

THE COMPOSITE PERFORMANCE & DESIGN (CPD) PROGRAMME

1997 - 2000

A MATERIALS METROLOGY RESEARCH PROGRAMME FUNDED
BY THE DEPARTMENT OF TRADE AND INDUSTRY

Project CPD2 - Report 3
LIFE ASSESSMENT AND PREDICTION

**Model of Composite Degradation
due to
Environmental Damage**

L N McCartney

September 1998

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August 1998

MODEL OF COMPOSITE DEGRADATION DUE TO ENVIRONMENTAL DAMAGE

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ABSTRACT

This report describes a new model that is able to predict the time-dependent axial strength of a unidirectional fibre reinforced composite arising from environmental exposure. The principal effect of the environment is assumed to be the time-dependent strength of individual fibres arising from stress corrosion cracking of small defects within the fibres. The initial strength of the fibres is assumed to be governed by a Weibull distribution which defines an initial statistical size distribution of fibre defects. Defect growth in the fibres, arising from environmental exposure, is assumed to be governed by a law where the growth rate is controlled by the stress intensity factor for the defect.

The principal conclusion to be drawn from the preliminary results presented in this report, is that the time dependence of the axial properties of a unidirectional fibre reinforced glass composite, subject to environmental exposure under fixed load, can be predicted using a parallel bar model of the composite where interface bonding is neglected. The model enables the prediction of the stress history of the fibre stress in surviving fibres from the point of first loading to the occurrence of catastrophic failure. The fibre stress is found to be almost independent of the matrix properties.

The model can also be used to predict the time dependence of the residual strength of the composite; a property which does show some dependence on matrix properties. However, it has been shown that, when the residual strength is divided by the static strength, the resulting residual strength ratio is virtually independent of the matrix properties. It is concluded that the residual strength ratio for a unidirectional composite is predictable (and therefore measurable) from the static strength of the composite, and the time dependence of the residual strength of a loose bundle of fibres. This major result of the modelling, suggests an efficient method for predicting and measuring the time dependence of the residual axial strength of unidirectional composites.

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ISSN 1361-4061

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Date..... 28/9/98

Date..... 24/9/98

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Nomenclature :

a	size of fibre defect
A	area of cross section of an individual fibre
A_b	area fraction of fibres in composite
A_m	area fraction of matrix in composite
α	the ratio $V_m E_m / V_f E_f$
E_f	axial Young's modulus of the fibres
E_m	Young's modulus of the matrix
$E_b(t)$	time dependent axial modulus of the fibre bundle
$E_c(t)$	time dependent axial modulus of the composite
$\varepsilon(t)$	time dependent axial strain of composite, surviving fibres and matrix
F	fixed load applied to the composite
F_{max}	maximum load carried by composite during static strength test
F_s	applied load when fibre stress has value s during residual strength test
$\hat{F}$	dimensionless load applied to composite defined by (19)
$\hat{F}_s$	normalised value of F_s defined by (43)
$F_b(t)$	time dependent load carried by surviving fibres of the bundle
$k(t)$	time dependent parameter defined by (42)
K	stress intensity factor for fibre defect
K_{Ic}	fracture toughness for fibre material
λ	parameter defined by (15) associated with fibre defect growth
Λ	parameter associated with the fibre defect growth law
m	Weibull modulus of fibre strength distribution
n	exponent appearing in fibre defect growth law
N_0	initial number of surviving fibres in the composite
$N(t)$	number of fibres in the composite surviving at time t
$\hat{N}_f$	the ratio $N(t_f)/N_0$
s	value of fibre stress during a residual strength test
$\hat{s}$	normalised value of s defined by (40)
s_i	initial strength of fibres failing during residual strength test at time t
$S(t)$	residual strength of composite at time t
$\hat{S}(t)$	normalised residual strength of composite at time t , defined by (48)
$\sigma(t)$	axial stress in each surviving fibre at time t
$\hat{\sigma}$	normalised value of $\sigma(t)$ defined by (19)
$\sigma_{app}(t)$	applied axial stress at time t
$\sigma_i(t)$	initial strength of fibres that fail at time t
$\sigma_m(t)$	axial stress in matrix at time t
$\sigma_{max}(t)$	maximum axial stress in fibre just before failure during a residual strength test
$\hat{\sigma}_{max}(t)$	normalised value of $\sigma_{max}(t)$ defined by (47)
σ_0	scaling parameter for the Weibull distribution
t_f	time to failure for composite subject to a fixed effective applied stress
V_f	fibre volume fraction
V_m	matrix volume fraction
y	parameter for stress intensity factor that accounts for defect geometry

1. INTRODUCTION

The axial strength of unidirectional fibre reinforced composites is controlled by the strength of the fibres. In cross-ply laminates, the axial strength of the laminate is controlled to a large degree by the strength of the fibres in the 0° plies. Fibre strength is statistical in nature due to the presence of defects both on fibre surfaces, and in their interior. This aspect has been described in the review [1]. The effect of interface properties on axial strength are of secondary importance, and modelling their effect on axial strength requires the use of sophisticated stress transfer models and Monte Carlo simulation techniques; topics to be considered in projects CPD2 and CPD3. For unprotected glass fibres, it is well known (see for example [2]) that the environmental exposure of the composite leads to time-dependent reductions in fibre strength. The strength reduction of the fibres results because of the progressive growth of fibre defects caused by stress corrosion cracking at a microscopic level.

Environmental exposure can lead to a deterioration in interface properties. Given that the axial strength of a unidirectional composite is not affected to a great extent by interface properties, it is reasonable to assume, when modelling the axial behaviour of a unidirectional composite, that the interfaces in the composite following prolonged exposure (to the point of saturation) have no 'strength'. This enables a relatively simple approach to be taken that will provide good insight into the axial behaviour of a composite when exposed to an aggressive environment.

Because of the dominance of fibre behaviour, earlier modelling work applied to glass fibre composites [3, 4] regarded the unidirectional composite as a loose bundle of parallel fibres having equal length, so that the relatively low load carrying capacity of the matrix was ignored. The glass fibres in the bundle were assumed to be attached to rigid supports which were able to share the applied load equally between all surviving fibres. The objective of this report is to extend the existing loose bundle model so that the load carrying capacity of the matrix is taken into account when considering the axial behaviour of glass fibre reinforced composites subject to environmental exposure.

2. MODEL GEOMETRY

Consider a unidirectional fibre reinforced composite having a fibre volume fraction V_f and matrix volume fraction V_m such that $V_f + V_m = 1$. The composite has been wholly immersed in an aggressive environment for a sufficient time for the composite to be fully saturated. The application of axial load to the composite leads to the environmental growth of defects in the fibres; a phenomenon well known to afflict glass fibres [2]. The interfaces between the fibres and matrix are regarded as being significantly weakened by the environment to the extent that it can be assumed that the fibres and matrix behave independently in regions of the composite that are well away from the uniaxial loading mechanism where clamping effects become important. This assumption means that the composite can be modelled as a parallel two bar model, as shown in Figure 1. The fibres in the composite are regarded as acting as a loose bundle forming one bar of the model. The matrix material in the composite is considered as being gathered together to form the other bar of the model which is regarded as homogeneous, i.e. the bar is solid. When a fibre fails the load it carried is shared between the surviving fibres in the bundle and the matrix in such a way that all surviving fibres and matrix experience the same axial strain increment. The fibres are assumed to have the same length so that each surviving fibre has the same stress throughout the progressive failure process. The composite is subject to a fixed applied load

F for all times $t > 0$, where $t = 0$ corresponds to the time when the fixed load F is first applied. Environmental defect growth in the fibres leads to progressive fibre failure until the bundle collapses. It is assumed that bundle collapse corresponds to the catastrophic failure of the composite, i.e. the matrix strength is insufficient to maintain the load when all the fibres have failed. The objective is to develop the parallel bar model of a composite so that it can predict the dependence of composite life t_f on the fixed applied load F , and the dependence of the residual strength $F^*(t)$ of the composite on elapsed time t from the instant of first loading.

The behaviour of bundles of loose fibres subject to environmental degradation has been modelled by Kelly & McCartney [3], and McCartney [4] for the case when the load applied to the bundle is fixed in time. For the parallel bar model of the composite, which is subject to a fixed load F , the progressive failure of fibres in the bundle leads to a time dependence of both the effective bundle stiffness and the load applied to the fibre bundle. Thus the earlier modelling [3, 4] requires modification if it is to be applied to the prediction of the behaviour of a uniaxially loaded unidirectional composite material having weak interfaces.

3. BASIC MECHANICS FOR PARALLEL BAR MODEL OF COMPOSITE

The analysis of the parallel bar model shown in Figure 1 will neglect any axial thermal stresses arising from a mismatch of the thermal expansion coefficients of the fibres and the matrix. The area fraction of all fibres in the bundle is denoted by A_b , and that of the matrix is A_m . It follows that

$$V_f = \frac{A_b}{A_b + A_m}, \quad V_m = \frac{A_m}{A_b + A_m} = 1 - V_f. \quad (1)$$

The load applied to the fibre bundle at time t is denoted by $F_b(t)$, the stress in each surviving fibre being denoted by $\sigma(t)$. The cross-sectional area of each of the fibres in the bundle is denoted by A , and the axial modulus of each fibres is denoted by E_f , which is assumed to be time independent. The axial stress at time t in the matrix is denoted by $\sigma_m(t)$. The modulus of the matrix is denoted by E_m which is assumed to be independent of time. A time dependence could be included to account for visco-elastic effects, or for time-dependence arising from matrix ageing.

