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**Time-Domain Measurement
and Analysis of Mechanical
Step Transitions in Mode-
Tuned Reverberation**

L. R. Arnaut

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Time-Domain Measurement and Analysis of Mechanical Step Transitions in Mode-Tuned Reverberation

L. R. Arnaut

06 October 2006

Abstract

This report presents experimental data and statistical analysis of time-domain measurements for a specific class of transient random electromagnetic fields. These transients are generated by mechanical (i.e., configurational) transitions between otherwise stationary states of a resonant cavity. They are produced by the discretized (stepwise) rotation of a mode tuner inside a reverberation chamber that is excited by a continuous time-harmonic source. Measured time-domain traces of the normalized amplitude of the received field during individual transitions are sampled, recorded, and transposed into stir sequences. Each stir sequence is stochastically wide-sense stationary and is characterized by an associated time delay, defining a specific age within the set of transitions. The cross-correlation function between sequences is found to permit an indirect detection of the average motion of the mode tuner, including mechanical oscillations, inertia, and backlash. From the sequences, we derive the temporal evolutions of the statistics of first order (probability distribution, mean value, standard deviation) and of second order (autocorrelation function, correlation length, modulation index), as well as extreme-value sample statistics (maximum value). These statistics show changes during the transitions that contain a deterministic (regular) contribution that can be correlated to the tuner motion. The measured fluctuations of the mean-normalized maximum received power are considerably larger than those estimated from measured fluctuations of the correlation length with presumed quasi-stationary ideal random fields. From the time-domain traces and stir sequences, the evolution of the rate of fluctuation and the modulation coefficient is determined. The influence of the operation frequency as well as various motor parameters is investigated. The effect of switching the source off/on (before/after each mechanical transition) on the maximum energy density is investigated, based on the measured chamber response to a pulse-amplitude modulated source field.

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Approved on behalf of the Managing Director of NPL by
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- 94 (color online) Maximum-to-mean ratio of P during transients as a function of time at $f = 1000$ MHz, for $M = 200$ steps ($N = 200$), with $A = 1000$ steps/s², $B = 600$ steps/s, $V = 2000$ steps/s: comparison between measured and estimated values. Dotted lines: semi-empirical estimates based on ideal χ_2^2 cdf $F_P(p)$, measured $\mu_P(t)$, and fluctuating measured $N_{\text{ind}}^{(\prime)}(t)$; dashed lines: fully-empirical estimates based on fluctuating measured cdfs $F_P(p)$ and fluctuating measured $N_{\text{ind}}^{(\prime)}(t)$; dot symbols: fully-empirical estimates based on fluctuating measured cdfs $F_P(p)$ and fixed measured $N_{\text{ind}}^{(\prime)}(t) = N_{\text{ind}}^{\text{ss}(\prime)}$. Black solid curve: measured values $\max[P(t)]/\mu_P(t)$; blue curves: estimated mean values $\langle \text{Max}[P(t)]/\mu_P(t) \rangle$; red curves: estimated median values $F_{\text{Max}[P(t)]/\mu_P(t)}^{-1}(1/2, t)$. Estimates of $N_{\text{ind}}^{(\prime)}(t)$ are based on (a) $N_{\text{ind}}(t)$ defined by (16), or (b) $N'_{\text{ind}}(t)$ defined by (17). 140
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99 (color online) Maximum-to-mean ratio of P during transients as a function of time at $f = 1000$ MHz, for $M = 400$ steps ($N = 100$), with $A = 100$ steps/s², $B = 600$ steps/s, $V = 2000$ steps/s: comparison between measured and estimated values. Dot lines: semi-empirical estimates based on fixed ideal χ_2^2 cdf $F_P(p, t)$ and fluctuating measured $N_{\text{ind}}^{(\prime)}(t)$; dashed lines: fully-empirical estimates based on fluctuating measured cdfs $F_P(p)$ and fluctuating measured $N_{\text{ind}}^{(\prime)}(t)$; dotted symbols: fully-empirical estimates based on fluctuating measured cdfs $F_P(p)$ and fixed measured $N_{\text{ind}}^{(\prime)}(t) = N_{\text{ind}}^{\text{ss}(\prime)}$. Black solid curve: measured values $\max[P(t)]/\mu_P(t)$; blue curves: estimated mean values $\langle \text{Max}[P(t)]/\mu_P(t) \rangle$; red curves: estimated median values $F_{\text{Max}[P(t)]/\mu_P(t)}^{-1}(1/2, t)$. Estimates of $N'_{\text{ind}}(t)$ are based on (17). 144

100 (color online) Maximum-to-mean ratio of P during transients as a function of time at $f = 1000$ MHz, for $M = 400$ steps ($N = 100$), with $A = 10$ steps/s², $B = 600$ steps/s, $V = 2000$ steps/s: comparison between measured and estimated values. Dot lines: semi-empirical estimates based on fixed ideal χ_2^2 cdf $F_P(p, t)$ and fluctuating measured $N_{\text{ind}}^{(\prime)}(t)$; dashed lines: fully-empirical estimates based on fluctuating measured cdfs $F_P(p)$ and fluctuating measured $N_{\text{ind}}^{(\prime)}(t)$; dotted symbols: fully-empirical estimates based on fluctuating measured cdfs $F_P(p)$ and fixed measured $N_{\text{ind}}^{(\prime)}(t) = N_{\text{ind}}^{\text{ss}(\prime)}$. Black solid curve: measured values $\max[P(t)]/\mu_P(t)$; blue curves: estimated mean values $\langle \text{Max}[P(t)]/\mu_P(t) \rangle$; red curves: estimated median values $F_{\text{Max}[P(t)]/\mu_P(t)}^{-1}(1/2, t)$. Estimates of $N'_{\text{ind}}(t)$ are based on (17). 145

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1 Introduction

Electromagnetic (EM) reverberation chambers are used in EMC for immunity, emissions, absorption, and shielding measurements and testing. In essence, reverberation chambers are highly overmoded EM cavities furnished with a mechanism for altering (mixing or “stirring”) the interior field in time, at any location inside the cavity. This stirring process can be realized either electronically (by varying the parameters of the excitation, e.g., frequency, modulation type, etc., or the local impedance boundary conditions) or mechanically (by varying the geometrical configuration), or as a combination of both methods. In this report, only mechanical boundary changes and time-harmonic excitation are considered (except in Sec. 8).

Reverberation chambers can be used in one of two regimes, viz., mode-stirred or mode-tuned operation. In *mode-stirred operation*, which has the longer history [1]–[5], the cavity boundaries are changed in a continuous fashion, typically by rotating or periodically moving a so-called mode stirrer (in the form of a reflective paddle wheel, diffusing wall, or other form of wave diffractor) with a constant angular velocity Ω . In practice, this continuous uniform motion can often be mimicked with the aid of a stepping motor, as an alternative to a variable-speed motor, provided the inertia of the mode tuner structure (governed by its mass, size, and rigidity) is sufficiently large, so that any mechanical oscillations and backlash occur on a time scale which is long compared to the step period T of the rotation of the motor axis. As a result, the discretized stepping of the motor is then effectively smoothened during the motion of the paddle wheel. By contrast, in *mode-tuned operation* [6], [7], the cavity is mechanically perturbed in a stepwise manner, by rotating the diffractor (now called mode tuner) through discrete, typically equiangular increments separated by dwell time intervals that are long compared to the mechanical time constant of the paddle wheel and the step period of the motor. In this way, starting at any one of N angular positions θ_n ($n = 0, 1, \dots, N - 1$), each (n th) tuner step transition is realized through a sequence of M incremental steps of the stepper motor, followed by a specified dwell time aimed at reaching an EM steady state at the next discrete angular position θ_{n+1} .

EMC standards covering the use of reverberation chambers [7]–[10] allow for mode stirring and/or mode tuning. The latter enables the interior field to be measured or electronic devices to be tested under mechanically static conditions, in principle. However, in the process of tuner stepping while the radio-frequency (RF) source is left switched on, an irregularly fluctuating EM transient field is generated because of the chamber’s high quality factor (Q) and, hence,

long reverberation (“ringing”) time. Due to the nonuniform motion of the boundaries during the transition under EM resonant conditions, tuner stepping gives rise to fluctuations of the instantaneous field and that are more complex or pronounced than those which exist when using mode stirring with a constant speed of rotation (uniform perturbation). As will be demonstrated, individual field transitions between tuner states appear as quasi-random events characterized by highly irregular and large fluctuations, even when successive steady-state field levels are strongly correlated for relatively small tuner step sizes. Of course, alternatives and remedies can be applied to reduce the adverse effects of such transients, whereby the RF source is switched off and back on again, before and after each step of the mode tuner, whether in a controlled or abrupt manner (source blanking, pulse shaping, etc.). Nevertheless, because of the high Q factor and overmoding, these methods introduce field dynamics and critical transient field characteristics of their own (instantaneous threshold crossings, field gradients, etc.), to a greater or lesser extent, which may or may not be acceptable in a particular practical test or application of interest.

In this report, we present measurement results and their statistical analysis for transient fields associated with mechanical stepping in a mode-tuned reverberation chamber excited by a continuous time-harmonic (CW) RF field between 100 and 1000 MHz. The amplitude of the transient received field associated with the mechanical transitions, normalized with respect to the instantaneous excitation field is sampled, recorded and grouped into tuner sequences, as will be specified in Sec. 4. Statistical analysis of an ensemble of transients provides an alternative to a case-by-case analysis of individual transitions (e.g., using wavelets, empirical signal decomposition based on envelope detection [11], etc.). Some preliminary measurements simulating switching effects are presented in Sec. 8.

The measurements presented in this report were conducted in the larger reverberation chamber at *NPL* during July 2002. Some preliminary results were presented in [12]. Related studies have been reported by other groups [13], [14]. A theory in support of the observed physical phenomena is under development [15], [16].

2 Measurement set-up and procedure

2.1 Configuration and instrumentation

2.1.1 Reverberation chamber

For our experiments, we used the *NPL* rectangular reverberation chamber ($6.55 \text{ m} \times 5.85 \text{ m} \times 3.5 \text{ m}$) in mode-tuned operation. The mode tuner is a paddle wheel of height 3 m and diameter 1.2 m, driven by a stepper motor with controller (PM341, McLennan Servo Supplies Ltd. [17]). The motor box is located outside the chamber, above the ceiling, and connected to the axis of the paddle wheel via a waveguide below cut-off¹. The shaft extends fully between the floor and the ceiling (height 3.5 m). The construction is supported on the floor using a bearing. One complete rotation is realized by N paddle wheel steps of equal angular size and corresponds to 40 000 motor steps. The number of motor steps M for each tuner transition is user-defined and determines the angular increment $\Delta\theta \equiv \theta_{n+1} - \theta_n = 2\pi M/40\,000 = 2\pi/N$ rad for the stepwise rotation of the paddle wheel. The overall time taken for one transition defines the stepping period, T .

2.1.2 Network analyzer

The dynamics of the interior field during paddle wheel transitions were measured using a vector network analyzer (VNA) with time-domain capability (HP 8573E). The input signal is generated by the VNA in its zero-span CW FREQ mode [18, p. 6-27] as a narrowband CW excitation signal centered at a specified operating frequency f with a constant and fixed bandwidth of 300 Hz. The temporal evolution of the amplitude of the transmission S-parameter, $|S_{21}(t)|$, was sampled at the maximum permitted resolution, viz., $L = 1601$ points t_ℓ ($\ell = -80, \dots, L - 81$) that are equally spaced across an interval $T = 10 \text{ s}$ ($\Delta t = T/L = 6.242 \text{ ms}$). This response was stored in a separate file after each transition of the mode tuner. Thus, for each angular starting position of the paddle wheel θ_n ($n = 0, \dots, N - 1$), we recorded the sampled 1D time-domain response $\{|S_{21}(t_\ell; \theta = \theta_n)|\}_{\ell=-80, \dots, L-81}$ which characterizes the amplitude fluctuations of the transmitted field during the mechanical transition of the paddle wheel, $\theta_n \rightarrow \theta_{n+1}$.

¹The possibility of the motor causing EMI in the received field was checked by measuring received power during the transitions at selected bands between 100 and 400 MHz, using a scalar power spectrum analyzer, without applying an RF excitation signal to the transmitting antenna. The noise measured during motor operation (rotation) was indistinguishable from background noise when the motor was idle, at all frequencies measured. This confirms that the motor does not interfere with the measured $|S_{21}(t)|$ signal inside the chamber.

Unlike a measurement of power received from an independent source (e.g., using a scalar power spectrum analyzer), the S-parameter measurement with a VNA enables fluctuations of the input power caused by real-time variations of the cavity input impedance [19] to be automatically accounted for, through the inherent normalization (division) by the instantaneous CW input signal. This fact is important when evaluating the origin and extent of fluctuations in real time. However, the measurement of the received power itself, in absolute terms (i.e., without accounting for real-time fluctuations of the input power), is also of merit because typical sources of radiation do not compensate for such fluctuations.

To monitor the initial EM steady state at θ_n , the recording of each time trace was started $80\Delta t \simeq 500$ ms before the controller sends a trigger signal to the motor to start a rotation step. The instance of the trigger signal is defined as $t_0 = 0$. The actual nominal duration of rotation for one tuner step depends on the chosen value of its size ($\Delta\theta$) and on the motor's control parameters, as detailed and discussed in Sec. 2.2. In our experiments, the duration of the axial rotation of the motor per tuner step is typically less than 3 s when $M = 400$ steps. Larger (smaller) values of M give rise to longer (shorter) epochs of angular rotation. To ensure capturing the complete time-domain response during each transition, the remaining 6.5 s of the interval records the approach toward the next steady-state field, i.e., at the next state θ_{n+1} .

2.1.3 Antennas, cables, connectors

For the measurements, a pair of commercially available and nominally identical bicone antennas² was employed to transmit and receive (Schwarzbeck UBA 9116, 160–1100 MHz [20]). Since no information about their impulse response was available, it is assumed that their response time is much smaller than the relaxation time of the chamber ($\tau_{\text{ch}} \sim \mu\text{s}$) and, *a fortiori*, the sampling time interval of the measurements $\Delta t = 6.242$ ms, so that their characteristic time constant can be neglected. The same assumptions are made about the cables, connectors, interconnects, etc. A detailed investigation would require a separate time-domain characterization of the individual mechanical components, e.g., inside an anechoic chamber.

²Although the antenna factor of the antennas increases with decreasing frequency below 160 MHz, this should not have a significant impact on the measurement of S_{21} .

2.2 Control parameters for stepper motor

The speeds, acceleration, deceleration, and dynamic constants of the motor are parameters that are under software control by the user [17, p. 24]. The definition and meaning of these parameters is illustrated in Fig. 1. All speeds are expressed in units of motor steps per second (steps/s), where Ω steps/s $\sim \pi\Omega/20\,000$ rad/s. Initially, the motor starts³ at a base speed of

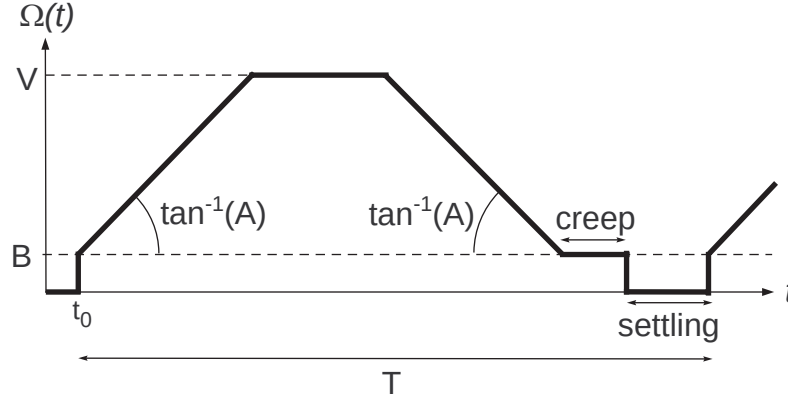


Figure 1: Velocity-time diagram with definition of motor parameters (after [17, p. 24]).

B steps/s and accelerates uniformly at a rate of A steps/s², until a specified slew speed of V steps/s is reached. Toward the end of a cycle, the motor decelerates uniformly at $-A$ steps/s², followed by a progression (creep) towards its final position across a specified number of steps M_C , at constant base speed B . This creep is intended to reduce mechanical backlash. Finally, a settling time t_s separates the end of the previous step from the beginning of the next one. Whilst the settling time was specified as 0.1 s in all measurements, this time is considerably longer in practice (of the order of 10 s), due to the large values of A or B used (cf. *infra*) and the artificial extension of this time beyond T , in order to enable the data to be written to file before the next cycle (step) is started. Thus, no EM transients caused by the previous step exist

³Stepper motors cannot be started from zero speed, for the sake of avoiding mechanical eigenresonances of the mode tuner governed by the overall mechanical load [21]. The motor controller performs an instantaneous acceleration to the base speed which is set above the resonance point. The speed accelerates from there. The acceleration required to reach this initial speed is governed by the current in the motor coils, which is set to be sufficiently large so as to avoid the driven mechanical construction passing through its resonance when increasing the motor speed from base speed B to slew speed V . This initial acceleration is beyond the control of the user. To achieve a smooth ramp starting from zero speed, a servo motor would be needed. With regard to the stop phase, the motor decelerates back to the base speed, performs a selected number of creep steps at this speed, and then stops abruptly.

at the start of each tuner step.

Parenthetically, in the present study, one objective was to intentionally *stimulate* excessive tuner backlash, in order to evaluate its effect on the transient cavity field. The rationale is as follows. In practical EMC testing, it is desirable to keep the rotation time of each tuner step to a minimum, in order to minimize the duration of the test cycle. This suggests choosing large values of A , B and/or V . However, realistic mode tuners – particularly heavy, suspended, and/or large-diameter designs exhibiting large momentum – often display a considerable degree of backlash, which increases the settling and, hence, overall test time. The mode tuner used in our experiment is exceptionally rigid, by way of its two-tier design using three honeycomb platform disks and floor bearing for the shaft. Therefore, backlash was artificially amplified by imposing abnormally high values of A and B to create a marked effect.

With the previous definitions, the profile of the tuner motion and the durations of its different stages (phases) can be determined. If the parameter values are chosen such that none of the motor phases during T has to be cut short (as would be the case when reaching the final position prematurely) then the time taken to reach the specified slew speed V starting from B is

$$t_A \equiv t_A - t_0 = \frac{V - B}{A} \quad (1)$$

after which the number of motor steps travelled is

$$M_A = B t_A + \frac{A t_A^2}{2} = \frac{V^2 - B^2}{2A}. \quad (2)$$

The same time and step count apply for the deceleration phase, from V reducing to B . The duration of the creep towards the final angular position of the motor axis (next angular state of the mode tuner) is

$$t_C = \frac{M_C}{B}. \quad (3)$$

The motor traverses the remaining $M_V = M - 2M_A - M_C$ steps at slew speed during a remaining time t_V , given by

$$t_V = \frac{M_V}{V} = \frac{M - \frac{V^2 - B^2}{A} - M_C}{V}. \quad (4)$$

The overall period for one tuner step, including the settling time t_s (which, in turn, incorporates the lead time t_0 for the next period), is

$$\begin{aligned} T - t_s &= t_L - t_0 = 2t_A + t_V + t_C \\ &= \frac{2(V - B)}{A} + \frac{M - \frac{V^2 - B^2}{A} - M_C}{V} + \frac{M_C}{B}. \end{aligned} \quad (5)$$

On the other hand, if the values of A and/or B are large to such an extent that the motor reaches its specified final position before its maximum speed V' reaches the specified V , i.e., $M - M_C < M_A/2$, then the overall travel time becomes

$$\begin{aligned}
 t_L - t_0 &= (t_{V'} - t_0) + (t_B - t_{V'}) + t_C \\
 &= \frac{B}{A} \left(\sqrt{1 + \frac{(M - M_C)A}{B^2}} - 1 \right) + \frac{V'}{A} \left(1 - \sqrt{1 - \frac{(M - M_C)A}{V'^2}} \right) + \frac{M_C}{B} \\
 &= \frac{2B}{A} \left(\sqrt{1 + \frac{(M - M_C)A}{B^2}} - 1 \right) + \frac{M_C}{B}
 \end{aligned} \tag{6}$$

where

$$V' \triangleq \max_{\{t\}} [\Omega(t)] = B \sqrt{1 + \frac{(M - M_C)A}{B^2}} \leq V. \tag{7}$$

For the nominal parameter values used in this study, viz., $A = 1000$ steps/s², $B = 600$ steps/s, $V = 2000$ steps/s, $M_C = 10$ steps, we obtain $M_A = 1820$ steps. If we choose $M = 400$ steps (i.e., $\Delta\theta = 3.6$ deg), then the motor starts decelerating before reaching the nominal slew speed (i.e., $V' = 866$ steps/s, $t_V = 0$), after $t_L - t_0 = (2 \times 0.266 + 0.017)$ s = 0.549 s. This agrees to within 0.1 s with the time interval measured by visible inspection of the physical motion of the mode tuner and stopwatch. For this choice of parameter values, a further 2 s is required for the (in this case clearly visible) tuner oscillations due to backlash to die out, again based on visual inspection. Other sets of (lower) parameter values were also investigated and are reported. The minimum settings used in our experiments are $A = 100$ steps/s², $B = 10$ steps/s, and $V = 200$ steps/s, yielding $t_L - t_0 = (2 \times 1.877 + 1)$ s = 4.755 s and $V' = 197.7$ steps/s for $M = 400$ steps. For these settings, visual inspection of the tuner motion revealed a smooth mechanical transition without any visible backlash or oscillation, even though the slew speed is still not reached in this case⁴.

The use of even smaller speeds and accelerations is hampered by the fact that the maximum permissible record length is 1601 samples, so that sufficiently dense⁵ sampling of the irregular fluctuations during very slow rotations covering each tuner transition in its entirety would not be achievable with the instrumentation used in this experiment. As will be shown, relatively sharp spikes of the field are always present during the transitions, particularly at lower frequencies, which may be inaccurately characterized or completely missed if sampling is too coarse. Our

⁴Measurement results will show that there is nevertheless a discernable effect of the value of V in this case.

⁵Although Δt can be adjusted, the relaxation time of the chamber remains a physical parameter.

chosen value of Δt constitutes a compromise. However, if one were interested only in a narrower subinterval (e.g., initial tuner acceleration phase), then it is possible to reduce Δt .

3 Measured data

The nominal values of the motor parameters were chosen to be $M = 400$ steps, $A = 1000$ steps/s², $B = 600$ steps/s, $V = 2000$ steps/s. These rather high values were chosen in order to generate substantial (viz., visible) backlash of the tuner during the start and stop phases. Sets of smaller values were also chosen for comparison, in order to generate seemingly smooth and oscillation-free rotations. The number of creep steps M_C is 10 throughout.

3.1 Transient tuner sweep data

3.1.1 Dependence on operating frequency

Figures⁶ 2 through 5(a) show the tuner transitions for one complete rotation as a concatenated 1D time series of transitions, for frequencies f between 100 MHz and 1000 MHz, with $M = 400$ steps ($N = 100$), $A = 1000$ steps/s², $B = 600$ steps/s, $V = 2000$ steps/s. The large fluctuations of $|S_{21}|$ between steady-state values are immediately apparent. Detailed inspection shows that the transitions become more irregular (quasi-random) but smoother when f increases. Note in particular that, for lower frequencies ($f \leq 200$ MHz), large spikes appear during each transition, in spite of the fact that consecutive steady-state values are then strongly correlated (for this value of M), for which it would normally be expected that the transitions are more gradual and the fluctuations smaller in magnitude. This suggests a distinct degree of nonstationarity during the transitions. The low frequency data also indicate that the stirrer's performance is not entirely uniform across one rotation, compare e.g. the steady-state levels for $t = 410 \dots 450$ s and $650 \dots 700$ s in Fig. 2(a). This suggests a certain degree of stir inhomogeneity even for the *steady-state* values, independent of the stochastic nonstationarity⁷ during the transitions.

⁶Use magnification of electronic PDF or Postscript version of this report to view details of the fluctuations during individual transitions, in particular for distinguishing between results for different sets of parameter values.

⁷(Non)stationarity is a well-defined concept in the theory of stochastic processes, and has strict-sense and wide-sense formulations. Briefly, one considers a large (theoretically infinite) number $\mathcal{N}$ of different realizations of a given random process $X(t)$, forming an ensemble, and calculates an ensemble statistic for a given time t across all these realizations, e.g., for the mean value, $\mu_X(t) = \sum_{n=1}^{\mathcal{N}} X_n(t)/\mathcal{N}$. A wide-sense stationary process is then defined as a process for which $\mu_X(t)$ and $\sigma_X(t)$ are independent of t ; by contrast, strict-sense stationarity holds if and only if this time-independence holds for *all* statistical moments, i.e., moments of all orders.

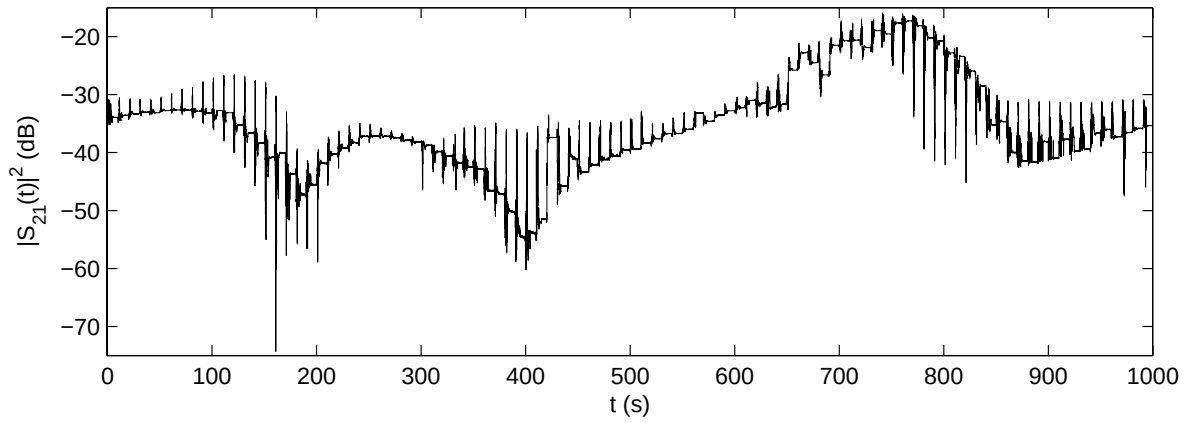
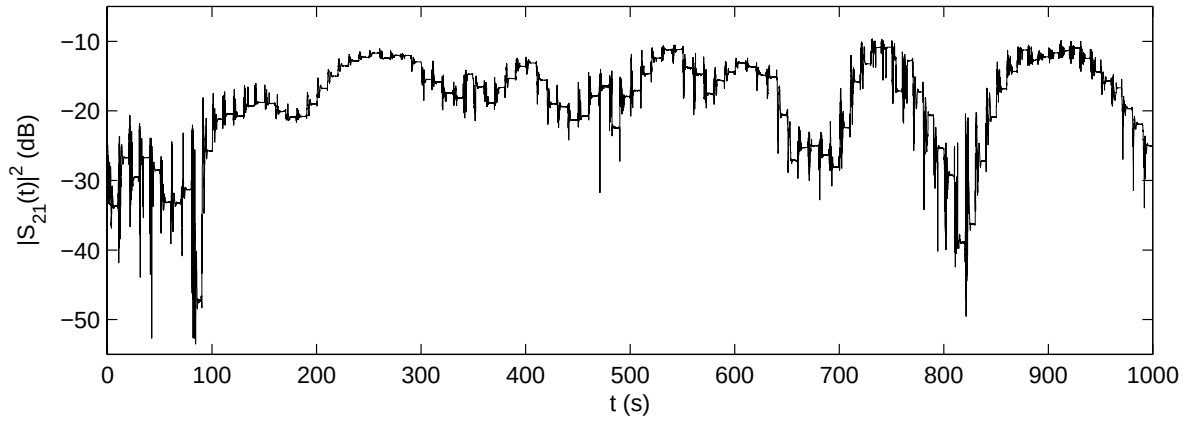
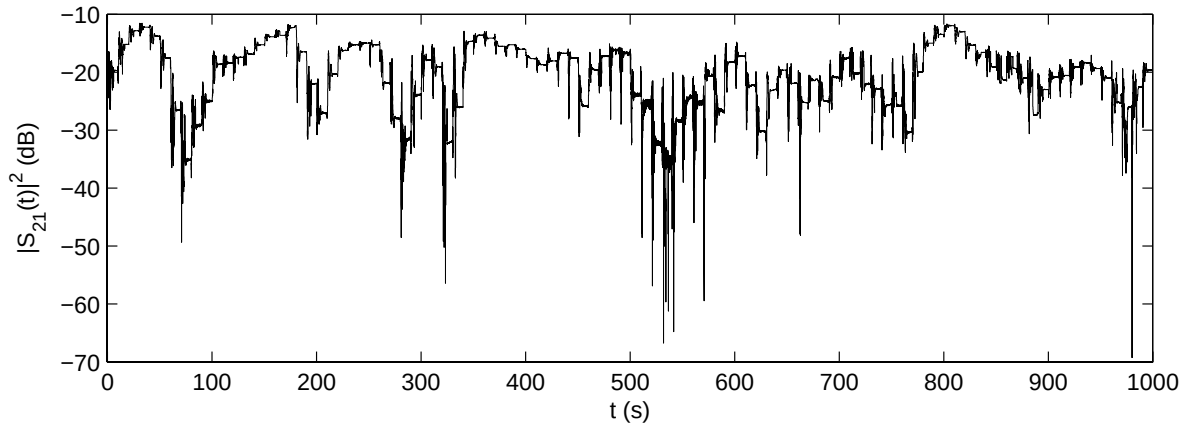
(a) $f = 100$ MHz(b) $f = 200$ MHz(c) $f = 300$ MHz

Figure 2: Time series representation of tuner transitions for $|S_{21}|^2$ as a function of time t , for all transitions during one complete rotation with $M = 400$ steps ($N = 100$), $A = 1000$ steps/s², $B = 600$ steps/s, $V = 2000$ steps/s, at (a) 100 MHz; (b) 200 MHz; (c) 300 MHz.

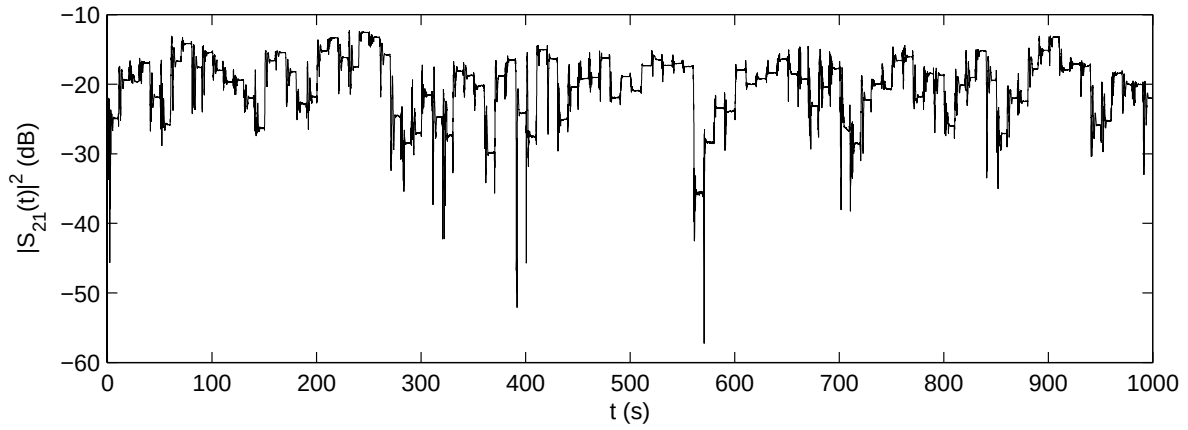
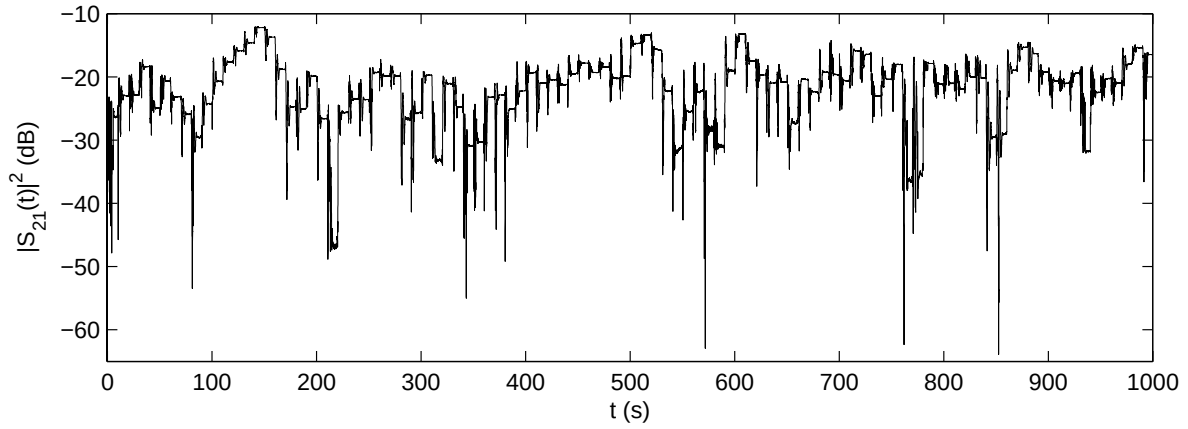
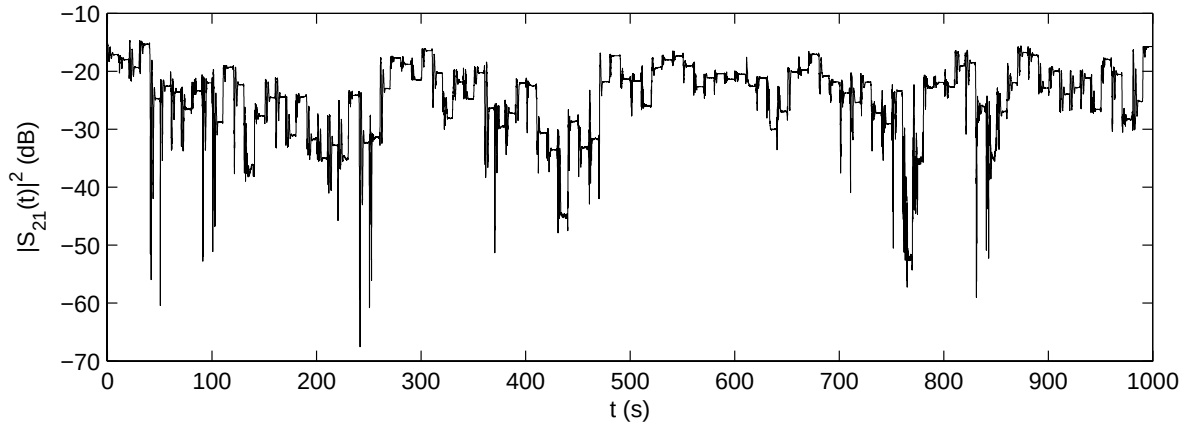
(a) $f = 400$ MHz(b) $f = 500$ MHz(c) $f = 600$ MHz

Figure 3: Time series representation of tuner transitions for $|S_{21}|^2$ as a function of time t , for all transitions during one complete rotation with $M = 400$ steps ($N = 100$), $A = 1000$ steps/s², $B = 600$ steps/s, $V = 2000$ steps/s, at (a) 400 MHz; (d) 500 MHz; (c) 600 MHz.

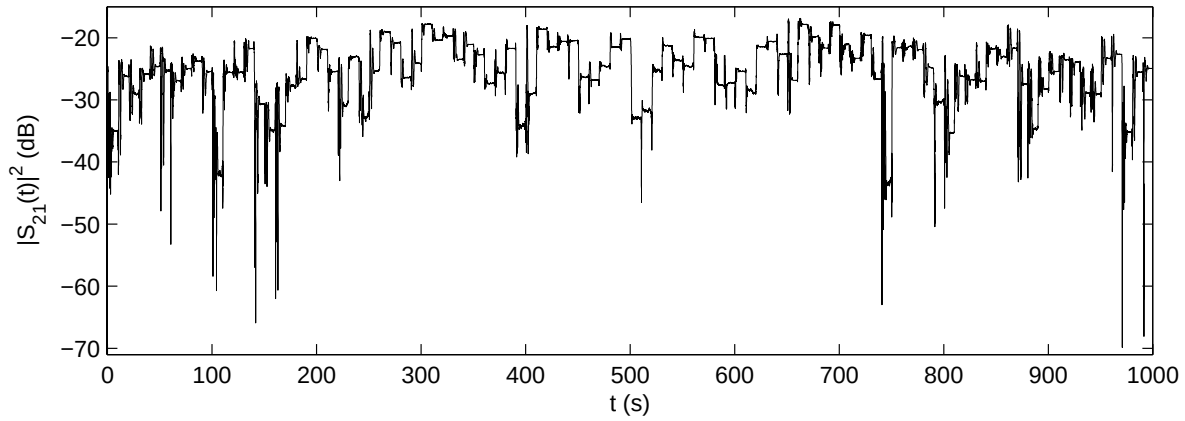
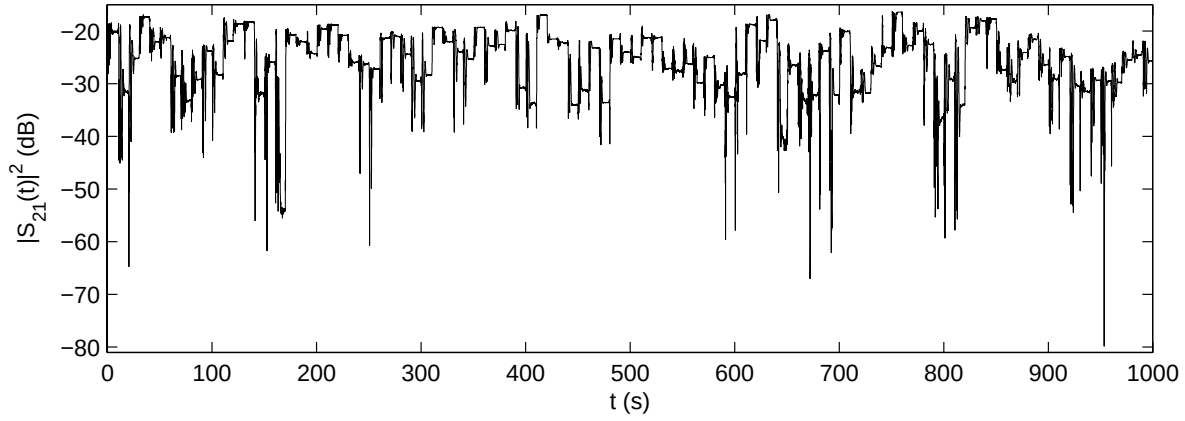
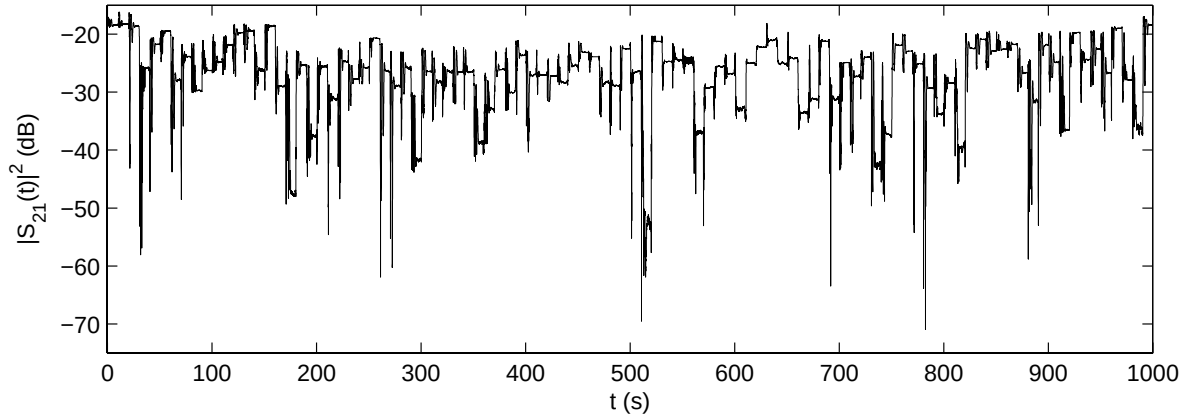
(a) $f = 700$ MHz(b) $f = 800$ MHz(c) $f = 900$ MHz

Figure 4: Time series representation of tuner transitions for $|S_{21}|^2$ as a function of time t , for all transitions during one complete rotation with $M = 400$ steps ($N = 100$), $A = 1000$ steps/s², $B = 600$ steps/s, $V = 2000$ steps/s, at (a) 700 MHz; (b) 800 MHz; (c) 900 MHz.

3.1.2 Dependence on motor acceleration and speed

Figures 5 through 6(a) show tuner transitions at $f = 1000$ MHz with $M = 400$ steps ($N = 100$), $B = 600$ steps/s, $V = 2000$ steps/s, for selected values of the motor acceleration A . Visual inspection of these graphs suggests a modest influence of A , at least for these values of B and V ; compare e.g. Fig. 5(a) with Fig. 6(a). The case $A = 1$ step/s² may be considered as a transition moving at essentially constant motor speed, because in this case $B \simeq V' = 600.3$ steps/s with $t_L - t_0 = 0.666$ s.

Figure 6 shows tuner transitions at $f = 1000$ MHz with $M = 400$ steps ($N = 100$), for selected values of A , B and V , by varying one parameter at a time. The results should also be compared to those for the nominal settings shown in Fig. 5(a). Compared to Figs. 5(a) and 6(a), one observes less abrupt transitions in Figs. 6(b) and (c), which are mainly due to the substantially lower value of B . This was confirmed by a visual inspection of the physical movement of the paddle wheel, which no longer showed discernable mechanical oscillations during the transitions for $A = 100$ steps/s², $B = 10$ steps/s. Most transitions now show a similar pattern, characterized by a single overshoot or undershoot at the start of the transition. On the other hand, the “plateaus” for steady state values of $|S_{21}|^2$ have now become more tilted for several transitions, indicating that in this case $T = 10$ s may be too short for reaching steady state with these settings.

On the other hand, Figs. 7(a) and (b) show transitions for low values of B (≤ 10 steps/s), but with higher values for A and V than in Fig. 6. The Figures show that the irregularities have now re-appeared. Thus, no single motor parameter is responsible for achieving or disrupting the smoothness of transitions. Instead, all parameter values should be kept as small as possible to achieve this. A search for (an) optimum triplet(s) (A, B, V) or any optimum relation between them, if any, is beyond the scope of the present study.

Finally, in order to appreciate the extent of the often considerable differences between the maximum transient field and the maximum steady-state field levels, Fig. 7(c) shows $|S_{21}(t)|^2$ of Fig. 5(a) but plotted on a linear scale. This issue will be further discussed in Sec. 6.4.

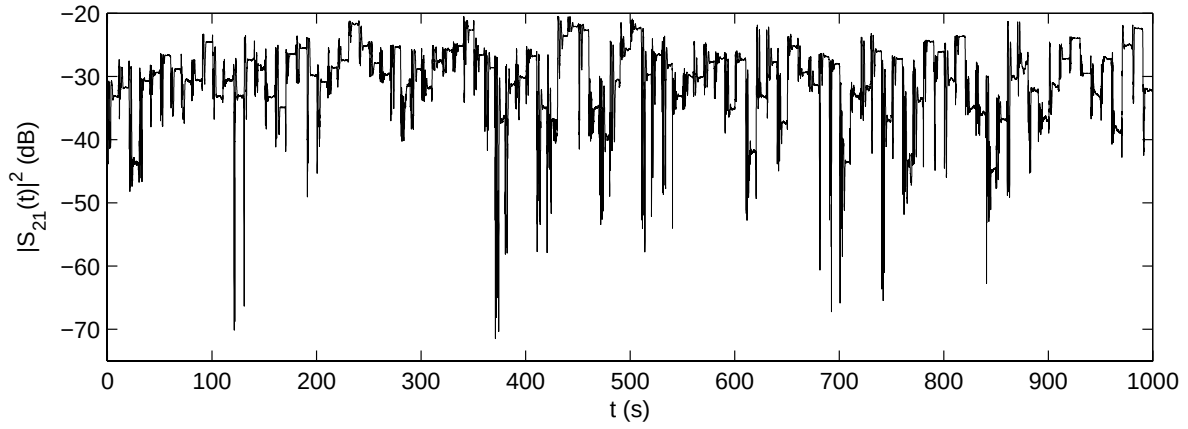
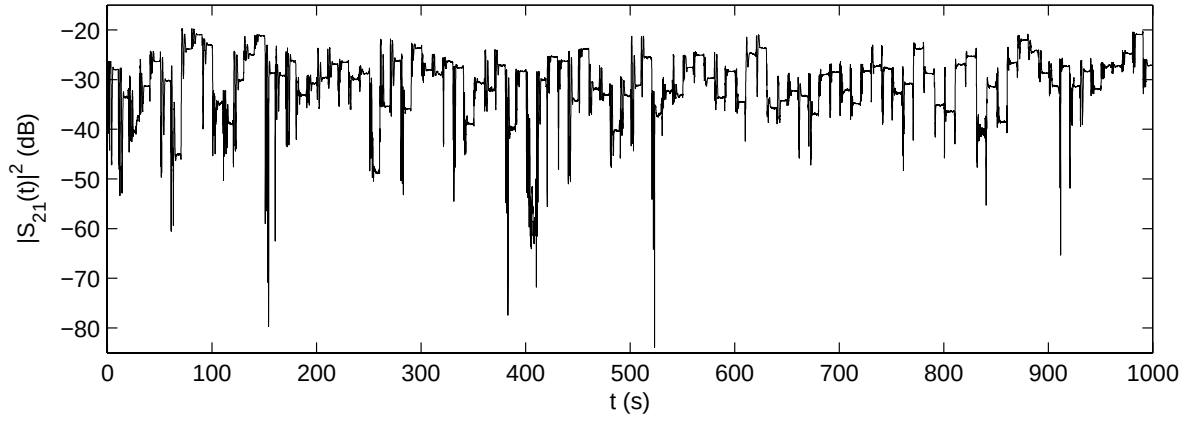
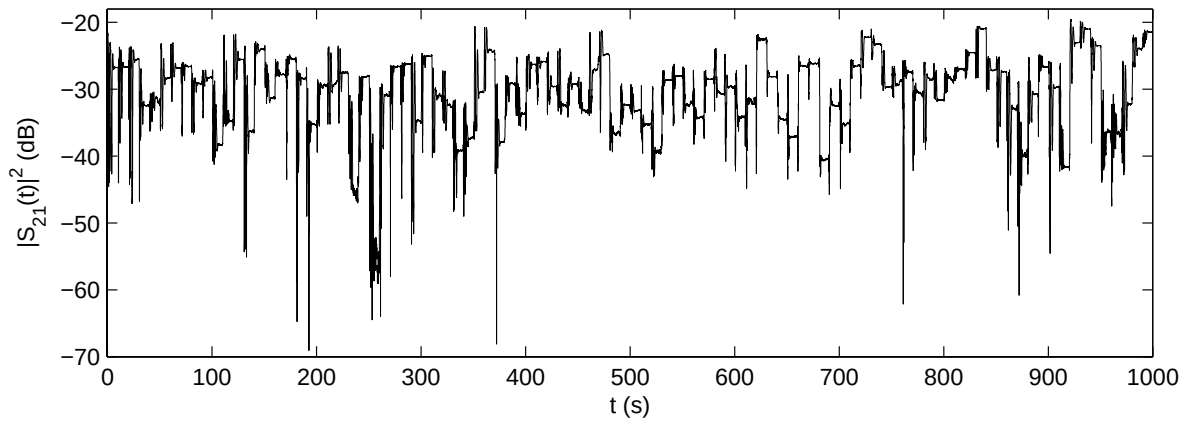
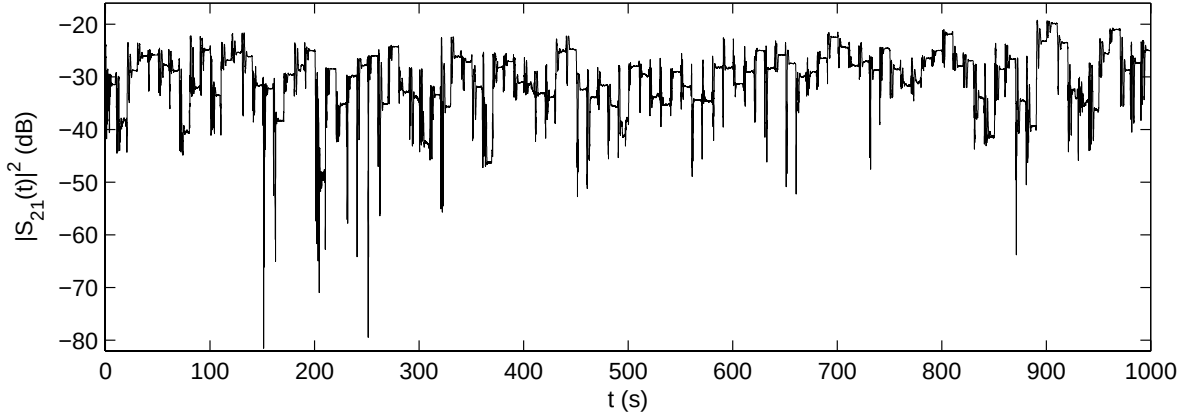
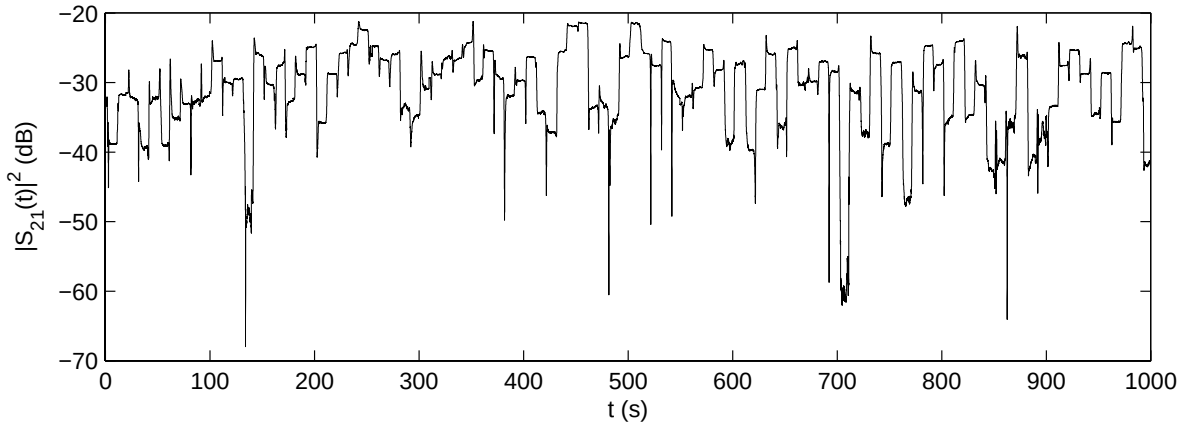
(a) $A = 1000 \text{ steps/s}^2$ (b) $A = 100 \text{ steps/s}^2$ (c) $A = 10 \text{ steps/s}^2$

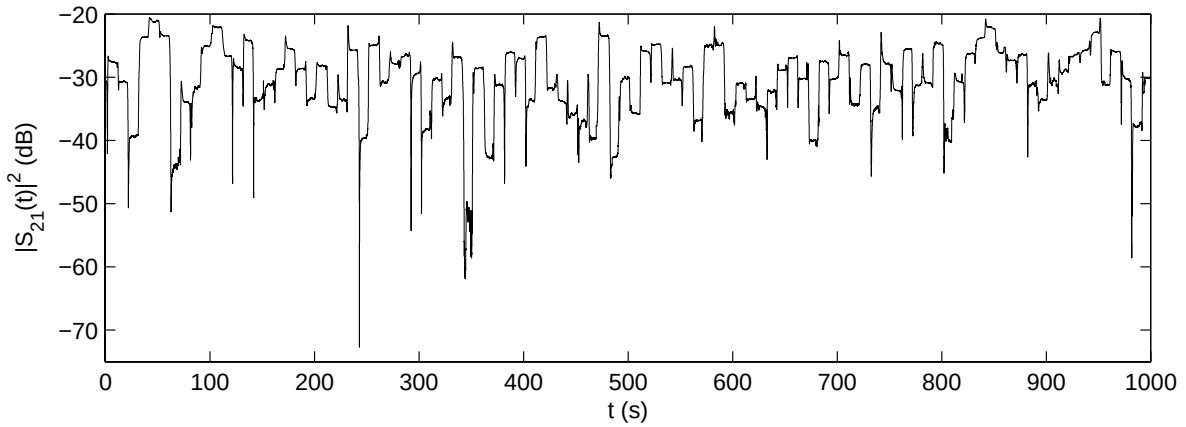
Figure 5: Time series representation of tuner transitions for $|S_{21}|^2$ as a function of time t , for all transitions during one complete rotation with $M = 400$ steps ($N = 100$), at $f = 1000$ MHz with $B = 600$ steps/s, $V = 2000$ steps/s, and (a) $A = 1000 \text{ steps/s}^2$; (b) $A = 100 \text{ steps/s}^2$; (c) $A = 10 \text{ steps/s}^2$.



(a) $A = 1 \text{ step/s}^2$, $B = 600 \text{ steps/s}$, $V = 2000 \text{ steps/s}$



(b) $A = 100 \text{ step/s}^2$, $B = 10 \text{ steps/s}$, $V = 2000 \text{ steps/s}$



(c) $A = 100 \text{ step/s}^2$, $B = 10 \text{ steps/s}$, $V = 200 \text{ steps/s}$

Figure 6: Time series representation of tuner transitions for $|S_{21}|^2$ as a function of time t , for all transitions during one complete rotation with $M = 400$ steps ($N = 100$), at $f = 1000$ MHz with (a) $A = 1 \text{ step/s}^2$, $B = 600 \text{ steps/s}$, $V = 2000 \text{ steps/s}$; (b) $A = 100 \text{ steps/s}^2$, $B = 10 \text{ steps/s}$, $V = 2000 \text{ steps/s}$; (c) $A = 100 \text{ steps/s}^2$, $B = 10 \text{ steps/s}$, $V = 200 \text{ steps/s}$.

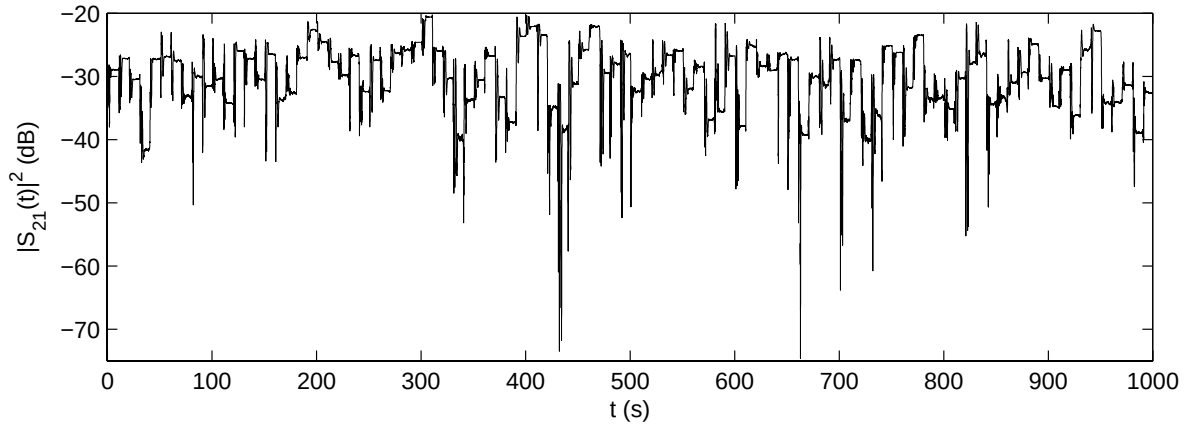
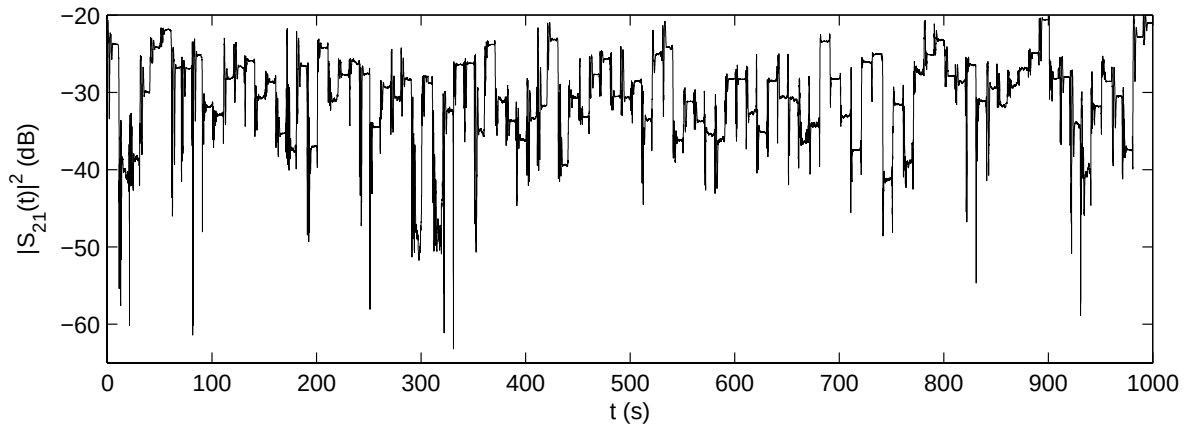
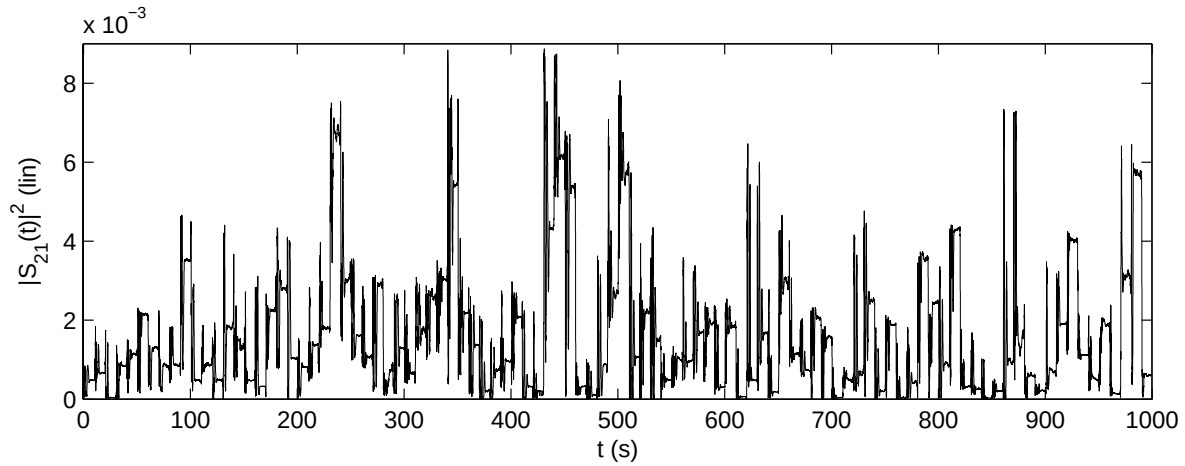
(a) $B = 10$ steps/s(b) $B = 1$ step/s(c) $A = 1000$ step/s², $B = 600$ steps/s, $V = 2000$ steps/s

Figure 7: Time series representation of tuner transitions for $|S_{21}|^2$ as a function of time t , for all transitions during one complete rotation with $M = 400$ steps ($N = 100$), at $f = 1000$ MHz with $A = 1000$ steps/s², $V = 2000$ steps/s, and (a) $B = 10$ steps/s; (b) $B = 1$ step/s. (c) Linear scale representation of Fig. 5(a).

3.1.3 Dependence on tuner step size

Next, we investigate the effect of changing the number of motor steps M per transition, governing the angle of rotation $\Delta\theta$ per transition (tuner step) and N . The results are shown in Fig. 8 (for selected values of M) and in Fig. 9 (for selected frequencies). At 100 MHz, the prolonged duration of the transitions and increased overall fluctuation in Fig. 8(c) compared to Fig. 8(a) suggests that the amplitude spikes are not merely a start/stop phenomenon (with theoretically infinite A), but that they can also appear during the continuous rotation (with finite A), *a fortiori* at higher instantaneous rotation speeds $\Omega(t)$. Note that the sampling rate has been maintained (1601 pts/10 s = 160.1 Hz), resulting in an effectively coarser sampling of the transient field, which may have caused certain intermittent spikes of still higher amplitudes to have been missed. Characteristic times, step counts, and motor speeds associated with the cases presented in Figs. 8 and 9 are given in Tbl. 1. Comparing Figs. 9(a) and (b) with Figs. 2(b) and (c), it can be seen that during this transition the field fluctuations are now quite noisy, also for overmoded operation, in spite of the still considerable degree of correlation between the steady-state values for consecutive tuner states. Comparing Fig. 9(a) with Fig. 9(b) suggests that this noise intensity increases with increasing frequency. Finally, comparing Fig. 9(c) with Fig. 5(a) shows that halving M and, hence, reducing V' suppresses the fluctuations quite considerably. This can be ascribed to the fact that the angular step size has become so small that the acceleration of the motor is being cut short still earlier, because of the small value of M .

M (steps)	t_A (s)	M_A (steps)	$t_L - t_0$ (s)	V' (steps/s)
200	0.142	95	0.300	742
400	0.266	195	0.549	866
800	0.472	395	0.961	1072
1200	0.645	595	1.307	1245

Table 1: Characteristic times, step counts, and speeds for $A = 1000$ steps/s², $B = 600$ steps/s, $V = 2000$ steps/s, at selected values of M .

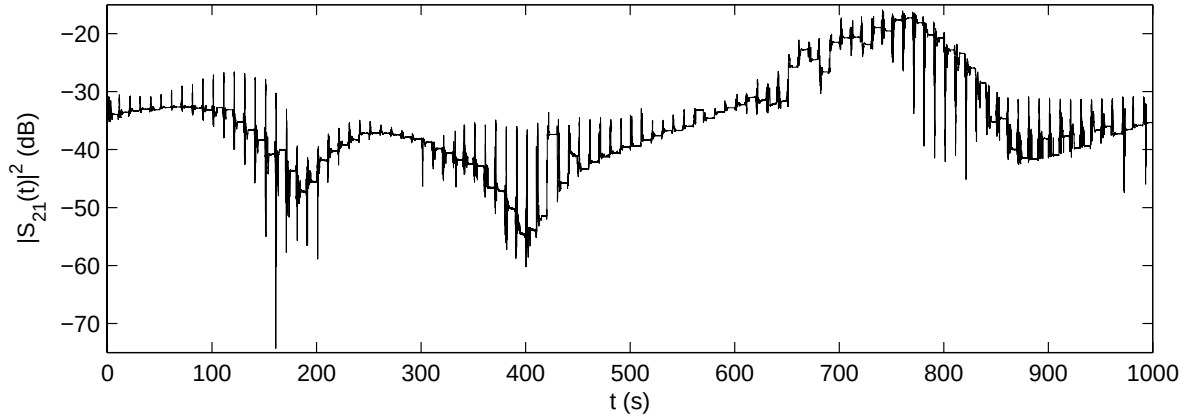
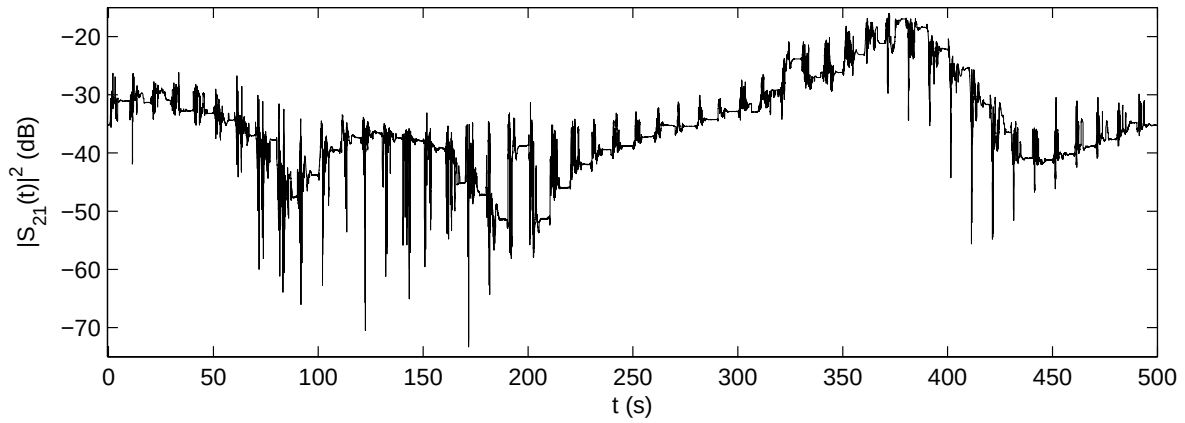
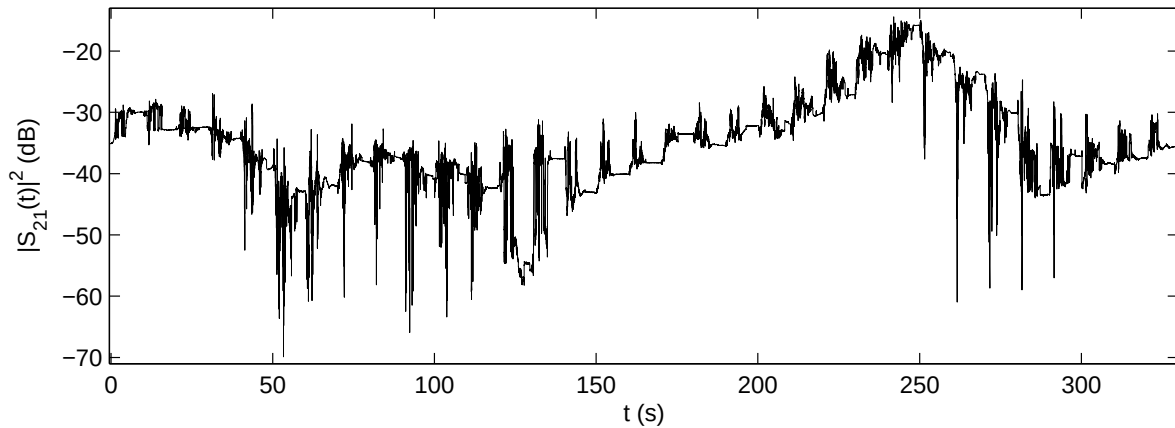
(a) $M = 400$ steps(b) $M = 800$ steps(c) $M = 1200$ steps

Figure 8: Time series representation of tuner transitions for $|S_{21}|^2$ as a function of time t , for all transitions during one complete rotation at $f = 100$ MHz, with $A = 1000$ steps/s², $B = 600$ steps/s, $V = 2000$ steps/s, and (a) $M = 400$ steps ($N = 100$); (b) $M = 800$ steps ($N = 50$); (c) $M = 1200$ steps ($N = 33$).

