

**Initial Stages in Disseminating the UK Mass Scale:
From the National Prototype Kilogram to the
Stainless Steel Reference Kilograms**

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ABSTRACT

This report describes the initial stages of the process whereby the UK mass scale was disseminated by NPL from the National Prototype Kilogram to the stainless steel reference kilograms, during the period from July 1984 to January 1991. This period is of particular interest as the mass value assigned to the National Prototype Kilogram (No 18) by BIPM was, for the first time, corrected by NPL for estimated mass regain after cleaning. Details are given of the weights involved, the weighing schemes, equipment and techniques used and the uncertainties of measurement estimated at that time.

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1 INTRODUCTION

The UK mass scale is derived from the International Prototype Kilogram, held by the International Bureau of Weights and Measures (BIPM), via the UK National Copy (No 18), referred to here as Kg 18. The National Physical Laboratory (NPL) is responsible for its custody and use. Kg 18 is periodically calibrated at BIPM against standards which are compared with the International Prototype. The traceability route and overall process are shown schematically in Figure 1, where the letters or numbers denote individual kilograms, with the exception of "WS" which is a group of working standards and official copies used by BIPM[1] to relate National Copies to the International Prototype Kilogram, K. Each box in the diagram represents a comparison stage.

The initial stages of disseminating the UK mass scale from Kg 18 are carried out by the NPL; these stages are as follows:

- (a) maintenance of Kg 18 and monitoring its mass;
- (b) calibration of Pt-Ir (and Pt) reference standards of 1 kg;
- (c) calibration of stainless steel reference standards of 1 kg;
- (d) calibration of stainless steel working standards of 1 kg;
- (e) calibration of stainless steel reference and working standards above and below 1 kg (ie propagation of the UK mass scale);
- (f) calibration of stainless steel working standards of various denominations for other organisations.

This report describes in detail the mass standards, the weighing schemes, the mass balance and the measurement techniques which were used for stages (a) to (c) during the period July 1984 to January 1991; stages (d) and (e) have already been described in a separate NPL publication[2]. The period covered by this report is of particular interest as, for the first time, the mass value assigned to Kg 18 was corrected for estimated mass regain after cleaning⁽¹⁾.

(1) The effects of cleaning Kg 18 and the corrections applied to its measured mass value to compensate for the estimated mass regain with time are described and discussed in detail in Reference 3.

2 DESCRIPTION AND BRIEF HISTORY OF THE NPL STANDARDS

2.1 GENERAL

A more detailed history of Kg 18 and the other important NPL kilogram standards (with the exception of Set NPLW 36) can be found in Reference 3. Included here are two sets of small auxiliary weights⁽²⁾ which were used to poise the loaded balance. Two further auxiliary weights, used to determine the balance sensitivity, are described with the balance in Section 3.1.

2.2 THE STANDARDS

2.2.1 Kilogram 18

The UK primary standard of mass is one of a group of forty weights which were made for BIPM by Johnson Matthey and Co (London) in 1884[4]. The alloy of 90%Pt-10%Ir is the same as that which was used for the metre standards. Its volume was measured at BIPM⁽³⁾ before it was polished, numbered and finally adjusted to $1 \text{ kg} \pm 1 \text{ mg}$. After extensive intercomparisons involving all forty kilograms and the International Prototype[6], Copy No 18 was received by the UK in 1889.

Kg 18 is in the form of a plain flat-ended cylinder, with height equal to diameter (39 mm) and having slightly chamfered edges. The surface condition is reasonable, showing a number of light irregular scratches only, not inconsistent with its limited use; there are no stains or visible films. The number 18 is marked on the cylindrical surface by slight depolishing. The BIPM calibration certificates give the volume at 0°C as 46.4149 cm^3 (equivalent to a density of $21.5448 \text{ g cm}^{-3}$) and the coefficient of cubical expansion between 0°C and $t^\circ\text{C}$ as $(25.863 + 0.00562t) \times 10^{-6} \text{ }^\circ\text{C}^{-1}$. The expansion coefficient was derived from measurements made at BIPM[7] of the linear expansion of Pt-Ir metre standards.

(2) Throughout this report the term "weight" is used to represent an artefact, not a physical quantity.

(3) ten determinations, using three different samples of distilled water; for details see Reference 5

2.2.2 Kilograms A and B

These one kilogram reference standards were made from a Pt-Ir (90%+10%) ingot produced by Johnson Matthey in 1979, compacted by extrusion into rod at NPL, then diamond-turned in 1982 at BIPM where the final weight adjustments and cleaning were carried out. They have the same cylindrical form as Kg 18 but an obviously superior finish. The identifying letter is marked on the side by slight depolishing. The density of each weight was determined at NPL at temperatures close to 20 °C, by hydrostatic weighing of the individual portions of the extruded rod after the initial skimming but before the diamond-turning and final adjustment of each weight. The coefficient of cubical expansion was assumed to be the same as for Kg 18, giving calculated volumes at 0°C of 46.4354 cm³ (Kg A) and 46.4399 cm³ (Kg B).

2.2.3 Kilogram E

This weight is distinctive for several reasons. It is the oldest of the group described here, having been bought by Professor W H Miller of the University of Cambridge from Gambey of Paris in 1844; it was made of pure platinum so is softer than the Pt-Ir weights. The surface of this plain cylindrical weight is in a rather poor condition, bearing many scratches due to its considerable use in the past. However, with present-day care the mass seems very stable. Miller[8] determined the density of Kg E in 1844, by hydrostatic weighing at 15 °C and assuming a value of $9.00 \times 10^{-6} \text{ }^{\circ}\text{C}^{-1}$ for the linear expansion coefficient to calculate the density and volume at 0°C, in present-day units: 21.13732 g cm⁻³ and 47.3096 cm³, respectively. Since about 1883 the volume at room temperature has been derived using $25.89 \times 10^{-6} \text{ }^{\circ}\text{C}^{-1}$ for the cubical expansion coefficient[9]. The use of different expansion coefficients introduced a small systematic error in the volume at room temperature and hence in the buoyancy corrections; this is discussed further in Section 5.3.

2.2.4 Kilogram PI

This is a group of four weights (500 + 200A + 200B + 100)g, collectively referred to as Kg PI or Σ PI. They are from a set of 23 weights ranging from 500 g to 1 mg (known as "PI grams") made by Oertling in 1894. The four weights comprising Kg PI are plain cylinders of Pt-Ir, the

surfaces being in a fair condition only, due to their age and usage. A density value at 0 °C of 21.3544 g cm⁻³ has been used for many years, this being the mean of determinations made at the Board of Trade⁽⁴⁾, but the exact source of this value and the uncertainty of measurement are not known. The coefficient of cubical expansion was assumed to be $25.70 \times 10^{-6} \text{ }^{\circ}\text{C}^{-1}$ [10].

Despite the drawback of its large surface-to-mass ratio making it more susceptible to mass instability, it is a useful member of the group of Pt-Ir reference standard kilograms.

2.2.5 Set NPLW 36

This set comprises two one-kilogram weights, referred to as Kg 36 and Kg 36D, which were, during the period covered by this report, the primary stainless steel reference standards at NPL. They were bought by NPL from Stanton Instruments Ltd in 1955. The material was described as a non-magnetic stainless steel, the composition of which was not known but is believed to be similar to Immaculate 5 (Firth Vickers) ie about 25% Cr, 20% Ni. They are cylindrical one-piece weights with integral knobs, well-rounded edges and slightly relieved bases; the well-polished surfaces were, at this time⁽⁵⁾, in good condition. The density was determined at NPL by hydrostatic weighing of the weights before final adjustments, giving a volume at 20 °C of 127.682 cm³. The coefficient of cubical expansion was assumed to be $45 \times 10^{-6} \text{ }^{\circ}\text{C}^{-1}$.

2.3 AUXILIARY SETS OF WEIGHTS

2.3.1 PI Grams

The smaller weights, from 1 mg up to 0.1 g, of the PI Grams set were used as "makeweights" to poise the loaded balance to within 1 mg. Weights up to and including the 10 mg are of aluminium, above this they are of Pt-Ir.

(4) corrected to g cm⁻³ from the original value of 21.355 g ml⁻¹

(5) Subsequent to the period covered by this report, the weights were accidentally damaged during an automatically controlled weighing.

2.3.2 NPLW 25

These weights were used as makeweights to poise the loaded balance to 50 µg or better. The set, acquired in 1932, originally comprised eleven wire weights, from 1.1 mg to 1.9 mg in 0.1 mg steps, plus two at 1.05 mg. In 1936 four more weights, from 1.01 mg to 1.04 mg in 0.01 mg steps, were included. Four of the set are 'Eureka' (a cupro-nickel alloy with a density of 8.9 g cm⁻³), the rest are of aluminium.

2.4 STORAGE

Since 1982, Kg 18 has been stored (and can be transported) in an enclosure of the type shown in Figures 2(a) and 2(b), consisting of a glass dome clamped onto a stainless steel base. A PTFE O-ring provides a seal between dome and base but air movement is possible via an outlet carrying two fine filters (0.2 µm pore size cellulose tri-acetate membranes) in series. The weight rests on several layers of acid-free tissue paper on a pad of washed chamois leather. During transportation all movement is restrained by three side-arms and a top pad, all cushioned by washed chamois-leather and layers of acid-free tissue paper. The enclosure is kept in a safe in a ventilated semi-subterranean vault maintained at a temperature close to 20° C. Kilograms A, B and E are stored in enclosures of the same type and under the same ambient conditions as Kg 18.

The weights of Kg PI are stored in a traditional wooden box with felt-lined cavities, which is kept with the other Pt-Ir weights. The weights Kg 36 and Kg 36D are stored in felt-lined cavities in a substantial wooden box which is kept in a balance room provided with filtered air and maintained near 20 °C.

2.5 CLEANING

Prior to the period covered by this report, Kg 18 had been cleaned a total of six times, the most recent being in February 1985 (by BIPM) after which its mass regain with time was studied.⁽⁶⁾ Kg A had not been cleaned since its first calibration at BIPM, but Kg B had been

(6) See Reference 3 for detailed information on the cleaning methods and mass changes with cleaning.

cleaned and washed at BIPM again in 1985. There is no record of Kg E having been cleaned or washed since Miller measured its density in 1844, but early records are insufficiently detailed to be absolutely certain. Likewise, the four weights in the group Kg PI had probably never been cleaned vigorously. Kgs 36 and 36D had not been cleaned or washed since the hydrostatic weighings to determine their density.

Apart from the occasions listed above, it was normal NPL practice to simply dust off all surfaces lightly with a camel-hair brush prior to use.

3 THE INTERCOMPARISON PROCESS

3.1 THE NPL 1 kg PRIMARY BALANCE

3.1.1 Role and general description

All comparisons of kilograms at the top of the NPL mass hierarchy were carried out on the balance which is described below and shown in Figure 3. The balance was also used with certain other stainless steel kilograms, which were reserved for stability checks, and for special tasks such as pound-kilogram comparisons. To minimise wear and because the balance was slow to use, it was not used for routine work.

The balance was originally designed and built at NPL in the 1930s. Its design incorporated a number of features which, combined with the attention given to various details, resulted in a significantly better performance than the best contemporary balances[11]. It was automated during the period 1978-1983. An outline is given below of the balance's features at the time covered by this report.

3.1.2 Basic Mechanical Features

It is a two-pan, equal-arm, undamped mechanical type, with knife-edges defining the beam arm lengths. Important features of the design include the following:

- (a) The pan suspension systems (see Figure 4) each provide three stages of decoupling,

to reduce sensitivity to eccentric loading of weights on the pans.

- (b) A remotely controlled weight interchange mechanism (partially shown in Figure 4) permits repeated "double weighing" cycles to be carried out without disturbing the settled conditions inside the weighing enclosure.
- (c) A "beam steady" device maintains contact under reduced load between the knives and planes while exchanging weights during double weighings (see Figure 5): a horizontal flexible metal strip, fixed to the lowest part of the beam, is "steadied" by application of light pressure from two agate points. This technique had been shown by Poynting[12] to significantly improve the repeatability of rest point determinations.
- (d) Two small wire weights (of about 0.5 mg) are applied, one over each pan in turn, to determine the balance sensitivity immediately after each complete set of double weighings.
- (e) All mechanical operations (after the initial loading) are controlled remotely by means of long shafts passing through a partition to an adjoining room, to minimize thermal disturbance from the operator. (See also Section 3.1.3.)
- (f) An optical lever system monitors the beam inclination. (The beam inclination does not normally exceed $\frac{1}{2}^\circ$ from the horizontal, and, as adjusted at this time, the period of swing was about 100 seconds with a 2 x 1 kg load.)

