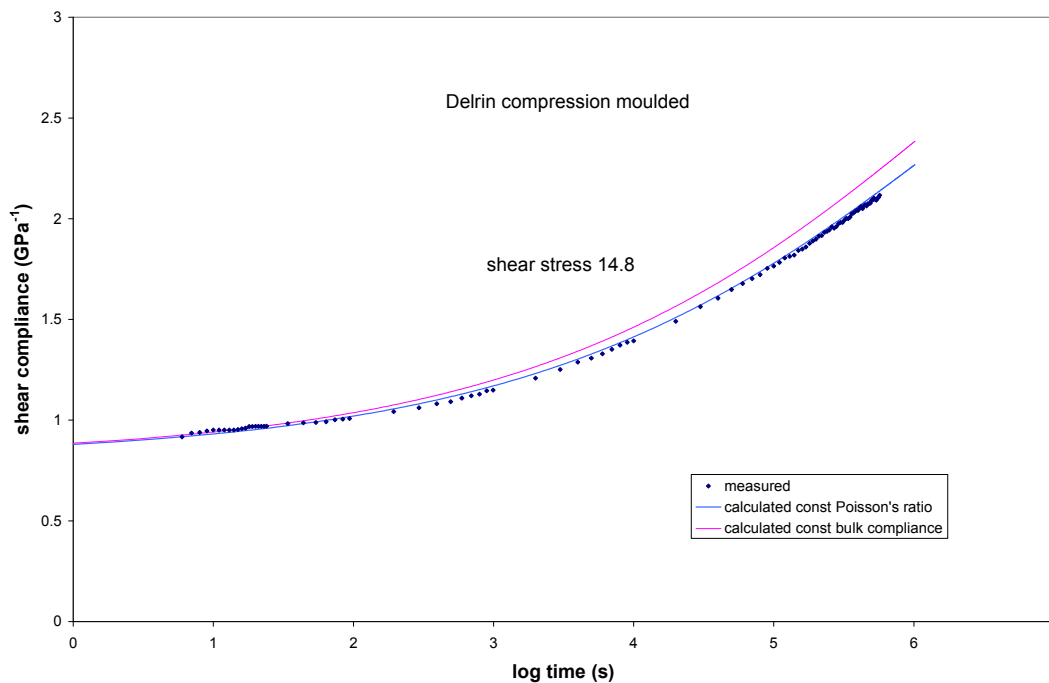


Figure 5 Comparison of measured creep data for Delrin in shear with predicted results.

5 APPLICATION OF THE CREEP MODEL UNDER TIME-VARYING STRESS

5.1 EXPERIMENTAL

In order to code the creep model described in section 4 into a finite element system, it is necessary to develop a theoretical framework that enables creep data to be used to predict time-varying strains in situations where the stress is not constant with time. This development has been supported using experiments where the stress has been held constant for a period and then increased or decreased in steps. These experiments have been carried out on specimens cut from the injection moulded plates of poly(oxymethylene) supplied by Ticona. Model parameters for this material were derived from tensile tests at different stress levels shown in figure 6. Parameter values are listed in table 3.