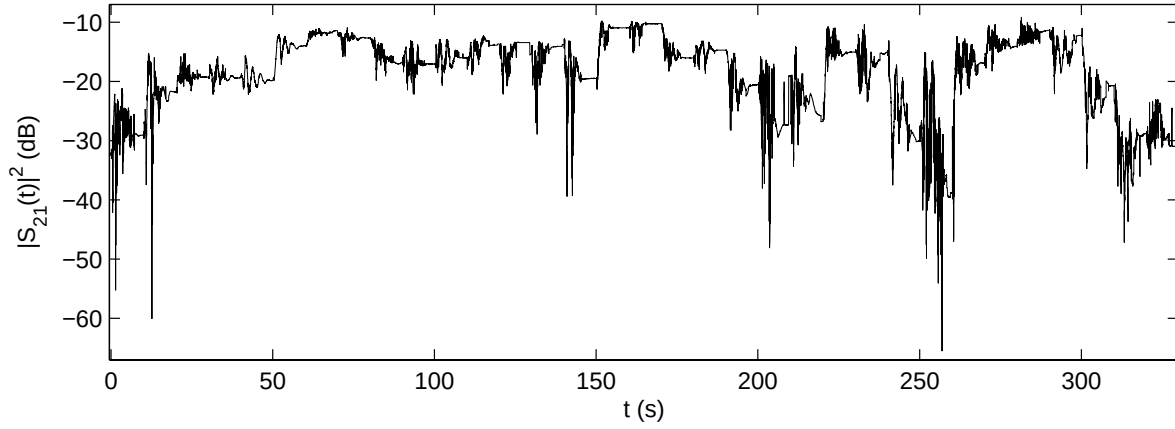
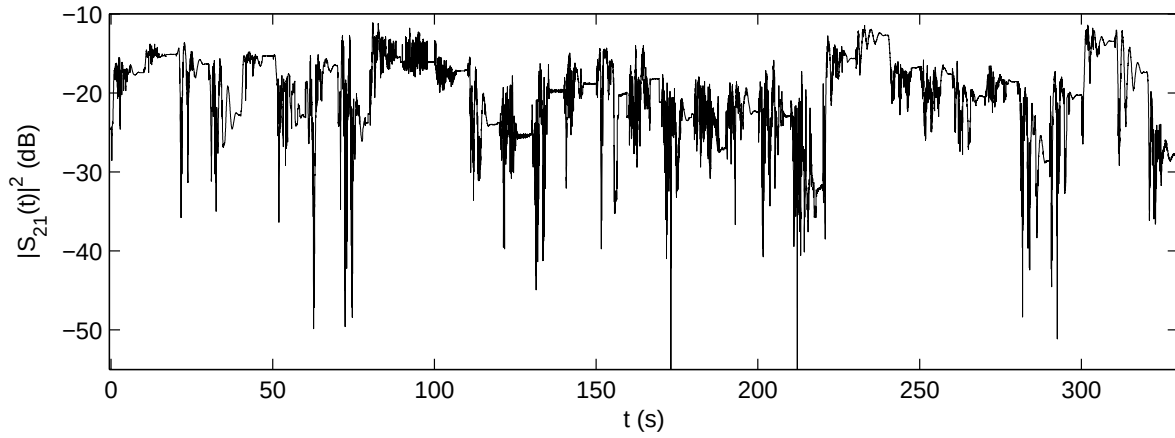
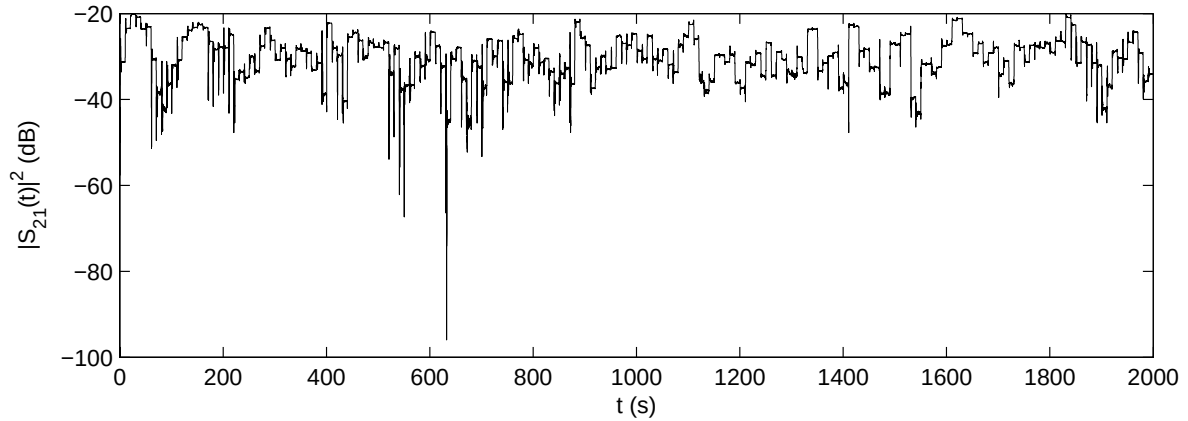
(a) $f = 200$ MHz(b) $f = 300$ MHz(c) $f = 1000$ MHz

Figure 9: Time series representation of tuner transitions for $|S_{21}|^2$ as a function of time t , for all transitions during one complete rotation, with $A = 1000$ steps/s², $B = 600$ steps/s, $V = 2000$ steps/s, and (a) $f = 200$ MHz, $M = 1200$ steps ($N = 33$); (b) $f = 300$ MHz, $M = 1200$ steps ($N = 33$); (c) $f = 1000$ MHz, $M = 200$ steps ($N = 200$).

4 Construction of transient stir sequences

4.1 Transposition of time traces into stir sequences

With a view to perform a statistical analysis of the quasi-random field fluctuations during the tuner transitions, the measured response is converted from a 1D time series into a 2D lattice process, as follows⁸. Provided⁹ the steady-state field values at different static tuner positions constitute a wide-sense stationary process (i.e., temporally homogeneous) across one full revolution¹⁰ (i.e., all tuner positions θ_n generate statistically equivalent steady-state fields, with no special physical significance attached to any arbitrarily chosen reference state $\theta_0 \equiv 0$), the individual traces capturing the real-time transitions between θ_n and θ_{n+1} represent N nominally¹¹ and statistically equivalent replicas of the tuner transitions. In other words, the time records generate N realizations from a *single* stochastic ensemble of statistically equivalent stepping transients. Using the instance of the controller's trigger command for the start of each tuner step as a datum line ($t_0 \equiv 0$ s), this permits the single 1D time series of concatenated transitions ($-0.5 \text{ s} \leq t < (10N - 0.5) \text{ s}$) [Fig. 10(a)] to be rearranged into a 2D discrete set of juxtaposed and synchronized transitions above the (t, θ) -plane, where $t_0 - 0.5 \text{ s} \leq t \leq t_0 + 9.5 \text{ s}$ and $0 \leq \theta < N\Delta\theta \equiv 2\pi$ [Fig. 10(b)¹²]. The steady-state end value of the n th trace equals the start value for the $(n + 1)$ th trace in the lattice, i.e.,

$$|S_{21}(t = t_0 + L\Delta t; \theta = \theta_n)| = |S_{21}(t = t_0; \theta = \theta_{n+1})|. \quad (8)$$

For brevity, we shall denote $|S_{21}(t; \theta)|^2$ as $P(t; \theta)$.

A 1D slice $\{P(t = t_\ell; n\Delta\theta)\}_{n=0, \dots, N-1}$, cut parallel to the θ -axis at an arbitrarily chosen instant (age) $t_\ell = t_0 + \ell\Delta t$ across all N transitions (cf. black circles in Fig. 10), results in a stir sequence that is then¹³ *stationary* with respect to θ , at $t = t_\ell$, because the start times of the tuner

⁸The method is an adaptation of a technique developed for analysis of pulse-amplitude modulated fields [22].

⁹Strictly, this should be verified quantitatively using a test of hypothesis for stationarity [25].

¹⁰In view of the observations from Fig. 2, Sec. 3.1.1, this is a good approximation, although possibly an idealization.

¹¹Small timing errors, variations in the extent of backlash, etc., may cause small differences in conditions. However, these differences are themselves random and hence the transitions are statistically equivalent. These random differences are part and parcel of the observed transient field fluctuations themselves.

¹²The traces are shown with respect to their *start* angle θ_n , rather than the instantaneous angle $\theta(t)$. For the latter case, owing to the accelerations and mechanical oscillations, the traces would have to be displayed in a nonuniformly skewed manner relative to both ordinate axes. However, the detailed $\theta(t)$ for individual transitions is unknown.

¹³The stationarity of each 1D slice presumes stationarity of the sequence of steady-state field levels.

rotations are synchronized. Each individual 1D slice cut in this way can therefore be analyzed using standard stochastic methods. By contrast, the time traces $\{P(\ell\Delta t; \theta = \theta_n)\}_{\ell=-80, \dots, L-81}$ along the t -direction represent transitory evolutions and are clearly nonstationary with respect to t .

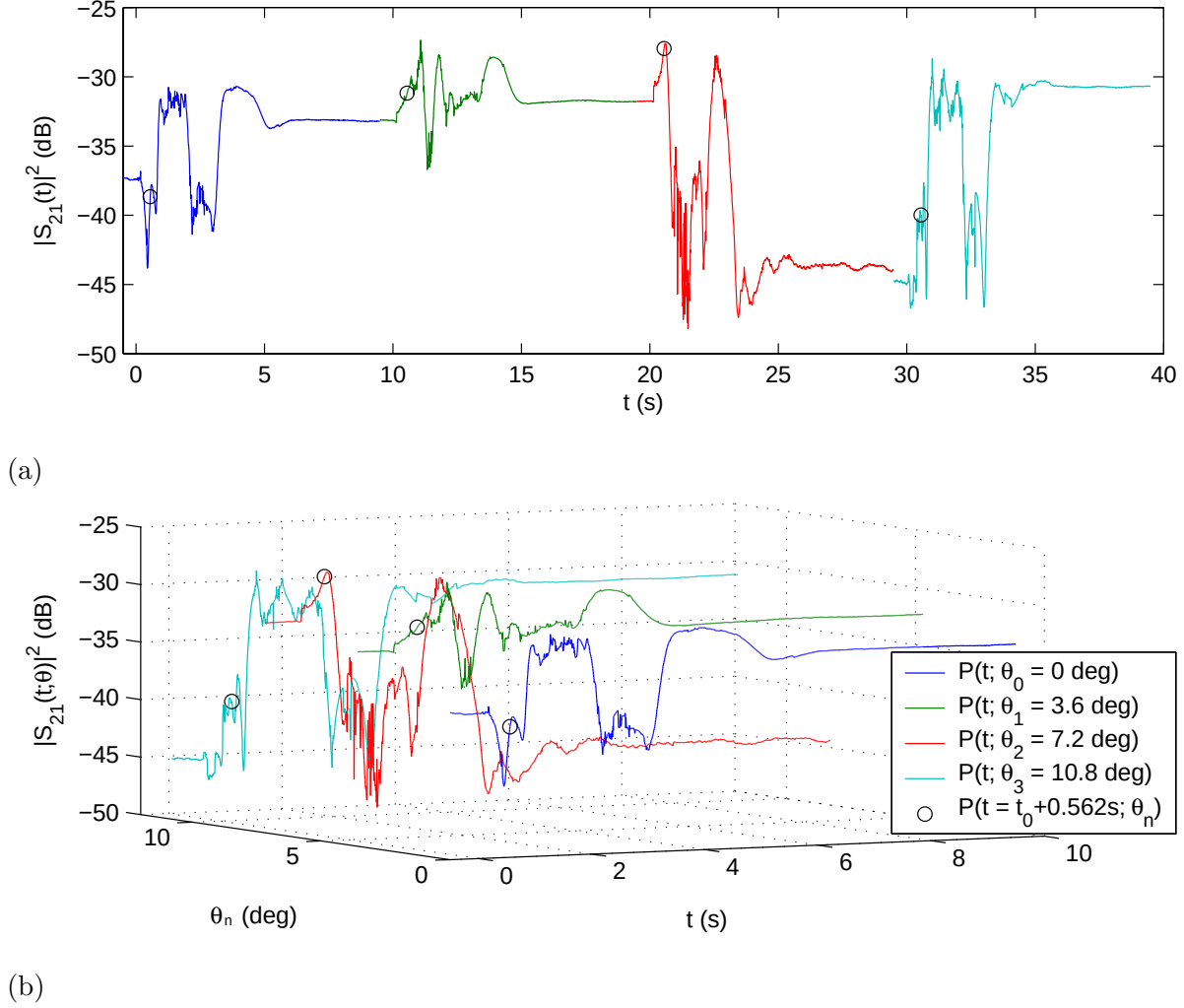


Figure 10: (color online) (a) 1D time series representation and (b) 2D lattice representation of the first four tuner transitions of $|S_{21}(t)|^2$ during one complete rotation of the mode tuner, as a function of time t and initial tuner state θ_n at $f = 1000$ MHz, with $M = 400$ steps ($N = 100$), $A = 1000$ steps/s², $B = 600$ steps/s, $V = 2000$ steps/s. Black circles indicate the first four data points ($n = 1, \dots, 4$) in the single stir sequence $\{P(t = t_0 + 0.562 \text{ s}; \theta_n)\}_{n=0, \dots, 999}$ associated with the instance $t = t_0 + 90\Delta t = t_0 + 0.562 \text{ s}$ in the lattice process $[t_0 = (10 \times n) \text{ s}]$.

5 Jitter of mechanical motion, onset and fan-out of transients

When transposing the time traces into stir sequences, mechanical jitter – i.e., small timing differences between the actual start times of the paddle wheel steps – may exist and cause mixing between two or more sequences belonging to two or more different t_ℓ . This is a particular issue for second-order statistics (cf. Secs. 6.5–6.7), which are more sensitive to jitter than first-order statistics, because mixing increases the spectral bandwidth of (and, hence, reduces the correlation within) the stir sequences. Unlike jitter in periodic binary signals, the tuner steps do not generate a two-level periodic sequence. Therefore, the time delays cannot be directly deduced from the measured S-parameters, because each step transition is between a pair of different steady-state levels. A proper investigation of jitter requires an encoded paddle wheel and investigation of its inertia and backlash from a purely mechanical point of view. This is outside the scope of the present study.

To investigate the effect of jitter on the field dynamics, we here introduce “fan-out” diagrams shown in Fig. 11, akin to the well-known “eye” diagrams used in the study of synchronization, phase delays and jitter in digital communication systems. The fan-out diagram demonstrates the onset of the field fluctuations for all N step transitions, following the trigger signal at $t = t_0 \triangleq 0$ s. This is achieved by plotting the deviation of $P(t)$ during each transition normalized by the steady-state starting value for that particular transition, i.e., $P(t)/P(t = -0.5 \text{ s})$, where $P(t)$ is expressed in linear units (not dB). Note that some of these steady-state values are very small (–40 dB or less), making them more sensitive to any small fluctuation than for higher steady-state values. Therefore, as a further rationalization, the fan-out diagram might instead be defined from the standardizations $[P(t) - P(t = -0.5 \text{ s})]/[P(t = 9.5 \text{ s}) - P(t = -0.5 \text{ s})]$ which always evolve from 0 to 1 for each transition. However, representations based on the latter definition have proved to be less illuminating and are therefore omitted here.

Fig. 11(a) shows that in the undermoded regime (100 MHz), where the number of excited modes within the cavity Q -bandwidth is very small at all times (occasionally zero for some tuner steps), the field increments form a blocked pattern (cf. stepwise changes between 0.05 s and 0.2 s in the Figure). These fluctuations can be interpreted as caused by a cavity mode moving in and out of the excitation band, producing a “crackling” of the resultant field when the average number of excited modes is small. The trigger delay is seen to be approximately 55 ms. At 200 MHz, shown in Fig. 11(b), the crackling is already much reduced, as most traces take off from the steady-state value in a smooth gradual manner. Occasionally, some of the “steady-state”

values are less stable, particularly when they are less than -30 dB. This is illustrated in the fan-out diagram for 300 MHz in Fig. 11(c), for which the transitions at $t = t_0 + 520$ s and $t = t_0 + 530$ s start from fluctuating levels. At 1000 MHz [Fig. 12(a)], the crackling has virtually completely disappeared.

A much reduced value of the motor acceleration has no discernable effect on the rate of the spreading and jitter in the fan-out [compare Fig. 12(b) vs. 12(a)]. By contrast, reducing the base speed delays the onset of the fan-out and smoothens the transitions [compare Fig. 12(c) vs. 12(a)]. This is *a fortiori* the case when both A and B have low values, as shown in Figs. 13(a) and (b).

Finally, halving the step size of the mode tuner decreases the average tuner speed, resulting in the fan-out “bundle” of traces to be more concentrated around the reference (0 dB) value [compare Fig. 13(c) vs. 12(a)].

For completeness, Fig. 14 shows two complete fan-out diagrams for the maximal (nominal) and minimal parameter values, respectively, indicating that the differences at steady-state are more or less symmetric (on a dB scale) with respect to 0 dB.

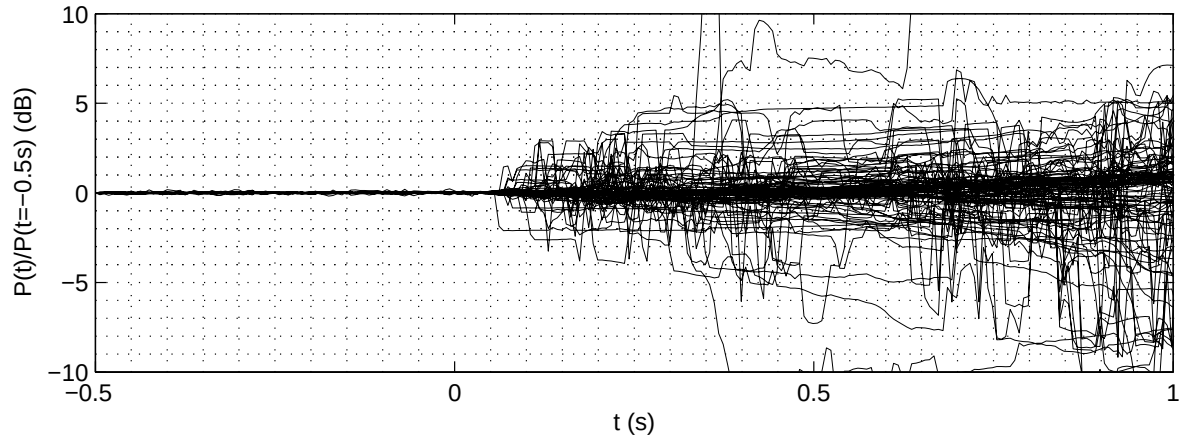
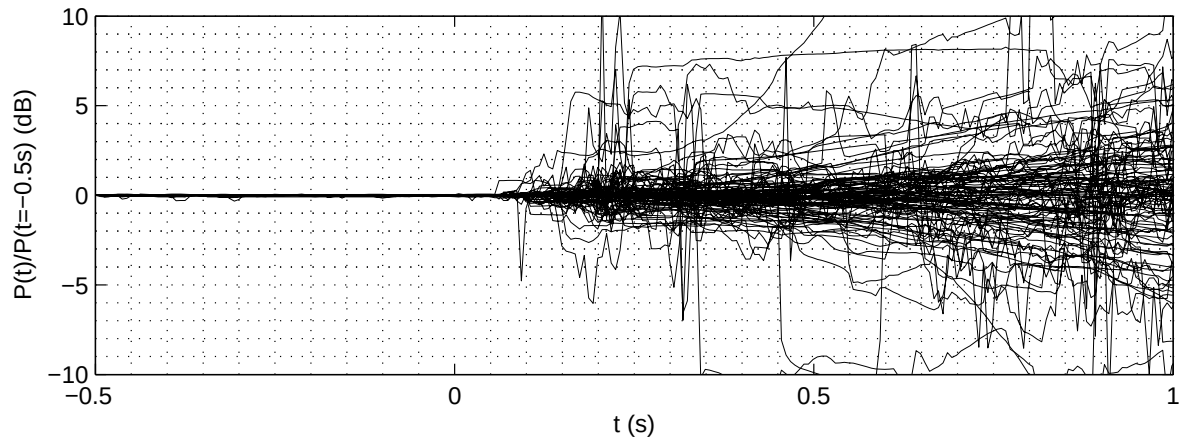
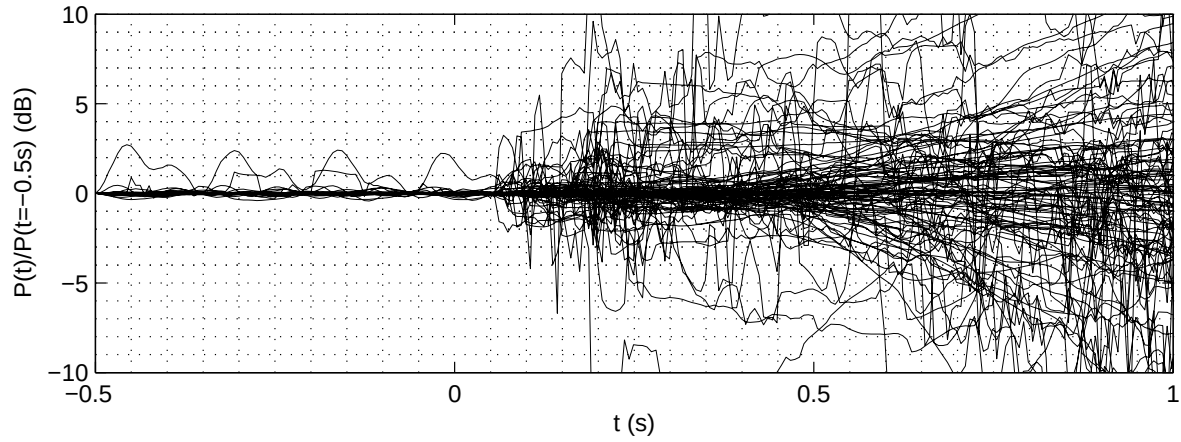
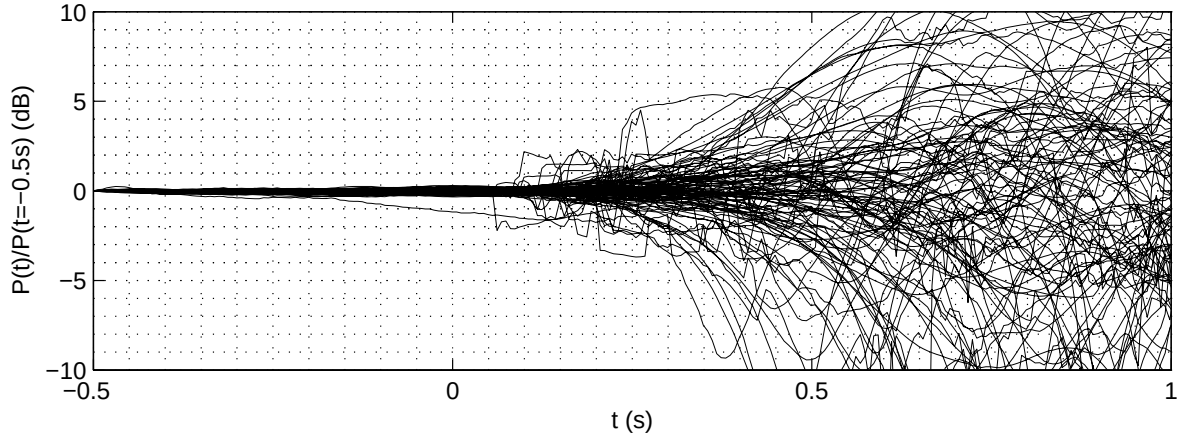
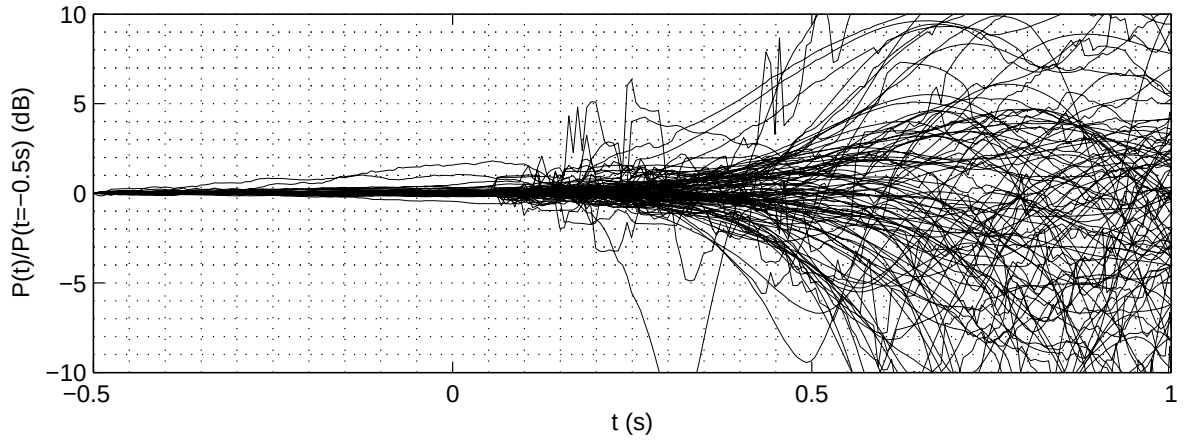
(a) $f = 100$ MHz(b) $f = 200$ MHz(c) $f = 300$ MHz

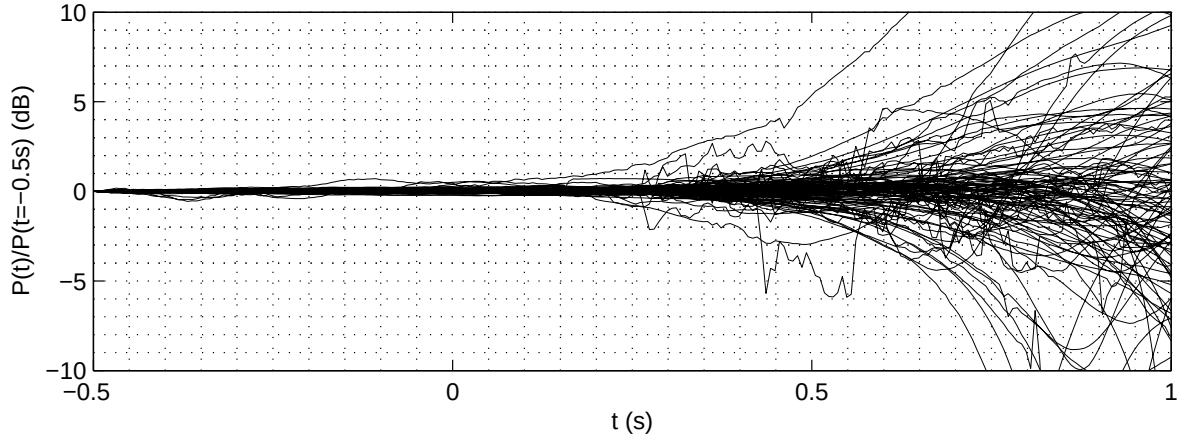
Figure 11: Fan-out diagrams of tuner transitions for $|S_{21}(t; \theta_n)|^2 / |S_{21}(t = -0.5 \text{ s}; \theta_n)|^2$ as a function of time t , for all N transitions during one complete rotation with $M = 400$ steps ($N = 100$), $A = 1000$ steps/s², $B = 600$ steps/s, $V = 2000$ steps/s, at (a) $f = 100$ MHz; (b) $f = 200$ MHz; (c) $f = 300$ MHz.



(a) $A = 1000 \text{ steps/s}^2$, $B = 600 \text{ steps/s}$

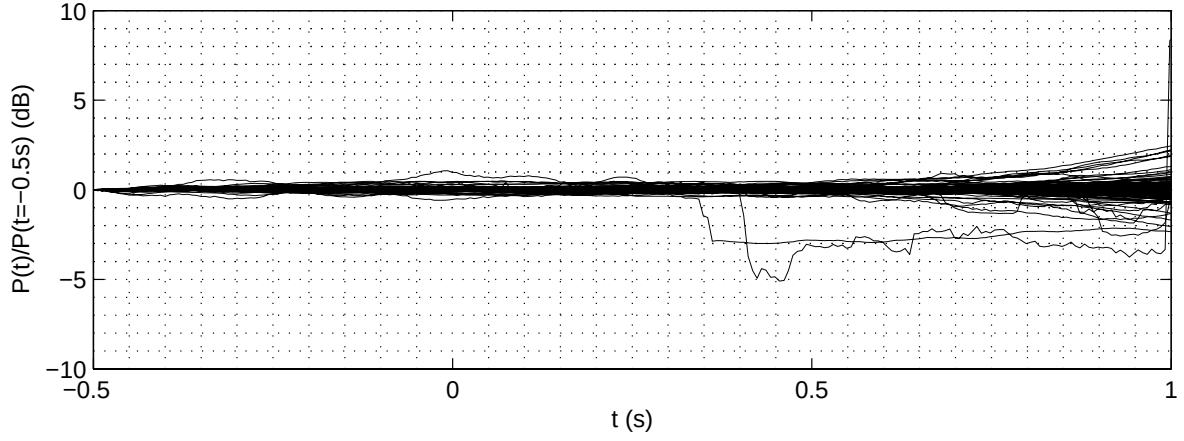


(b) $A = 1 \text{ step/s}^2$, $B = 600 \text{ steps/s}$

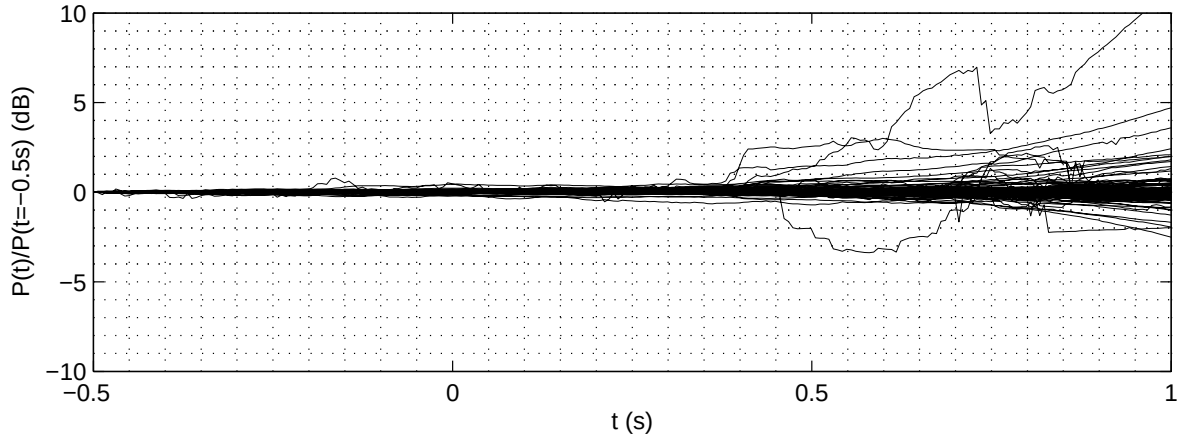


(c) $A = 1000 \text{ steps/s}^2$, $B = 1 \text{ step/s}$

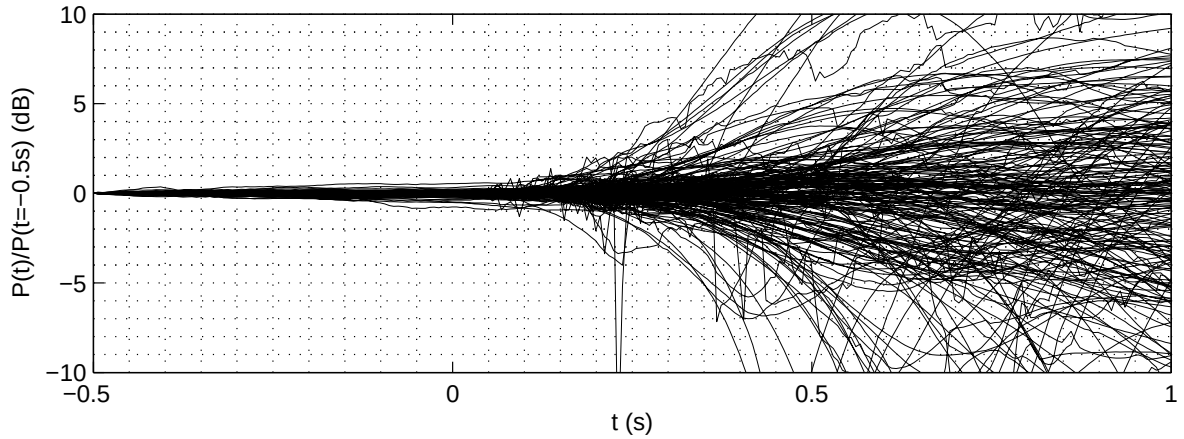
Figure 12: Fan-out diagrams of tuner transitions for $|S_{21}(t; \theta_n)|^2 / |S_{21}(t = -0.5 \text{ s}; \theta_n)|^2$ as a function of time t , for all N transitions during one complete rotation, at $f = 1000 \text{ MHz}$ with $M = 400 \text{ steps}$ ($N = 100$), $V = 2000 \text{ steps/s}$, and (a) $A = 1000 \text{ steps/s}^2$, $B = 600 \text{ steps/s}$; (b) $A = 1 \text{ step/s}^2$, $B = 600 \text{ steps/s}$; (c) $A = 1000 \text{ steps/s}^2$, $B = 1 \text{ step/s}$.



(a) $M = 400$ steps ($N = 100$), $A = 100$ steps/s², $B = 10$ steps/s, $V = 2000$ steps/s

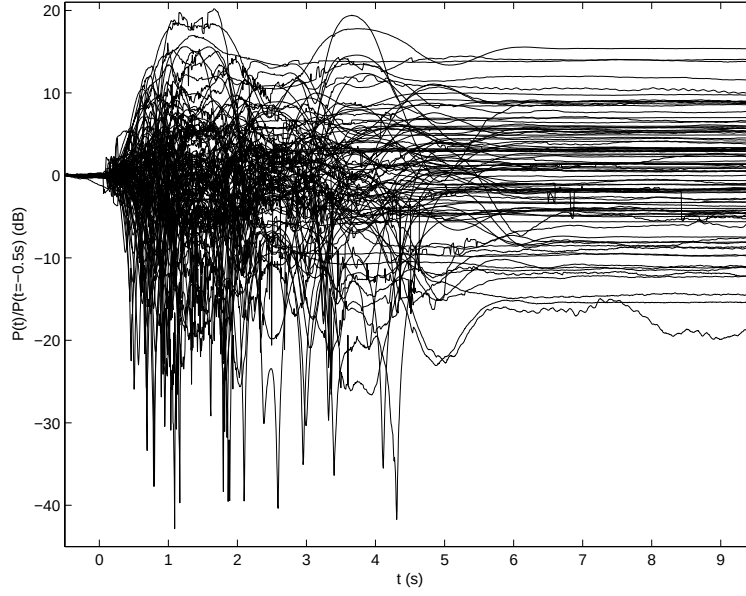


(b) $M = 400$ steps ($N = 100$), $A = 100$ steps/s², $B = 10$ steps/s, $V = 200$ steps/s

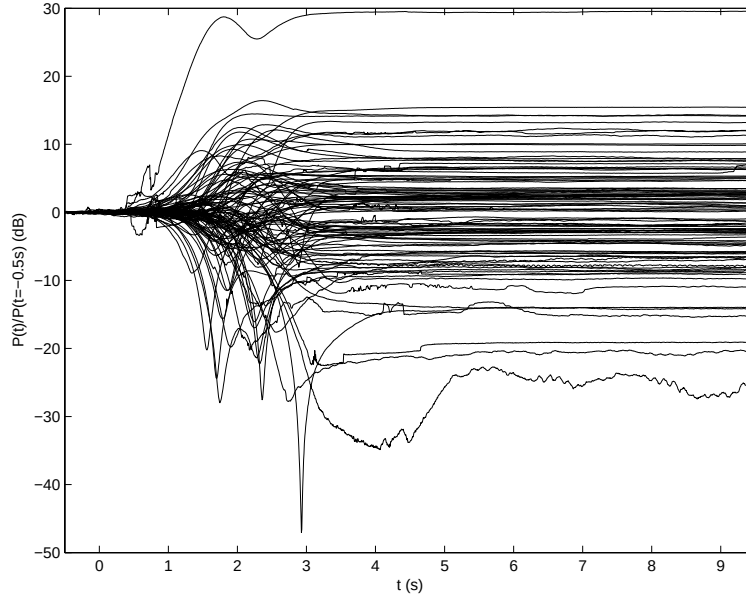


(c) $M = 200$ steps ($N = 200$), $A = 1000$ steps/s², $B = 600$ steps/s, $V = 2000$ steps/s

Figure 13: Fan-out diagrams of tuner transitions for $|S_{21}(t; \theta_n)|^2 / |S_{21}(t = -0.5 \text{ s}; \theta_n)|^2$ as a function of time t , for all N transitions during one complete rotation, at $f = 1000$ MHz with (a) $M = 400$ steps ($N = 100$), $A = 100$ steps/s², $B = 10$ steps/s, $V = 2000$ steps/s; (b) $M = 400$ steps ($N = 100$), $A = 100$ steps/s², $B = 10$ steps/s, $V = 200$ steps/s; (c) $M = 200$ steps ($N = 200$), $A = 1000$ steps/s², $B = 600$ steps/s, $V = 2000$ steps/s.



(a) $A = 1000 \text{ steps/s}^2$, $B = 600 \text{ steps/s}$, $V = 2000 \text{ steps/s}$



(b) $A = 100 \text{ steps/s}^2$, $B = 10 \text{ steps/s}$, $V = 200 \text{ steps/s}$

Figure 14: Fan-out diagram of tuner transitions for $|S_{21}(t; \theta_n)|^2 / |S_{21}(t = -0.5 \text{ s}; \theta_n)|^2$ as a function of time t , for all N transitions during one complete rotation, at $f = 1000 \text{ MHz}$ with $M = 400 \text{ steps}$ ($N = 100$) and (a) $A = 1000 \text{ steps/s}^2$, $B = 600 \text{ steps/s}$, $V = 2000 \text{ steps/s}$; (b) $A = 100 \text{ steps/s}^2$, $B = 10 \text{ steps/s}$, $V = 200 \text{ steps/s}$.

6 Statistical data analysis

6.1 Temporal evolution of cross-correlation between stir sequences

6.1.1 Dependence on frequency

Figure 15 shows the cross-correlation function $\varrho_P(t_0, \ell\Delta t) \triangleq \varrho_{P_0, P_\ell}(t_0, t_\ell = t_0 + \ell\Delta t)$ of $P(t; \theta)$ between the stir sequence formed by the starts of the tuner motions, viz., $\{P(t = t_0; \theta_n)\}_{n=0, \dots, N-1}$ and any intermediate sequence $\{P(t = t_\ell = t_0 + \ell\Delta t; \theta_n)\}_{n=0, \dots, N-1}$ where $\ell = -80, \dots, L - 81$ and $N = 100$. Formally, this function is defined by

$$\varrho_P(t_0, \ell\Delta t) \triangleq \frac{\langle P(t_0)P(t_\ell = t_0 + \ell\Delta t) \rangle - \langle P(t_0) \rangle \langle P(t_\ell = t_0 + \ell\Delta t) \rangle}{\sqrt{\langle P^2(t_0) \rangle - \langle P(t_0) \rangle^2} \sqrt{\langle P^2(t_\ell) \rangle - \langle P(t_\ell) \rangle^2}} \quad (9)$$

where t_0 is arbitrary for a stationary sequence, but here chosen as $t_0 = 0$. For each frequency, a distinct damped oscillatory behaviour towards a frequency-dependent steady-state value (asymptotic cross-correlation coefficient) is observed. The smoothness and amplitude of $\varrho_P(t)$, relative to its steady-state value, increase with increasing frequency. During early times, the values of ϱ_P for $f \geq 400$ MHz are relatively tightly grouped (e.g., $\varrho_P \sim 0.15$ at $t = 1.5$ s), whereas beyond the first quasi-period of the damped oscillation ($t \geq 2.5$ s) their values have spread across a larger range between different frequencies. The oscillations of the $\varrho_P(t)$ exhibit the same quasi-period for all frequencies, and its value is considerably longer than the time scale of the intrinsic cavity relaxation ($\sim \mu\text{s}$) at any frequency. This indicates that the oscillations in $\varrho_P(t)$ represent mechanical oscillations caused by tuner backlash (more precisely, they represent the effect that these mechanical oscillations have on the interior field), rather than the beating of cavity modes during the transition, caused by changing eigenmodes, which occurs on a much smaller time scale. It is possible, however, that the latter are responsible for the fine structure of $\varrho_P(t)$, as supported by the fact that a decrease of frequency results in an increase of relaxation times (causing prolonged effects of the “previously” existing modes) and decreasing mode density (causing proportionally larger effects of changes of a given number of modes). However, more extensive measurements with smaller Δt are needed to investigate this further. For $f = 1000$ MHz, the anomalous behaviour near $\varrho_P = 0$ may be caused by limited resolution near zero and inaccuracy due to the relatively short length of the sequence ($N = 100$).

The significance of the time variation of ϱ_P is that it provides an indirect method for detecting the nominal (average) tuner dynamics during the transitions. The damped mechanical oscillation of the tuner cannot be discerned in individual field transients, because the individual time-

domain traces of transients are dominated by irregular fluctuations [Figs. 2 to 5(a)]. Physically, these irregular fluctuations can be a result of a combination of migration, mutation, bunching, and beating of “previous”, “current”, and “future” cavity modes (temporal interference), as a result of the finite relaxation time and dynamic eigenspectrum. This interference gives rise to *deterministic* instantaneous mixing and randomization of the field (as opposed to a mode tuned field, which is merely *statistically* mixed and randomized, across the tuner sweep interval) and produces a dominant “fine” structure riding on top of the damped oscillation in the time-domain response. Significantly, the cross-correlation analysis enables EM fluctuations to be filtered from the average trend of $|S_{21}|$, as shown most dramatically by $\varrho_P(t)$ for $f \leq 300$ MHz in Fig. 15.

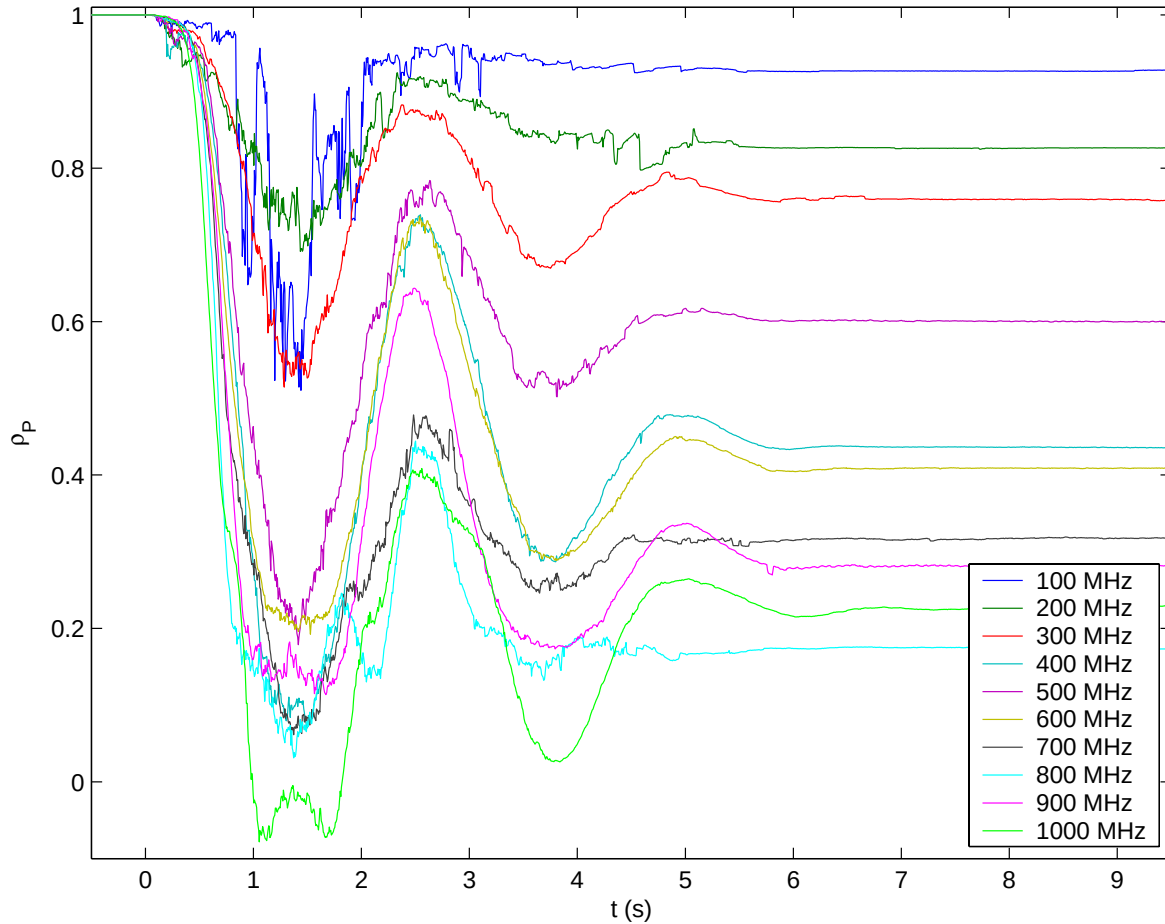


Figure 15: (color online) Cross-correlation coefficient ϱ_P as a function of transition time $t = t_\ell - t_0 = \ell \Delta t$ at selected frequencies f , with $M = 400$ steps ($N = 100$), $A = 1000$ steps/s², $B = 600$ steps/s, $V = 2000$ steps/s.

6.1.2 Dependence on motor parameters

In view of the deduced relationship between the tuner motion and $\varrho_P(t)$, it is of interest to investigate the effect of the various control parameters for the stepper motor on $\varrho_P(t)$. To this end, we vary one parameter at a time.

Figure 16 shows the effect of varying the motor acceleration, A , on $\varrho_P(t)$. The graph indicates that, as expected, the smallest acceleration produces the most regular motion and smoothest transients, particularly during the early part of the tuner motion, as well as giving the best repeatability for the steady-state value of ϱ_P . (Differences in the steady-state value are a combined result of friction, inertia, and inherent statistical fluctuations of random fields.)

Figure 17 shows the effect of varying the base speed, B , again indicating that a smaller parameter value yields a more regular motion.

Figure 18 shows the effect of varying the slew speed, V . Even for the lowest values $A = 100$ steps/s² and $B = 10$ steps/s, the lower value of V has a noticeable effect, in that it causes the duration of the acceleration to become much smaller than the duration of the complete transition. As a result, the oscillation in $\varrho_P(t)$ for $V = 2000$ steps/s turns into a relaxation towards the final state for $V = 200$ steps/s.

Finally, Fig. 19 shows the effect of varying the number of motor steps M per angular step of the paddle wheel, i.e., varying $\Delta\theta$. While the quasi-period is equal in both cases, halving angular steps size (200 steps/s) leads to the steady state being reached sooner, because the maximum speed is smaller in this case.

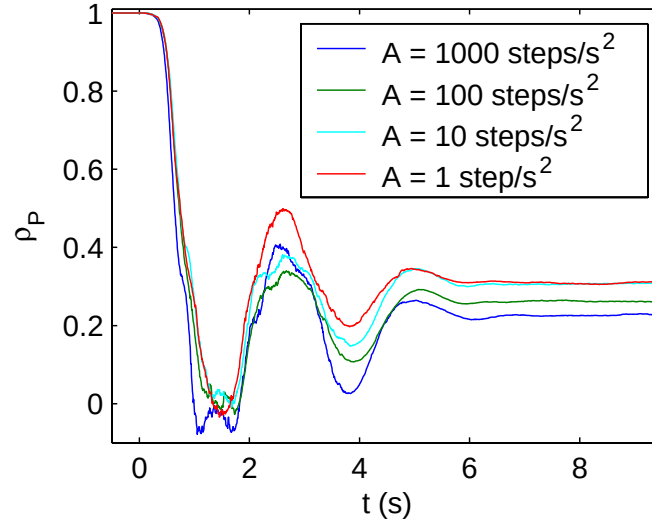


Figure 16: (color online) Cross-correlation coefficient ρ_P as a function of transition time $t = t_\ell - t_0 = \ell\Delta t$ at $f = 1000$ MHz, for selected values of A , with $M = 400$ steps ($N = 100$), $B = 600$ steps/s, $V = 2000$ steps/s.

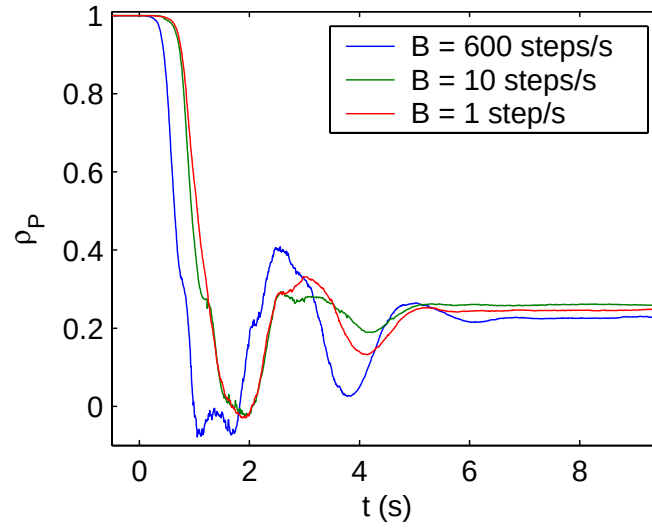


Figure 17: (color online) Cross-correlation coefficient ρ_P as a function of transition time $t = t_\ell - t_0 = \ell\Delta t$ at $f = 1000$ MHz, for selected values of B , with $M = 400$ steps ($N = 100$), $A = 1000$ steps/s², $V = 2000$ steps/s.

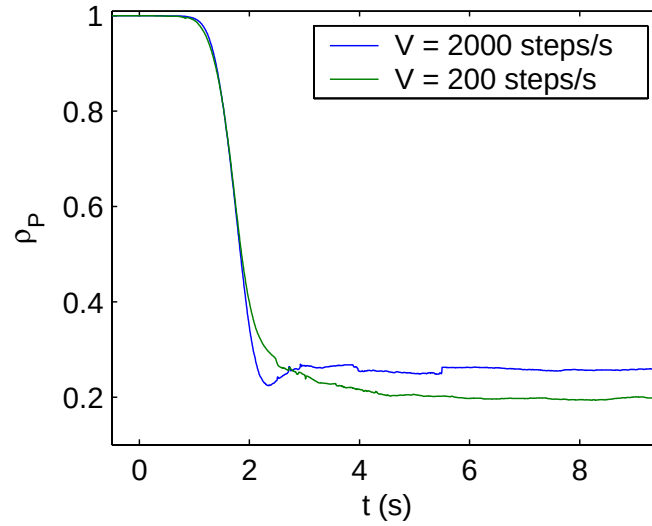


Figure 18: (color online) Cross-correlation coefficient ϱ_P as a function of transition time $t = t_\ell - t_0 = \ell\Delta t$ at $f = 1000$ MHz, for selected values of V , with $M = 400$ steps ($N = 100$), $A = 100$ steps/s², $B = 10$ steps/s.

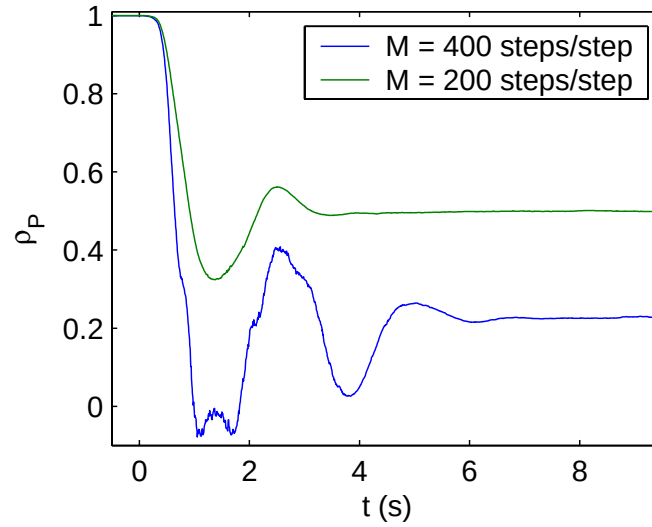


Figure 19: (color online) Cross-correlation coefficient ϱ_P as a function of transition time $t = t_\ell - t_0 = \ell\Delta t$ at $f = 1000$ MHz, for selected values of M , with $A = 1000$ steps/s², $B = 600$ steps/s, $V = 2000$ steps/s.

6.2 Temporal evolution of mean value, standard deviation and coefficient of variation

6.2.1 Mean value

6.2.1.1 Dependence on frequency Figure 20 compares instantaneous sample average values $\mu_P(t)$ between different frequencies, denoted and defined by

$$\mu_P(t) \equiv \langle |S_{21}(t)|^2 \rangle = \sum_{n=0}^{N-1} \frac{|S_{21}(t; \theta_n \rightarrow \theta_{n+1})|^2}{N} \quad (10)$$

after normalization with respect to their steady-state values $\mu_P(0)$. Typically, $\mu_P(t)/\mu_P(0)$ varies by 5...15% during the transition, for any frequency. At 100 MHz (undermoded regime), the fluctuations are somewhat larger. This degree of random fluctuations for $\mu_P(t)$ may be expected on account of the properties of sample statistics: the variance of the *sample* mean for a χ^2_2 -distributed $P(t)$ is¹⁴ $\sigma_P(t)/\sqrt{N_{\text{ind}}} \sim \sigma_P(t)/\sqrt{N} = \mu_P(t)/\sqrt{N}$, while for $N_{\text{ind}} < N$ (i.e., at lower frequencies) it is somewhat higher. However, as will be demonstrated in Sec. 6.2.1.2 through a parametric study, the fluctuations of $\mu_P(t)$ are not as random as may seem at first. This latter observation carries important implications with regard to the statistical-physical properties of the random field.

6.2.1.2 Dependence on motor parameters Figure 21 shows $\mu_P(t)$ at $f = 1000$ MHz for selected values of A . At this relatively high frequency, the extent of variations of $\mu_P(t)$ is not particularly sensitive to the value of A . The graphs indicate that $\mu_P(t)$ exhibits a weak but distinct quasi-periodicity for any value of A , on average, first with increasing and then decreasing instantaneous amplitude, suggesting that the local energy density during the transition is not constant¹⁵ but becomes bunched in time (modulated) due to the mechanical motion. It also serves as a further indication that the number of participating modes (i.e., modes within the cavity excitation bandwidth) at any one time fluctuates during the transition, as a result of the coexistence of, and coupling between “previous”- and “next”-state cavity modes in the course of the transition owing to finite relaxation times, as already mentioned. The variations indicate that

¹⁴Here, $\sigma_P(t)$ represents the ensemble standard deviation.

¹⁵Note that *irregular* fluctuations of $\mu_P(t)$ are to be expected in any case, on account of the fact that each stir sequence is a single realization, thus producing a single *sample* value $\mu_P(t)$ whose sample standard deviation is $s_P = \sigma_P/\sqrt{N}$. The key point is the fact that here the observed variations also contain a *regular* component, which cannot be explained by the fact that sample statistics for finite M are considered, which by themselves exhibit purely *random* fluctuations.

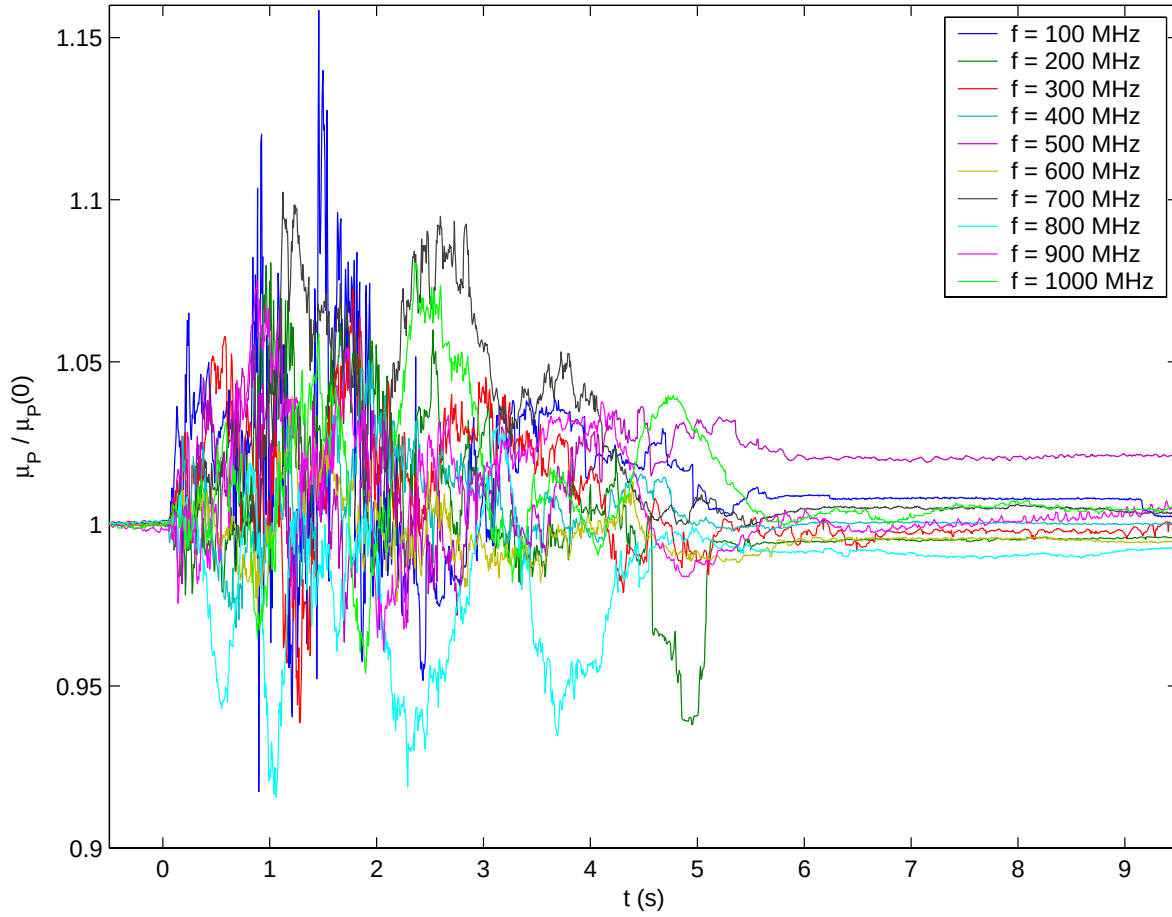


Figure 20: (color online) Normalized average value $\mu_P(t)/\mu_P(0) \equiv \langle |S_{21}(t)|^2 \rangle / \langle |S_{21}(t_{0-})|^2 \rangle$ as a function of transition time $t = t_\ell - t_0 = \ell \Delta t$, at selected frequencies f for $M = 400$ steps ($N = 100$), $A = 1000$ steps/s², $B = 600$ steps/s, $V = 2000$ steps/s.

the assumption of temporal homogeneity (EM quasi-stationarity) is at best an approximation. Except for their first quasi-peak, the $\mu_P(t)$ reach local maximum values near the turning points, i.e., near the local maxima and minima of the tuner angular position, as follows by comparing Fig. 21 with Fig. 16. Consequently, the quasi-period of $\mu_P(t)$ is about half the value of the quasi-period for $\varrho_P(t)$. Unlike the continuously damped $\varrho_P(t)$, the fluctuations of $\mu_P(t)$ reach a maximum amplitude about halfway during the transitional motion, reaching an overall quasi-peak around $2 \dots 2.5$ s.

Similar effects are observed when changing B or V instead of A . Fig. 22 shows that, unlike when reducing A (compare Fig. 21) for which quasi-oscillatory behaviour of $\mu_P(t)$ is maintained throughout, reducing B rapidly reduces the quasi-periodic behaviour while still permitting significant fluctuations, indicating that the tuner oscillations in the latter case are reduced. Fig. 23 shows that oscillations in $\mu_P(t)$ are much reduced when choosing $A = 100$

steps/s². In particular, $\mu_P(t)$ in Fig. 23 follows the triangular velocity profile of the motor, qualitatively: for this choice of parameter values (cf. Sec. 2.2), the motor reaches its maximum speed ($V' = 197.7$ steps/s) at 1.88 s, returns to base speed after 3.76 s, and stops after 4.76 s. The effect of motor acceleration/deceleration on $\mu_P(t)$ can be clearly witnessed, suggesting that the average instantaneous $\mu_P(t)$ during the transition is roughly proportional to the instantaneous speed of the mode tuner and reaches a maximum when the instantaneous speed is largest. The small time delays of the points on $\mu_P(t)$ where transitions in $\langle |S_{21}|^2 \rangle$ occur, compared to the transition points for the motor speed itself, may be ascribed to inertia of the paddle wheel. Note that Fig. 23 contains almost no irregular sample-statistical variations at all anymore: the variability of $\mu_P(t)$ has now become almost purely deterministic.

Values of $\mu_P(t)$ for different tuner step sizes, for which results are shown in Fig. 24, indicate that the maximum fluctuation in $\mu_P(t)$ is now reduced (although the quasi-periodicity remains about the same). This is also partially due to the resulting increase in N .

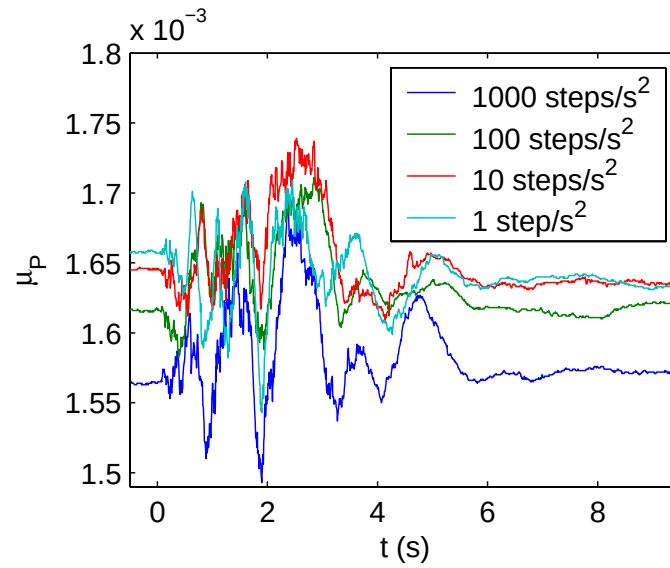


Figure 21: (color online) Average value μ_P as a function of transition time $t = t_\ell - t_0 = \ell \Delta t$ at $f = 1000$ MHz, for selected values of A , with $M = 400$ steps ($N = 100$), $B = 600$ steps/s, $V = 2000$ steps/s.

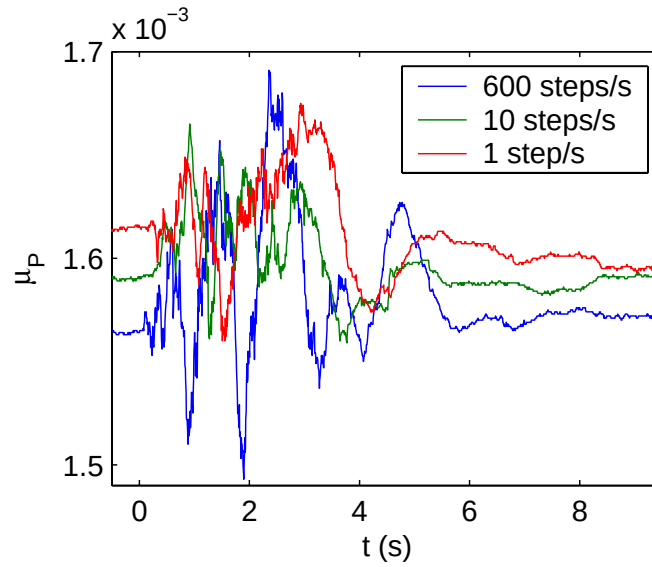


Figure 22: (color online) Average value μ_P as a function of transition time $t = t_\ell - t_0 = \ell \Delta t$ at $f = 1000$ MHz, for selected values of B , with $M = 400$ steps ($N = 100$), $A = 1000$ steps/s², $V = 2000$ steps/s.

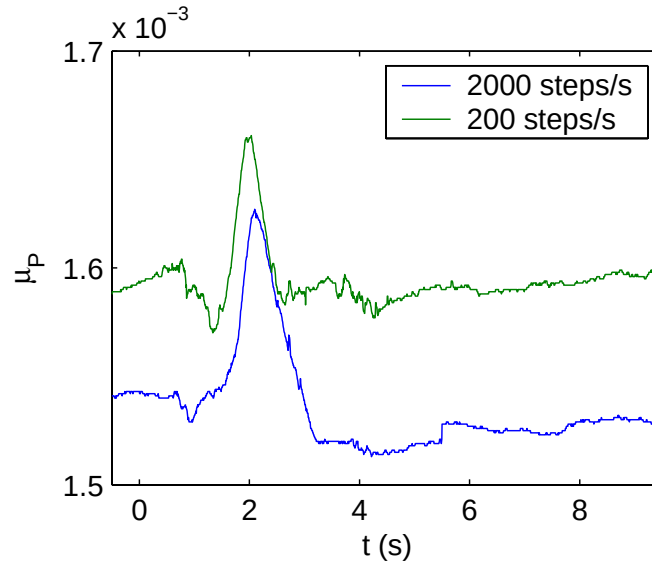


Figure 23: (color online) Average value μ_P as a function of transition time $t = t_\ell - t_0 = \ell \Delta t$ at $f = 1000$ MHz, for selected values of V , with $M = 400$ steps ($N = 100$), $A = 100$ steps/s², $B = 10$ steps/s.