The beam is of gun-metal (a type of bronze), with arms 15 cm long. All the planes are of agate, except the central plane which is of synthetic sapphire. The three main knives are unusual, being of gun-metal with chromium-plated facets (0.2 mm plating, ground and lapped to about 0.1 mm), the fulcrum knife being 44 mm long. Other knives are of agate. The pan suspension systems (stirrups and pan hooks) are mainly of aluminium alloy but the pan plates are of nickel-chromium alloy (80-20). The substantial balance cover is of gun-metal (1.3 cm thick) with three plate glass windows, and is raised by a hydraulic hoist. Particular care was taken to avoid magnetic materials in the balance, in its supporting pedestal and in

the balance room; the chromium-plating on the main knives has been shown to have a negligible effect. Further details of the design and adjustment of the balance in its original (manual) form are given in Reference 11.

3.1.3 Automation

The balance was automated between 1978 and 1983, subsequently allowing weighings to be made at any time with no operator present. This necessitated changes to the optical lever system, motorization of the mechanical controls and the introduction of sensors for air temperature, pressure and humidity which could be read automatically by a controlling microcomputer (NASCOM-2). The sensors used for air density measurement and their associated equipment are described in Section 3.2; the other changes are described below.

The optical lever system which monitored the inclination of the balance beam is shown in Figures 6(a) and 6(b). The linear photodiode array was aligned with the path swept by the light spot reflected from the concave mirror mounted on the balance beam. This sensor array (which consisted of 1024 photodiodes, each 25 μm square, and had an overall length 26 mm) was fixed on top of the balance cover. The associated electronics provided a digital output which indicated the position of the centre of the light spot to one part in 10^4 of the array length but the true resolution of the system was about one part in 2000. The output was fed to the microcomputer and to a digital display, which could be operator-selected to show either the instantaneous position of the spot or the maximum and minimum readings only (corresponding to the turning-points of the swings). The diode array signal was also processed and displayed on an oscilloscope so that the operator could monitor the alignment of the array with the spot's path.

The main beam arrest control remained manually-operated but the four other remote control shafts and the sensitivity weight controls had been fitted with stepper or synchronous motors as appropriate. These motors were coupled to the microcomputer, which thus controlled the following mechanical and data-gathering operations:

- (1) receiving initial data input and commands from the operator
- (2) release and application of the pan stops
- (3) release and application of the beam steady
- (4) raising, interchanging and lowering the pan plates carrying the weights
- (5) application and removal of the sensitivity weights
- (6) monitoring and recording the position of the beam (turning-points of the swings)
- (7) monitoring and recording the temperature, pressure and humidity within the balance enclosure
- (8) determining the time of all operations and observations
- (9) part-processing the data
- (10) transmitting data, results and messages to a monitor screen and to a printer plus paper-tape punch (Teletype ASR33) for subsequent data storage and processing by the mainframe computer.

The complete system, shown schematically in Figure 7, was installed in a linear suite of three small rooms situated near the centre of a large semi-subterranean basement. The end, or inner, room housed the balance mounted on an anti-vibration pedestal, the middle room contained the motorized controls and instruments for measuring air pressure and CO₂ content, while the remaining control, measuring and recording equipment was in the outer room. Figure 8 gives a general view of the balance and motorized controls from the middle room and Figure 9 shows the equipment in the outer room.

The natural distribution of temperature within the suite was quite uniform and steady, the surrounding walls being about 60 cm thick, but the instrumentation introduced for automation generated appreciable heat, particularly in the outer room. A powered flow of filtered air into the inner balance room was extracted via a fan in the outer room. The air flow helped to remove the heat, moisture and CO₂ generated by the operator and the monitoring equipment.

The balance originally attained an accuracy of about $\pm 1 \mu\text{g}$ but the performance had deteriorated slightly in recent years. This may be due to wear of the knives and planes, accelerated by the increased use of the balance following automation; other factors which may contribute were (possibly) less steady temperature conditions and the introduction of

certain changes in the operating procedure.

3.2 AIR DENSITY DETERMINATION

3.2.1 Method

The air density during weighings was calculated from measured values of temperature, pressure and humidity using the formula recommended by the BIPM[13]. Normally the CO₂ content was assumed to be 400 ppm; it was measured only when comparing Pt-Ir and stainless steel kilograms, ie when large volume difference (hence buoyancy corrections) were involved. Values of the three main parameters were recorded periodically throughout each weighing run.

3.2.2 Temperature

A 25 ohm 4-terminal platinum resistance thermometer (PRT) (Rosemount type E109) was mounted vertically inside the balance case, the sensing region being in approximately the same horizontal plane as the kilograms on the balance pans. The PRT resistance was measured by a self-balancing AC bridge (Tinsley "Ambassador" type 5840C), the last digit of the resistance readout being equivalent to approximately 0.1 mK. The resistance reading was transmitted to the controlling microcomputer via an IEEE-488 interface. The bridge's internal reference resistor could be checked and adjusted at any time (outside a run) using a calibrated external standard resistor. The PRT was calibrated periodically and the constants stored in the microcomputer and main computer programs. The uncertainty in the PRT calibration (with the sensor in a liquid) was ± 3 mK, but when used in air the uncertainty of temperature measurement was estimated to be ± 5 mK.

3.2.3 Pressure

The balance case was connected via polythene tubing to a quartz bourdon tube barometer with electromagnetic nulling (Ruska DDR-6000), having a resolution of 1 Pa. The voltage output was measured by a high precision digital voltmeter (Datron 1061A) which was read by the microcomputer. The two instruments had been calibrated as a combined unit, and the

constants stored in the microcomputer and main computer programs. To correct for slow zero drift of the barometer, a particularly well-calibrated manually-operated precision aneroid barometer was used to determine, in situ, the small correction to the reading of the automatic barometer. Normally this correction needed to be determined only weekly, but when comparing Pt-Ir and stainless steel kilograms the correction was measured before or after each run. Using the more frequent zero correction procedure, the uncertainty in the pressure measurement was estimated to be ± 10 Pa.

3.2.4 Humidity

The use of a dew-point hygrometer gives rise to air movement which would be intolerable during weighing, therefore a humidity meter (Ancom RH 2000 DT) was used, its polystyrene-coated sensor being mounted inside the balance case, and changes in electrical impedance measured. The % RH was displayed and an analogue voltage output was measured by the digital voltmeter (see Figure 7). The humidity meter was checked regularly against a calibrated dew point hygrometer (Michell series 3000), reading to 0.1 °C. Air was drawn from the balance case through stainless steel tubing to the dew point meter head, situated close to the balance cover, then back via a pump and a flow meter to the case. Air temperature (t) was also recorded so that % RH could be calculated from the dew point using

$$\% \text{ RH} = [(\text{vapour pressure at dew point})/(\text{vapour pressure at } t)] \times 100$$

Normally the resulting correction to the observed RH meter reading was obtained weekly, but when Pt-Ir and stainless steel kilograms were being compared it was determined before or after each run. Allowing for uncertainties arising from the dew point hygrometer calibration, the method used to calibrate the RH meter and drift of both instruments, an overall uncertainty of $\pm 2\%$ in the RH was considered to be realistic when the more frequent checks were made.

3.2.5 Carbon Dioxide Content

When comparing Pt-Ir and stainless steel kilograms, this parameter was measured immediately before or after each run, using a direct-reading infra-red gas analyzer (SIEGER model IRGA 120) readable to 5 ppm. Air was pumped from the balance enclosure through

polythene tubing to the gas analyzer then returned to the balance. Before use the instrument was adjusted to give correct readings using a zero reference gas (oxygen-free grade nitrogen) and a reference gas mixture (400 ppm CO₂ in nitrogen). The uncertainty of a measurement was determined mainly by the accuracy of the reference mixture ($\pm 5\%$), but allowing for the instrument maker's specification for repeatability ($\pm 1\%$), linearity ($\pm 2\%$) and 24 hour stability ($\pm 1\%$) raised the total uncertainty to $\pm 6\%$, or ± 24 ppm at 400 ppm.

3.3 WEIGHING METHOD

The balance was used as a mass comparator, with the two weights being compared being poised very closely (normally to within about 25 μg) using makeweights from the auxiliary sets NPLW 25 and PI grams, and the residual mass difference measured. The residual mass difference was obtained, using a double weighing technique, from the change of rest-point and the balance sensitivity and by applying appropriate air buoyancy corrections. To determine and correct for unavoidable drift of the rest-points, a repeated double weighing procedure was used, each such procedure being referred to as a weighing run. As the interchange of the weights on the beam was achieved by interchanging the pan-plates, the mass difference obtained included that of the pans; to eliminate the mass difference of the pans the contents of the pans had to be transposed⁽⁷⁾ and the double weighing process (run) repeated. A more detailed description is given in the following paragraphs.

For each weighing run, the procedure was as follows. After loading and poising the balance, the beam arrest was released and the balance was left, with the "beam steady" applied, for a minimum of 8 to 10 hours to stabilize. The zeros and calibration of the pressure, humidity and CO₂ instruments were checked as described in Section 3.2. Before weighings started, the controlling program began with a preliminary interchange (lift, rotate, lower) of the pans plus their contents in configuration X.

For each comparison (ie rest-point determination) the beam steady was released, the first two turning-points were ignored and the next seven (from which the rest-point was calculated)

(7) The arrangements of the pan contents are denoted as configurations X and Y.
For configuration X: weight A plus makeweight W_A on pan 1, weight B plus makeweight W_B on pan 2
For configuration Y: weight A plus makeweight W_A on pan 2, weight B plus makeweight W_B on pan 1

were recorded. The beam was steadied, the pans plus contents were interchanged and the next rest-point was determined. These interchanges and comparisons were repeated until thirteen rest-points for the X-configuration had been obtained, ie seven rest-points with A plus W_A (on pan 1) on the right, alternating with six rest-points with A plus W_A (on pan 1) on the left. From these two sets of rest-points best fit curves are calculated. The offset between these two curves, $R(X)$, after correcting for buoyancy at each comparison, gives a drift-corrected measure of the mass difference between the pans plus contents in the X-configuration. Thus

$$(m_A + w_A + p_1) - (m_B + w_B + p_2) = R(X)/2S + b_i \quad (i = 1 \text{ to } 13) \quad (1)$$

where m_A and m_B are the masses of weights A and B, w_A and w_B are the masses of makeweights W_A and W_B , p_1 and p_2 are the masses of the pans, S is the sensitivity and b_i is the nett buoyancy correction for the i th comparison due to the volume difference between the pans' contents.

Re-arranging (1) gives

$$(m_A - m_B) + (p_1 - p_2) = R(X)/2S + b_i - (w_A - w_B) = m(X_i) \quad (2)$$

Immediately after the thirteenth comparison, without disturbing the weights on the pans and while the beam was still swinging freely, the two sensitivity weights (one at each end of the beam) were in turn applied then removed three times, recording the turning-points and calculating the rest-points as before. From the twelve rest-points obtained three curves were calculated, corresponding to "on" and "off" for each sensitivity weight. The difference between associated curves gives a drift-corrected measure of the left and right sensitivities and hence the mean sensitivity (S). The buoyancy correction for the i th comparison (b_i) is given by $\rho_i (v_A - v_B)$, where v_A and v_B are the volumes of weights A and B including the associated makeweights⁽⁸⁾ and ρ_i is the air density at the time of the i th comparison. The temperature, pressure and humidity within the balance case were automatically recorded regularly

(8) As the pans are closely equal in mass (hence volume), the difference between their buoyancy corrections is small and will vary by significantly less than $1 \mu\text{g}$ for any likely variation in air density during measurements. The mass difference ($p_1 - p_2$) is eliminated by the procedure used, so the virtually constant nett buoyancy correction for the pans can be ignored.

throughout⁽⁹⁾ and the air density was calculated, assuming CO₂ content of 400 ppm. Air density at the time of each comparison was calculated by linear interpolation⁽¹⁰⁾, thus allowing b_i for each comparison to be calculated and hence $m(X_i)$ in equation (2) to be evaluated. The complete routine described thus far, including the sensitivity and air density determinations, constituted a weighing run. Only at the end of the run was the beam arrested.

