

# Report

**NPL REPORT  
DEPC-EM 009**

The Effect of Hydraulic Fluids  
on Pressure Balances

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John Greenwood**

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February 2007



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## The Effect of Hydraulic Fluids on Pressure Balances

Claudiu Giusca, Ian Severn, John Greenwood  
DEPC

### ABSTRACT

This work was initiated as a response to growing concerns about the effect of using different oils in Pressure Balances, which are used to realise the pressure scale both as national standards and as calibration instruments in the industry.

We report, for the first time, systematic errors in the effective area of a Pressure Balance that are directly attributable to the use of different oils as pressure transmitting fluid. To avoid the cancelling effects of the traditional cross-floating technique, an alternative technique is reported in which the measurements are made using two separate oils simultaneously. The influence of four lubricating oils on three nominal sizes of piston-cylinders assemblies, covering the pressure range 2.8 MPa to 200 MPa, has been investigated.

The change in effective area relative to operation with a sebacate oil was also found to be dependent on the pressure and therefore on gap size, however at this time we cannot make any quantitative predictions relating the size of the effect to the gap.

Although a difference related to the liquid viscosity (due to its contribution to the pressure distribution in the piston-cylinder inner space) was intuitively expected, the measurements instead appear to correlate more closely with the liquid density.

This work has been performed under the National Measurement System Directorate Engineering Measurement Programme, Project 2.19: *Improved Realisation of conventional high-pressure scale*.

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ISSN 1744-0254

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Approved on behalf of the Managing Director, NPL  
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# 1 INTRODUCTION

At NPL the aim of high-pressure metrology is to establish standards, based on fundamental units, to meet the demands of science and industry for accurate measurements.

One of the methods by which pressure can be measured is in terms of the force that acts on a surface of known area. The instrument using this approach is known as the pressure balance [1], in which the downward force, mainly due to known masses supported on the cylindrical piston, balances the pressure acting on the other end of the piston [2].

Pressure balances are widely used in the range 3.5 kPa to 500 MPa. They are used to realise the pressure scale at all stages of the traceability chain, both as national standards and as calibration instruments in industry.

A formal theory of the pressure balance, based on the laws of fluid dynamics, was developed in the mid 1960s [3]. This theory has proved to be extremely successful, however, in recent years, with the introduction of modern manufacturing techniques and closer tolerances, a certain amount of anomalous behaviour has been observed. These anomalies have appeared in particular when using different gas species [4,5] as the pressure transmitting fluid and during international comparisons done using oil as the transmitting fluid at pressures above 100 MPa [6,7]. The most likely explanation is that the assumptions used to formulate the theoretical model for the pressure balance are no longer adequate.

In particular, the basic theory does not predict a change of the piston-cylinder's effective area when using different liquids as pressure transmitting media, whereas a difference related to the liquid viscosity, due to its contribution to the pressure distribution in the piston-cylinder gap, is expected. As indicated in earlier work [1], if any such changes do occur, they will affect the effective area of the piston-cylinder at high pressures, thus having an effect on the 'distortion coefficient' of the effective area.

In general, calibration laboratories use sebacate oil (di-2-ethylhexylsebacate) in hydraulically operated devices, but some laboratories also operate and calibrate instruments using different oils. The common approach is to calibrate the hydraulic pressure balance (HPB) piston-cylinder assembly (PCA) in the same fluid as is used for the standard instrument.

The current work within the scope of National Measurement System Directorate Engineering Measurement Programme, Project 2.19: *Improved Realisation of Conventional High Pressure*

*Scale* which was initiated as a response to growing concerns about the use of different oils in pressure balances. The project started in September 2005 and was initially targeted towards UKAS accredited laboratories, however there was also an input from academia and industry. Initially, more than 20 individuals expressed an interest in the outcome of this study and supported the project: N Buckeridge (GE Sensing), N Morgan (Chamois Metrology), T Johnson (DACO Scientific), D Highton (ANTECH Calibration Services), R Hewett (SIRA Test & Certification Ltd), S Tyson (MOD), M Parkinson (DH Budenberg), P Clow (UKAS), T Thompson (UKAS), G Brown (Sonardyne International Ltd), R Bilham (R B Consultants), C. Dingle (PCD), K Moore (Short Brothers Plc), K Scrimshaw (KHS Engineering Developments), J Tyrrell (Variomh Components), S Ogg (AMEC Internal Asset Management Ltd), N Maplesden (Interleck Testing – ITEX), D Byrne, J C Radon, B Hariyanto (Indonesia), C L Ik (Nigeria), Ambareshudu (India), Lene Schou (Denmark), Shuang Jake Yang (University of Maryland), Makio Iino (Japan).