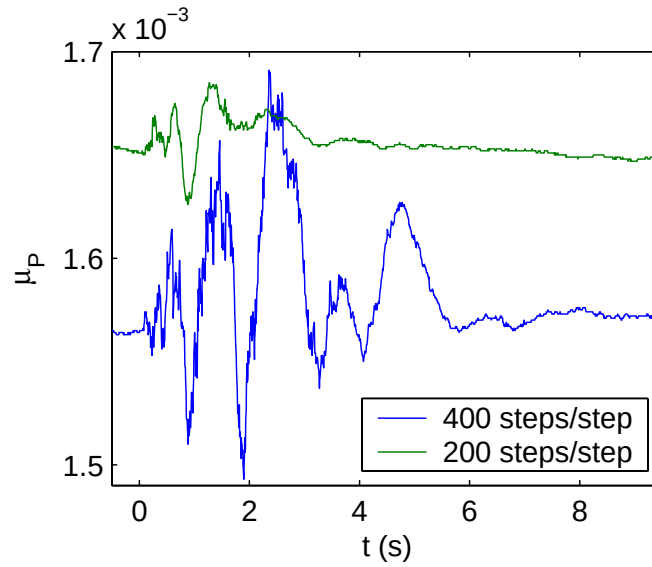


Figure 24: (color online) Average value μ_P as a function of transition time $t = t_\ell - t_0 = \ell \Delta t$ at $f = 1000$ MHz, for selected values of M , with $A = 1000$ steps/s², $B = 600$ steps/s, $V = 2000$ steps/s.

6.2.2 Standard deviation

6.2.2.1 Dependence on frequency Figure 25 compares values for the sample¹⁶ standard deviation $\sigma_P(t)$ defined by

$$\sigma_P(t) \triangleq \sqrt{\langle P^2(t) \rangle - \langle P(t) \rangle^2} \equiv \sqrt{\langle |S_{21}(t)|^4 \rangle - \langle |S_{21}(t)|^2 \rangle^2} \quad (11)$$

and normalized with respect to its steady-state value $\sigma_P(0)$, for different frequencies. Typical variations of $\sigma_P(t)/\sigma_P(0)$ are seen to be between 10 and 15% during the transition, i.e., roughly of the same order of magnitude as those for $\mu_P(t)$.

6.2.2.2 Dependence on motor parameters Figure 26 shows that $\sigma_P(t)$, like $\mu_P(t)$, exhibits quasi-periodicity and, on average, higher values during the transition than in steady state, for any value of A . On average, the increase of $\sigma_P(t)$ during the transitions is significantly larger and positively biased compared to $\mu_P(t)$ (cf. Fig. 21). Again, similar effects occur when changing B , V or M instead of A , as shown in Figs. 27, 28 and 29, respectively.

¹⁶Here, “sample” refers to the finite value of N at arbitrary t .

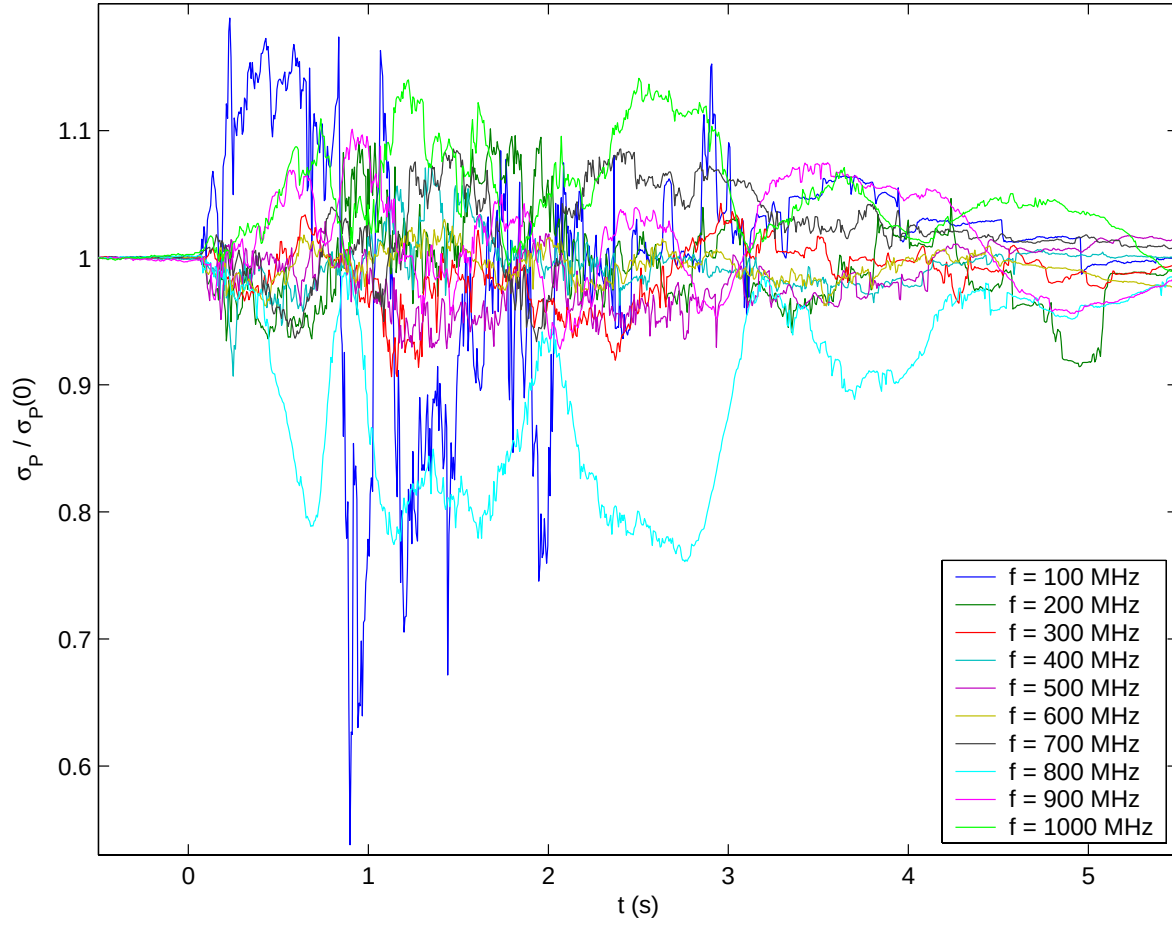


Figure 25: (color online) Normalized standard deviation $\sigma_P(t)/\sigma_P(t_{0-})$ as a function of transition time $t = t_\ell - t_0 = \ell\Delta t$ at selected frequencies f , with $M = 400$ steps ($N = 100$), $A = 1000$ steps/s², $B = 600$ steps/s, $V = 2000$ steps/s.

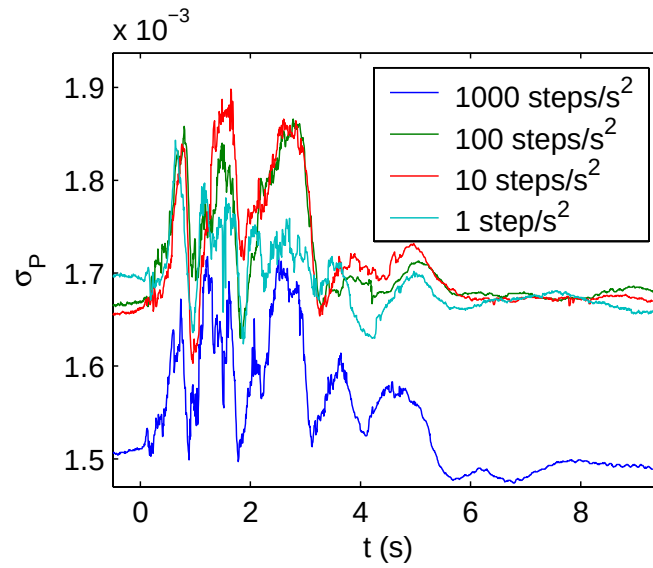


Figure 26: (color online) Standard deviation σ_P as a function of transition time $t = t_\ell - t_0 = \ell\Delta t$ at $f = 1000$ MHz, for selected values of A , with $M = 400$ steps ($N = 100$), $B = 600$ steps/s, $V = 2000$ steps/s.

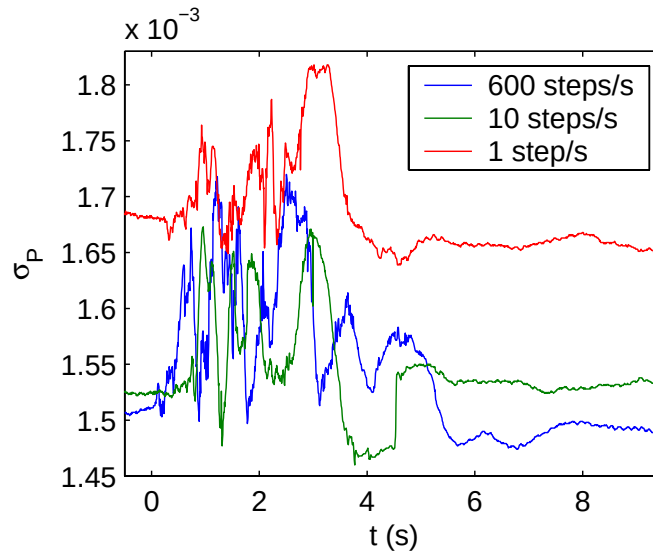


Figure 27: (color online) Standard deviation σ_P as a function of transition time $t = t_\ell - t_0 = \ell\Delta t$ at $f = 1000$ MHz, for selected values of B , with $M = 400$ steps ($N = 100$), $A = 1000$ steps/s², $V = 2000$ steps/s.

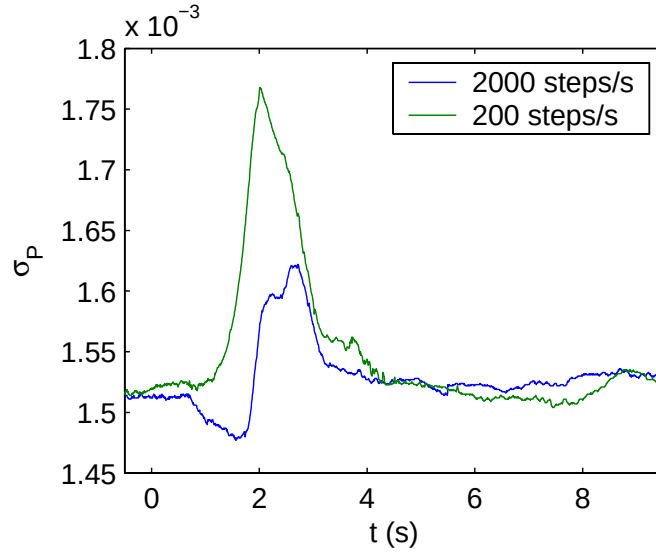


Figure 28: (color online) Standard deviation σ_P as a function of transition time $t = t_\ell - t_0 = \ell\Delta t$ at $f = 1000$ MHz, for selected values of V , with $M = 400$ steps ($N = 100$), $A = 100$ steps/s², $B = 10$ steps/s.

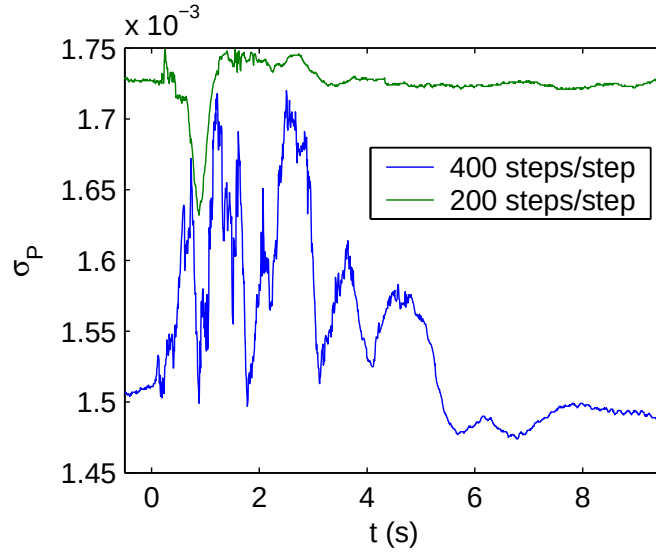
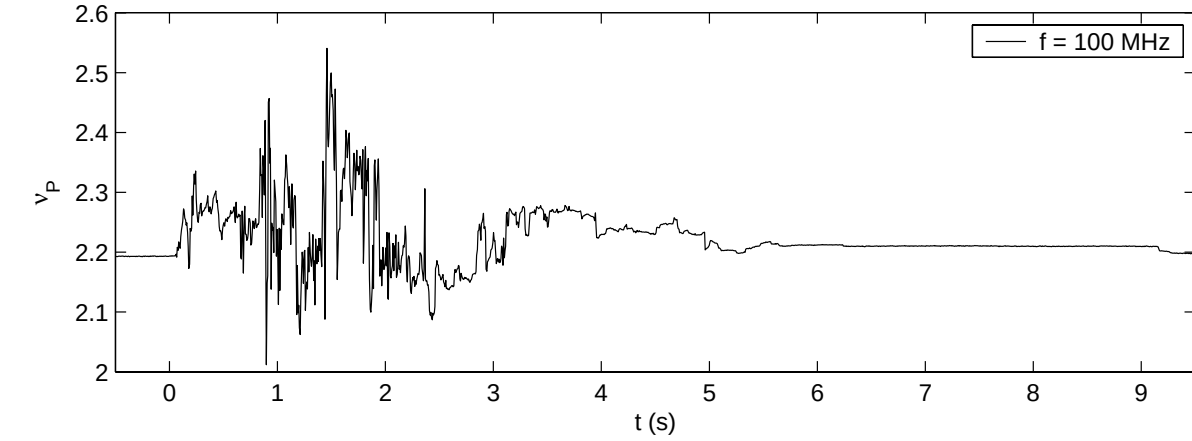


Figure 29: (color online) Standard deviation σ_P as a function of transition time $t = t_\ell - t_0 = \ell\Delta t$ at $f = 1000$ MHz, for selected values of M , with $A = 1000$ steps/s², $B = 600$ steps/s, $V = 2000$ steps/s.

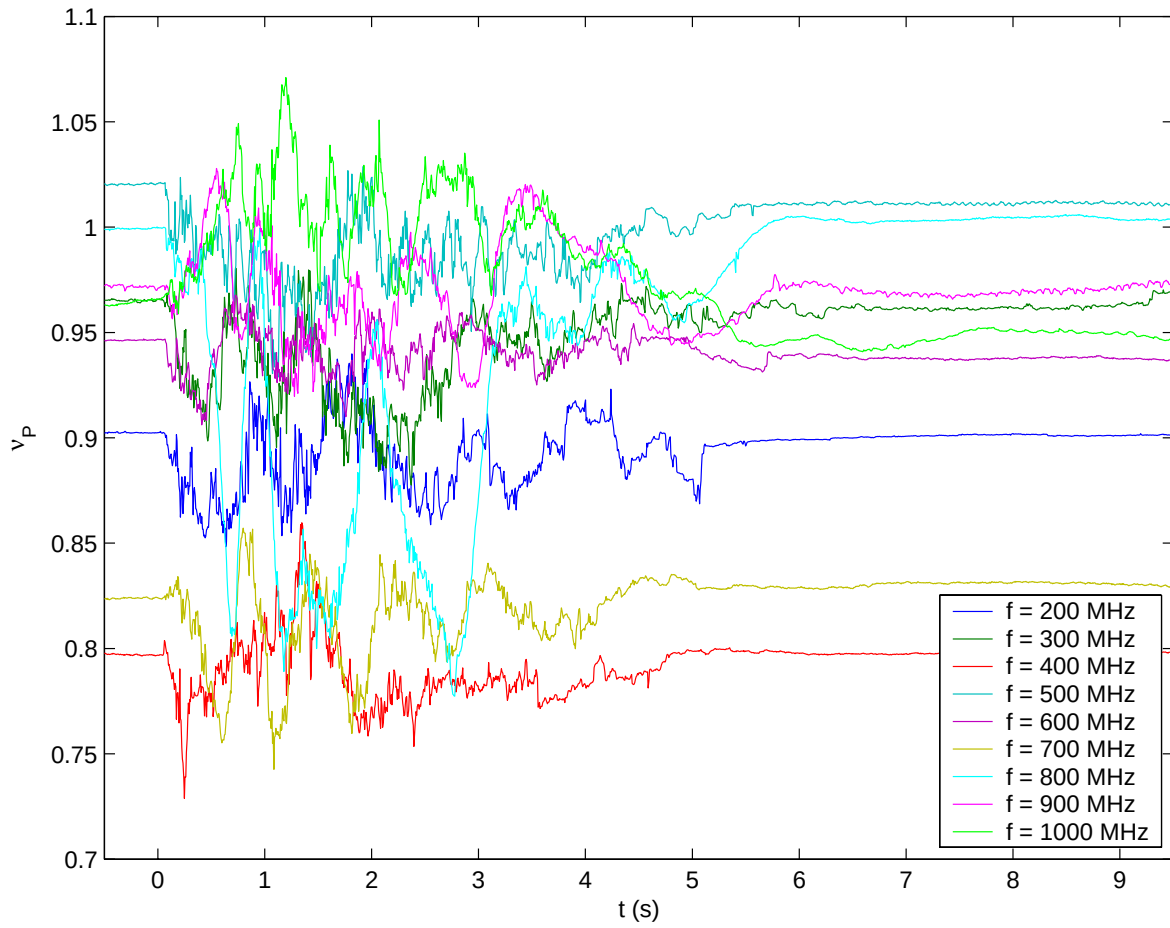
6.2.3 Coefficient of variation

6.2.3.1 Dependence on frequency Figure 30 compares the coefficient of variation $\nu_P(t) \equiv \sigma_P(t)/\mu_P(t)$ (non-normalized) for different frequencies. After normalizing each curve by its starting value $\nu_P(0)$, it is found that $\nu_P(t)/\nu_P(0)$ typically varies again between 10 and 15%, but tends to exhibit a more pronounced bias with respect to 1 across the transition for a given frequency. It is confirmed that the steady-state value of $\nu_P(t)$ approaches unity, in the mean, when frequency is increased, as expected for χ^2_2 distributions of quasi-stationary ideal random fields. The relative fluctuations of $\nu_P(t)$ are somewhat smaller than those for $\sigma_P(t)$ and are more uniform across frequency.

6.2.3.2 Dependence on motor parameters Figure 31 shows $\nu_P(t)$ for selected values of A at $f = 1000$ MHz. Quasi-periodicity is again observed during the transition. Corresponding evolutions for B , V , and M as parameters are shown in Figs. 32, 33, and 34, respectively. For the dependence on V , the transitions are again seen to be significantly smoother. Figures 33 and 34 suggest that the magnitude and rate of fluctuations of $\nu_P(t)$ are strongly correlated with the value of the instantaneous motor speed.



(a)



(b)

Figure 30: (color online) Coefficient of variation ν_P as a function of transition time $t = t_\ell - t_0 = \ell \Delta t$ at selected frequencies f , with $M = 400$ steps ($N = 100$), $A = 1000$ steps/s², $B = 600$ steps/s, $V = 2000$ steps/s, at (a) $f = 100$ MHz; (b) $f = 200$ to 1000 MHz.

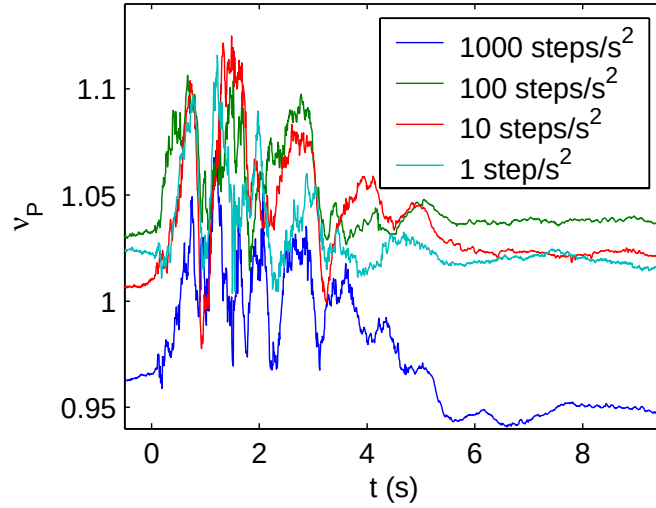


Figure 31: (color online) Coefficient of variation ν_P as a function of transition time $t = t_\ell - t_0 = \ell \Delta t$ at $f = 1000$ MHz, for selected values of A , with $M = 400$ steps ($N = 100$), $B = 600$ steps/s, $V = 2000$ steps/s.

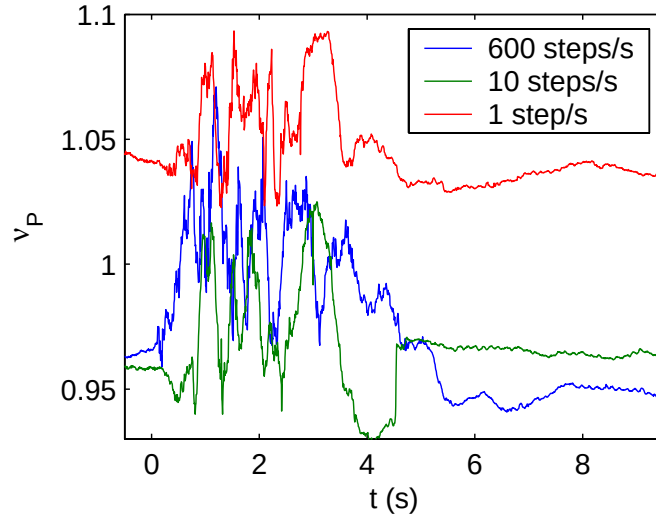


Figure 32: (color online) Coefficient of variation ν_P as a function of transition time $t = t_\ell - t_0 = \ell \Delta t$ at $f = 1000$ MHz, for selected values of B , with $M = 400$ steps ($N = 100$), $A = 1000$ steps/s², $V = 2000$ steps/s.

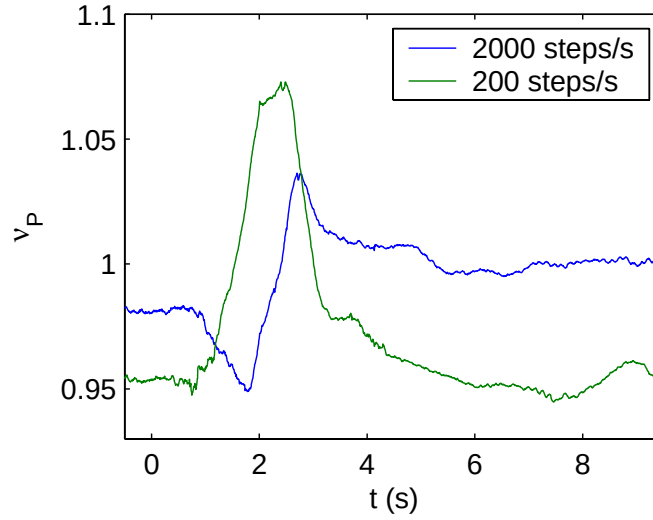


Figure 33: (color online) Coefficient of variation ν_P as a function of transition time $t = t_\ell - t_0 = \ell\Delta t$ at $f = 1000$ MHz, for selected values of V , with $M = 400$ steps ($N = 100$), $A = 100$ steps/s², $B = 10$ steps/s.

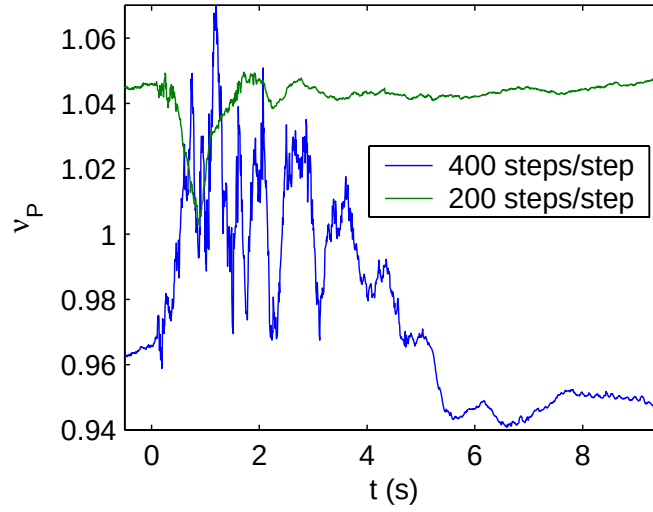


Figure 34: (color online) Coefficient of variation ν_P as a function of transition time $t = t_\ell - t_0 = \ell\Delta t$ at $f = 1000$ MHz, for selected values of M , with $A = 1000$ steps/s², $B = 600$ steps/s, $V = 2000$ steps/s.

6.3 Temporal evolution of probability distribution

6.3.1 Dependence on frequency

Complete information on all first-order (i.e., instantaneous) statistics is contained within the probability density function (pdf), $f_P[p(t)]$ or, equivalently, the cumulative distribution function (cdf), $F_P[p(t)]$ of $P(t) \equiv |S_{21}(t)|^2$. For a quasi-stationary ideal random field, P is known to exhibit a χ_2^2 (exponential) distribution. Upon normalizing the values of $P(t)$ by the instantaneous mean value $\langle P(t) \rangle$, we can thus monitor any qualitative change in its distribution as a function of time.

Figures 35(a)(b)–44(a)(b) show sets of measured instantaneous sample cdfs $F_P(p, t)$ for $N = 100$ at selected frequencies. The left hand diagrams show cdfs during the early transition, viz., $-10\Delta t \leq t \leq 50\Delta t$, i.e., $-0.062 \text{ s} \leq t \leq 0.312 \text{ s}$; the right hand diagrams are for a later and wider time interval covering the medium and late transition ($51\Delta t \leq t \leq 700\Delta t$, i.e., $0.318 \text{ s} \leq t \leq 4.369 \text{ s}$). For completeness, we also plot all sample cdfs, i.e., taken across the full transition interval¹⁷ ($-80\Delta t \leq t \leq 1520\Delta t$, i.e., $-0.5 \text{ s} \leq t \leq 9.5 \text{ s}$). For $t > 4.369 \text{ s}$, there are no significant further changes. These sample cdfs are compared against the exponential cdf, also shown in the plots. Figures 35(c)–44(c) show the complete evolutions as meshed three-dimensional plots with flat color shading. Note that here the probabilities $F_P(p, t)$ have been directed along one of the horizontal axes, in order to increase legibility of the graphs. The white “speckle” in the surfaces, which indicates minute temporal variations between consecutive cdfs, becomes more pronounced for decreasing frequencies. Note that even at low frequencies, the temporal variations are generally small for most values of $P(t)/\langle P(t) \rangle$, except at and near the sample maximum values of the cdfs.

It is seen that evolutionary changes are typically small, throughout the transition. This suggests that, except for the tail of the distribution during the early transition, the underlying (ensemble) instantaneous cdfs at times t do not significantly change in the course of the tuner stepping process, and that the spread between different experimental cdfs is to a considerable extent due to the limited sample size N , causing sample variations. These observations are in marked contrast to the cdfs for pulse modulated reverberation fields, which exhibit a distinct evolution during the rise time of the pulse due to nonergodicity [22]. The spread of the experimental cdfs here suggests that it may be beneficial to characterize the experimental distribution

¹⁷Nevertheless, visual comparison with the 2D plots for $-10\Delta t \leq t \leq 700\Delta t$ showed no visible differences.

by a family of theoretical distributions (e.g., Meijer G , Bessel K , Weibull–Brody dfs) governed by distribution parameters that vary *randomly*, in order to capture the fluctuations of the cdfs and their effect on the overall uncertainty and confidence.

Nevertheless, notable systematic deviations between the experimental and χ^2_2 distributions exist, particularly in their upper tail, also at relatively high frequencies. These are a result of physical imperfections of the reverberant field compared to an ideal random field, whether quasi-stationary or nonstationary, and are caused by the finite mode density at finite frequencies. Recall that for a χ^2_2 distribution, the pdf follows immediately from the plots as $f_P[p(t)] = 1 - F_P[p(t)]$. As a result, the diagrams indicate that the experimental pdfs have a consistently heavier upper tail than a χ^2_2 pdf has.

Although the plots for $N = 100$ coupled with large set size of time (and, hence, distribution) samples ($L = 1601$) provides a good basis for comparison, evaluation at any single instance requires a larger number of tuner steps N in order to increase accuracy and confidence in a comparison. To this end, Figure 51 compares the instantaneous experimental cdfs for $N = 200$. It is seen that systematic deviations remain for the upper tail between the experimental and χ^2_2 distributions.

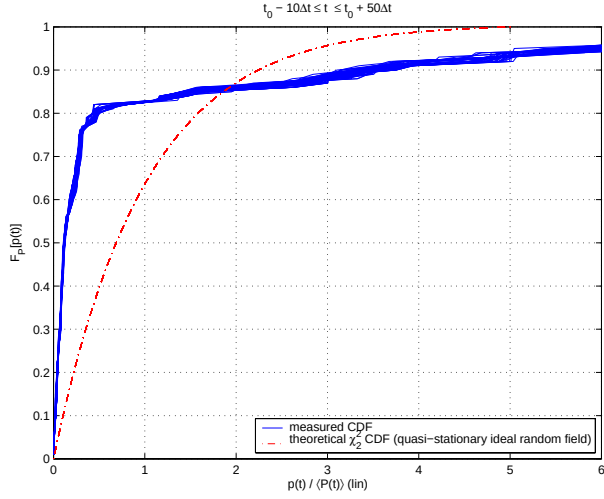
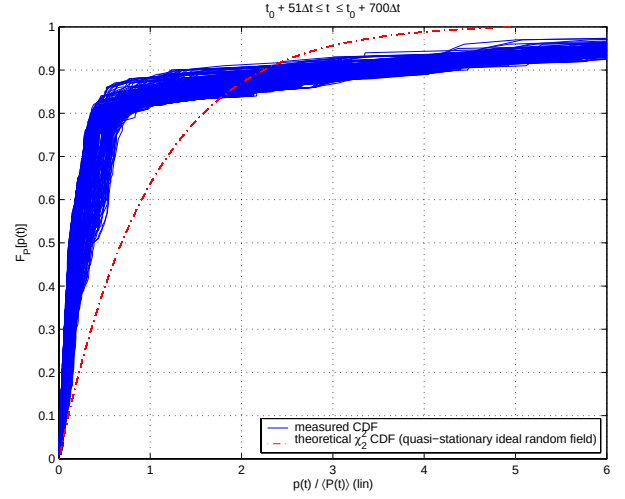
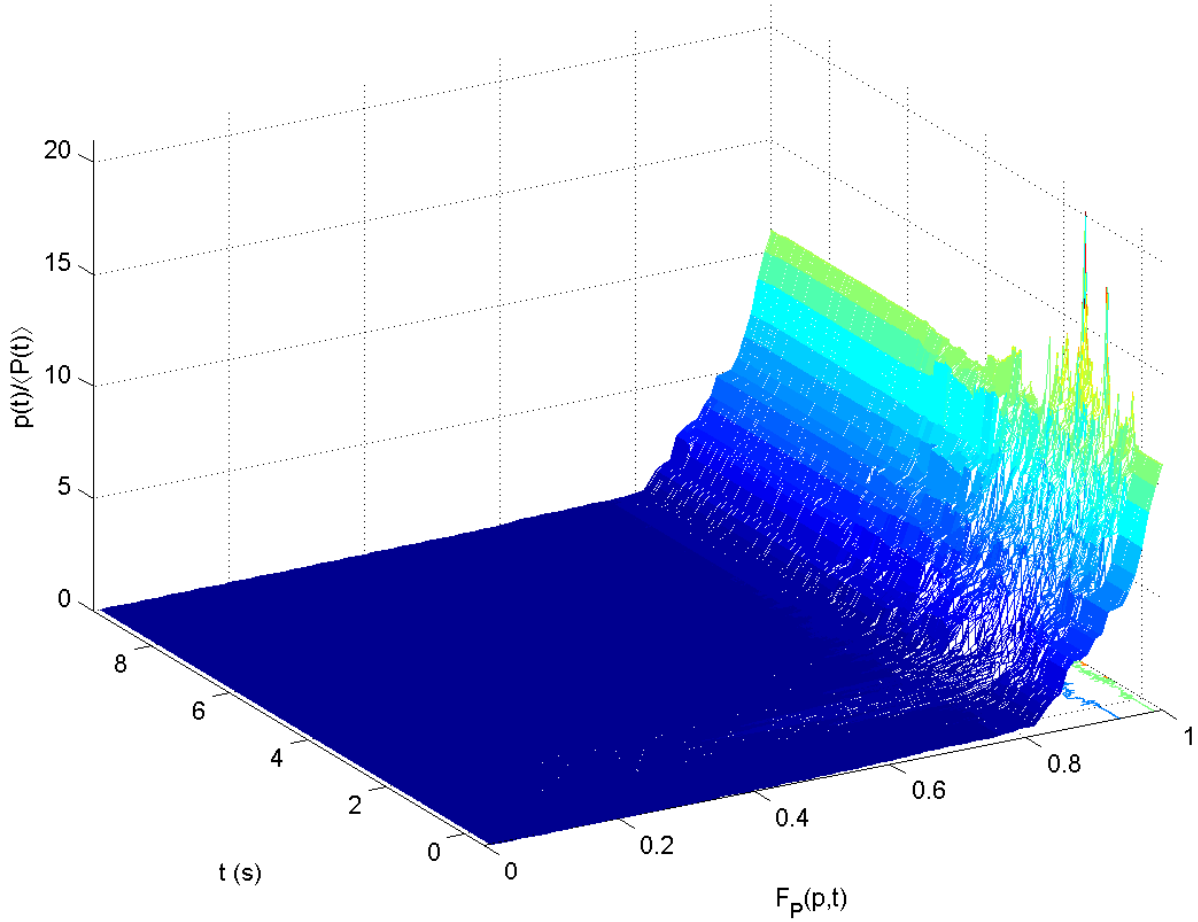
(a) $t_0 - 10\Delta t \leq t \leq t_0 + 50\Delta t$ (b) $t_0 + 51\Delta t \leq t \leq t_0 + 700\Delta t$ (c) $t_0 - 80\Delta t \leq t \leq t_0 + 1520\Delta t$

Figure 35: (color online) Sample cumulative distribution functions $F_P[p(t)]$ of mean-normalized values of $P(t) \equiv |S_{21}(t)|^2$ at $f = 100$ MHz, with $M = 400$ steps ($N = 100$), $A = 1000$ steps/s², $B = 600$ steps/s, $V = 2000$ steps/s, for transition times (a) $t_0 - 10\Delta t \leq t \leq t_0 + 50\Delta t$ (61 curves); (b) $t_0 + 51\Delta t \leq t \leq t_0 + 700\Delta t$ (650 curves); (c) complete evolution.

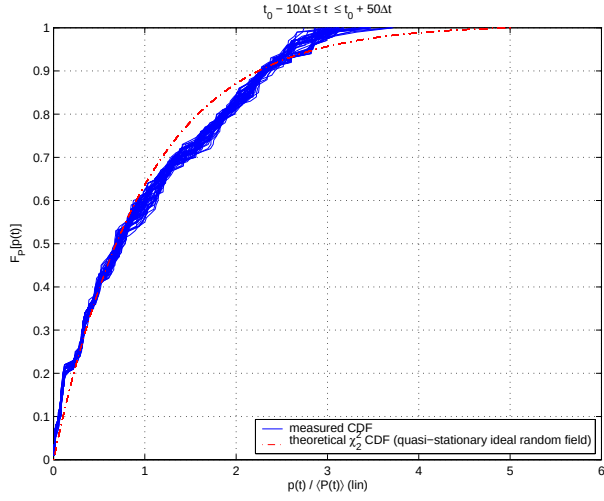
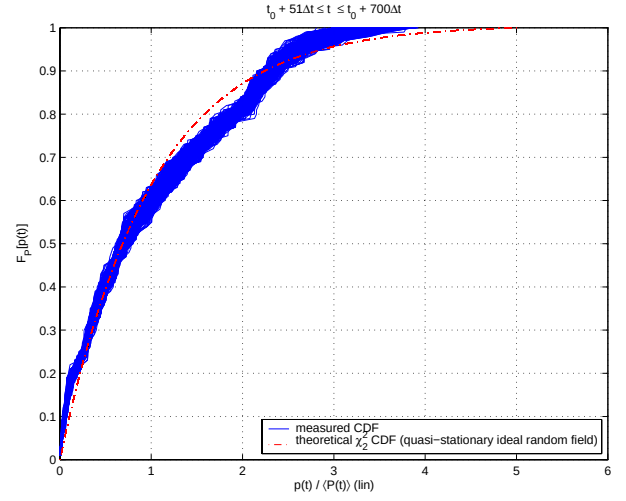
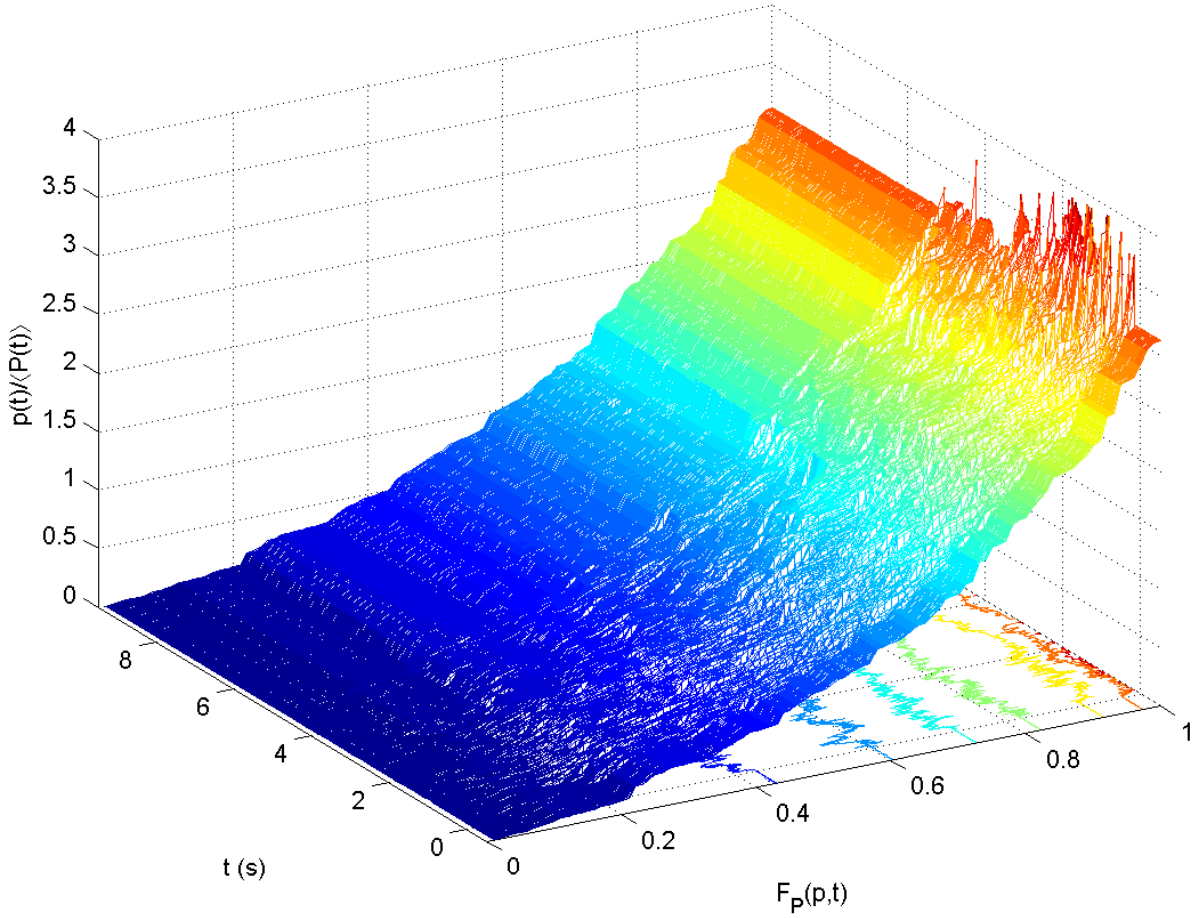
(a) $t_0 - 10\Delta t \leq t \leq t_0 + 50\Delta t$ (b) $t_0 + 51\Delta t \leq t \leq t_0 + 700\Delta t$ (c) $t_0 - 80\Delta t \leq t \leq t_0 + 1520\Delta t$

Figure 36: (color online) Sample cumulative distribution functions $F_P[p(t)]$ of mean-normalized values of $P(t) \equiv |S_{21}(t)|^2$ at $f = 200$ MHz, with $M = 400$ steps ($N = 100$), $A = 1000$ steps/s², $B = 600$ steps/s, $V = 2000$ steps/s, for transition times (a) $t_0 - 10\Delta t \leq t \leq t_0 + 50\Delta t$ (61 curves); (b) $t_0 + 51\Delta t \leq t \leq t_0 + 700\Delta t$ (650 curves); (c) complete evolution.

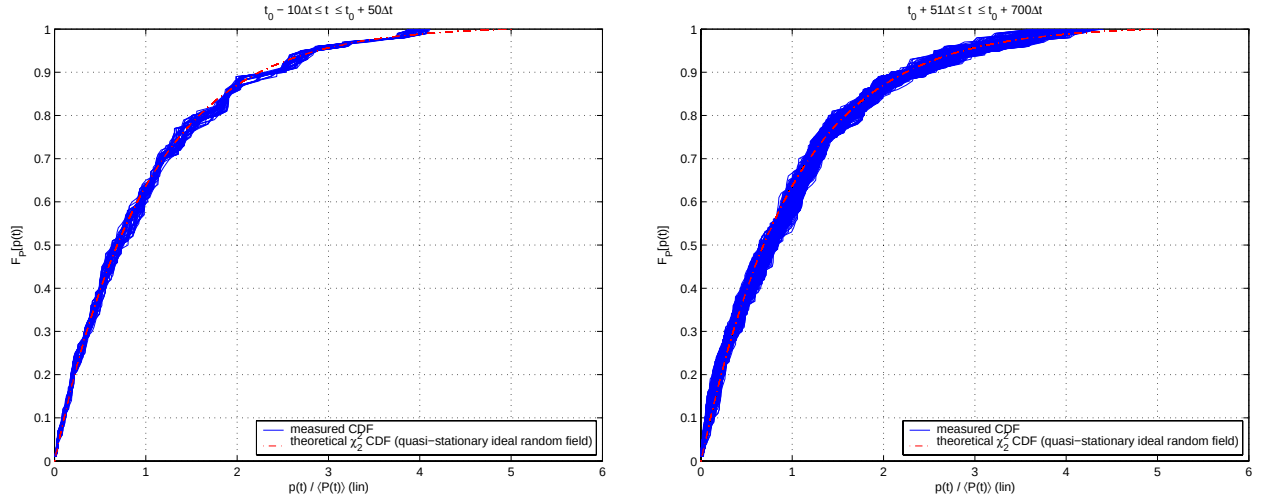
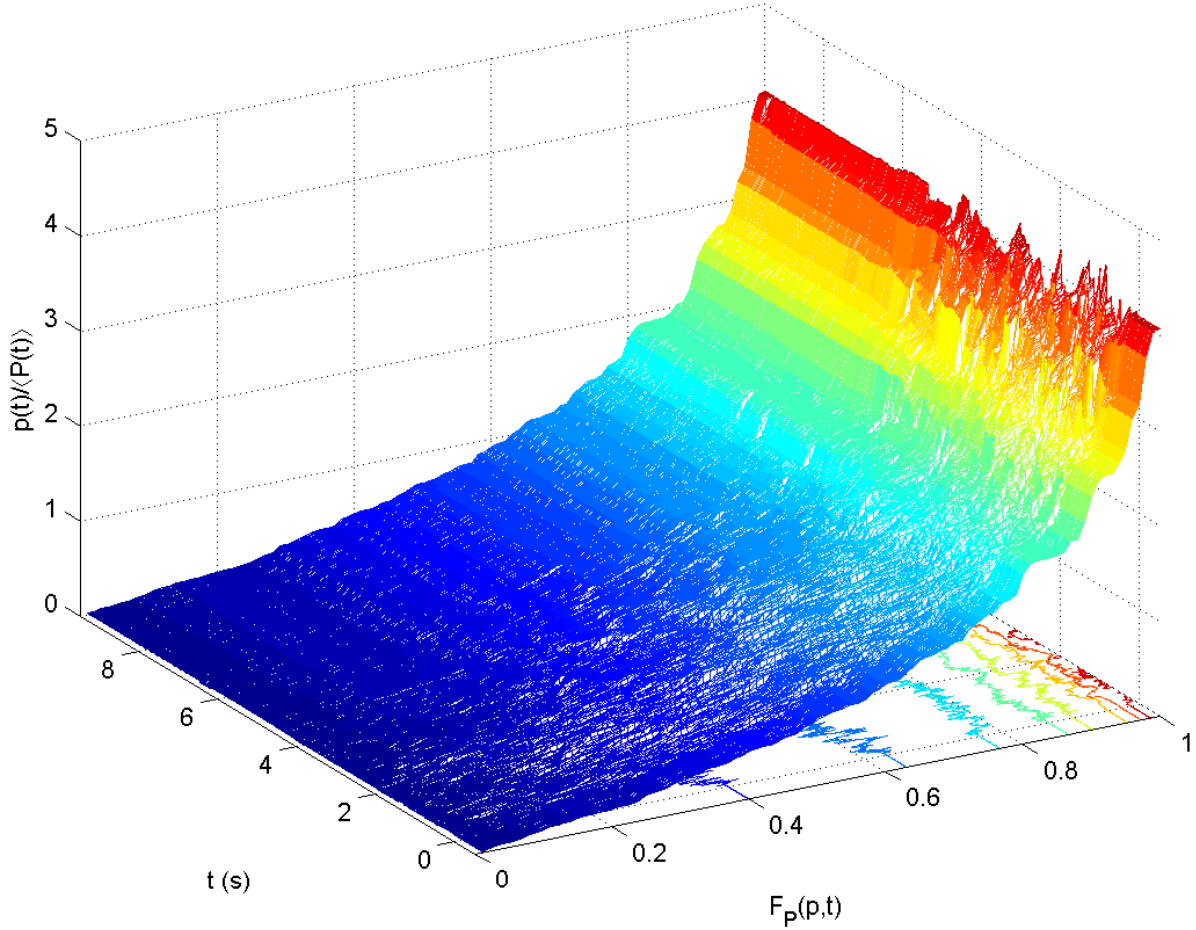
(a) $t_0 - 10\Delta t \leq t \leq t_0 + 50\Delta t$ (b) $t_0 + 51\Delta t \leq t \leq t_0 + 700\Delta t$ (c) $t_0 - 80\Delta t \leq t \leq t_0 + 1520\Delta t$

Figure 37: (color online) Sample cumulative distribution functions $F_P[p(t)]$ of mean-normalized values of $P(t) \equiv |S_{21}(t)|^2$ at $f = 300$ MHz, with $M = 400$ steps ($N = 100$), $A = 1000$ steps/s², $B = 600$ steps/s, $V = 2000$ steps/s, for transition times (a) $t_0 - 10\Delta t \leq t \leq t_0 + 50\Delta t$ (61 curves); (b) $t_0 + 51\Delta t \leq t \leq t_0 + 700\Delta t$ (650 curves); (c) complete evolution.

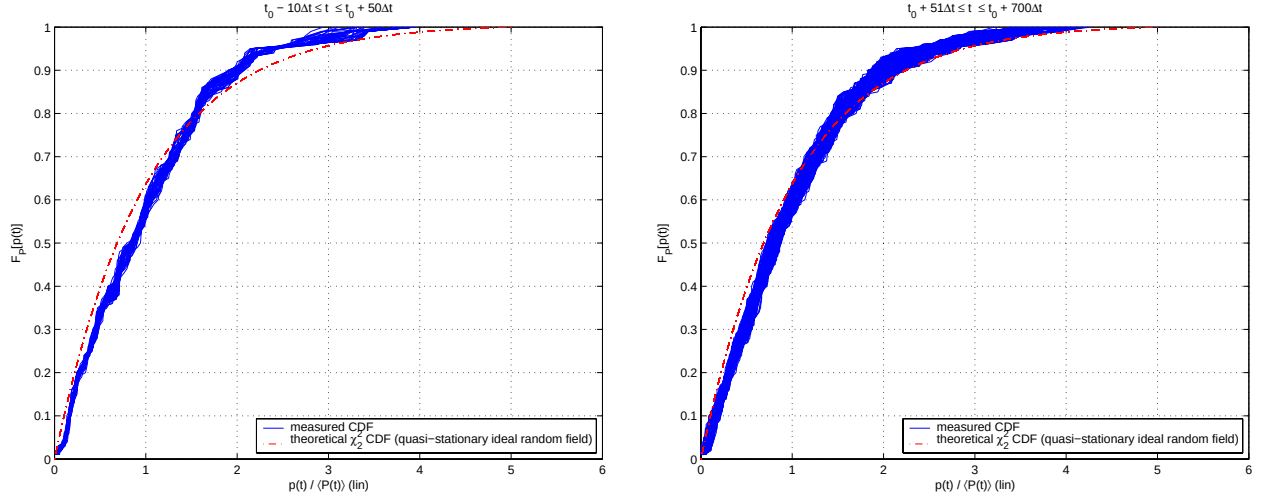
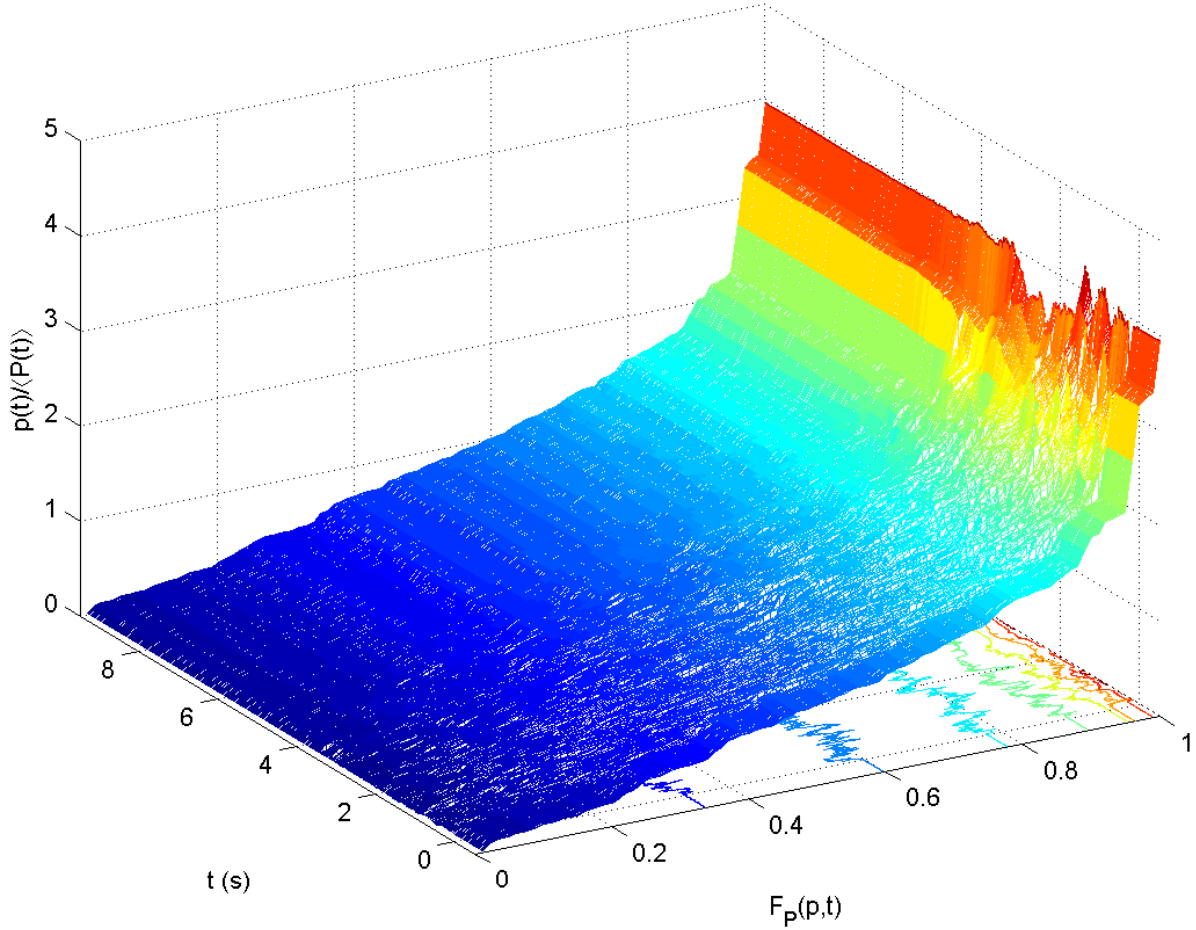
(a) $t_0 - 10\Delta t \leq t \leq t_0 + 50\Delta t$ (b) $t_0 + 51\Delta t \leq t \leq t_0 + 700\Delta t$ (c) $t_0 - 80\Delta t \leq t \leq t_0 + 1520\Delta t$

Figure 38: (color online) Sample cumulative distribution functions $F_P[p(t)]$ of mean-normalized values of $P(t) \equiv |S_{21}(t)|^2$ at $f = 400$ MHz, with $M = 400$ steps ($N = 100$), $A = 1000$ steps/s², $B = 600$ steps/s, $V = 2000$ steps/s, for transition times (a) $t_0 - 10\Delta t \leq t \leq t_0 + 50\Delta t$ (61 curves); (b) $t_0 + 51\Delta t \leq t \leq t_0 + 700\Delta t$ (650 curves); (c) complete evolution.

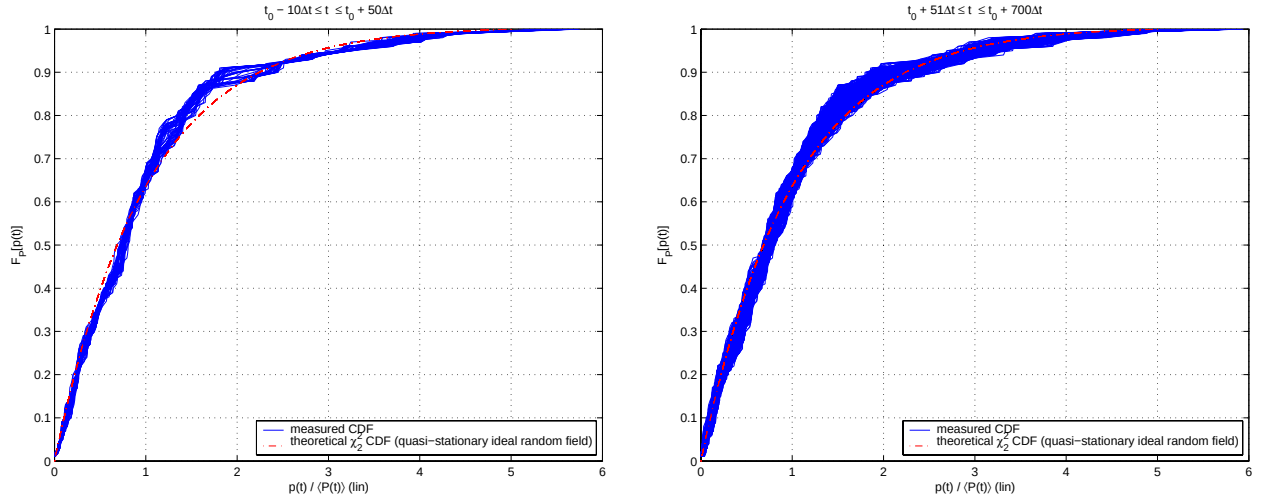
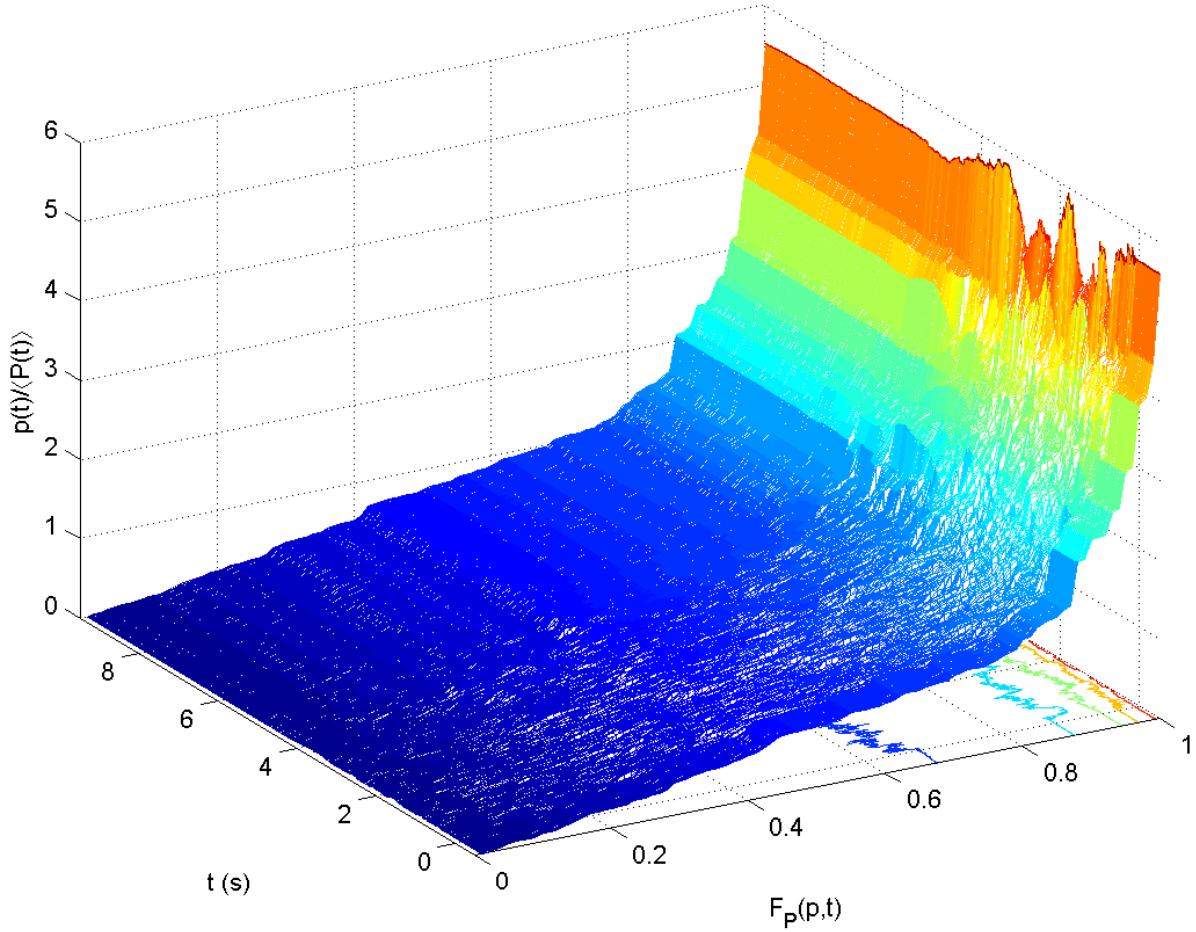
(a) $t_0 - 10\Delta t \leq t \leq t_0 + 50\Delta t$ (b) $t_0 + 51\Delta t \leq t \leq t_0 + 700\Delta t$ (c) $t_0 - 80\Delta t \leq t \leq t_0 + 1520\Delta t$

Figure 39: (color online) Sample cumulative distribution functions $F_P[p(t)]$ of mean-normalized values of $P(t) \equiv |S_{21}(t)|^2$ at $f = 500$ MHz, with $M = 400$ steps ($N = 100$), $A = 1000$ steps/s², $B = 600$ steps/s, $V = 2000$ steps/s, for transition times (a) $t_0 - 10\Delta t \leq t \leq t_0 + 50\Delta t$ (61 curves); (b) $t_0 + 51\Delta t \leq t \leq t_0 + 700\Delta t$ (650 curves); (c) complete evolution.

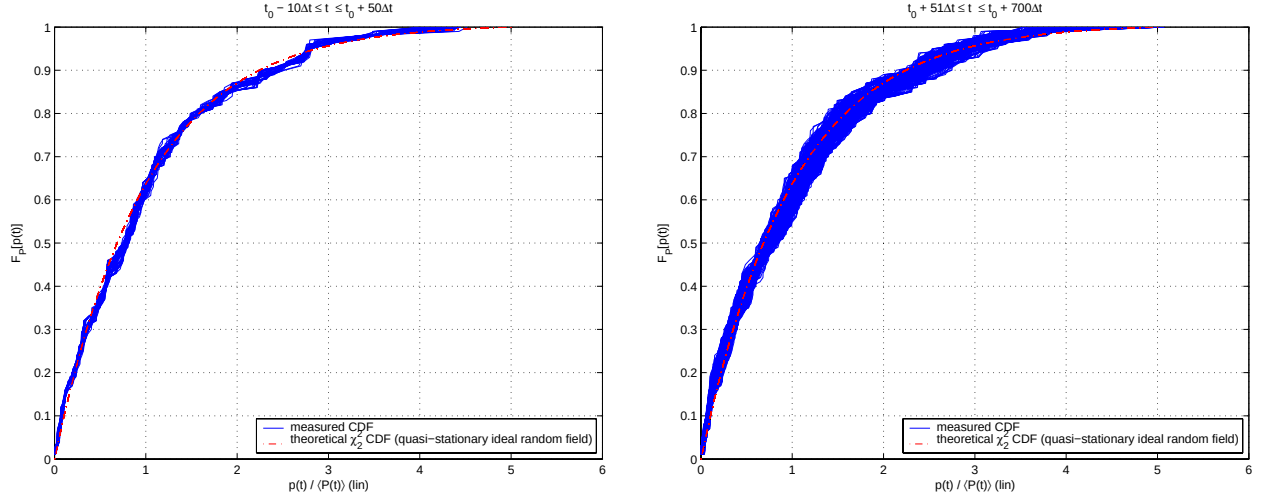
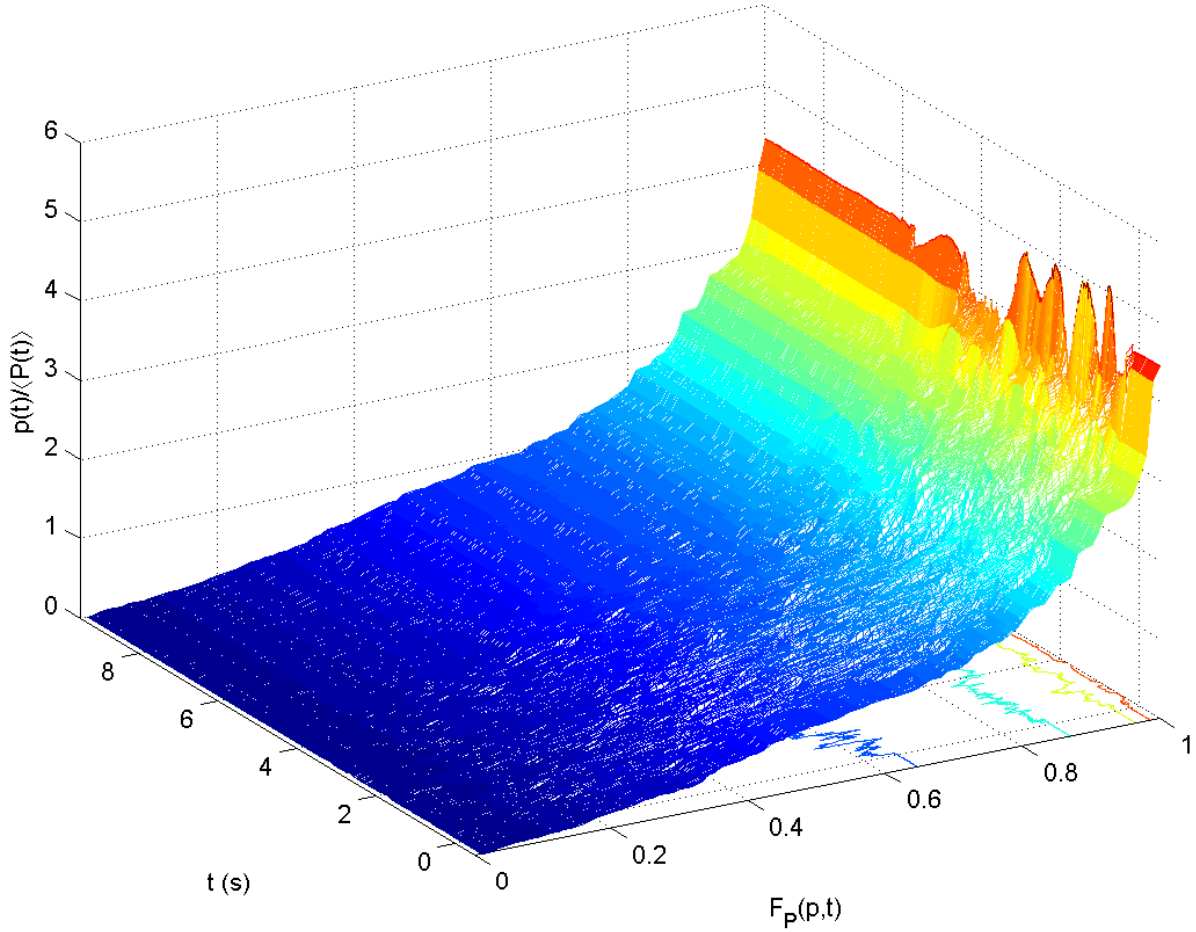
(a) $t_0 - 10\Delta t \leq t \leq t_0 + 50\Delta t$ (b) $t_0 + 51\Delta t \leq t \leq t_0 + 700\Delta t$ (c) $t_0 - 80\Delta t \leq t \leq t_0 + 1520\Delta t$

Figure 40: (color online) Sample cumulative distribution functions $F_P[p(t)]$ of mean-normalized values of $P(t) \equiv |S_{21}(t)|^2$ at $f = 600$ MHz, with $M = 400$ steps ($N = 100$), $A = 1000$ steps/s², $B = 600$ steps/s, $V = 2000$ steps/s, for transition times (a) $t_0 - 10\Delta t \leq t \leq t_0 + 50\Delta t$ (61 curves); (b) $t_0 + 51\Delta t \leq t \leq t_0 + 700\Delta t$ (650 curves); (c) complete evolution.