The axial strain in all surviving fibres of the bundle and the matrix has the same time dependent value that is denoted by $\epsilon(t)$. As thermal expansion mismatch effects are neglected it follows that

$$\epsilon(t) = \frac{\sigma(t)}{E_f} = \frac{\sigma_m(t)}{E_m}. \quad (2)$$

The balance of forces in the parallel bar model leads to the equilibrium relation

$$F_b(t) + A_m \sigma_m(t) = F. \quad (3)$$

The number of surviving fibres in the bundle at time t is denoted by $N(t)$ so that the load applied to the bundle at time t may be written

$$F_b(t) = N(t)A\sigma(t). \quad (4)$$

On substituting (4) into (3) followed by the elimination of $\sigma_m(t)$ using (2) it is easily shown that the number of fibres surviving at time t is related to the fibre stress $\sigma(t)$ through the following relation that quantitatively characterises the load sharing that occurs when fibres in the composite fail

$$\left[\frac{N(t)}{N_0} + \alpha \right] \sigma(t) = \frac{F}{A_b}, \quad \alpha = \frac{V_m E_m}{V_f E_f}, \quad (5)$$

where $N_0 = N(0)$ and $A_b = N_0 A$. This load sharing rule for a composite generalises a relation used when modelling a loose bundle of fibres subject to environmental degradation [3, 4] which is recovered from (5) on taking the limit $\alpha \rightarrow 0$.

It is useful to relate the number of fibres surviving in the bundle at time t to the effective axial modulus of the bundle $E_b(t)$. The effective stress applied to the bundle is defined by

$$\sigma_b(t) = \frac{F_b(t)}{A_b}, \quad (6)$$

and since the axial strain of the bundle and the individual fibres has the value $\varepsilon(t)$

$$\varepsilon(t) = \frac{\sigma(t)}{E_f} = \frac{\sigma_b(t)}{E_b(t)} = \frac{F_b(t)}{A_b E_b(t)} = \frac{N(t)}{N_0} \frac{\sigma(t)}{E_b(t)}, \quad (7)$$

where use has been made of (4) and (6). Clearly the effective axial modulus of the fibre bundle is given by

$$E_b(t) = \frac{N(t)}{N_0} E_f. \quad (8)$$

The effective stress σ_{app} applied to the composite is defined by

$$\sigma_{app} = \frac{F}{A_b + A_m}, \quad (9)$$

and it can be shown from (1) and (5), together with the fact that $A_b = N_0 A$, that

$$\sigma_{app}(t) = E_c(t) \varepsilon(t), \quad \text{where} \quad E_c(t) = V_f E_b(t) + V_m E_m, \quad (10)$$

and where $E_c(t)$ is the effective axial modulus of the composite defined by the rule of mixtures, as to be expected.

4. PREDICTING COMPOSITE DEGRADATION

The objective here is to show how the analysis of Kelly & McCartney [3], developed for loose bundles exposed to an aggressive environment, must be modified for application to a unidirectional composite having weak interfaces. The analysis is based on the assumption that the strength of individual fibres is determined by surface defects whose effective size and distribution along the fibre surface is statistically distributed.

4.1 Accounting for growth of fibre defects

Fibre failure is assumed to be governed by a Griffith type of failure criterion having the form

$$K^2 = y^2 \sigma^2 a = K_{lc}^2, \quad (11)$$

where K is an effective stress intensity factor for a fibre defect of effective size a subject to a fibre stress σ , K_{lc} is the effective fracture toughness of the fibre material, and where y is a dimensionless parameter designed to account for defect geometry. Exposure to an aggressive environment leads to defect growth when the fibre is under load. Such defect growth is assumed to be governed by a growth law of the form

$$\frac{da}{dt} = \Lambda K^n, \quad (12)$$

where Λ and n are material constants.

When a constant load is applied to a unidirectional composite, exposed to an aggressive environment to the point of saturation, the fibre defects grow in size according to the growth law (12) eventually leading to fibre failure when the failure criterion (11) is satisfied. Thus fibres progressively fail and the load carried by failed fibres is, for the parallel bar model under discussion, transferred to the surviving fibres and matrix using the load sharing rule (5). The stress in each fibre of the system is thus time dependent. It is useful to present here the relationship that determines the initial defect size $X_0(t)$ that requires a time t to grow to the critical size $a_c(t)$, at which the fibre stress is $\sigma(t)$, under the influence of a time dependent fibre stress history $\sigma(\tau)$; $0 < \tau < t$. The critical defect size at time t is predicted by (11) to be

$$a_c(t) = \left[\frac{K_{lc}}{y\sigma(t)} \right]^2, \quad (13)$$

and it can be shown on integrating (12) between the limits $X_0(t)$ and $a_c(t)$ that

$$X_0(t) = \frac{K_{lc}^2}{y^2} \left[\sigma^{n-2}(t) + (n-2) \lambda \int_0^t \sigma^n(\tau) d\tau \right]^{\frac{2}{2-n}}, \quad (14)$$

where

$$\lambda = \frac{1}{2} \Lambda K_{lc}^{n-2} y^2. \quad (15)$$

On using (11) it follows from (14) that the initial strength $\sigma_i(t)$ of the fibres, that fail at time t when their stress is $\sigma(t)$, is given by

$$\sigma_i(t) = \left[\sigma^{n-2}(t) + (n-2) \lambda \int_0^t \sigma^n(\tau) d\tau \right]^{\frac{1}{n-2}}. \quad (16)$$

The cross-sectional area of the sample of unidirectional composite is assumed to be large enough for there to be a very large number of fibres. It can then be assumed that the bundle of fibres used in the parallel bar model contains every possible fibre strength that can arise in the statistical distribution. It is assumed that the strength distribution of the

fibres is given by the two parameter Weibull distribution [5] so that, for a large bundle of N_0 fibres, the expected number of fibres N surviving when the stress in each fibre is σ is given by

$$N = N_0 \exp \left[- \left(\frac{\sigma}{\sigma_0} \right)^m \right], \quad (17)$$

where σ_0 is a scaling parameter that will depend on the length of the composite.

4.2 Prediction of static strength

It is useful to investigate the prediction of the static strength of a unidirectional composite assuming that the parallel bar model is valid. When using (17) in conjunction with (5) it is easily shown that the total load F carried by the composite, when the stress in surviving fibres has the value $\sigma(t)$, may be obtained from

$$\hat{F} = \hat{\sigma} \left[\alpha + e^{-\hat{\sigma}^m} \right], \quad (18)$$

where $\hat{F}$ and $\hat{\sigma}$ are a dimensionless normalised load and stress respectively defined by

$$\hat{F} = \frac{F}{N_0 \sigma_0 A}, \quad \hat{\sigma} = \frac{\sigma(t)}{\sigma_0}. \quad (19)$$

The static strength of the composite is the maximum value of the effective applied stress ($= F / (N_0 A)$) that can be carried by the composite. The maximum stress occurs when $\hat{F}$ has a local maximum when plotted as a function of $\hat{\sigma}$. The maximum fibre stress $\sigma_{\max}$ satisfies the transcendental equation

$$m \hat{\sigma}_{\max}^m = 1 + \alpha e^{\hat{\sigma}_{\max}^m}. \quad (20)$$

The corresponding static strength $F_{\max} / (N_0 A)$ for the composite is then obtained using

$$\frac{F_{\max}}{N_0 \sigma_0 A} = \hat{F}_{\max} = \hat{\sigma}_{\max} \left[\alpha + e^{-\hat{\sigma}_{\max}^m} \right] = \alpha \frac{m \hat{\sigma}_{\max}^{m+1}}{m \hat{\sigma}_{\max}^m - 1}, \quad (21)$$

which is consistent with the known result for a loose bundle [3] in the limit $\alpha \rightarrow 0$. The equation (20) governing the maximum fibre stress does not always have a solution as is seen by examining the form of the LHS and RHS of (20). On letting $x = \hat{\sigma}^m$ the critical conditions defining the limit of solutions to (20) may be written

$$mx = 1 + \alpha e^x, \quad m = \alpha e^x. \quad (22)$$

These conditions correspond to the touching of the curves $y = mx$ and $y = \alpha e^x$. It is easily seen that the critical condition occurs when

$$x = \ln \frac{m}{\alpha} = \frac{1+m}{m} = \hat{\sigma}^m. \quad (23)$$

It is concluded that the equation (20) has a solution only if

$$\alpha < m e^{-\left(1 + \frac{1}{m}\right)} . \quad (24)$$

If this condition is not satisfied then it is deduced that the fibres progressively fail until there is just one surviving fibre which will then fail, i.e. the bundle does not suddenly collapse. The value of the Weibull modulus m for fibres of interest is usually such that the condition (24) is satisfied so that bundle collapse is always expected in practice.