A questionnaire was designed to discover what types of lubricating liquids are used as pressure transmitting fluids by different laboratories. Following an analysis of the survey results four oils were selected for evaluation (see Table 1).

In this report, Section 2 begins with a statement of the measurement equation for a pressure balance operated in gauge mode. It goes on to show how the change in effective area is calculated when the working fluid is changed in one of a pair of cross-floated pressure balances. Section 3 then describes the experimental arrangement and procedures that were used to make such cross-float measurements. The results are presented in Section 4 and are summarised in Section 5. The uncertainty associated with the change in effective area is described extensively in the Appendix to this report.

## 2 MEASUREMENT EQUATION

The effective area of a ‘*Test*’ pressure balance is usually determined by cross-floating the balance against a similar ‘*Standard*’ pressure balance. In this technique the balances can be operated in either ‘absolute’ or ‘gauge’ mode (where, above the piston, there is vacuum or atmospheric pressure respectively)

The pressure generated by a pressure balance in gauge mode [1] is given by:

$$p = \frac{\left(1 - \frac{1.2}{8000}\right) g \sum_j m_j \left(1 - \frac{\rho_a - 1.2}{\rho_j}\right) + \gamma C - (\rho_l - \rho_a) V g}{A_0 (1 + \lambda p) [1 + \alpha (t - 20)]} + (\rho_l - \rho_a) g h \quad \dots(1)$$

Where, unless specified otherwise, SI units are used throughout and the symbols have the following meaning:

- $p$  - gauge pressure generated by a hydraulic pressure balance
- $A_0$  - effective area of the PCA at null pressure and 20 °C
- $\lambda$  - distortion coefficient of the PCA
- $\alpha$  - combined temperature coefficient of the PCA
- $t$  - temperature of the PCA/ °C
- $m_j$  - conventional mass of the load
- $\rho_j$  - density of the mass artefacts
- $C$  - circumference of the floating element of the PCA at the point where it emerges from the pressure transmitting liquid
- $V$  - volume of floating element submerged in the pressure transmitting liquid that is subject to a liquid buoyancy correction
- $h$  - height difference between the reference level of the pressure balance and arbitrary reference level where the generated pressure is reported
- $\gamma$  - surface tension of pressure transmitting liquid
- $\rho_l$  - density of pressure transmitting liquid

$\rho_a$  - density of the ambient air

$g$  - local value of acceleration due to gravity

During cross-float both instruments, ‘*Test*’ and ‘*Standard*’, are pressurised to the same nominal pressure. At this pressure the loaded mass is adjusted until the floating elements of both pressure balances are falling at their natural rate [1]. The pressure thus generated by the ‘*Standard*’ instrument is equal to the pressure generated by the ‘*Test*’ instrument and a state of equilibrium is achieved. Therefore we have:

$$P_{\text{Standard}} = P_{\text{Test}} \quad \dots(2)$$

For the sake of clarity, all equations in the remainder of this report will use subscript “*St*” for ‘*Standard*’ and “*T*” for ‘*Test*’.

In all of the experiments reported here the cross-float technique was used to determine the effective area of different piston-cylinders assemblies in different oils and at different pressures.

Initially, the effective area of the ‘*Test*’ pressure balance was determined by cross floating against a ‘*Standard*’ pressure balance using the same oil (Sebacate supplied by C P Hall). The ‘*Test*’ effective area is given by:

$$A_T^i = A_{St}^i \times \frac{1 + \alpha_{St} (t_{St}^i - 20)}{1 + \alpha_T (t_T^i - 20)} \times \frac{\sum_j m_{Tj}^i b_{Tj}^i + \frac{\gamma_T^i C_T}{g} - (\rho_{iT}^i - \rho_a^i) V_T}{\sum_j m_{Stj}^i b_{Stj}^i - \frac{\gamma_{St}^i C_{St}}{g} - (\rho_{iSt}^i - \rho_a^i) V_{St} + [(\rho_{iSt}^i - \rho_a^i) h_{St}^i - (\rho_{iT}^i - \rho_a^i) h_T^i] A_{St}^i} \quad \dots(3)$$

Where superscript “*i*” denotes the *initial* calibration and:

$$A_{St}^i = A_{St0} (1 + \lambda_{St} p^i) \quad \dots(4)$$

$$b^i = \left(1 - \frac{1.2}{8000}\right) \left(1 - \frac{\rho_a^i - 1.2}{\rho}\right) \quad \dots(5)$$

With  $\rho = \rho_{St}$  or  $\rho = \rho_T$  as required.