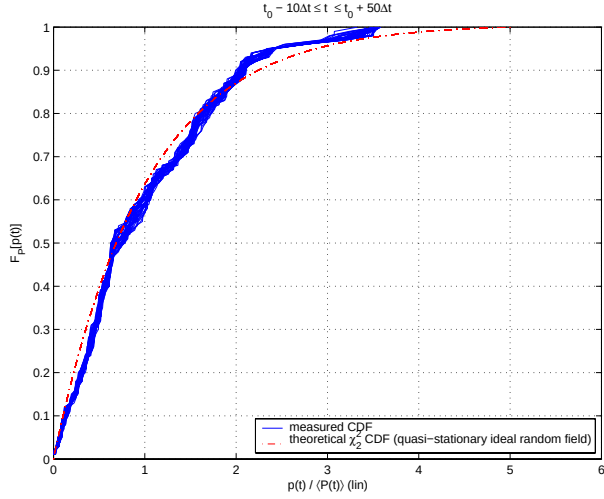
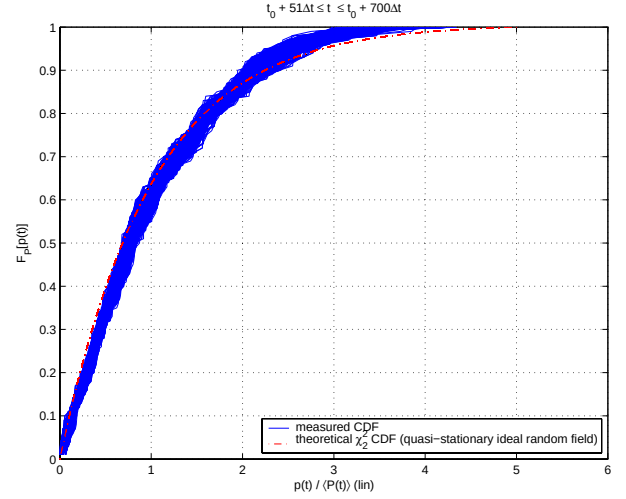
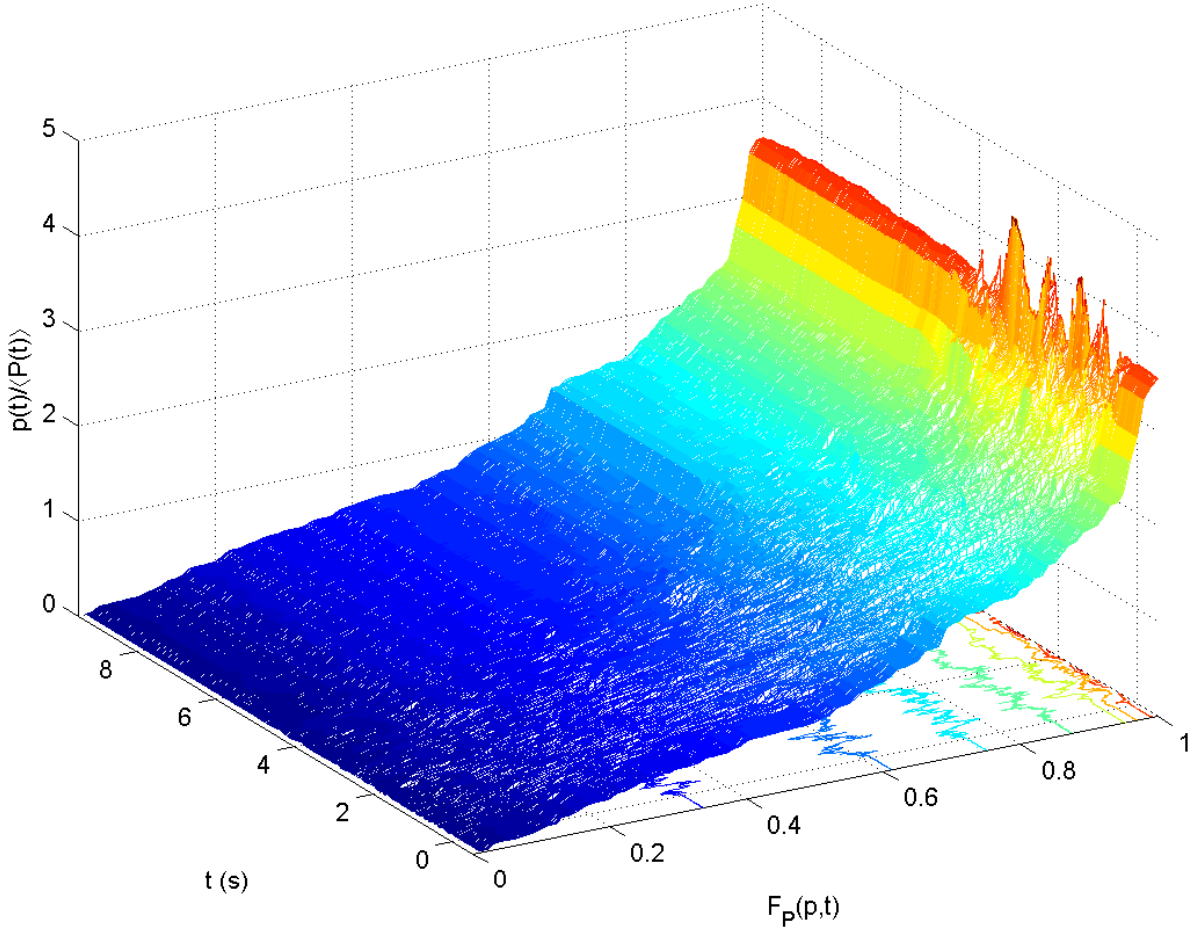
(a) $t_0 - 10\Delta t \leq t \leq t_0 + 50\Delta t$ (b) $t_0 + 51\Delta t \leq t \leq t_0 + 700\Delta t$ (c) $t_0 - 80\Delta t \leq t \leq t_0 + 1520\Delta t$

Figure 41: (color online) Sample cumulative distribution functions $F_P[p(t)]$ of mean-normalized values of $P(t) \equiv |S_{21}(t)|^2$ at $f = 700$ MHz, with $M = 400$ steps ($N = 100$), $A = 1000$ steps/s², $B = 600$ steps/s, $V = 2000$ steps/s, for transition times (a) $t_0 - 10\Delta t \leq t \leq t_0 + 50\Delta t$ (61 curves); (b) $t_0 + 51\Delta t \leq t \leq t_0 + 700\Delta t$ (650 curves); (c) complete evolution.

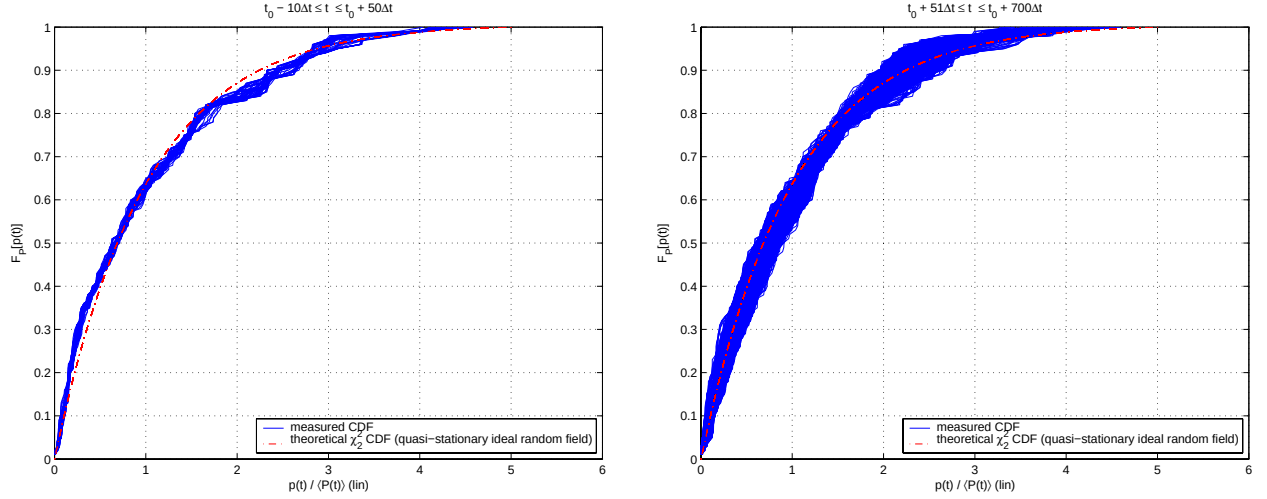
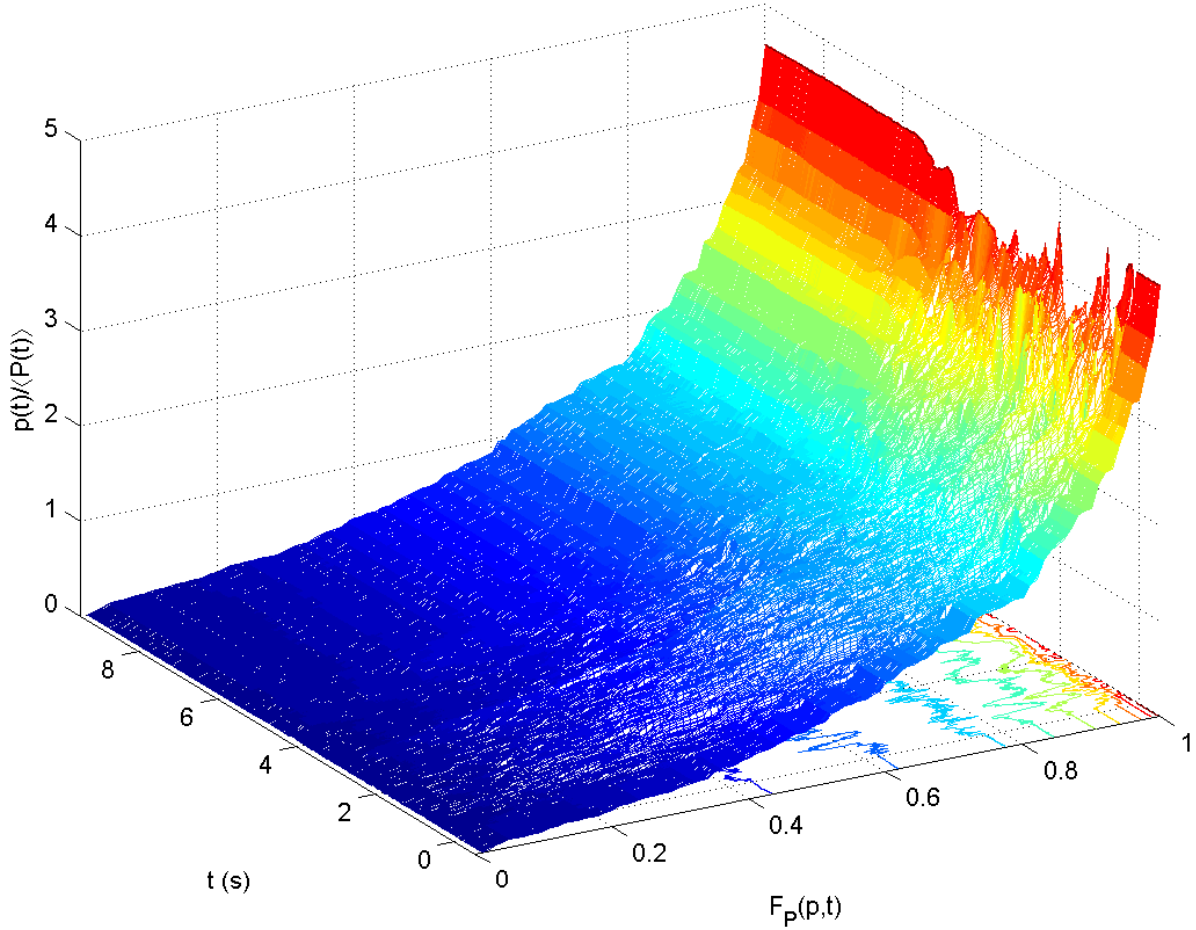
(a) $t_0 - 10\Delta t \leq t \leq t_0 + 50\Delta t$ (b) $t_0 + 51\Delta t \leq t \leq t_0 + 700\Delta t$ (c) $t_0 - 80\Delta t \leq t \leq t_0 + 1520\Delta t$

Figure 42: (color online) Sample cumulative distribution functions $F_P[p(t)]$ of mean-normalized values of $P(t) \equiv |S_{21}(t)|^2$ at $f = 800$ MHz, with $M = 400$ steps ($N = 100$), $A = 1000$ steps/s², $B = 600$ steps/s, $V = 2000$ steps/s, for transition times (a) $t_0 - 10\Delta t \leq t \leq t_0 + 50\Delta t$ (61 curves); (b) $t_0 + 51\Delta t \leq t \leq t_0 + 700\Delta t$ (650 curves); (c) complete evolution.

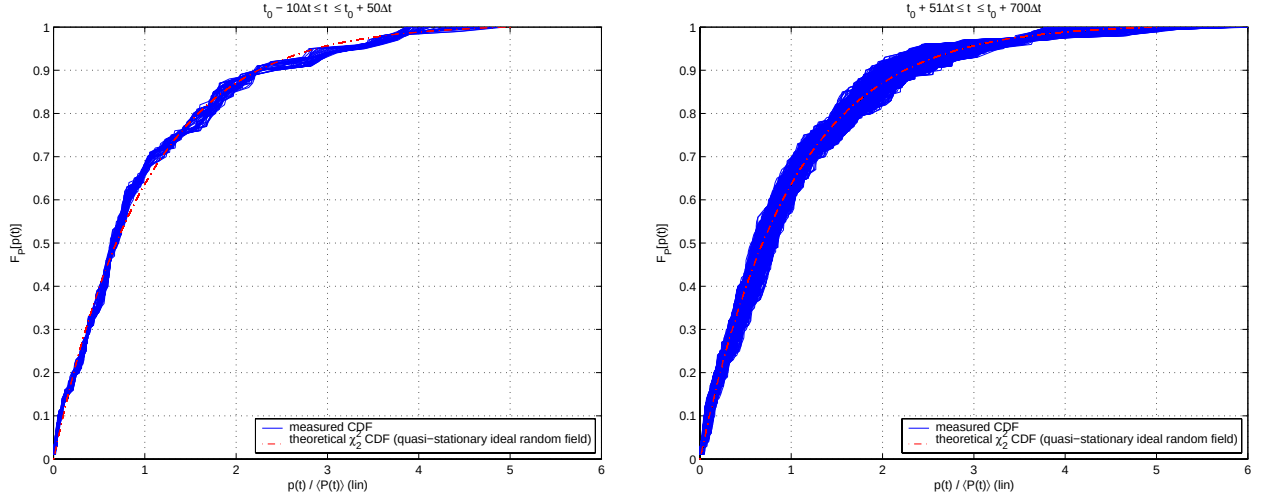
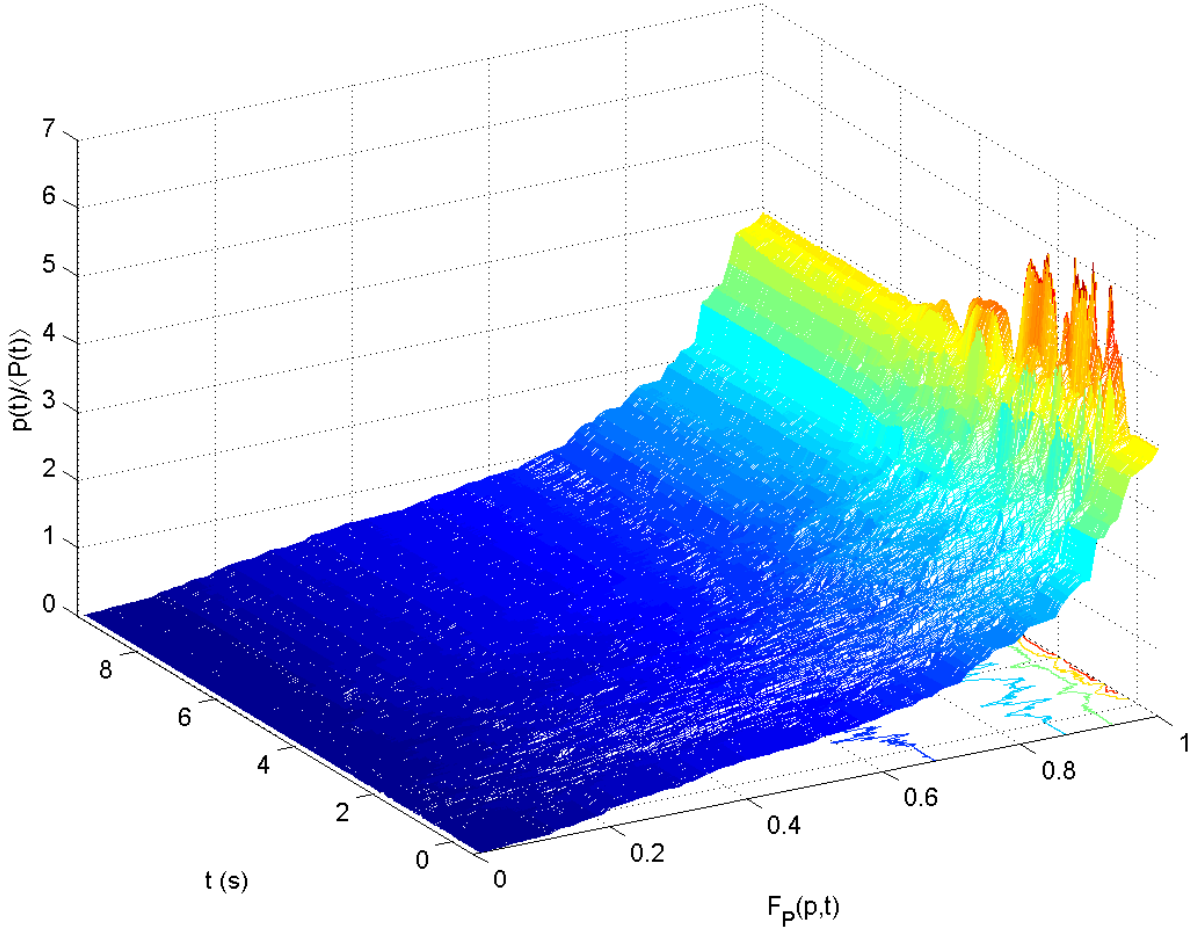
(a) $t_0 - 10\Delta t \leq t \leq t_0 + 50\Delta t$ (b) $t_0 + 51\Delta t \leq t \leq t_0 + 700\Delta t$ (c) $t_0 - 80\Delta t \leq t \leq t_0 + 1520\Delta t$

Figure 43: (color online) Sample cumulative distribution functions $F_P[p(t)]$ of mean-normalized values of $P(t) \equiv |S_{21}(t)|^2$ at $f = 900$ MHz, with $M = 400$ steps ($N = 100$), $A = 1000$ steps/s², $B = 600$ steps/s, $V = 2000$ steps/s, for transition times (a) $t_0 - 10\Delta t \leq t \leq t_0 + 50\Delta t$ (61 curves); (b) $t_0 + 51\Delta t \leq t \leq t_0 + 700\Delta t$ (650 curves); (c) complete evolution.

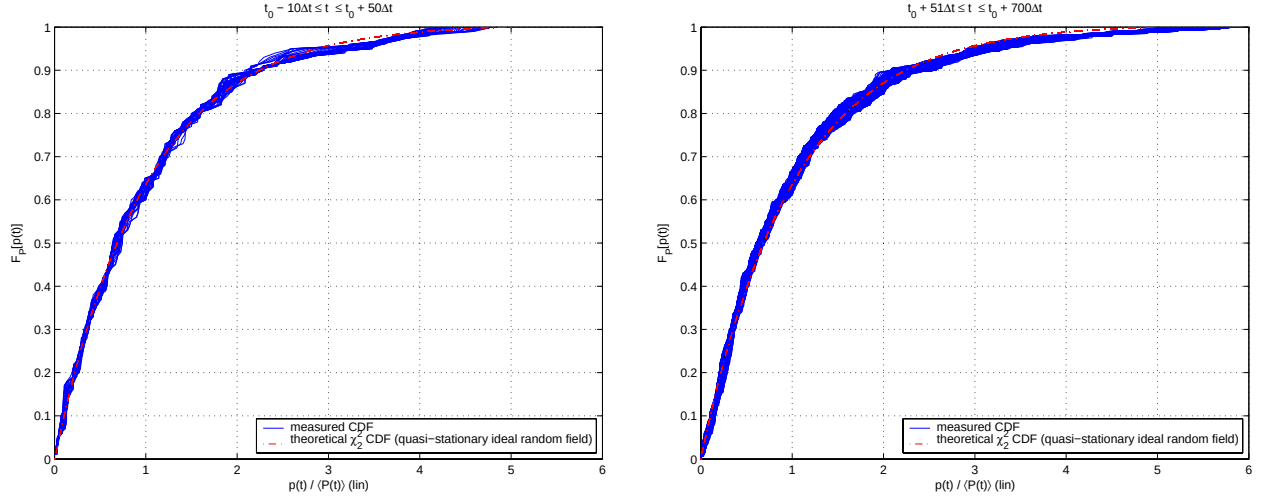
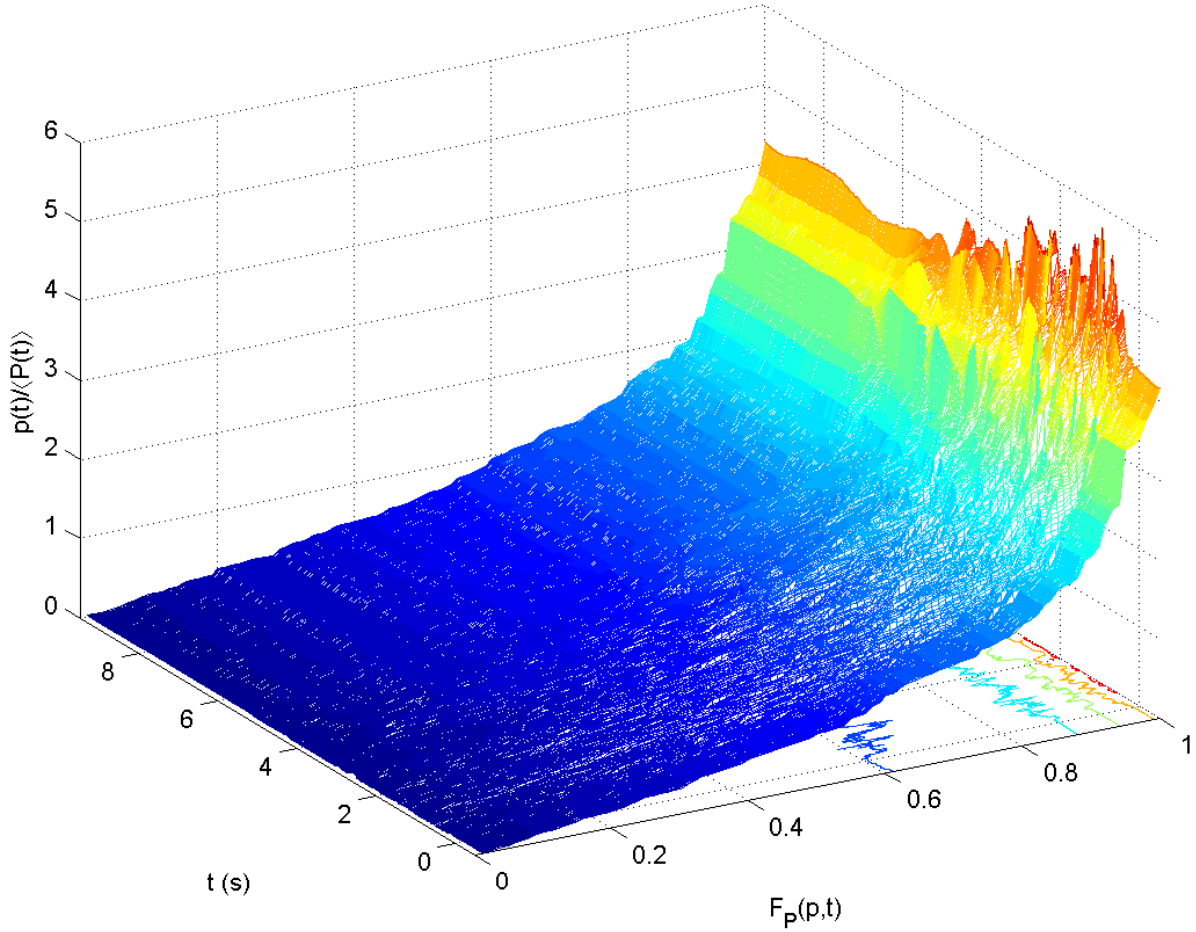
(a) $t_0 - 10\Delta t \leq t \leq t_0 + 50\Delta t$ (b) $t_0 + 51\Delta t \leq t \leq t_0 + 700\Delta t$ (c) $t_0 - 80\Delta t \leq t \leq t_0 + 1520\Delta t$

Figure 44: (color online) Sample cumulative distribution functions $F_P[p(t)]$ of mean-normalized values of $P(t) \equiv |S_{21}(t)|^2$ at $f = 1000$ MHz, with $M = 400$ steps ($N = 100$), $A = 1000$ steps/s², $B = 600$ steps/s, $V = 2000$ steps/s, for transition times (a) $t_0 - 10\Delta t \leq t \leq t_0 + 50\Delta t$ (61 curves); (b) $t_0 + 51\Delta t \leq t \leq t_0 + 700\Delta t$ (650 curves); (c) complete evolution.

6.3.2 Dependence on motor parameters

Figures 45(a)(b)–49(a)(b) show corresponding measured cdfs for different sets of parameter values. Smaller values of B and V (i.e., slower rotation) are seen to result in markedly smaller fluctuations of the experimental cdfs across a given time interval. In general, the spread on the sample distributions increases for higher values of A , B , or V .

Figures 45(c)–49(c) show the complete evolutions of the empirical cdf as three-dimensional plots, for selected motor parameter variations.

6.3.3 Dependence on tuner step size

For completeness, Fig. 51 shows results for the case of a larger number of smaller tuner steps ($M = 200$, $N = 200$). The effect on smaller sample variations between cdfs due to the larger value of N follows from comparing with Figure 44.

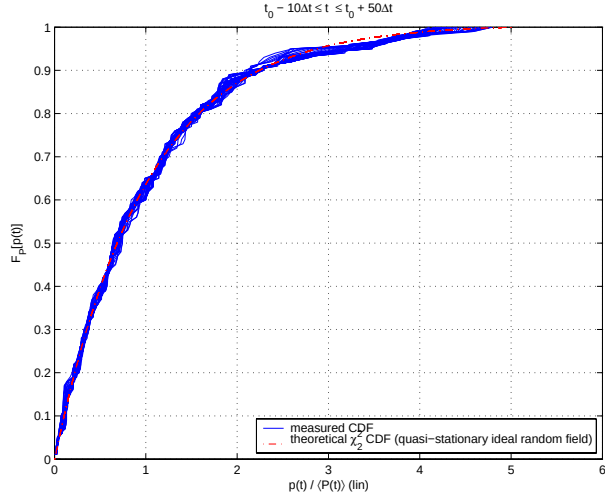
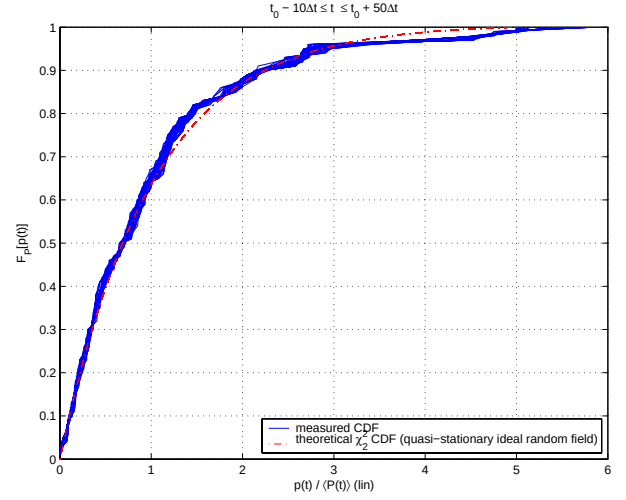
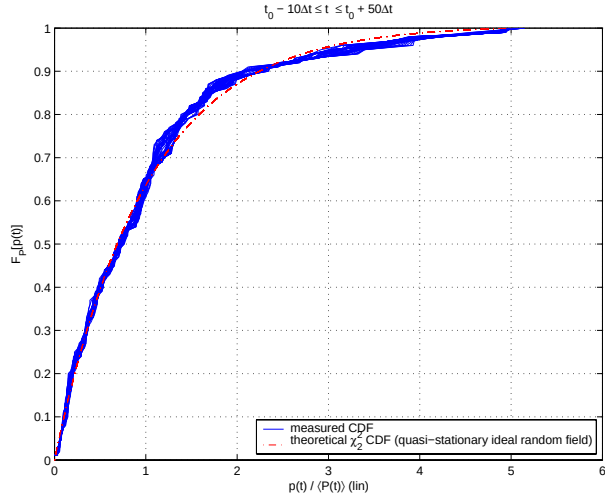
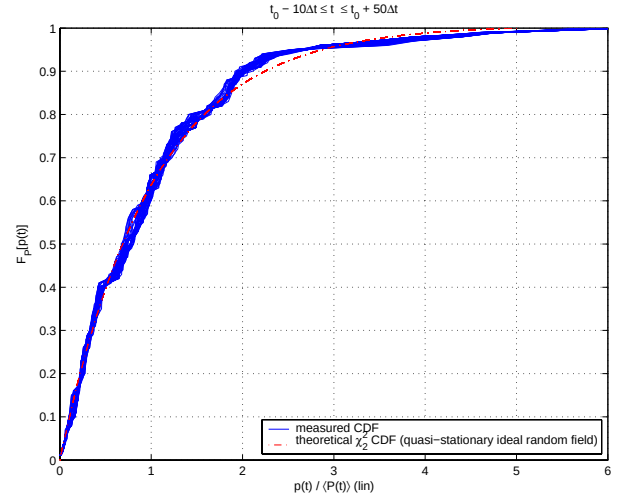
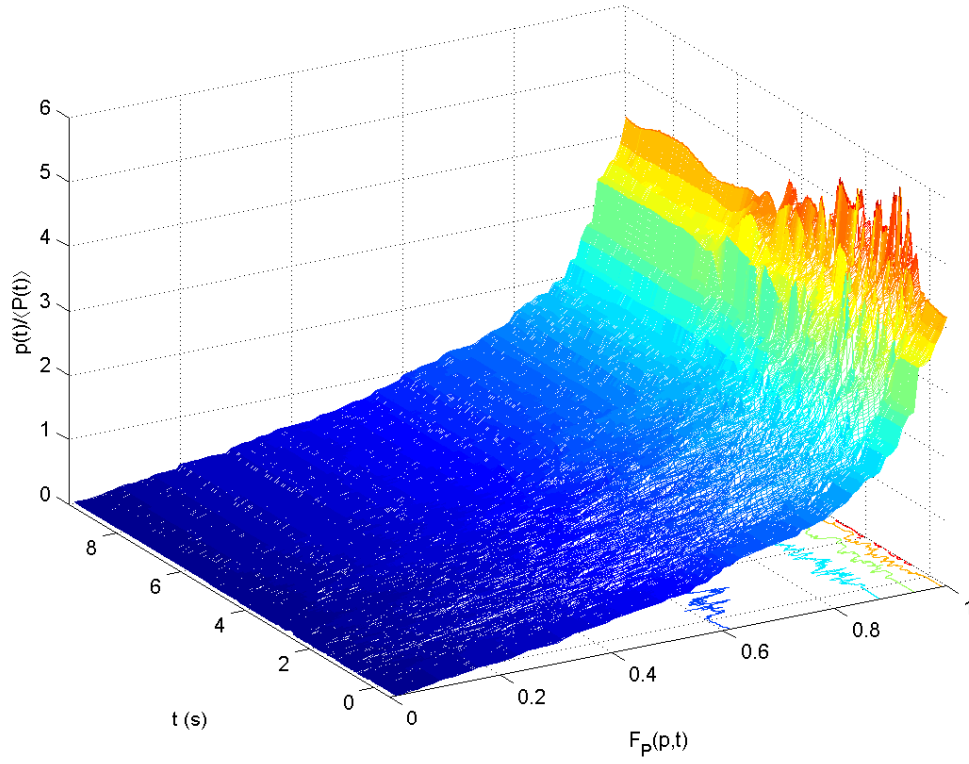
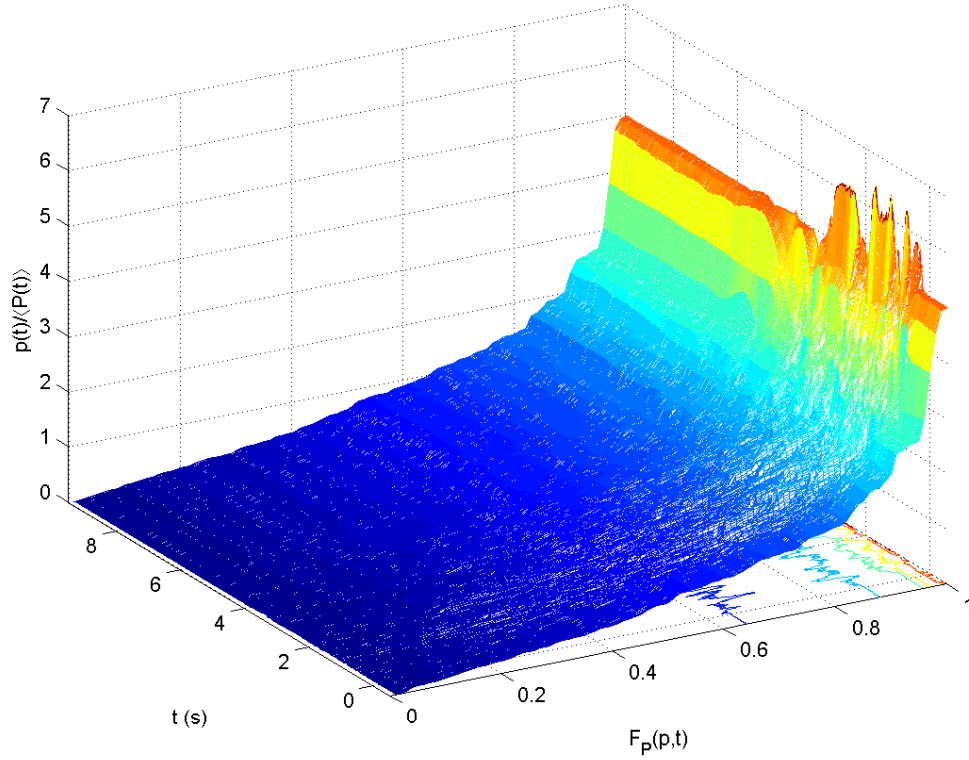
(a) $A = 1000 \text{ steps/s}^2$ (b) $A = 100 \text{ steps/s}^2$ (c) $A = 10 \text{ steps/s}^2$ (d) $A = 1 \text{ step/s}^2$

Figure 45: (color online) Sample cumulative distribution functions $F_P[p(t)]$ of mean-normalized values of $P(t) \equiv |S_{21}(t)|^2$ for transition times $t_0 - 10\Delta t \leq t \leq t_0 + 50\Delta t$ (61 curves) at $f = 1000 \text{ MHz}$, with $M = 400 \text{ steps}$ ($N = 100$), $B = 600 \text{ steps/s}$, $V = 2000 \text{ steps/s}$, and (a) $A = 1000 \text{ steps/s}^2$; (b) $A = 100 \text{ steps/s}^2$; (c) $A = 10 \text{ steps/s}^2$; (d) $A = 1 \text{ step/s}^2$.

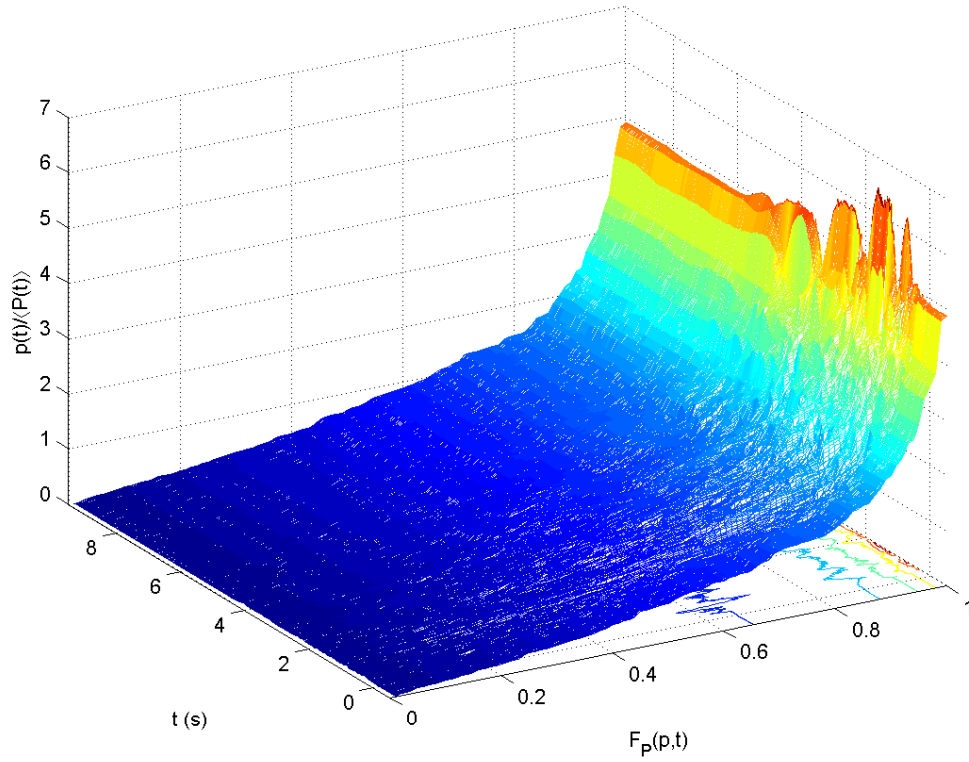


(a) $A = 1000 \text{ steps/s}^2$, $B = 600 \text{ steps/s}$, $V = 2000 \text{ steps/s}$

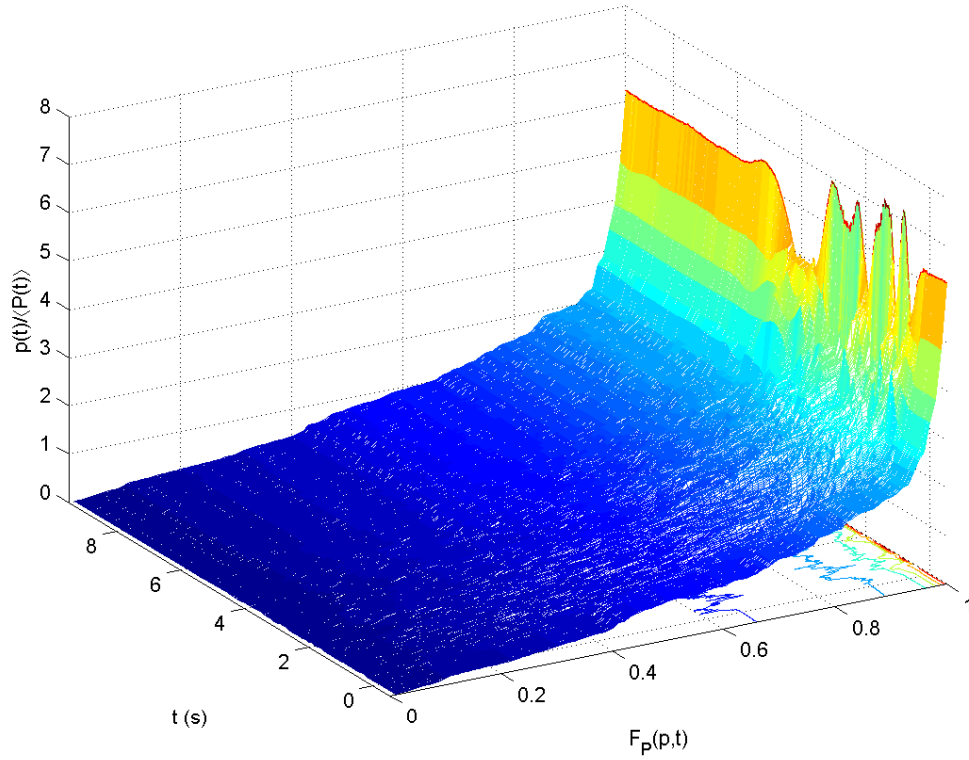


(b) $A = 100 \text{ steps/s}^2$, $B = 600 \text{ steps/s}$, $V = 2000 \text{ steps/s}$

Figure 46: (color online) Temporal evolution of sample cumulative distribution function $F_P(p, t)$ at $f = 1000 \text{ MHz}$, with $M = 400 \text{ steps}$ ($N = 100$), $B = 600 \text{ steps/s}$, $V = 2000 \text{ steps/s}$, and (a) $A = 1000 \text{ steps/s}^2$; (b) $A = 100 \text{ steps/s}^2$.



(a) $A = 10 \text{ steps/s}^2$, $B = 600 \text{ steps/s}$, $V = 2000 \text{ steps/s}$



(b) $A = 1 \text{ step/s}^2$, $B = 600 \text{ steps/s}$, $V = 2000 \text{ steps/s}$

Figure 47: (color online) Temporal evolution of sample cumulative distribution function $F_P(p, t)$ at $f = 1000 \text{ MHz}$, with $M = 400 \text{ steps}$ ($N = 100$), $B = 600 \text{ steps/s}$, $V = 2000 \text{ steps/s}$, and (a) $A = 10 \text{ steps/s}^2$; (b) $A = 1 \text{ step/s}^2$.

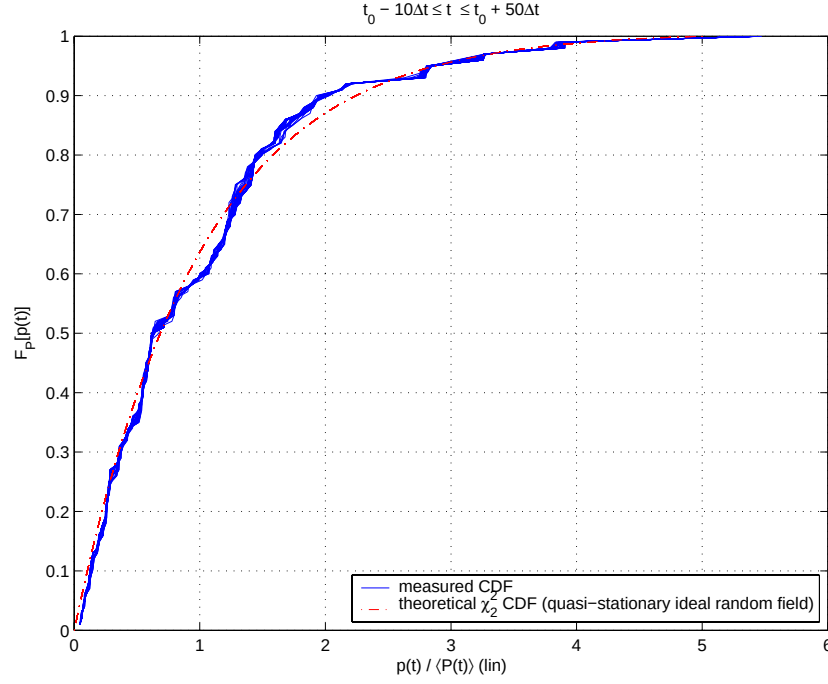
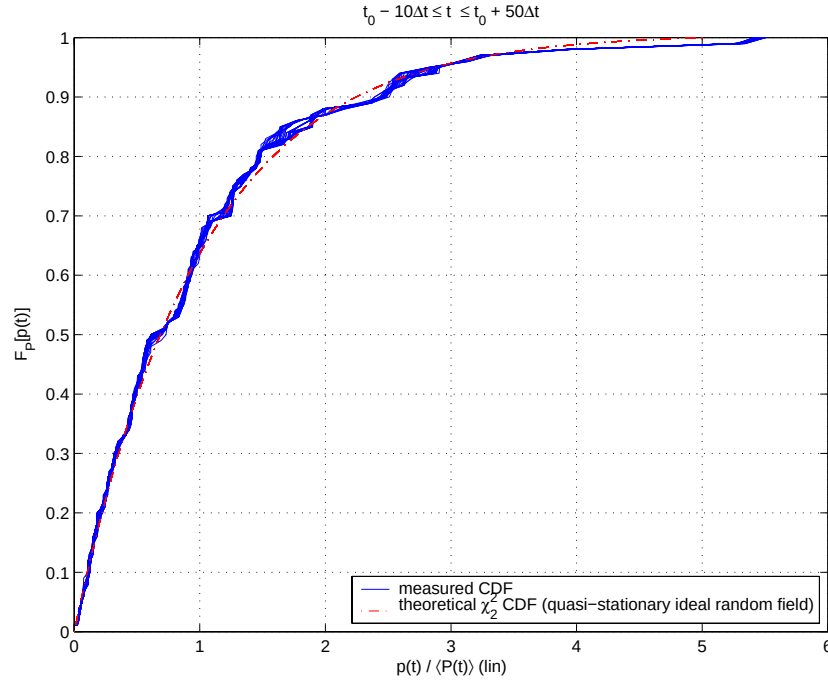
(a) $B = 10$ steps/s(b) $B = 1$ step/s

Figure 48: (color online) Sample cumulative distribution functions $F_P[p(t)]$ of mean-normalized values of $P(t) \equiv |S_{21}(t)|^2$ for transition times $t_0 - 10\Delta t \leq t \leq t_0 + 50\Delta t$ (61 curves) at $f = 1000$ MHz, with $M = 400$ steps ($N = 100$), $A = 1000$ steps/s², $V = 2000$ steps/s, and (a) $B = 10$ steps/s; (b) $B = 1$ step/s.

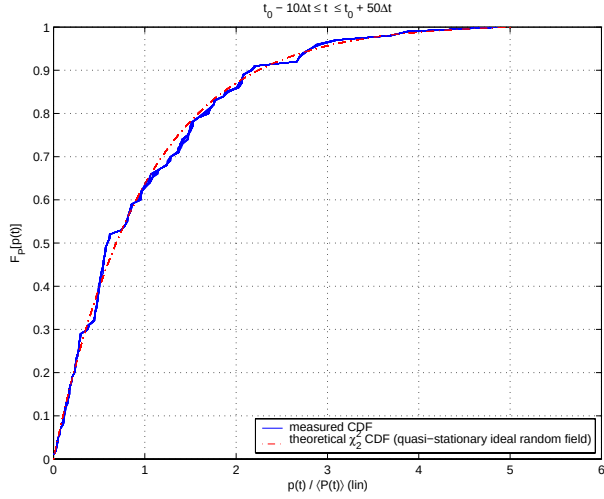
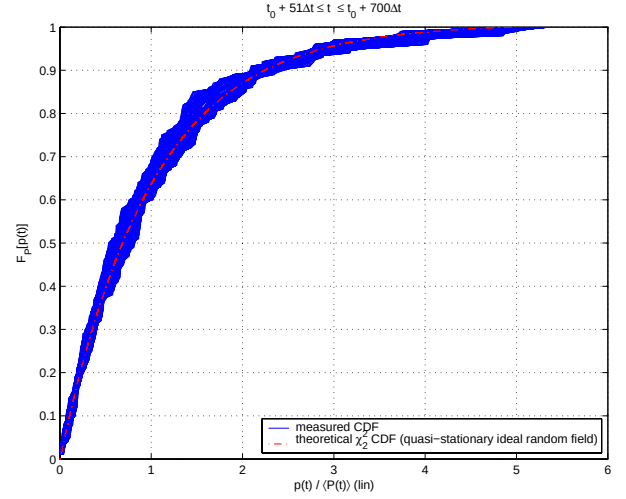
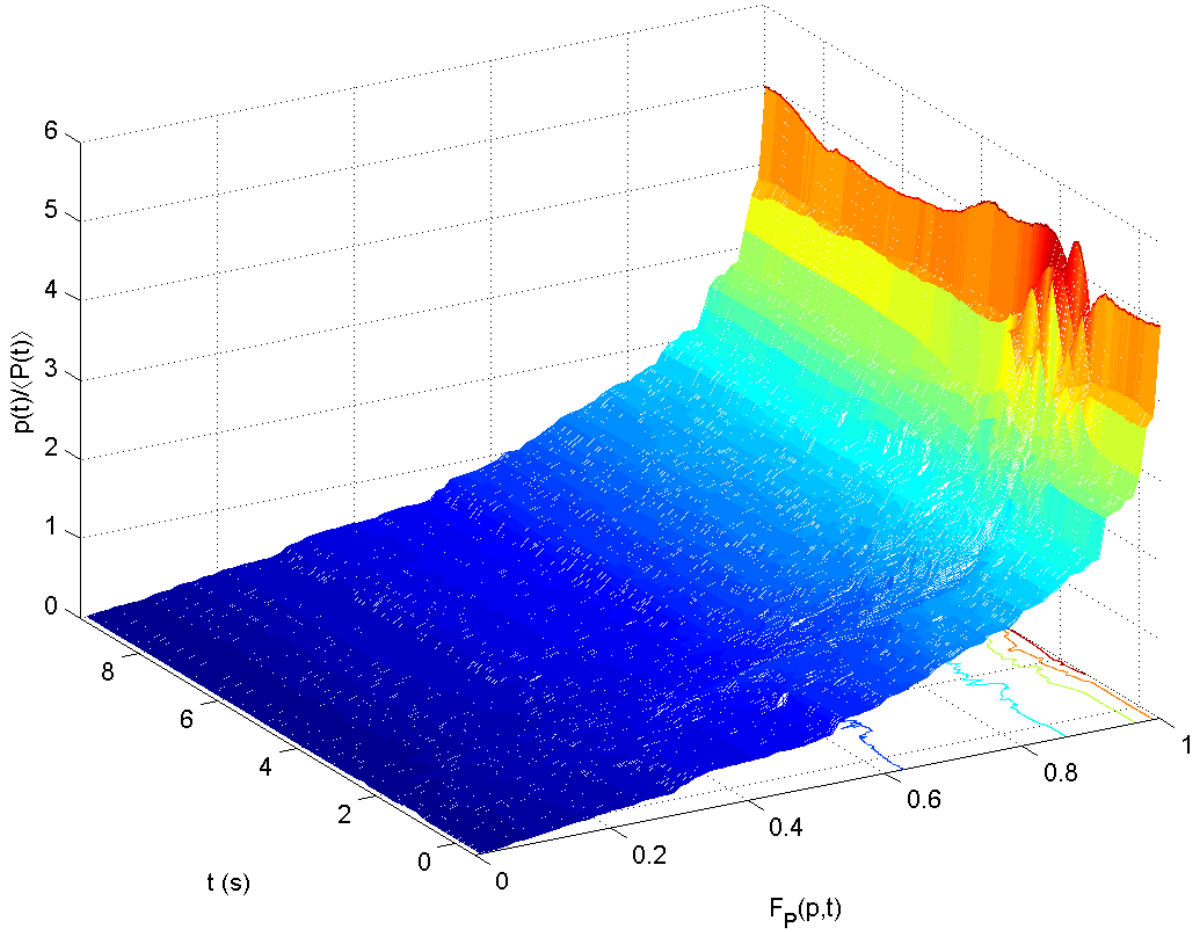
(a) $t_0 - 10\Delta t \leq t \leq t_0 + 50\Delta t$ (b) $t_0 + 51\Delta t \leq t \leq t_0 + 700\Delta t$ (c) $t_0 - 80\Delta t \leq t \leq t_0 + 1520\Delta t$

Figure 49: (color online) Sample cumulative distribution functions $F_P[p(t)]$ of mean-normalized values of $P(t) \equiv |S_{21}(t)|^2$ at $f = 1000$ MHz, with $M = 400$ steps ($N = 100$), $A = 100$ steps/s², $B = 10$ steps/s, and $V = 200$ steps/s, for transition times (a) $t_0 - 10\Delta t \leq t \leq t_0 + 50\Delta t$ (61 curves); (b) $t_0 + 51\Delta t \leq t \leq t_0 + 700\Delta t$ (650 curves); (c) complete evolution.

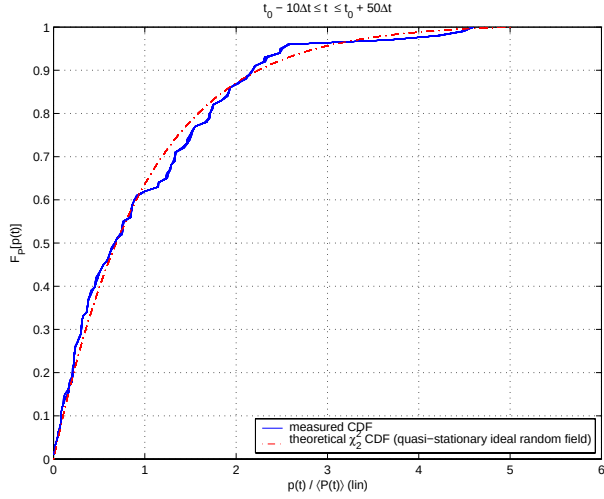
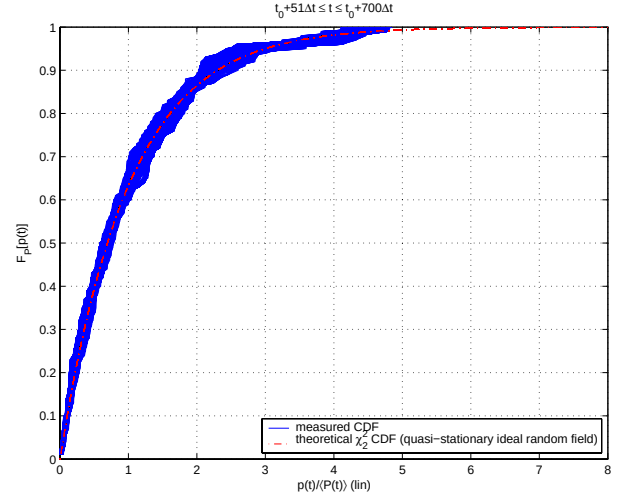
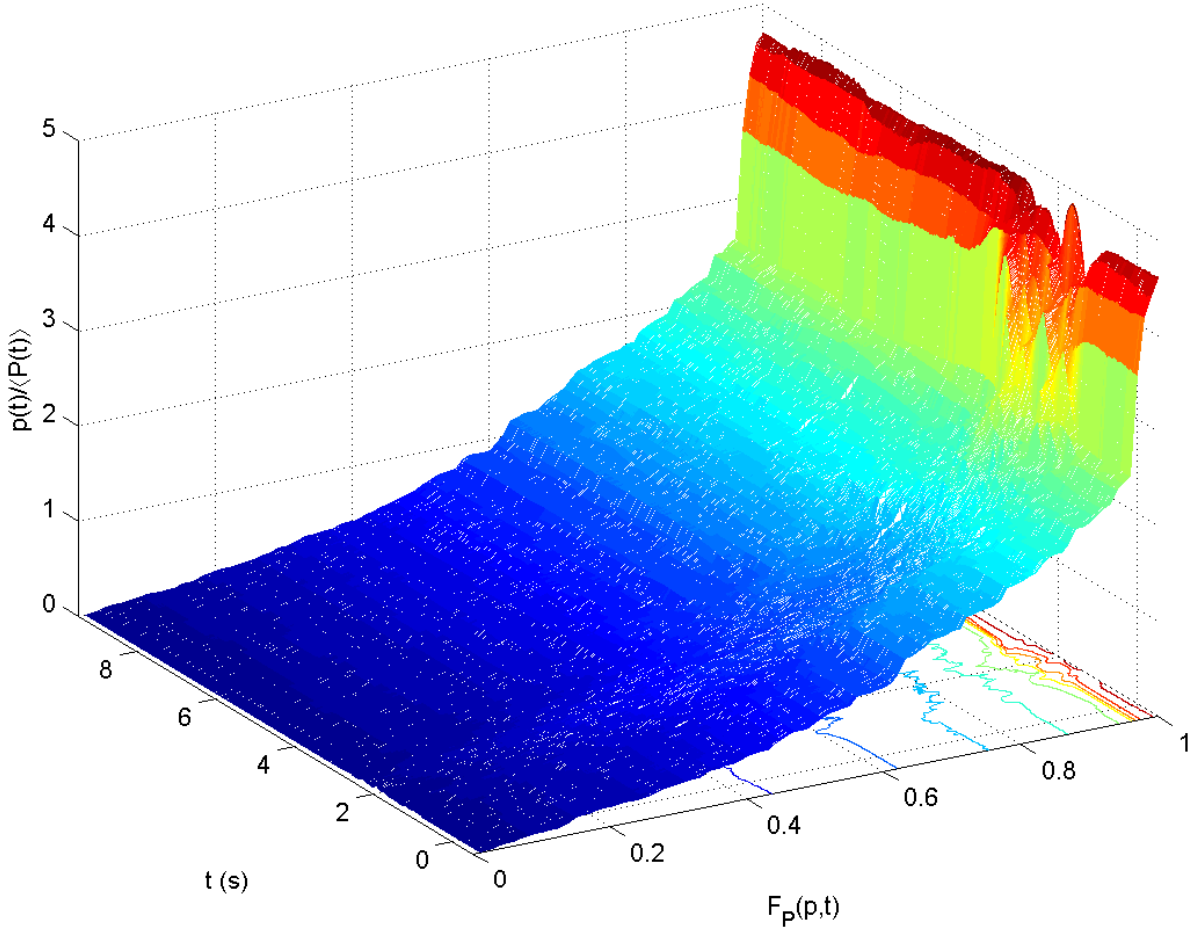
(a) $t_0 - 10\Delta t \leq t \leq t_0 + 50\Delta t$ (b) $t_0 + 51\Delta t \leq t \leq t_0 + 700\Delta t$ (c) $t_0 - 80\Delta t \leq t \leq t_0 + 1520\Delta t$

Figure 50: (color online) Sample cumulative distribution functions $F_P[p(t)]$ of mean-normalized values of $P(t) \equiv |S_{21}(t)|^2$ at $f = 1000$ MHz, with $M = 400$ steps ($N = 100$), $A = 100$ steps/s², $B = 10$ steps/s, and $V = 2000$ steps/s, for transition times (a) $t_0 - 10\Delta t \leq t \leq t_0 + 50\Delta t$ (61 curves); (b) $t_0 + 51\Delta t \leq t \leq t_0 + 700\Delta t$ (650 curves); (c) complete evolution.

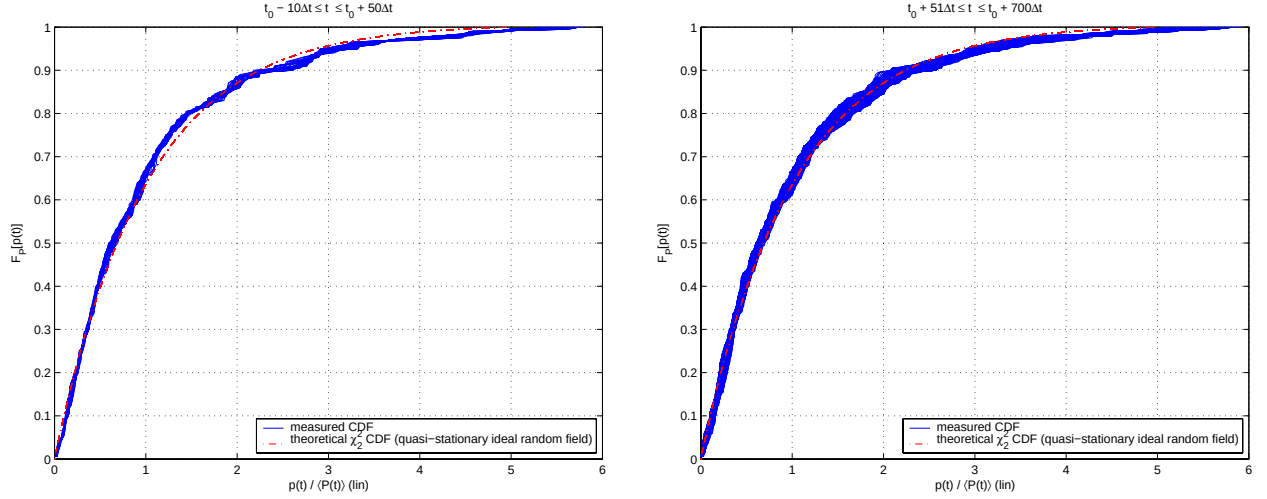
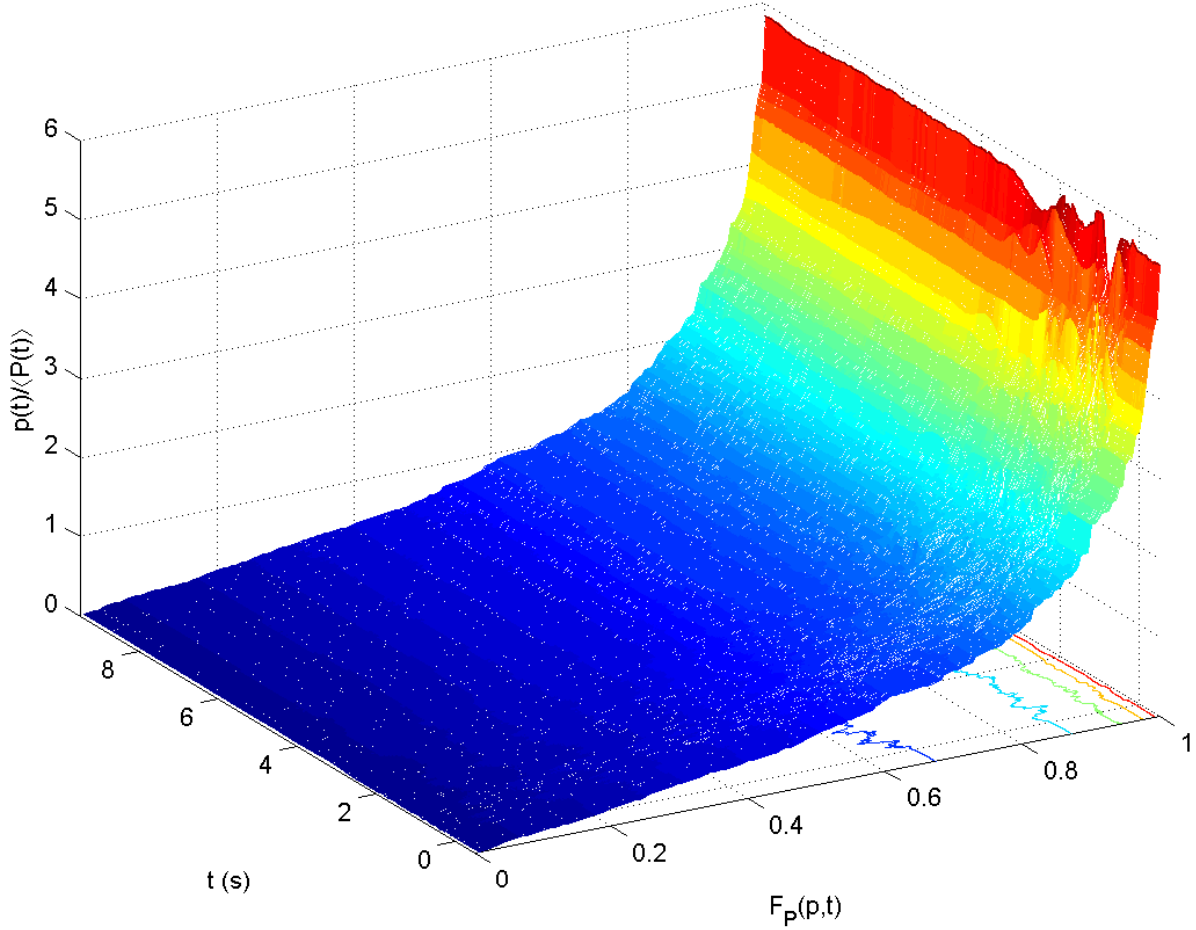
(a) $t_0 - 10\Delta t \leq t \leq t_0 + 50\Delta t$ (b) $t_0 + 51\Delta t \leq t \leq t_0 + 700\Delta t$ (c) $t_0 - 80\Delta t \leq t \leq t_0 + 1520\Delta t$

Figure 51: (color online) Sample cumulative distribution functions $F_P[p(t)]$ of mean-normalized values of $P(t) \equiv |S_{21}(t)|^2$ at $f = 1000$ MHz, with $M = 200$ steps ($N = 200$), $A = 1000$ steps/s², $B = 600$ steps/s, $V = 2000$ steps/s, for transition times (a) $t_0 - 10\Delta t \leq t \leq t_0 + 50\Delta t$ (61 curves); (b) $t_0 + 51\Delta t \leq t \leq t_0 + 700\Delta t$ (650 curves); (c) complete evolution.

6.4 Temporal evolution of maximum value and maximum-to-mean ratio

The maximum-to-average ratio of $P(t) \equiv |S_{21}(t)|^2$ is a quantity of fundamental importance in EMC immunity testing. For economic reasons, the number of selected steady-state tuner steps in an EMC test is often significantly smaller than the maximum number of independent states N_{ind} that a mode tuner is capable of generating, inside a given cavity at a given frequency of operation. If the RF source remains uninterrupted during the tuner transitions, then it remains to be established whether or not the maximum field value achieved during the transition is *significantly* different from the registered (sample) maximum value at the steady-state positions. This is particularly important in the testing of susceptible electronic equipment whose response and/or cycle times are short (or unknown) compared to the time scale of the rate of fluctuation of the field during the tuning or stirring process.

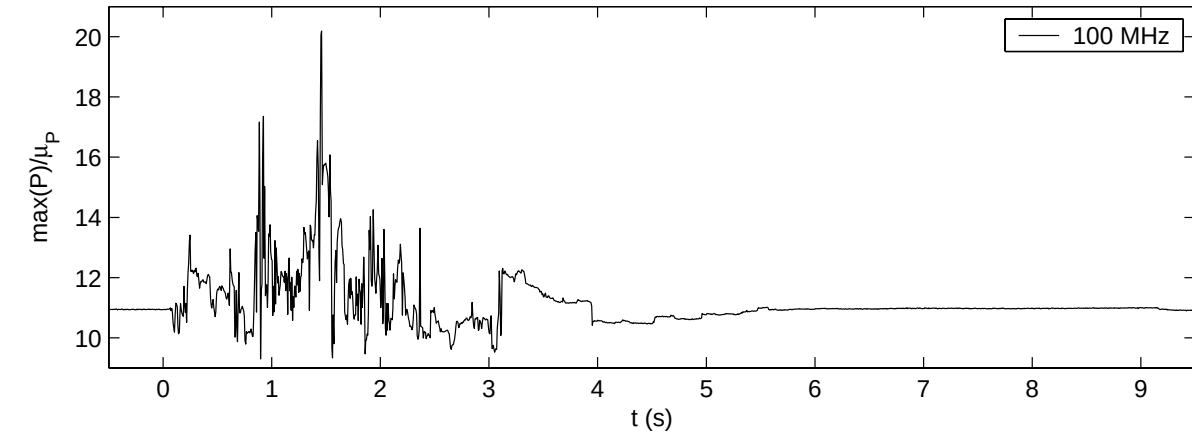
6.4.1 Dependence on frequency

Figure 52 shows the evolution of the measured maximum-to-mean ratio $\text{Max}[P(t)]/\mu_P(t)$ at selected frequencies, for $M = 400$ steps ($N = 100$), with $A = 1000$ steps/s², $B = 600$ steps/s, $V = 2000$ steps/s. Since N was chosen to be constant for each frequency, the steady-state values of the maximum-to-mean ratio ought to be similar as well provided the field is sufficiently close to ideal random. At the lower frequencies ($f \leq 200$ MHz), however, there are significantly less than 100 independent samples generated in steady state. This is reflected in large variations of the steady-state values of the measured maximum-to-mean ratio with respect to frequency in this frequency range (larger uncertainties for smaller N_{ind}), e.g., compare $\text{max}[P(t)]/\mu_P(t) = 11$ at 100 MHz vs. 3.15 at 200 MHz. Since higher frequencies permit larger values of N_{ind} , the maximum-to-mean ratio increases during the transitions, on average, with increasing frequency; compare e.g. values for $0 \leq t \leq 3$ s at 900 and 1000 MHz to the steady-state values at these frequencies. Thus, in the overmoded regime, the higher the frequency the more likely the upper limit of some $\eta\%$ -confidence interval for the maximum-to-average ratio will be approached or exceeded during one or more transitions.

