

**NPL REPORT
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**Calibration of two Spinning Rotor
Gauges as a bilateral comparison
between NPL and INRIM in the range
 $3 \cdot 10^{-4}$ Pa to $9 \cdot 10^{-2}$ Pa**

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DEPC

ABSTRACT

This report details measurements made at NPL and INRIM between July and August 2005 which form a comparison to determine the degree of equivalence of absolute pressure standards at these national metrology institutes.

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Approved on behalf of the Managing Director, NPL
by Susan Evans, Director, DEPC

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1. Introduction

This report details measurements made at NPL and INRIM between July and August 2005 which form a comparison to determine the degree of equivalence of absolute pressure standards at these national metrology institutes.

The format and certain general portions of the text of the report follow the EUROMET comparison report (EUROMET.M.P-K1.b, Project 442B, July 2000 [1]) and a subsequent bilateral comparison between PTB and CENAM (key comparison SIM-EUROMET.M.P-BK3, August 2004 [2]).

The transfer standard package consisted of two spinning rotor gauges (SRGs). The gauges were supplied by PTB and are those used in both EUROMET.M.P-K1b [1] and SIM-EUROMET.M.P-BK3 [2]. These have been shown to have excellent stability during comparisons, particularly due to the novel transportation mechanism which allows the rotors to travel in a fixed position under vacuum [3]. For the purposes of this bi-lateral comparison NPL acted as the pilot laboratory.

Comparison measurements at NPL were performed simultaneously for the two spinning rotor gauges, each being operated by a MKS SRG2 controller with a SH-700 sensing head. INRIM has only one MKS SRG2-CE control unit so their calibrations were performed independently.

Data was collected and analysed according to the protocol of the SIM-EUROMET.M.P-BK3 that was completed between April 2002 and June 2002 [2].

At both NPL and INRIM, the measurements were performed on consecutive days using the standards described below. The following target pressures were used for the calibration: $3 \cdot 10^{-4}$ Pa, $9 \cdot 10^{-4}$ Pa, $3 \cdot 10^{-3}$ Pa, $9 \cdot 10^{-3}$ Pa, $2 \cdot 10^{-2}$ Pa and $9 \cdot 10^{-2}$ Pa.

The report begins with a brief description of the vacuum standards used and their principle of operation. The measurements are then described and summarised. Finally, the evaluation of various uncertainty estimates is explained.

2. Participating laboratories and their standards

Laboratory	Standard	Character of Standard	Traceable to	CMC Listed
NPL	Static expansion system	Primary	independent	yes
INRIM	Continuous expansion system	Primary	independent	yes

Table 1. Details of participating standards.

2.1 The NPL series expansion system

The NPL series expansion system SEA2 used for this comparison is a CMC listed, independent primary standard which is shown schematically below.

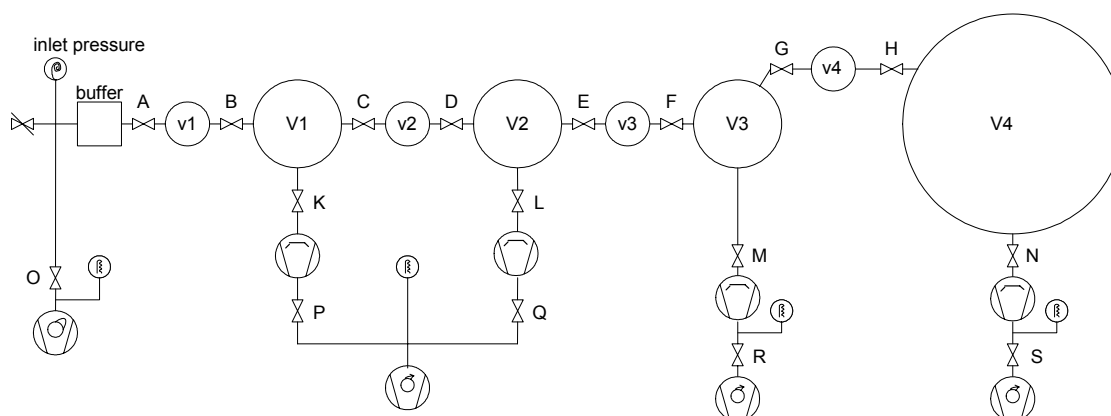


Figure 1. Schematic representation of the vacuum standard SEA2

Using this apparatus, calculable pressures between $5 \cdot 10^{-7}$ Pa and 900 Pa may be generated using the series expansion method.

In operation, a sample of gas is contained in one of the small vessels (i.e. v1, v2, v3 or v4) and then expanded into the next large and small vessels, which have been previously evacuated to a high vacuum. This procedure is then repeated using subsequent expansion stages until the gas is expanded into the calibration vessel (V4). The pressure of the initial gas sample is measured using a calibrated resonant silicon

gauge. By varying the initial pressure and the number of expansion stages, a range of pressures may be generated in the calibration vessel. The pressure generated is calculated from knowledge of the initial pressure, the gas temperature and the ratio of the relevant volumes e.g. $(v_3+V_3+v_4)/v_3$.