Terms  $\sum_j m_{Tj}^i b_{Tj}^i$  and  $\sum_j m_{Sj}^i b_{Sj}^i$  include the trim mass used to bring the system to equilibrium. After the initial calibration, the oil used by the ‘*Test*’ pressure balance was changed and the new effective area is given by the equation (6), where superscript “*f*” denotes the *final* calibration.

$$A_T^f = A_{St}^f \times \frac{1 + \alpha_{St} (t_{St}^f - 20)}{1 + \alpha_T (t_T^f - 20)} \times \frac{\sum_j m_{Tj}^f b_{Tj}^f + \frac{\gamma_T^f C_T}{g} - (\rho_{IT}^f - \rho_a^f) V_T}{\sum_j m_{Sj}^f b_{Sj}^f - \frac{\gamma_{St}^f C_{St}}{g} - (\rho_{IS}^f - \rho_a^f) V_{St} + [(\rho_{IS}^f - \rho_a^f) h_{St}^f - (\rho_{IT}^f - \rho_a^f) h_t^f] A_{St}^f} \quad \dots(6)$$

The differences in effective area when using different oils will then be given by:

$$\Delta A_T = A_T^f - A_T^i \quad \dots(7)$$

Some observations can be made to simplify the calculus of  $\Delta A_T$ :

- i. By the nature of the experiment, some of the input parameters used in the *initial* calibration, equation (3), are identical with the corresponding input parameters for the *final* calibration, equation (6).
- ii. Each ‘*Test*’ pressure balance was initially cross-floated in CP Hall Sebacate oil against the ‘*Standard*’ at several nominal pressures. After changing the oil, the ‘*Test*’ effective area was determined at the same nominal pressures and using the same mass sequences for ‘*Test*’ and for ‘*Standard*’ respectively. As the relative variation of nominal pressure was no bigger than 100 ppm, equation (4) tells us that the ‘*Standard*’ effective area at the same nominal pressure  $p$  and at 20 °C does not vary by more than  $10^{-9}$  ppm between the *initial* and *final* calibrations (based on a maximum distortion coefficient of 3.5 ppm MPa<sup>-1</sup>, determined from instrument history).
- iii. The formula that expresses the dependency of liquid density on pressure is a fourth order polynomial equation, while for the temperature a linear dependency is found to be sufficient [9]. For CP Hall Sebacate, the density (kg m<sup>-3</sup>) is [10]:

$$\rho(P, t) = 930.54 - 0.794 \times t + 5.746 \times 10^{-1} \times P - 1.278 \times 10^{-3} \times P^2 + 2.227 \times 10^{-6} \times P^3 - 1.5 \times 10^{-9} \times P^4 \quad \dots(8)$$

In equation (8)  $t$  is the oil temperature expressed in °C and  $P$  is the oil pressure expressed in MPa.

At the same nominal pressure, any difference in CP Hall Sebacate density between two consecutive calibrations is mainly caused by the difference in temperature. A maximum difference of 3.5 °C was registered for CP Hall Sebacate temperature throughout the experiment corresponding to a variation of 2.8 kg m<sup>-3</sup> in density. This difference is included in the uncertainty estimate for CP Hall Sebacate density, therefore we can consider that the density of CP Hall Sebacate remains unchanged throughout the experiment.

Based on the assumptions introduced above, the following notation will be used from here onwards:

$$\begin{aligned} m_{Sij}^i &= m_{Sij}^f = m_{Sij}, & m_{Tj}^i &= m_{Tj}^f = m_{Tj}, & A_{St}^i &\cong A_{St}^f = A_{St}, \\ \rho_{lSt}^i &= \rho_{lT}^i \cong \rho_{lSt}^f = \rho_{lSt}, & \gamma_{St}^i &= \gamma_{St}^f = \gamma_T^i = \gamma_{St}, & & \\ \rho_{lT}^f &= \rho_{lT}, & \gamma_T^f &= \gamma_T \end{aligned} \quad \dots(9)$$