4.3 Prediction of progressive damage

At time t the fibres that survive in the composite are those whose initial strengths were greater than $\sigma_i(t)$ defined by (16). It then follows from (17) that the expected number of surviving fibres $N(t)$ at time t is given by

$$\frac{N(t)}{N_0} = \hat{N}(t) = \exp \left[- \left(\frac{\sigma_i(t)}{\sigma_0} \right)^m \right] . \quad (25)$$

On substituting (16) in (25)

$$\left(\ln \frac{1}{\hat{N}(t)} \right)^{\frac{n-2}{m}} = \hat{\sigma}^{n-2}(t) + (n-2)\eta \int_0^t \hat{\sigma}^n(\tau) d\tau , \quad (26)$$

where

$$\eta = \lambda \sigma_0^2 = \frac{1}{2} \Lambda K_{lc}^{n-2} y^2 \sigma_0^2 , \quad (27)$$

and where use has been made of the definitions (19), which when applied to the load sharing rule (5) lead to

$$\hat{N}(t) = \frac{\hat{F}}{\hat{\sigma}(t)} - \alpha . \quad (28)$$

On substituting (28) into (26)

$$\left(\ln \frac{\hat{\sigma}(t)}{\hat{F} - \alpha \hat{\sigma}(t)} \right)^{\frac{n-2}{m}} = \hat{\sigma}^{n-2}(t) + (n-2)\eta \int_0^t \hat{\sigma}^n(\tau) d\tau . \quad (29)$$

On differentiating (29) with respect to t , the following differential equation governing the time dependence of the normalised fibre stress $\hat{\sigma}(t)$ is obtained

$$\left[\frac{1}{m} \frac{\hat{F}}{\hat{F} - \alpha \hat{\sigma}(t)} \left(\ln \frac{\hat{\sigma}(t)}{\hat{F} - \alpha \hat{\sigma}(t)} \right)^{\frac{n-m-2}{m}} - \hat{\sigma}^{n-2}(t) \right] \frac{d\hat{\sigma}(t)}{d(\eta t)} = \hat{\sigma}^{n+1}(t) . \quad (30)$$

This differential equation is solved by standard numerical techniques [6] subject to the initial condition

$$\hat{\sigma}(0) = s_0, \quad (31)$$

where s_0 is the solution of the transcendental equation

$$\hat{F} = s_0 \left[\alpha + e^{-s_0^m} \right], \quad (32)$$

corresponding to (18), that must be solved numerically.

4.4 Predicting the failure stress and time to failure

The structure of the differential equation is such that $d\hat{\sigma}/dt \rightarrow \infty$ when $\hat{\sigma}(t) \rightarrow \hat{\sigma}_f$, occurring when $t \rightarrow t_f$, where

$$\frac{1}{m} \frac{\hat{F}}{\hat{F} - \alpha \hat{\sigma}_f} \left(\ln \frac{\hat{\sigma}_f}{\hat{F} - \alpha \hat{\sigma}_f} \right)^{\frac{n-m-2}{m}} = \hat{\sigma}_f^{n-2}. \quad (33)$$

The stress $\hat{\sigma}_f$ in the surviving fibres when the composite fails can thus be determined using numerical methods without having to solve the differential equation (30). It should be noted that when $\hat{F} = \hat{F}_{\max}$ the solution of (33) is given by $\hat{\sigma}_f = \hat{\sigma}_{\max}$ where $\hat{F}_{\max}$ and $\hat{\sigma}_{\max}$ are given by (21) and (20) respectively. The time to failure for the composite is denoted by t_f .

The number of surviving fibres just before composite failure is obtained using (17) and is given by

$$\frac{N(t_f)}{N_0} = \hat{N}_f = e^{-\hat{\sigma}_f^m}. \quad (34)$$

The transcendental equation (33), that usually must be solved numerically, involves the dimensionless loading parameter $\hat{F}$ in a complicated way. It is useful to unravel the dependence on this parameter by expressing (33) in terms of N_f as follows

$$\hat{F}^{n-2} = \frac{1}{m} \frac{[\hat{N}_f + \alpha]^{n-1}}{\hat{N}_f} \left(\ln \frac{1}{\hat{N}_f} \right)^{\frac{n-m-2}{m}}. \quad (35)$$

Having solved (35) to find $\hat{N}_f$ using numerical methods, the normalised failure stress in the fibres is obtained, on making use of (5), from the relation

$$\hat{\sigma}_f = \frac{\hat{F}}{\hat{N}_f + \alpha}. \quad (36)$$

The time to failure t_f can be predicted only by solving the differential equation (30) in the normalised stress range $s_0 \leq \hat{\sigma}(t) \leq \hat{\sigma}_f$.

4.5 Predicting residual strength

A key requirement concerning the effects of environment on composite degradation is the prediction of the time dependence of the residual strength of a composite. This has already been considered for the case of a loose bundle of fibres [4]. The objective now is to extend the analysis, and simplify it as far as is possible, so that the residual strength of a unidirectional composite with weak interfaces can be predicted. After an elapsed time t from the application of a fixed load F , the load is instantaneously increased until the composite fails catastrophically. Just before the load is suddenly increased the stress in the surviving fibres has the value $\sigma(t)$ and at any stage during the subsequent instantaneous load increase the value of the stress in the fibres is denoted by s . When the fibre stress has the value s the critical defect size has the following value specified by (11)

$$a_c^* = \left(\frac{K_{Ic}}{ys} \right)^2. \quad (37)$$

It is necessary to calculate the original size X^* of the critical defect using (11) and (12). It can be shown that

$$(X^*)^{\frac{2-n}{2}} = (a_c^*)^{\frac{2-n}{2}} + \frac{1}{2} \Lambda(n-2) y^n \int_0^t \sigma^n(\tau) d\tau. \quad (38)$$

On using (11) the initial strength of the fibres that are critical at time t when the fibre stress has the value s is denoted by s_i and is given, on using (38), by the relation

$$\hat{s}_i^{n-2} = \hat{s}^{n-2} + (n-2)\eta \int_0^t \hat{\sigma}^n(\tau) d\tau, \quad (39)$$

where use has been made of (27) and where

$$\hat{s}_i = \frac{s_i}{\sigma_0}, \quad \hat{s} = \frac{s}{\sigma_0}. \quad (40)$$

On using (29) the relation (39) may be written in the form

$$\hat{s}_i^{n-2} = \hat{s}^{n-2} - k(t), \quad (41)$$

where

$$k(t) = \hat{\sigma}^{n-2}(t) - \left[\ln \frac{\hat{\sigma}(t)}{\hat{F} - \alpha \hat{\sigma}(t)} \right]^{\frac{n-2}{m}}. \quad (42)$$

The load applied to the composite F_s , when the fibre stress has the value s , is obtained from (19) and (28) so that

$$\frac{F_s}{N_0 \sigma_0 A} = \hat{F}_s = \hat{s} [\alpha + \hat{N}_s] , \quad (43)$$

where $\hat{F}_s$ is the normalised applied load and where $\hat{N}_s$ is the normalised number of surviving fibres when the load on the composite is such that the fibre stress has the value s . It follows from (17) that

$$\hat{N}_s = e^{-\hat{s}_1^m} . \quad (44)$$

On substituting (44) into (43) the following expression is derived for the normalised load applied to the composite during a residual strength test

$$\hat{F}_s = \hat{s} [\alpha + e^{-\hat{s}_1^m}] . \quad (45)$$

The residual strength $S(t)$ of the composite at time t is the maximum value of $F_s/(N_0 A)$ when s is varied, or alternatively the maximum value of $\hat{F}_s$ when $\hat{s}$ is varied. Noting that $k(t)$ is independent of $\hat{s}$, the maximum value of $\hat{F}_s$ occurs when $\hat{s}_1 = x(t)$ which satisfies the transcendental equation

$$1 + \alpha e^{x^m(t)} = m x^m(t) \left(1 + \frac{k(t)}{x^{n-2}(t)} \right) . \quad (46)$$

On using (41) the stress $\sigma_{\max}(t)$ in the surviving fibres just before the composite fails during a residual strength test is obtained from

$$\frac{\sigma_{\max}(t)}{\sigma_0} = [x^{n-2}(t) + k(t)]^{\frac{1}{n-2}} = \hat{\sigma}_{\max}(t) . \quad (47)$$

It then follows from (41) and (45) that the residual strength of the composite $S(t)$ is obtained using

$$\frac{S(t)}{\sigma_0} = \hat{\sigma}_{\max}(t) [\alpha + e^{-x^m(t)}] = \hat{S}(t) . \quad (48)$$

When $t = 0$ it can be shown using (29) that $k(0) = 0$ in which case the transcendental equation (46) reduces to the form (20) which needs to be solved when calculating the static strength of the composite.

5. PRELIMINARY RESULTS

In order to assess the properties of the model some preliminary predictions have been made to illustrate the principal characteristics. There are four parameters that need to be given in order to obtain preliminary predictions :

1. the Weibull exponent m characterising the strength distribution of the fibres before environmental exposure. This parameter, which often has values in the range 4 - 8, appears in the relation (17) defining the expected number of fibre failures for a given fibre stress. Most predictions will assume that $m = 8$, although lifetime predictions will consider the case when $m = 4$. The value of m is usually obtained from single fibre strength tests,
2. the exponent n appearing in the defect growth law (12). This parameter, which usually has values in the range 3 - 30, is obtained from stress corrosion cracking tests carried out using monolithic glass testpieces. Predictions of behaviour will be made for the three values $n = 3, 15$ and 30 ,
3. the ratio α defined by (5) which takes account of the properties of the fibre and matrix, and also of the volume fraction. For many glass fibre composites of interest the value of α lies in the range 0 - 0.1. Predictions will be made for the three values $\alpha = 0, 0.05$ and 0.1 . When $\alpha = 0$ the composite is being modelled in exactly the same way as a loose bundle of fibres,
4. the level of loading applied to the composite where the model assumes that the ratio F/F_m is given where F is the axial load applied to the composite and F_m is the static strength of the composite, i.e. the strength before environmental exposure. The value of the ratio F/F_m always lies in the range 0 - 1.