2.2 The INRIM continuous expansion standard

The INRIM continuous expansion system involved in this comparison is based on the expansion of a pure gas, which is pumped through a fixed conductance.

Figure 2. Schematic representation of the INRIM vacuum standard.

The fixed conductance was evaluated by both analytical and Monte Carlo methods.

The effective pumping speed is computed from the values of calculated conductance and the ratio of the conductance to the pumping speed (periodically measured).

The gas flow-rate device used to generate the pressures into the fixed conductance IMGC-CNR-D2 system is based on the constant pressure – variable volume method. The upper limit of the molecular range of the system is 10^{-1} Pa.

3. Transfer standards

The spinning rotor gauge is widely accepted as a transfer standard due to its measurement accuracy and long-term and transport stability. Two devices were used in order to reduce the influence of transport instabilities, to provide redundancy and to increase the accuracy of the comparison.

There has still been no published evidence to indicate that a change of controller and sensing head has an influence on the calibration results of a rotor/finger combination. It was therefore permissible for each laboratory to use their own controllers.

The details of the two spinning rotor gauges used as transfer standards are shown in Table 2.

Transfer Standard	Rotor 1	Rotor 2
Identifier	25 UM	23 UM
Material	Etched Stainless Steel	Invar
Nominal diameter	4.762 mm	4.50 mm
Nominal density	7.715 g /cm ³	8.2 g /cm ³

Table 2 Transfer standards used for comparison.

Data exists from 1994 onwards for the calibration of each rotor at PTB. [2]

4. Calibration constant

As reported elsewhere [1], [2], the value to be calibrated by each laboratory j for each rotor i was the effective accommodation coefficient σ_{ij} which is mainly determined by the tangential momentum accommodation coefficient of the gas molecules to the rotor, and partly by the energy accommodation factor and additionally by using nominal values for diameter and density of the rotors instead of the real ones.

σ_{ij} was determined by the following equation:

$$\sigma_{ij} = \sqrt{\frac{8kT_j}{\pi m}} \frac{\pi d_i \rho_i}{20 p_{stj}} \left(\left(\frac{\dot{\omega}}{\omega} \right)_i - RD_i(\omega) \right) \quad (1)$$

where :

p_{st} = is the generated pressure in the standard

T_j = the temperature of the gas in the calibration vessel

d_i = the nominal diameter of the rotor i

ρ_i = the nominal density of the rotor i

m = the molecular mass of nitrogen

$(\dot{\omega}/\omega)_i$ = the relative deceleration rate of the rotor frequency ω (DCR)

RD_i = pressure independent residual drag

RD is a function of ω , $RD(\omega)$, so whenever a determination of σ_{ij} is made it was necessary to measure the value of ω in order to subtract the correct $RD(\omega)$ in Eq.(1).

For measuring $RD(\omega)$ two different approaches were used by the participants.

At INRIM : Before starting the calibrations the rotor frequency dependence of the residual drag (in units $DCR = s^{-1}$) was measured over a long period of time. Furthermore, before each of the three calibration cycles of each rotor, the behaviour of the offset signal was checked to verify that it was comparable with the estimate of $RD(\omega)$ obtained by the previous “long” acquisition. The rotor frequency covered the full range that may occur during the calibration. The residual pressure in the standard was about $1 \cdot 10^{-6}$ Pa.

At NPL : $RD(\omega)$ was determined during the course of the calibrations by pumping down the vacuum system to residual pressure conditions after each target pressure point.

In both cases a linear least square fit is applied to obtain the function $RD(\omega)$.

5. Organisation of the comparison and schedule

Two spinning rotor gauges were used as transfer standards as described in Section 3. An initial calibration was completed at NPL before the standards were transported by courier to INRIM. The novel transport mechanism designed by PTB enabled the transportation of the rotors in fixed positions whilst under vacuum [3].

Following calibration at INRIM, the standards were returned to NPL for a repeat calibration.

To monitor any transport instability, a third spinning rotor gauge (check standard) was calibrated by NPL at the same time as the transfer gauges. It was not removed from the NPL primary standard throughout the duration of the comparison.

The chronology of the calibrations is shown below in Table 3.

Calibrating Laboratory	Dates	Notes
NPL	July 12, 13, 14 2005	Afterwards, sent by courier to INRIM
INRIM	July 27, 28, 29 and August 2, 3, 4 2005	Afterwards, sent by courier to NPL
NPL	August 24, 25, 26 2005	Comparison Finished

Table 3. Chronology of measurements

6. Calibration procedure and reported results

The calibration procedure agreed upon before the comparison was similar to that of the SIM-EUROMET.M.P-BK3 bilateral comparison [2] : Each laboratory calibrated the two spinning rotor gauges at 6 nominal target pressures p_t in ascending order: $3 \cdot 10^{-4}$ Pa, $9 \cdot 10^{-4}$ Pa, $3 \cdot 10^{-3}$ Pa, $9 \cdot 10^{-3}$ Pa, $2 \cdot 10^{-2}$ Pa and $9 \cdot 10^{-2}$ Pa..