Table 3 Values for creep model parameters for injection-moulded Hostaform

D_0 (GPa ⁻¹)	ΔD (GPa ⁻¹)	n	A (s)	α (MPa ⁻¹)
0.32	1.0	0.26	$6.1 \cdot 10^7$	0.16

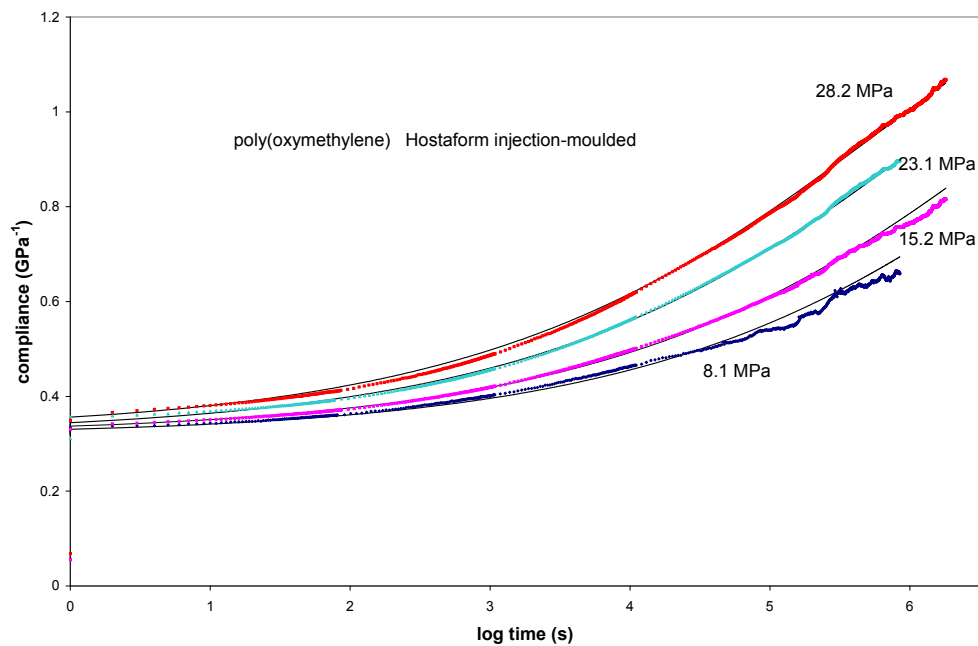


Figure 6 Tensile creep compliance curves for injection moulded Hostaform at different stress levels.

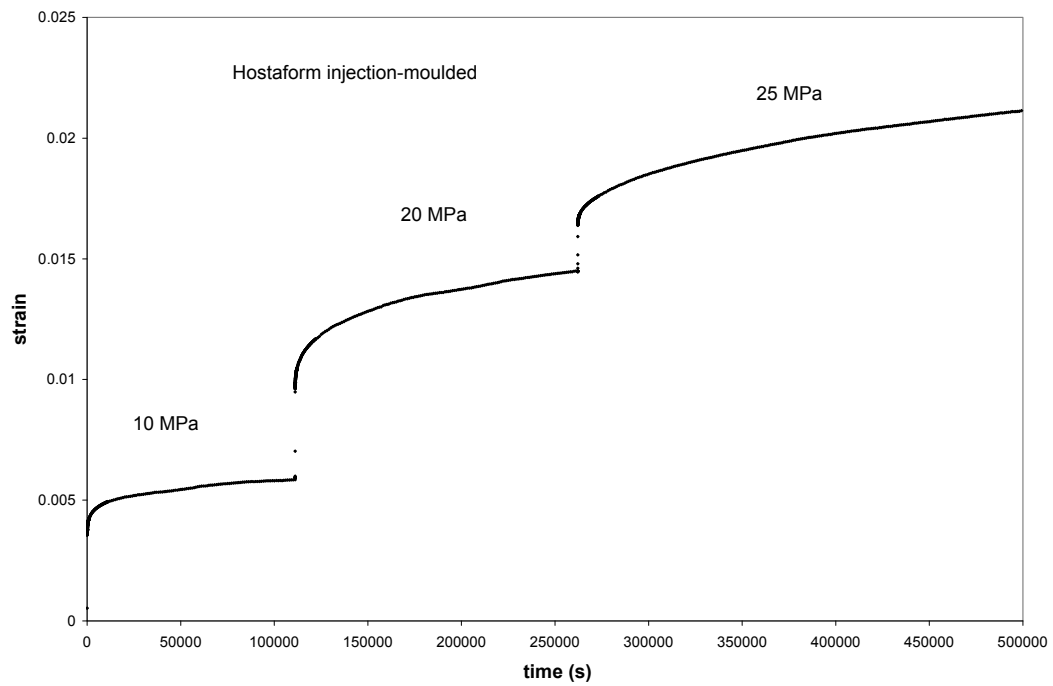


Figure 7 Measured values of strain against time obtained from a step loading test on injection moulded Hostaform.