The Euler/Richardson solution technique [6] is used to solve the differential equation (30) where the dimensionless time ηt is regarded as the unknown function of $\hat{\sigma}$. The initial condition is specified by (31) and (32) and the range $s_0 \leq \hat{\sigma} \leq \hat{\sigma}_f$ is subdivided into 100 equal intervals when solving the differential equation. The upper bound $\hat{\sigma}_f$ is determined by the relations (35) and (36). Figure 2 shows the result of solving the differential equation (30) to find the dimensionless lifetime ηt_f for the case when $\alpha = 0.1$ and $n = 15$ for various values of loading parameter F/F_m , and for values $m = 4$ and 8 of the Weibull exponent. The parameter η is defined by (27). It is seen that as F/F_m tends to unity the lifetime tends to zero (the logarithm tending to $-\infty$). Also, it is clear that the effect of varying the Weibull exponent in the range $4 \leq m \leq 8$ leads to only slight differences in predictions. For this reason all further predictions assume the single value $m = 8$.

For the cases $m = 8$, $F/F_m = 0.6$ and $n = 3, 15, 30$, Figures 3-5 show respectively the predictions for the normalised fibre stress $\hat{\sigma}$, defined by (19), as a function of elapsed time ratio t/t_f . Each figure shows predictions for the three values $\alpha = 0, 0.05$ and 0.1 . On comparing Figures 3-5 it is seen that increasing the value of the parameter n leads to larger values for the normalised fibre stress as composite failure is approached. In addition, larger values of n lead to a more sudden failure. The effect of varying the parameter α is not very significant, i.e. the matrix is not having much effect on the stress history of the fibres in the composite.

For the cases $m = 8$, $F/F_m = 0.6$ and $n = 3, 15, 30$, Figures 6-8 show respectively the predictions for the normalised residual strength $\hat{S}$, defined by (48), as a function of elapsed time ratio t/t_f . Each figure shows predictions for the three values $\alpha = 0, 0.05$ and 0.1 . On comparing Figures 6-8 it is seen that increasing the value of the parameter n leads to curves (Figure 6) which vary from almost linear having slightly different gradients for the three values of α , to curves (Figure 8) which are almost parallel for the three values of α , but which exhibit a sudden loss of strength as $t/t_f \rightarrow 1$. The effect of varying the parameter α

is significant, i.e. the matrix is having an effect on the time dependence of the residual strength of the composite.

On re-plotting the results shown in Figures 6-8, so that the residual strength at time t is normalised by dividing the residual strength by the static strength of the composite, it is seen from Figures 9-11 that there is hardly any dependence of the residual strength ratio on the parameter α . This result is found for all of the values of $n = 3, 15$ and 30 .

While the model described here has been developed for unidirectional composites having very weak interfaces arising from environmental exposure, the model can be used to take account, in an approximate way, of the effects of strong fibre/matrix interfaces. When the interface is strong the effects of fibre fracture remain local to the broken fibre. One way of dealing with this is to model the unidirectional composite as a series of much shorter two bar models. The length of each two bar system would be related to the stress transfer length for isolated fibre fractures in a unidirectional composite, which is usually between three and ten fibre diameters. The only parameter that would change is the scaling parameter σ_0 appearing in (17) which appears in the Weibull distribution. The assumption that the number of fibres is large enough for the relation (17) to be valid means that the behaviour of each two bar model in the series is virtually identical; a situation that is not particularly realistic. An alternative approach is to adopt a Monte Carlo modelling approach to the prediction of the failure of composite materials, and to modify the methodology to take account of the effect of environmental exposure. This will be the subject of a future study.

6. CONCLUSION

The principal conclusion to be drawn from the preliminary results presented in this report, is that the time dependence of the axial properties of a unidirectional fibre reinforced glass composite, subject to environmental exposure under fixed load, can be predicted using a parallel bar model of the composite where interface bonding is neglected. The model enables the prediction of the stress history of the fibre stress in surviving fibres from the point of first loading to the occurrence of catastrophic failure. The fibre stress is found to be almost independent of the matrix properties.

The model can also be used to predict the time dependence of the residual strength of the composite; a property which does show some dependence on matrix properties. However, it has been shown that, when the residual strength is divided by the static strength, the resulting residual strength ratio is virtually independent of the matrix properties. It is concluded that the residual strength ratio for a unidirectional composite is predictable (and therefore measurable) from the static strength of the composite, and the time dependence of the residual strength of a loose bundle of fibres. This major result of the modelling, suggests an efficient method for predicting and measuring the time dependence of the residual axial strength of unidirectional composites.

ACKNOWLEDGEMENT

The report was prepared as part of the research undertaken for the Department of Trade and Industry funded project on "Composites Performance and Design - Life Assessment and Prediction".

REFERENCES

1. Broughton, W.R. and McCartney, L.N., 'Predictive models for assessing long-term performance of polymer matrix composites', NPL Report CMMT(A)95, April 1998.
2. Metcalfe, A.G., Gulden, M.E. and Schmitz, G.K., *Glass technology*, 12, (1971), 15-23.
3. Kelly A. and McCartney, L.N., 'Failure by stress corrosion of bundles of fibres', *Proceedings of the Royal Society of London*, A374, (1981), 475-489.
4. McCartney, L.N., 'Time dependent strength of large bundles of fibres loaded in corrosive environments', *Fibre Science & Technology*, 16, (1982), 95-109.
5. Weibull, W., 'A statistical distribution function of wide applicability', *Journal of Applied Mechanics*, 19, (1951), 293-297.
6. Churchhouse, R.F., 'Handbook of Applicable Mathematics', Vol. 3 on 'Numerical Methods', series edited by W Ledermann, John Wiley and Sons, Chichester - New York - Brisbane - Toronto, 1981, pp. 319-321.

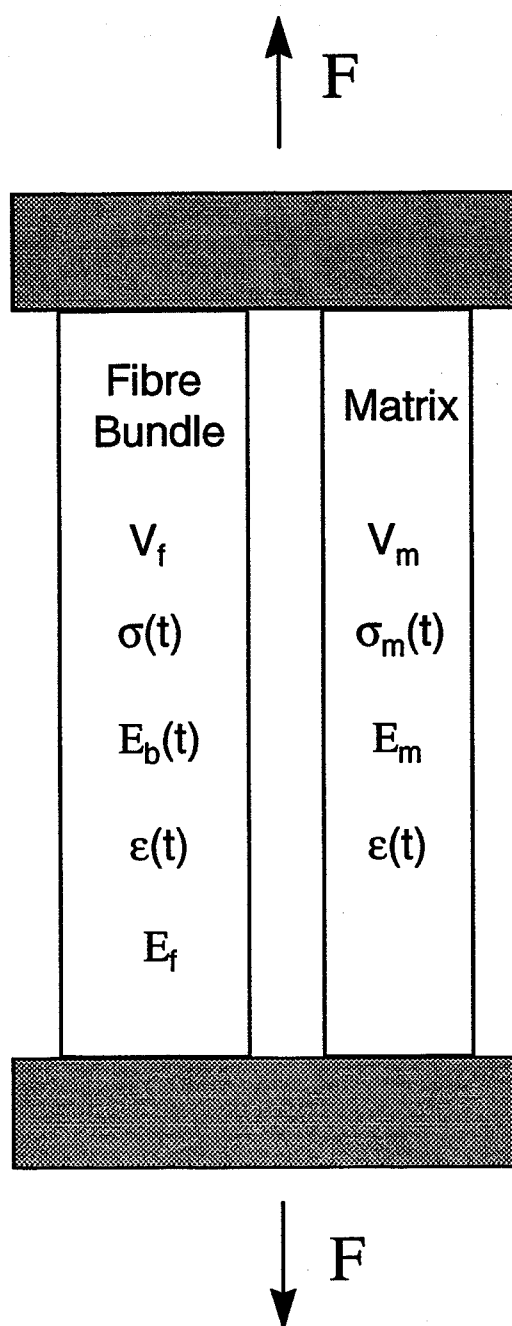


Figure 1 : Schematic diagram of the parallel bar model of a unidirectional composite that is used to predict the effects of environmental exposure on axial composite properties.