Hence:

$$\begin{aligned} \Delta A_T &= A_{St} \times \\ &\left\{ \frac{1 + \alpha_{St}(t_{St}^f - 20)}{1 + \alpha_T(t_T^f - 20)} \left[ \sum_j m_{Tj} b_{Tj}^f + \Delta m_T b_T + \frac{\gamma_T C_T}{g} - (\rho_{lT} - \rho_a^f) V_T \right] \right. \\ &\quad \left. - \frac{\sum_j m_{Sij} b_{Sij}^f - \frac{\gamma_{St} C_{St}}{g} - (\rho_{lSt} - \rho_a^f) V_{St} + \left[ (\rho_{lSt} - \rho_a^f) h_{St}^f - (\rho_{lT} - \rho_a^f) h_t^f \right] A_{St}}{\sum_j m_{Sij} b_{Sij}^i - \frac{\gamma_{St} C_{St}}{g} - (\rho_{lSt} - \rho_a^i) V_{St} + \left[ (\rho_{lSt} - \rho_a^i) h_{St}^i - (\rho_{lSt} - \rho_a^i) h_t^i \right] A_{St}} \right. \\ &\quad \left. - \frac{1 + \alpha_{St}(t_{St}^i - 20)}{1 + \alpha_T(t_T^i - 20)} \left[ \sum_j m_{Tj} b_{Tj}^i + \frac{\gamma_{St} C_T}{g} - (\rho_{lSt} - \rho_a^i) V_T \right] \right\} \quad \dots(10) \end{aligned}$$

Where  $\Delta m_T$  is the difference in the trim mass due to the difference in effective area of the ‘Test’ pressure balance when changing the oil.

### 3 EXPERIMENT

#### 3.1 EQUIPMENT

The oils used in this project, together with some of the relevant characteristics (density and viscosity), are listed in Table 1.

**Table 1** – *Oils used in this work*

| Fluid                                                                                     | Fluid density<br>at 20 °C and 1 013.25 Pa<br>/(kg m <sup>-3</sup> ) | Viscosity<br>/(mPa s) |
|-------------------------------------------------------------------------------------------|---------------------------------------------------------------------|-----------------------|
| DH-Budenberg<br>HP Fluid                                                                  | 989                                                                 | 131                   |
| Hydraulic Mineral Oil<br>(Shell Tellus T15)<br>ISO VG15                                   | 862                                                                 | 26                    |
| Hydraulic Mineral Oil<br>(Shell Tellus T37)<br>ISO VG37                                   | 870                                                                 | 76                    |
| Univis P12 (early NPL oil)<br>(di-2-ethylhexylsebacate<br>supplied by Esso)               | 921                                                                 | 22.5                  |
| CP Hall Sebacate (current<br>NPL oil)<br>(di-2-ethylhexylsebacate<br>supplied by CP Hall) | 915                                                                 | 21.5                  |

As described in the previous section, the experiment involved calibrating several test instruments using the oils mentioned in Table 1 as the pressure-transmitting medium, against a standard instrument, which used CP Hall Sebacate.

Although it is an established technique to use differential pressure transducers when characterizing pneumatic pressure balances, until recently no sufficiently accurate sensor has been available to be used for oil at high pressure. Oil pressure transducer technology has developed rapidly in recent years, and now high-pressure sensors with ppm resolution are available. However, rather than using a differential sensor, the setup consists of two independent pressure systems linked together via two constant-volume valves (V1 and V2) as shown in Fig. 1, in which the two valves kept the oils separated during tests.

Two high-pressure sensors (HPS), DH Instruments model RPM4, were used having sensor ranges 0 MPa to 7 MPa and 0 MPa to 200 MPa. Measurements were always made using the most sensitive range available.