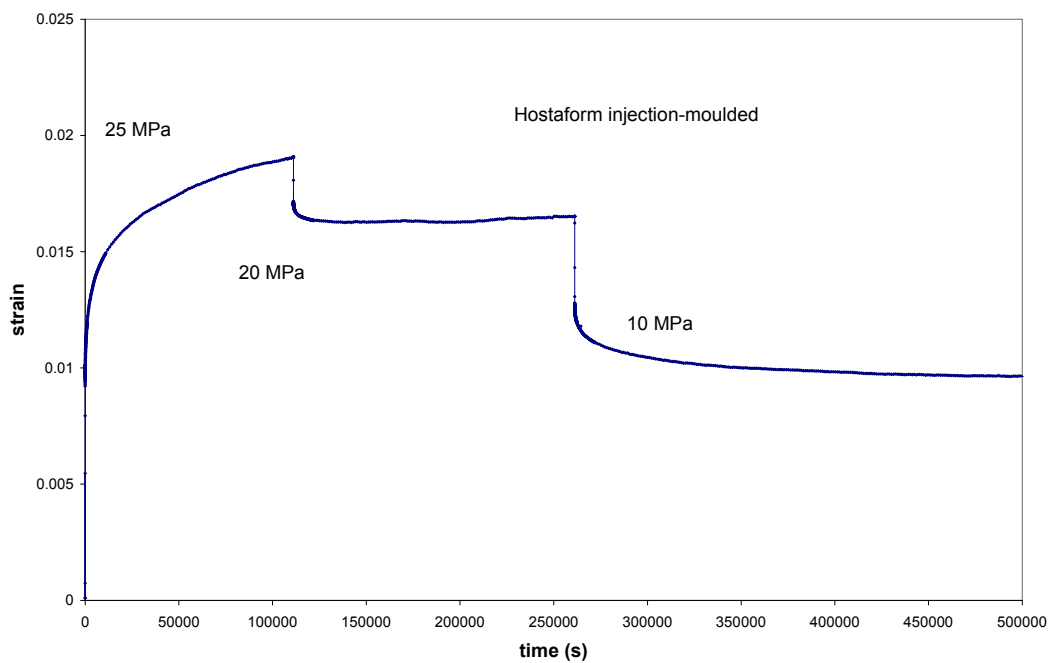


Figure 8 Measured values of strain against time obtained from a step load removal test on injection moulded Hostaform.

For the step-loading and step load-removal tests, the variation of strain with time was measured. The results from these tests and the stress levels and durations are shown in figures 7 and 8.

5.2 USE OF NON-LINEAR CREEP MODELS IN THE FINITE ELEMENT SYSTEM ABAQUS

In order to explore methods for using creep data to predict deformation under a time-varying stress, it is instructive to investigate how a finite element system such as ABAQUS deals with this problem. Non-linear creep models in ABAQUS are based on time-dependent plasticity and do not accurately model the creep behaviour of polymers over long periods of creep time. However, if the creep time is relatively short, say in the region of 10^5 s, and if the stress range is fairly narrow, a simple model in ABAQUS can be used to give an approximate description of creep deformation.

Attempts to model the creep results at the stress of 10 MPa in figure 7 and 25 MPa in figure 8 were made using a simple creep function in ABAQUS that is used to describe non-linear creep of metals at elevated temperatures. Under uniaxial tension, this takes the form

$$\dot{\epsilon}(t) = C \sigma_T^m t^p \quad (16)$$

where C , m and p are material parameters. When integrated, this gives the creep equation

$$\epsilon(t) = \epsilon_0 + \frac{C}{p+1} \sigma_T^m t^{p+1} \quad (17)$$

where $\epsilon_0 = D_0 \sigma_T$ is the initial strain produced by application of the stress. For any applied constant stress, a plot of $\log(\epsilon(t) - \epsilon_0)$ against $\log t$ should therefore be linear with a gradient $p+1$ independent of stress. The creep data at 10 MPa and 25 MPa are plotted on axes of log strain difference vs log time in figure 9. Best fits to the data are shown having the same gradient $p+1$. From the intercepts of the linear fits, it is possible

to determine values for the other parameters C and m in equation (16). Values for these parameters are given in table 4.

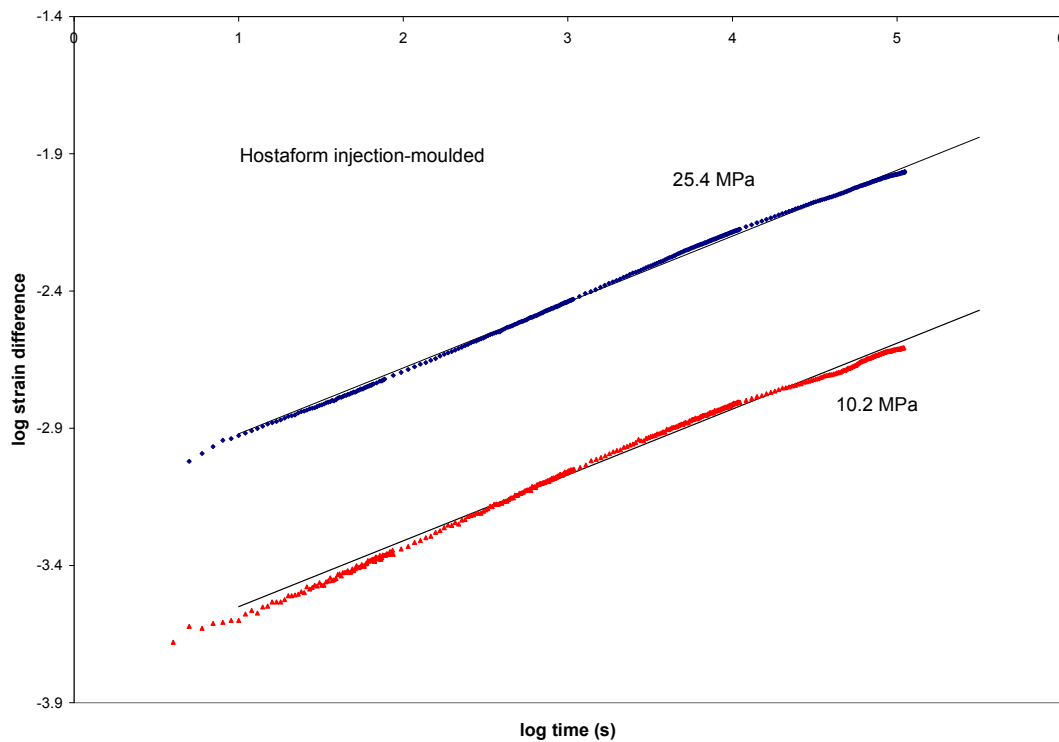


Figure 9 Plot of log strain difference against log time for the results of the first step of the test data shown in figures 7 and 8.