A tolerance of $\pm 10\%$ in establishing the nominal pressure was accepted for $p_t < 9 \cdot 10^{-2}$ Pa and $\pm 5\%$ for $9 \cdot 10^{-2}$ Pa. Each target pressure had to be generated 3 times. After a measurement at the target point, the system was pumped down to residual pressure conditions and the same point re-generated. In total 18 measurements were made in each calibration sequence. It was required that this calibration sequence be repeated on three separate days.

The number of readings and sample interval times for each point were as follows:

- 30s with 10 repeat readings for the target points $3 \cdot 10^{-4}$ Pa, $9 \cdot 10^{-4}$ Pa and $3 \cdot 10^{-3}$ Pa.
- 30s with 5 repeat readings for $9 \cdot 10^{-3}$ Pa, $2 \cdot 10^{-2}$ Pa and $9 \cdot 10^{-2}$ Pa

It was required that the offset be measured during the calibration no matter what method had been chosen for offset determination:

- at the beginning of the calibration sequence.
- after the target pressures $3 \cdot 10^{-4}$ Pa, $9 \cdot 10^{-4}$ Pa, $3 \cdot 10^{-3}$ Pa and $9 \cdot 10^{-2}$ Pa (at the end of the calibration sequence).

It was agreed that no bake-out should be performed as part of the calibration sequence since it is both time consuming and may also affect the accommodation coefficient.

At the end of this calibration procedure, for each generated p_{stj} near the respective target point and for each rotor i and for each of the calibration sequences a value for σ_{ij} existed. Together with the value of σ_{ij} , each laboratory also reported the standard uncertainty $u(p_{stj})$ of p_{stj} .

7. Uncertainties of primary standards

Table 4 shows the relative standard uncertainties due to Type B uncertainties for the two primary standards. Type A uncertainties of the standards will be incorporated in the scatter of data for repeat measurements.

p_{st} (Pa)	NPL	INRIM
3.00E-04	3.04E-03	7.30E-03
9.00E-04	2.98E-03	6.40E-03
3.00E-03	2.97E-03	4.70E-03
9.00E-03	2.97E-03	3.60E-03
2.00E-02	2.97E-03	3.40E-03
9.00E-02	2.58E-03	3.20E-03

Table 4 Relative standard uncertainties of generated pressures due to systematic effects (Type B) as reported by participants.

8. Uncertainties of reported measured values

Again, following the analysis of key comparison SIM-EUROMET.M.P-BK3 [2], we introduce:

$$K_i = \sqrt{\frac{8k \cdot 296.15}{\pi m}} \frac{\pi d_i \rho_i}{20}, \quad (2)$$

and

$$DCR_i = \left(\frac{\dot{\omega}}{\omega} \right)_i \quad (3)$$

The effective accommodation coefficient has been measured $n = 9$ times σ_{ijk} , $k=1 \dots n$, therefore we can write :

$$\sigma_{ijk} = \sqrt{\frac{T_{jk}}{296.15}} \cdot \frac{K_i}{p_{stjk}} (DCR_{ik} - RD_{ik}(\omega)) \quad (4)$$

and Eq. (1) can be replaced by taking the mean of the repeated measurements to give

$$\sigma_{ij} = \frac{1}{n} \sum_{k=1}^n \sigma_{ijk} \quad (5)$$

The same values for k , m , d_i , and ρ_i were used by each laboratory and the K_i were therefore fully correlated, so that in effect no uncertainty needed to be attributed to K_i (a systematic error in d_i for example, would be calibrated into σ_{ij} in the same way at each laboratory and would have no effect on the result of the comparison). All type A uncertainties of values on the right hand side of Eq. (4) will contribute to the scatter of σ_{ijk} and hence the standard deviation of σ_{ij} . Therefore since Type A uncertainties are accounted for by the standard deviation, for calculation of the overall uncertainty of σ_{ij} only the type B uncertainties of values on the right hand side of Eq. (4) have to be evaluated for inclusion. For DCR_i there is no such uncertainty. $RD(\omega)$ is not determined at the same time as DCR_i , but before or after the measurement, and so therefore has a Type B uncertainty. Also the gas temperature and the generated pressure p_{stj} will have Type B uncertainties which are known before the measurements.

For this reason the variance in σ_{ij} is given by:

$$u_{\sigma_{ij}} = \frac{n-1}{n-3} s_{\sigma_{ij}}^2 + \left(\frac{\partial \sigma_{ij}}{\partial RD_i} \right)^2 u_{RD_i}^2 + \left(\frac{\partial \sigma_{ij}}{\partial T_j} \right)^2 u_{T_j}^2 + \left(\frac{\partial \sigma_{ij}}{\partial p_{stj}} \right)^2 u_{p_{stj}}^2, \quad (6)$$

where $s_{\sigma_{ij}}^2$ is the square of the standard deviation of the mean of the repeat measurements σ_{ijk} and where we understand that all standard uncertainties u are due to systematic effects which do not contribute to the scatter of σ_{ij} . Since only 9 measurements were taken, $s_{\sigma_{ij}}$ was multiplied by $\sqrt{(n-1)/(n-3)}$, i.e. 1.33 for $n = 9$ as suggested by Kacker and Jones [4]. The last term in Eq. (6) is due to the generated

pressure as described in section 7, all other terms are due to uncertainties of the transfer standard.