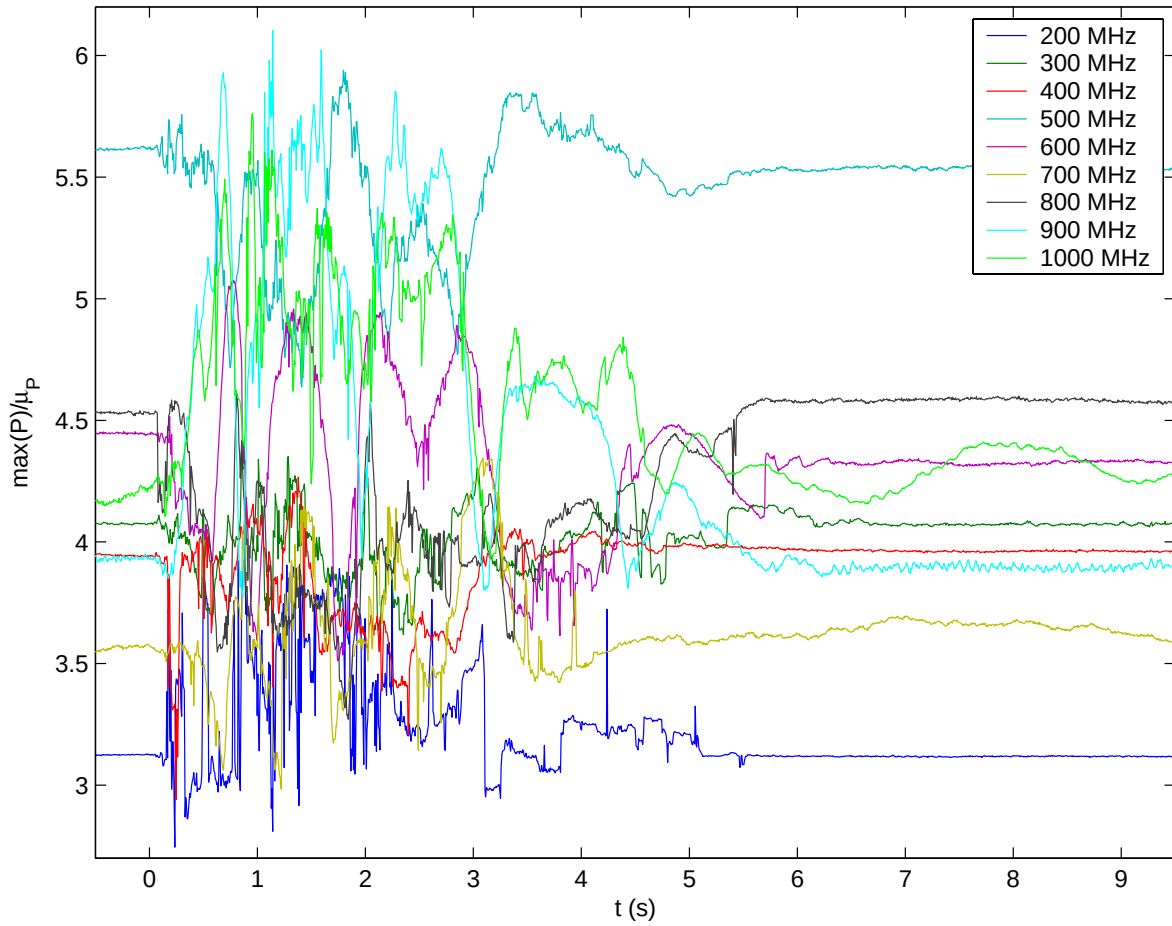
The ‘spiky’ nature of the fluctuations in Fig. 52 at lower frequencies mirrors a similar feature of the actual $|S_{21}(t)|^2$ themselves during the transitions (cf. Fig. 8). At first sight, this behaviour is surprising, on account of the fact that stronger correlation between steady states at lower frequencies suggest smoother and smaller fluctuations in between. The observed result again suggests a manifestation of nonstationary behaviour at lower frequencies, and the

fact that fluctuations of the small number of participating cavity modes during transitions have a comparatively large impact on the instantaneous $P(t)$ and its maximum value. At higher frequencies, the fluctuations of $\max[P(t)]/\mu_P(t)$, while still irregular, become increasingly smoother.

The spikes during transitions may occasionally cause the maximum field occurring in the course of the transients to be generated near tuner positions that are relatively far apart from those for which the measured steady-state values reach their maximum. For example, in Fig. 2(a), the maximum maximum is reached near $t = t_0 + 740$ s, whereas the steady-state maximum is reached near $t = t_0 + 765$ s, i.e., separated by $\Delta\theta \simeq 9$ deg. See also Fig. 7(c), showing a difference of approximately 25% during three individual transients at 1000 MHz.



(a)



(b)

Figure 52: (color online) Maximum-to-mean ratio of P as a function of time $t = t_\ell - t_0 = \ell \Delta t$ at selected frequencies, with $M = 400$ steps ($N = 100$), $A = 1000$ steps/s², $B = 600$ steps/s, $V = 2000$ steps/s, at (a) $f = 100$ MHz; (b) $f = 200$ to 1000 MHz.

6.4.2 Dependence on motor parameters

Figure 53 shows the evolution of the measured $\max[P(t)]$ and $\max[P(t)]/\mu_P(t)$ at $f = 1000$ MHz, for selected values of A . Unlike for quasi-stationary fields and steady-state stir sequences, the maximum and the maximum-to-average ratio are no longer scaled replicas of each other because $\mu_P(t)$ is not constant throughout the measured transitions. Nevertheless, Figs. 53(a) and 53(b) indicate that the fluctuations of $\max[P(t)]$ are more than twice as large as those of $\mu_P(t)$, yet $\max[P(t)]$ and $\max[P(t)]/\mu_P(t)$ are very similar in appearance. Provided A and B are not both very large, fluctuations of the maximum value are produced on a similar time scale as that of the oscillations of the mode stirrer, on average, with about the same periodicity as $\mu_P(t)$. This, again, suggests a mode bunching effect caused by tuner motion. Differences between steady-state values for different values of A may be due to the following reasons: (a) the inherent and relatively large uncertainty of the maximum value as an order statistic, for fixed N_{ind}^{ss} (where N_{ind}^{ss} represents the value of $N_{\text{ind}}(t)$ associated with the steady states), or (b) fluctuations of N_{ind} during the transition. This issue will be further investigated in Sec 6.7. For the sampling rate used in the present experiments ($\Delta t = T/L = 6.242$ ms) and the listed values of B and V , Fig. 53(b) shows that the measured maximum maximum is up to 30% higher than the measured steady-state maximum value.

Comparison of Fig. 53 with the measured $\max[P(t)]/\mu_P(t)$ for different values of B (cf. Fig. 54) shows that the sensitivity to these parameters is qualitatively similar but less pronounced than for A . The dependence on V (cf. Fig. 55) shows again a characteristic dip near $t = 1.5$ s, followed by a sharp maximum maximum. On the other hand, the absence of tuner oscillations results in fewer local maxima later on.

The true, i.e., physical value of the actual maximum maximum of the continuous trace is likely to be still larger and to become detectable when the sampling rate is increased beyond $1/\Delta t = 160.1$ Hz. To investigate this conjecture, we analyze a tuner sweep with double the number of tuner steps ($M = 200$, $N = 200$), which was shown in Fig. 9(c). By preserving the sample count ($L = 1601$), the angular sampling rate per transition is maintained while, at the same time, the halving of $\Delta\theta$ reduces the maximum speed V' and its effect on $\max[P(t)]$. The result is shown in Fig. 56. As expected the steady-state value of the maximum has increased because of the larger N . It is noteworthy that, in spite of the reduced velocities, the measured $\max[P(t)]$ and $\max[P(t)]/\mu_P(t)$ during the transitions¹⁸ are now still larger (even

¹⁸The increase of steady-state values is purely statistical and therefore not relevant here.

though only slightly so) than for $N = 100$. This means that it is exceedingly difficult, within the limitations on sampling count (and, hence, sampling rate) to accurately estimate the increase of the maximum maximum during transitions. Also, during the transitions, the ratio does no longer show a distinct positive bias compared to the steady-state value. Nevertheless, because the maximum maximum values are similar in both cases, the difference is within the (relatively large) inherent statistical uncertainty of the maximum value as a sample statistic. For reference, the theoretical ratio $\text{Max}[P(t)]/\mu_P(t)$ for $N_{\text{ind}} = 100$ or 200 for a quasi-stationary ideal random field has expected values 5.19 or 5.88 and standard deviations 1.635 or 1.640, respectively.

In summary, the fluctuations of the measured sample values of the maximum value or maximum-to-mean ratio are always larger during the transition than at steady state. Typically, the values during the transition are also larger than the theoretical expected steady-state value associated with N_{ind}^{ss} . The results show that N_{ind} cannot be chosen during a test (whereas N can) but that N_{ind} is given by its maximum number, i.e., $N_{\text{ind}}(t)$. Results for still higher sampling rates are needed for high speeds, in order to establish whether a yet significantly higher transitory maximum may have gone undetected as a result of the sampling still being too coarse, in view of the large and irregular fluctuations of $\text{Max}[P(t)]$ during the transition.

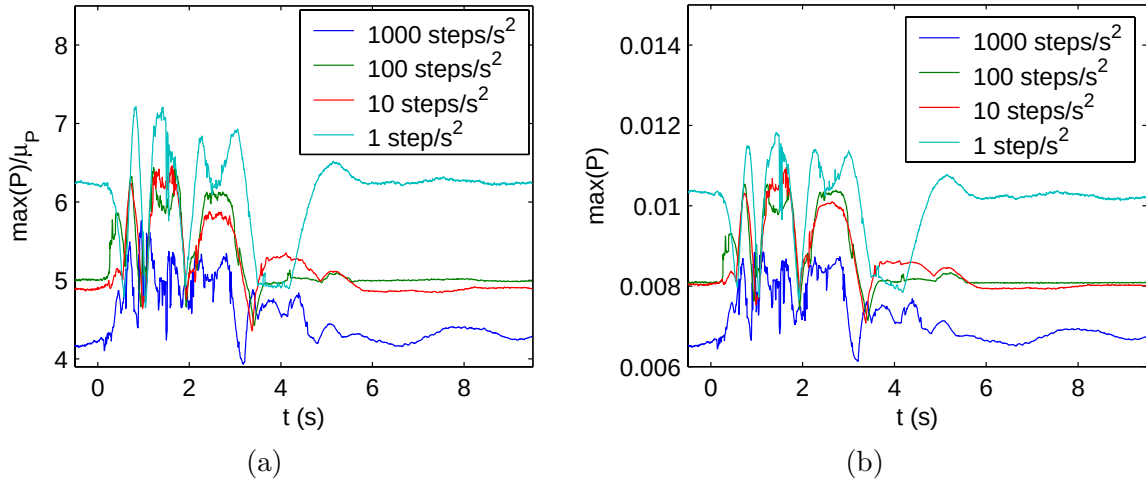


Figure 53: (color online) (a) Maximum-to-mean ratio and (b) absolute maximum of P as a function of time at $f = 1000$ MHz, for selected values of A , with $M = 400$ steps ($N = 100$), $B = 600$ steps/s, $V = 2000$ steps/s.

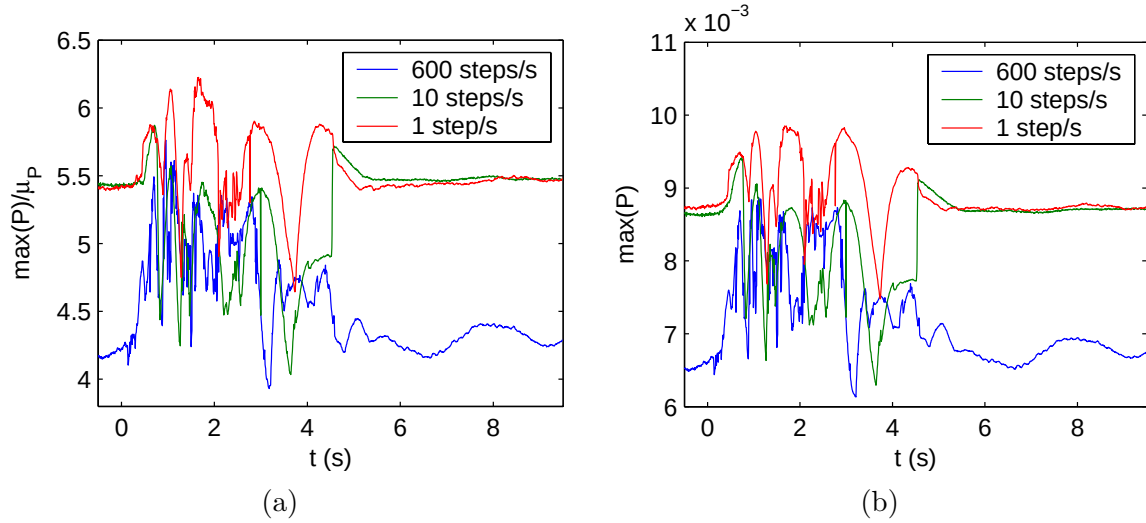


Figure 54: (color online) (a) Maximum-to-mean ratio and (b) absolute maximum of P as a function of time at $f = 1000$ MHz, for selected values of B , with $M = 400$ steps ($N = 100$), $A = 1000$ steps/s², $V = 2000$ steps/s.

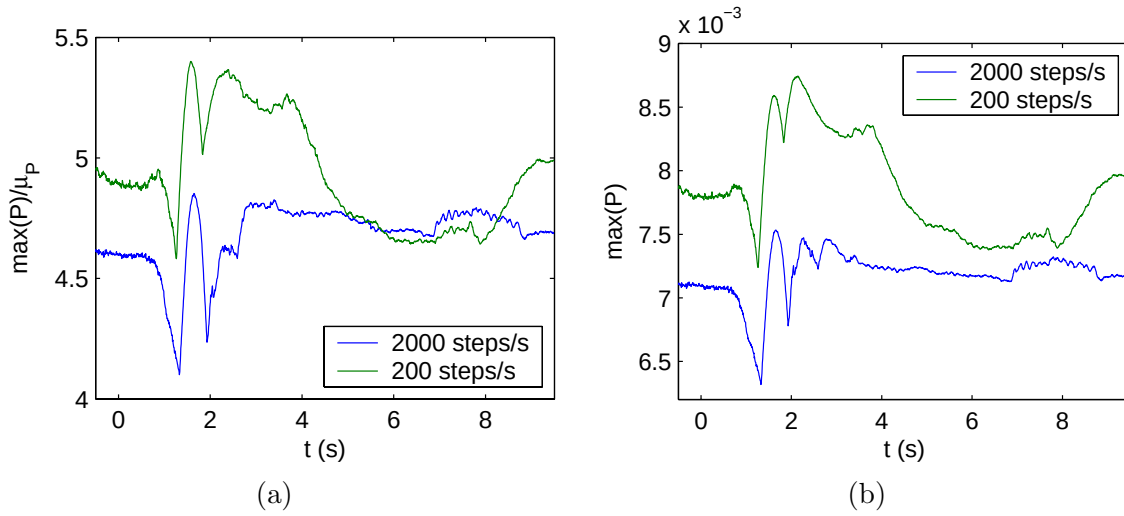


Figure 55: (color online) (a) Maximum-to-mean ratio and (b) absolute maximum of P as a function of time at $f = 1000$ MHz, for selected values of V , with $M = 400$ steps ($N = 100$), $A = 100$ steps/s², $B = 10$ steps/s.

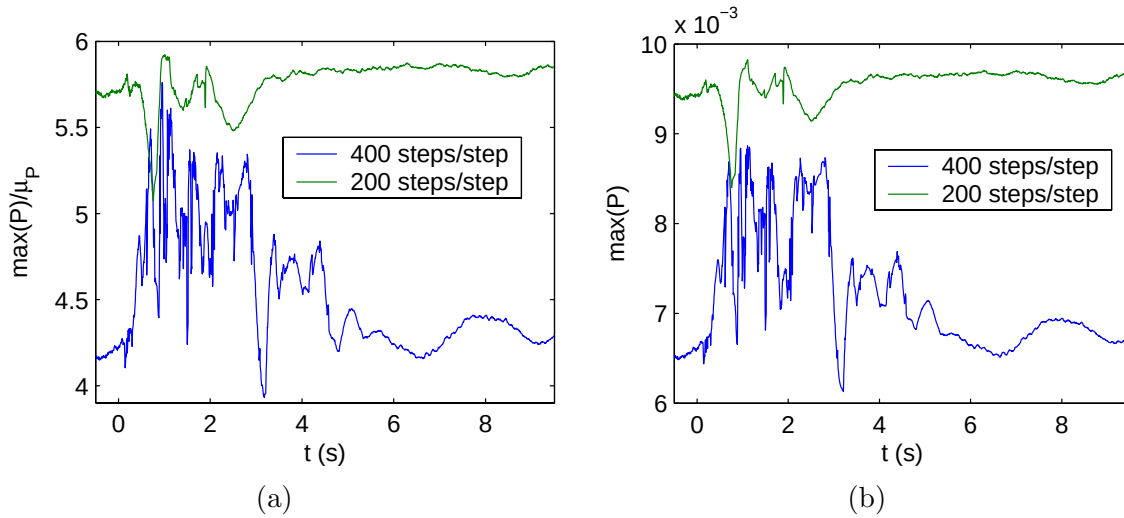


Figure 56: (color online) Maximum-to-mean ratio as a function of time at $f = 1000$ MHz, for selected values of motor steps M per paddle wheel step, with $A = 1000$ steps/s², $B = 600$ steps/s, $V = 2000$ steps/s.

6.5 Temporal evolution of rate of fluctuation

So far, the calculated statistics of the stir sequences for arbitrary t_ℓ were of first order with respect to time, i.e., local (instantaneous) with respect to the age $t - t_0$ of the transient. In general, for homogeneous or inhomogeneous random fields, second-order spatial or temporal statistics provide information about the wave structure (in space) or evolution (in time), i.e., about its rate of fluctuation, in contrast to the magnitude and range of fluctuation governing first-order statistics.

In order to investigate more systematically the smoothness of the fluctuations during transitions, we wish to analyze the evolution of the (nonlocal) gradient. For discretized (sampled) measured data, this can be done approximately by means of the magnitude of the quotient of finite differences $|\Delta P(t)/\Delta t|$ as a discrete approximation to the continuous first-order time derivative $|dP(t)/dt|$. To characterize the statistics of $|\Delta P(t)/\Delta t|$, we construct sequences of temporal differences $\{|P(t_{\ell+1}; \theta = \theta_n) - P(t_\ell; \theta = \theta_n)|\}_{n=0, \dots, N-1}$ for $\ell = -80, \dots, L - 81$. In other words, these difference sequences are obtained by grouping (cross-referencing) corresponding differences $|P(t_{\ell+1}) - P(t_\ell)|$ within each (n th) time trace across all N tuner transitions $\theta_n \rightarrow \theta_{n+1}$ into $L - 1$ sequences of size (sample length) N . The pdf of ΔP for the general case of spatially or temporally inhomogeneous fields is derived in Appendix A.

The extent to which $|\Delta P(t)/\Delta t|$ is representative of $|dP(t)/dt|$ when $\Delta t \rightarrow 0$ depends on the relative sampling rate: the smoother the physical time traces, the closer the statistics of $|\Delta P(t)/\Delta t|$ resemble those for the asymptotic $|dP(t)/dt|$. Figure 57 compares the sampling of a single transition at 100 MHz with that at 1000 MHz, for $M = 400$ steps, $A = 1000$ steps/s², $B = 600$ steps/s, $V = 2000$ steps/s, $L = 1601$. Most fluctuations are captured faithfully, although there are some notable exceptions, particularly at the lower frequency, where changes can be abrupt and result in local undersampling affecting the statistics of $|\Delta P/\Delta t|$.

Since synchronization is established by the controller's trigger signal to the motor, rather than by triggering with respect to a fixed amplitude level, small timing errors (jitter) between the traces may affect the results. Such jitter gives rise to additional noise in the temporal dependence of difference statistics.

6.5.1 Dependence on frequency

Figure 58 shows the evolution of the estimated average rate of fluctuation, $\langle |\Delta P(t)/\Delta t| \rangle$, with respect to θ at selected frequencies. First, it is noted that for $t < t_0$ and for $t \gg t_0 + 6.5$ s, in spite

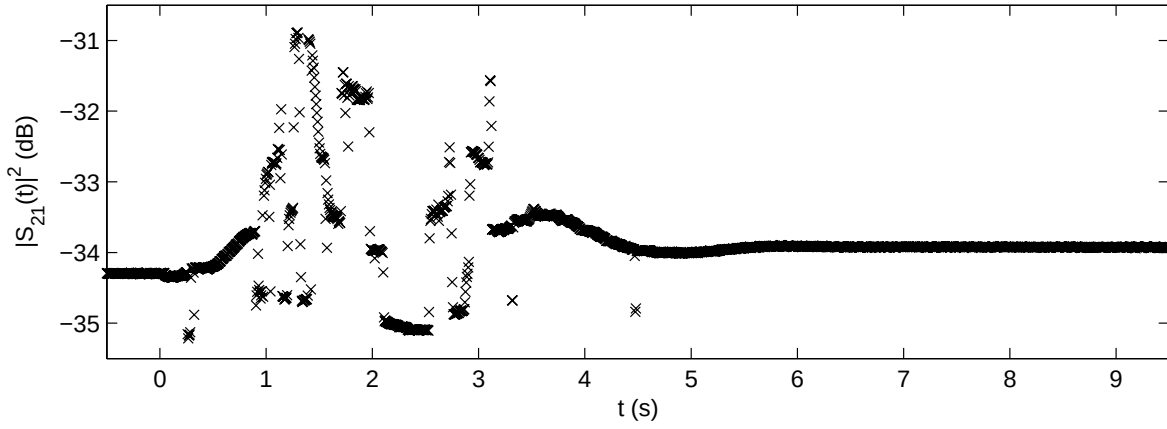
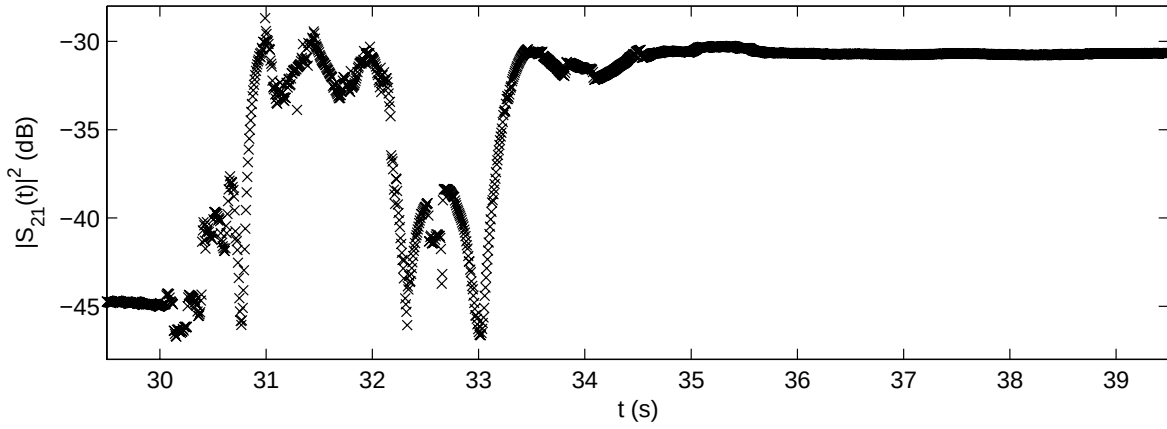
(a) $f = 100$ MHz(b) $f = 1000$ MHz

Figure 57: Sampling of a single tuner transition, with $L = 1601$, $M = 400$ steps ($N = 100$), $A = 1000$ steps/s², $B = 600$ steps/s, $V = 2000$ steps/s, at (a) $f = 100$ MHz; (b) $f = 1000$ MHz.

of the paddle wheel then being mechanically static at these times in principle, the steady-state value of $\langle |\Delta P(t)/\Delta t| \rangle$ exhibits large noisy fluctuations. These fluctuations are believed to be primarily due to the residual mechanical vibrations, forces exerted on the paddle wheel by the air flow of the air-conditioning system, residual EMI, etc., which all cause variations of input and/or transmitted power at the microscopic level. These fluctuations determine the accuracy with which statistics of $|\Delta P(t)/\Delta t|$ can be evaluated experimentally. The de-noised steady-state value is frequency dependent because $\langle |S_{21}(t)|^2 \rangle$ decreases with increasing frequency (increasing insertion loss).

Regarding $\langle |\Delta P(t)/\Delta t| \rangle$ during the transition, because of inertia and mechanical oscillations, the driven paddle wheel has an “output” velocity profile that is distorted compared to the “input” profile imposed by the motor itself. Recalling Fig. 1, we know that, at the start of the

transition ($t = t_{0+}$), the instantaneous motor velocity exhibits a sharp initial increase, caused ideally by a “step” change of the speed of the motor toward base speed B at $t = t_0$, followed by a linear increase of the instantaneous velocity at a rate A . This change in instantaneous velocity is reflected, *mutatis mutandis*, in the change of $\langle |\Delta P(t)/\Delta t| \rangle$. Figure 58 indicates a precursor-like behaviour for $0 < t - t_0 < 0.5$ s, in response to the step change. The change is most abrupt at lower frequencies. Since the chosen nominal values of the motor parameters do not result in a plateau of constant slew speed, $\langle |\Delta P(t)/\Delta t| \rangle$ does not exhibit a plateau either. Instead, $\langle |\Delta P(t)/\Delta t| \rangle$ peaks near $t - t_0 = 0.266$ s, i.e., near the time when the motor reaches its maximum velocity in this case. However, whereas the motor itself decelerates after this time, $\langle |\Delta P(t)/\Delta t| \rangle$ increases further, until about $t - t_0 = 0.55$ s when the motor speed has returned to B , reaching in fact its maximum near twice this time. The decrease of $\langle |\Delta P(t)/\Delta t| \rangle$ is, on average, approximately exponential for any frequency, with a superimposed quasi-periodicity, as a manifestation of oscillatory tuner motion.

The profile of $\langle |\Delta P(t)/\Delta t| \rangle$ for the undermoded case (100 MHz), also shown in 58, is distinctly different from that for the overmoded regime. The noise in $\langle |\Delta P(t)/\Delta t| \rangle$ increases with decreasing frequency. The duration of the decay of $\langle |\Delta P(t)/\Delta t| \rangle$ appears to shorten with decreasing frequency, despite the fact that the chamber relaxation time then increases. For the sampling rate used in our experiments ($1/\Delta t = 160.1$ Hz), the dynamic range of $\langle |\Delta P(t)/\Delta t| \rangle$ spans approximately 2.5 dB/ms. However, higher sampling rates may well reveal a higher value that asymptotically approaches $\langle |dP(t)/dt| \rangle$ in the limit $\Delta t \rightarrow 0$.

Figure 59 shows the corresponding standard deviation, $\sigma_{|\Delta P/\Delta t|}(t)$, which exhibits a similar time dependence and dynamic range as $\langle |\Delta P(t)/\Delta t| \rangle$, but with a somewhat broader time interval of high values for $\sigma_{|\Delta P/\Delta t|}(t)$.

Figure 60 shows the maximum-to-mean ratio of $|\Delta P(t)/\Delta t|$ at selected frequencies. During the transition, the ratio is seen to decrease, on average, when frequency is increased, from about 16 dB at 100 MHz to values oscillating around 8 dB at 1000 MHz. A similar difference exists for the steady-state ratios (i.e., for $t = t_0 > 6.5$ s) between frequencies. The effect of tuner oscillations on the maximum-to-mean ratio becomes more pronounced as frequency increases. The actual maximum value of $|\Delta P(t)/\Delta t|$, shown in Fig. 61, exhibits a similar behaviour as that of $\langle |\Delta P(t)/\Delta t| \rangle$. Its dynamic range across the transition is again approximately 2.5 dB/ms.

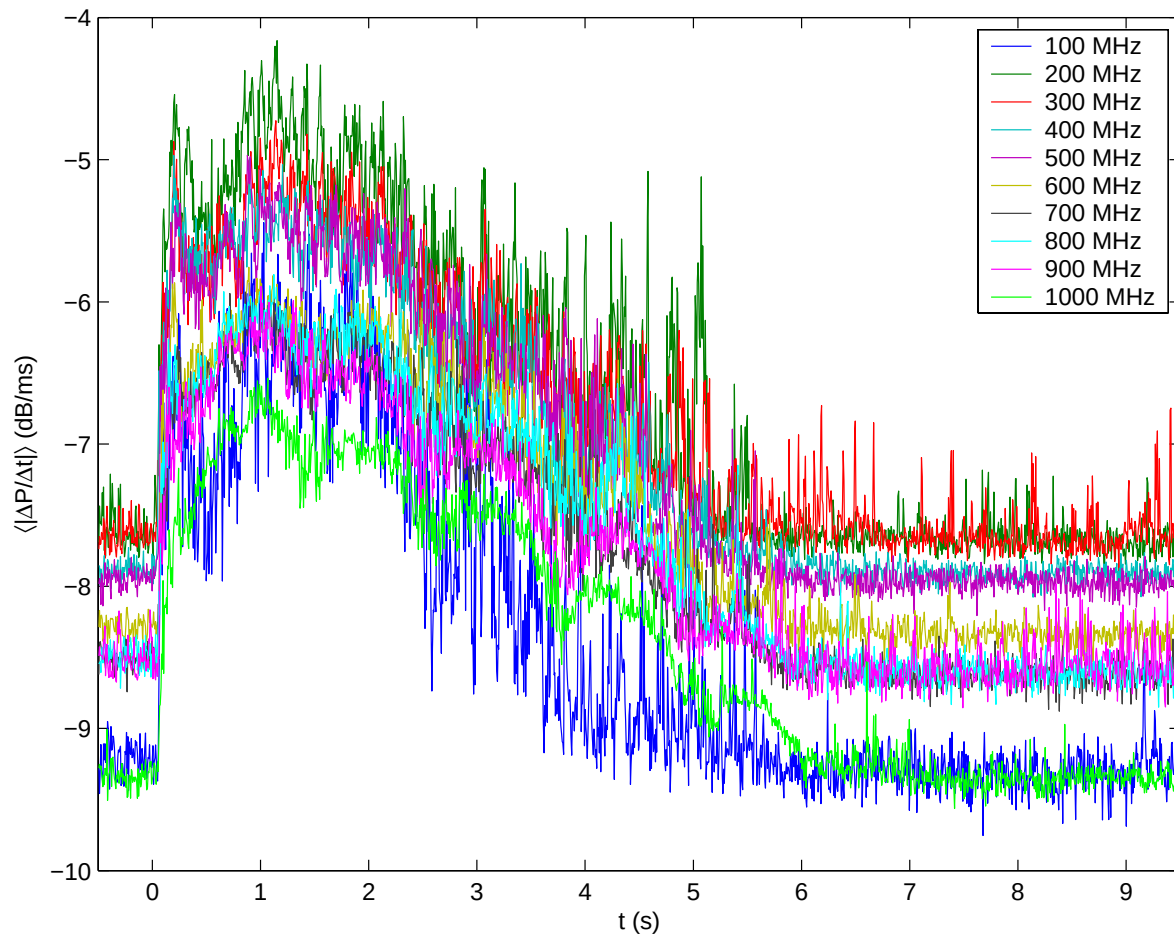


Figure 58: (color online) Average value of the rate of fluctuation of received power $\langle |\Delta P / \Delta t| \rangle$ as a function of time, at selected frequencies, with $M = 400$ steps ($N = 100$), $A = 1000$ steps/s², $B = 600$ steps/s, $V = 2000$ steps/s.

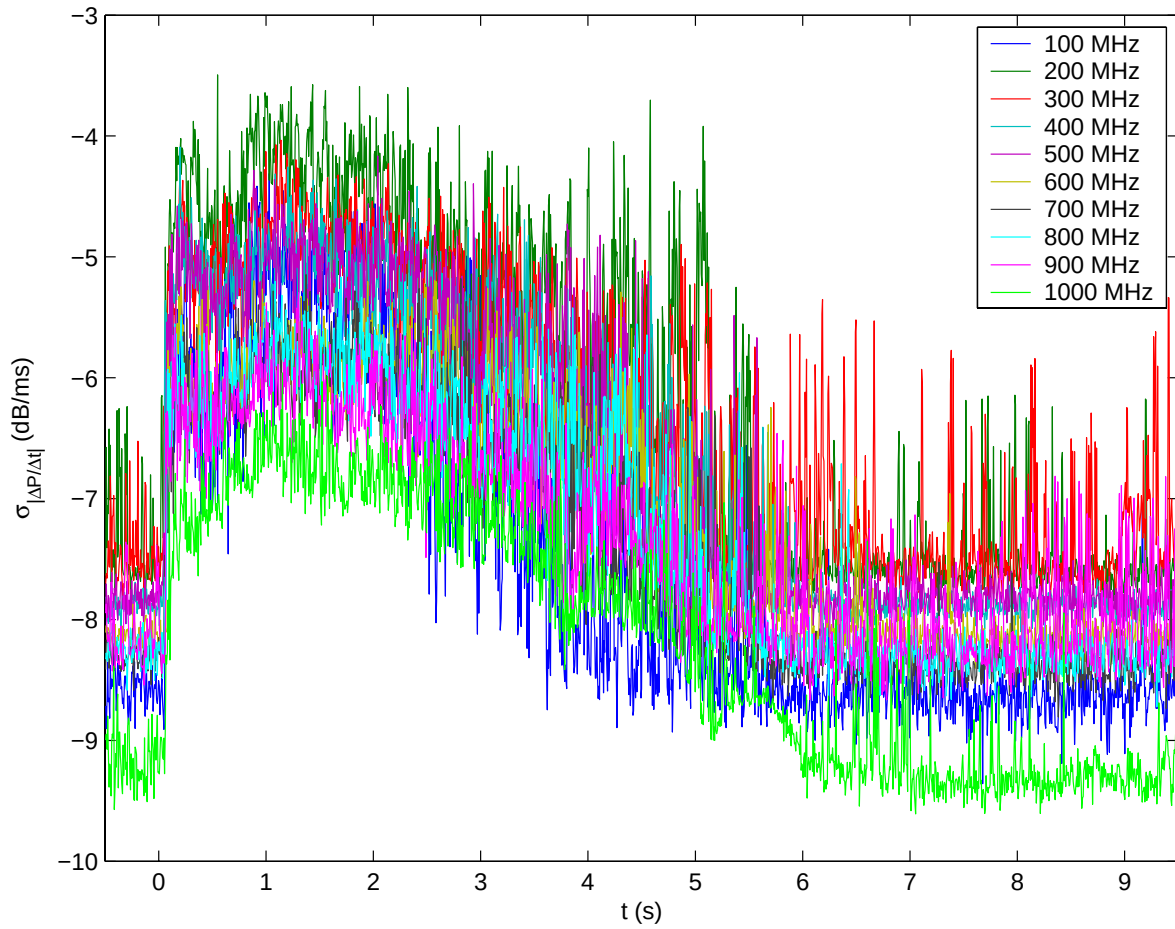


Figure 59: (color online) Standard deviation of the rate of fluctuation $\sigma_{|\Delta P(t)/\Delta t|}$ as a function of time, at selected frequencies, with $M = 400$ steps ($N = 100$), $A = 1000$ steps/s², $B = 600$ steps/s, $V = 2000$ steps/s.

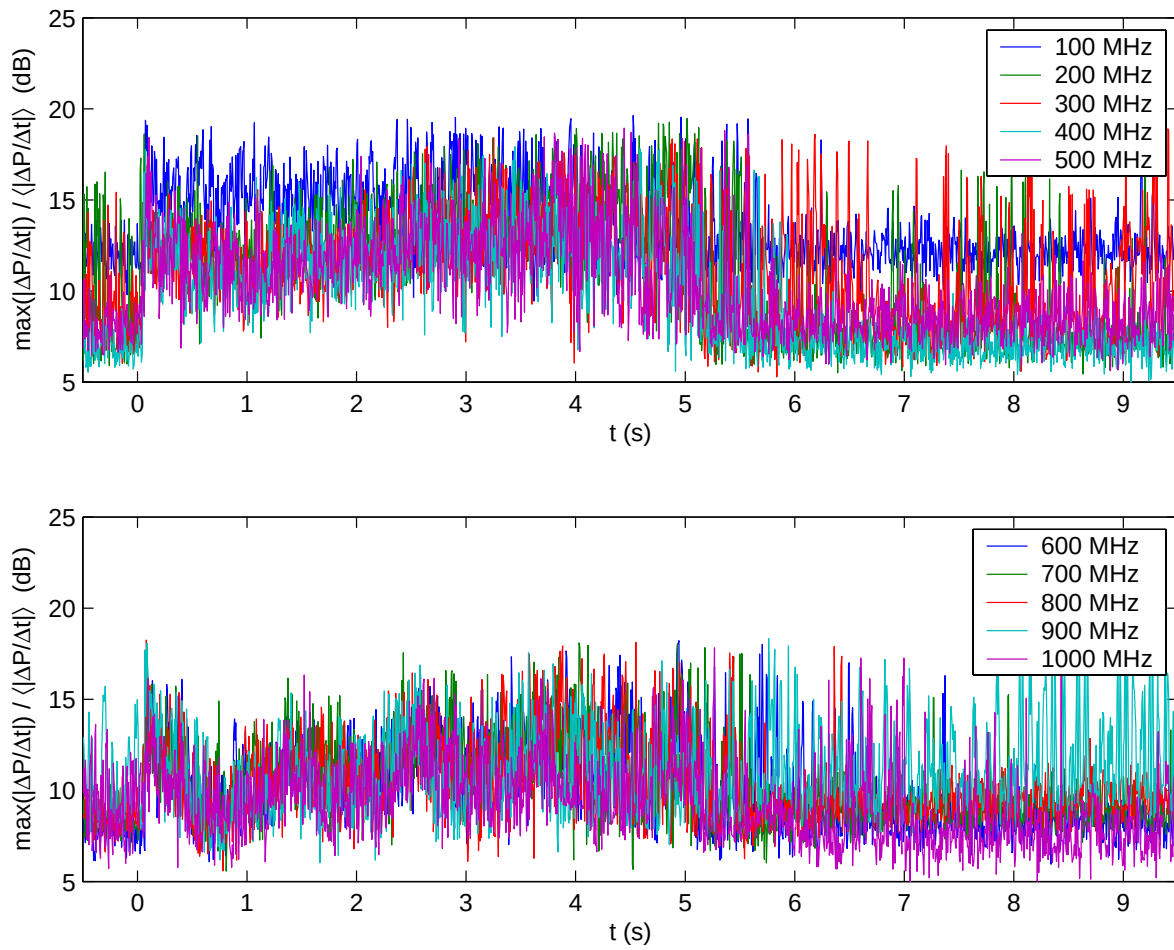


Figure 60: (color online) Maximum-to-average ratio of rate of fluctuation $\max(|\Delta P(t)/\Delta t|)/\langle |\Delta P/\Delta t| \rangle$ as a function of time at selected frequencies, with $M = 400$ steps ($N = 100$), $A = 1000$ steps/s², $B = 600$ steps/s, $V = 2000$ steps/s.

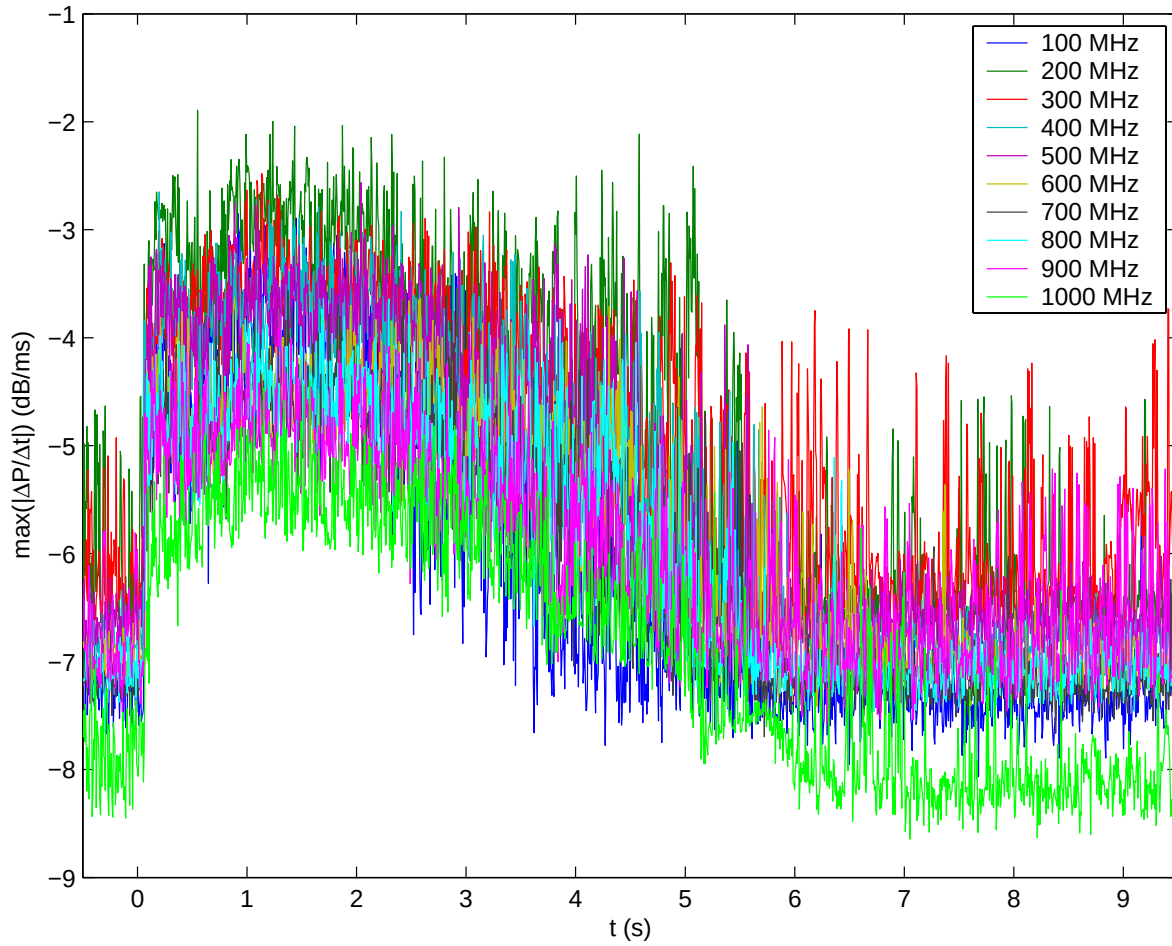


Figure 61: (color online) Maximum rate of fluctuation $\max(|\Delta P(t)/\Delta t|)$ as a function of time at selected frequencies, for $M = 400$ steps ($N = 100$), $A = 1000$ steps/s², $B = 600$ steps/s, $V = 2000$ steps/s.

6.5.2 Dependence on motor parameters

Figures 62 and 63 show the effect of the value of A on $\langle |\Delta P/\Delta t| \rangle$ and $\sigma_{|\Delta P/\Delta t|}$, indicating that there is relatively little influence. The same is true for $\max(|\Delta P/\Delta t|)$, shown in Fig. 64.

On the other hand, the effect of slower and more gradual motor rotations on $\langle |\Delta P/\Delta t| \rangle$, $\sigma_{|\Delta P/\Delta t|}$, and $\max(|\Delta P/\Delta t|)$ is clearly noticed by comparing Figs. 65–67 with Figs. 62–64. As expected, the increases during the transition are now much reduced, and their maximum values are now about 1 dB/ms lower than in the previous cases. Their steady-state value is reached significantly earlier, despite the longer rotation time. Note that the graphs in Fig. 65 closely match the triangular velocity profile of the motor: the motor reaches its maximum speed at 1.88 s, reduces to base speed at 3.76 s, and stops after 4.76 s for this set of parameters (cf. Sec. 2.2); these instances also mark special points on the profile of $\langle |\Delta P/\Delta t| \rangle$. By contrast, the graphs in Fig. 62 have profiles that are far from symmetric-triangular, because of the effect of the mechanical oscillations.

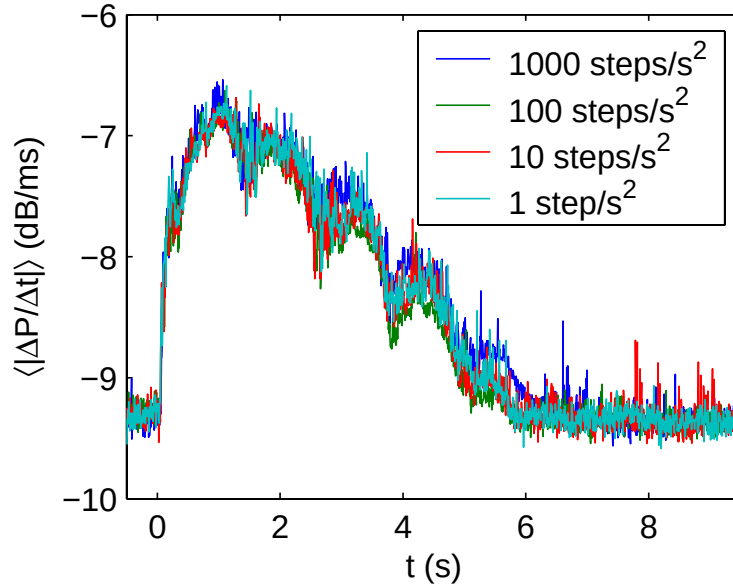


Figure 62: (color online) Average rate of fluctuation of receiver power $\langle |\Delta P/\Delta t| \rangle$ as a function of time at $f = 1000$ MHz, for selected values of A , with $M = 400$ steps ($N = 100$), $B = 600$ steps/s, $V = 2000$ steps/s.

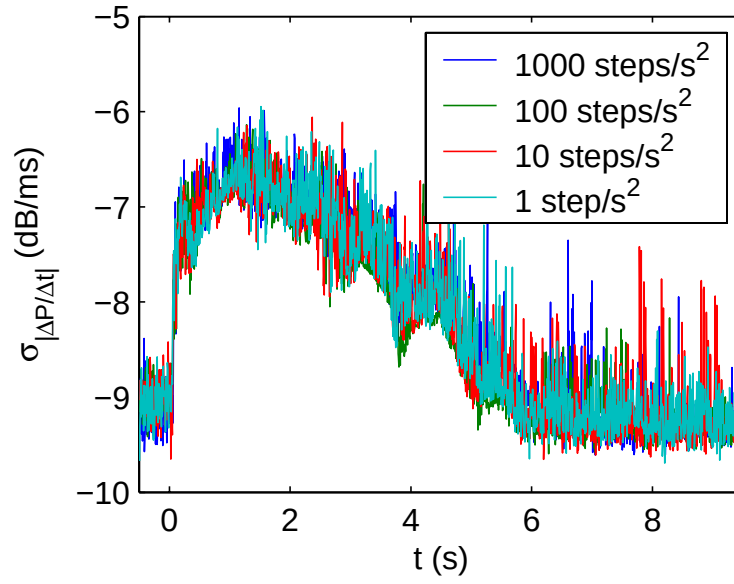


Figure 63: (color online) Standard deviation of fluctuation of received power $\sigma_{|\Delta P/\Delta t|}$ as a function of time at $f = 1000$ MHz, for selected values of A , with $M = 400$ steps ($N = 100$), $B = 600$ steps/s, $V = 2000$ steps/s.

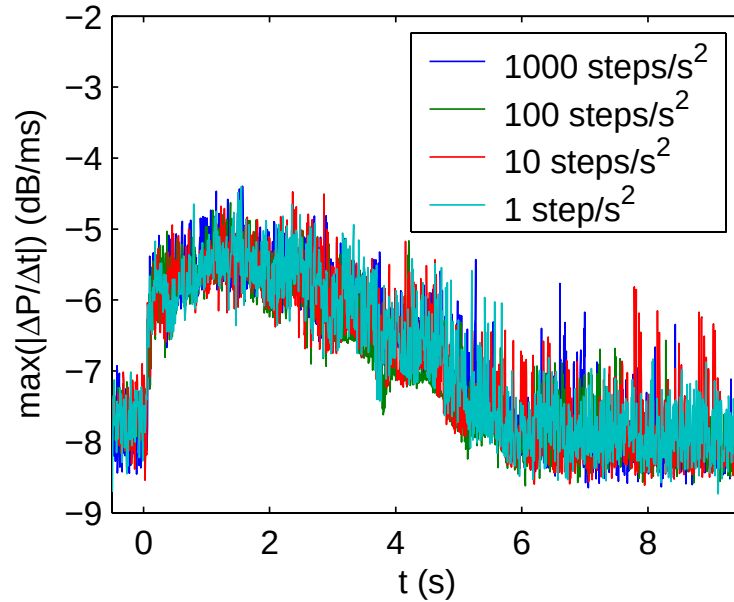


Figure 64: (color online) Maximum rate of fluctuation of received power $\max(|\Delta P/\Delta t|)$ as a function of time at $f = 1000$ MHz, for selected values of A , with $M = 400$ steps ($N = 100$), $B = 600$ steps/s, $V = 2000$ steps/s.

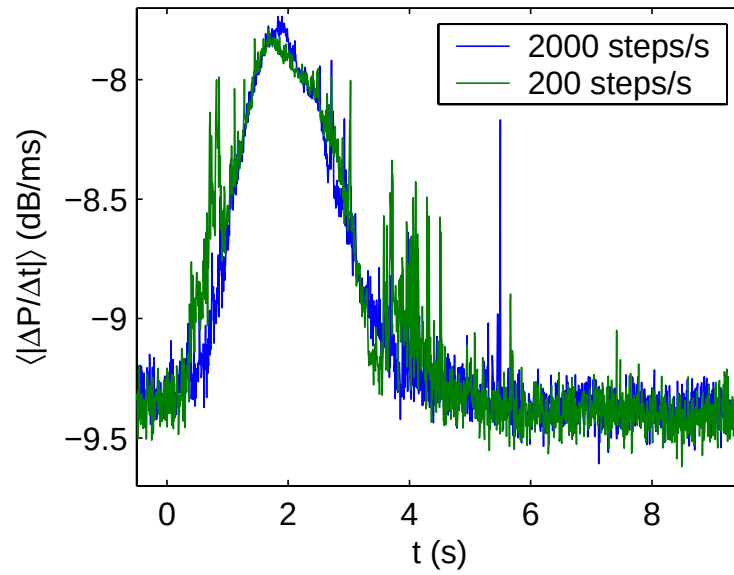


Figure 65: (color online) Average rate of fluctuation of received power $\langle |\Delta P / \Delta t| \rangle$ as a function of time at $f = 1000$ MHz, for selected values of V , $M = 400$ steps ($N = 100$), $A = 100$ steps/s², $B = 10$ steps/s.

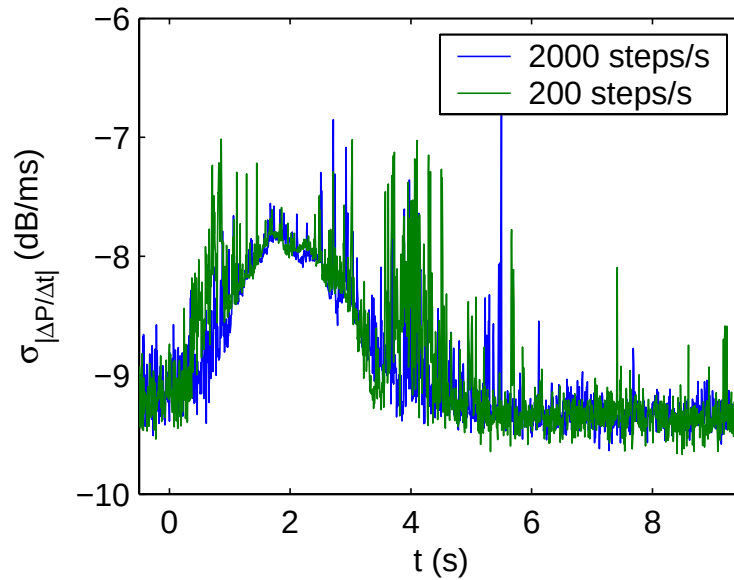


Figure 66: (color online) Standard deviation of fluctuation of received power $\sigma_{|\Delta P / \Delta t|}$ as a function of time at $f = 1000$ MHz, for selected values of V , $M = 400$ steps ($N = 100$), $A = 100$ steps/s², $B = 10$ steps/s.

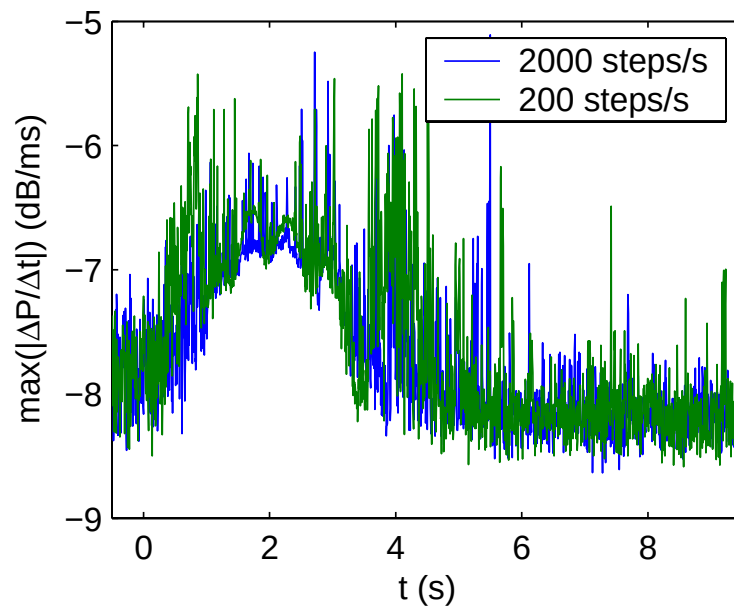


Figure 67: (color online) Maximum rate of fluctuation of received power $\max(|\Delta P/\Delta t|)$ as a function of time at $f = 1000$ MHz, for selected values of V , $M = 400$ steps ($N = 100$), $A = 100$ steps/s², $B = 10$ steps/s.

6.6 Temporal evolution of modulation index and modulation coefficient

In the study of the temporal rate of fluctuation, the instantaneous *modulation index* $\mathcal{M}(t)$ is a variate of fundamental interest. It can be obtained [15] as a simple generalization of the familiar modulation index for modulated signals, which is a constant in case of deterministic signals. The random modulation index for the random energy density or $|S_{21}(t)|^2$ is twice that for the field magnitude $\mathcal{A}(t) \propto |S_{21}(t)|$:

$$\mathcal{M}(t) \triangleq \frac{P'(t)}{P(t)} = \frac{2\mathcal{A}(t)\mathcal{A}'(t)}{\mathcal{A}^2(t)} = 2\frac{\mathcal{A}'(t)}{\mathcal{A}(t)}. \quad (12)$$

When expressed in terms of finite differences, for practical use with sampled time series, the functional form of the modulation index resembles that of previously introduced field anisotropy and field inhomogeneity coefficients, if we approximate $P(t)$ by the mid-point value $\bar{P}(t, \Delta t) \triangleq [P(t + \Delta t) + P(t)]/2$:

$$\mathcal{M}(t, \Delta t) \triangleq \frac{2}{\Delta t} \times \frac{P(t + \Delta t) - P(t)}{P(t + \Delta t) + P(t)} \equiv \frac{\left[\frac{P(t + \Delta t) - P(t)}{\Delta t} \right]}{\left[\frac{P(t + \Delta t) + P(t)}{2} \right]} \xrightarrow{\Delta t \rightarrow 0} \frac{P'(t)}{P(t)}. \quad (13)$$

[Since we are primarily interested in the rate of change in absolute terms, irrespective of whether this change is positive or negative at any one time, we can consider $[P'(t)/P(t)]^2$ or $|P'(t)/P(t)|$ instead of $P'(t)/P(t)$.] The instantaneous *modulation coefficient* is here introduced and defined as

$$M(t, \Delta t) \triangleq \mathcal{M}(t, \Delta t) \frac{\Delta t}{2} = \frac{P(t + \Delta t) - P(t)}{P(t + \Delta t) + P(t)}. \quad (14)$$

Its statistics follow by rescaling $\mathcal{M}(t)$. Although being a dimensionless quantity, $M(t, \Delta t)$ depends implicitly on the chosen sampling scale Δt . Again, $M^2(t, \Delta t)$ or $|M(t, \Delta t)|$ can be studied instead. Here, we focus on $|M(t)|$ for fixed Δt . Corresponding values for $\mathcal{M}(t, \Delta t)$ follow by dividing $M(t, \Delta t)$ by $\Delta t/2 = 3.121$ ms.

6.6.1 Dependence on frequency

Figures 68–70 show the evolution of the average value, standard deviation and maximum value of $|M(t)|$ for selected frequencies. Their values increase by up to 15...20 dB during the early part of the transition, relative to their steady-state values. A steep increase of approximately 10 dB occurs near $t = t_0$. The corresponding increases for $\langle |\mathcal{M}(t)| \rangle$ (not shown) are of the order of 3 dB and reach up to -2 dB/ms, -2 dB/ms, and 0 dB/ms, respectively. Compared to $|\Delta P(t)/\Delta t|$ for the same frequencies (Figs. 58–61), the steady-state values for $|M(t)|$ are more

tightly grouped, and there is no clear trend in their frequency dependence. Combined with the fact that $M(t)$ is a dimensionless quantity, this suggests that the modulation coefficient is a more universal measure than the rate of fluctuation, for sufficiently small Δt .

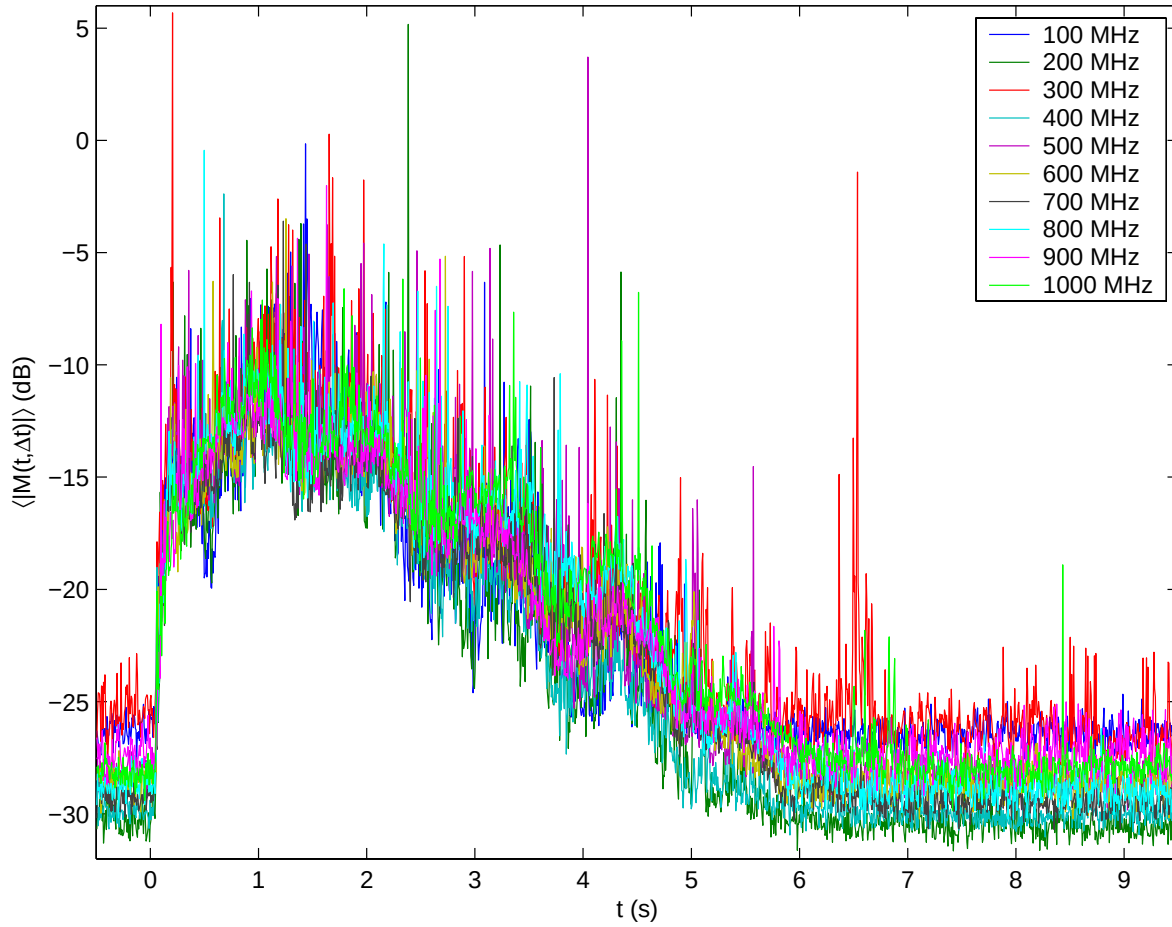


Figure 68: (color online) Average modulation coefficient $\langle |M(t)| \rangle$ as a function of time at selected frequencies, with $M = 400$ steps ($N = 100$), $A = 1000$ steps/s², $B = 600$ steps/s, $V = 2000$ steps/s, $\Delta t = 6.242$ ms.

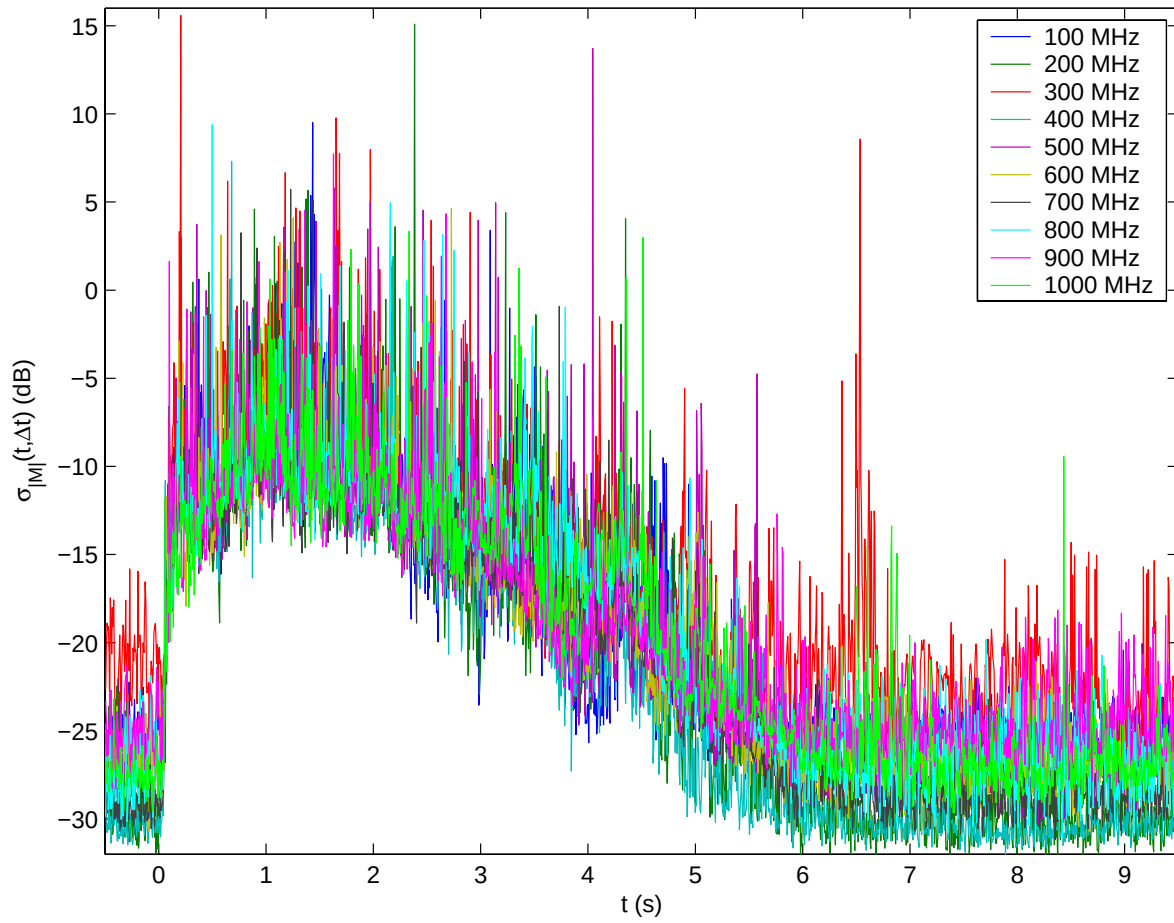


Figure 69: (color online) Standard deviation of modulation coefficient $\sigma_{|M|}(t)$ as a function of time at selected frequencies, with $M = 400$ steps ($N = 100$), $A = 1000$ steps/s², $B = 600$ steps/s, $V = 2000$ steps/s, $\Delta t = 6.242$ ms.