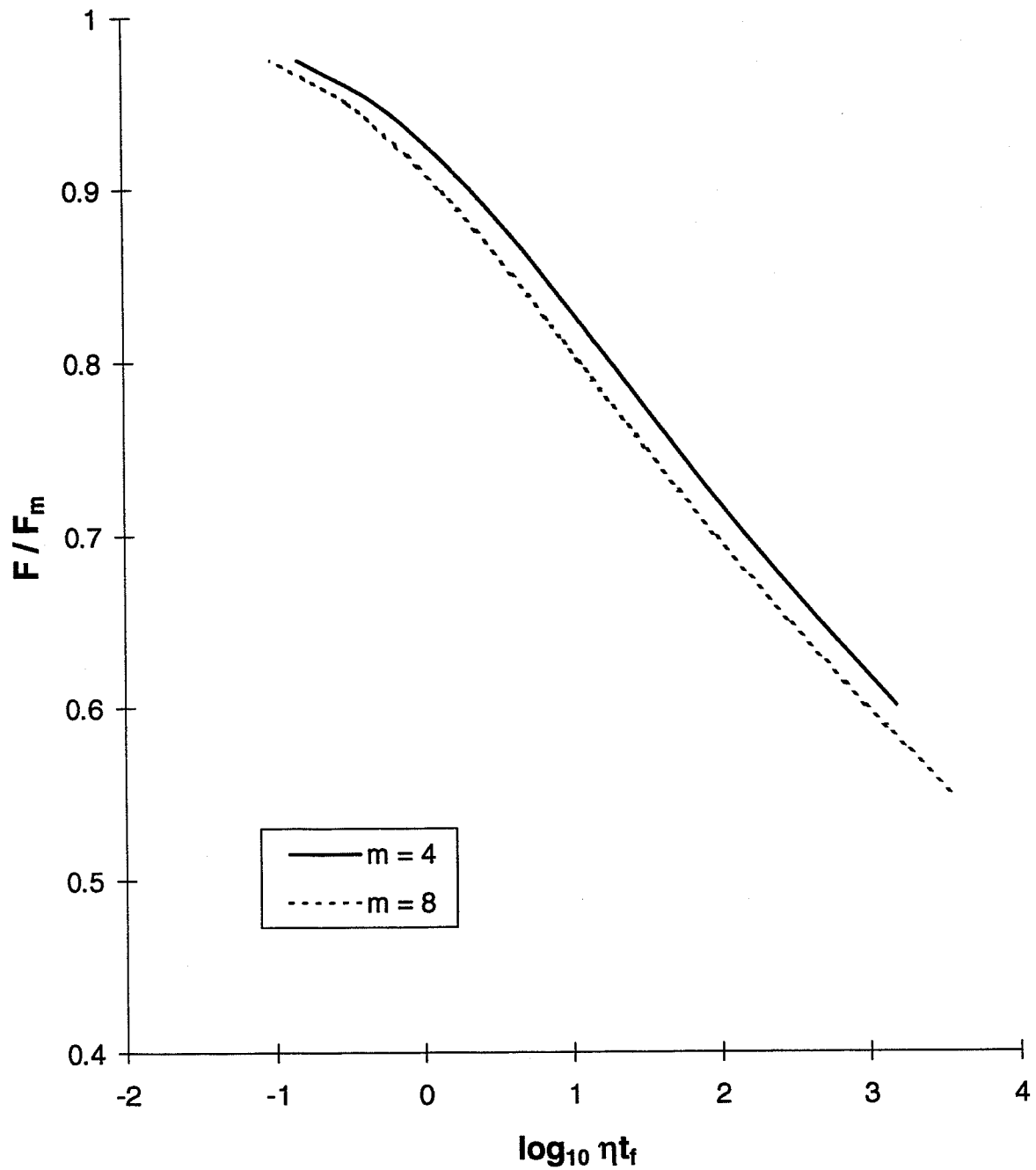


Figure 2 : The dependence of dimensionless composite lifetime (ηt_f) on the axial loading parameter (F/F_m) for the case when $\alpha = 0.1$ and $n = 15$.

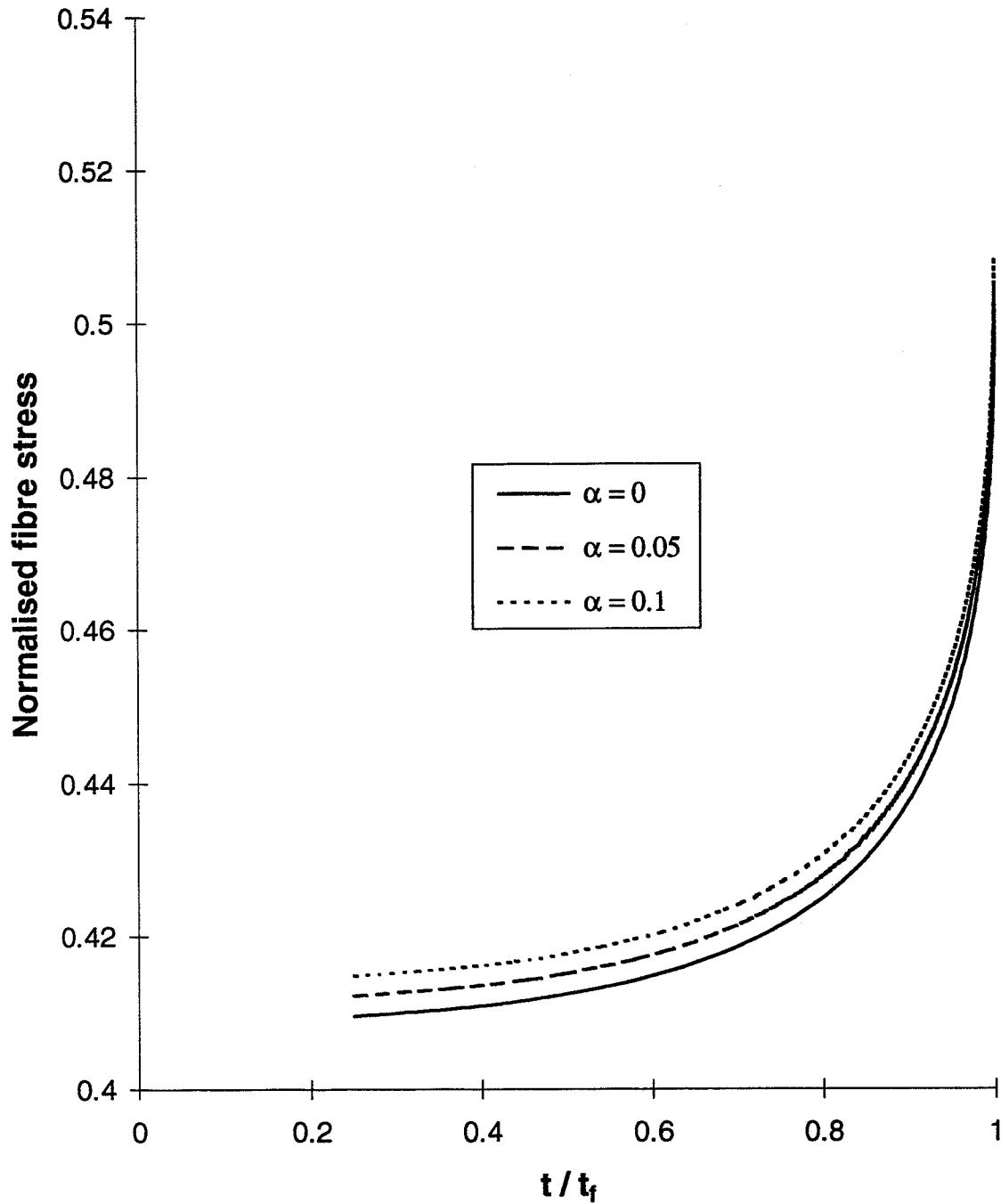


Figure 3 : The dependence of the normalised fibre stress ($\hat{\sigma}$) on elapsed time ratio (t/t_f) for the case when $F/F_m = 0.6$, $m = 8$ and $n = 3$.

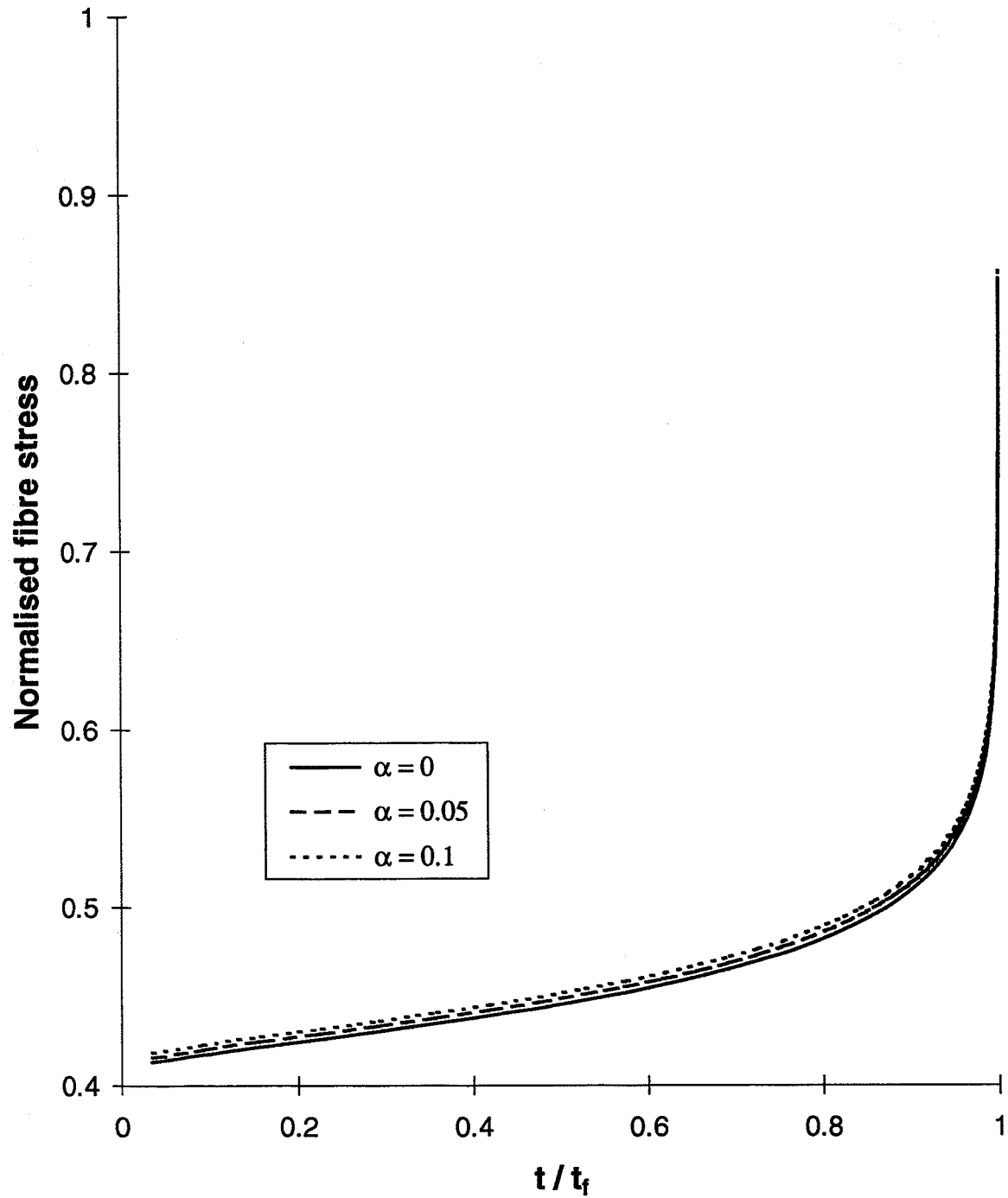


Figure 4 : The dependence of the normalised fibre stress ($\hat{\sigma}$) on elapsed time ratio (t/t_f) for the case when $F/\bar{F}_m = 0.6$, $m = 8$ and $n = 15$.