The sensitivity coefficients are:

$$\left(\frac{\partial \sigma_{ij}}{\partial RD_i} \right) = -\frac{K_i}{p_{stj}} \cdot \sqrt{\frac{T_j}{296.15}} \quad (7)$$

$$\left(\frac{\partial \sigma_{ij}}{\partial p_{stj}} \right) = -\frac{K_i}{p_{stj}^2} \cdot \sqrt{\frac{T_j}{296.15}} (DCR_i - RD_i) \quad (8)$$

$$\left(\frac{\partial \sigma_{ij}}{\partial T_j} \right) = \frac{1}{2} \frac{K_i}{p_{stj}} \cdot \frac{1}{\sqrt{296.15}} \cdot \frac{1}{\sqrt{T_j}} (DCR_i - RD_i) \quad (9)$$

$u(p_{stj})$ and $u(T_j)$ were reported by each laboratory, where it is assumed that $T_{jk} \approx T_j$ (constant temperature of standard during 3 repeat measurements). The following effects may contribute to the uncertainty of the residual drag RD_i :

- the scatter of the measurement results,
- the imprecisely known frequency dependence of the offset,
- a possible drift of the offset values between it's determination and the time of calibration,

9. Reported results of the laboratories

As NPL had carried out two calibration sequences and INRIM one, it was possible to check to see if a significant change could be observed between σ_{ij} for each sequence.

No shift in the mean σ_{ij} value was seen at NPL between the results collected in July 2005 and August 2005 for both of the spinning rotor gauges used as transfer standards.

Therefore the mean value of all data for a single rotor at a single target pressure could be taken for data reduction:

$$\sigma_{ij} = \frac{1}{n} \sum_{k=1}^n \sigma_{ijk} \quad n = 9 \quad (5)$$

The results reported by each laboratory and the corresponding uncertainties according to Eq. (6) are shown in the following Tables and Figures.

P_t /Pa	NPL 1	Jul 05	INRIM 1	Jul/Aug 05	NPL 2	Aug 05
	σ_1	$u(\sigma_1)$	σ_1	$u(\sigma_1)$	σ_1	$u(\sigma_1)$
3.00E-04	1.1467	0.0145	1.1657	0.0088	1.1461	0.0171
9.00E-04	1.1461	0.0072	1.1661	0.0075	1.1455	0.0079
3.00E-03	1.1494	0.0039	1.1645	0.0055	1.1469	0.0040
9.00E-03	1.1499	0.0036	1.1626	0.0042	1.1485	0.0036
2.00E-02	1.1504	0.0036	1.1617	0.0040	1.1491	0.0036
9.00E-02	1.1505	0.0031	1.1509	0.0037	1.1484	0.0031

Table 5. The mean values σ_{2j} of the reported results for Rotor 1 (25 UM) and the uncertainties as calculated by Eq.(6).

Rotor 1 (25 UM)