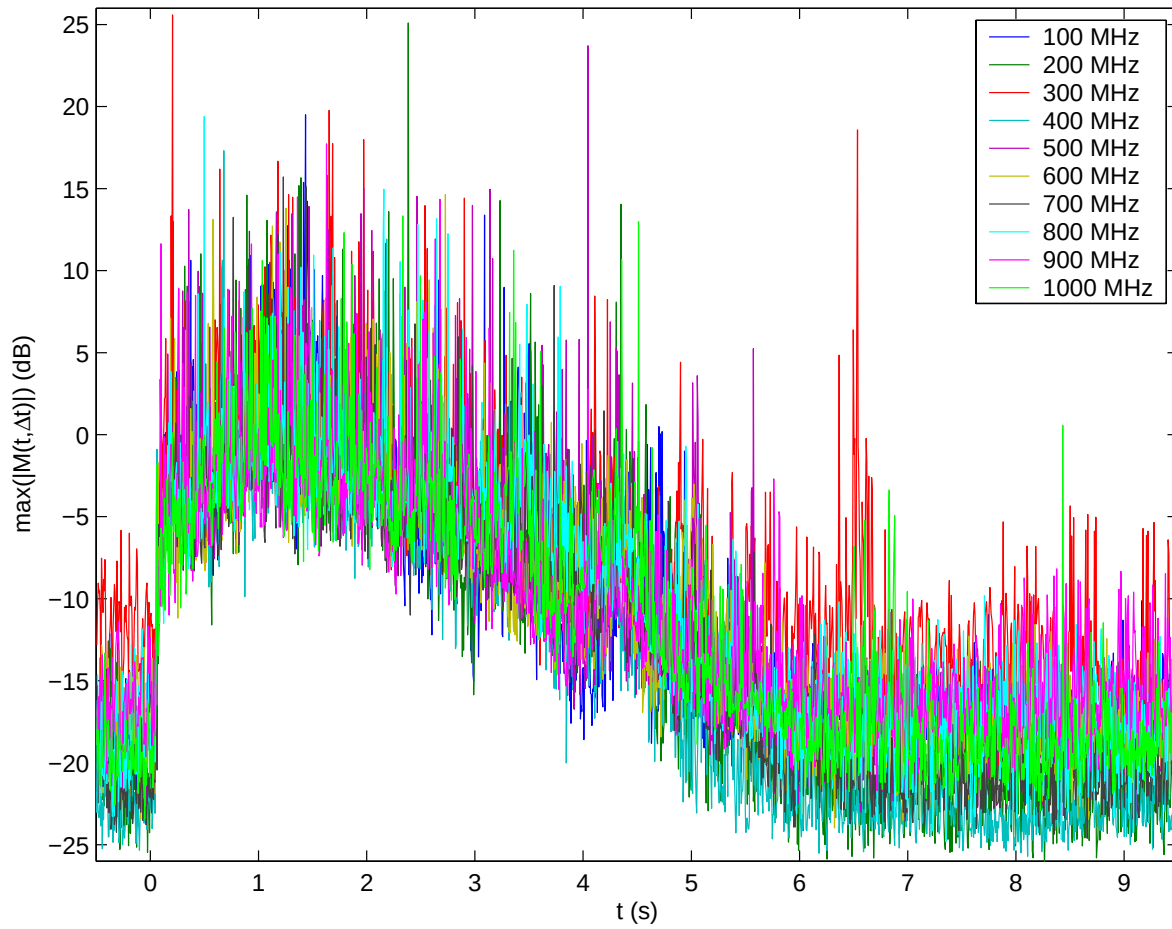


Figure 70: (color online) Maximum modulation coefficient $\max(|M(t)|)$ as a function of time at selected frequencies, with $M = 400$ steps ($N = 100$), $A = 1000$ steps/s², $B = 600$ steps/s, $V = 2000$ steps/s, $\Delta t = 6.242$ ms.

6.6.2 Dependence on motor parameters

The dependence of $|M(t)|$ on the motor acceleration A is again weak, as was the case for $|\Delta P(t)/\Delta t|$. Therefore, we only give results for slow gradual movements of the paddle wheel, in Figs. 71–74. The increase of $\langle|M(t)|\rangle$ during the transition is about 20 dB. The increase of $\langle|P'(t)/P(t)|\rangle$ (not shown) is considerably smaller, viz., of the order of 1.5 dB. The correspondence with the velocity profile of the motor is again noteworthy. The standard deviation and maximum value exhibit a number of distinct peaks during the transition, with a local minimum reached at the tuner's maximum speed, near $t = 1.88$ s.

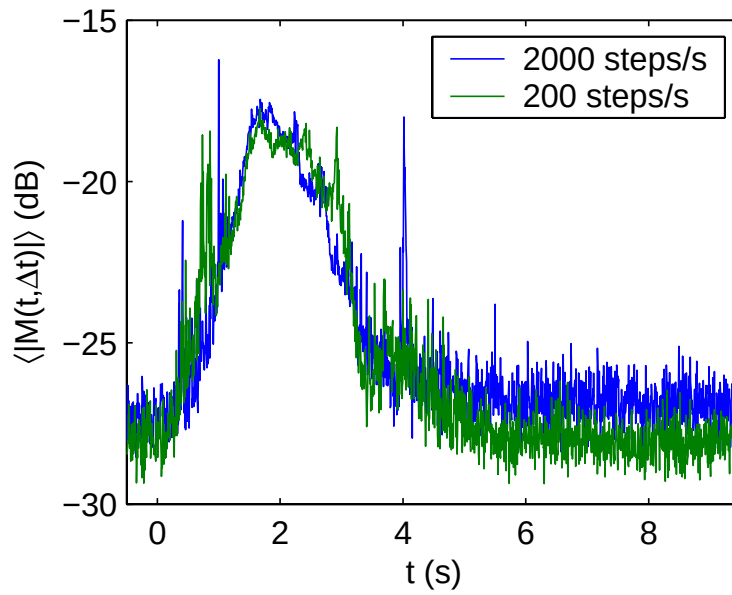


Figure 71: (color online) Average modulation coefficient $\langle|M(t)|\rangle$ as a function of time at $f = 1000$ MHz, for selected values of V , with $M = 400$ steps ($N = 100$), $A = 100$ steps/s², $B = 10$ steps/s, $\Delta t = 6.242$ ms.

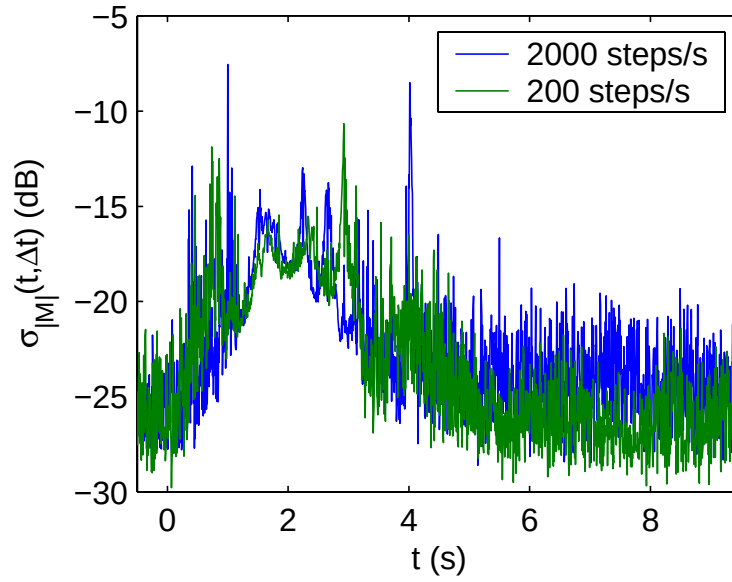


Figure 72: (color online) Standard deviation of modulation coefficient $\sigma_{|M|}(t)$ as a function of time at $f = 1000$ MHz, for selected values of V , with $M = 400$ steps ($N = 100$), $A = 100$ steps/s², $B = 10$ steps/s, $\Delta t = 6.242$ ms.

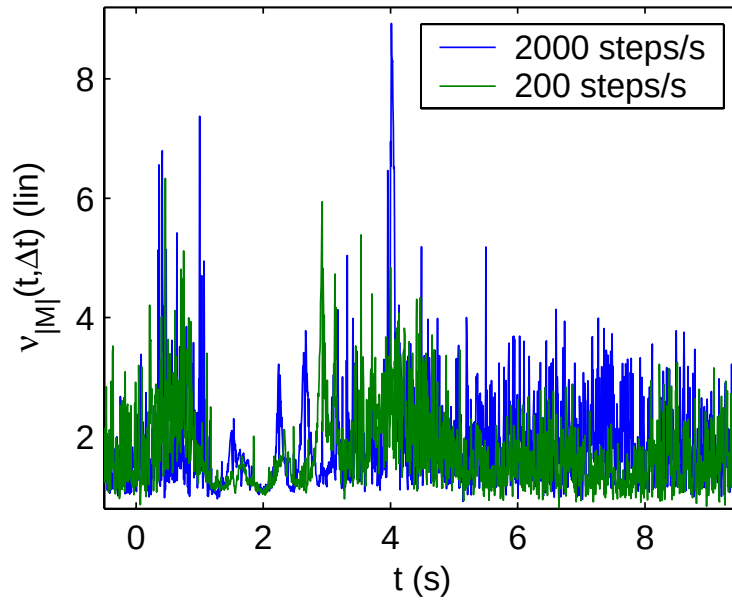


Figure 73: (color online) Coefficient of variation of modulation coefficient $\nu_{|M|}(t)$ as a function of time at $f = 1000$ MHz, for selected values of V , with $M = 400$ steps ($N = 100$), $A = 100$ steps/s², $B = 10$ steps/s, $\Delta t = 6.242$ ms.

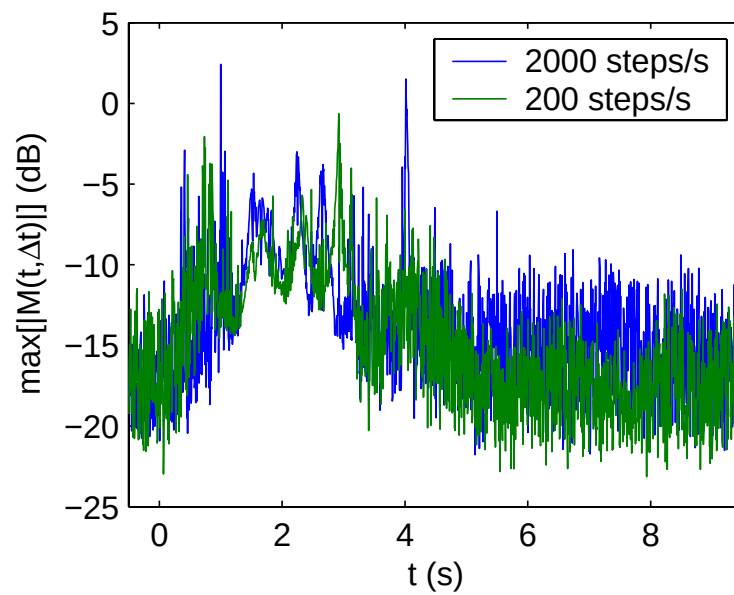


Figure 74: (color online) Maximum modulation coefficient $\max(|M(t)|)$ as a function of time at $f = 1000$ MHz for selected values of V , with $M = 400$ steps ($N = 100$), $A = 100$ steps/s², $B = 10$ steps/s, $\Delta t = 6.242$ ms.

6.7 Temporal evolution of correlation length of stir sequences

Having investigated second-order statistics with respect to time, we now turn attention to second-order statistics with respect to stir state, θ_n . The correlation length of the stir sequences is a quantity of basic interest. As is well known, for steady-state random fields ($t \rightarrow +\infty$), the correlation length ℓ_θ calculated from the autocorrelation function (acf) $\rho_P(\theta)$ for a steady-state stir sequence, i.e.,

$$\rho_P(\Delta\theta) = \frac{\langle [P(\theta + \Delta\theta) - \langle P(\theta + \Delta\theta) \rangle] [P(\theta) - \langle P(\theta) \rangle] \rangle}{\sqrt{\langle [P(\theta + \Delta\theta) - \langle P(\theta + \Delta\theta) \rangle]^2 \rangle} \sqrt{\langle [P(\theta) - \langle P(\theta) \rangle]^2 \rangle}} \quad (15)$$

is instrumental in estimating the number of equivalent statistically independent samples, N_{ind}^{ss} , that the tuner can generate across one full rotation. In this Section, we investigate how ℓ_θ , and hence $N_{\text{ind}}(t)$, evolves in the course of the tuner transitions. The findings will enable us to compare measured maximum-value sample statistics during the transition with their estimated evolutions in Sec. 7.

Numerous definitions for the correlation length exist. Here, we shall compare results for two widely used definitions, viz.,

- the integral metric

$$\ell_\theta(t) \triangleq \int_0^\pi |\rho_P(t; \theta)|^2 d\theta, \quad (16)$$

where the integration of the finite-length discrete (sampled) acf $\rho_P(t; \theta)$ at $t = t_\ell$ is implemented in practice as a Riemann partial summation $\sum_{i=0}^{N/2-1} |\rho_P(t; \theta_i)|^2 \Delta\theta$, i.e., without taking the limit $\Delta\theta \rightarrow 0$. [In (16), the upper limit of the integration is taken as π instead of $+\infty$ because of the finite arc length of a tuner rotation.];

- the threshold metric

$$\ell'_\theta(t) \triangleq \rho_P^{-1}(t; \rho_P = 1/e), \quad (17)$$

representing the tuner angle increment (with reference to the start position θ_0) at which $\rho_P(\theta)$ first crosses $1/e$ downward. Its value is calculated via linear interpolation of the sampled values $\rho_P(\theta_n)$ at the discrete steps n . [In (17), the prime does not represent differentiation.]

Care must be exercised in interpreting the thus obtained $\ell_\theta^{(j)}(t)$. Unlike first-order statistics, which only depend on the magnitude of fluctuations, second-order statistics are also sensitive

to the rate of fluctuation, which may have various origins. In particular, jitter – as analyzed in Sec. 5 – may give rise to an apparent whitening, i.e., increase of the apparent rate of fluctuation within the sequences when they have become temporally mixed¹⁹ This increased apparent rate causes an apparent reduction of their correlation length, whereas the actual reason in this case is, in fact, a loss of (mechanical) synchronization.

6.7.1 Dependence on frequency

Figure 75 shows $\ell_\theta(t)$, in units $\Delta\theta$, as calculated from (16) at selected frequencies. No clear trend with respect to time is observed, although an on-average small decrease during the transitions is suggested at most frequencies. The fluctuations in $\ell_\theta(\tau)$ become smaller at higher frequencies. On the other hand, $\ell'_\theta(t)$ as calculated from (17) and shown in Fig. 76, exhibits somewhat smaller fluctuations in overmoded regime and, in absolute terms, smaller values than $\ell_\theta(t)$.

6.7.2 Dependence on motor parameters

Figure 77 shows $\ell_\theta^{(j)}(t)$ at $f = 1000$ MHz at selected values of A . Both definitions (16) and (17) yield an on-average decreasing $\ell_\theta^{(j)}$ coupled with strong oscillations during the transitions. This suggests an on-average increase of the equivalent number of independent samples $N_{\text{ind}}(t)$ but with nevertheless large variations. In general, the smaller A , the more regular the oscillations become between the steady-state value and another, lower, value. The time instances of the extrema of these oscillations can be traced back more or less to turning points of the paddle wheel, although less clearly so than in the case of first-order statistics. Qualitatively similar results occur when selecting different values of B , as shown in Fig. 78.

For relatively slow motion of the paddle wheel during transitions, Fig. 79 indicates a marked increase of $\ell_\theta^{(j)}(t)$ followed by a decrease; in other words, a compression followed by a dilation of fluctuations within the stir sequences as time evolves. This suggests a resonant-like evolution of the equivalent number of independent samples during transitions. The $\ell_\theta^{(j)}(t)$ appear to go through their steady-state level at the turning point of $\Omega(t)$, i.e., near $t - t_0 = 1.88$ s.

Finally, the effect of halving the tuner step ($M = 200$ motor steps) is shown in Fig. 80. As expected, the smaller step causes higher levels of correlation and, hence, a larger correlation

¹⁹By mixing of sequences, we mean that values $P(t = t_{\ell\pm 1}, \theta \neq \theta_n)$ or $P(t = t_{\ell\pm 2}, \theta \neq \theta_n)$, etc. are being grouped with $P(t = t_\ell, \theta = \theta_n)$ into the ℓ th sequence $\{P(t = t_\ell, \theta_n)\}_{n=0, \dots, N-1}$, as a direct result of variations of t_0 with θ_n .

length. On the other hand, smaller steps yield smaller velocities resulting in smaller fluctuations in $\ell_\theta^{(i)}(t)$.

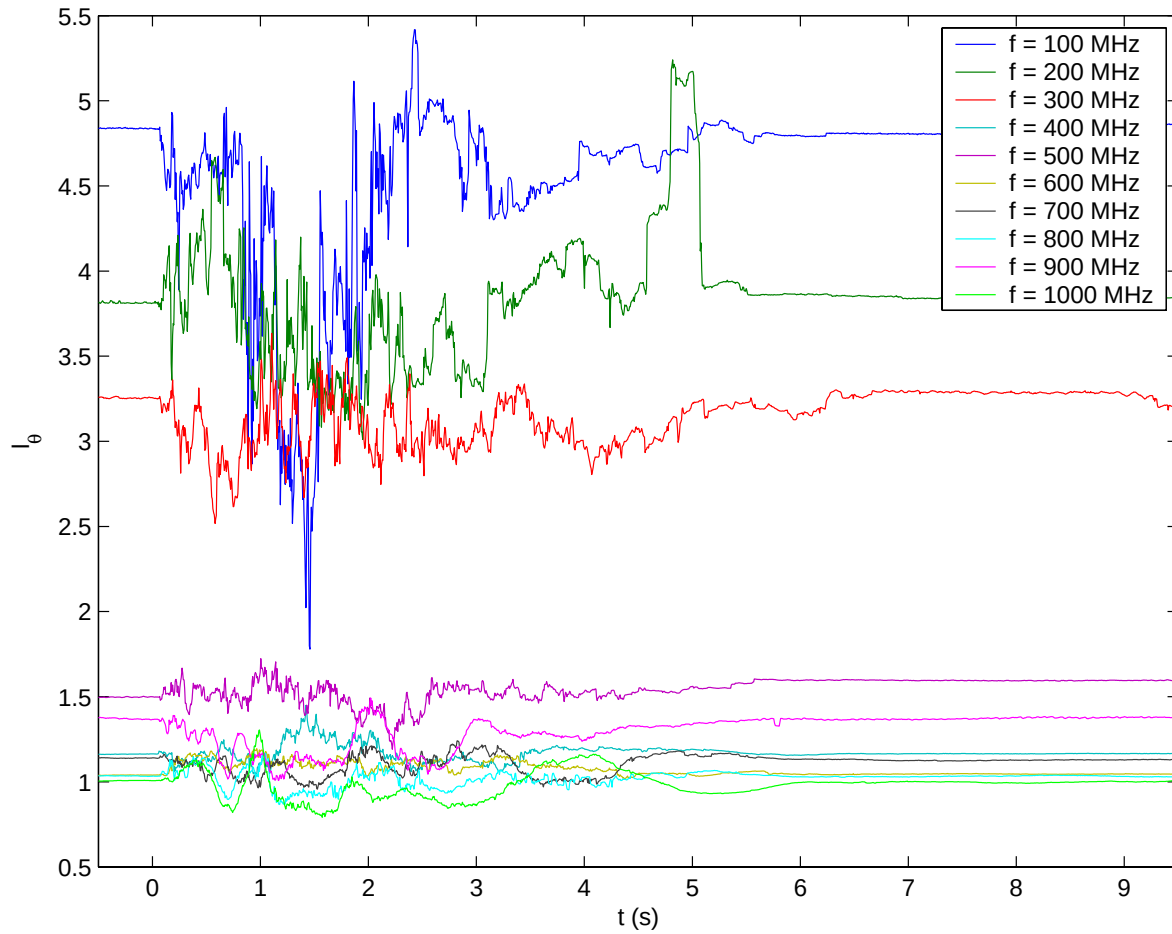


Figure 75: (color online) Correlation length ℓ_θ of stir sequences, in units $\Delta\theta$, as a function of time at selected frequencies, using definition (16), with $M = 400$ steps ($N = 100$), $A = 1000$ steps/s², $B = 600$ steps/s, $V = 2000$ steps/s.

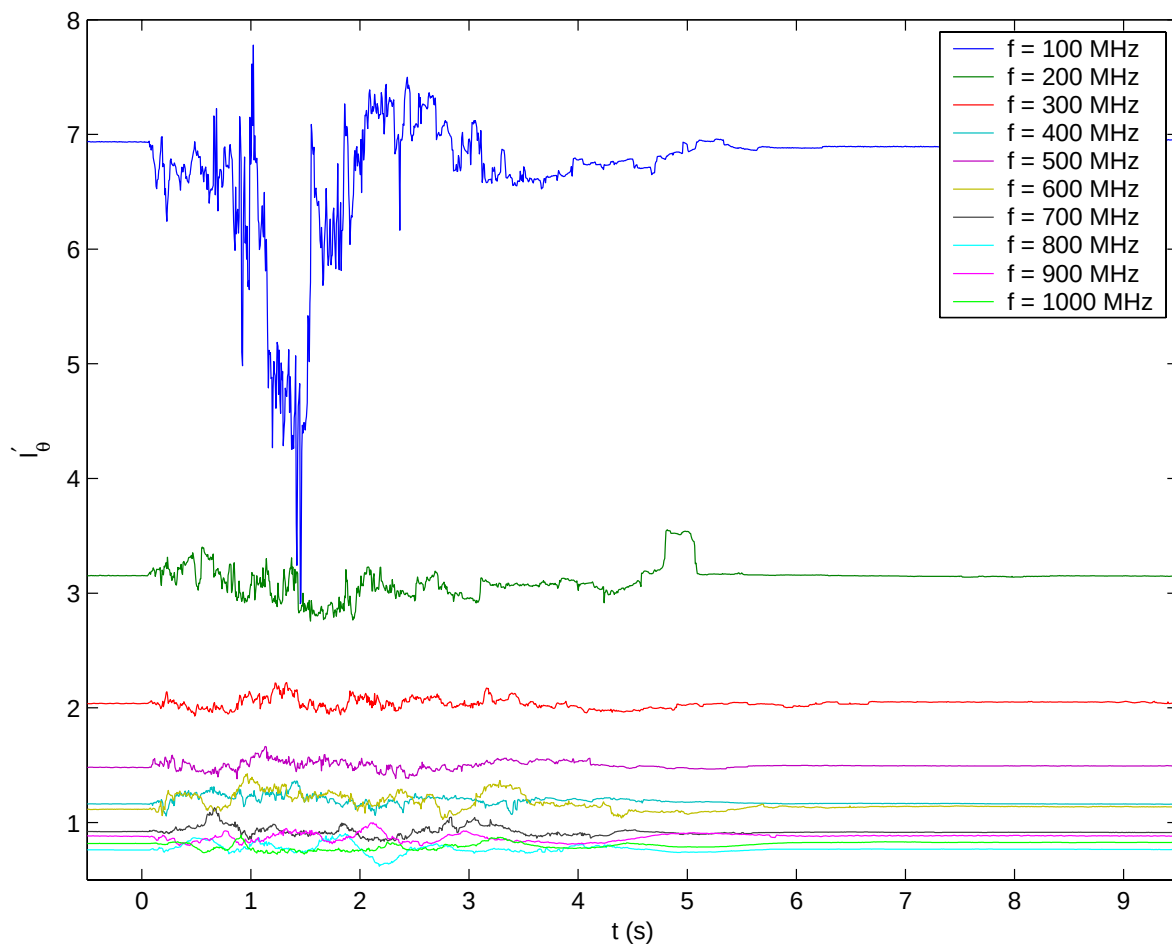


Figure 76: (color online) Correlation length ℓ'_θ of stir sequences, in units $\Delta\theta$, as a function of time at selected frequencies, using definition (17), with $M = 400$ steps ($N = 100$), $A = 1000$ steps/s², $B = 600$ steps/s, $V = 2000$ steps/s.

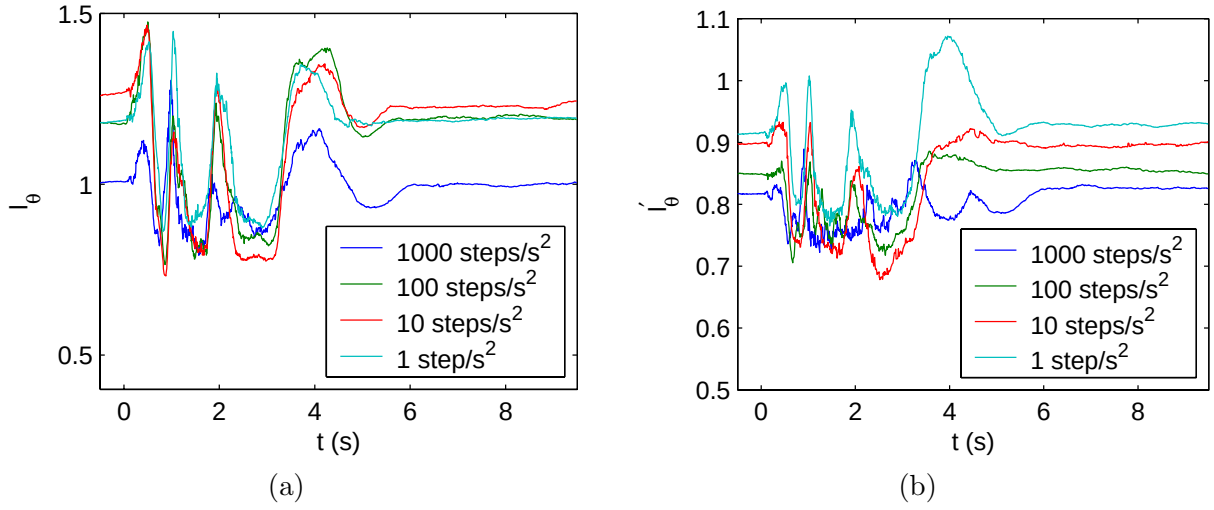


Figure 77: (color online) Correlation lengths $\ell_\theta^{(i)}$ of stir sequences as a function of time at $f = 1000$ MHz, for selected values of A , with $M = 400$ steps ($N = 100$), $B = 600$ steps/s, $V = 2000$ steps/s: (a) for $\ell_\theta(t)$ defined by (16); (b) for $\ell'_\theta(t)$ defined by (17).

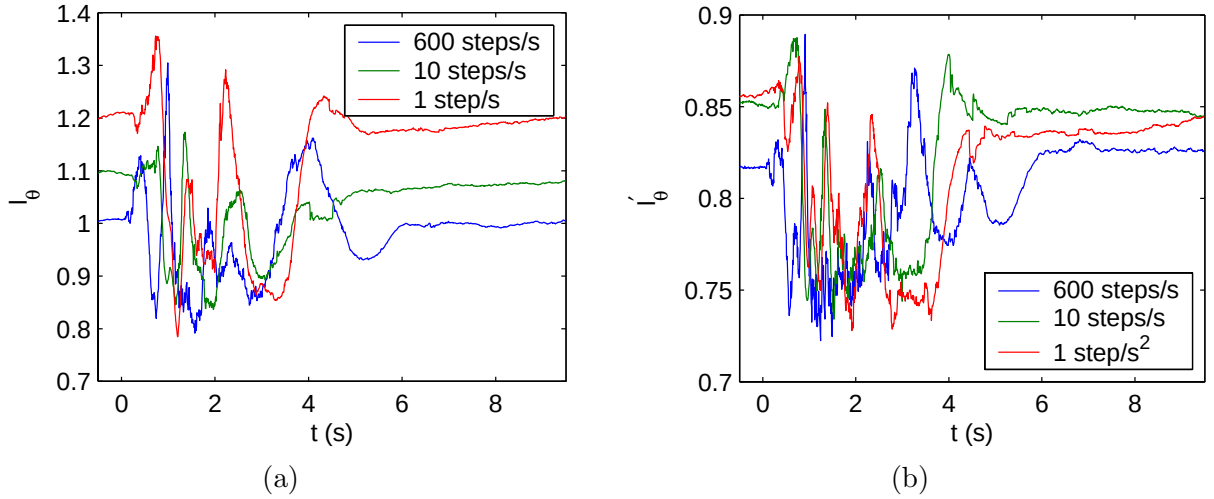


Figure 78: (color online) Correlation lengths $\ell_\theta^{(i)}$ of stir sequences as a function of time at $f = 1000$ MHz, for selected values of B , with $M = 400$ steps ($N = 100$), $A = 1000$ steps/s², $V = 2000$ steps/s: (a) for $\ell_\theta(t)$ defined by (16); (b) for $\ell'_\theta(t)$ defined by (17).

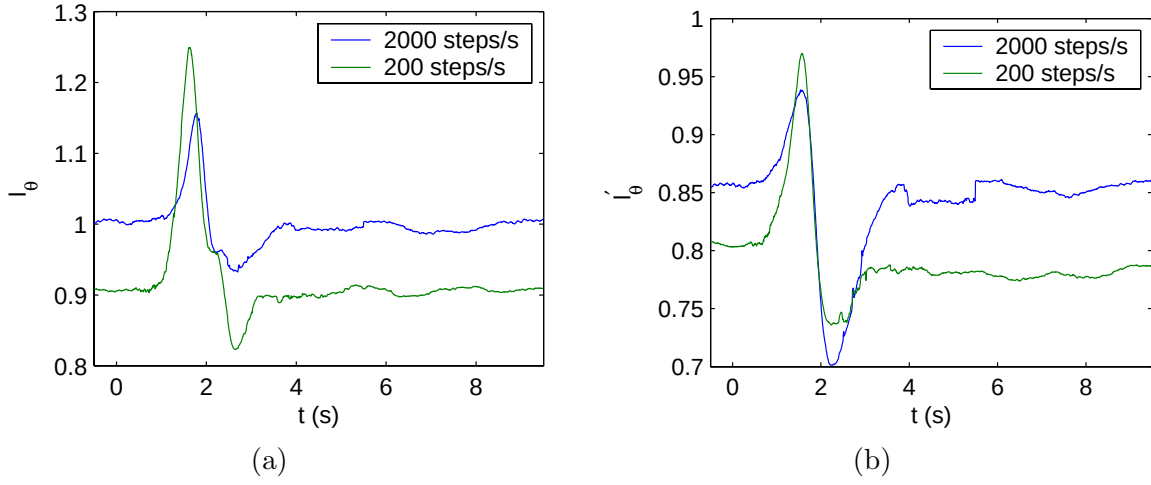


Figure 79: (color online) Correlation lengths $\ell_\theta^{(i)}$ of stir sequences as a function of time at $f = 1000$ MHz, for selected values of V , with $M = 400$ steps ($N = 100$), $A = 100$ steps/s², $B = 10$ steps/s: (a) for $\ell_\theta(t)$ defined by (16); (b) for $\ell'_\theta(t)$ defined by (17).

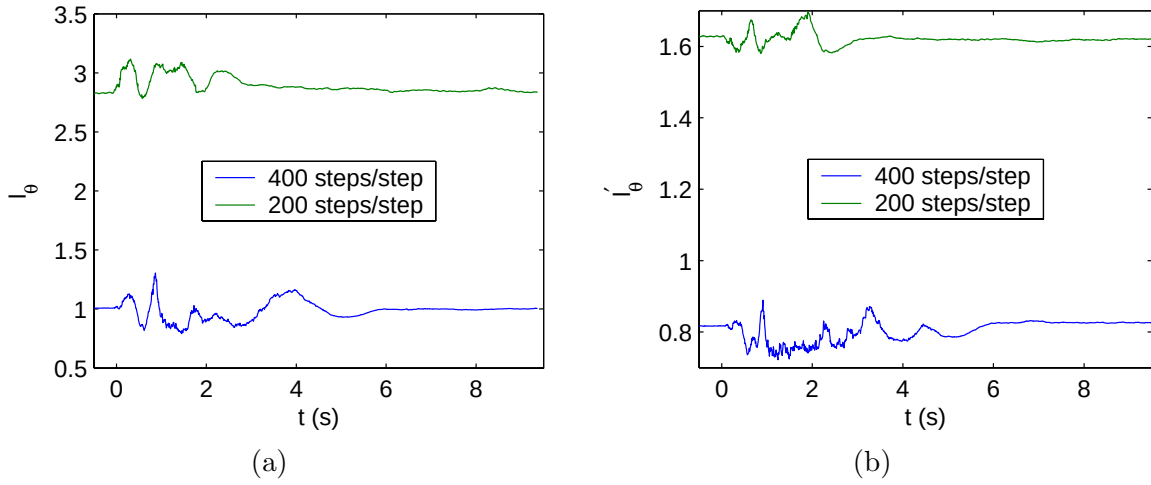


Figure 80: (color online) Correlation lengths $\ell_\theta^{(i)}$ of stir sequences as a function of time at $f = 1000$ MHz, for selected values of M , with $A = 1000$ steps/s², $B = 600$ steps/s, $V = 2000$ steps/s: (a) for $\ell_\theta(t)$ defined by (16); (b) for $\ell'_\theta(t)$ defined by (17).

7 Comparison of measured vs. estimated evolution of the maximum energy density

7.1 Theoretical, semi-empirical, and fully-empirical estimates

Having obtained experimental values for the autocorrelation length $\ell_\theta^{(\iota)}(t)$ of $P(t)$, we are now in a position to estimate the maximum-to-mean ratio $\text{Max}[P(t)]/\mu_P(t)$ as a sample statistic, and compare with its directly measured values, $\text{max}[P(t)]/\mu_P(t)$, during the transients. From the theory of extreme value distributions, it follows that this estimation requires knowledge of the number of independent samples $N_{\text{ind}}^{(\iota)}(t)$, which can be estimated from

$$N_{\text{ind}}^{(\iota)}(t) \simeq \frac{N}{\ell_\theta^{(\iota)}(t)} \quad (18)$$

based on the definitions (16) or (17). In view of the weak global²⁰ changes of $F_P(p)$ observed during its evolution $F_P(p, t)$ (cf. Sec. 6.3) we can assume that, to first approximation, the *type* of the underlying cdf is sufficiently close to the ideal χ_2^2 cdf (except at 100 MHz) and remains invariant during the stepping transitions, while only the value of its (single) distribution parameter changes in time. Therefore, we approximate the underlying cdf $F_P(p, t)$ at all times by $F_P(p, t) = 1 - \exp[-p(t)/\mu_P(t)]$ valid for a quasi-stationary ideal random field but with time-varying mean value $\mu_P(t)$ [or $\sigma_P(t)$]. The expected value $\langle \text{Max}[P(t)]/\mu_P(t) \rangle$ and standard deviation $\sigma_{\text{Max}(P)/\mu_P}(t)$ of $\text{Max}[P(t)]/\mu_P(t)$ are then estimated as²¹

$$\left\langle \frac{\text{Max}[P(t)]}{\mu_P(t)} \right\rangle \simeq \gamma + \ln \left[N_{\text{ind}}^{(\iota)}(t) + 1 \right] - \frac{1}{2 \left[N_{\text{ind}}^{(\iota)}(t) + 1 \right]} \quad (19)$$

$$\sigma_{\text{Max}(P)/\mu_P}(t) \simeq \sqrt{\frac{\pi^2}{6} - \frac{1}{N_{\text{ind}}^{(\iota)}(t) + 1}} \simeq 1.2825 \left[1 - \frac{0.3040}{N_{\text{ind}}^{(\iota)}(t) + 1} \right] \quad (20)$$

where $\gamma = 0.57722\dots$ is the Euler–Mascheroni constant. We shall further denote this method as the *semi-empirical estimation* of $\text{Max}[P(t)]/\mu_P(t)$, because the estimation is based on the theoretical $F_P(p)$ but measured $\mu_P(t)$ and measured $N_{\text{ind}}^{(\iota)}(t)$.

While the use of the χ_2^2 cdf serves as a reasonable first approximation, it should be remembered that any small discrepancy between the upper tails of the ideal theoretical χ_2^2 distribution and the actual experimental (i.e., sample) underlying distribution (Fig. 51) are being amplified

²⁰i.e., when considered across the full range of $P(t)$

²¹For lower values of $N_{\text{ind}}^{(\iota)}(t)$, the expression (19) yields somewhat more accurate values than $\gamma + \ln[N_{\text{ind}}^{(\iota)}(t)] + 1/[2N_{\text{ind}}^{(\iota)}(t)]$.

in the calculation of the distribution of $\text{Max}[P(t)]$, because of the typically large value of $N_{\text{ind}}^{(\iota)}(t)$ appearing as an exponent in

$$f_{\text{Max}(P)/\mu_P}(\text{max}(P)/\mu_P, t) = N_{\text{ind}}^{(\iota)}(t) [F_P(\text{max}(P)/\mu_P, t)]^{N_{\text{ind}}^{(\iota)}(t)-1} f_P(\text{max}(P)/\mu_P, t) \quad (21)$$

$$F_{\text{Max}(P)/\mu_P}(\text{max}(P)/\mu_P, t) = [F_P(\text{max}(P)/\mu_P, t)]^{N_{\text{ind}}^{(\iota)}(t)}. \quad (22)$$

To better account for these discrepancies, one may instead base the estimation on the *experimental* instantaneous cdf $F_P(p, t)$. We shall further denote this alternative method as the *fully-empirical estimation* of $\text{Max}[P(t)]/\mu_P(t)$, signifying that estimation is then based on the experimental instantaneous $F_P(p, t)$ (both in terms of type and parameters) and the experimental instantaneous $N_{\text{ind}}^{(\iota)}(t)$. The associated time-varying median value can be scaled by the mean-to-median ratio of the ideal distribution of the maximum-to-mean ratio for a χ_2^2 -distributed $P(t)$, i.e.,

$$\left\langle \frac{\text{Max}[P(t)]}{\mu_P(t)} \right\rangle \simeq F_{\text{Max}(P)/\mu_P}^{-1}(1/2, t) \times \frac{\text{mean} \left\{ \text{Max} \left[P_{\chi_2^2}(t) \right] / \mu_{P_{\chi_2^2}}(t) \right\}}{\text{median} \left\{ \text{Max} \left[P_{\chi_2^2}(t) \right] / \mu_{P_{\chi_2^2}}(t) \right\}}, \quad (23)$$

in which $P_{\chi_2^2}(t)$ represents an ideal χ_2^2 -distributed $P(t)$ and where²²

$$\frac{\text{mean} \left\{ \text{Max} \left[P_{\chi_2^2}(t) \right] / \mu_{P_{\chi_2^2}}(t) \right\}}{\text{median} \left\{ \text{Max} \left[P_{\chi_2^2}(t) \right] / \mu_{P_{\chi_2^2}}(t) \right\}} \simeq \frac{\gamma + \ln \left[N_{\text{ind}}^{(\iota)}(t) \right]}{-\ln [\ln(2)] + \ln \left[N_{\text{ind}}^{(\iota)}(t) \right]} = 1 + \frac{0.2107}{0.3665 + \ln \left[N_{\text{ind}}^{(\iota)}(t) \right]}. \quad (24)$$

The thus obtained (23) serves as an estimate for the fully-empirical mean value $\langle \text{Max}[P(t)]/\mu_P(t) \rangle$, instead of the semi-empirically estimated mean value (19). For $10 \leq N_{\text{ind}}^{(\iota)}(t) \leq 10\,000$, the mean-to-median ratio (24) ranges between 1.079 and 1.022, i.e., a relatively small adjustment. The experimental median $F_{\text{Max}(P)/\mu_P}^{-1}(1/2, t)$ can be obtained by linear interpolation on the empirical $F_{\text{Max}(P)/\mu_P}(\text{max}(P)/\mu_P, t)$. The rationale for introducing the median value here is that – unlike the mean value – it does not require deducing and approximating the experimental pdf $f_P[p(t)]$ by numerically differencing $F_P[p(t)]$ (or a curve fitted to it) – whether for a Fourier or Fourier–Stieltjes integral expression for the mean value – which would introduce a numerical inaccuracy. As a bonus, the median – unlike the mean – is a robust statistic, i.e., its sample value is less sensitive to outliers and to limited sample size, thus providing a more stable measure of centrality. This is especially important because the number of sample points (stationary tuner

²²Here, the median is approximated using $F_{\text{Max}(P)/\mu_P}(x, t) = [1 - \exp(-x)]^{N_{\text{ind}}^{(\iota)}(t)} = 1/2$ as $x \simeq -\ln \left[\ln(2)/N_{\text{ind}}^{(\iota)}(t) \right]$ for $x \gg 1$.

steps) that are used to generate these cdfs is relatively small in our experiments ($N = 100$ or 200).

For completeness, a purely *theoretical estimate* of $\langle \text{Max}[P(t)]/\mu_P(t) \rangle$ can be obtained by using a theoretical value for $N_{\text{ind}}^{(\prime)}(t)$ in (19) (or in a corresponding expression for any other assumed theoretical pdf instead of χ_2^2). To this end, one may use the estimate [23]

$$N_{\text{ind}}^{(\prime)} = \frac{V}{(\lambda/2)^3} - \frac{L_1 + L_2 + L_3}{\lambda} \quad (25)$$

for a rectangular empty cavity of volume $V = L_1 L_2 L_3$ at wavelength λ . Typically, however, (25) would considerably overestimate the actual value of $N_{\text{ind}}^{(\prime)}(t)$. Also, this estimate represents an average value (based on the spectrally averaged mode density, as expressed by the generalized Weyl formula) that does not account for any dynamic mode bunching/dilation effects occurring during transients (cf. *infra*). Similar difficulties exist with alternative independent theoretical estimates, e.g., based on values of $\ell_\theta^{(\prime)}$ obtained from an analytic model for the stir-domain ensemble correlation function for quasi-stationary fields [24] and used with (18).

7.2 Dependence on frequency

In Figs. 81–90, semi- and fully-empirical estimates of the average and median values of maximum-to-mean ratio $\text{Max}[P(t)]/\mu_P(t)$ are compared with the directly measured experimental values $\text{max}[P(t)]/\mu_P(t)$, for frequencies between 100 and 1000 MHz. The values of the motor parameters are $M = 400$ steps ($N = 100$), $A = 1000$ steps/s², $B = 600$ steps/s, $V = 2000$ steps/s at all frequencies. Estimates of $N'_{\text{ind}}(t)$ are based on the threshold metric (17) and are frequency dependent. The results show that fully-empirical estimates are nearly always much closer than semi-empirical ones to the measured values, both qualitatively and quantitatively. This indicates the paramount importance of experimentally characterizing and using the sample cdf $F_P(p, t)$ and its time variations when estimating maximum-value statistics. The *direct* effect of the variations of $N'_{\text{ind}}(t)$ on the estimates is (very) small compared to the effect of variations of $F_P(p, t)$ (compare dot symbols with long-dashed lines in the Figures). In fact, one may safely replace $N_{\text{ind}}^{(\prime)}(t)$ in (21)–(22) by the steady-state value $N_{\text{ind}}^{ss(\prime)} \equiv N_{\text{ind}}^{(\prime)}(t < t_0)$ without much effect. However, the temporal rate of fluctuation of $\text{max}[P(t)]/\mu_P(t)$ is seen to be similar (if not identical) to that of $N'_{\text{ind}}(t)$, and this time scale becomes larger with increasing frequency²³, yielding progressively smoother fluctuations of $\langle \text{Max}[P(t)]/\mu_P(t) \rangle$. This correspondence suggests that $N'_{\text{ind}}(t)$

²³Resolution at the lowest frequencies is hardware limited by the relatively large sample time interval Δt .

affects the empirical $F_P(p, t)$ indirectly, thus providing a connection between the number of independent samples (sample distribution characteristic) and the number of degrees of freedom (ensemble distribution characteristic). Physically, this suggests that the transient fluctuations of the maximum-to-mean ratio are caused – at least in part – by variations in the number of instantaneously participating modes (bunching and dilation), manifesting itself in variations of $N'_{\text{ind}}(t)$ (cf. dotted lines in the Figures).

Above 500 MHz, the measured quasi-oscillations of the transient $\langle \text{Max}[P(t)]/\mu_P(t) \rangle$ are markedly larger than (but in phase with) the oscillations of the fully-empirical estimates. The maximum maximum values during the transients are always larger – often significantly so (up to²⁴ 50%) – than the steady-state (quasi-stationary) values of the maximum-to-mean ratio, particularly at the lower and the higher frequencies investigated here.

Although good overall agreement between measured and fully-empirical estimates is obtained, several significant features of the measured $\text{max}[P(t)]/\mu_P(t)$ remain unexplained. For example, whereas the large-scale changes in $N'_{\text{ind}}(t)$ are, on average, in phase with those of the measured $\text{max}[P(t)]/\mu_P(t)$ at the lower and higher frequencies (compare dotted lines vs. black solid line in the Figures), the changes in $N'_{\text{ind}}(t)$ and $F_P(p, t)$ are in distinctly opposite phase at intermediate frequencies 400 MHz and, especially, 500 MHz (Figs. 84 and 85). Also, the transient at $f = 1000$ MHz does not appear to follow the trend of increased smoothing with increasing frequency, on average, despite all other parameters remaining equal. Finally, the quantitative difference between the local maxima of $\text{max}[P(t)]/\mu_P(t)$ and those of the fully-empirical $\langle \text{Max}[P(t)]/\mu_P(t) \rangle$ are often large, despite good qualitative agreement (e.g., Figs. 87 and 89).

7.3 Dependence on tuner step size

Different angular step sizes for the mode tuner give rise to different velocity profiles and different maximum speeds. This may affect the transient field and its maximum amplitude. In order to investigate this, Figs. 91–93 show values for $M = 1200$ steps ($N = 33.3$) at $f = 100, 200$ and 300 MHz, respectively, i.e., a threefold increase of tuner step size compared to $M = 400$ steps as shown in Figs. 81–83, respectively. In addition, Fig. 94 shows values for $M = 200$ steps ($N = 200$) at $f = 1000$ MHz, i.e., a halved step size compared to $M = 400$ steps in Fig.

²⁴Note that, since our chosen sampling interval Δt is much larger than the chamber relaxation time τ_{ch} , it is likely that even higher maximum maximum transient values occur in reality.

90. To some extent, rotations over larger angles resemble mode-stirred operation more closely, although it should be remembered that the instantaneous speed remains nonuniform (triangular velocity-time profile) and, also, the value of the maximum speed is different (congruent triangular profiles); cf. Tbl 1. Comparisons suggest that, at relatively low frequencies, the larger M the more faithful the fully-empirical prediction of the transient maximum-to-mean ratio. At $f = 1000$ MHz (Fig. 94), the time scale of fluctuation is larger because of the smaller maximum speed and – despite the proportionally larger effect of accelerations due to starting/stopping – remains qualitatively accurately predictable for $t_0 < t < t_0 + 2.5$ s.

7.4 Dependence on choice of definition of correlation length

Figure 94 also compares estimates based on $N_{\text{ind}}(t)$ as defined by (16) with estimates based on $N'_{\text{ind}}(t)$ defined by (17). The differences are seen to be relatively small and to affect mainly the semi-empirical estimates, as expected. Comparisons at other frequencies and parameter values (not shown) lead to similar conclusions.

7.5 Dependence on motor parameter values

In Figs. 95–98, semi-empirical estimates of $\langle \text{Max}[P(t)]/\mu_P(t) \rangle$ are compared with directly measured values, for different choices of motor parameter values. In Figs. 99–105, measured values are compared with both semi- and fully-empirical estimates for these parameter values. In the following discussion, we focus on the influence of A (Fig. 95), but results and observations for the other parameterizations are similar.

Both the semi-empirically estimated and measured values of $\text{Max}[P(t)]/\mu_P(t)$ suggest that, in the majority of cases, its values during the transitions are predominantly well above the steady-state level, in terms of both duration (excursion length) and amplitude. Recall from the theory of extreme value statistics that, for a chosen definition of correlation length, the difference between measured and estimated steady-state levels of $\text{Max}(P)/\mu_P$ is due to the inherent statistical uncertainty (spread) of $\text{Max}(P)/\mu_P$ as a sample statistic: repeated tuner sweep realizations and measurements at different antenna locations and orientations yield different sampled maximum-to-mean ratios, causing *irregular (random)* fluctuations between the steady-state levels and, consequently, during transients. The range of these fluctuations is a slowly decreasing function of the tuner sample size ($N = 100$ or 200 in our experiments), as indicated by (20). Since this difference is inherent to sample statistics, however – even when the underlying ensemble pdf

and $N_{\text{ind}}^{(l)}(t)$ were known with perfect accuracy – it is of no concern in the analysis of transients of $\text{Max}[P(t)]/\mu_P(t)$: the graphs can be simply shifted upward or downward in order to match estimated and measured steady-state levels with each other. Instead, as before, we focus on the qualitative and quantitative transient fluctuations of $\text{Max}[P(t)]/\mu_P(t)$ relative to such (shifted) steady-state levels.

In all cases shown in Figs. 95–105, there is again good qualitative agreement with the measured $\text{max}[P(t)]/\mu_P(t)$ [i.e., fluctuations taking place in the same sense (up/down) and on the same time scale (rate of fluctuation)]. Detailed inspection shows that the maximum maximorum can be more accurately estimated if A and/or B are relatively small. This is not surprising, because one expects a deterministic (physical) basis for the experimentally observed fluctuations of the maxima: when the mode tuner is rotated arbitrarily slowly with uniform speed, the transients necessarily disappear. The fluctuations of $\text{max}[P(t)]/\mu_P(t)$ are at least partially due to temporal fluctuations of: (a) $N_{\text{ind}}^{(l)}$ during the transitions, as indicated by (18)–(19) and Figs. 75–80, or (b) the sample distributions with respect to $f_P(p, t)$, e.g., as a manifestation of the underlying number of degrees of freedom M_n of the underlying non-ideal pdf $f_P(p, t)$.

Figures 97 and 104–105 show results for lower-speed rotations ($A = 100$ steps/s², $B = 10$ steps/s). The now reduced variability of the measured maximum-to-mean ratio allows for an even better overall correlation with the fully-empirical estimates. Nevertheless, certain discrepancies that are particularly poignant remain unexplained. For example, there is an apparent temporal *shift* of about 0.3 s in Fig. 105 between measured and fully-empirically estimated values. From an alternative point of view, this discrepancy represents an *opposing* trend between measured vs. estimated curves that occurs at the two turning points bounding the main lobe in Fig. 105. The measured main lobe, centered at $t = 1.65$ s, is *large and smooth (deterministic)* and coincides timewise with a reduction of the values of $N'_{\text{ind}}(t)$, also in Fig. 104 (compare semi-empirical and measured transients for $t_0 + 1.329$ s $< t < t_0 + 1.928$ s in the Figs. 104 and 105) but remains poorly predictable: while changes in $N'_{\text{ind}}(t)$ predict a relatively small negative effect, the measured effect is instead large and positive. Whereas $\rho_P(\Delta t)$ in Fig. 18 did not suggest any peculiar behaviour in the tuner motion during this interval, the mean value of $P(t)$, its average rate of fluctuation, and modulation coefficient all exhibit large *positive* increases during this interval (cf. Figs. 23, 65 and 71). The most remarkable correspondence for these lobes at low speeds is with $\ell'_\theta(t)$, which shows a resonant *increase* during this interval (Fig. 79). Thus, fully-empirical estimation based on $N'_{\text{ind}}(t) = N_{\text{ind}}^{ss'}$ does not fully resolve the discrepancies within

this interval, even at these relatively low acceleration and speed.

To investigate the origins of this anomalous lobe of Fig. 104 further, the evolution of the empirical cdf during its epoch is shown in Fig. 106(a). The Figure shows the cdfs at three characteristic points of the lobe for $\max[P(t)]/\mu_P(t)$, i.e., at the two local minima ($t = t_0 + 1.329$ s and $t = t_0 + 1.928$ s) and at the maximum ($t = t_0 + 1.647$ s). Around $t = t_0 + 1.647$ s, the values of the sample maximum are unusually large (outliers), and staircaising (“crooking”) of the cdf occurs in the heavy upper tail, indicating clustering of the high values of $p(t)/\langle P(t) \rangle$. Such features are largely absent at other times during the complete transient, in particular at $t_0 + 1.329$ s and $t_0 + 1.928$ s for which the cdf indicates a deficiency of large values. To demonstrate this more clearly, Fig. 106(b) shows the corresponding complementary cdf $1 - F_P(p, t)$. As a function of time, the line segment connecting the largest two sample values in Fig. 106(b) changes in slope in a continuous manner, from the blue, over the red, back to the green curve, with the “hook” remaining centered at $p(t)/\langle P(t) \rangle \simeq 3$ across the mean lobe. The complete evolution of the cdf for this parameter set was already shown in Fig. 50(c). It is worth comparing these observations for the measured $\max[P(t)]/\mu_P(t)$ with corresponding ones for special points on the *estimated* $\langle \text{Max}[P(t)]/\mu_P(t) \rangle$. Here, the lobe appears to be a compressed and damped version of the measured version, with local minima now at $t = t_0 + 1.329$ s and $t = t_0 + 1.679$ s, and a weaker local maximum now at $t = t_0 + 1.498$ s. The results are shown in Fig. 107. Clearly, the fully-empirical estimation does not keep sufficiently well track of the changes of the tail of the empirical cdf with regard to characterizing the anomalous lobe. Similar forms of the cdf were also²⁵ observed for $A = 100$ steps/s², $B = 10$ steps/s, $V = 200$ steps/s (not shown).

The characteristic forms of the cdf at the local minima and maxima of $\max[P(t)]/\mu_P(t)$ can also be clearly observed for higher speeds and accelerations. As an example example, Fig. 108 shows the corresponding plots at $f = 900$ MHz. The cdfs associated with the local maxima (red curves) resemble those for the undermoded regime (“flat” cdfs; cf. Fig. 35), whereas those for the local minima (blue curves) show “steep” cdfs.

In summary, nonstationarity manifests itself by anomalous discrepancies between measured and fully-empirical estimates of the instantaneous cdf during certain time intervals. It results in an outlier-type maximum value and a crooked empirical $F_P(p, t)$ [heavy-tailed $f_P(p, t)$] near local maxima, while yielding curtailed $f_P(p, t)$ near local minima. Additional measurements

²⁵In this case, crooking of the cdf and clustering of its tail values occur for $t_0 + 1.260$ s $\leq t \leq t_0 + 1.835$ s, with a maximum at $t = t_0 + 1.579$ s.

with larger values of N , smaller Δt , and smaller A and B are needed for characterizing the cdf more accurately during the nonstationary period.

The precise physical reason(s) and quantification of the discrepancies, even at relatively low speeds, remain an open question at the moment and require further investigation. It is possible that the basis on which way $N_{\text{ind}}(t)$ is estimated from $\rho_P(t, \Delta\theta)$ during transients may require revision for transients, as a result of the mixing of “previous” with “present” cavity modes during transitions caused by nonzero reverberation times (mode bunching/dilation). Another possible reason is that the simple estimation of $\text{Max}[P(t)]/\mu_P(t)$ from $P(t)$ at any t based on (21)–(22) may no longer hold during transitions because of nonstationarity, on the basis that the underlying random-phaser model for the random field is then no longer valid because the phasors become time-variant (specifically, nonstationarity affects the coefficients of the Fokker–Planck equation governing the instantaneous pdf and causes “slipping” in the stirred field [16]). Physically, nonstationarity (time-variance) as the result of the moving walls plus fixed antenna is electromagnetically indistinguishable from the scenario of a fixed wall and moving antenna, both causing a Doppler shift and resulting in a purely sinusoidal excitation being converted into an oscillatory process [25], i.e., a sinusoid at the original frequency but with time-varying amplitude (at an average rate of half the Doppler frequency). Nonstationarity as a source of discrepancies for estimates of $\text{max}[P(t)]/\mu_P(t)$, if confirmed, would not be altogether unexpected, because of known results for the closely related problem of threshold crossing statistics of nonstationary vs. stationary Rayleigh processes (cf., e.g., [26, Sec. 4(ii)]). Even when quasi-stationarity still holds, the sample cdfs exhibit fluctuations around the ensemble cdf (cf. Fig. 51), which increases the overall uncertainty of $\text{Max}[P(t)]/\mu_P(t)$ within which the main lobe falls.

The close overall correspondence between measured and fully-empirically estimated values is appreciated by remembering that estimating the median from $F_{\text{Max}(P)/\mu_P}(\text{max}[P(t)]/\mu_P(t), t)$ involves linear interpolation between the last two tail points of $F_P(p, t)$. At these points, the increase of $F_{\text{Max}(P)/\mu_P}(\text{max}[P(t)]/\mu_P(t), t)$ to 1 is sharp because $N_{\text{ind}}^{(t)}(t)$ is large, whence the accuracy of the estimation of the median value is necessarily limited.

To summarize, the primary reason for the measured variations the maximum-to-mean values during the stepping transition is the temporal variation of the underlying distribution of $P(t) \equiv |S_{21}(t)|^2$. This suggests that a two-parameter distribution with adaptive tail (e.g., Bessel-K, powered gamma, etc., distribution [27]) but with a time-varying number of degrees of freedom M_n is feasible in order to accurately characterize and estimate the temporal vari-

ations of the maximum value at steady-state and during stepping transitions. Nevertheless, the sample variability of the sample distribution of the maximum value has a dominant effect. Certain measured features, in particular the main lobe observed in tuner transitions for low rotation speeds and accelerations and the opposite tendencies (turning) at its bounding points, remain unexplained with the available data, methods of analysis for quasi-stationary fields, and uncertainties due to limited sample size (tuner steps).

Finally, we discuss the extent to which any jitter may influence the rate of fluctuation of $\max[P(t)]/\mu_P(t)$. Mixing of stir sequences at an arbitrary t_ℓ caused by jitter renders these sequences to become always more randomized, i.e., yielding an apparent decrease of the correlation length at any t_ℓ . From (19), such a decrease is expected to give rise to an increase of the estimated $N_{\text{ind}}^{(\prime)}(t_\ell)$ and, therefore, an increase of the local maxima of the estimated $\max[P(t)]/\mu_P(t)$. Thus, this yields large oscillations of the estimated ratio. The fan-out diagrams [e.g., Fig. 11(a)] suggest that the effect of any mechanical jitter occurs on a time scale of the order of tens of ms at most. This time scale is much smaller than the quasi-period of the undulations in Fig. 95. On the other hand, mixing of sequences as a result of jitter gives rise to “smearing” in time (temporal equalization) of the correlation functions and, hence, of $\ell_\theta^{(\prime)}(t)$, thus giving rise to less faithful tracking of the temporal fluctuations of the maximum-to-mean ratio.

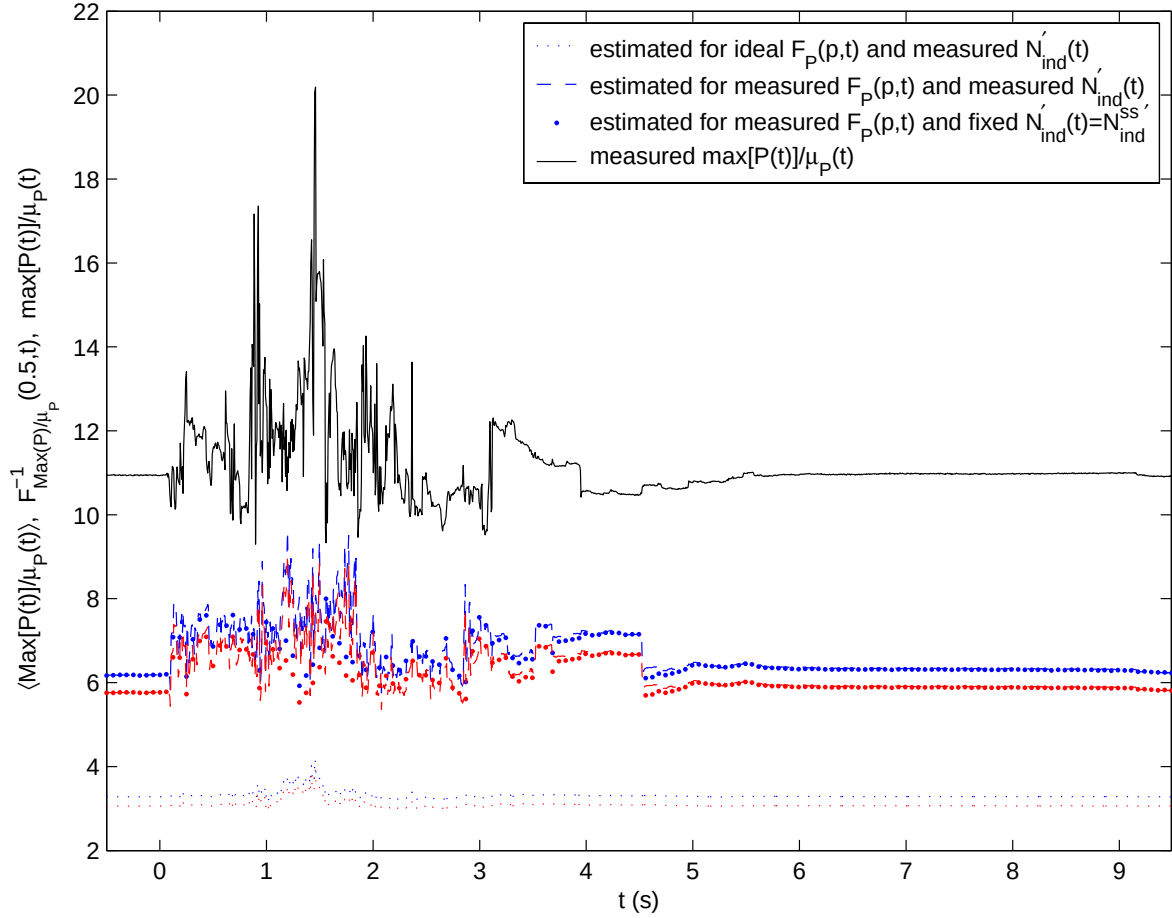


Figure 81: (color online) Comparison of measured vs. estimated transient maximum-to-mean ratio of P as a function of time at $f = 100$ MHz, with $M = 400$ steps ($N = 100$), $A = 1000$ steps/s², $B = 600$ steps/s, $V = 2000$ steps/s. Dotted lines: semi-empirical estimates based on ideal χ_2^2 cdf $F_P(p)$, measured $\mu_P(t)$, and measured $N'_{\text{ind}}(t)$; dashed lines: fully-empirical estimates based on fluctuating measured cdfs $F_P(p, t)$ and fluctuating measured $N'_{\text{ind}}(t)$; dot symbols: fully-empirical estimates based on fluctuating measured cdfs $F_P(p, t)$ and fixed measured $N_{\text{ind}}^{(l)}(t) = N_{\text{ind}}^{\text{ss}l}$. Black solid curve: measured values $\text{max}[P(t)]/\mu_P(t)$; blue curves: estimated mean values $\langle \text{Max}[P(t)]/\mu_P(t) \rangle$; red curves: estimated median values $F_{\text{Max}[P(t)]/\mu_P(t)}^{-1}(1/2, t)$. Estimates of $N'_{\text{ind}}(t)$ are based on (17).

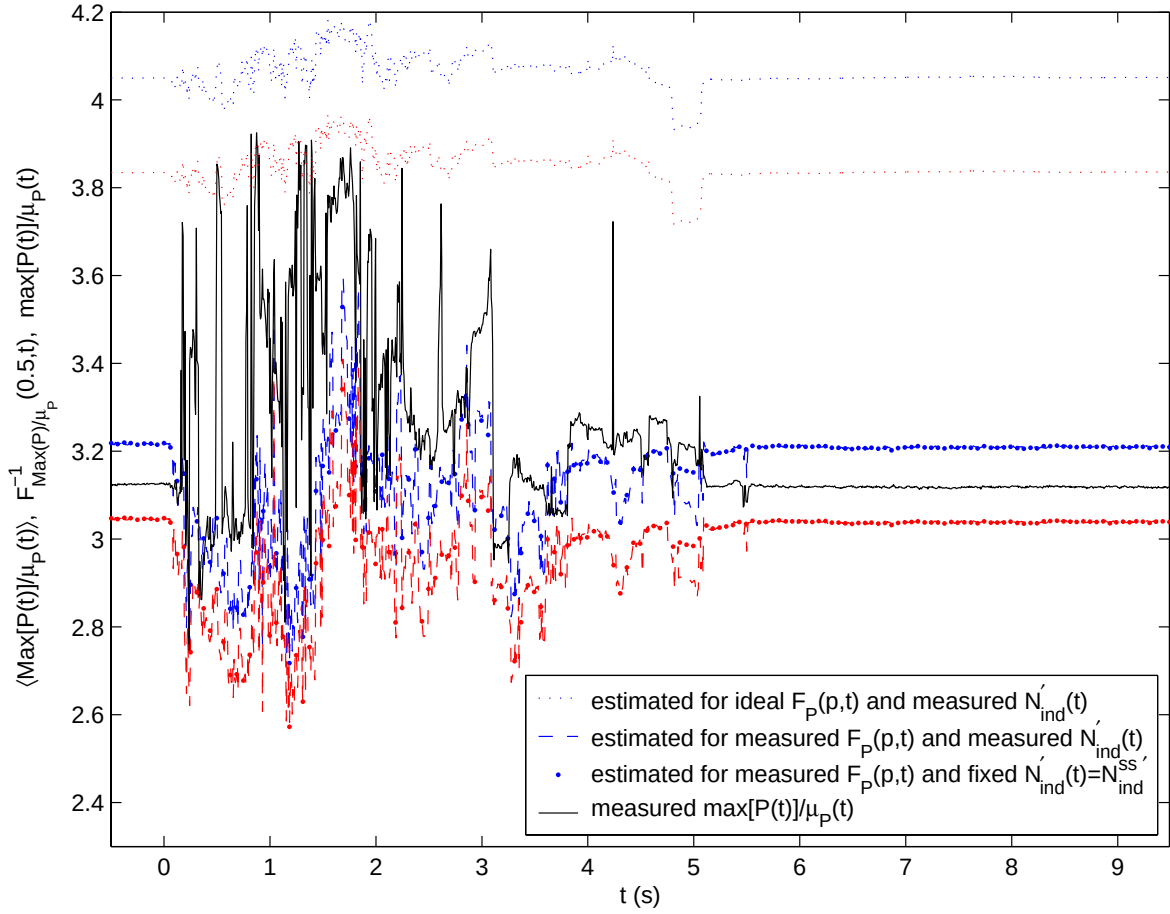


Figure 82: (color online) Comparison of measured vs. estimated transient maximum-to-mean ratio of P as a function of time at $f = 200$ MHz, with $M = 400$ steps ($N = 100$), $A = 1000$ steps/s², $B = 600$ steps/s, $V = 2000$ steps/s. Dotted lines: semi-empirical estimates based on ideal χ^2_2 cdf $F_P(p)$, measured $\mu_P(t)$, and measured $N'_{ind}(t)$; dashed lines: fully-empirical estimates based on fluctuating measured cdfs $F_P(p, t)$ and fluctuating measured $N'_{ind}(t)$; dot symbols: fully-empirical estimates based on fluctuating measured cdfs $F_P(p, t)$ and fixed measured $N'_{ind}(t) = N_{ind}^{ss}$. Black solid curve: measured values $\max[P(t)]/\mu_P(t)$; blue curves: estimated mean values $\langle \max[P(t)]/\mu_P(t) \rangle$; red curves: estimated median values $F_{\max[P(t)]/\mu_P(t)}^{-1}(1/2, t)$. Estimates of $N'_{ind}(t)$ are based on (17).