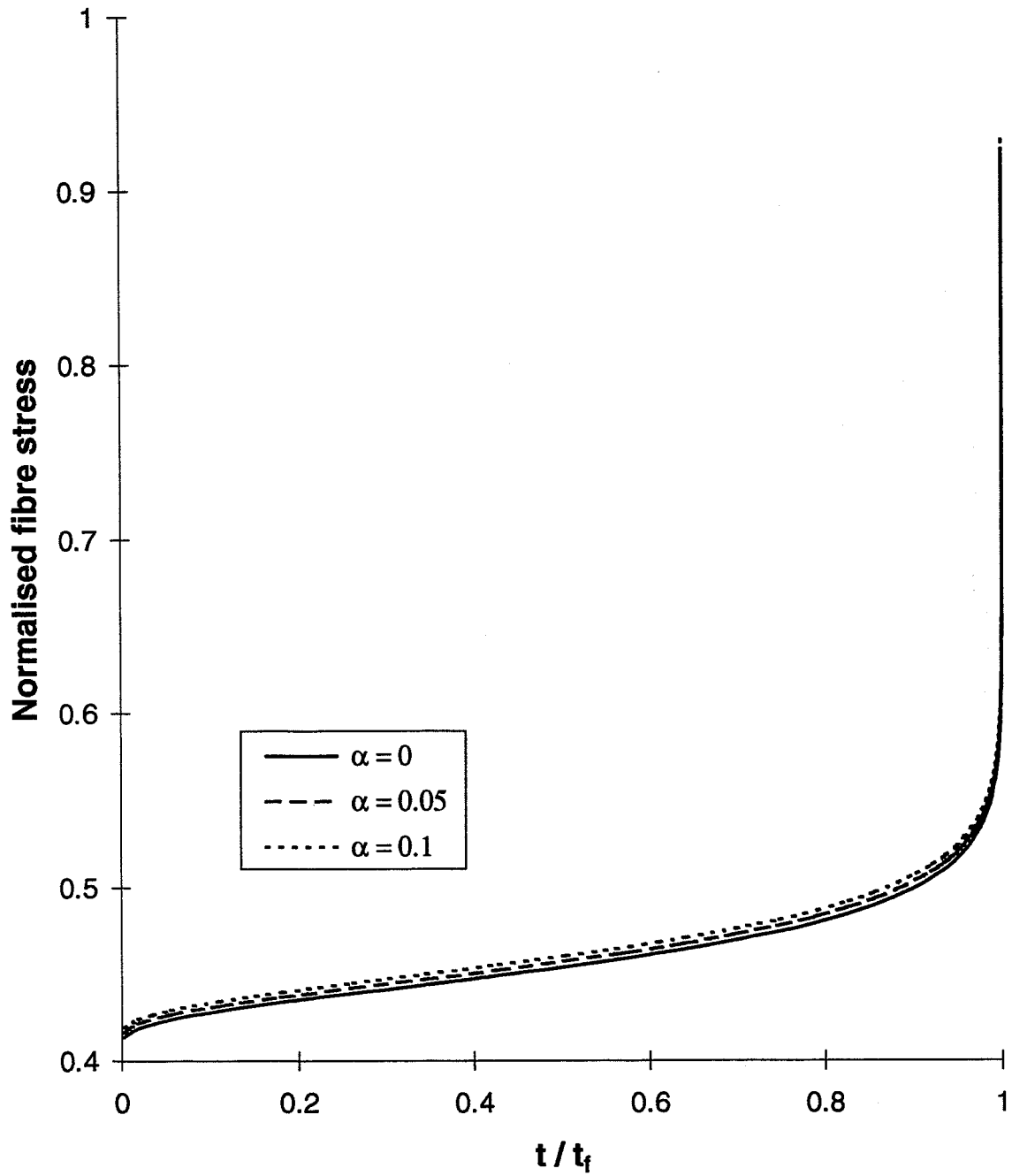


Figure 5 : The dependence of the normalised fibre stress ($\hat{\sigma}$) on elapsed time ratio (t/t_f) for the case when $F/F_m = 0.6$, $m = 8$ and $n = 30$.

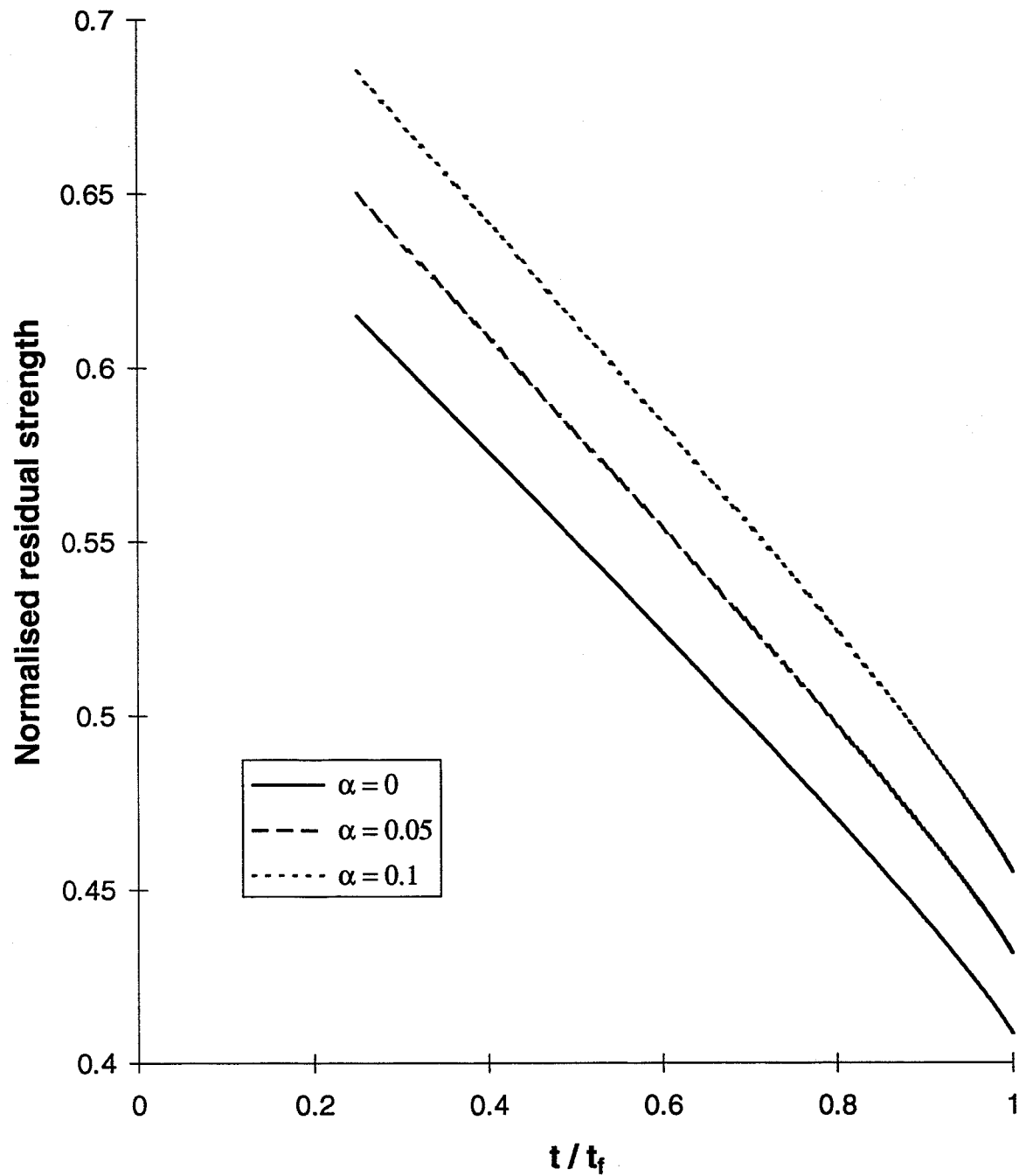


Figure 6 : The dependence of the normalised residual strength ($\hat{S}$) on elapsed time ratio (t/t_f) for the case when $F/F_m = 0.6$, $m = 8$ and $n = 3$.