With the weights remaining in configuration X the run was repeated to provide a duplicate result, $m(X_2)$; from these two results a mean, $m(\bar{X})$, was obtained. Hence

$$(m_A - m_B) + (p_1 - p_2) = m(\bar{X}) \quad (3)$$

The contents of the pans were then transposed (giving configuration Y) and the whole process described above for configuration X repeated, giving results $m(Y_1)$ and $m(Y_2)$. For configuration Y equation (2) becomes

$$(m_B - m_A) + (p_1 - p_2) = R(Y)/2S - b_i + (w_A - w_B) = m(Y) \quad (4)$$

and equation (3) becomes

$$(m_B - m_A) + (p_1 - p_2) = m(\bar{Y}) \quad (5)$$

Subtracting (5) from (3) eliminates the masses of the pans:

$$(m_A - m_B) = (m(\bar{X}) - m(\bar{Y}))/2 \quad (6)$$

The complete process, comprising four runs, constituted one "weighing". A series of such weighings, intercomparing the weights in a group, forms a "weighing scheme". Recent schemes that have been used to establish mass traceability from Kg 18 to the stainless steel reference standards (NPLW36 and 36D) are discussed in Section 3.5.

3.4 PROCESSING THE BALANCE DATA

Some of the data from runs on the primary balance was handled by the controlling microcomputer, but it was all finally processed on the mainframe computer using the

(9) nine times: at the start of the run, after each even-numbered comparison, between the left and right sensitivity measurements and at the end of the run

(10) Reference 14, page 3

software "Runbal"[14]. This was originally run in Algol-60 on an ICL 2972 but was converted to Fortran-77 by May 1989 before transferring to the replacement mainframe (Digital VAX 6230). After August 1989 only the Fortran version, run on the VAX 6230, was used to process data from runs. The software conversion was verified by processing data from twenty-seven runs on both computing systems; this verification showed that the change in software introduced changes in the value obtained for the measured mass differences of less than $0.8 \mu\text{g}^{(11)}$. The main operations carried out by the "Runbal" software are listed in Appendix 1.

The air density computed by "Runbal" assumes CO_2 content is 400 ppm. When kilograms of stainless steel and Pt-Ir (or Pt) were compared, the small correction for the actual CO_2 content was applied to the mass difference result obtained from "Runbal" as follows:

Let the average air density computed by "Runbal" for the run be ρ , then the air density correction, $c(\rho)$, for a measured CO_2 content of κ ppm is given by

$$c(\rho) = \frac{1.44 \rho}{3.48353} (\kappa - 400) \times 10^{-6}$$

The resulting adjustment to the air buoyancy correction is given by

$c(\rho) \times (v_1 - v_2)$ where v_1 and v_2 are the volumes of the weights on the pans. Hence,

$$\text{corrected mass difference} = \text{"Runbal" mass difference} + c(\rho) \times (v_1 - v_2)$$

3.5 WEIGHING SCHEMES

When a group of kilogram reference standards are intercompared it is normal NPL practice to perform a complete comparison "square", ie to weigh each weight against each of the others in the group in turn; when dissimilar weights are involved (kilograms and pounds) a different weighing scheme is necessary. The weighing schemes used in between July 1984 and February 1990 are given in Appendix 2. One of the weighing schemes used also included

(11) Slight differences in the mainframe computers' library routines called up for the curve-fitting were considered to be the most likely cause.

those imperial reference standards (in Pt-Ir or Pt) that are held at NPL; these comprise the National Pound Standard (PS), the Parliamentary Copy No 5 (PC5) and the composite pound (Σ PI grains) which is assembled from three weights of the set "PI grains".

3.6 EVALUATING THE RESULTS FROM WEIGHING SCHEMES

3.6.1 The two methods

When a group of similar weights have been intercompared and each weight has been weighed against each of the others (ie a complete "square" has been obtained), the results can be evaluated by simple calculation as indicated in 3.6.2 below.

Under some circumstances results were evaluated using the program "Massrun", described in 3.6.3. For example this was necessary when: dissimilar weights had been used, requiring two or more weights on one balance pan; a complete "square" of weighings had not been obtained; or, additional statistical information was required.

3.6.2 Calculation By "Square"

This method is best described by means of a hypothetical example involving a group of, say, four weights (A,B,C,D). Let the final result given by equation (6), be expressed in the form $A = (B + d)$, where d is the measured mass difference between the masses of the weights A and B; the results of the six required weighings can then be represented by the equations:

$$\begin{array}{ll} A = B + d_1 & B = C + d_4 \\ A = C + d_2 & B = D + d_5 \\ A = D + d_3 & C = D + d_6 \end{array}$$

where d_1 to d_6 are the measured mass differences. These results are arranged in a square:

	A	B	C	D
A		$-d_1$	$-d_2$	$-d_3$
B	d_1		$-d_4$	$-d_5$
C	d_2	d_4		$-d_6$
D	d_3	d_5	d_6	
Sum	Σ_A	Σ_B	Σ_C	Σ_D

The results in each column of the square are summed, giving:

$$\Sigma_A = + d_1 + d_2 + d_3$$

$$\Sigma_B = - d_1 + d_4 + d_5$$

$$\Sigma_C = - (d_2 + d_4) + d_6$$

$$\Sigma_D = - (d_3 + d_5 + d_6)$$

The adjusted mass differences (δ_1 to δ_6) are given by:

$$\delta_1 = (\Sigma_A - \Sigma_B)/4$$

$$\delta_2 = (\Sigma_A - \Sigma_C)/4$$

$$\delta_3 = (\Sigma_A - \Sigma_D)/4$$

$$\delta_4 = (\Sigma_B - \Sigma_C)/4$$

$$\delta_5 = (\Sigma_B - \Sigma_D)/4$$

$$\delta_6 = (\Sigma_C - \Sigma_D)/4$$

With one weight taken as the standard, the values of the others are derived directly from the adjusted equations ($A = B + \delta_1$, etc) that include the standard.

The residuals (r) are calculated from $r_i = \delta_i - d_i$ ($i = 1$ to 6 in this example).

The standard deviation, σ , of the measured results is given by

$$\sigma = [2\Sigma r^2 / (M - 1)(M - 2)]^{1/2} \quad (7)$$

and the standard deviation of the adjusted results is given by

$$\sigma = [4\Sigma r^2 / M(M - 1)(M - 2)]^{1/2} \quad (8)$$

where M is the number of weights ($M = 4$ in this example).

3.6.3 "Massrun" Program

Like the "Runbal" program, the "Massrun" software was originally written in Algol-60 and run on the ICL 2972 mainframe; it was converted to Fortran-77 and transferred to the VAX 6230 mainframe by mid-1989. The program is described below; more detailed information is given in References 15 and 16.

The series of equations that represents the results from a weighing scheme is arranged into two matrices: one represents the weighing scheme, ie it defines the N weighings performed, and the other contains the corresponding measured mass differences. A third matrix holds the required values (deviations from nominal value) of the M weights involved. These matrices are related by:

$$X\beta = y \quad (9)$$

where X is the $N \times M$ matrix of weighings, β is the $M \times 1$ vector containing the required deviations from nominal for the M weights and y is the $N \times 1$ vector of the measured mass differences of the N equations.

Note that $N > M$ and that M is the number of weights excluding any taken as standard, (therefore such weights do not appear in matrices X or β .) For equations involving a standard weight the program incorporates the standard by correcting the measured mass difference, d , to take into account the deviation of the standard from its nominal value. Thus

$$y = z - h \quad (10)$$

where y becomes the vector incorporating the corrected values of d , z is the vector of the uncorrected (measured) values of d and h is a vector containing the deviation(s) from the nominal value of the standard(s) (where they have been used).

Least-squares analysis of the system of equations represented by equation (9) gives a best estimate of β , ($\hat{\beta}$).

$$\hat{\beta} = (X^T X)^{-1} X^T y \quad (11)$$

From these values of the weights, predicted mass differences for each equation are calculated; comparison of the predicted and measured differences gives the residual for each equation $\hat{r}$:

$$\hat{r} = X\hat{\beta} - y \quad (12)$$

(Inspection of these residuals will identify inconsistent or suspect equations, which may need to be repeated.)

From the matrix of residuals an unbiased estimate of the variance ($\hat{\sigma}^2$) of the least-squares solution is calculated:

$$\hat{\sigma}^2 = \frac{\hat{r}^T \hat{r}}{N-M} \quad (13)$$

An estimate of the variance-covariance matrix for $\hat{\beta}$ is then calculated from:

$$\hat{\sigma}^2 (X^T X)^{-1}$$

The diagonal elements of this matrix give the variances of $\hat{\beta}$ ($\text{var } \hat{\beta}_i$, $i = 1$ to M). Finally, the uncertainties for the individual weights are calculated from:

$$\pm t_{N-M} (\text{var } \hat{\beta}_i)^{1/2}$$

where t_{N-M} is the Student-t value for $(N-M)$ degrees of freedom at the 95% confidence level. Additionally, the operator can opt to have the effect of certain systematic errors analysed and for further matrices to be output.

4 MASS VALUES OF THE STANDARD WEIGHTS

The confidence that can be placed in the UK mass scale depends on the uncertainties associated with the standards involved and those introduced by the weighing process; however, the former depends not only on the uncertainty associated with the determination

of the mass of the weight at the time of its calibration, but also upon the stability of its mass whilst in use. A detailed study of the history and the stability of Kg 18 and the other important kilograms is given in Reference 3. A brief outline is given in this Section of the basis of the values used for Kg 18 and the other kilogram standards during the period under consideration here.

For about the first sixty years of its existence (ie until about 1950) Kg 18 was presumed to maintain a constant value between calibrations by BIPM; the differences observed were probably considered to be within the uncertainties and as having little significance. However, more recent studies have shown that the contamination of mass standards and their re-contamination after cleaning cause mass changes which are far from insignificant.

The mass regain of Kg 18 immediately after cleaning was measured by BIPM over a period of 14 months during 1985-6. In this study the four weights involved were left undisturbed on the balance pans, to provide an uninterrupted series of values. The data obtained on Kg 18[17][18] are given in Table 1 and plotted in Figure 10. Following this study an important change was introduced by NPL; instead of using the BIPM-certified post-cleaning value for Kg 18⁽¹²⁾, values were assigned to it by extrapolating the BIPM study data. The dates of use of Kg 18 and the bases of the NPL-assigned values are given below.

Oct-Nov 1986 NPL-assigned value: Kg 18 = 1 kg + 64 µg.

This was based on the last value reported by BIPM (1 kg + 63.1 µg), adjusted for a rate of increase in mass of 0.22 µg/month for a period of 5 months. The rate of increase was based on the gain in mass of 17 µg measured by BIPM between April 1978 and November 1984, the previous washing being 79 months earlier; the rate of increase was assumed to be linear.

Jan-Mar 1988 NPL-assigned value: Kg 18 = 1 kg + 65 µg.

Linear extrapolation using a rate of increase in mass of 0.22 µg/month (as above) gave Kg 18 = 1 kg + 68 µg; this was rejected because it gave values for Kgs A, B, E and PI which were considered to be rather different from their expected values. Instead a rate of 1 µg/year, which had been found by BIPM[1][20] to be more appropriate to later in the post-cleaning

(12) Under previous practice only one value would have been obtained immediately after cleaning, ϵ_6 Kg 18 = 1 kg + 57 µg.

period, was applied to the 15 months following the previous scheme.

Jan-Feb 1990 NPL-assigned value Kg 18 = 1 kg + 67 µg.

The 1 µg/year rate was applied for the two years after January 1988.

The above-derived values for Kg 18 were used to calculate the mass values for Kgs A, B, E and PI given in Tables 2 and 3; for completeness and comparison, the earlier values obtained by BIPM for Kgs A and B are also included. Kgs B and E were subsequently used to derive the mass values for Kgs 36 and 36D, given in Table 4.

5 UNCERTAINTIES

5.1 GENERAL

The total uncertainty at any stage of the mass calibration chain was considered as arising from two sources: the standard weight(s) and the intercomparison process. These are examined below and the component uncertainties are summarised in Table 5. (It should be noted that these values are those which were adopted during the period of the report, some of which have subsequently been revised.)