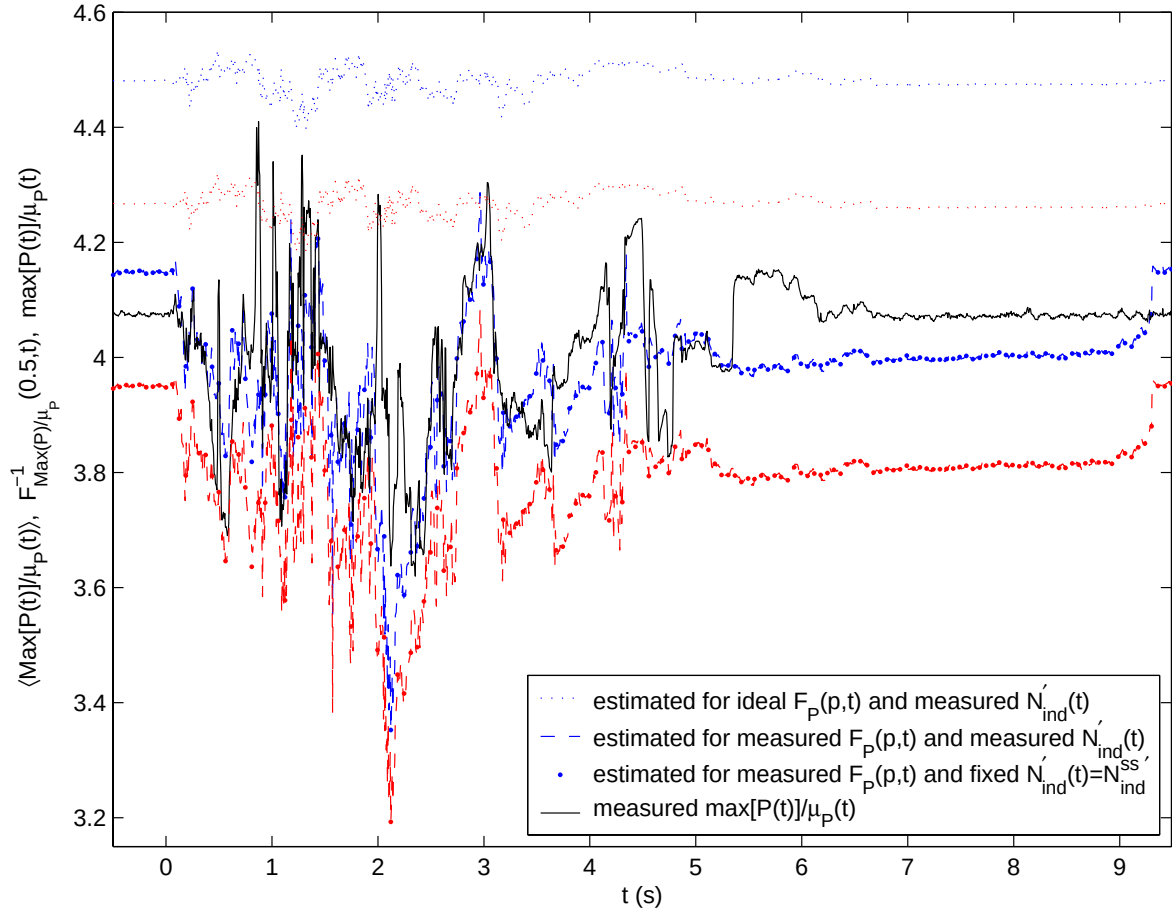


Figure 83: (color online) Comparison of measured vs. estimated transient maximum-to-mean ratio of P as a function of time at $f = 300$ MHz, with $M = 400$ steps ($N = 100$), $A = 1000$ steps/s², $B = 600$ steps/s, $V = 2000$ steps/s. Dotted lines: semi-empirical estimates based on ideal χ_2^2 cdf $F_P(p)$, measured $\mu_P(t)$, and measured $N'_{\text{ind}}(t)$; dashed lines: fully-empirical estimates based on fluctuating measured cdfs $F_P(p, t)$ and fluctuating measured $N'_{\text{ind}}(t)$; dot symbols: fully-empirical estimates based on fluctuating measured cdfs $F_P(p, t)$ and fixed measured $N'_{\text{ind}}(t) = N_{\text{ind}}^{\text{ss}}$. Black solid curve: measured values $\text{max}[P(t)]/\mu_P(t)$; blue curves: estimated mean values $\langle \text{Max}[P(t)]/\mu_P(t) \rangle$; red curves: estimated median values $F_{\text{Max}[P(t)]/\mu_P(t)}^{-1}(1/2, t)$. Estimates of $N'_{\text{ind}}(t)$ are based on (17).

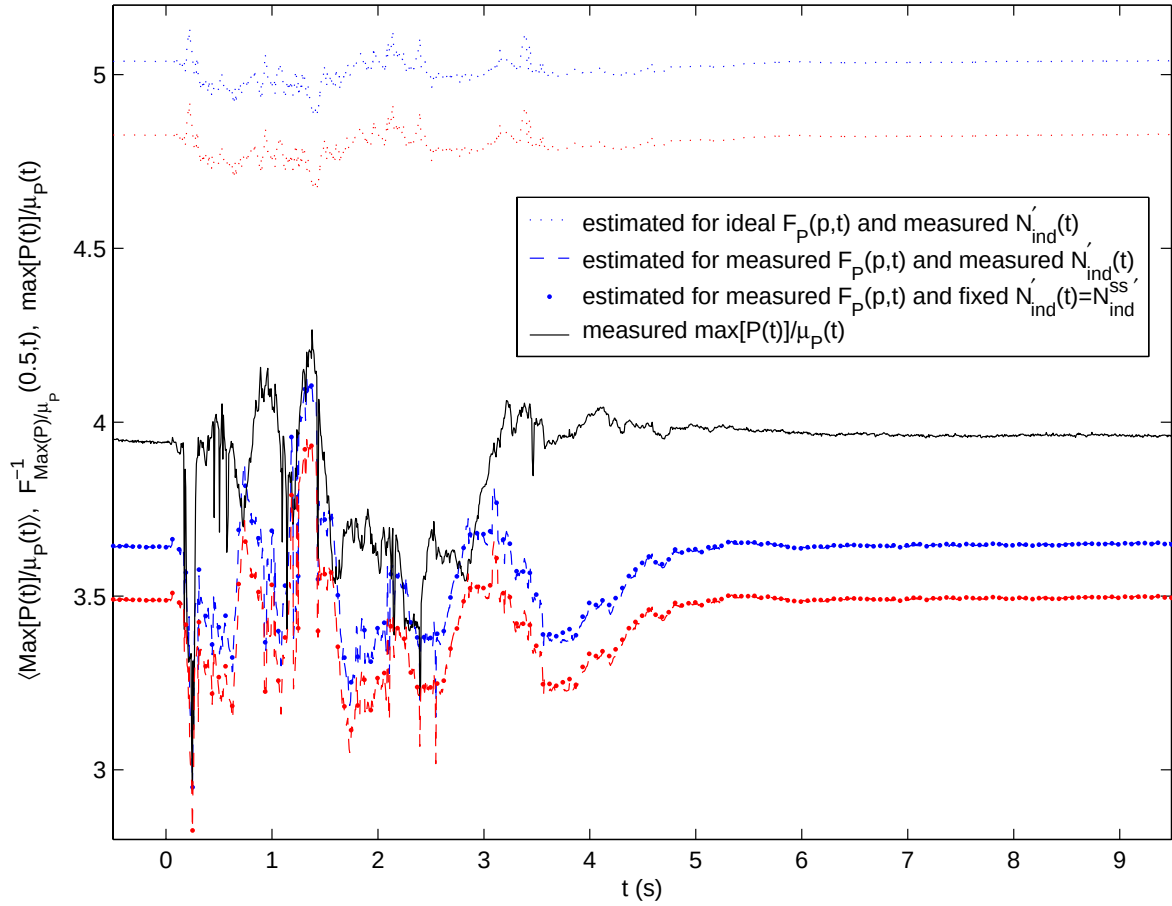


Figure 84: (color online) Comparison of measured vs. estimated transient maximum-to-mean ratio of P as a function of time at $f = 400$ MHz, with $M = 400$ steps ($N = 100$), $A = 1000$ steps/s², $B = 600$ steps/s, $V = 2000$ steps/s. Dotted lines: semi-empirical estimates based on ideal χ^2_2 cdf $F_P(p)$, measured $\mu_P(t)$, and measured $N'_{\text{ind}}(t)$; dashed lines: fully-empirical estimates based on fluctuating measured cdfs $F_P(p, t)$ and fluctuating measured $N'_{\text{ind}}(t)$; dot symbols: fully-empirical estimates based on fluctuating measured cdfs $F_P(p, t)$ and fixed measured $N_{\text{ind}}^{(l)}(t) = N_{\text{ind}}^{ss'}$. Black solid curve: measured values $\text{max}[P(t)]/\mu_P(t)$; blue curves: estimated mean values $\langle \text{Max}[P(t)]/\mu_P(t) \rangle$; red curves: estimated median values $F_{\text{Max}[P(t)]/\mu_P(t)}^{-1}(1/2, t)$. Estimates of $N'_{\text{ind}}(t)$ are based on (17).

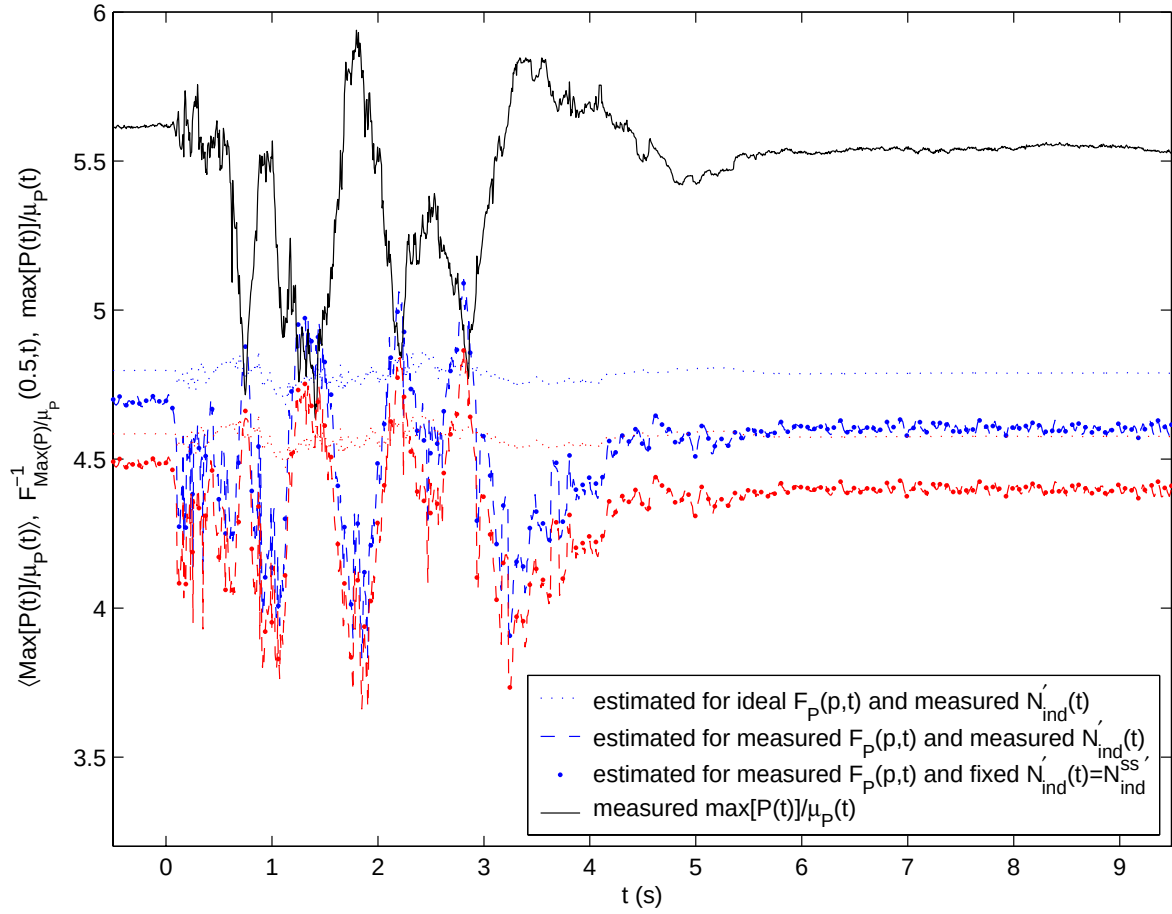


Figure 85: (color online) Comparison of measured vs. estimated transient maximum-to-mean ratio of P as a function of time at $f = 500$ MHz, with $M = 400$ steps ($N = 100$), $A = 1000$ steps/s², $B = 600$ steps/s, $V = 2000$ steps/s. Dotted lines: semi-empirical estimates based on ideal χ_2^2 cdf $F_P(p)$, measured $\mu_P(t)$, and measured $N'_{\text{ind}}(t)$; dashed lines: fully-empirical estimates based on fluctuating measured cdfs $F_P(p, t)$ and fluctuating measured $N'_{\text{ind}}(t)$; dot symbols: fully-empirical estimates based on fluctuating measured cdfs $F_P(p, t)$ and fixed measured $N'_{\text{ind}}(t) = N_{\text{ind}}^{\text{ss}}$. Black solid curve: measured values $\text{max}[P(t)]/\mu_P(t)$; blue curves: estimated mean values $\langle \text{Max}[P(t)]/\mu_P(t) \rangle$; red curves: estimated median values $F_{\text{Max}[P(t)]/\mu_P(t)}^{-1}(1/2, t)$. Estimates of $N'_{\text{ind}}(t)$ are based on (17).

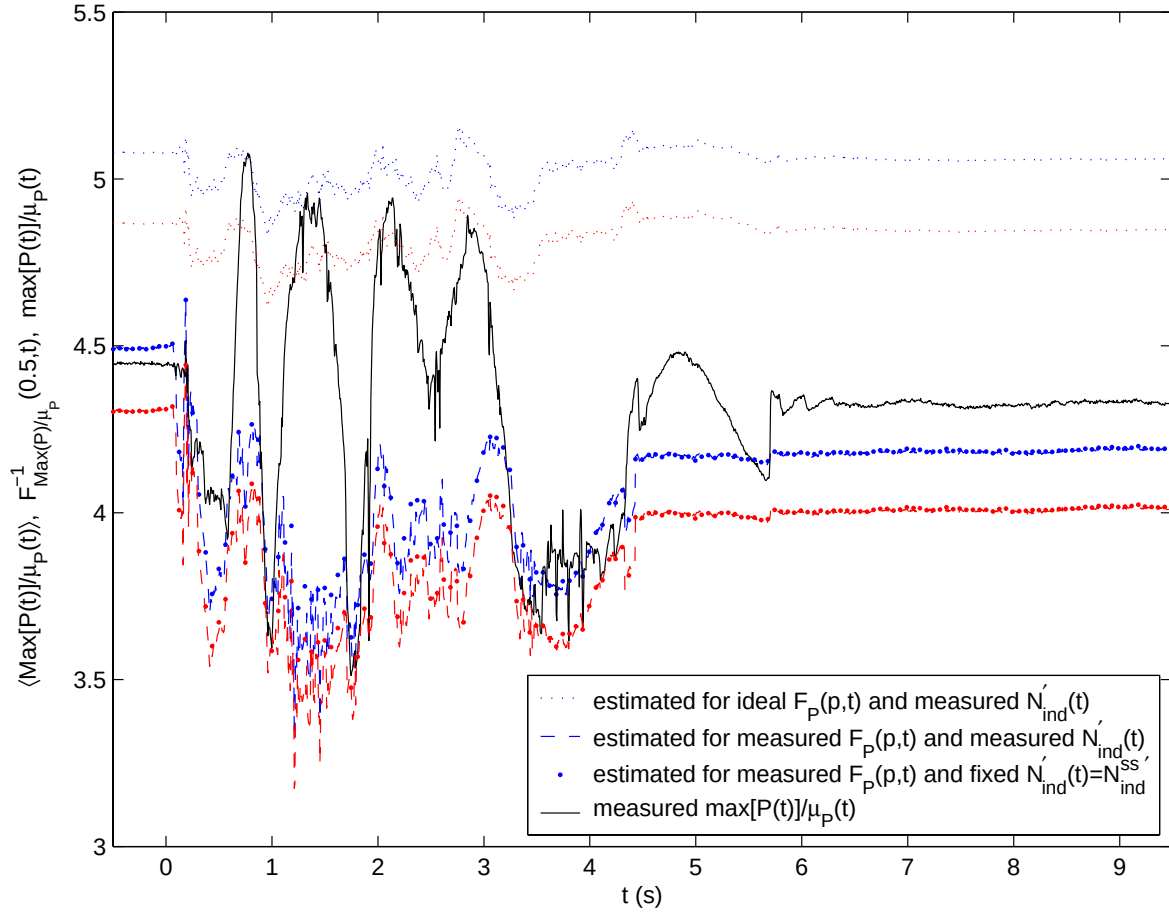


Figure 86: (color online) Comparison of measured vs. estimated transient maximum-to-mean ratio of P as a function of time at $f = 600$ MHz, with $M = 400$ steps ($N = 100$), $A = 1000$ steps/s², $B = 600$ steps/s, $V = 2000$ steps/s. Dotted lines: semi-empirical estimates based on ideal χ^2_2 cdf $F_P(p)$, measured $\mu_P(t)$, and measured $N'_{\text{ind}}(t)$; dashed lines: fully-empirical estimates based on fluctuating measured cdfs $F_P(p, t)$ and fluctuating measured $N'_{\text{ind}}(t)$; dot symbols: fully-empirical estimates based on fluctuating measured cdfs $F_P(p, t)$ and fixed measured $N'_{\text{ind}}(t) = N_{\text{ind}}^{\text{ss}}$. Black solid curve: measured values $\text{max}[P(t)]/\mu_P(t)$; blue curves: estimated mean values $\langle \text{Max}[P(t)]/\mu_P(t) \rangle$; red curves: estimated median values $F_{\text{Max}[P(t)]/\mu_P(t)}^{-1}(1/2, t)$. Estimates of $N'_{\text{ind}}(t)$ based on (17).

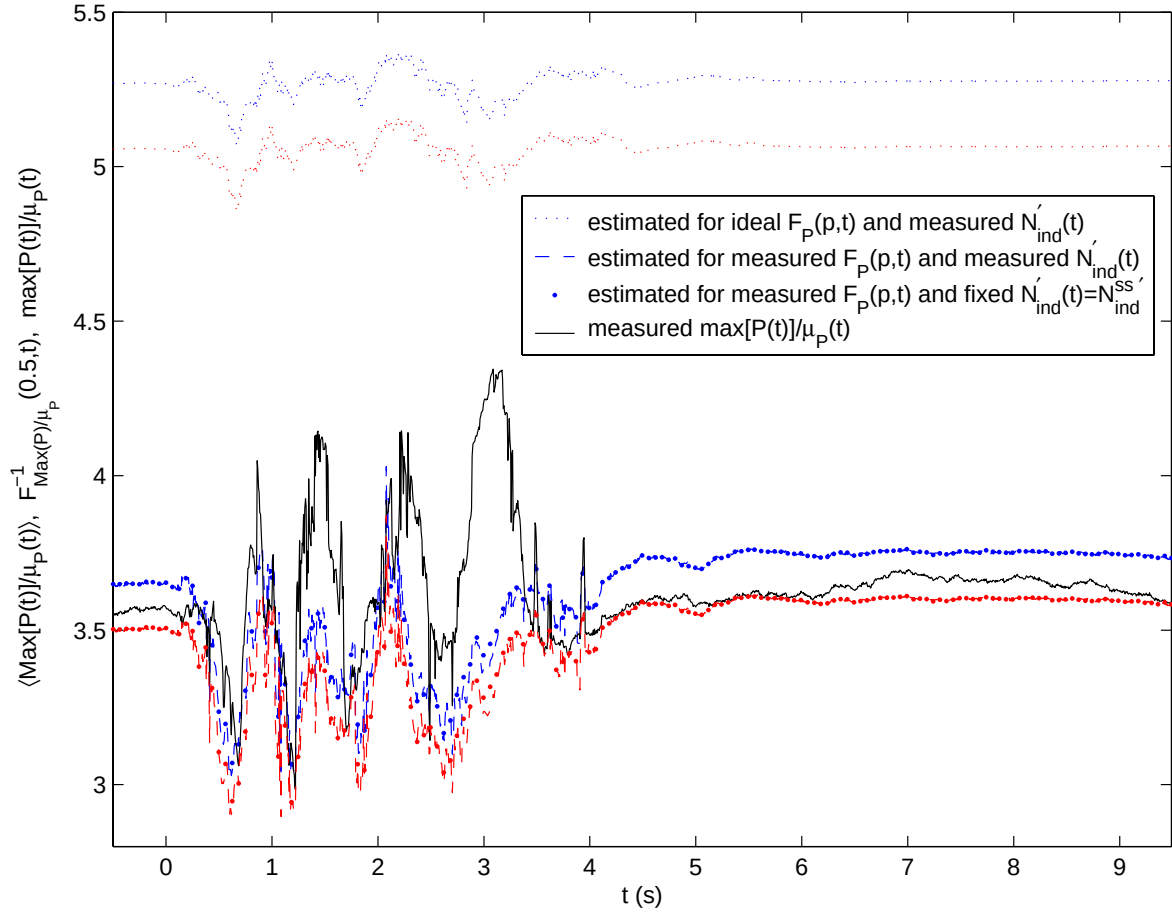


Figure 87: (color online) Comparison of measured vs. estimated transient maximum-to-mean ratio of P as a function of time at $f = 700$ MHz, with $M = 400$ steps ($N = 100$), $A = 1000$ steps/s², $B = 600$ steps/s, $V = 2000$ steps/s. Dotted lines: semi-empirical estimates based on ideal χ_2^2 cdf $F_P(p)$, measured $\mu_P(t)$, and measured $N'_{\text{ind}}(t)$; dashed lines: fully-empirical estimates based on fluctuating measured cdfs $F_P(p, t)$ and fluctuating measured $N'_{\text{ind}}(t)$; dot symbols: fully-empirical estimates based on fluctuating measured cdfs $F_P(p, t)$ and fixed measured $N'_{\text{ind}}(t) = N_{\text{ind}}^{\text{ss}}$. Black solid curve: measured values $\text{max}[P(t)]/\mu_P(t)$; blue curves: estimated mean values $\langle \text{Max}[P(t)]/\mu_P(t) \rangle$; red curves: estimated median values $F_{\text{Max}[P(t)]/\mu_P(t)}^{-1}(1/2, t)$. Estimates of $N'_{\text{ind}}(t)$ are based on (17).

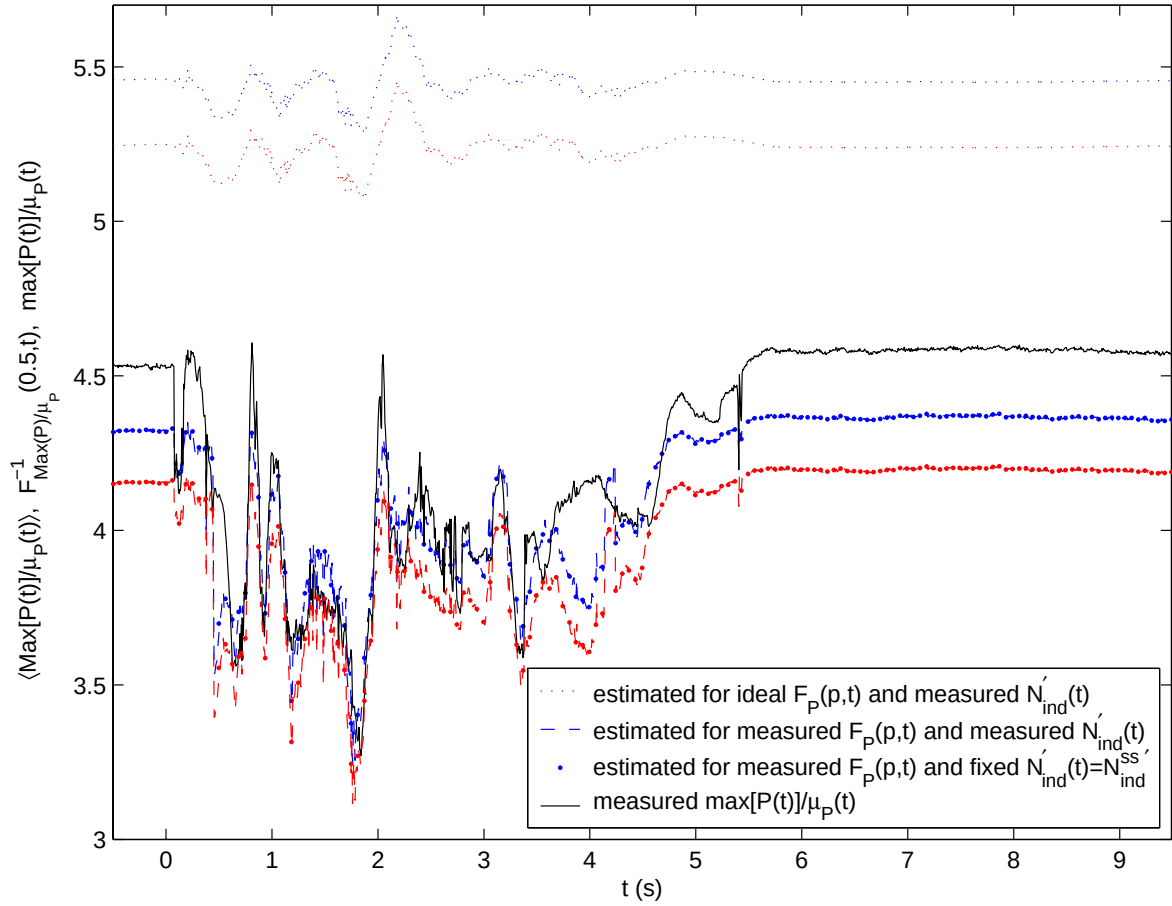


Figure 88: (color online) Comparison of measured vs. estimated transient maximum-to-mean ratio of P as a function of time at $f = 800$ MHz, with $M = 400$ steps ($N = 100$), $A = 1000$ steps/s², $B = 600$ steps/s, $V = 2000$ steps/s. Dotted lines: semi-empirical estimates based on ideal χ^2_2 cdf $F_P(p)$, measured $\mu_P(t)$, and measured $N'_{\text{ind}}(t)$; dashed lines: fully-empirical estimates based on fluctuating measured cdfs $F_P(p, t)$ and fluctuating measured $N'_{\text{ind}}(t)$; dot symbols: fully-empirical estimates based on fluctuating measured cdfs $F_P(p, t)$ and fixed measured $N_{\text{ind}}^{(t)}(t) = N_{\text{ind}}^{\text{ss}}$. Black solid curve: measured values $\text{max}[P(t)]/\mu_P(t)$; blue curves: estimated mean values $\langle \text{Max}[P(t)]/\mu_P(t) \rangle$; red curves: estimated median values $F_{\text{Max}[P(t)]/\mu_P(t)}^{-1}(1/2, t)$. Estimates of $N'_{\text{ind}}(t)$ are based on (17).

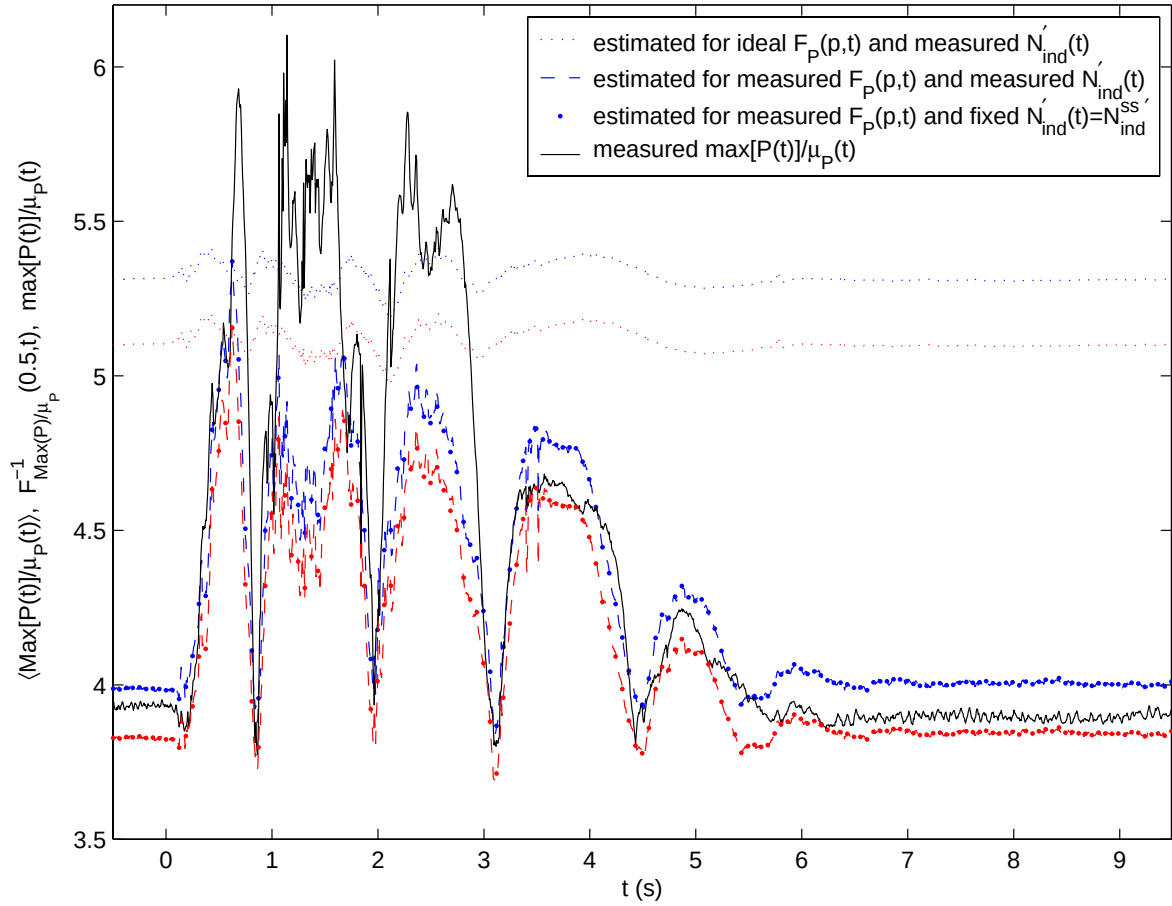


Figure 89: (color online) Comparison of measured vs. estimated transient maximum-to-mean ratio of P as a function of time at $f = 900$ MHz, with $M = 400$ steps ($N = 100$), $A = 1000$ steps/s², $B = 600$ steps/s, $V = 2000$ steps/s. Dotted lines: semi-empirical estimates based on ideal χ^2_2 cdf $F_P(p)$, measured $\mu_P(t)$, and measured $N'_{\text{ind}}(t)$; dashed lines: fully-empirical estimates based on fluctuating measured cdfs $F_P(p, t)$ and fluctuating measured $N'_{\text{ind}}(t)$; dot symbols: fully-empirical estimates based on fluctuating measured cdfs $F_P(p, t)$ and fixed measured $N^{(l)}_{\text{ind}}(t) = N^{ssl}_{\text{ind}}$. Black solid curve: measured values $\max[P(t)]/\mu_P(t)$; blue curves: estimated mean values $\langle \max[P(t)]/\mu_P(t) \rangle$; red curves: estimated median values $F^{-1}_{\max[P(t)]/\mu_P(t)}(1/2, t)$. Estimates of $N'_{\text{ind}}(t)$ are based on (17).

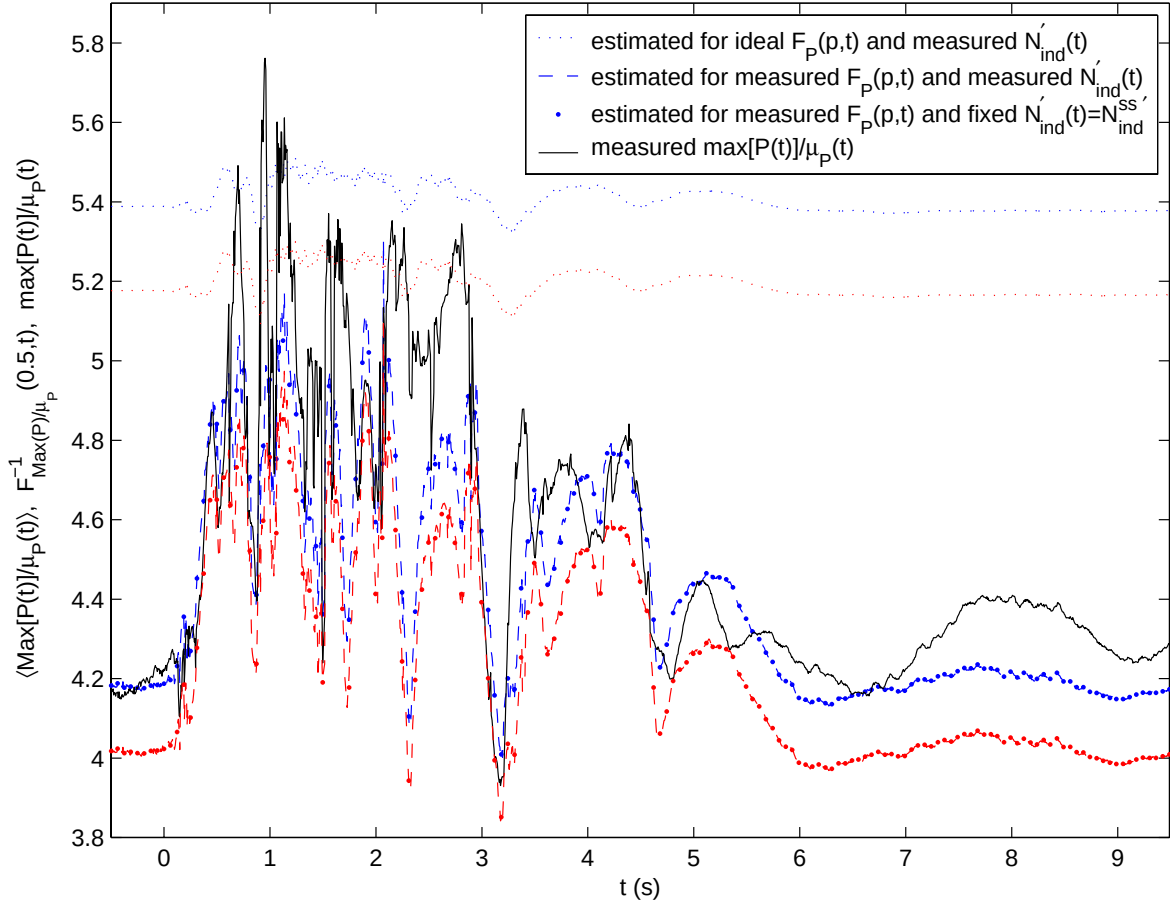


Figure 90: (color online) Comparison of measured vs. estimated transient maximum-to-mean ratio of P as a function of time at $f = 1000$ MHz, for $M = 400$ steps ($N = 100$), with $A = 1000$ steps/s², $B = 600$ steps/s, $V = 2000$ steps/s: comparison between measured and estimated values. Dotted lines: semi-empirical estimates based on ideal χ^2_2 cdf $F_P(p)$, measured $\mu_P(t)$, and fluctuating measured $N_{\text{ind}}^{(l)}(t)$; dashed lines: fully-empirical estimates based on fluctuating measured cdfs $F_P(p, t)$ and fluctuating measured $N_{\text{ind}}^{(l)}(t)$; dot symbols: fully-empirical estimates based on fluctuating measured cdfs $F_P(p)$ and fixed measured $N_{\text{ind}}^{(l)}(t) = N_{\text{ind}}^{\text{ss}(l)}$. Black solid curve: measured values $\text{max}[P(t)]/\mu_P(t)$; blue curves: estimated mean values $\langle \text{Max}[P(t)]/\mu_P(t) \rangle$; red curves: estimated median values $F_{\text{Max}[P(t)]/\mu_P(t)}^{-1}(1/2, t)$. Estimates of $N_{\text{ind}}^{(l)}(t)$ are based on (17).

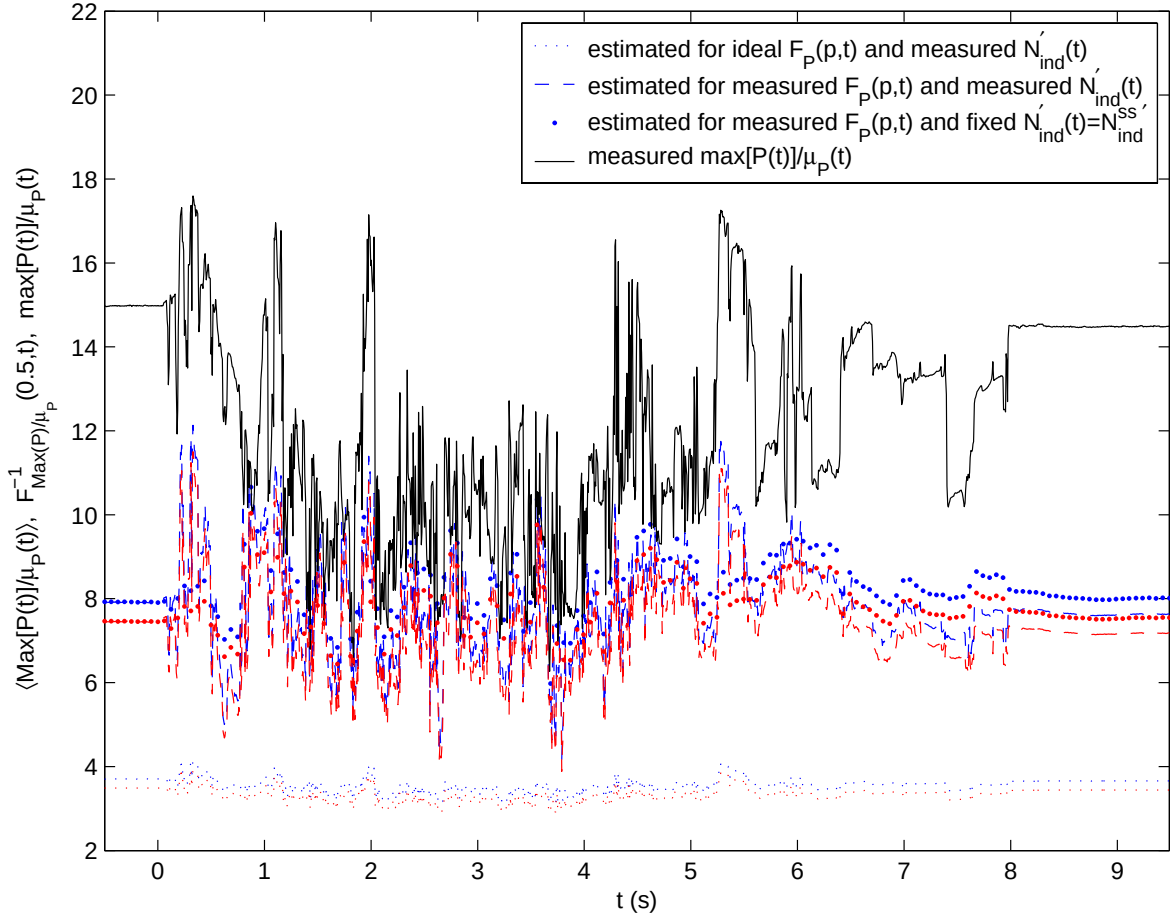


Figure 91: (color online) Maximum-to-mean ratio of P during transients as a function of time at $f = 100$ MHz, for $M = 1200$ steps ($N = 33.3$), with $A = 1000$ steps/s², $B = 600$ steps/s, $V = 2000$ steps/s: comparison between measured and estimated values. Dotted lines: semi-empirical estimates based on ideal χ^2_2 cdf $F_P(p)$, measured $\mu_P(t)$, and fluctuating measured $N'_{\text{ind}}(t)$; dashed lines: fully-empirical estimates based on fluctuating measured cdfs $F_P(p)$ and fluctuating measured $N'_{\text{ind}}(t)$; dot symbols: fully-empirical estimates based on fluctuating measured cdfs $F_P(p)$ and fixed measured $N'_{\text{ind}}(t) = N_{\text{ind}}^{\text{ss}}$. Black solid curve: measured values $\text{max}[P(t)]/\mu_P(t)$; blue curves: estimated mean values $\langle \text{Max}[P(t)]/\mu_P(t) \rangle$; red curves: estimated median values $F_{\text{Max}[P(t)]/\mu_P(t)}^{-1}(1/2, t)$. Estimates of $N'_{\text{ind}}(t)$ are based on (17).

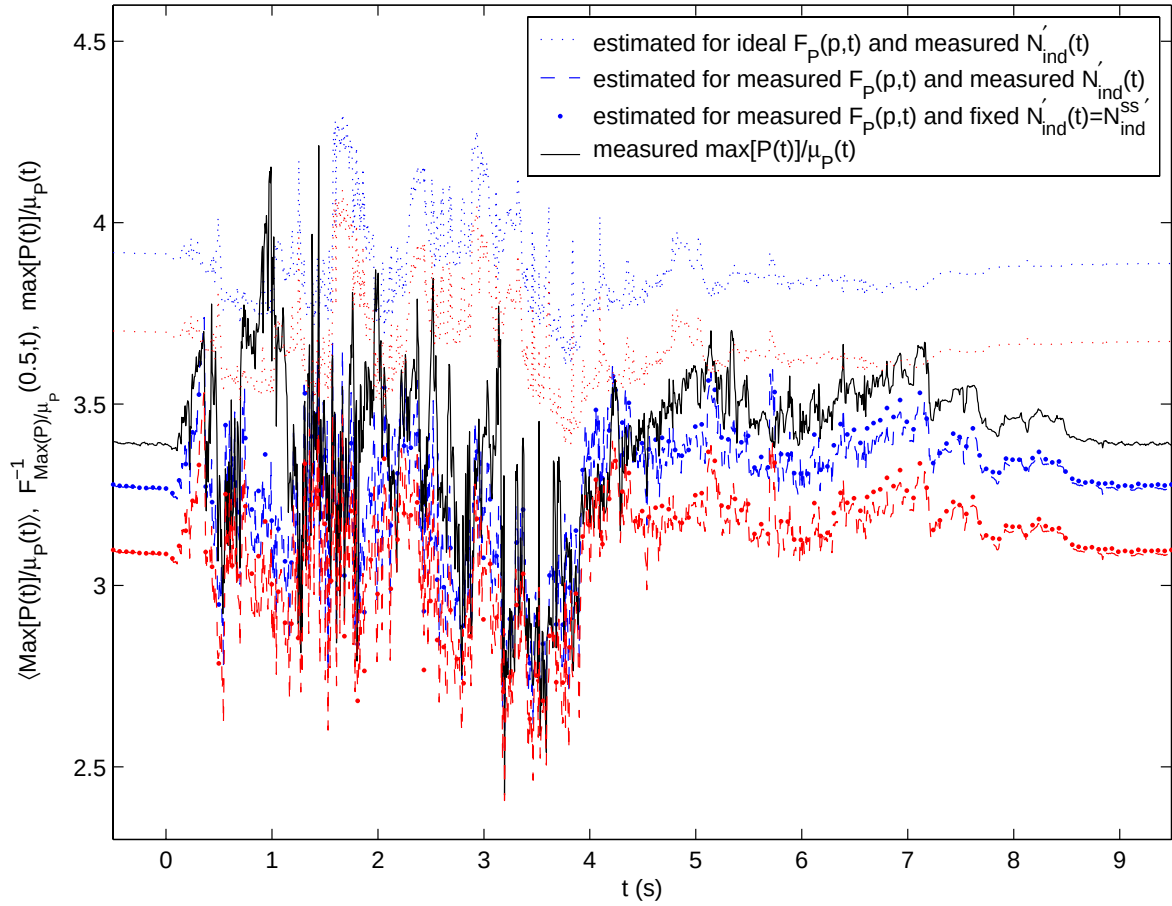


Figure 92: (color online) Maximum-to-mean ratio of P during transients as a function of time at $f = 200$ MHz, for $M = 1200$ steps ($N = 33.3$), with $A = 1000$ steps/s², $B = 600$ steps/s, $V = 2000$ steps/s: comparison between measured and estimated values. Dotted lines: semi-empirical estimates based on ideal χ^2_2 cdf $F_P(p)$, measured $\mu_P(t)$, and fluctuating measured $N'_{\text{ind}}(t)$; dashed lines: fully-empirical estimates based on fluctuating measured cdfs $F_P(p)$ and fluctuating measured $N'_{\text{ind}}(t)$; dotted symbols: fully-empirical estimates based on fluctuating measured cdfs $F_P(p)$ and fixed measured $N'_{\text{ind}}(t) = N_{\text{ind}}^{\text{ss}'}$. Black solid curve: measured values $\text{max}[P(t)]/\mu_P(t)$; blue curves: estimated mean values $\langle \text{Max}[P(t)]/\mu_P(t) \rangle$; red curves: estimated median values $F_{\text{Max}[P(t)]/\mu_P(t)}^{-1}(1/2, t)$. Estimates of $N'_{\text{ind}}(t)$ are based on (17).

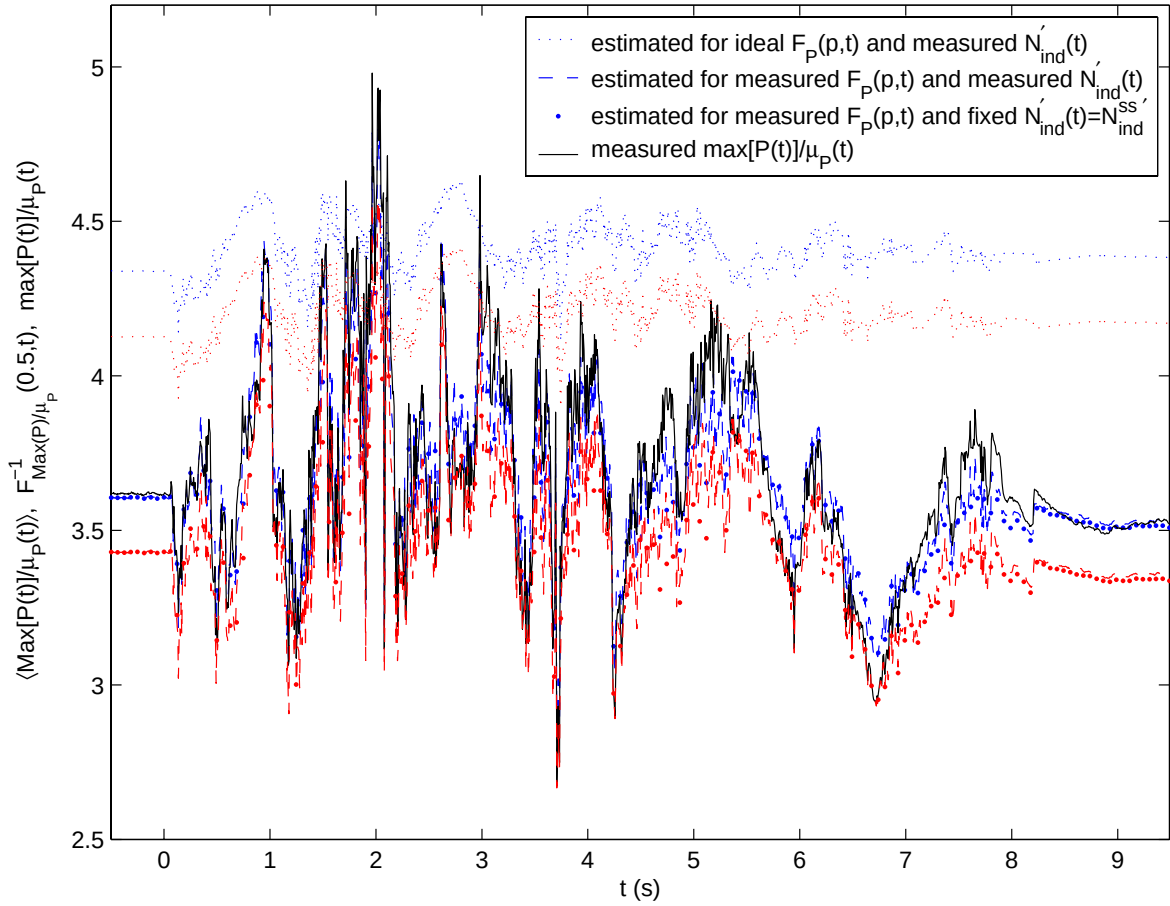


Figure 93: (color online) Maximum-to-mean ratio of P during transients as a function of time at $f = 300$ MHz, for $M = 1200$ steps ($N = 33.3$), with $A = 1000$ steps/s², $B = 600$ steps/s, $V = 2000$ steps/s: comparison between measured and estimated values. Dotted lines: semi-empirical estimates based on ideal χ^2_2 cdf $F_P(p)$, measured $\mu_P(t)$, and fluctuating measured $N'_{\text{ind}}(t)$; dashed lines: fully-empirical estimates based on fluctuating measured cdfs $F_P(p)$ and fluctuating measured $N'_{\text{ind}}(t)$; dot symbols: fully-empirical estimates based on fluctuating measured cdfs $F_P(p)$ and fixed measured $N'_{\text{ind}}(t) = N_{\text{ind}}^{\text{ss}'}$. Black solid curve: measured values $\text{max}[P(t)]/\mu_P(t)$; blue curves: estimated mean values $\langle \text{Max}[P(t)]/\mu_P(t) \rangle$; red curves: estimated median values $F_{\text{Max}[P(t)]/\mu_P(t)}^{-1}(1/2, t)$. Estimates of $N'_{\text{ind}}(t)$ are based on (17).

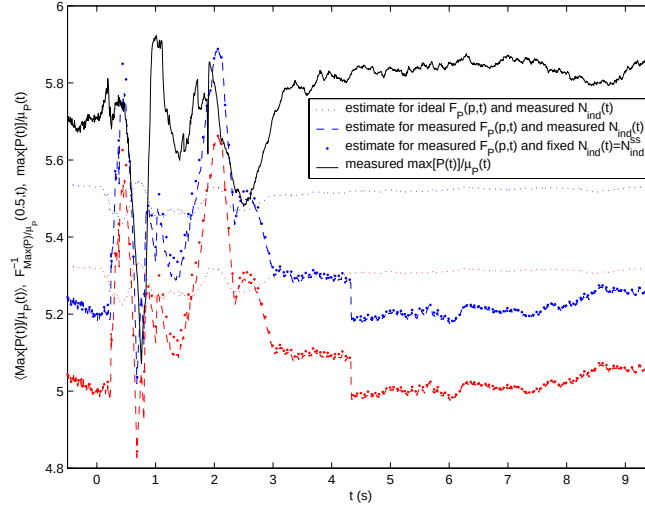
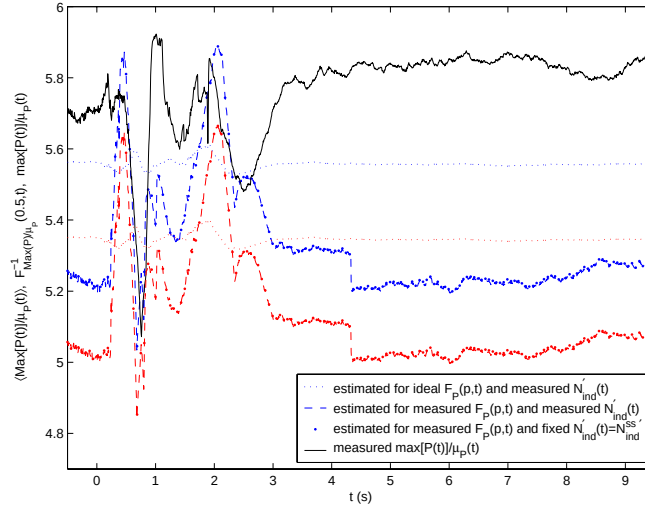
(a) for $N_{\text{ind}}(t)$ based on (16)(b) for $N'_{\text{ind}}(t)$ based on (17)

Figure 94: (color online) Maximum-to-mean ratio of P during transients as a function of time at $f = 1000$ MHz, for $M = 200$ steps ($N = 200$), with $A = 1000$ steps/s², $B = 600$ steps/s, $V = 2000$ steps/s: comparison between measured and estimated values. Dotted lines: semi-empirical estimates based on ideal χ^2_2 cdf $F_P(p)$, measured $\mu_P(t)$, and fluctuating measured $N_{\text{ind}}^{(l)}(t)$; dashed lines: fully-empirical estimates based on fluctuating measured cdfs $F_P(p)$ and fluctuating measured $N_{\text{ind}}^{(l)}(t)$; dot symbols: fully-empirical estimates based on fluctuating measured cdfs $F_P(p)$ and fixed measured $N_{\text{ind}}(t) = N_{\text{ind}}^{\text{ss}}$. Black solid curve: measured values $\text{max}[P(t)]/\mu_P(t)$; blue curves: estimated mean values $\langle \text{Max}[P(t)]/\mu_P(t) \rangle$; red curves: estimated median values $F_{\text{Max}[P(t)]/\mu_P(t)}^{-1}(1/2, t)$. Estimates of $N_{\text{ind}}^{(l)}(t)$ are based on (a) $N_{\text{ind}}(t)$ defined by (16), or (b) $N'_{\text{ind}}(t)$ defined by (17).

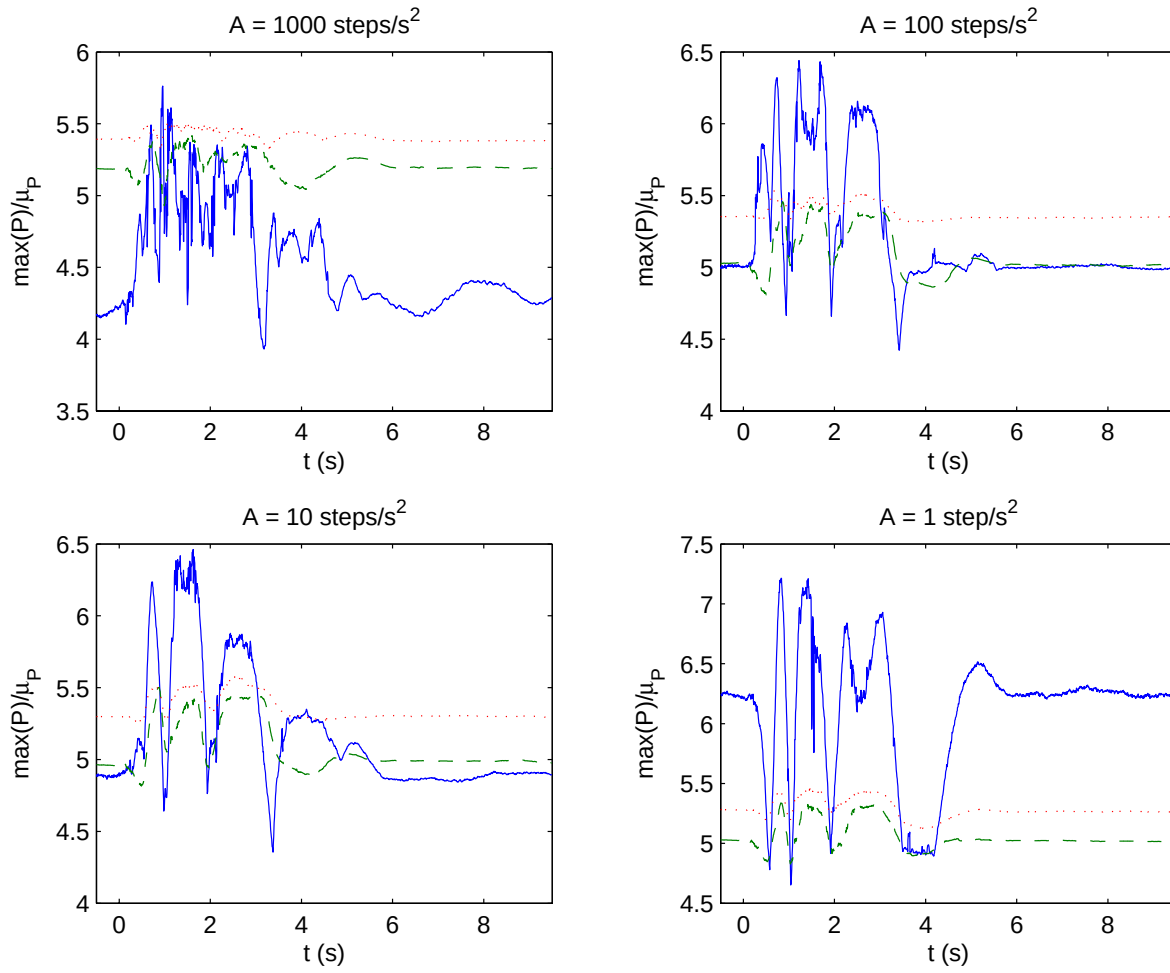


Figure 95: (color online) Comparison between measured and semi-empirically estimated values of maximum-to-mean ratio of P as a function of time at $f = 1000 \text{ MHz}$, for selected values of A , with $M = 400 \text{ steps}$ ($N = 100$), $B = 600 \text{ steps/s}$, $V = 2000 \text{ steps/s}$. Blue solid lines: measured data; green dashed lines: semi-empirical estimates based on (16); red dotted lines: semi-empirical estimates based on (17).

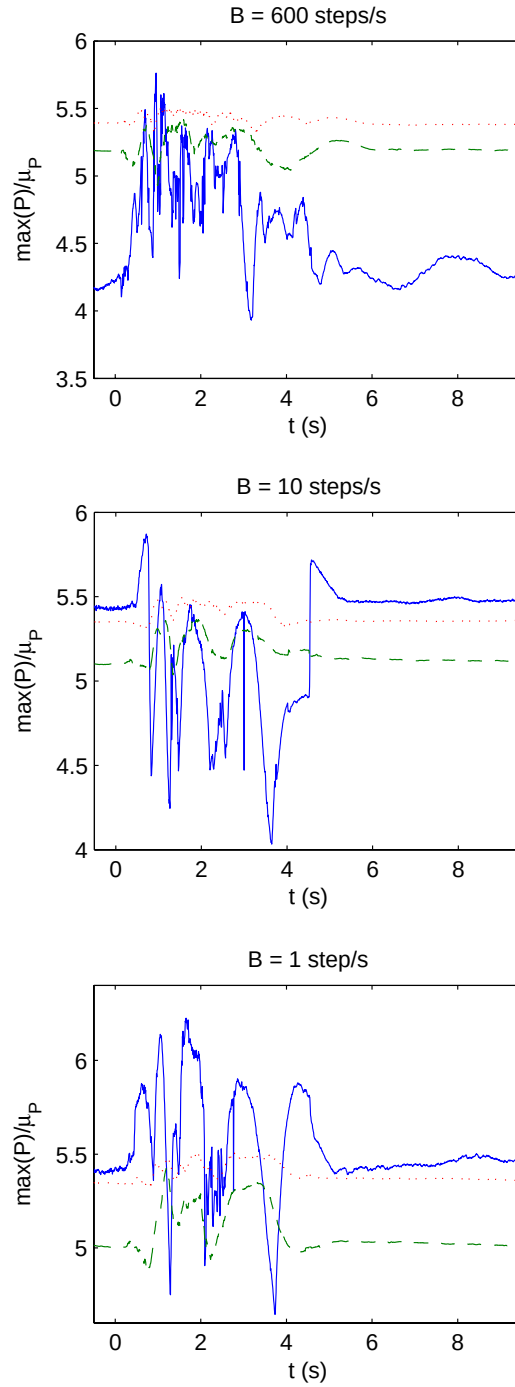


Figure 96: (color online) Comparison between measured and semi-empirically estimated values of maximum-to-mean ratio of P as a function of time at $f = 1000$ MHz, for selected values of B , with $M = 400$ steps ($N = 100$), $A = 1000$ steps/s², $V = 2000$ steps/s. Solid blue lines: measured data; dashed green lines: semi-empirical estimates based on (16); dotted red lines: semi-empirical estimates based on (17).

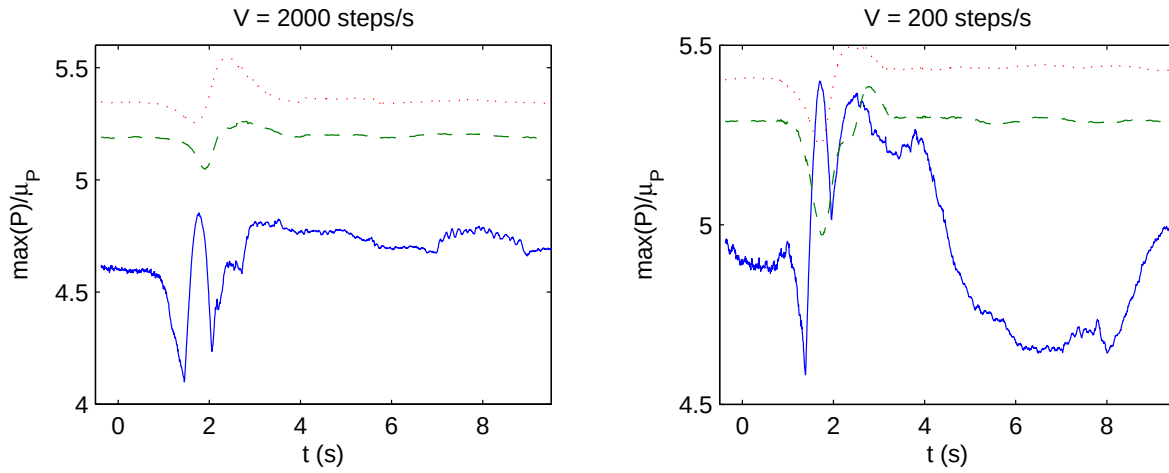


Figure 97: (color online) Comparison between measured and semi-empirically estimated values of maximum-to-mean ratio of P as a function of time at $f = 1000$ MHz, for selected values of V , with $M = 400$ steps ($N = 100$), $A = 100$ steps/s², $B = 10$ steps/s. Solid blue lines: measured data; dashed green lines: semi-empirical estimates based on (16); dotted red lines: semi-empirical estimates based on (17).

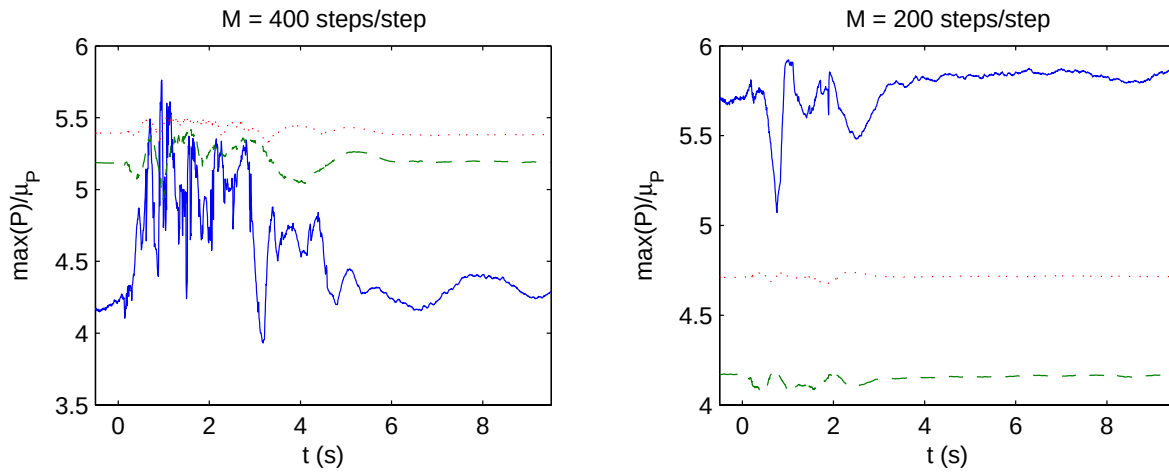


Figure 98: (color online) Comparison between measured and semi-empirically estimated values of maximum-to-mean ratio of P as a function of time at $f = 1000$ MHz, for selected values of M , with $A = 1000$ steps/s², $B = 600$ steps/s, $V = 2000$ steps/s. Solid blue lines: measured data; dashed green lines: semi-empirical estimates based on (16); dotted red lines: semi-empirical estimates based on (17).

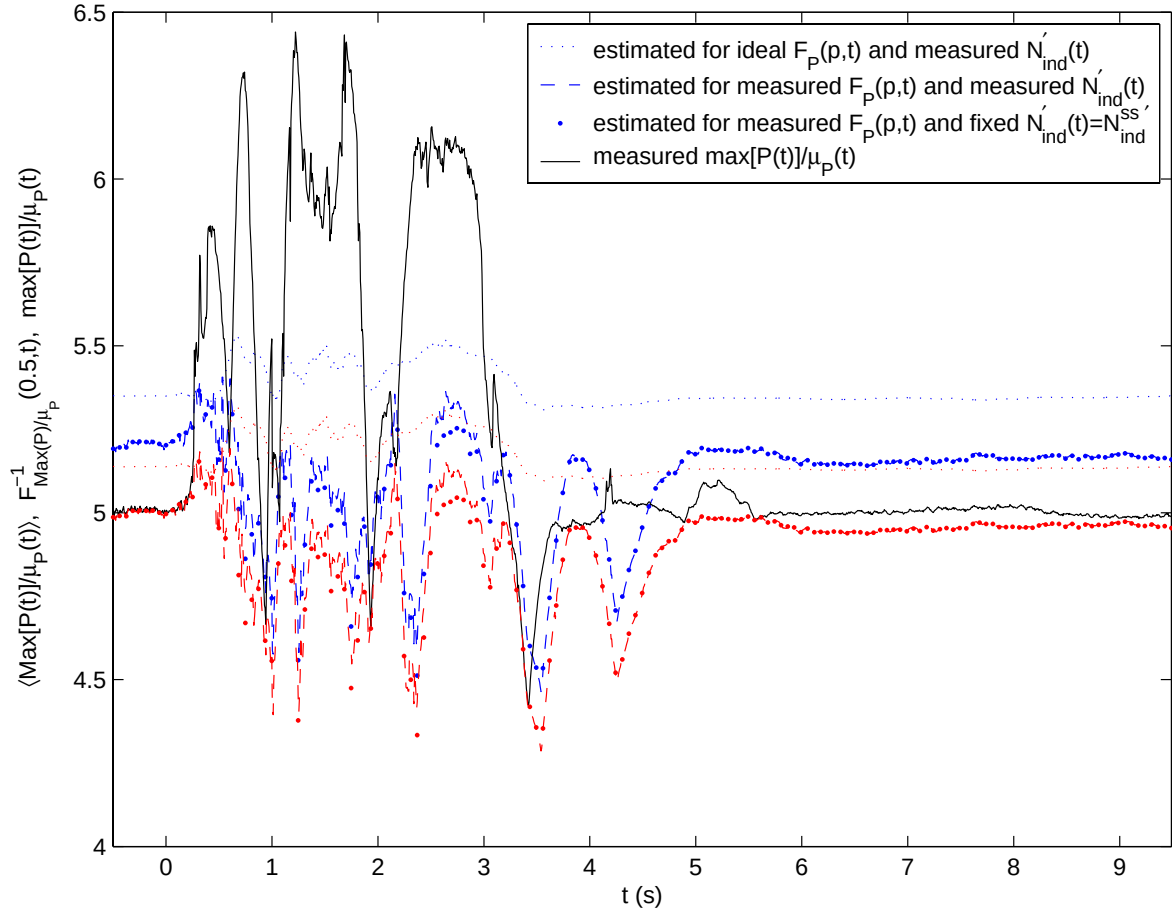


Figure 99: (color online) Maximum-to-mean ratio of P during transients as a function of time at $f = 1000$ MHz, for $M = 400$ steps ($N = 100$), with $A = 100$ steps/s², $B = 600$ steps/s, $V = 2000$ steps/s: comparison between measured and estimated values. Dot lines: semi-empirical estimates based on fixed ideal χ^2_2 cdf $F_P(p, t)$ and fluctuating measured $N^{(r)}_{\text{ind}}(t)$; dashed lines: fully-empirical estimates based on fluctuating measured cdfs $F_P(p)$ and fluctuating measured $N^{(r)}_{\text{ind}}(t)$; dotted symbols: fully-empirical estimates based on fluctuating measured cdfs $F_P(p)$ and fixed measured $N^{(r)}_{\text{ind}}(t) = N^{ss(r)}_{\text{ind}}$. Black solid curve: measured values $\max[P(t)]/\mu_P(t)$; blue curves: estimated mean values $\langle \max[P(t)]/\mu_P(t) \rangle$; red curves: estimated median values $F^{-1}_{\max[P(t)]/\mu_P(t)}(1/2, t)$. Estimates of $N'_{\text{ind}}(t)$ are based on (17).