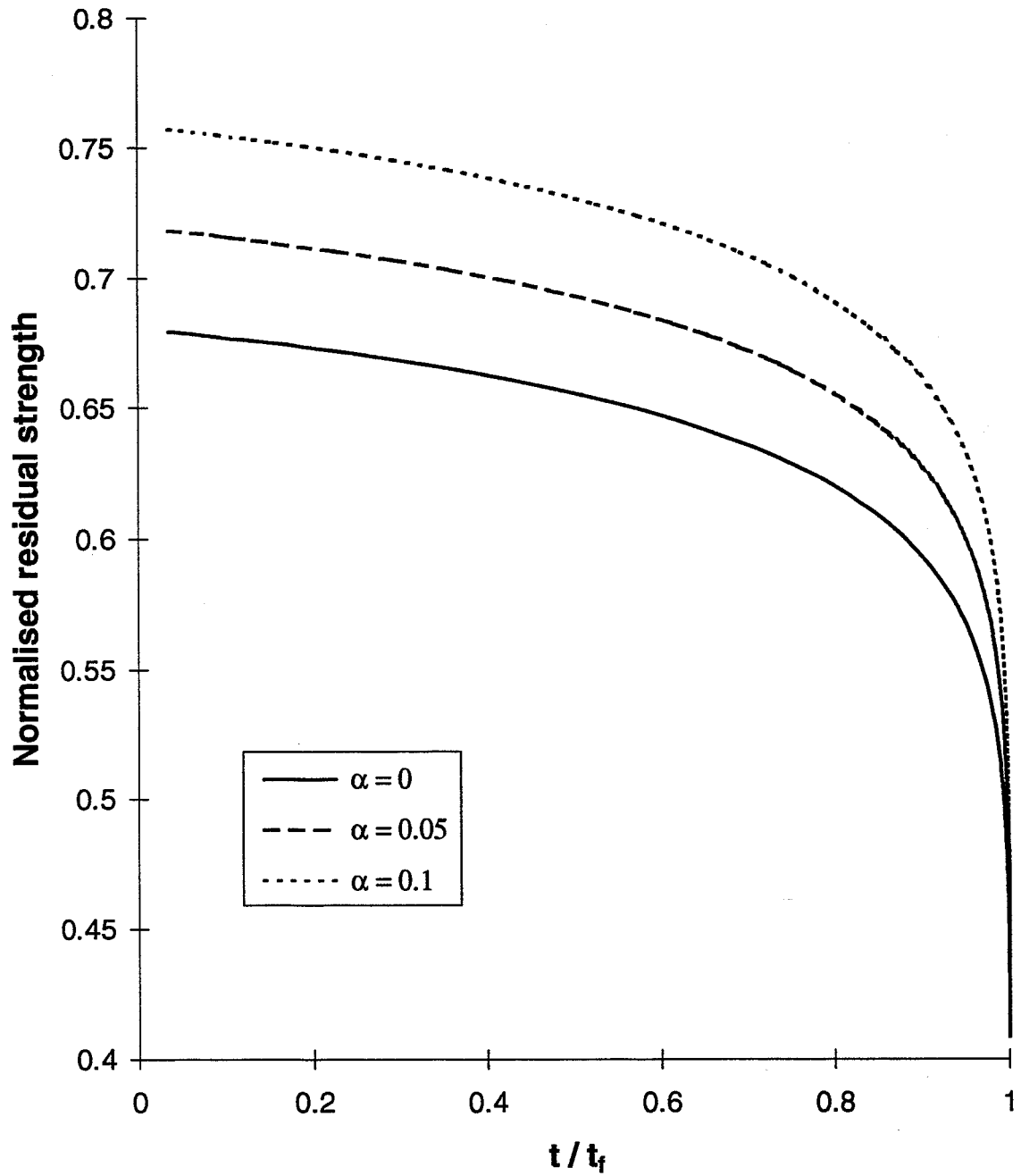


Figure 7 : The dependence of the normalised residual strength ($\hat{S}$) on elapsed time ratio (t/t_f) for the case when $F/F_m = 0.6$, $m = 8$ and $n = 15$.

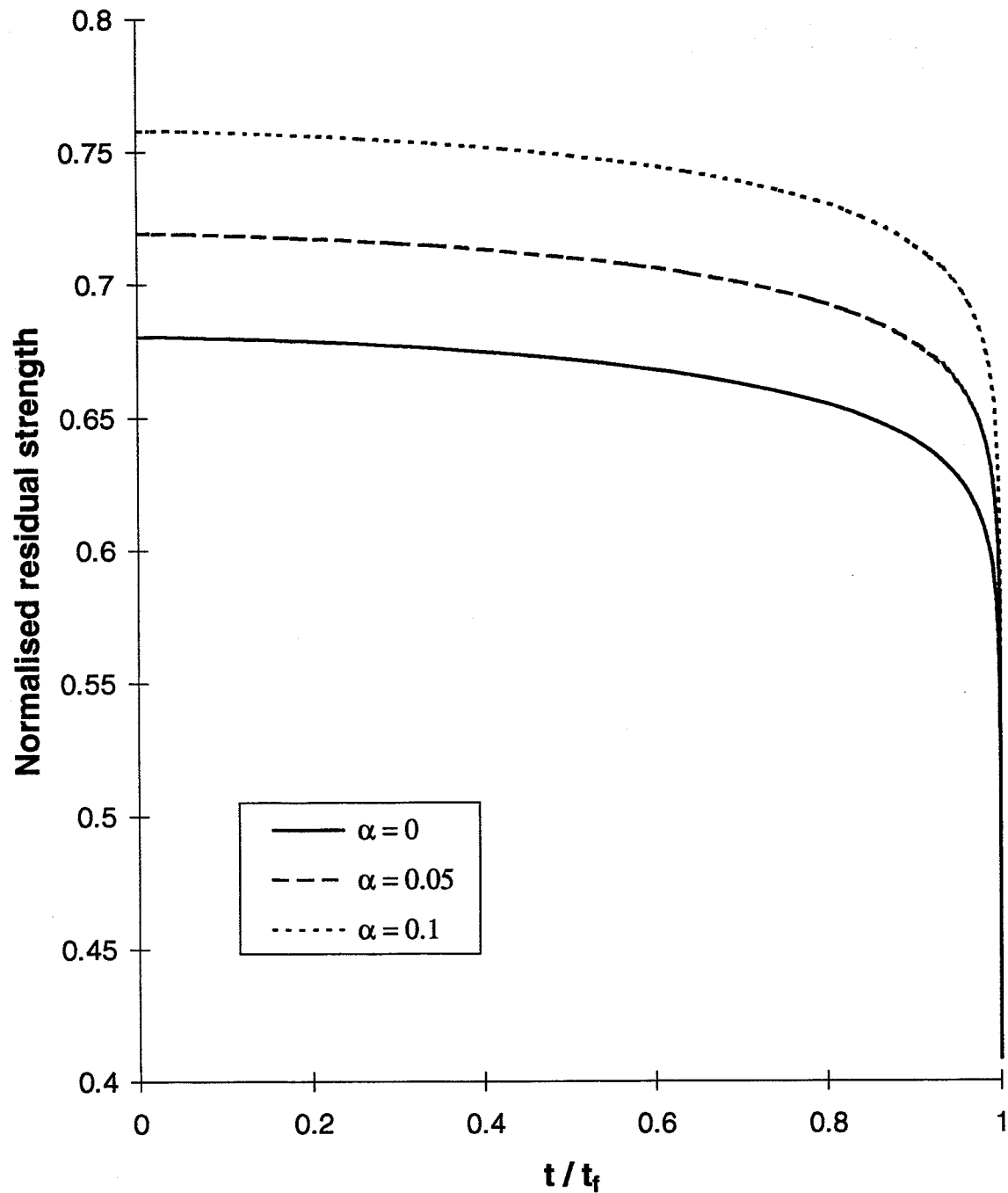


Figure 8 : The dependence of the normalised residual strength ($\hat{S}$) on elapsed time ratio (t/t_f) for the case when $F/F_m = 0.6$, $m = 8$ and $n = 30$.