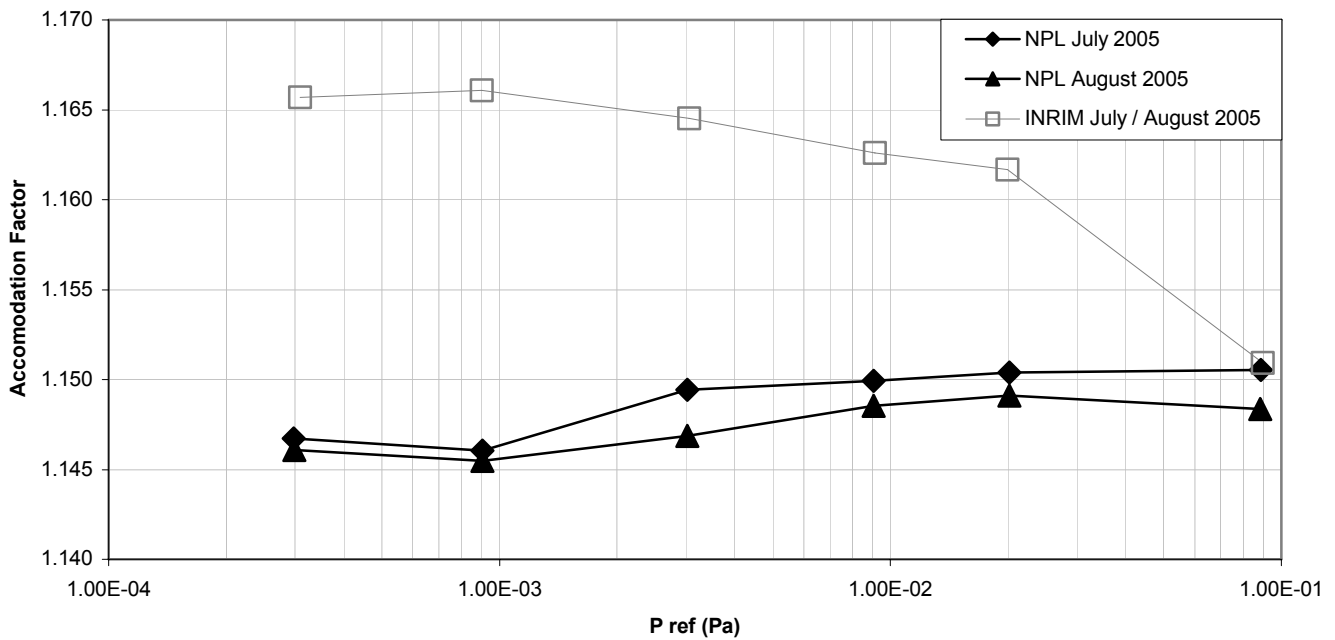


Figure 3. Graphical presentation of the results of Rotor 1 (Table 6). Uncertainties are not shown for better visibility and can be taken from table 5.

P_t / Pa	NPL 1	Jul 05	INRIM 1	Jul/Aug 05	NPL 2	Aug 05
	σ_2	$u(\sigma_2)$	σ_2	$u(\sigma_2)$	σ_2	$u(\sigma_2)$
3.00E-04	1.1061	0.0346	1.1191	0.0084	1.1062	0.0441
9.00E-04	1.1112	0.0120	1.1231	0.0072	1.1108	0.0148
3.00E-03	1.1131	0.0051	1.1245	0.0053	1.1125	0.0056
9.00E-03	1.1136	0.0038	1.1228	0.0041	1.1129	0.0037
2.00E-02	1.1138	0.0036	1.1217	0.0039	1.1133	0.0035
9.00E-02	1.1141	0.0030	1.1121	0.0036	1.1126	0.0030

Table 6. The mean values σ_{ij} of the reported results for Rotor 2 and the uncertainties as calculated by Eq.(6)

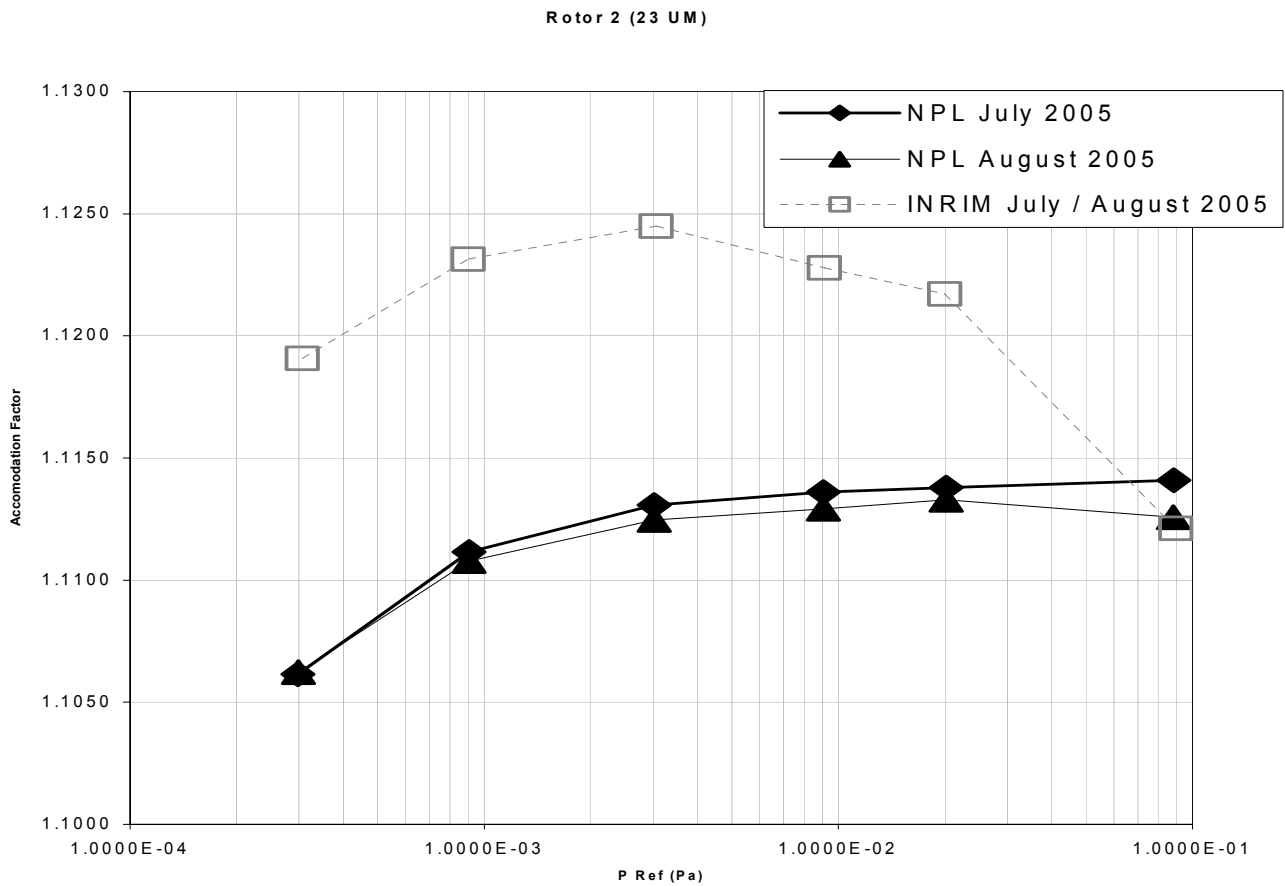


Figure 4. Graphical presentation of the results of Rotor 2 (23 UM) (Table 5). Uncertainties are not shown for better visibility and can be taken from table 6.