Table 4 Values for the parameters in equation (16) deduced from the results in figure 9.

$C \text{ (MPa}^{-m} \cdot \text{s}^{-(p+1)})$	m	p
$0.97 \cdot 10^{-6}$	1.59	-0.76

These parameters have been used in ABAQUS to predict the strains produced by the step loading and unloading histories used to obtain the experimental results shown in figures 7 and 8. The predictions are compared with the experimental results in figures 10 and 11. In figure 10, it can be seen that the ABAQUS predictions fall below measured values at creep times just after application of the steps in stress. In figure 11, it can be seen that the predictions are unable to capture the stress recovery following the reductions in stress. These results indicate that this approach to predicting the response to a time-varying stress, using a non-linear creep model, is not applicable to deformation in polymeric materials.

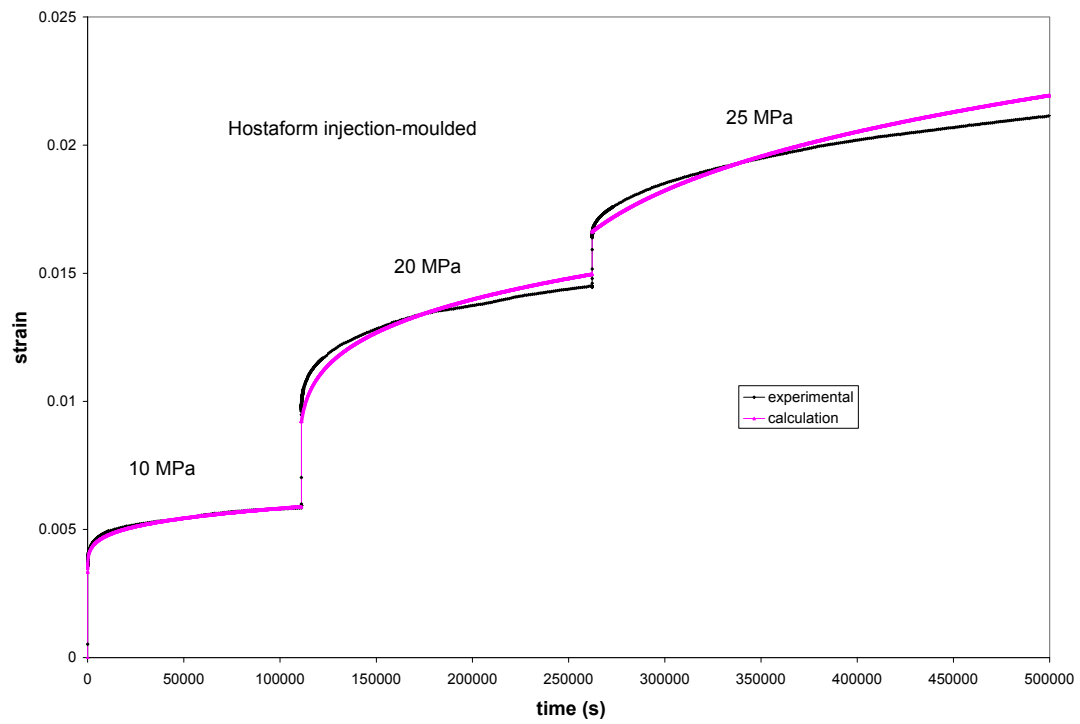


Figure 10 Comparison of measurements of strain under step loading in figure 7 with values calculated using a creep model in ABAQUS.