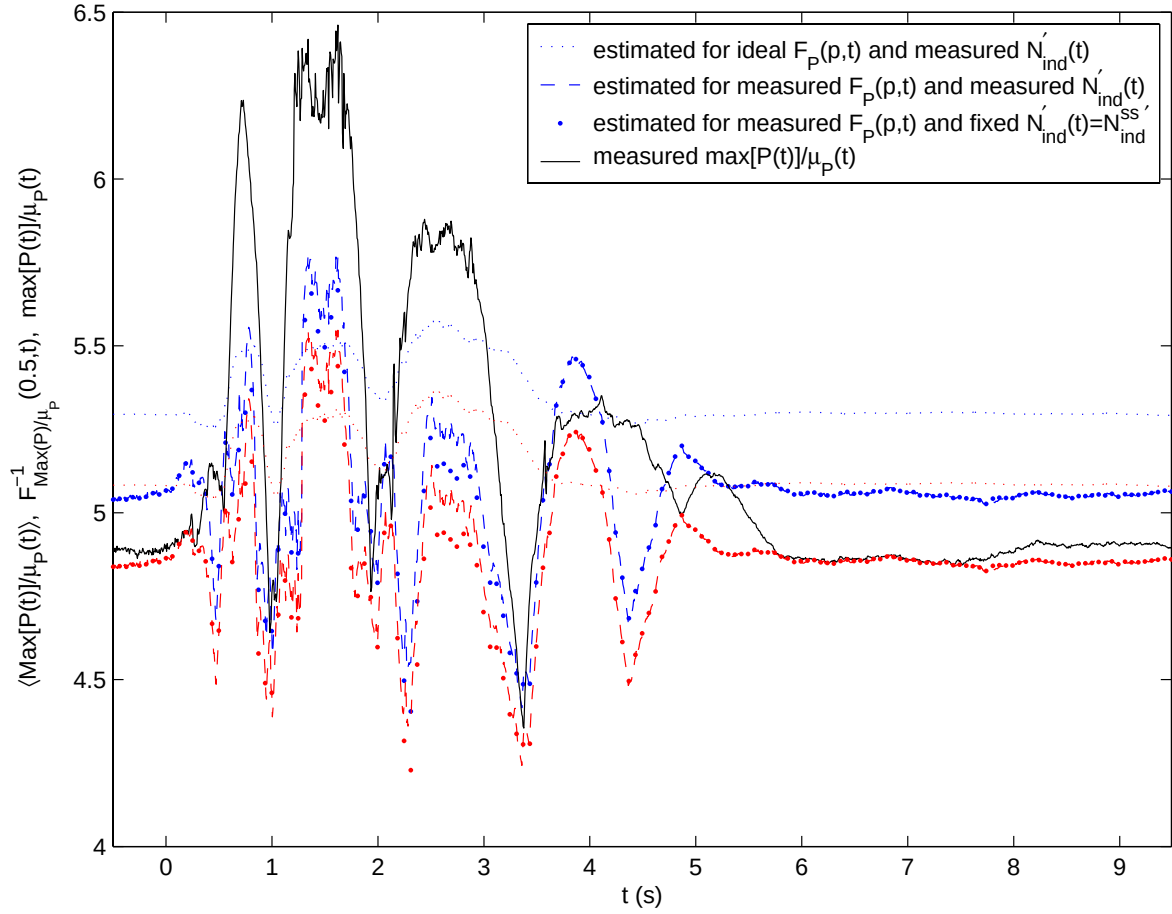


Figure 100: (color online) Maximum-to-mean ratio of P during transients as a function of time at $f = 1000$ MHz, for $M = 400$ steps ($N = 100$), with $A = 10$ steps/s², $B = 600$ steps/s, $V = 2000$ steps/s: comparison between measured and estimated values. Dot lines: semi-empirical estimates based on fixed ideal χ^2_2 cdf $F_P(p, t)$ and fluctuating measured $N_{\text{ind}}^{(l)}(t)$; dashed lines: fully-empirical estimates based on fluctuating measured cdfs $F_P(p)$ and fluctuating measured $N_{\text{ind}}^{(l)}(t)$; dotted symbols: fully-empirical estimates based on fluctuating measured cdfs $F_P(p)$ and fixed measured $N_{\text{ind}}^{(l)}(t) = N_{\text{ind}}^{\text{ss}(l)}$. Black solid curve: measured values $\max[P(t)]/\mu_P(t)$; blue curves: estimated mean values $\langle \max[P(t)]/\mu_P(t) \rangle$; red curves: estimated median values $F_{\max[P(t)]/\mu_P(t)}^{-1}(1/2, t)$. Estimates of $N_{\text{ind}}^{(l)}(t)$ are based on (17).

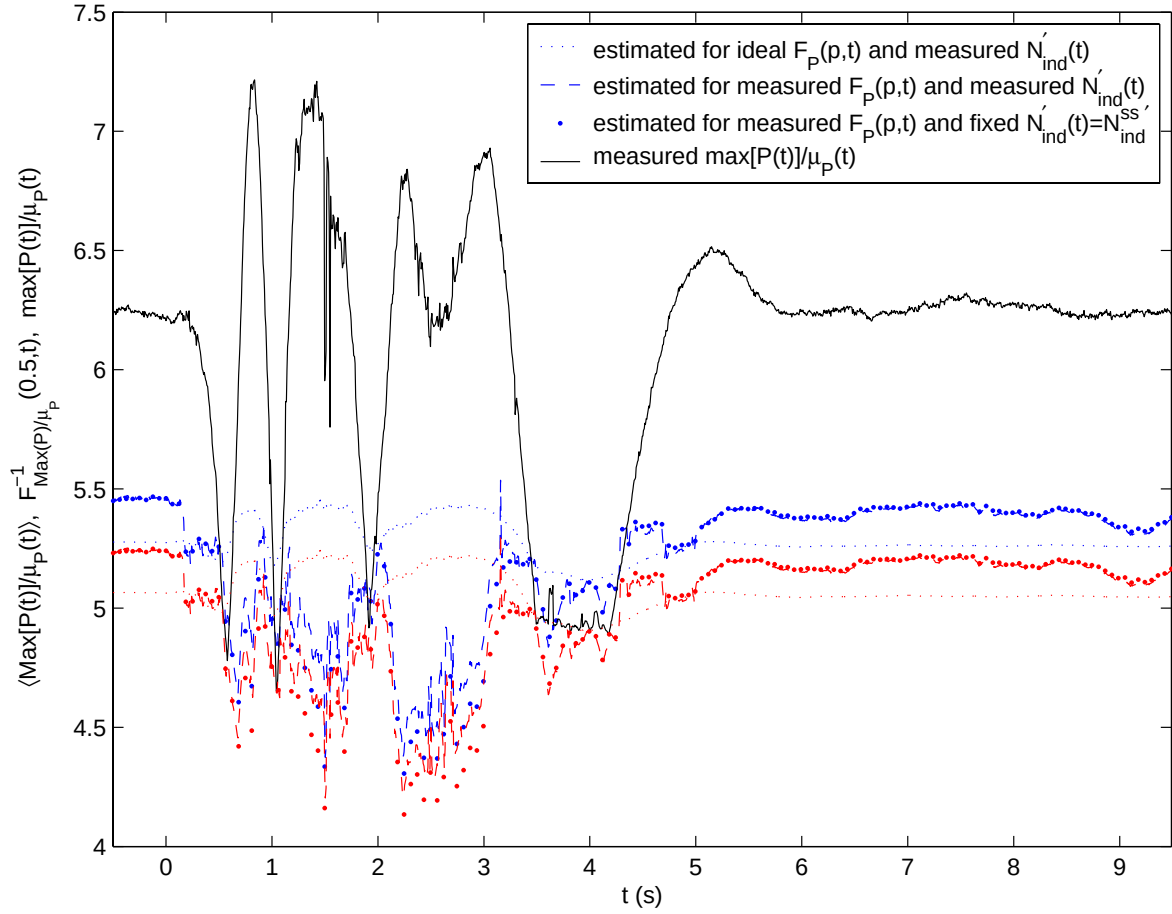


Figure 101: (color online) Maximum-to-mean ratio of P during transients as a function of time at $f = 1000$ MHz, for $M = 400$ steps ($N = 100$), with $A = 1$ step/s², $B = 600$ steps/s, $V = 2000$ steps/s: comparison between measured and estimated values. Dotted lines: semi-empirical estimates based on fixed ideal χ^2_2 cdf $F_P(p, t)$ and fluctuating measured $N_{\text{ind}}^{(r)}(t)$; dashed lines: fully-empirical estimates based on fluctuating measured cdfs $F_P(p)$ and fluctuating measured $N_{\text{ind}}^{(r)}(t)$; dot symbols: fully-empirical estimates based on fluctuating measured cdfs $F_P(p)$ and fixed measured $N_{\text{ind}}^{(r)}(t) = N_{\text{ind}}^{\text{ss}(r)}$. Black solid curve: measured values $\text{max}[P(t)]/\mu_P(t)$; blue curves: estimated mean values $\langle \text{Max}[P(t)]/\mu_P(t) \rangle$; red curves: estimated median values $F_{\text{Max}[P(t)]/\mu_P(t)}^{-1}(1/2, t)$. Estimates of $N'_{\text{ind}}(t)$ are based on (17).

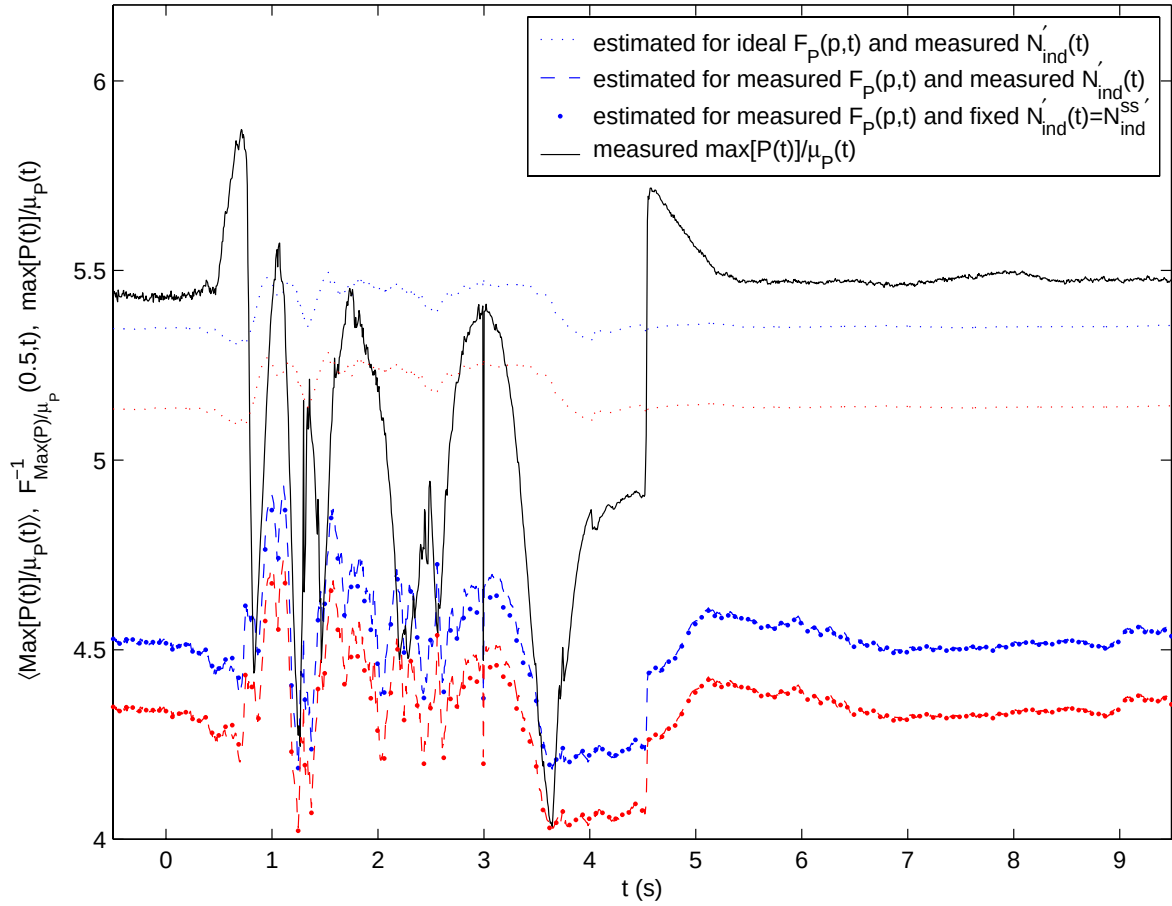


Figure 102: (color online) Maximum-to-mean ratio of P during transients as a function of time at $f = 1000$ MHz, for $M = 400$ steps ($N = 100$), with $A = 1000$ steps/s², $B = 10$ steps/s, $V = 2000$ steps/s: comparison between measured and estimated values. Dotted lines: semi-empirical estimates based on fixed ideal χ^2_2 cdf $F_P(p, t)$ and fluctuating measured $N_{\text{ind}}^{(t)}(t)$; dashed lines: fully-empirical estimates based on fluctuating measured cdfs $F_P(p)$ and fluctuating measured $N_{\text{ind}}^{(t)}(t)$; dot symbols: fully-empirical estimates based on fluctuating measured cdfs $F_P(p)$ and fixed measured $N_{\text{ind}}^{(t)}(t) = N_{\text{ind}}^{\text{ss}(t)}$. Black solid curve: measured values $\text{max}[P(t)]/\mu_P(t)$; blue curves: estimated mean values $\langle \text{Max}[P(t)]/\mu_P(t) \rangle$; red curves: estimated median values $F_{\text{Max}[P(t)]/\mu_P(t)}^{-1}(1/2, t)$. Estimates of $N_{\text{ind}}^{(t)}(t)$ are based on (17).

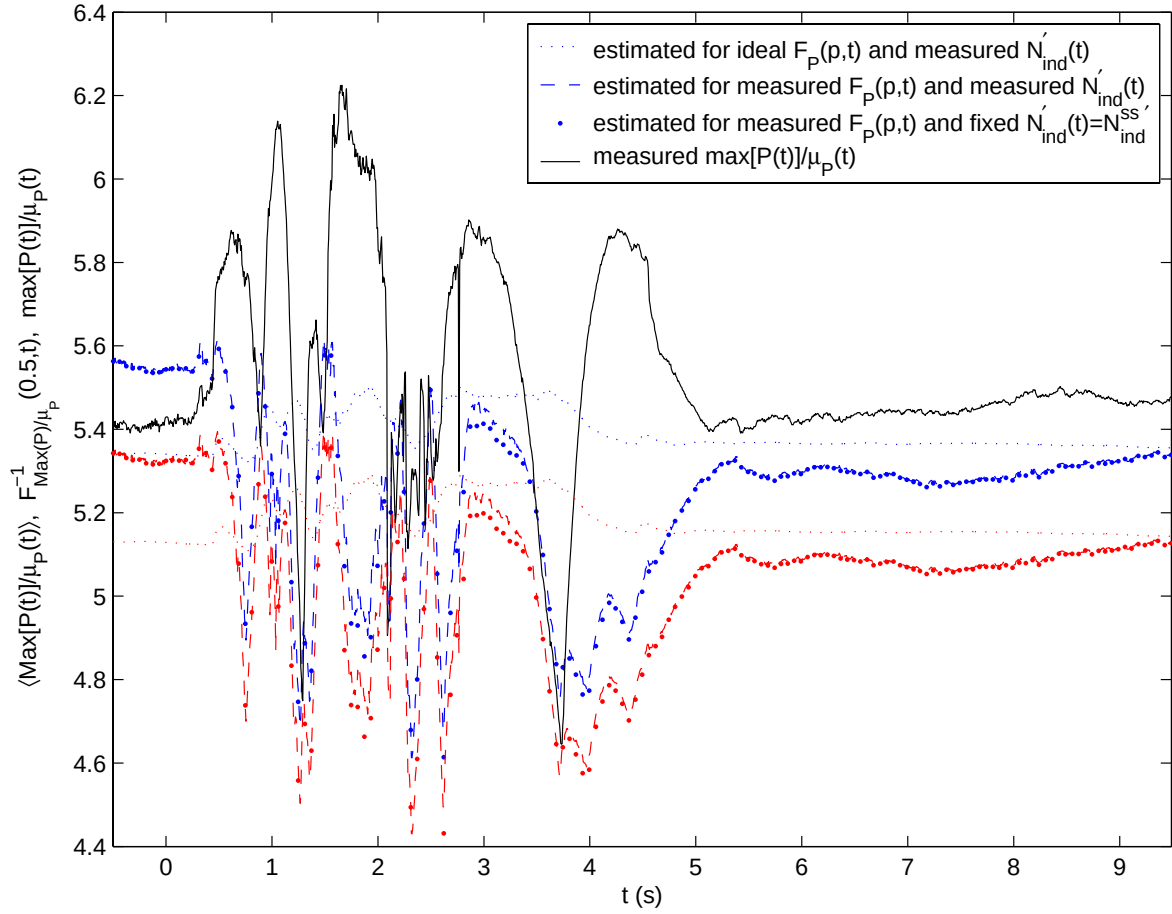


Figure 103: (color online) Maximum-to-mean ratio of P during transients as a function of time at $f = 1000$ MHz, for $M = 400$ steps ($N = 100$), with $A = 1000$ steps/s², $B = 1$ step/s, $V = 2000$ steps/s: comparison between measured and estimated values. Dotted lines: semi-empirical estimates based on fixed ideal χ^2_2 cdf $F_P(p, t)$ and fluctuating measured $N_{\text{ind}}^{(r)}(t)$; dashed lines: fully-empirical estimates based on fluctuating measured cdfs $F_P(p)$ and fluctuating measured $N_{\text{ind}}^{(r)}(t)$; dot symbols: fully-empirical estimates based on fluctuating measured cdfs $F_P(p)$ and fixed measured $N_{\text{ind}}^{(r)}(t) = N_{\text{ind}}^{\text{ss}(r)}$. Black solid curve: measured values $\text{max}[P(t)]/\mu_P(t)$; blue curves: estimated mean values $\langle \text{Max}[P(t)]/\mu_P(t) \rangle$; red curves: estimated median values $F_{\text{Max}[P(t)]/\mu_P(t)}^{-1}(1/2, t)$. Estimates of $N_{\text{ind}}^{(r)}(t)$ are based on (17).

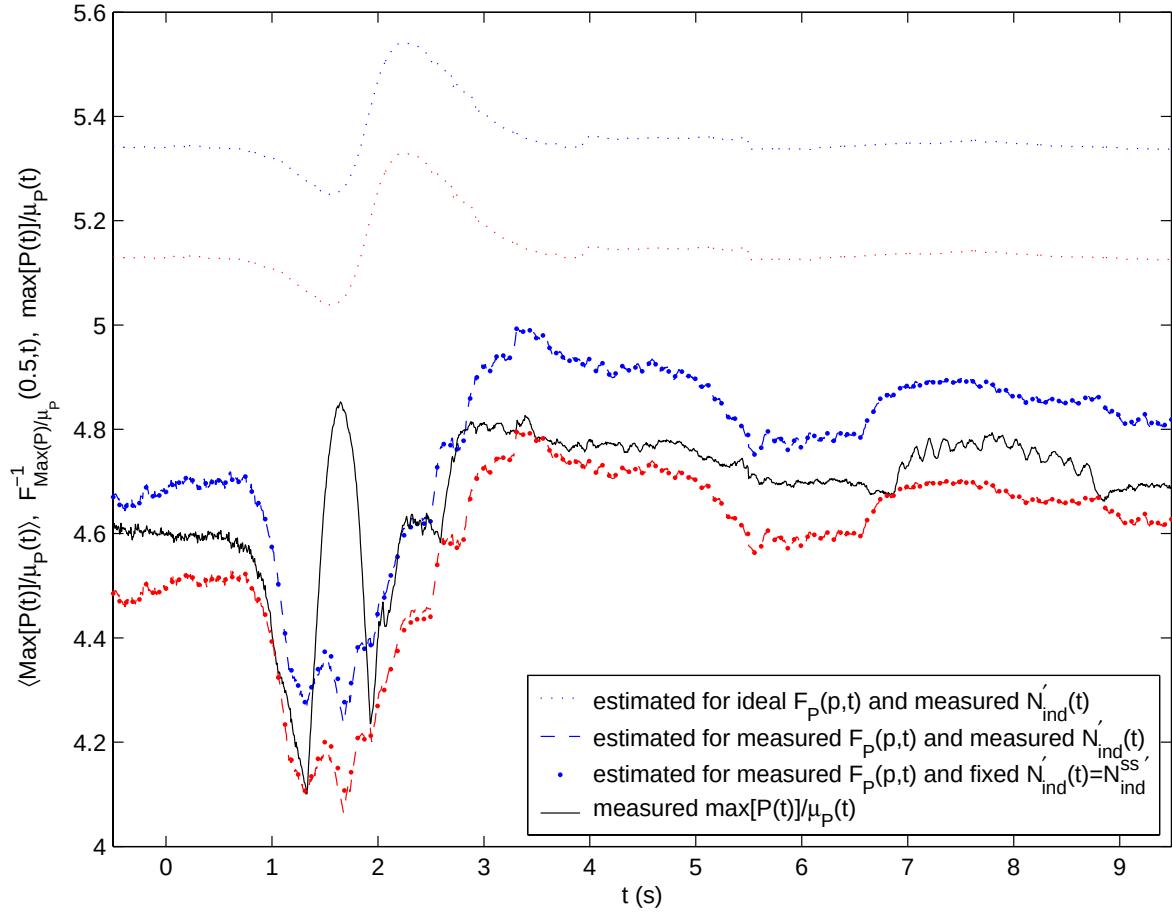


Figure 104: (color online) Maximum-to-mean ratio of P during transients as a function of time at $f = 1000$ MHz, for $M = 400$ steps ($N = 100$), with $A = 100$ steps/s², $B = 10$ steps/s, $V = 2000$ steps/s: comparison between measured and estimated values. Dotted lines: semi-empirical estimates based on fixed ideal χ_2^2 cdf $F_P(p, t)$ and fluctuating measured $N_{\text{ind}}^{(r)}(t)$; dashed lines: fully-empirical estimates based on fluctuating measured cdfs $F_P(p)$ and fluctuating measured $N_{\text{ind}}^{(r)}(t)$; dot symbols: fully-empirical estimates based on fluctuating measured cdfs $F_P(p)$ and fixed measured $N_{\text{ind}}^{(r)}(t) = N_{\text{ind}}^{\text{ss}(r)}$. Black solid curve: measured values $\text{max}[P(t)]/\mu_P(t)$; blue curves: estimated mean values $\langle \text{Max}[P(t)]/\mu_P(t) \rangle$; red curves: estimated median values $F_{\text{Max}[P(t)]/\mu_P(t)}^{-1}(1/2, t)$. Estimates of $N_{\text{ind}}^{(r)}(t)$ are based on (17).

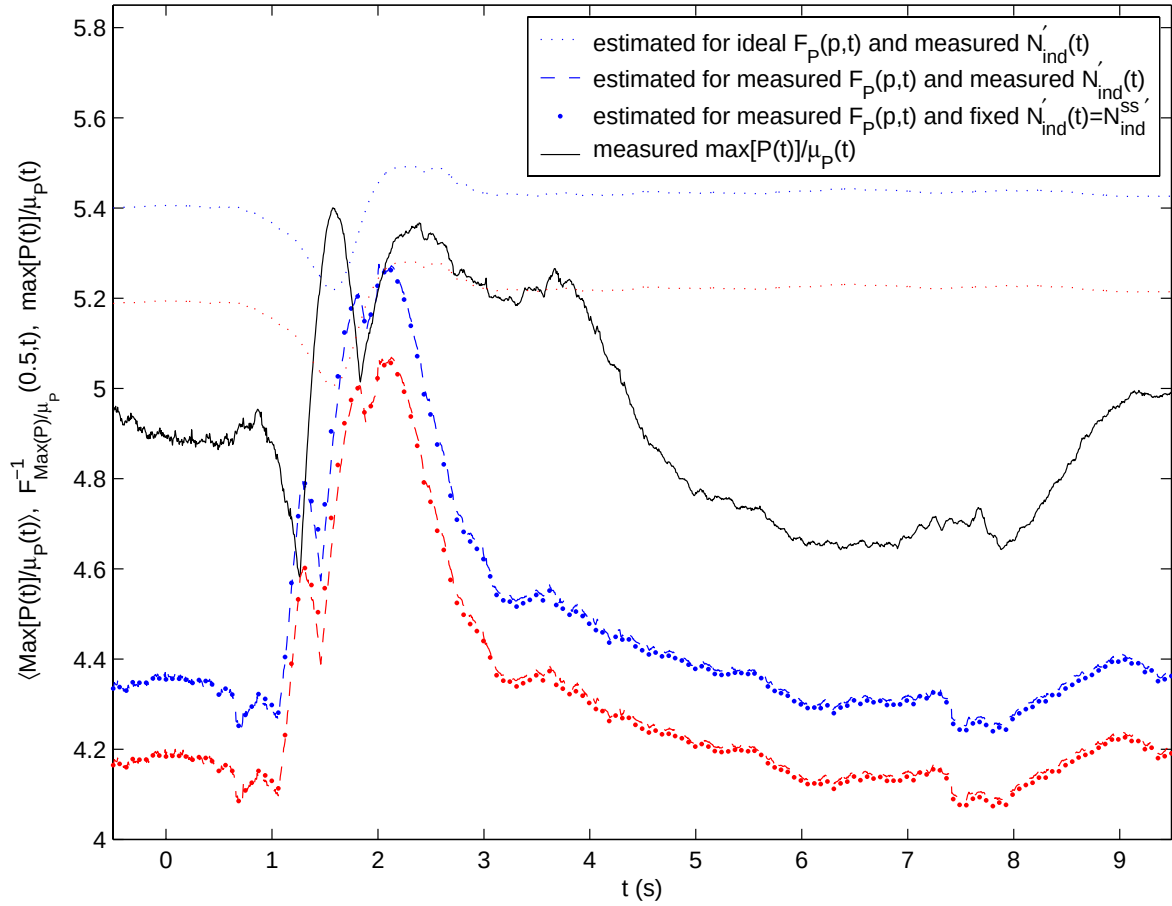


Figure 105: (color online) Maximum-to-mean ratio of P during transients as a function of time at $f = 1000$ MHz, for $M = 400$ steps ($N = 100$), with $A = 100$ steps/s², $B = 10$ steps/s, $V = 200$ steps/s: comparison between measured and estimated values. Dotted lines: semi-empirical estimates based on fixed ideal χ^2_2 cdf $F_P(p, t)$ and fluctuating measured $N_{\text{ind}}^{(l)}(t)$; dashed lines: fully-empirical estimates based on fluctuating measured cdfs $F_P(p)$ and fluctuating measured $N_{\text{ind}}^{(l)}(t)$; dot symbols: fully-empirical estimates based on fluctuating measured cdfs $F_P(p)$ and fixed measured $N_{\text{ind}}^{(l)}(t) = N_{\text{ind}}^{\text{ss}(l)}$. Black solid curve: measured values $\text{max}[P(t)]/\mu_P(t)$; blue curves: estimated mean values $\langle \text{Max}[P(t)]/\mu_P(t) \rangle$; red curves: estimated median values $F_{\text{Max}[P(t)]/\mu_P(t)}^{-1}(1/2, t)$. Estimates of $N_{\text{ind}}^{(l)}(t)$ are based on (17).

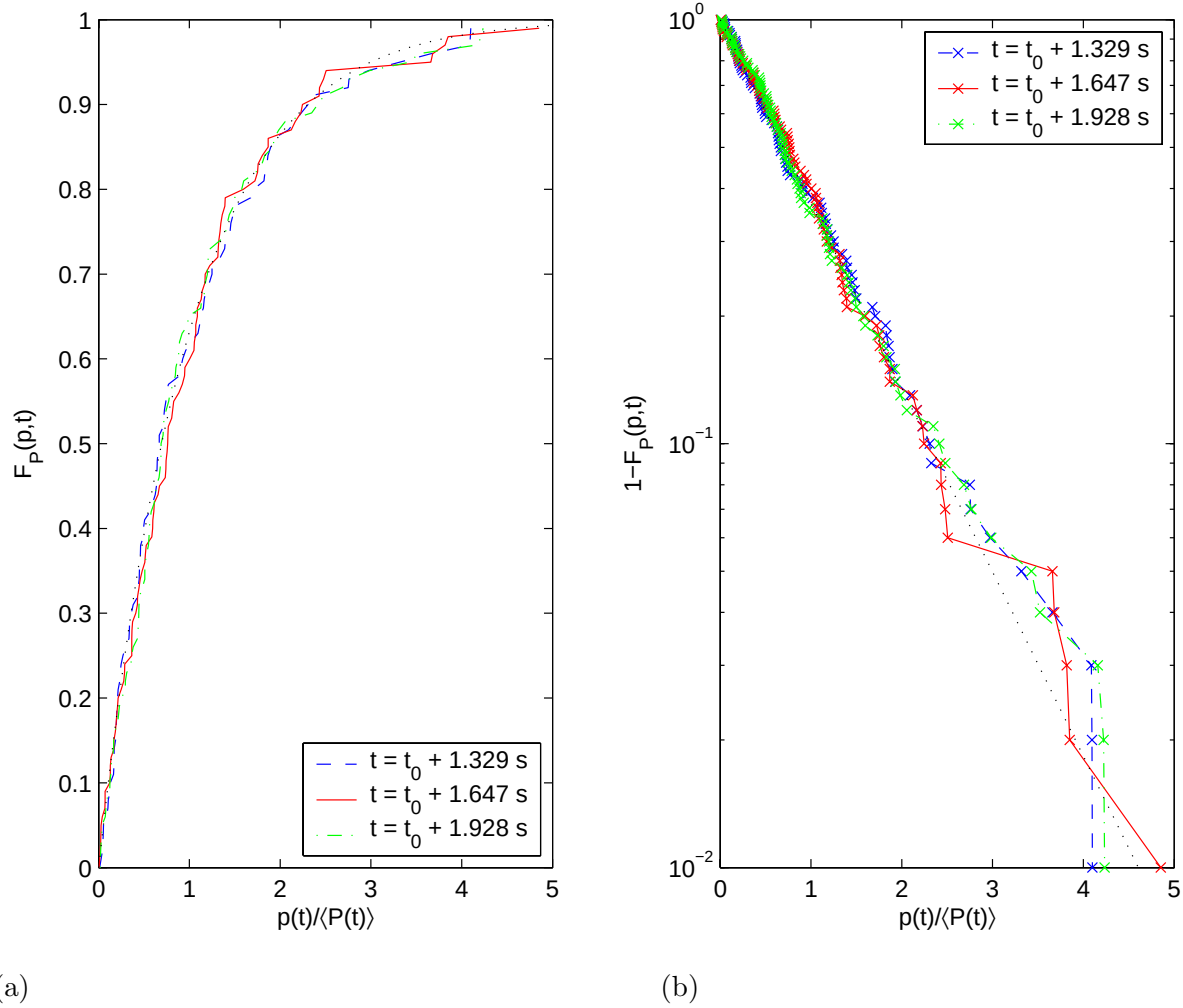


Figure 106: (color online) Cumulative distribution function of P at extremal points of anomalous lobe of measured transient $\max[P(t)]/\mu_P(t)$ at $f = 1000$ MHz, for $M = 400$ steps ($N = 100$), with $A = 100$ steps/s², $B = 10$ steps/s, $V = 2000$ steps/s: comparison between ideal quasi-stationary χ^2_2 distribution (black dotted line) and measured distributions at the local minima (at $t = t_0 + 1.329$ s and $t = t_0 + 1.928$ s) and maximum (at $t = t_0 + 1.647$ s) of the experimental lobe shown in Fig. 104: (a) cumulative distribution; (b) complementary cumulative distribution.

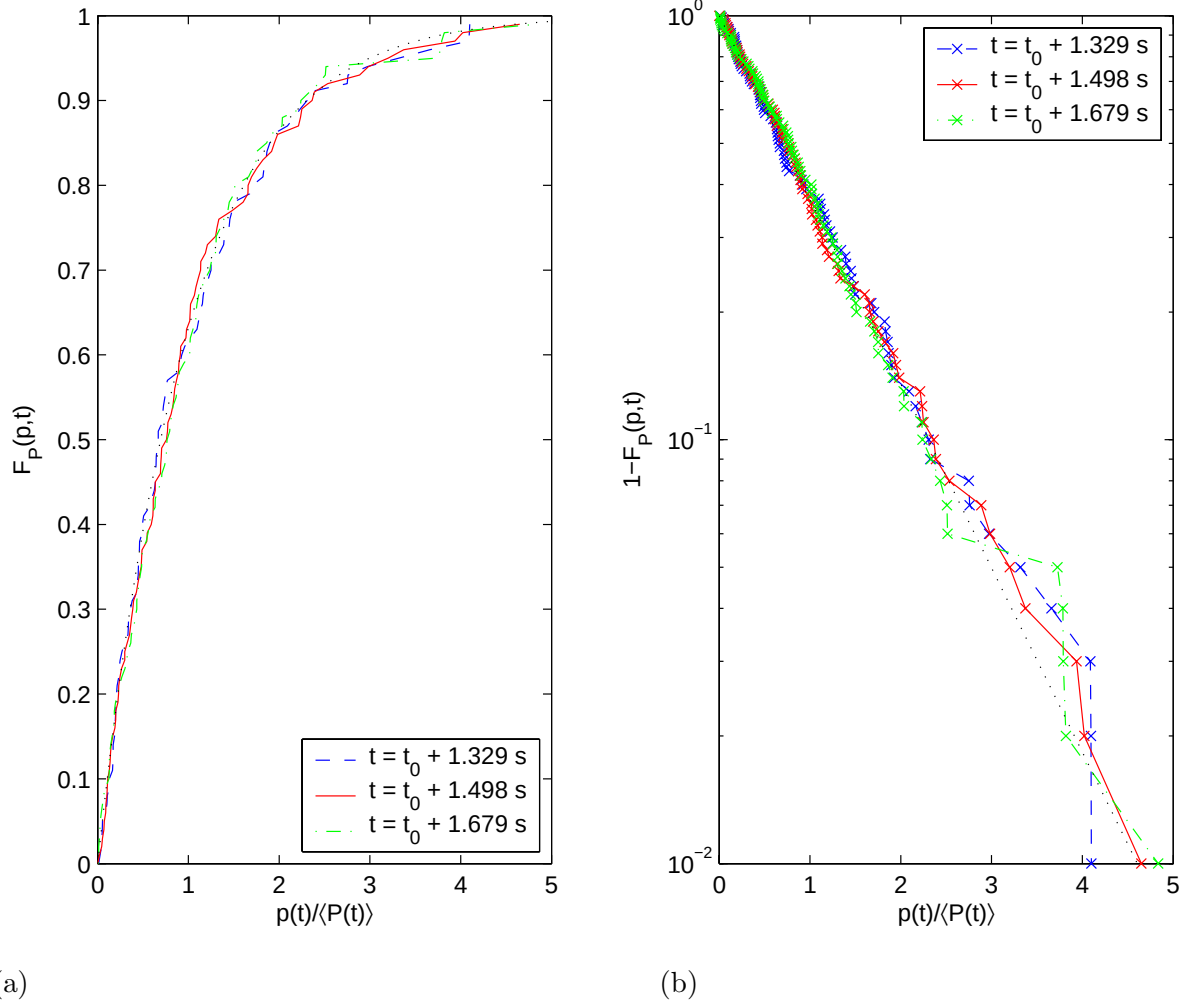


Figure 107: (color online) Cumulative distribution function of P at extremal points of anomalous lobe of fully-empirically estimated transient $\max[P(t)]/\mu_P(t)$ at $f = 1000$ MHz, for $M = 400$ steps ($N = 100$), with $A = 100$ steps/s², $B = 10$ steps/s, $V = 2000$ steps/s: comparison between ideal quasi-stationary χ^2_2 distribution (black dotted line) measured distributions at the local minima (at $t = t_0 + 1.329$ s and $t = t_0 + 1.679$ s) and maximum (at $t = t_0 + 1.498$ s) of the fully-empirically estimated lobe shown in Fig. 104: (a) cumulative distribution; (b) complementary cumulative distribution.

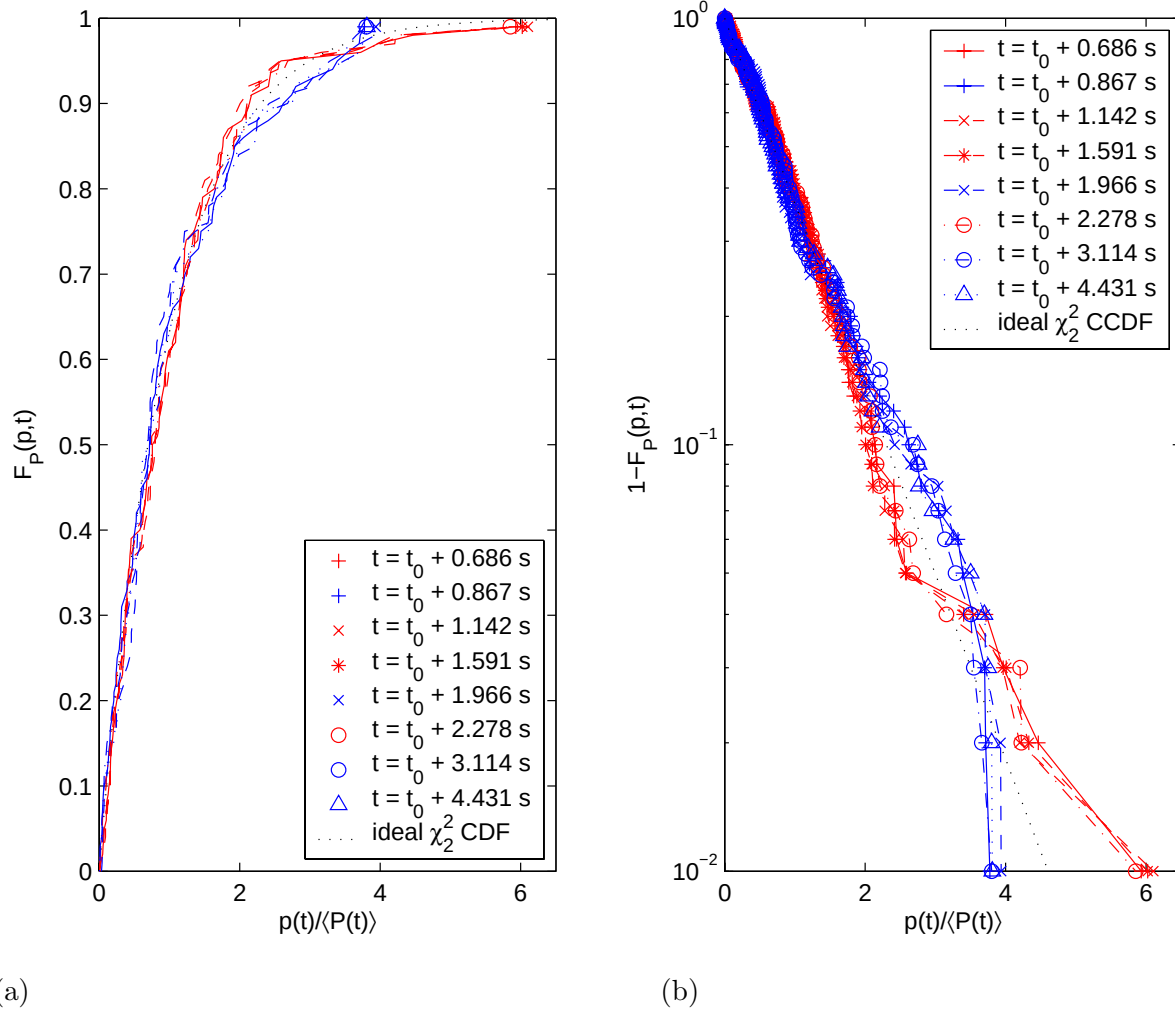


Figure 108: (color online) Cumulative distribution function of P at extremal points of anomalous lobe of measured transient $\max[P(t)]/\mu_P(t)$ at $f = 900$ MHz, for $M = 400$ steps ($N = 100$), with $A = 1000$ steps/s², $B = 600$ steps/s, $V = 2000$ steps/s: comparison between ideal quasi-stationary χ^2_2 distribution (black dotted line) and measured distributions at the local maxima (red curves) and minima (blue curves) of the experimental lobe shown in Fig. 89: (a) cumulative distribution; (b) complementary cumulative distribution.

8 Effect of source switching on maximum test level

In Sec. 1, it was noted that source switching – as part of a procedure of source blanking aimed at mitigating the effects of transient fields during paddle wheel movement – may introduce unwanted field dynamics and EM transient fields of its own. Here, we illustrate the possible effect of such transients on the maximum received power for a pulse-amplitude modulated input signal. In this way, we mimic the switch-on or switch-off of an otherwise steady-state CW excitation signal.

Because of the strongly dispersive nature of a reverberation chamber, substantial pulse distortion will occur if the pulse rise time is sufficiently short. For example, the field decay after switch-off may start with a significant overshoot above the steady-state level before the average decay sets in. This is illustrated in Fig. 109, showing an extreme (but by no means exceptional!) example of overshoots in both the leading and trailing edges of the envelope of a pulse response. This response was measured using a power spectrum analyzer and a separate pulsed source producing a pulse amplitude modulated RF input signal with carrier frequency 1000 MHz, duty cycle 40%, and rise time 150 ns.

Figure 110 shows the maximum values for the measured pulse response across $N = 1000$ mechanically steady-state paddle wheel positions at 200, 800, and 1000 MHz, after synchronization with respect to their starting times. The diagrams indicate that overshoots during switching of pulses are the rule rather than the exception, at any frequency. It may be expected that increasing the rise/fall times of the input pulse decreases the magnitude of these overshoots. However, because of the stochastic nature of the received power, overshoots can never be completely eliminated. Thus, the analysis of transients of CW fields in mode-tuned operation is a problem of general and fundamental interest. In [13]–[16], several aspects associated with the effect of transients and more general nonstationarity for discrete or continuous rotation of the paddle wheel are discussed.

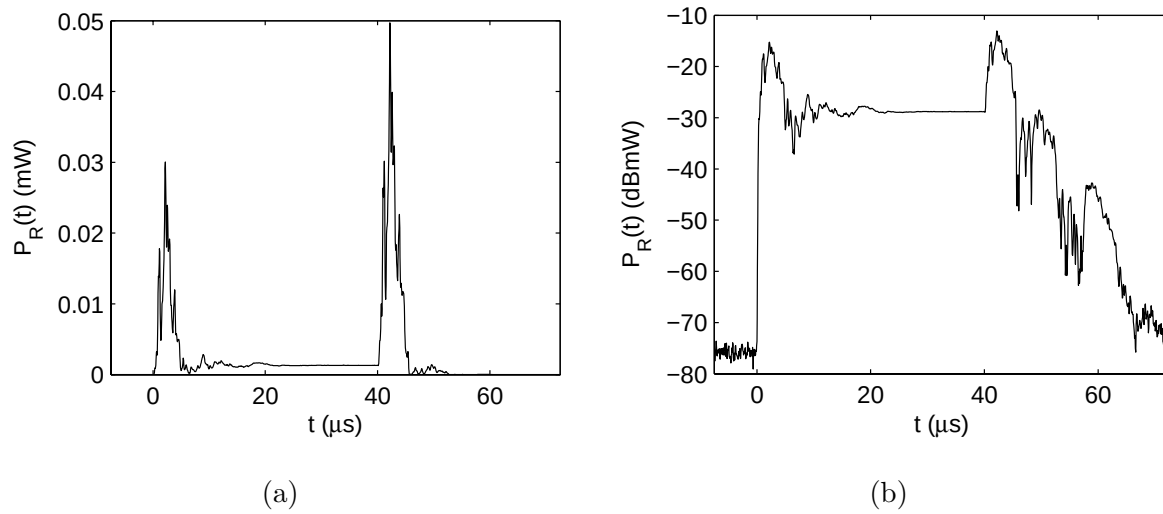


Figure 109: Example of measured chamber response (envelope $P_R(t)$ of received power) for a pulse-amplitude modulated RF excitation signal at $f = 1000$ MHz, for an arbitrary position of the mode tuner: (a) linear scale, (b) logarithmic scale (data courtesy of D. A. Knight).

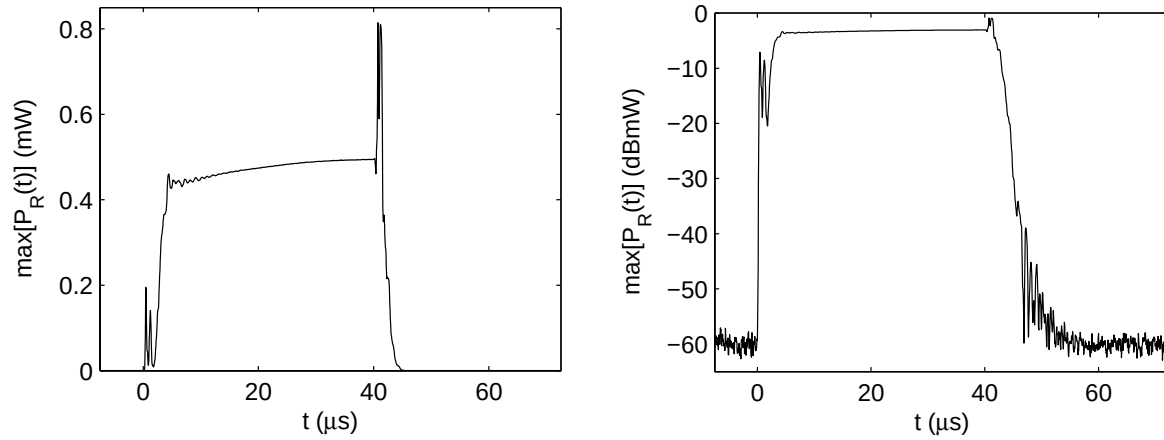
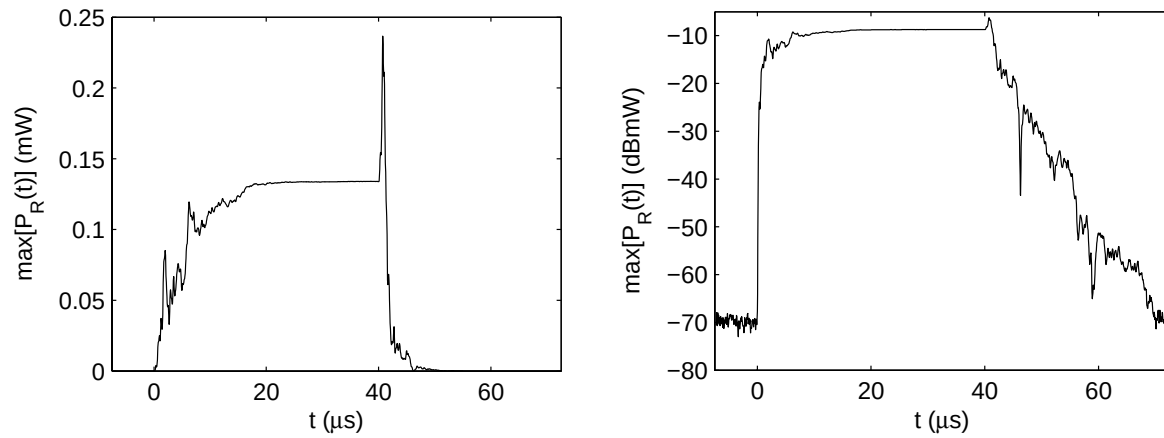
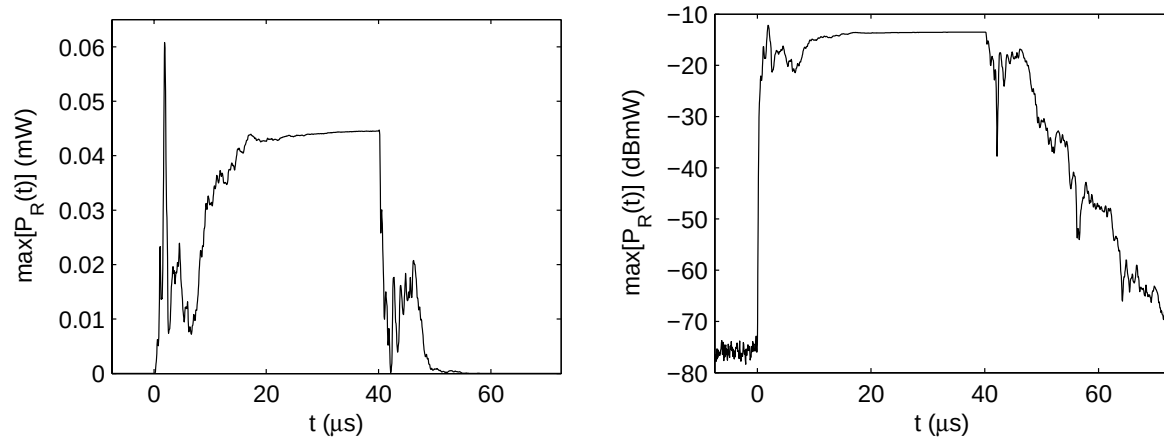
(a) $f = 200$ MHz(b) $f = 800$ MHz(c) $f = 1000$ MHz

Figure 110: Maximum measured chamber response over all N envelopes $P_R(t; \theta_n)$ of received power, for a pulse-amplitude modulated RF excitation signal and arbitrary position of the mode tuner, at (a) 200 MHz, (b) 800 MHz, (c) 1000 MHz, in linear (left) and logarithmic (right) scales (data courtesy of D. A. Knight).

9 Conclusions

In this report, we presented measured data with a statistical analysis for field transients generated by the stepwise rotation of a mode tuner (reflective large paddle wheel) inside a reverberation chamber. The analysis was based on time-domain characterization of the scattering transmission parameter $|S_{21}(t)|$ across the tuner transitions, sampled at 160.1 Hz. On account of the statistical equivalence of the tuner transitions at any age $t = t_\ell - t_0$ during the steps, wide-sense stationary stir sequences were extracted by transposition of the measured time traces.

Obviously, in the limit when the maximum instantaneous rotation speed during stepping or stirring becomes arbitrarily slow, i.e., $\max_{\{t\}}[\Omega(t)] \rightarrow 0$, all mechanical and electromagnetic transitions and their time traces become quasi-stationary and smooth, irrespective of the operating frequency. For $\Omega(t) \neq 0$, however, a number of observations and conclusions follow from our experiments:

1. In the undermoded regime ($f \leq 200$ MHz), large spikes occur near the start of each tuner transition, even when consecutive steady-state values [which produce time-domain plateaus in Fig. 2(a), etc.] are strongly correlated. This produces a “crackling” effect, i.e., any single mode migration during any transition producing a comparatively large effect on the transient response in the traces, which can be attributed to the relatively small modal density in the undermoded regime. Crackling was confirmed by calculating fan-out diagrams for the transitions, showing stepwise increments (Figs. 11–14).
2. Also in undermoded regime, the jumps between steady-state levels (i.e., in the typical tuner sweep data for stationary positions of the mode tuner) are not entirely regular as a function of time, for uniform sampling [Fig. 2(a)]. This suggests field inhomogeneity (nonstationarity) with respect to the stir states θ_n .
3. Although no single motor parameter is solely responsible for smoothing or disrupting field transitions, significantly less abrupt transitions are obtained (Fig. 6) by lowering the instantaneous speed $\Omega(t)$, which is governed mainly by the value of the base speed B of the motor and, to a lesser extent, its acceleration A .
4. Calculation of the cross-correlation coefficient between pairs of equal-time stir sequences, as a function of elapsed time, enables the detection of the oscillatory motion of the mode tuner, as measured through its effect on the value of $|S_{21}|^2$ (Fig. 15). The period of

the damped oscillations is independent of the frequency of the RF source signal. Steady state is reached most rapidly for the *slowest* rotations, despite the then longer rotation time, because posterior mechanical oscillations are then minimized. The smoothness and amplitude of the cross-correlation function increases with increasing excitation frequency.

5. Generally, the mean value and standard deviation of $|S_{21}(t)|^2$ show fluctuations of the order of 5...15% during the transitions, for the chosen sample size (Figs. 20–29). This level of variability is of the same order of magnitude as what can be expected for the variability (*random* fluctuation) on the basis of the standard deviation of the sample mean ($= \sigma_P(t)/\sqrt{N_{\text{ind}}} \approx \mu_P(t)/\sqrt{N_{\text{ind}}} \sim \mu_P(t)/\sqrt{N}$). Fluctuations in the coefficient of variation are comparable, although somewhat smaller (Figs. 30–34).
6. For the set of parameters yielding the slowest tuner transition in our experiments (viz., acceleration 100 steps/s², base speed 10 steps/s), the mean value of $|S_{21}|^2$ exhibits distinctly different values during the transition, showing regular smooth temporal variations that closely follows the instantaneous velocity profile (Fig. 23). The comparatively slow nonuniformity of the mean value points at nonstationarity of the transient field.
7. The measured sample distributions (cdfs) during the transitions all fall within a relatively narrow range, suggesting that the type (but not the statistics) of the distribution does not change dramatically during the stepping transitions (Figs. 35–44). Similar to the analysis of the quasi-static distributions [27], compared to an ideal χ^2_2 exponential distribution, a systematic difference (longer (shorter) PDF tails) is observed at transition times corresponding to local maxima (minima), even at relatively high frequencies (Fig. 51).
8. The measured maximum-to-mean ratio shows distinct fluctuations during the tuner transitions, in the form of smooth (deterministic, long-term) variations that cannot be attributed to the statistical uncertainty of the field (random, short-term fluctuations). These variations can be witnessed from the different time scales of fluctuations that are involved (order of seconds of transition time, compared to ms for sampling interval and μ s for mode relaxation, respectively): the timings of marked changes in the measured maximum-to-mean ratio (Figs. 52–56) match closely those of the turning points in the mechanical motion, either for its position or acceleration. The variations of the maximum-to-mean ratio are, on average, in phase with those in the correlation length of the stir sequence, but are quantitatively much larger than predicted semi-empirically. This suggests the ex-

istence of a complicated mode bunching/dilation effect during the transitions caused by distinct phases in the tuner motion and tuner acceleration. Unless both the acceleration and base speed are high (in which case these fluctuations manifest themselves as quasi-random, at any time scale), the oscillations in the tuner movement can be observed in the time evolution of the maximum-to-mean ratio (Figs. 53–55). This ratio always shows markedly larger values during the transition than in mechanical steady state, particularly at lower frequencies (Figs. 81–83), but the fairly coarse time step used in our experiments (6.242 ms) has prevented a precise quantification of the extent of this increase at these low frequencies. Specifically, in the overmoded regime, the higher the acceleration, the more likely the upper limit of a specified $\eta\%$ -confidence interval for the maximum-to-mean ratio will be approached or exceeded during the transition (Figs. 86–90). The detailed variations of the maximum-to-mean ratio were shown to be mainly due to changes of the underlying sample distribution for $P(t)$ (Figs. 81–90). They are also caused by variations of the correlation length of stir sequences, showing good qualitative agreement of timings for moderately rapid rotations (Figs. 95–96). Certain experimentally observed features, however, cannot be completely tracked or explained, neither qualitatively nor quantitatively, in the framework of quasi-stationarity and effects of limited sample size of the sequences, even at the lowest speeds investigated here (cf. lobes in Figs. 104–105). A preliminary investigation involving tracking and inspection of the underlying cdf for $P(t)$ during these anomalies, at the lowest speeds used in this study, suggests the occurrence of a distinctly greater deviation from the ideal χ^2_2 cdf, with sample values in the upper tail becoming more clustered (Fig. 106). The extent of the variations of the observed maximum-to-mean ratio is of the same order as the variations induced by changes in the tail of the experimental cdf. Additional measurements involving a larger number of tuner steps, slower speeds and accelerations, and smaller sample time step are needed for a more detailed investigation.

9. For large accelerations and/or base speeds, the average, standard deviation, and maximum value of the rate of fluctuation of the field (Figs. 58–59, 61–64), its modulation coefficient (Figs. 68–70) or modulation index all show a sharp rise at the start of the motion followed by a first-order (exponential) decay, with superimposed oscillations caused by the tuner oscillations. For lower speeds, these statistics show again a symmetric triangular pattern which matches the velocity profile qualitatively (Figs. 65–67, 71–74).

10. The theoretical distribution of the rate of fluctuation for finite differences assuming ideal isotropic and generally inhomogeneous random fields has been derived (Appendix A).
11. Preliminary measurements using pulse-modulated signals for mechanical steady states indicate that overshoots are more the rule than the exception when switching the RF source on or off (Figs. 109–110). This poses particular requirements on the rate with which an excitation signal can be switched off and back on again, as part of a source blanking procedure to mitigate the effect of stepping transitions, digitally modulated fields, etc.

10 Acknowledgements

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A Appendix: Distribution of Correlated Sample Differences for Homogeneous or Inhomogeneous Energy Density

We derive the pdf of the sample difference $|E_1|^2 - |E_2|^2$ between the normalized local electric energy densities $|E(\underline{r}_i)|^2 \triangleq |E_i|^2$ at two locations $\underline{r}_1$ and $\underline{r}_2$ in close vicinity in space or time. The underlying complex fields $E_i \equiv E'_i - jE''_i$ are assumed to be centered circular Gauss normal ($\mu_{E'_i} = \mu_{E''_i} = 0$, $\sigma_{E'_i} = \sigma_{E''_i} \triangleq \sigma_i$), but not necessarily identically distributed unless the field is statistically homogeneous, in which case $\sigma_{E_1^{(\prime)}} = \sigma_{E_2^{(\prime)}} \triangleq \sigma$. The statistical dependence between the two fields is expressed by their mutual correlation coefficient ρ , i.e.,

$$\rho_{E'_1 E'_2} = \rho_{E''_1 E''_2} \triangleq \rho, \quad \rho_{E'_1 E''_2} = \rho_{E''_1 E'_2} = 0. \quad (26)$$

Since the real and imaginary parts per location are mutually independent anyway, we give details for the calculation for the real parts only. Using the identities

$$(E_1 \mp E_2)(E_1 \pm E_2)^* = |E_1|^2 - |E_2|^2 \mp j2\Im(E_1 E_2^*) \quad (27)$$

the sample difference of the energy densities can be expanded in terms of sums and differences of components of the underlying fields as

$$|E_1|^2 - |E_2|^2 = \frac{1}{2} [(E_1 - E_2)(E_1 + E_2)^* + (E_1 + E_2)(E_1 - E_2)^*] \quad (28)$$

$$= (E'_1 + E'_2)(E'_1 - E'_2) + (E''_1 + E''_2)(E''_1 - E''_2). \quad (29)$$

Defining

$$U_{\pm} \triangleq E'_1 \pm E'_2 \quad (30)$$

we have

$$f_{U_{\pm}}(u_{\pm}) = \int_{-\infty}^{+\infty} f_{E'_1, E'_2}(u_{\pm} \mp e'_2, e'_2) de'_2 = \frac{\exp\left(-\frac{u_{\pm}^2}{2\sigma_{U_{\pm}}^2}\right)}{\sqrt{2\pi\sigma_{U_{\pm}}^2}} \quad (31)$$

in which $\sigma_{U_{\pm}}^2 = \sigma_{E'_1}^2 + \sigma_{E'_2}^2 \pm 2\rho\sigma_{E'_1}\sigma_{E'_2}$ and where

$$f_{E'_1, E'_2}(e'_1, e'_2) = \frac{\exp\left[-\frac{1}{2(1-\rho^2)}\left(\frac{e'^2_1}{\sigma_{E'_1}^2} + \frac{e'^2_2}{\sigma_{E'_2}^2} - \frac{2\rho e'_1 e'_2}{\sigma_{E'_1}\sigma_{E'_2}}\right)\right]}{2\pi\sqrt{1-\rho^2}\sigma^2} \quad (32)$$

is the joint pdf for two correlated centered Gauss normally distributed E'_1 and E'_2 . Thus, as may be expected on account of linearity, U_+ and U_- are themselves centered Gauss normal ($\langle U_+ \rangle = \langle U_- \rangle = 0$). If $\sigma_{E_1^{(\prime)}} = \sigma_{E_2^{(\prime)}} = \sigma$, then $\sigma_{U_{\pm}} = \sqrt{2(1 \pm \rho)}\sigma$.

Next, with the aid of [28, eq. (3.471.9)], we obtain the pdf of $X \triangleq U_+ \cdot U_-$ as

$$\begin{aligned}
 f_X(x) &= \int_{-\infty}^{+\infty} |u|^{-1} f_{U_+, U_-} \left(u, \frac{x}{u} \right) du = \frac{1}{2\pi \sqrt{1 - \rho_{U_+ U_-}^2} \sigma_{U_+} \sigma_{U_-}} \\
 &\times \int_{-\infty}^{+\infty} \exp \left[-\frac{1}{2(1 - \rho_{U_+ U_-}^2)} \left(\frac{u^2}{\sigma_{U_+}^2} + \frac{x^2}{u^2 \sigma_{U_-}^2} - \frac{2\rho_{U_+ U_-}}{\sigma_{U_+} \sigma_{U_-}} x \right) \right] \frac{du}{|u|} \\
 &= \frac{\exp \left[\frac{\rho_{U_+ U_-} x}{(1 - \rho_{U_+ U_-}^2) \sigma_{U_+} \sigma_{U_-}} \right] K_0 \left[\frac{|x|}{(1 - \rho_{U_+ U_-}^2) \sigma_{U_+} \sigma_{U_-}} \right]}{\pi \sqrt{1 - \rho_{U_+ U_-}^2} \sigma_{U_+} \sigma_{U_-}}, \quad -\infty < x < +\infty \quad (33)
 \end{aligned}$$

in which $K_0(\cdot)$ represents the modified Bessel function of the second kind (Macdonald's function) of order zero, and where

$$\rho_{U_+ U_-} = \frac{\sigma_{E'_2}^2 - \sigma_{E'_1}^2}{\sqrt{(\sigma_{E'_1}^2 + \sigma_{E'_2}^2)^2 - (2\rho \sigma_{E'_1} \sigma_{E'_2})^2}}. \quad (34)$$

For homogeneous fields, $\rho_{U_+ U_-} = 0$, as a result of $\sigma_{E'_1} = \sigma_{E'_2}$ and irrespective of the value of ρ , whence $\langle U_+ U_- \rangle = 0$. The pdf of $Y \triangleq (E''_1 + E''_2)(E''_1 - E''_2)$ is identical to $f_X(x)$ because the real and imaginary field components are identically distributed and identically correlated.

The pdf of $|E_1|^2 - |E_2|^2 \triangleq Z = X + Y$ finally follows as

$$\begin{aligned}
 f_Z(z) &= \int_{-\infty}^{+\infty} f_X(z - y) f_Y(y) dy = \frac{\exp \left[\frac{\rho_{U_+ U_-} z}{(1 - \rho_{U_+ U_-}^2) \sigma_{U_+} \sigma_{U_-}} \right]}{(2\pi \sqrt{1 - \rho_{U_+ U_-}^2} \sigma_{U_+} \sigma_{U_-})^2} \\
 &\times \int_{-\infty}^{+\infty} K_0 \left[\frac{|z - y|}{(1 - \rho_{U_+ U_-}^2) \sigma_{U_+} \sigma_{U_-}} \right] K_0 \left[\frac{|y|}{(1 - \rho_{U_+ U_-}^2) \sigma_{U_+} \sigma_{U_-}} \right] dy. \quad (35)
 \end{aligned}$$

In the limit where both locations and, hence, their associated local energy densities approach each other arbitrarily closely in space or time, i.e., $|\Delta r| \equiv ||\underline{r}_1 - \underline{r}_2|| \rightarrow 0$ but $\rho \rightarrow 0$ and $\sigma_{E'_1} \rightarrow \sigma_{E'_2}$, the scaled sample difference $Z/|\Delta r|$ approaches the local derivative $|\dot{E}|^2 \equiv d(|E|^2)/dr$ (tacitly assuming that the random field is mean-square differentiable). The pdf of this derivative for Cartesian field components is known to be a Laplace two-sided negative exponential distribution [24, eq. (87)]:

$$f_{|\dot{E}|^2}(|\dot{e}|^2) = \frac{\exp \left(-\frac{\sqrt{2}|\dot{e}|^2}{\sigma_{|\dot{E}|^2}} \right)}{\sqrt{2}\sigma_{|\dot{E}|^2}}, \quad -\infty < |\dot{e}|^2 < +\infty \quad (36)$$

as a limiting case of (35) in which $\rho \rightarrow 1$ which constitutes a singularity. Its corresponding expression for the total (vector) field is also known [24, eq. (91)].