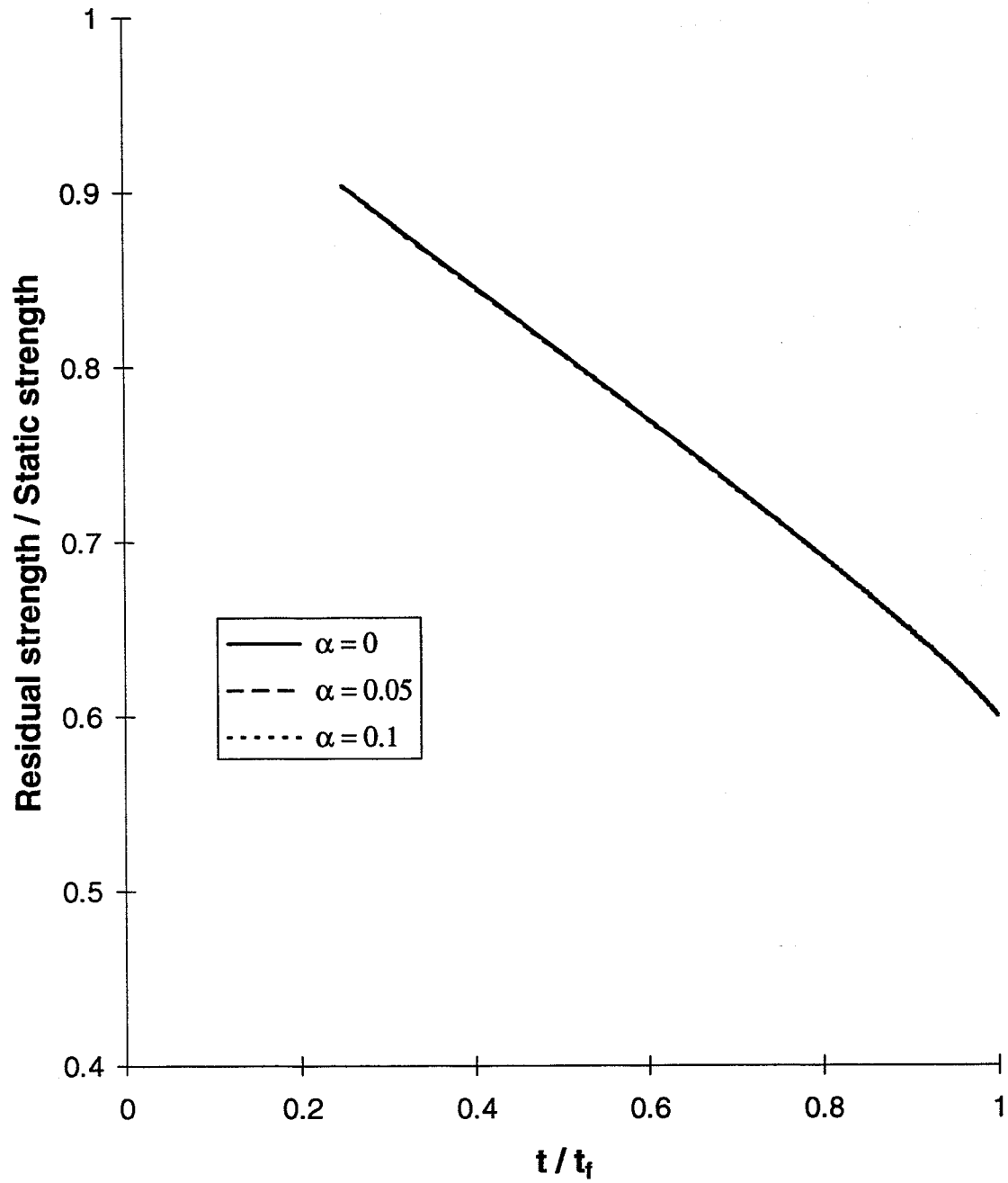


Figure 9 : The dependence of the residual strength ratio on elapsed time ratio (t/t_f) for the case when $F/F_m = 0.6$, $m = 8$ and $n = 3$.

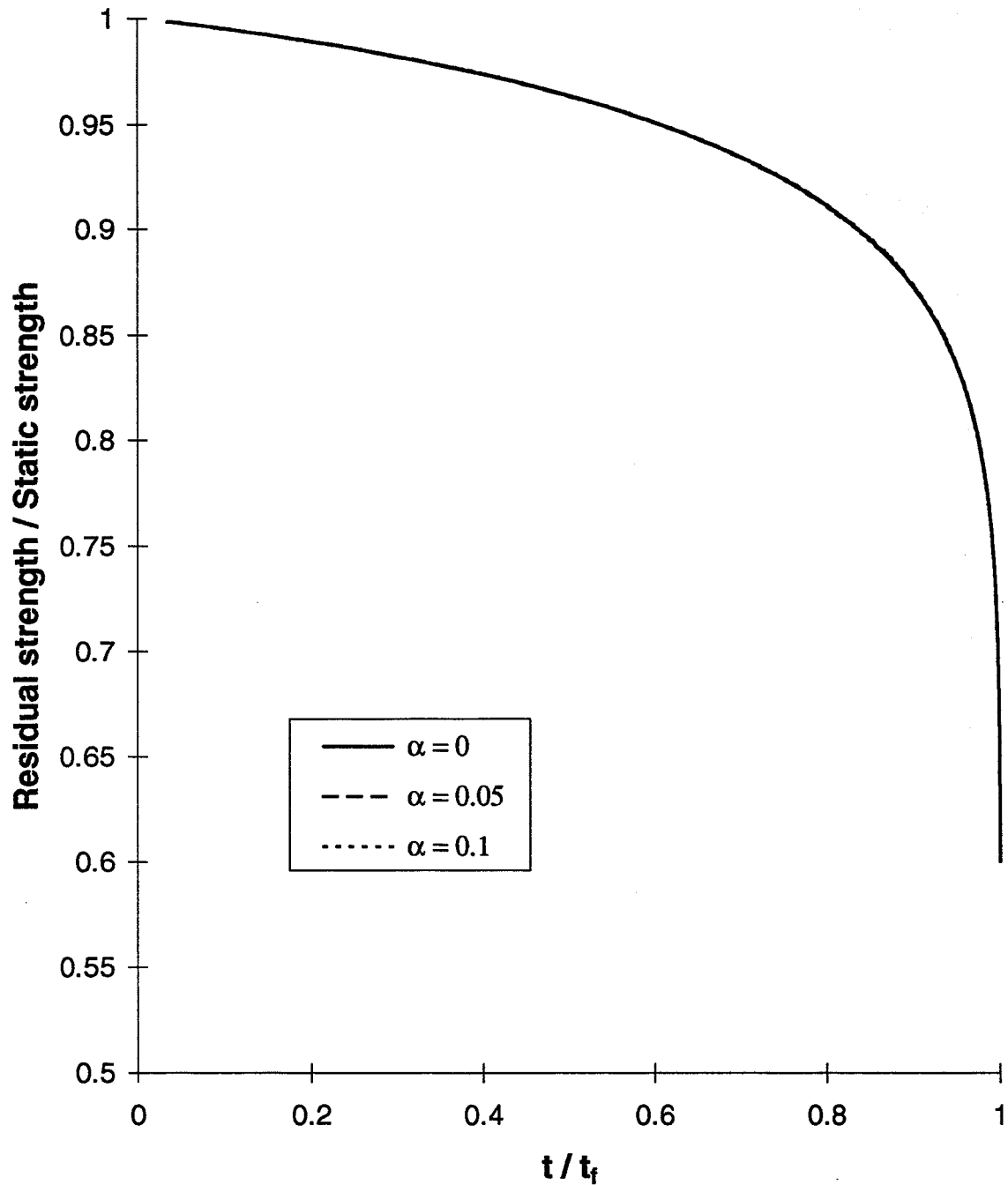


Figure 10 : The dependence of the residual strength ratio on elapsed time ratio (t/t_f) for the case when $F/F_m = 0.6$, $m = 8$ and $n = 15$.

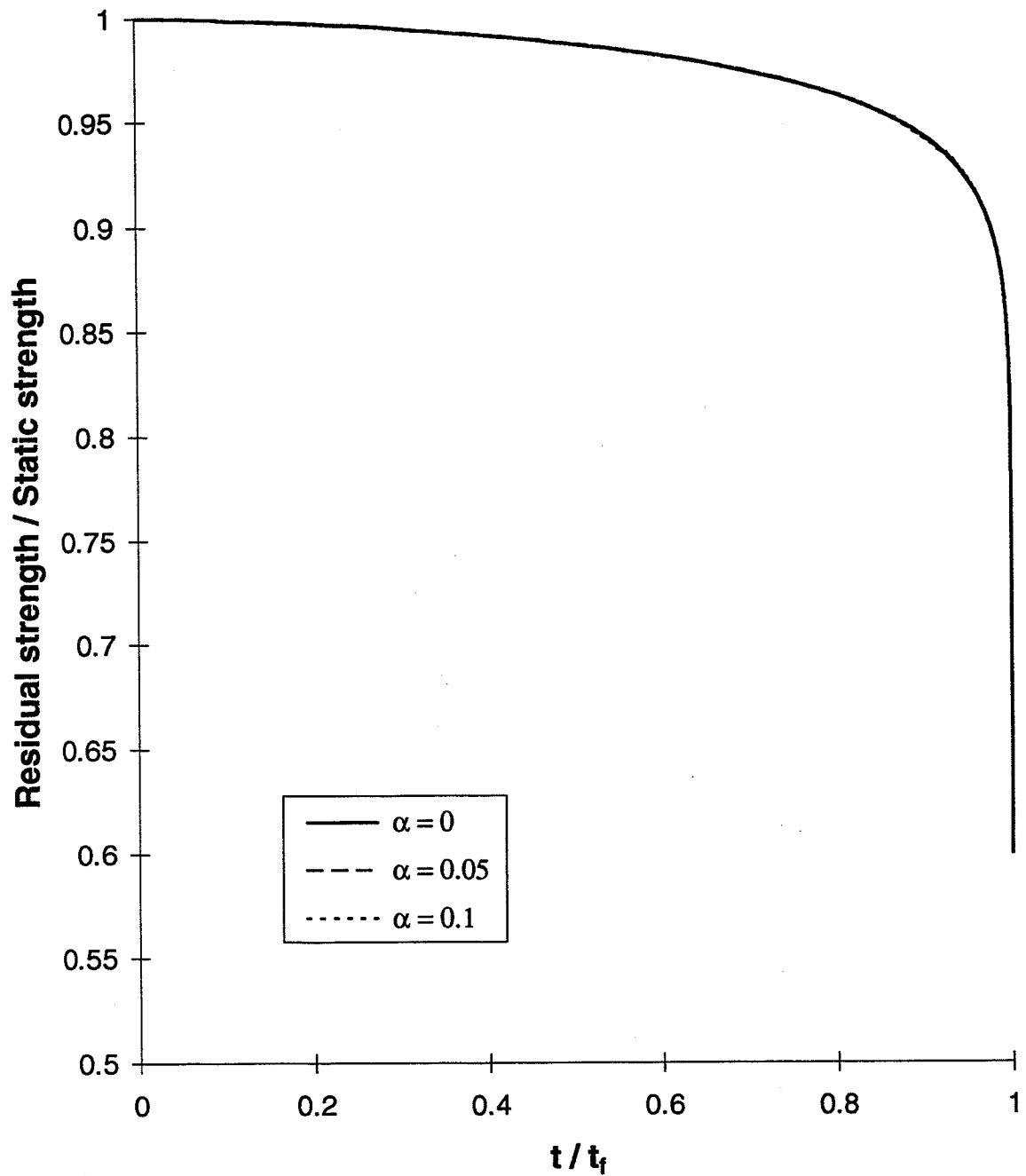


Figure 11 : The dependence of the residual strength ratio on elapsed time ratio (t/t_f) for the case when $F/F_m = 0.6$, $m = 8$ and $n = 30$.