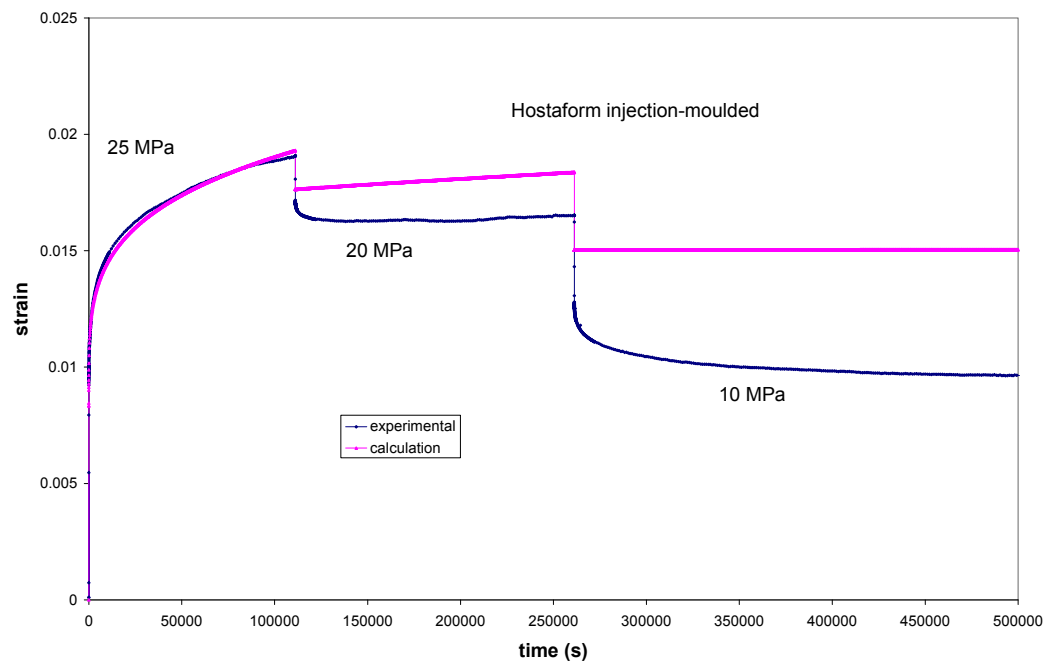


Figure 11 Comparison of measurements of strain for the step load removal test in figure 8 with values calculated using a creep model in ABAQUS.

5.3 PROPOSED APPLICATION OF THE CREEP MODEL FOR POLYMERS

With polymers, the conventional method for using creep data to predict the response to a stress that varies with time is through superposition [12]. Thus, under a stress history comprising step increases in a tensile stress where

$$\sigma = \sigma_0 \text{ for times } 0 \leq t \leq t_1$$

$$\sigma = \sigma_1 \text{ for times } t_1 < t \leq t_2$$

$$\text{and } \sigma = \sigma_2 \text{ for times } t > t_2$$

the total strain at times $t > t_2$ is given by adding the strain responses to each stress change as follows

$$\varepsilon(t) = \sigma_0 D(t) + (\sigma_1 - \sigma_0) D(t - t_1) + (\sigma_2 - \sigma_1) D(t - t_2) \quad (18)$$

Here $D(t-t_1)$ is the tensile creep compliance corresponding to the stress increment at time t_1 . The equation is valid for decreases as well as increases in stress. If the creep behaviour is linear, this process of summation is acceptable since the parameters in the creep model, in particular the time parameter, will be the same independent of stress. However, for non-linear behaviour, any change in stress will influence the subsequent contribution to the total strain from all previous stresses owing to the influence of the stress level on the time parameter t_0 . Whenever the stress changes, the time parameter $t_0(\sigma_i)$ in each of the compliance functions in equation (18) should take on the value corresponding to the new stress σ_i acting at that time. Thus to take proper account of changes in t_0 with stress, it is proposed that the compliance functions in equation (18) should be written

$$D(t) = D_0 + \Delta D \left[1 - \exp \left[- \left(\frac{t_1}{t_0(\sigma_0)} + \frac{t_2 - t_1}{t_0(\sigma_1)} + \frac{t - t_2}{t_0(\sigma_2)} \right)^n \right] \right]$$

$$D(t - t_1) = D_0 + \Delta D \left[1 - \exp \left[- \left(\frac{t_2 - t_1}{t_0(\sigma_1)} + \frac{t - t_2}{t_0(\sigma_2)} \right)^n \right] \right] \quad (19)$$

$$\text{and } D(t-t_2) = D_0 + \Delta D \left[1 - \exp\left(-\left(\frac{t-t_2}{t_0(\sigma_2)}\right)^n\right) \right]$$

The validity of equations (18) and (19) is now explored through comparisons of predictions using these equations with the experimental results in figures 7 and 8. Values for the model parameters recorded in table 3 were used, and the predictions are compared with experiment in figures 12 and 13, respectively. It can be seen that the agreement is very good, especially for the case where the load is increasing. In the stress removal test, figure 13, the predictions capture strain recovery, in contrast to the ABAQUS results in figure 11, although there is a significant discrepancy between predictions and measurement in the final step to 10 MPa. The reason for this is not clear at present and should be studied in future work.