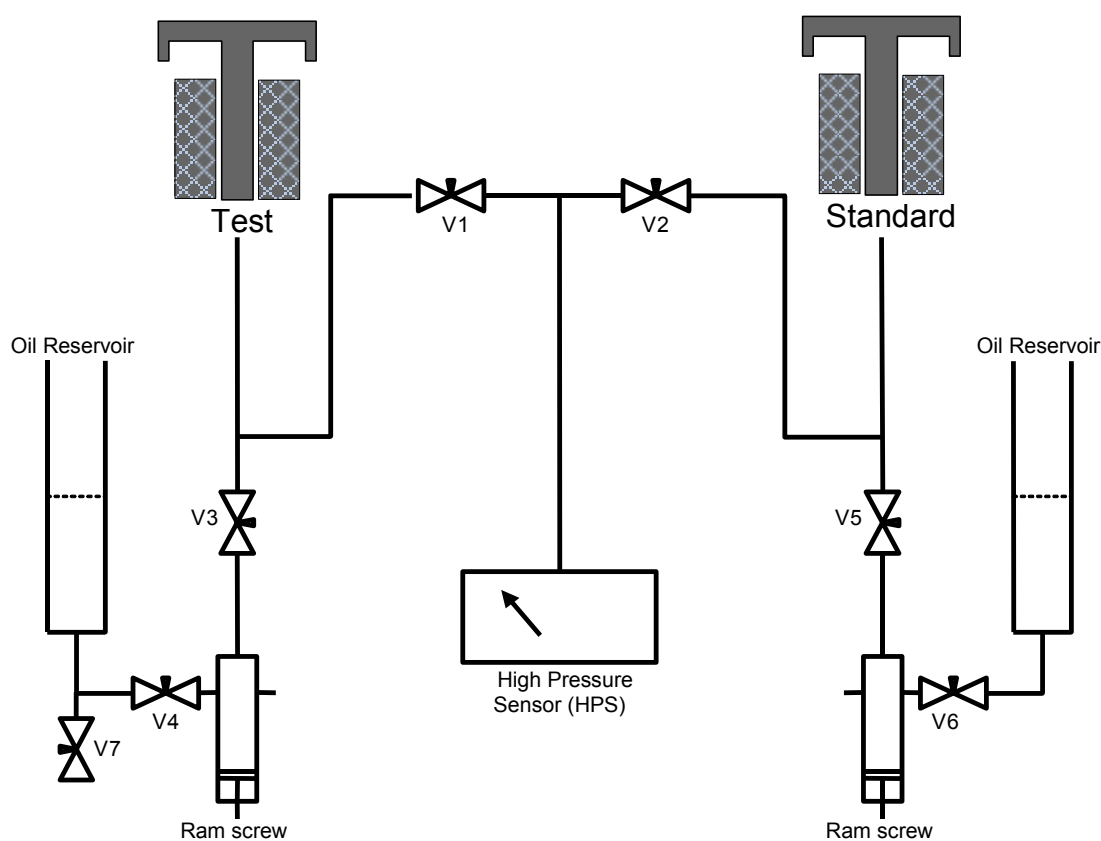


**Fig. 1** – *Experimental setup (V1 to V7 are isolating valves)*

Valves V3 and V4, the ram screw and oil reservoir at the left of valve V1 were used to create pressure in the ‘Test’ system isolated by valve V1. The valve V7 was used to drain the oil in the ‘Test’ system.

Valves V5 and V6, the ram screw and oil reservoir at the right of valve V2 were used to create pressure in the '*Standard*' system. The HPS and the space between V1 and V2 was filled with CP Hall Sebacate and was initially pressurised with V2 open and V1 closed.

The fluid head was measured using a cathetometer. The ambient temperature and piston cylinder assembly temperature were monitored using Edale (thermistor) temperature probes. The sensors monitoring the PCA temperature were placed so that an optimum thermal contact between PCA and the temperature probes was obtained. The barometric pressure was monitored using a digital barometer, model DPI145 manufactured by GE Sensing. The relative humidity was monitored using a digital hygrometer model MIK 3000 manufactured by Vaisala.

### 3.2 PISTON-CYLINDER ASSEMBLY SELECTION

The experiment used nine PCAs (Ruska 2481) in three nominal sizes of effective area ( $84 \text{ mm}^2$ ,  $16.8 \text{ mm}^2$ ,  $8.4 \text{ mm}^2$ ), covering the hydraulic pressure range 2.8 MPa to 280 MPa.

The serial numbers of the PCAs available for experiment were:

B1–821, B1–822, B1–823 nominal area of  $8.4 \text{ mm}^2$

B2–811, B2–812, B2–813 nominal area of  $16.8 \text{ mm}^2$

B3–824, B3–826, B3–827 with nominal area of  $84 \text{ mm}^2$

Commonly the PCA serial number is stamped on both elements, piston and cylinder, but in the case of PCA B1–821, “B1–821” was stamped on the piston and “B1–824” on the cylinder.

For each nominal size, one PCA was selected as ‘*Standard*’ instrument and two assemblies as ‘*Test*’ instruments. Two configurations were used to make the PCAs cover as a wide range of gap sizes as possible:

Configuration 1. The pressure balance is used, as originally designed, in “re-entrant” mode as shown in Fig. 2.a). The re-entrant configuration makes use of an o-ring in the upper part of the cylinder which seals the system, allowing the pressure fluid to get past the bottom of the cylinder and act on the cylinder’s outer diameter. In this case we obtain a negative value for the PCA distortion coefficient.