Uncertainties are classified as type A or B and are identified and evaluated separately. When required, uncertainties were combined by "quadratic summation", as recommended by the BIPM[21]. The magnitude of some uncertainties depend on the individual weight, weighing scheme, etc; this is indicated where significant. In particular, the values in Table 5 relate to the two stages of the major intercomparison exercise at NPL in 1988:

First stage: Kgs 18, A, B, E, PI (weighing scheme of Jan-Mar 1988)

Second stage: Kgs B, E, 36, 36D (weighing scheme of May-July 1988, Nov 1989)

Following NPL practice, uncertainties are evaluated and presented at the 2σ level.

5.2 THE PRIMARY STANDARD

The BIPM calibrations of Kg 18 quoted a total uncertainty (at the 1σ level) of $\pm 8 \mu\text{g}$. This was composed of type A components ($\pm 1 \mu\text{g}$) arising from the BIPM weighing process, plus type B ($\pm 8 \mu\text{g}$) associated with their working standards. However, for the work reported here the total uncertainty was considered to be type B and a 2σ value of $\pm 16 \mu\text{g}$ was adopted.

The additional uncertainty arising from the progressive change in mass of Kg 18 following cleaning was minimised by the extrapolation of the reported mass values. The uncertainty associated with the extrapolated value increased with time; it was estimated to be $\pm 3 \mu\text{g}$ for February 1988.

5.3 BUOYANCY CORRECTION

There are two components to the buoyancy correction (BC), the air density (ρ_a) and the difference in volume of the weights (Δv), related by $\text{BC} = \rho_a \Delta v$. The total uncertainty in BC is given by

$$U(\text{BC}) = [(\rho_a U(\Delta v))^2 + (\Delta v U(\rho_a))^2]^{1/2} \quad (14)$$

where $U(\Delta v)$ and $U(\rho_a)$ are the total uncertainties for Δv and ρ_a . Note that $U(\text{BC})$ is type B, any type A elements being assimilated into the overall type A uncertainty given by the "Massrun" program.

The uncertainty in the air density $U(\rho_a)$ arises from uncertainties in the measured parameters that are involved in calculating ρ_a (temperature, pressure, humidity and CO_2 content) and the uncertainty due to the BIPM equation itself[13]. Estimated values for these are given in Table 6, together with the resulting uncertainties in ρ_a . With the exception of temperature, where the uncertainty has been doubled to allow for temperature inhomogeneity within the balance enclosure, the uncertainties in the measured parameters were taken from Section 3.2.

Table 7 lists the volumes, v , and the estimated uncertainties, $U(v)$, for the kilogram standards.

The factors affecting v and hence $U(v)$ are the density, the coefficient of cubical expansion, α , and the actual temperature of the weight. In use, the volumes of the weights were calculated for the temperature at the start of a run; by the end of the run⁽¹³⁾, drifts up to about 0.08 °C were recorded. At 20 °C this temperature change corresponded to a volume change of about $\pm 0.0001 \text{ cm}^3$ for Pt-Ir kilograms (which could be neglected) and about $\pm 0.0005 \text{ cm}^3$ for the stainless steel kilograms. The value of α for Kgs 18, A and B is known with high accuracy as it was derived from linear expansion measurements at BIPM on metre bars of very similar composition to the kilograms; the effect of the uncertainty in α did not exceed $\pm 0.0001 \text{ cm}^3$ and could be neglected. For Kgs E and PI the values of α were taken from a BoT document of 1883[9][10]; the estimated uncertainty in the determination of α was $\pm 1\%$, giving a $U(v)$ of less than $\pm 0.0003 \text{ cm}^3$, which could also be neglected. However, for Kg E a systematic error had been introduced into the calculation of its volume due to the adoption of a different value of α in about 1883 (see below). For the stainless steel kilograms 36 and 36D an assumed value for α of $45 \times 10^{-6} \text{ }^\circ\text{C}^{-1}$ was used, with an estimated uncertainty of $\pm 3 \times 10^{-6} \text{ }^\circ\text{C}^{-1(14)}$; as the difference between the temperature at which the density was determined and the temperature of use was within 4 °C, the maximum effect of the uncertainty in α was a $U(v)$ of $\pm 0.0015 \text{ cm}^3$.

The estimates of the uncertainty given in Table 7 for the measured density (hence volume, $U(v)$) of each kilogram were derived as follows:

Kg 18: Twice the standard deviation of the original ten volume determinations at BIPM[4] ($\pm 0.0006 \text{ cm}^3$) was used.

Kg A: Duplicate density measurements at NPL on Kgs A and B differed by 5 to 10 ppm, equivalent to 0.0005 cm^3 ; twice this figure was used.

Kg B: As for Kg A.

Kg E: The estimated uncertainty in the measured density was ± 4 in 10^5 , equivalent to $\pm 0.0020 \text{ cm}^3$. The change in α from the value originally used ($27.00 \times 10^{-6} \text{ }^\circ\text{C}^{-1}$ in the density determination to the current value of $25.89 \times 10^{-6} \text{ }^\circ\text{C}^{-1}$) introduced a volume

(13) the duration of a run was approximately four hours

(14) this assumption was supported by determinations made at NPL and elsewhere on samples of similar composition to Set NPLW36 (see Reference 22)

error⁽¹⁵⁾ of 0.0017 cm³, which remained uncorrected during the period covered by this report. It has therefore been treated here as a type B uncertainty and combined (by quadratic summation) with the other $U(v)$ component to give a total $U(v)$ of ± 0.0026 cm³.

Kg PI: The estimated uncertainty in the density is ± 5 in 10^5 , equivalent to ± 0.0023 cm³.

Kg 36: The components of $U(v)$ were: ± 0.0051 cm³ due to the estimated uncertainty in density of ± 4 in 10^5 , ± 0.0015 cm³ due to α and ± 0.0005 cm³ due to temperature.

Quadratic summation gave a total $U(v)$ of ± 0.0053 cm³.

Kg 36D: As for Kg 36.

For each weighing, $U(\Delta v)$ was calculated by quadratic summation of the two $U(v)$ components. Inserting this value and the appropriate value of Δv into equation (14), together with $\rho_a = 1200$ $\mu\text{g cm}^{-3}$ and $U(\rho_a) = 0.284$ $\mu\text{g cm}^{-3}$, gave $U(\text{BC})$ for that weighing. Results for a selection of weighings are given in Table 8. For the first stage, $U(\text{BC})$ lies in the range ± 1.7 μg to ± 4.2 μg , depending on the weights involved; an average value of ± 3.0 μg has been used in Table 5. For weighings in the second stage, $U(\text{BC})$ was close to ± 24.0 μg .

5.4 INTERCOMPARISON PROCESS

The Massrun program derived the uncertainties due to the fit of the weighing data, which included all the type A uncertainties arising from the weighing process itself and the weighing scheme used. These were typically ± 5.0 μg and ± 12.8 μg , for the first and second stages respectively; the larger uncertainty for the second stage arises from the variations of air density during a weighing run coupled with the much larger buoyancy correction required.

The type B uncertainties arising from the uncertainties in the values of the weights used to poise the balance and the balance sensitivity were as follows:

(15) This figure includes the effect of changing from the water density values originally used in the density determination to those in current use.

(a) Makeweights

Uncertainties on the makeweights ($\pm 0.3 \mu\text{g}$ to $\pm 0.5 \mu\text{g}$) will be encompassed by the fitting of the data: this is because each weighing or "equation" in a weighing scheme will generally use different makeweights; their uncertainties will contribute to the equations' residuals and finally be included in the uncertainty calculated by "Massrun" for the weights in that scheme.

(b) Balance Sensitivity

The balance sensitivity was estimated to have an uncertainty of $\pm 5\%$. For the first stage the balance was poised to within $50 \mu\text{g}$, the maximum uncertainty from this source was therefore $\pm 2.5 \mu\text{g}$ (5% of $50 \mu\text{g}$). During the second stage the large volume difference between stainless steel and Pt-Ir kilograms rendered the state of poise very sensitive to small air density changes during a run; therefore the estimated uncertainty increased to $\pm 5 \mu\text{g}$.

(c) Height Correction

As the earth's gravitational field decreases by 3.14 parts in 10^7 for each metre height increase near sea-level, when comparing a pair of weights any height difference between their centres of gravity will require a correction to be applied to the mass difference obtained. Kgs PI and Set NPLW36 are of sufficiently different geometry from the other kilograms to give rise to significant corrections; these are $- 2.0 \mu\text{g}$ for Kg PI and $+3.1 \mu\text{g}$ for Kgs 36 and 36D⁽¹⁶⁾. Neither correction was applied during this period, thus introducing a systematic error.

5.5 SUMMARY

The estimated total uncertainties in the determination of the mass values of the kilograms discussed in this report are summarized in Table 5, which relates specifically to the 1988 determinations. These estimates (at the 2σ level) are $\pm 18 \mu\text{g}$ for the noble metal kilograms and $\pm 33 \mu\text{g}$ for the stainless steel kilograms, Kgs 36 and 36D. The uncertainty assessment made in 1988 did not include all of the sources considered here⁽¹⁷⁾ and some sources were assigned slightly different values; however, the total uncertainties used at that time were

(16) based on centres of gravity above their base of 13.2 mm for Kg PI, 30 mm for Set NPLW36 and 19.5 mm for all the other kilograms

(17) For example, the error in the volume of Kg E was discovered during researches for the preparation of Reference 3.

substantially the same as those given here. Indeed, the recognition of and correction for the instability in the mass of the National Prototype Kilogram, Kg 18, during this period represents an important improvement in the estimate of the actual mass value and its associated uncertainty.

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TABLE 1: MASS REGAIN OF Kg 18 AFTER CLEANING AND WASHING AT BIPM

DATE	MASS* /μg
As measured by BIPM:	
19 FEB 1985	3rd cleaning and washing†
14-19 MAR 1985	56.9
13-14 MAY 1985	58.1
24-26 JUN 1985	59.8
10-11 SEP 1985	60.2
17-25 OCT 1985	61.3
25-26 NOV 1985	60.8
11-12 DEC 1985	62.1
18-19 FEB 1986	59.5
7-8 APR 1986	62.4
5-6 MAY 1986	63.1
Values assigned by NPL by extrapolation of the BIPM results:	
OCT-NOV 1986	64
JAN-MAR 1988	65
JAN-FEB 1990	67

* Deviation from 1 kg

† See Reference 19 for details

TABLE 2: MASS VALUES OF Kg A AND Kg B

DATE	OBSERVER	BASIS	MASS* /μg		COMMENTS (and see notes below)
			Kg A	Kg B	
AUG 1982	BIPM	Kg 9, Kg 31	68	176	
NOV 1984	BIPM	Kg 9, Kg 31	-	175	Before cleaning
MAR 1985	BIPM	Kg 9, Kg 31	-	167	After cleaning at BIPM (a)
NOV 1986	NPL	Kg 18 = 1 kg + 64 μg	66	172	Note adjustments to Kg 18 value; see Note (b)
MAR 1988	NPL	Kg 18 = 1 kg + 65 μg	76	179	
FEB 1990	NPL	Kg 18 = 1 kg + 67 μg	-	179	

* Deviation from 1 kg

Notes:

- (a) Rubbed with chamois leather soaked in benzene then with chamois leather soaked in ethanol, followed by washing with jets of steam. Whole operation performed three times.
- (b) Values for Kg 18 extrapolated from March 1985 - May 1986 mass regain data, see Section 4.

TABLE 3: MASS VALUES OF Kgs E AND PI

DATE	BASIS	MASS* /μg	
		E	PI
SEP 1984	Kg 18 = 1 kg + 59 μg	- 2147†	-
NOV 1986	Kg 18 = 1 kg + 64 μg	- 2131	- 44028
MAR 1988	Kg 18 = 1 kg + 65 μg	- 2128	- 44027
FEB 1990	Kg 18 = 1 kg + 67 μg	- 2129	-

* Deviation from 1 kg

† This value is based on an un-adjusted value of Kg 18, prior to the BIPM study; adoption of the pre-lavage value of Nov 1984 of 1 kg + 76 μg gives a value for Kg E of 1 kg - 2130 μg.