5.4 GENERALISATION TO AN ARBITRARY STRESS OR STRAIN HISTORY.

The substitution of equation (19) into equation (18) gives the change of strain with time in response to discrete increases in stress at times $t = 0, t_1$, and t_2 . For a situation where the stress changes continuously with time, the summation in equation (18) can be replaced by an integral over incremental changes in stress. For each change in stress, there is a change in the parameter t_0 , so the summation involving terms in time in equation (19) must also be replaced by an integral. For an arbitrary stress history $\sigma(\tau)$, equation (18) becomes

$$\varepsilon(t) = \int_0^t \dot{\sigma}(\tau) \left[D_0 + \Delta D \left(1 - \exp\left(-\left(\int_{\tau}^t \frac{du}{t_0(\sigma(u))}\right)^n\right) \right) \right] d\tau \quad (20)$$

Here $\sigma(\tau)$ defines the stress history and $\sigma(u)$ defines the stress history for times between τ and t . A Fortran routine has been written to obtain solutions to equation (20) for the time-varying strain under prescribed stress histories. For the step loading and unloading stress histories depicted in figures 7 and 8, the solution to equation (20) is clearly identical to those given by equations (18) and (19) and shown in figures 12 and 13.

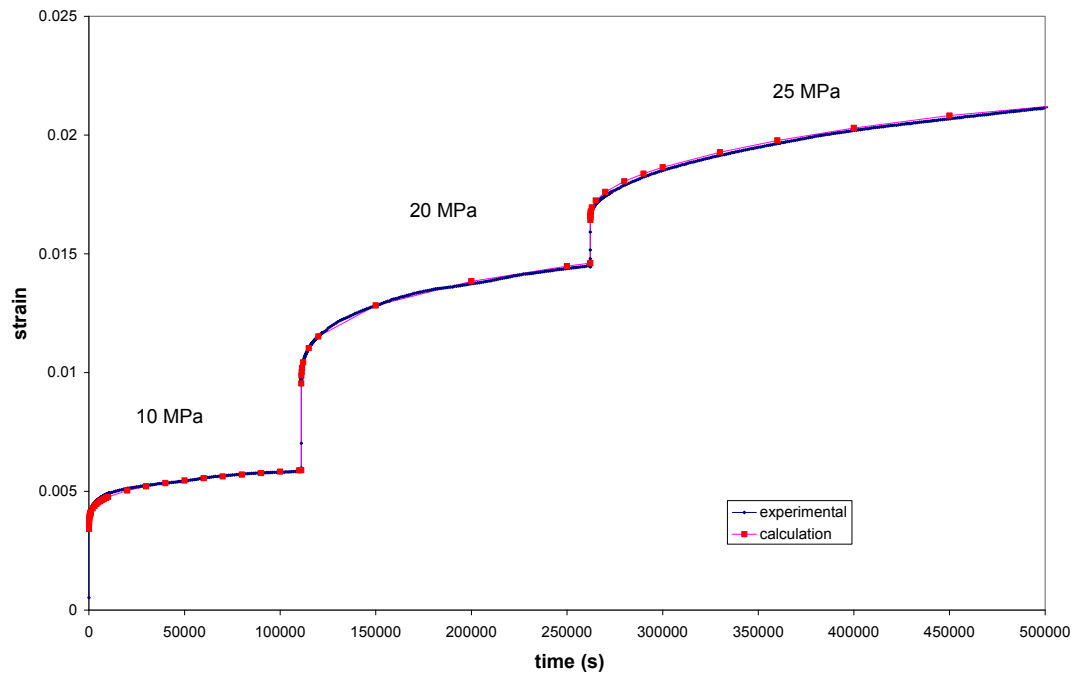


Figure 12 Comparison of measurements of strain under step loading in figure 7 with values calculated using equations (18) and (19).