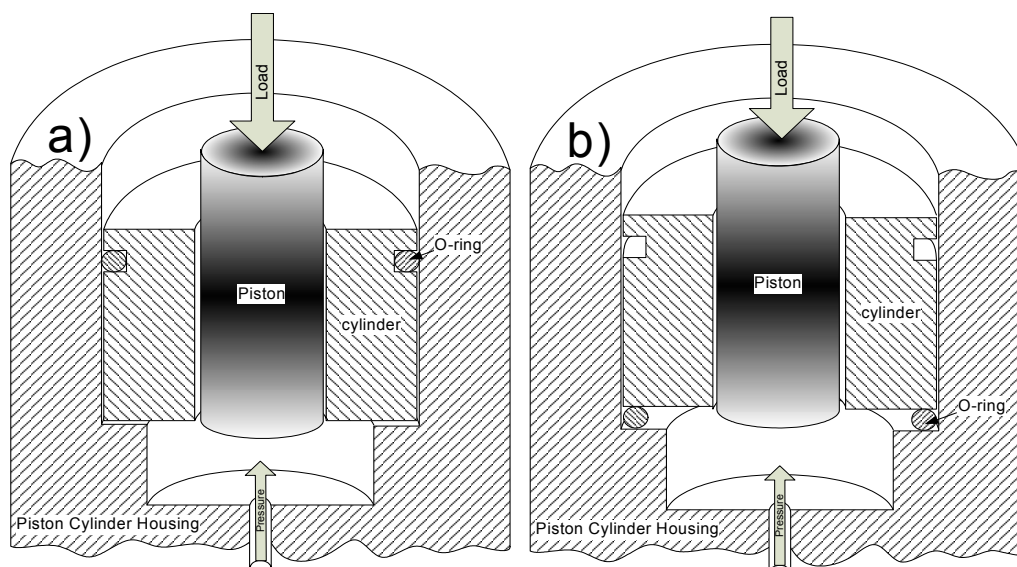


**Fig. 2** – Ruska pressure balance model 2481: **a)** re-entrant mode, **b)** free-deformation mode

Configuration 2. Placing an o-ring at the bottom of the cylinder rather than on the upper side of the cylinder as shown in Fig. 2.b), the PCA operates in “free

deformation” mode. Having the system sealed at the bottom of the cylinder, the pressure fluid does not act on the outside wall of the cylinder but only on the inner wall of the cylinder, leading to larger annular gaps at high pressures. As a consequence, a positive distortion coefficient is obtained. (As a safety measure, the o-ring from the upper part of the cylinder was eliminated when the PCA was used in the free deformation mode.)

An additional means to achieve a wide range of annular gaps is to find the original assembly with the biggest and smallest effective areas and swap the pistons. The PCAs available for experiment were used prior to 2004 as NPL high-pressure standards and are well characterised for CP Hall Sebacate oil. The effective area of each assembly at zero pressure ( $A_0$ ) is listed in Table 2.

**Table 2** – *Assemblies used in this work and their effective area*

| PCA S/No | $A_0/(\text{mm}^2)$ | PCA S/No | $A_0/(\text{mm}^2)$ | PCA S/No | $A_0/(\text{mm}^2)$ |
|----------|---------------------|----------|---------------------|----------|---------------------|
| B1–821   | 8.399 87            | B2–811   | 16.801 61           | B3–824   | 84.016 5            |
| B1–822   | 8.401 20            | B2–812   | 16.802 50           | B3–826   | 84.007 8            |
| B1–823   | 8.399 83            | B2–813   | 16.803 10           | B3–827   | 84.007 6            |

For each family of PCAs having a particular nominal area the assembly calibrated in re-entrant mode consisted of the *piston* taken from the assembly with the biggest effective area and the *cylinder* from the assembly with the smallest effective area. The *piston* from the assembly with the smallest area and the *cylinder* associated with the biggest area, was calibrated in the free deformation mode, and the remaining PCA was operated in re-entrant mode. This assembly was used as the ‘*Standard*’.

To summarise:

a. For the 8.4 mm<sup>2</sup> family

B1–823 was used as ‘*Standard*’ in re-entrant mode, now identified as B1<sub>St</sub>

B1–824cylinder(c) – 822piston(p) was used in re-entrant mode, now identified as B1<sub>T1</sub>

B1–822c–821p was used in free-deformation mode, now identified as B1<sub>T2</sub>

Measurements were made at the nominal pressures: 28 MPa, 84 MPa, 138 MPa and 194 MPa.

b. For the 16.8 mm<sup>2</sup> family

B2–812 was used as '*Standard*' in re-entrant mode, identified as B2<sub>St</sub>

B2–811c–814p was used in re-entrant mode, identified as B2<sub>T1</sub>

B2–814c–811p was used in free-deformation mode, identified as B2<sub>T2</sub>

Measurements were made at the nominal pressures: 14 MPa, 56 MPa, 96 MPa and 138 MPa.