TABLE 4: MASS VALUES OF Kg 36 AND Kg 36D

DATE	BASIS	MASS* /μg		MASS DIFFERENCE /μg
		Kg 36	Kg 36D	36 - 36D
SEP 1984	Kg B = 1 kg + 176 μg	2521	1320	1201
JUL 1988/ NOV 1989	Kg B = 1 kg + 179 μg Kg E = 1 kg - 2128 μg	2507	1309	1198

* Deviation from 1 kg

TABLE 5: SUMMARY OF UNCERTAINTIES

Source of Uncertainty	Uncertainty /± μg		
	Type A	Type B	Total
Kgs A, B, E, PI			
Kg 18: BIPM value		16	
Kg 18: NPL adjustment†		3	
Comparison (data fit)	5.0		
Balance sensitivity		2.5	
Buoyancy correction		3.0	
Totals	5.0	16.7	17.5
Kgs 36 and 36D			
Standards	5.0	16.7	
Comparison (data fit)	12.8		
Balance sensitivity		5	
Buoyancy correction		24	
Height correction		3.1	
Totals	13.7	29.8	32.9

† For Feb 1988 (this uncertainty increases with time)

TABLE 6: Uncertainty in determination of air density

Source of uncertainty	Uncertainty	$U(\rho_a)/\pm \mu\text{g cm}^{-3}$
Temperature	$\pm 0.01\text{ }^{\circ}\text{C}$	0.044
Pressure	$\pm 10\text{ Pa}$	0.119
Humidity	$\pm 2\%$	0.209
CO ₂ content	$\pm 24\text{ ppm}$	0.012
BIPM equation†	$\pm 12 \times 10^{-5}$	0.144
TOTAL‡		0.284

† This is discussed in Reference 13, which summarises various authors' assessments of, and ways of combining, both types of uncertainty and gives maximum, minimum and mean values. The value used here is a mean of the type B assessments with quadratic summation of component uncertainties, giving 6×10^{-5} at the 1σ level for the case $\rho_a = 1200\text{ }\mu\text{g cm}^{-3}$.

‡ Obtained by quadratic summation of individual components tabulated above, which is not strictly correct: as ρ_a is a non-linear function of the parameters listed, the method used for U(BC) (see equation 14) should be applied; however the simplified approach adopted here is considered to be of adequate accuracy.

TABLE 7: Uncertainty in volumes of standard kilograms

Kg	$v/\text{cm}^3(20^{\circ}\text{C})$	$U(v)/\pm \text{cm}^3$
18	46.439	0.0012
A	46.460	0.0010
B	46.464	0.0010
E	47.334	0.0026
PI	46.851	0.0023
36, 36D	127.682	0.0053

TABLE 8: Uncertainties arising from volume differences

Weighing	Δv $/\text{cm}^3$	$U(\Delta v)$ $/\pm \text{cm}^3$	$U(\text{BC})^*$ $/\pm \mu\text{g}$
A = B	0.0045	0.0014	1.68
18 = B	0.025	0.0016	1.92
18 = E	0.895	0.0029	3.49
E = PI	0.483	0.0035	4.20
36 = B	81.22	0.0054	24.0
36 = E	80.35	0.0059	23.9

* Based on nominal value of air density of $1200\text{ }\mu\text{g cm}^{-3}$ and an estimated uncertainty in air density determination of $\pm 0.284\text{ }\mu\text{g cm}^{-3}$.

APPENDIX 1: MAIN OPERATIONS CARRIED OUT BY "RUNBAL" SOFTWARE

The "Runbal" software performs the following main operations on the data from each "run":

- (1) Read initial information from input file. This includes:
 - (a) date and identification of the run; identification of the weights and makeweights;
 - (b) details of the weighings (the contents of each pan, the number of weighings for each pan position during the comparisons and the sensitivity determinations, the number of interchanges, turning-points and damping cycles used);
 - (c) volumes of the weights and makeweights;
 - (d) weight in air values of the sensitivity weights;
 - (e) operator's remarks;
- (2) Read measurement data from the input file. This comprises:
 - (a) nine sets of readings from the pressure, temperature and humidity sensors and the corresponding times;
 - (b) seven balance readings (turning-points) plus the corresponding times, for each of the thirteen comparisons;
 - (c) as (b) for each of the twelve rest-points in the left and right sensitivity determinations.
- (3) Convert the air parameter sensors' readings to pressure, temperature and relative humidity values using stored values of the instruments' calibration constants. Calculate an air density value, using the BIPM formula, for each time from (2) (a). Calculate, by linear interpolation between these values, an air density value at the time of each of the thirteen weight comparisons and the twelve sensitivity determination rest-points.
- (4) Estimate, from each set of seven turning-points, a rest-point and its variance. This is done by a least squares fitting of two exponential curves to the two sub-sets of turning-points (1,3,5,7, and 2,4,6), then obtaining the convergence value at infinite time.
- (5) Calculate the "left" and "right" sensitivities, S_L and S_R . The three sets of rest-points involved in the sensitivity determinations (corresponding to left weight loaded, right weight loaded, neither loaded) are related to S_L and S_R and the other parameters (air density, values of the sensitivity weights, volumes of the weights on the pans) by three sets of equations. All these equations include a term representing the zero reading (z) of the unloaded balance. To allow for drift of z with time (t) a quadratic function is used: $z(t) = c_0 + c_1t + c_2t^2$ where c_0 , c_1 , c_2 are the "drift coefficients". The whole system of equations (usually thirteen) is solved by a least-squares procedure to give values for S_L , S_R the drift coefficients and the variances. A mean sensitivity, S , is also calculated.
- (6) Calculate the mass difference between the pans plus contents. The two sets of rest-points resulting from the thirteen comparisons in a run are modelled by two sets of equations of the form:

$$\text{for pan 1 on the right } R_{R,i} = [(m_2 - m_1) - \rho_{R,i}(v_2 - v_1)]S + z(t_{R,i})$$

$$i = 1, \dots, n+1$$

and

for pan 1 on the left $R_{L,i} = [(m_1 - m_2) - \rho_{L,i}(v_1 - v_2)]S + z(t_{L,i})$

$i = 1, \dots, n$

where R_i is the rest-point reading for the i th comparison, m_1 and m_2 are the masses of pans 1 and 2 plus their contents, v_1 and v_2 are the volumes of the weights on the pans, ρ_i is the air density at the time of the i th comparison, S is the balance sensitivity, $z(t_i)$ is the "zero" reading, as described in (5) above, at the time of the i th comparison and n is the number of comparisons with pan 1 on the left. The system composed of the above two sets of equations is solved by a least-squares procedure which yields values for the mass difference $m_1 - m_2$, the drift coefficients defining $z(t)$ and the variances.

(7) Write to the output file the following:

- (a) the initial information as listed in (1)
- (b) for each set of air density parameter readings, the calculated values of pressure, temperature, relative humidity, air density and the associated time;
- (c) for each rest-point, the calculated rest-point value, the interpolated air density and the associated time;
- (d) the left, right and mean sensitivity values, the drift coefficients and the variances;
- (e) the mass difference ($m_1 - m_2$), the drift coefficients and the variances;
- (f) for each rest-point involved in the sensitivity determinations, the residual associated with the best-fit function generated by the solution mentioned in (5);
- (g) for each rest-point involved in the weight comparisons, the residual associated with the best-fit function generated by the solution mentioned in (6).

The mass difference between the two pans plus their contents ($m_1 - m_2$), is printed out with the other information listed in (7), with the exception that standard deviations, not variances, are printed.

APPENDIX 2: WEIGHING SCHEMES (cont)

MAY - JUL 1988 and NOV 1989⁽¹⁾

Weights used: Kgs B, E, 36, 36D

Weighings:	B = 36	E = 36
	B = 36D	E = 36D
	36 = 36D	

Evaluation by "Massrun"

Standard deviation of adjusted results = $\pm 6.6 \mu\text{g}$

JAN - FEB 1990

Weights used: Kgs 18, B, E.

Weighings:	18 = E	18 = B	E = B
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Evaluation by "square"

Standard deviation of adjusted results = $\pm 1.7 \mu\text{g}$

(1) The original weighing B = 36D yielded an unusually high residual and was repeated later (Nov 1989). As this reduced, by a factor of 4, that equation's residual and the overall standard deviation, the repeat result was used instead of the original in the final evaluation.

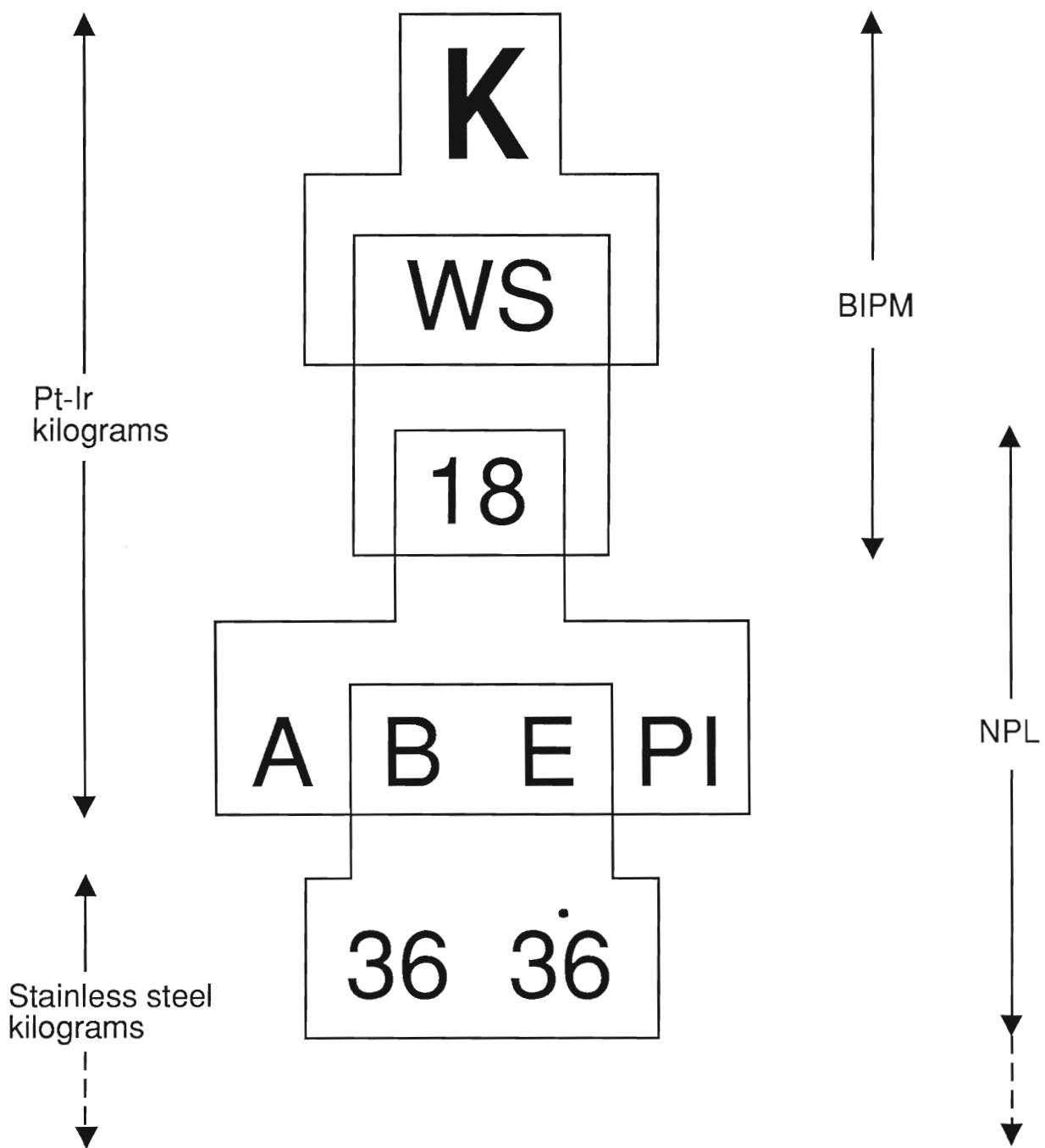


Figure 1: Initial stages of the mass calibration traceability route



Figure 2(a): Enclosure for platinum-iridium kilogram standards

KEY:

- A Bolts
- B Glass dome
- C Metal retaining ring
- D Collars
- E Hexagonal nut
- F Chamois leather covered arm - restrains laterally
- G Screw-knob for clamping N
- H Top-plate
- J Collar
- K Tommy-bar hole
- L Lower nut
- M Pillar assembly
- N Chamois covered pad - restrains vertically
- P Hexagonal nut