The results show that the values obtained at INRIM are systematically higher than those measured at NPL for all but the highest test pressure.

Rotor 1 shows a difference in accommodation factor ranging from 1.8 % to 1.0 % in the molecular regime between NPL and INRIM values, except for the target pressure $9 \cdot 10^{-2}$ Pa, characterized by a difference of about 0.1 % . Rotor 2 shows a maximum difference in accommodation factor of around 1.2 % in the molecular regime between NPL and INRIM values.

Good transport stability is shown by the rotors as the values of accommodation factor for NPL2 are within 0.25% of those recorded during NPL1.

10. Stability of Transfer gauges

In order to monitor the transport stability of the calibration constant of the three rotors during the course of the calibration, the mean values of σ_1 and σ_2 between $9 \cdot 10^{-4}$ Pa and $2 \cdot 10^{-2}$ Pa of the pilot laboratory were calculated. Rotor 3 was the check standard which did not travel.

	σ_1	$s(\sigma_1)$	σ_2	$s(\sigma_2)$	σ_3	$s(\sigma_3)$
NPL1	1.1489	0.0010	1.1129	0.0006	1.0028	0.0008
NPL2	1.1475	0.0016	1.1124	0.0006	1.0011	0.0010

Table 7 The mean values of σ_1 and σ_2 between $9 \cdot 10^{-4}$ Pa and $2 \cdot 10^{-2}$ Pa for the two NPL calibrations and the respective scatter (standard deviation s) of the mean. Rotor 3 did not travel and served as a check standard for NPL.

As was argued in SIM-EUROMET.M.P-BK3 [2], the impossibility of being able to calibrate the transfer standards simultaneously on the different primary standards requires the mean value of the pilot laboratory calibrations to be calculated. This gives an approximation of a coincident time at which both participants values can be compared.

The measured values between NPL1 and NPL2 are within the combined standard deviation for the two rotors which travelled and also for the check standard which did not travel.

It is therefore reasonable to assume that both the effective accommodation factors did not change during this bilateral comparison.

11. Data reduction and evaluation of the degree of equivalence

As outlined in the previous section, it is reasonable to assume that the transport instability of the two spinning rotor gauges was zero during this comparison. For this reason all values taken at the pilot laboratory NPL can be assumed to belong to the same parent population and the mean of all this data can be taken.

$$\sigma_{ij} = \frac{1}{n} \sum_{k=1}^n \sigma_{ijk} \quad n = 18 \quad (10)$$

The situation is similar to the SIM-EUROMET.M.P-BK3 [2] therefore the analysis presented in that paper can be followed. Since one of the goals of this comparison is to compare the generated pressures of the two standards, a pressure value has to be generated from the σ_{ij} .

For the data from each laboratory and for each spinning rotor gauge i a value of indicated pressure p_{ij} for a common hypothetical target pressure p_t can be calculated with the following equation:

$$p_{ij} = p_t \cdot \sigma_{ij} \quad i = 1,2 \quad j = 1,2 \quad (11)$$

σ_{ij} denotes the mean accommodation coefficient according to Eq.(5) of spinning rotor gauge i as determined by INRIM ($j=1$) or according to Eq (10) of SRG i as determined by NPL ($j=2$).

The method adopted here is to use σ_{ij} to predict gauge readings that would be observed when the different standards are set to generate pressures of the same value at a coincident time. The difference in the predicted gauge readings is taken as an indicator of the difference between the true pressures actually realised by the different standards (“generated pressures”) when the two laboratories state the same calculated value. This latter difference between calculated pressures, when the standards are set to produce exactly the same transfer gauge reading near the target pressure, is to a

very good approximation (provided the differences are small) equal to the difference in the predicted gauge readings but of opposite sign.

It was found by Jousten [5] that σ_{ij} may be slightly temperature dependent. The size of this effect varies from rotor to rotor and is also dependent on the specific surface condition of a single rotor. Relative temperature changes of $(\Delta\sigma_{eff} / \Delta T) / \sigma_{eff} = -1 \cdot 10^{-4} / \text{K}$ to $-4 \cdot 10^{-4} / \text{K}$ were found around room temperature.