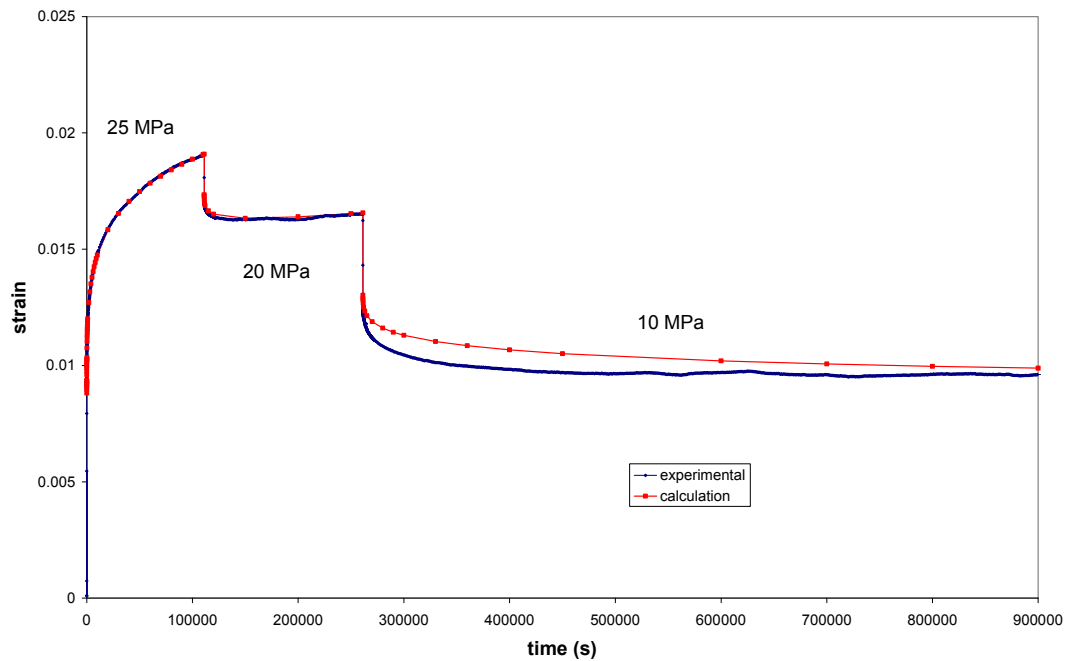


Figure 13 Comparison of measurements of strain for the step load removal test in figure 8 with values calculated using equations (18) and (19).

The above routine has been extended to give solutions for the stress $\sigma(t)$ under a prescribed strain history. This has been used for the more challenging task of predicting the results of a stress relaxation test, where the strain is held constant with time. The stress relaxation test was carried out on a specimen cut from the same material as used for the step loading experiments. A tensile strain was applied to the specimen by moving the cross-head of a universal testing machine at a speed of 1 mm/minute. When the strain reached 0.01, it was held constant for the remaining duration of the test. The strain history is shown in figure 14. The measured stress is compared with the solution obtained using equation (20) in figure 15. The agreement is excellent and gives further support to the validity of equation (20) for using creep data to predict the stress or strain response to known uniaxial strain or stress histories.

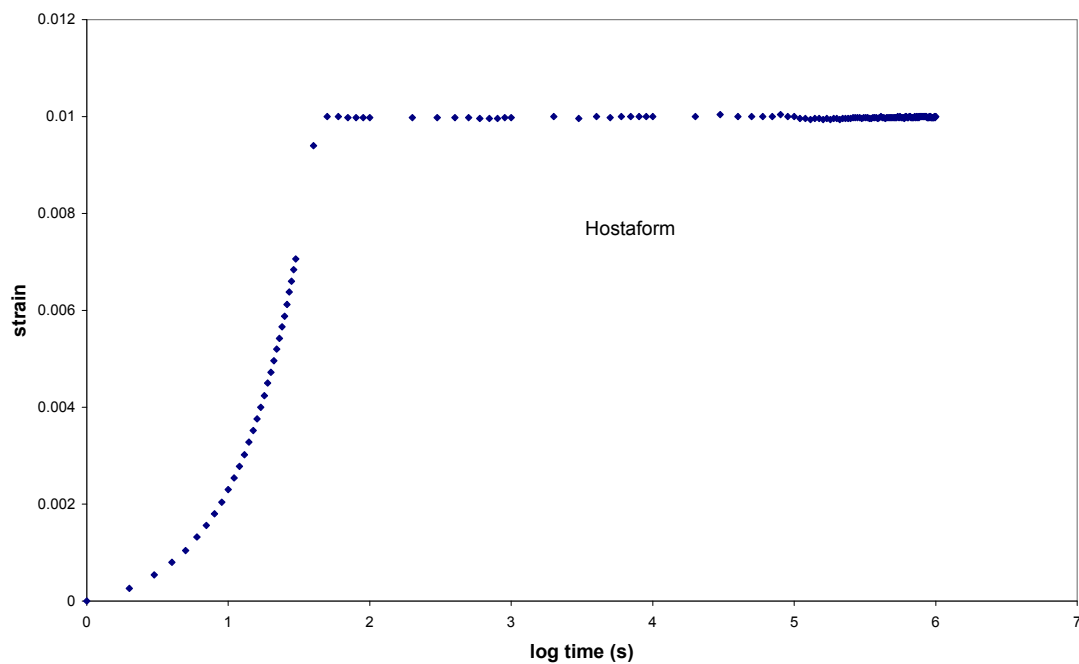


Figure 14 Strain history for the stress relaxation test on Hostaform.