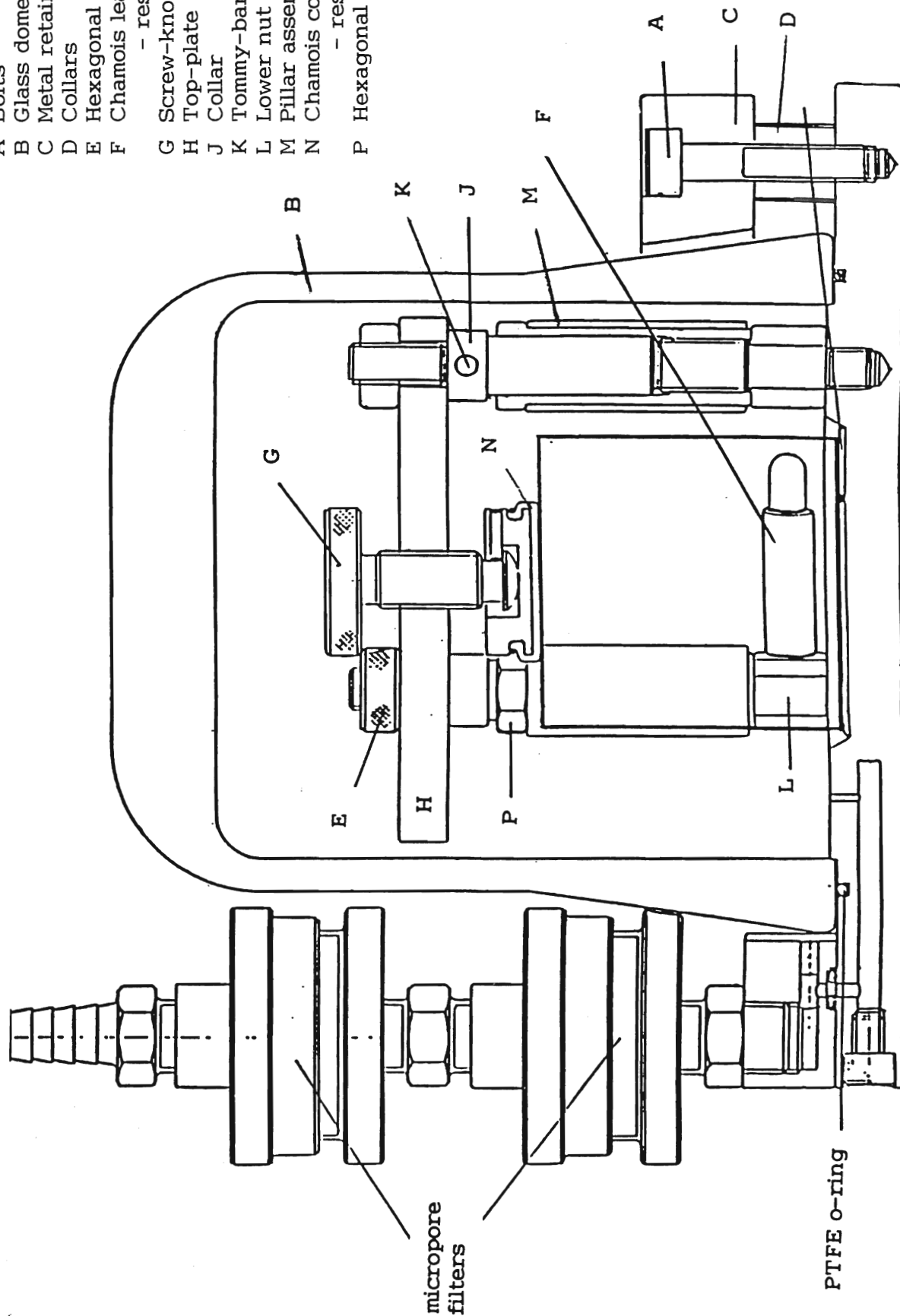


Figure 2(b): Enclosure for platinum-iridium kilogram standards

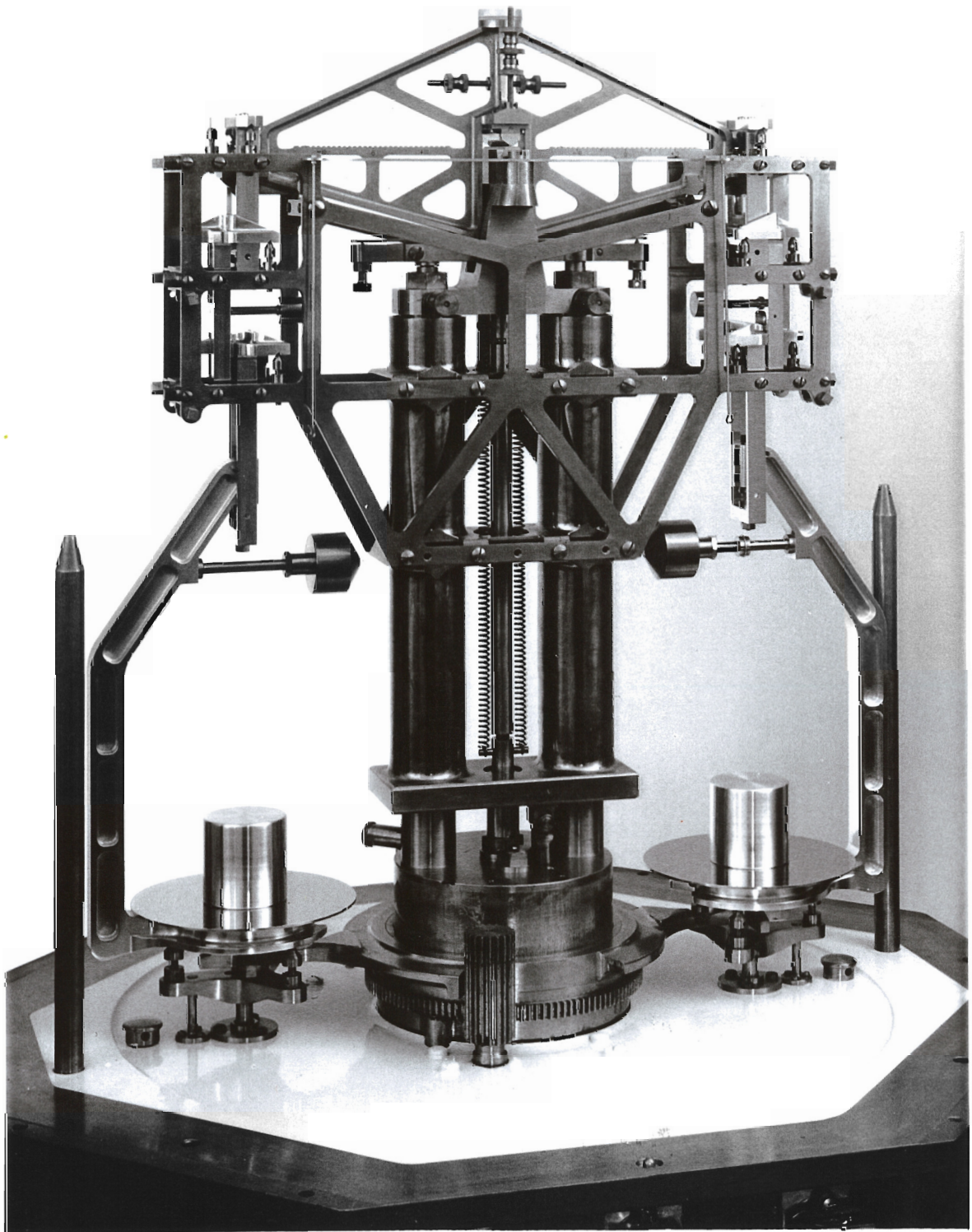


Figure 3: The NPL 1 kg primary balance

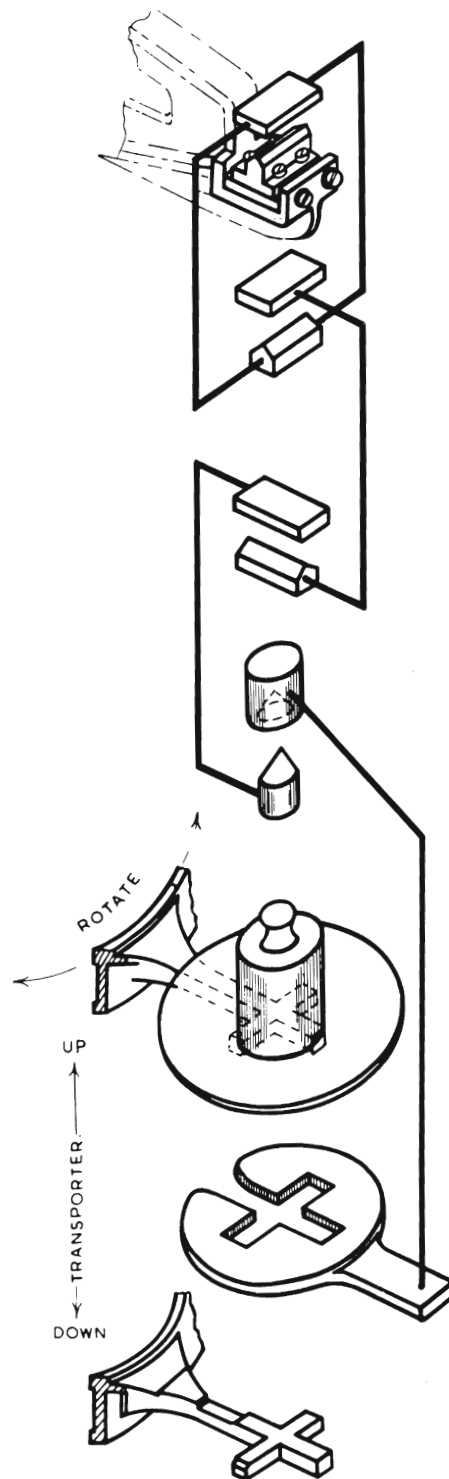


Figure 4: Pan suspension and weight interchange systems of the balance

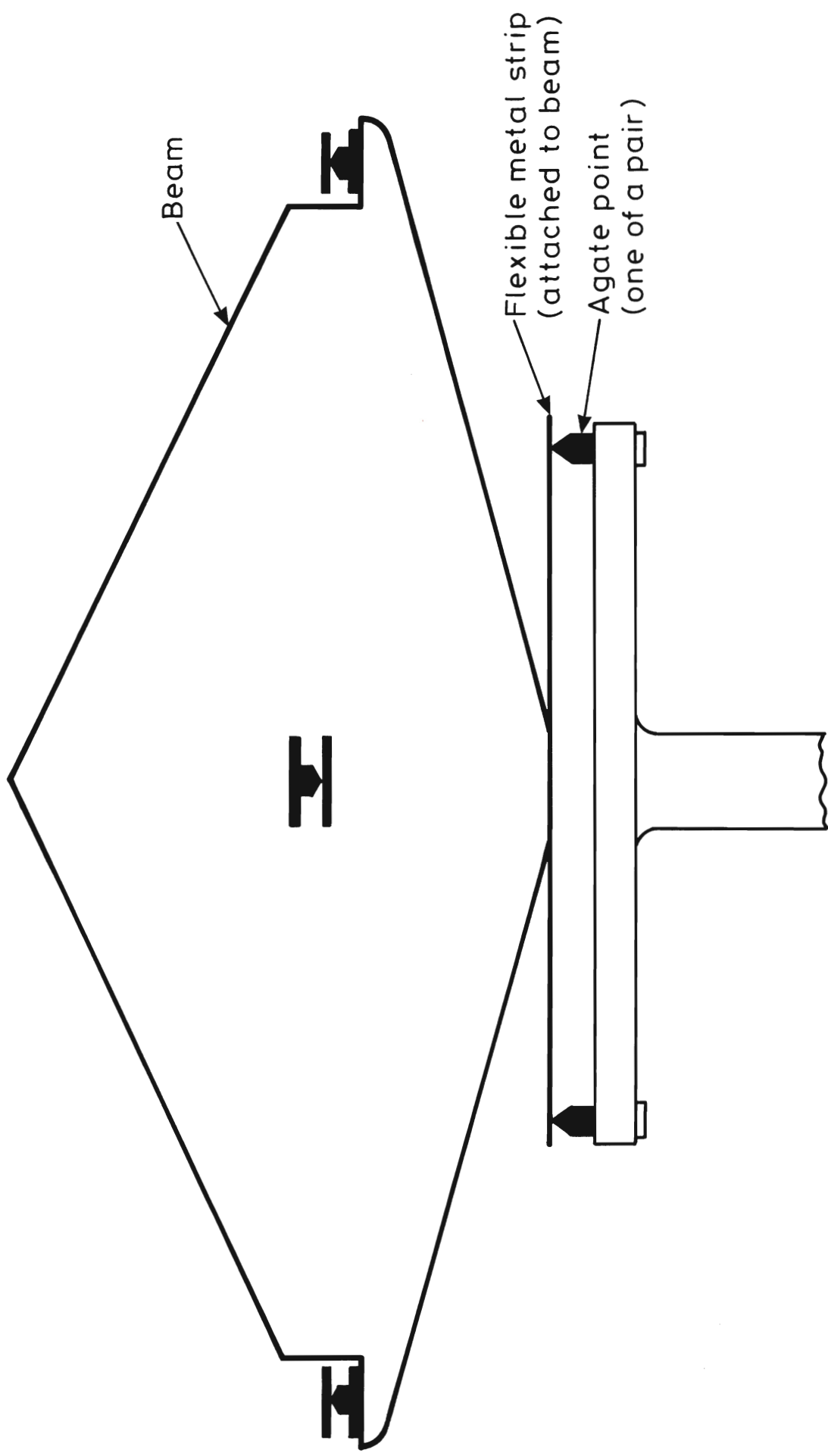


Figure 5: Device for steadying the beam of the primary balance