The mean temperatures during calibrations at NPL (293.08 K) and INRIM (293.65 K) differed by 0.57 K. Since the temperature dependence of σ_{eff} of the transfer standards was not measured and may also have changed during the comparison, the only possibility to consider this effect is to add an uncertainty for the p_{ij} in Eq. (11)

If we assume a mean temperature of 293.365 K so that each laboratory differs by 0.285 K from this temperature, and a possible temperature dependence of $(\Delta\sigma_{eff} / \Delta T) / \sigma_{eff} = -2 \cdot 10^{-4} / \text{K}$ to calculate the standard uncertainty of the temperature effect for the p_{ij} in Eq. (11)

$$u_{T\sigma}(p_{ij}) = 0.57 \cdot 10^{-4} p_{ij} \quad (12)$$

Since p_t is simply a numerical value without uncertainty, the uncertainty of p_{ij} is calculated from the following equation:

$$u(p_{ij}) = p_{ij} \sqrt{\left(\frac{u(\sigma_{ij})}{\sigma_{ij}}\right)^2 + \left(\frac{u_{T\sigma}(p_{ij})}{p_{ij}}\right)^2} \quad (13)$$

$u(\sigma_{ij})$ was given in Eq. (6) and $u_{T\sigma}(p_{ij})$ in Eq. (12)

The following tables show the results for the p_{ij} obtained from Eqs. (11) and (13).

P_t (Pa)	P_{1NPL} (Pa)	$u(P_{1NPL})$	P_{1INRIM} (Pa)	$u(P_{1INRIM})$
3.0E-04	3.439E-04	3.356E-06	3.497E-04	2.641E-06
9.0E-04	1.031E-03	4.817E-06	1.049E-03	6.791E-06
3.0E-03	3.444E-03	8.464E-06	3.494E-03	1.655E-05
9.0E-03	1.034E-02	2.304E-05	1.046E-02	3.813E-05
2.0E-02	2.299E-02	5.047E-05	2.323E-02	7.994E-05
9.0E-02	1.035E-01	1.991E-04	1.036E-01	3.363E-04

Table 8 : The predicted gauge readings and their uncertainties as obtained from Eqs. (11) and (13) for rotor 1.

P_t (Pa)	P_{2NPL} (Pa)	$u(P_{2NPL})$	P_{2INRIM} (Pa)	$u(P_{2INRIM})$
3.0E-04	3.319E-04	8.403E-06	3.357E-04	2.517E-06
9.0E-04	9.999E-04	8.595E-06	1.011E-03	6.504E-06
3.0E-03	3.338E-03	1.128E-05	3.373E-03	1.594E-05
9.0E-03	1.002E-02	2.395E-05	1.011E-02	3.674E-05
2.0E-02	2.227E-02	5.006E-05	2.243E-02	7.725E-05
9.0E-02	1.002E-01	1.941E-04	1.001E-01	3.238E-04

Table 9 : The predicted gauge readings and their uncertainties as obtained from Eqs. (11) and (13) for rotor 2.

The degree of equivalence between the two standards can be tested using [6] :

$$d_i = p_{iINRIM} - p_{iNPL} \quad (14)$$

for each target point and each transfer standard i . The uncertainty $u(d_i)$ is given by

$$u(d_i) = \sqrt{u^2(p_{iINRIM}) + u^2(p_{iNPL})} \quad (15)$$

However, since this comparison covered 3 orders of magnitude of pressure, the use of a ratio is more appropriate in order to get a clear visualisation of the results. This ratio is defined as:

$$r_i = \frac{p_{iNRIM}}{p_{iNPL}} \quad (16)$$

with

$$u(r_i) = \sqrt{\left(\frac{u(p_{iNRIM})}{p_{iNRIM}}\right)^2 + \left(\frac{u(p_{iNPL})}{p_{iNPL}}\right)^2} \quad (17)$$

P_t (Pa)	r_1	r_2	$u(r_1)$	$u(r_2)$	$u'(r_1)$	$u'(r_2)$
3.0E-04	1.0168	1.0117	0.0123	0.0264	0.0097	0.0253
9.0E-04	1.0177	1.0110	0.0080	0.0107	0.0043	0.0084
3.0E-03	1.0143	1.0105	0.0053	0.0058	0.0014	0.0027
9.0E-03	1.0116	1.0086	0.0043	0.0044	0.0009	0.0012
2.0E-02	1.0104	1.0073	0.0041	0.0041	0.0008	0.0010
9.0E-02	1.0013	0.9989	0.0038	0.0038	0.0008	0.0008

Table 10 The ratios $r_i = p_{iNRIM} / p_{iNPL}$ and their uncertainties as determined by Eq.(16) and (17). See below for u' description.

If r_1 and r_2 are of the approximate same value significantly different from 1, this would clearly indicate a difference in the true generated pressures of the two standards. Generally the values of r_1 and r_2 will be slightly different due to the scatter of the data. r_1 and r_2 are correlated, because the same standards j were used to determine σ_{1j} and σ_{2j} . In the appendix of reference [2] is outlined a way to consider this correlation by omitting $u(p_{stj})$ in Eq.(6) for the determination of $u(p_{ij})$ in Eq.(13).