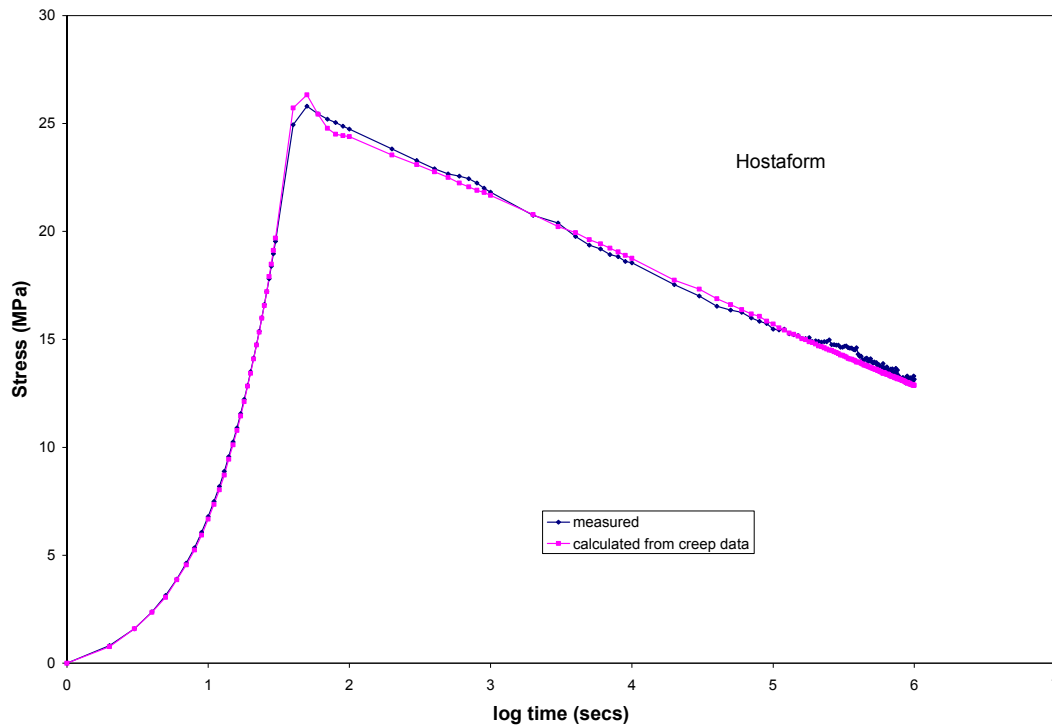


Figure 15 Comparison of measured values of stress against log time from the stress relaxation test on Hostaform with values calculated from creep data.

6 FURTHER WORK

The main objective of this work was to implement a valid creep model for polymers into a finite element system to enable stress analyses to be carried out on manufactured products that take account of the time-dependent properties of the polymer. This will require equation (20) to be generalised to 3 dimensions so that strain components are related to the components of a multiaxial stress. It will then be necessary to write code that will enable this tensor equation to be solved using a finite element system. The performance of the code should then be evaluated by comparing predictions of the deformation of some test specimens of simple geometry with measurements. Suitable examples might be a rectangular beam under flexure and the shear specimen reported in Section 3. If the flexure test is carried out under 3-point bending, the magnitude and the state of stress will vary through the specimen thickness. If the induced stresses are in the region of non-linear creep behaviour, there will be a redistribution of stress with time in all of the elements in the specimen. This will influence the predicted deformation of the specimen which can be compared with experiment. The situation is similar in the shear specimen where the stress magnitude and state will vary with

position in the plane of the specimen. With this test however, the stress state is predominantly shear, so comparisons of measured and predicted deformations here will yield a more in-depth evaluation of the method proposed in Section 4.4 for describing creep under a multiaxial stress. As part of this evaluation, the opportunity should be taken to explore the validity of assumptions of the Poisson's ratio and the bulk modulus being independent of time.

7 CONCLUSIONS

The tensile creep behaviour of the poly(oxymethylene) materials studied here has been modelled using a stretched exponential function with 4 material parameters. For creep times up to 10^6 s and at ambient temperature, 3 of the parameters are independent of stress. The fourth parameter, which represents a mean retardation time for the relaxation process responsible for creep, decreases with increasing stress and gives rise to non-linearity in creep behaviour. Non-linearity in creep behaviour in poly(oxymethylene) is evident at stress levels above 5 MPa.

Creep tests under uniaxial compression reveal that the magnitude of the retardation time parameter depends on the stress magnitude as well as the stress state. A relationship has been proposed relating the time parameter to an effective stress which is a linear combination of the shear and hydrostatic components of the stress. This has enabled expressions to be proposed relating components of strain and stress under a multiaxial creep stress. Since accurate measurements of creep behaviour have only been made under a uniaxial tensile stress, derivation of the expressions for creep under multiaxial stress has required the assumption that Poisson's ratio does not change with time. The validity of this assumption should be explored in further work which should also explore the assumption of a time independent bulk modulus.

To enable the creep model to be implemented in a finite element system, a theoretical expression has been proposed relating time-dependent stresses and strains in situations where the stress is not constant with time. This has been evaluated through comparisons of predicted and measured strains in experiments where the stress is increased or decreased in discrete steps and where the strain is held constant with time. Very good

agreement has been obtained which supports the further development of this expression for the preparation of code for implementing the creep model in a finite element system.

ACKNOWLEDGEMENT

This work was funded by the Department for Innovation Universities and Skills as part of the Characterisation Programme.

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