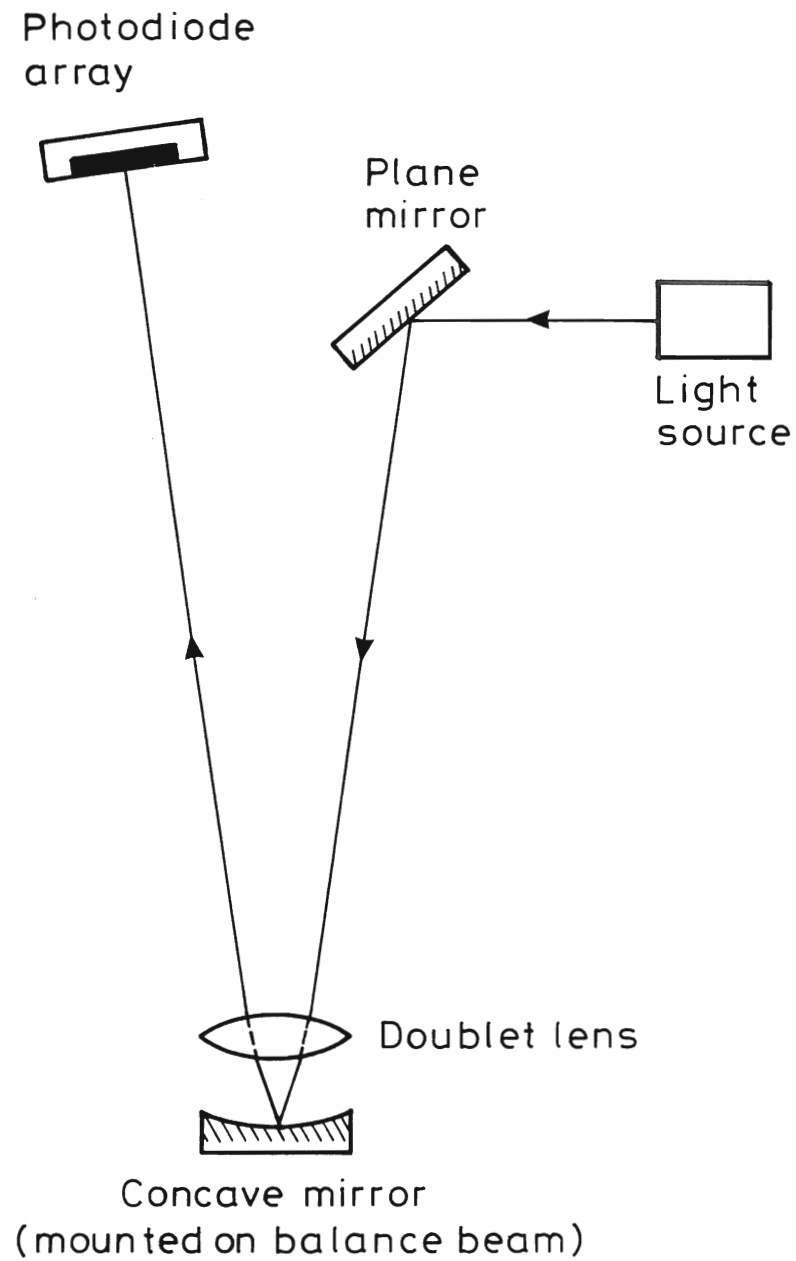


Figure 6(a): Optical lever system used to measure the balance beam inclination

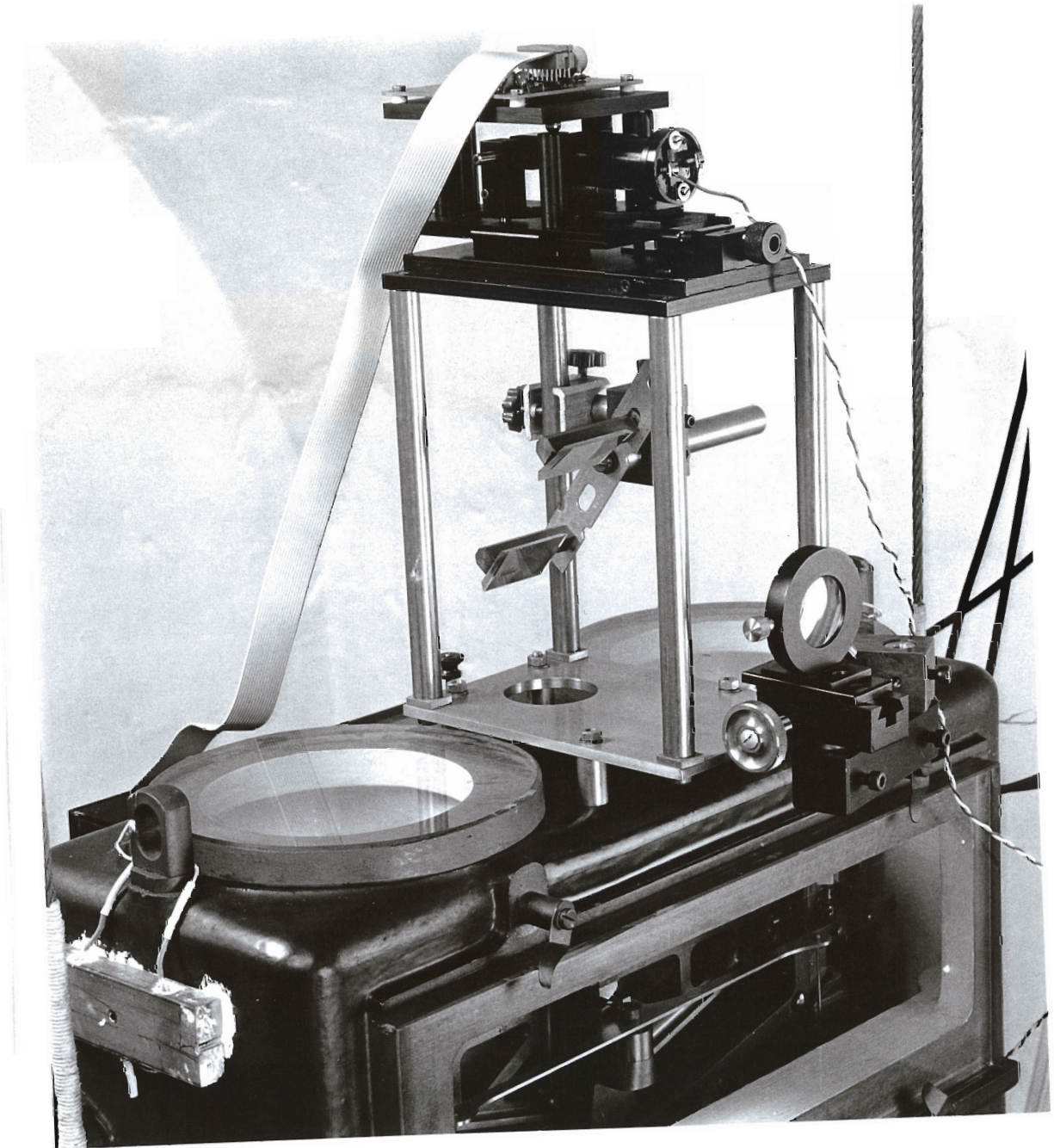


Figure 6(b): The balance optical lever system

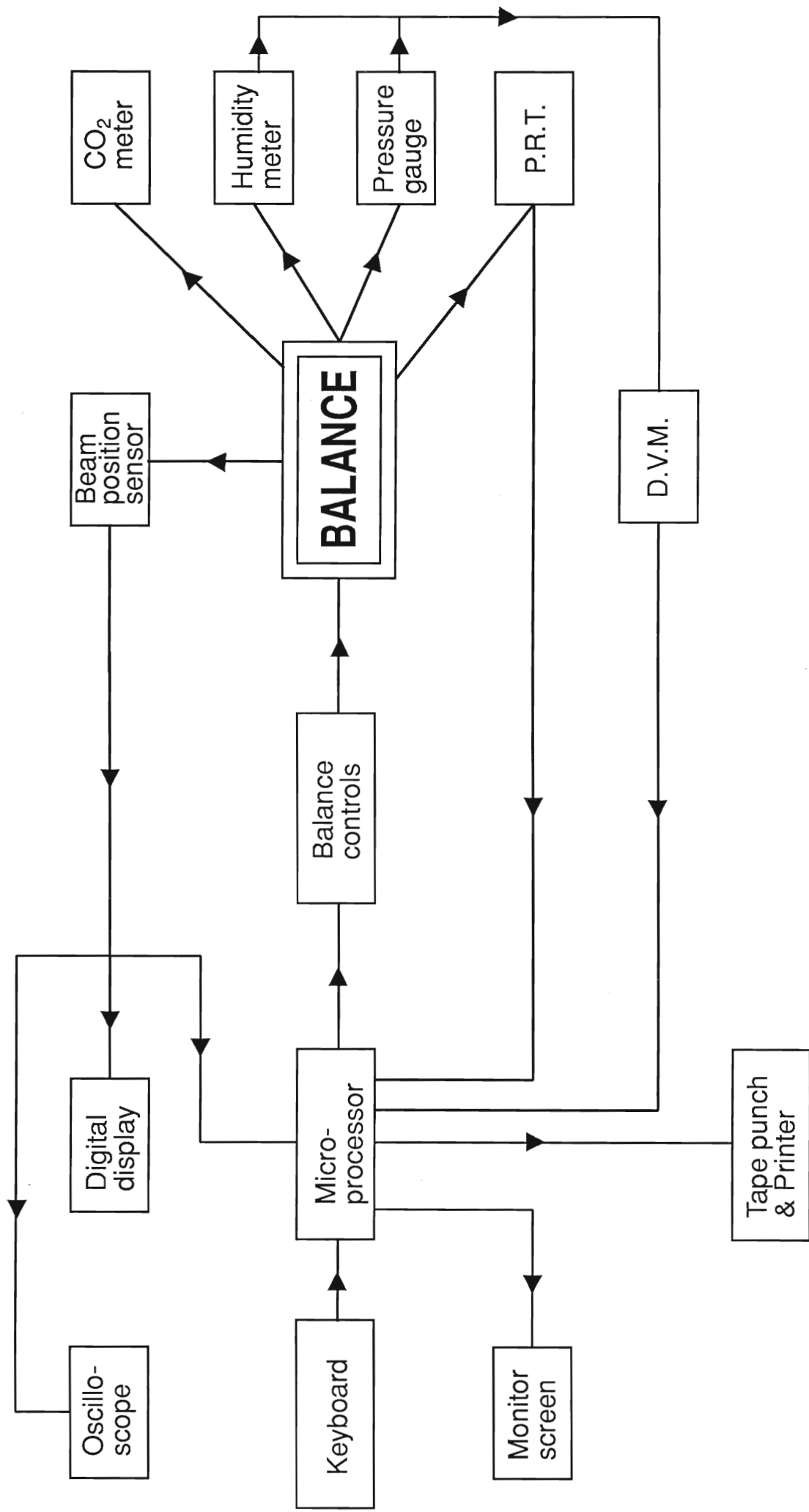


Figure 7: Control, measuring and recording system for the balance

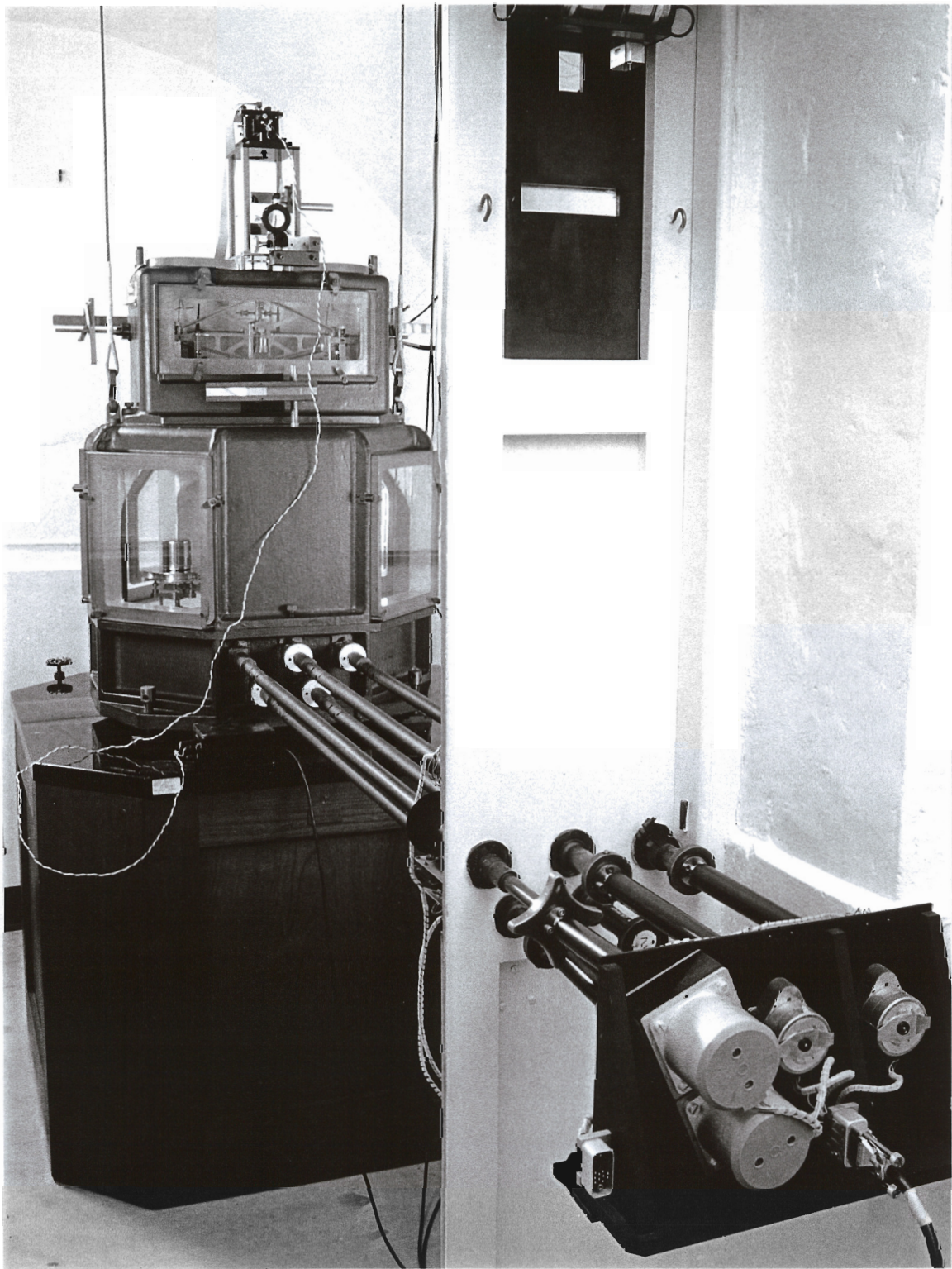