c. For the 8.4 mm<sup>2</sup> family

B3–826 was used as '*Standard*' in re-entrant mode, B3<sub>St</sub>

B3–827c–824p was used in re-entrant mode, B3<sub>T1</sub>

B3–824c–827p was used in free-deformation mode, B3<sub>T2</sub>

Measurements made at the nominal pressures: 2.8 MPa, 12 MPa, 20 MPa and 28 MPa

### 3.3 MEASUREMENT PROCEDURE

After floating both pistons at the required pressure, the calibration technique involved the following steps:

1. V1 – closed and V2 – open, the HPS reading of the ‘*Standard*’ ( $I_{St-1}$ ) was recorded
2. V1 – open and V2 – closed, the HPS reading of the ‘*Test*’ ( $I_{T-1}$ ) was recorded
3. V1 – closed and V2 – open,  $I_{St-2}$  was recorded
4. V1 – open and V2 – closed,  $I_{T-2}$  was recorded
5. V1 – closed and V2 – open,  $I_{St-3}$  was recorded
6. V1 – closed and V2 – open, a step weight (5 g) was added on the ‘*Standard*’, and  $I_{St-4}$  was recorded.

The effective trim mass has been calculated using the next equation:

$$\Delta m = \text{Step Weight} \times \frac{\frac{I_{T-1} + I_{T-2}}{2} - \frac{I_{St-1} + I_{St-2} + I_{St-3}}{3}}{I_{St-4} - I_{St-3}} \quad \dots(11)$$

and the effective area was calculated using (10).

At each pressure point, 4 repeated measurements of the effective area were performed, two with rising pressure and two with falling pressure. In all cases, the floating element for both assemblies, ‘*Standard*’ and ‘*Test*’, was rotated clock-wise at approximately 30 rpm.

## 4 RESULTS

Care was taken to ensure experimental conditions reflected those typical for a pressure calibration laboratory. The ambient temperature was controlled to a nominal value of  $20\text{ }^{\circ}\text{C} \pm 2\text{ }^{\circ}\text{C}$  and the humidity to a nominal relative value of  $50\% \pm 10\%$ . Due to a fault of the air conditioning unit two calibrations were performed at a nominal temperature of  $23\text{ }^{\circ}\text{C}$  increasing the uncertainty associated with results obtained under those environmental conditions.

All piston cylinder assemblies under test were cross-floated initially in CP Hall Sebacate and subsequently in the oils mentioned in Table 1. For reconfirmation, a final calibration was performed using CP Hall Sebacate. The initial calibration results using CP Hall Sebacate in both PCAs were used as reference values.

Six calibrations were therefore performed for each PCA at the same four nominal pressures. At each pressure point the '*Test*' effective area was calculated using equation (3). The trim mass was determined using the method described in section 3.3.

Before each calibration the '*Standard*' and '*Test*' piston cylinder assemblies were cleaned and mounted in their corresponding base. Both systems, '*Test*' and '*Standard*', were levelled within 30 seconds of arc using a spirit level mounted on top of the weight stack while floating the pistons.

When calculating the pressure generated by '*Standard*' the results are reported relative to the horizontal line described by the horizontal pressure pipe that connected the '*Test*' system to the '*Standard*' system via valves V1 and V2. The effective area of '*Test*' was referenced to the bottom of the piston in its mid-operating position.

The results obtained during the experimental work are presented in the following graphs and tables, which also include estimated uncertainties that were determined using the arguments presented in the Appendix.