The respective value is called u' which is also listed in Table 10. The weighted mean r of r_1 and r_2 is then calculated for each target point by

$$r = \frac{r_1 / u'(r_1)^2 + r_2 / u'(r_2)^2}{1/u'(r_1)^2 + 1/u'(r_2)^2} \quad (18)$$

The standard uncertainty of r at the respective target pressure p_t is

$$u(r) = \sqrt{\left(\frac{u(p_{st,INRIM})}{p_{st,INRIM}}\right)^2 r^2 + \left(\frac{u(p_{st,NPL})}{p_{st,NPL}}\right)^2 r^2 + 2 \frac{(u_{T\sigma}(p_t))^2}{p_t^2} + \left(\frac{1}{u'^2(r_1)} + \frac{1}{u'^2(r_2)}\right)^{-1}} \quad (19)$$

The first two terms under the square root describe the influence of the uncertainty of the standard pressures p_{stj} that correlate to r_1 and r_2 , the third term is the uncertainty contribution due to the temperature dependence of σ_{eff} as described before Eq. (12), and the last term in the bracket is due to all other influences of Type A, which are due to the rotor instability, offset determination RD , temperature and scatter of data. Also for $u_{T\sigma}$ it is conservatively assumed that r_1 and r_2 and respectively σ_{1j} and σ_{2j} are correlated by their temperature dependence of σ_{eff} .

The quantity

$$d = r - 1 \quad (20)$$

describes the relative difference between the two primary standards in this comparison, where

$$u(d) = u(r) \quad (21)$$

Equivalence is generally assumed if

$$d \leq 2u(d) = U(d) \quad (22)$$

or

$$|E_n| \leq 1 \quad (23)$$

with

$$E_n = \frac{d}{U(d)} \quad (24)$$

The values generated from Eqs. (20) to (24) are summarised in Table 16 below.

P_t (Pa)	r	d	$U(d)$	E_n
3.0E-04	1.0162	0.0162	0.0242	0.67
9.0E-04	1.0163	0.0163	0.0163	1.00
3.0E-03	1.0135	0.0135	0.0116	1.16
9.0E-03	1.0105	0.0105	0.0096	1.10
2.0E-02	1.0091	0.0091	0.0092	0.99
9.0E-02	1.0001	0.0001	0.0083	0.01

Table 11 Summary of data as a result of this comparison. r is the ratio of the true generated pressures in the two standards (Eq.(18)), d is the relative difference between the pressures in the two standards (Eq.(20)), $U(d)$ is the expanded uncertainty ($k=2$) of d (Eqs. (20), (21) and (22)) and E_n is defined by Eq. (24).

12. Discussion and conclusions

The comparison has tested the degree of equivalence between the pressures generated at the primary vacuum standards of NPL and INRIM.

The results show that the equivalence between the pressures generated at the primary vacuum standards of NPL and INRIM cannot be demonstrated for the target pressures $3 \cdot 10^{-3}$ Pa and $9 \cdot 10^{-3}$ Pa at the $k=2$ level of expanded uncertainty; for the other pressures the equivalence is demonstrated, even if for the target pressures $9 \cdot 10^{-4}$ Pa and $2 \cdot 10^{-2}$ Pa the E_n values are high, being respectively equal to 1.00 and 0.99.

In December 2005 INRIM started a series of investigations of its continuous expansion standard. The procedure followed in the comparison was globally considered. The effect of different geometries of the tubing connecting the rotor to the calibration chamber and the effect of the residual pressure in the calibration chamber were investigated. The whole system was shown to be leak-tight following extensive leak detection.

Possible errors associated with flow rate measurements by the flowmeter were assessed. The stability of the capacitance diaphragm gauges used for the inlet pressure

measurement, the effect of the initial position of the flowmeter piston and the leak tightness of the piston's Teflon seal were checked. The effects of the bypass valve closure between the reference and measurement chambers on the zero signal of the differential capacitance diaphragm gauge and the temperature drift between the two chambers of the flowmeter were also all investigated. The results obtained have shown that all these effects have a negligible influence on the generated standard pressure.

Finally the effect of non-molecular flow [7] at the highest generated standard pressure ($9 \cdot 10^{-2}$ Pa) was considered. The implementation of a correction taking into account this effect, as suggested by Jousten in a private communication with INRIM, would increase the accommodation coefficient value by about 1% for the target pressure $9 \cdot 10^{-2}$ Pa. Applying this correction to all the INRIM accommodation coefficient values in the considered pressure range for both rotors would show a pressure independent systematic shift compared to the corresponding values of NPL. This effect could be due to the conductance between the calibration and pumping chamber. Consequently, the geometry of the INRIM orifice conductance will be re-evaluated in the near future.

As a consequence of this comparison, the expanded uncertainty associated to the INRIM CMC entry for the pressure range from $9 \cdot 10^{-4}$ Pa to $9 \cdot 10^{-2}$ Pa has been increased from $1.5 \cdot 10^{-2} \cdot p / \text{Pa}$ to $3 \cdot 10^{-2} \cdot p / \text{Pa}$ to include the shift.

The INRIM CMC entries related to the continuous expansion system pressure range will be revaluated as soon as other results of internal and bilateral comparisons are available.

13. References

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