Figure 8: General view of the balance and its motorized remote controls

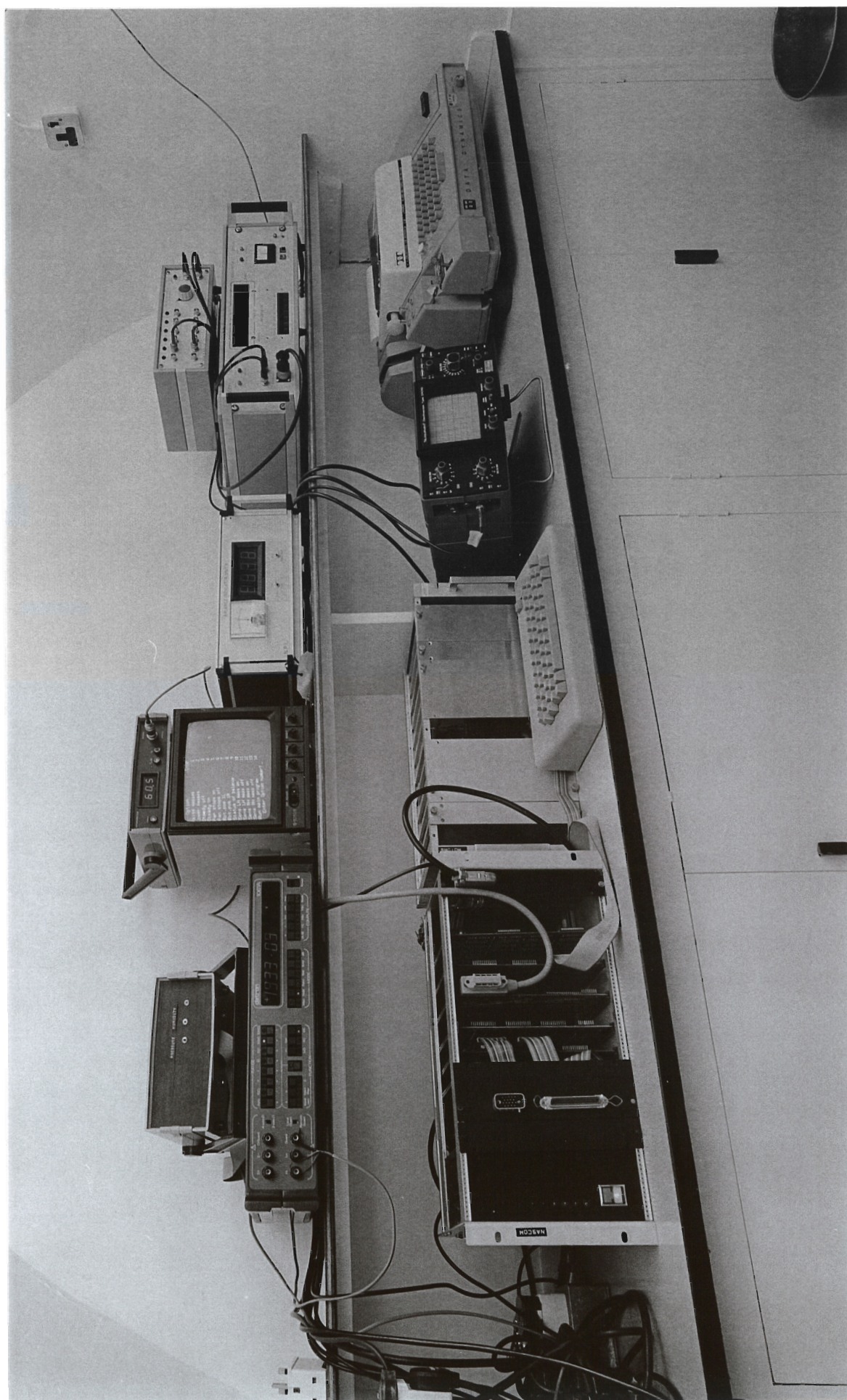


Figure 9: View of the control, measuring and recording equipment for the balance

Kg 18, Mass regain after cleaning
Date of cleaning: 19 February 1985

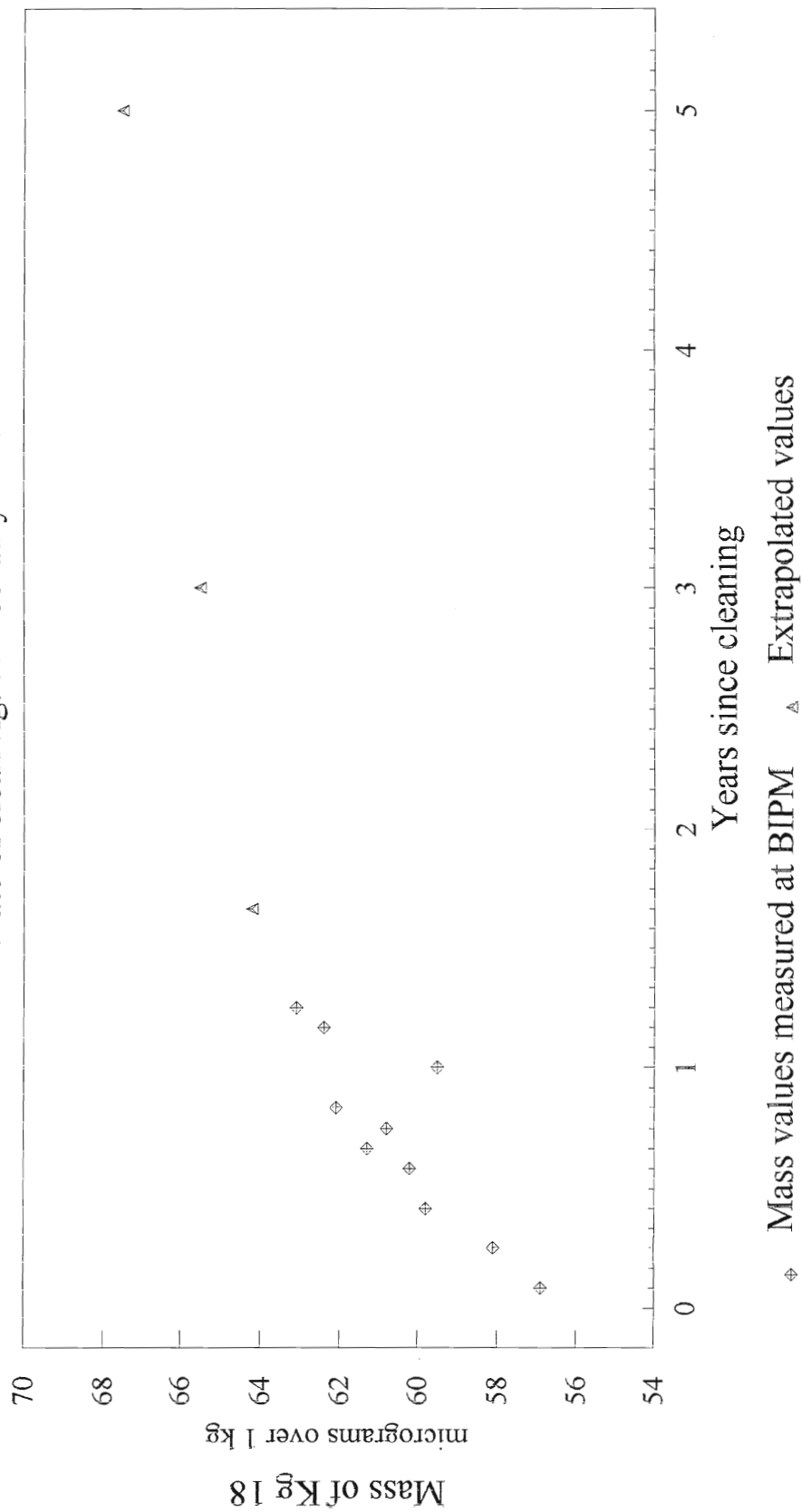


Figure 10: Mass regain of Kg 18 after cleaning in 1985 by BIPM