## 4.1 RESULTS FOR B3 FAMILY

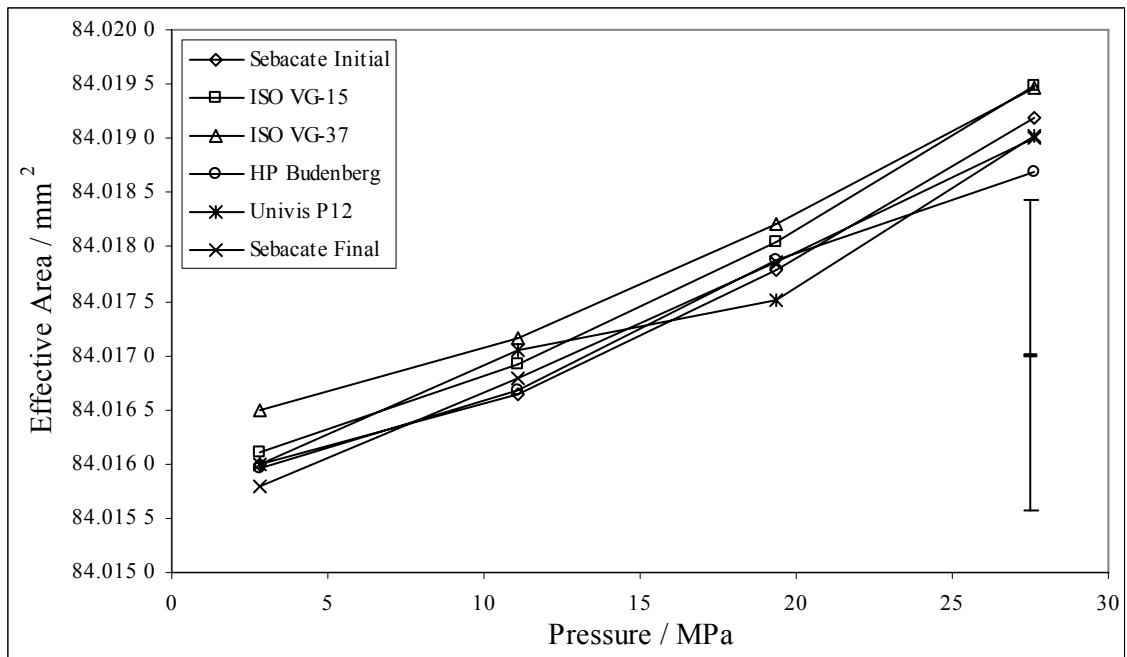


**Fig. 3 -** *Effective area of  $B3_{T2}$  assembly when calibrated in different oils (typical error bar shown at  $k = 1$ )*

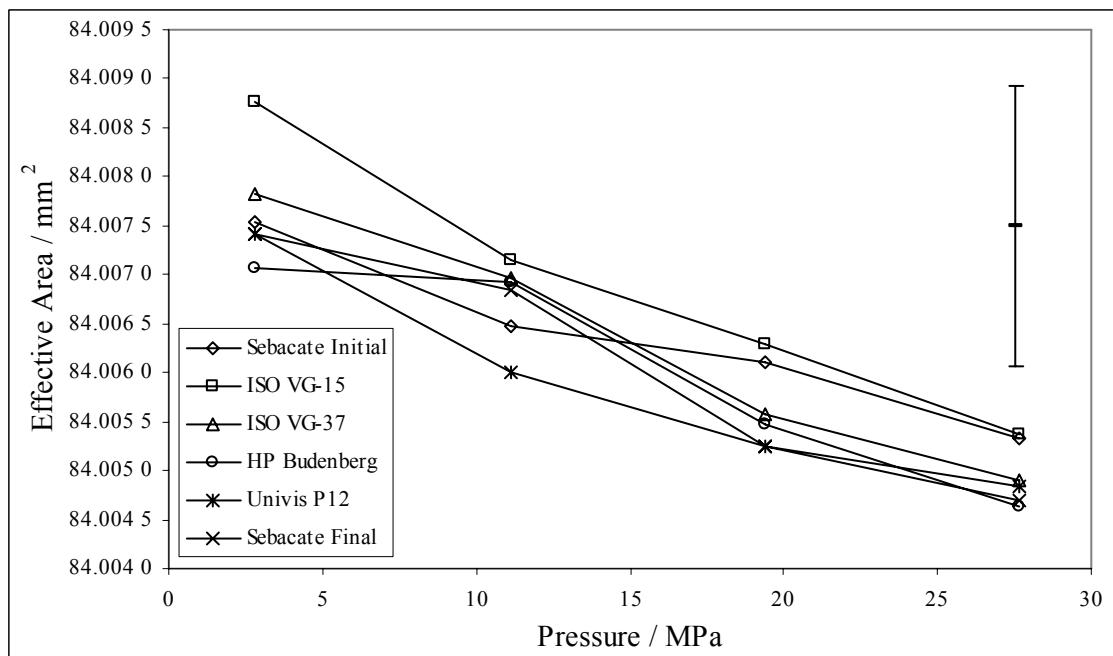


**Fig. 4 -** *Effective area of  $B3_{T3}$  assembly when calibrated in different oils (typical error bar shown at  $k = 1$ )*

**Table 3** - Results for  $B3_{T2}$  v  $B3_{St}$ 

| Pressure            | ISO VG-15                                                                                           | ISO VG-37 | HP Budenberg | Univis P12 |
|---------------------|-----------------------------------------------------------------------------------------------------|-----------|--------------|------------|
| /MPa                | Difference in effective area ( $\Delta A_T$ ) w.r.t. CP Hall Sebacate initial/(ppm of nominal area) |           |              |            |
| $2.826 \times 10^0$ | 1.3                                                                                                 | 5.9       | -0.5         | 0.1        |
| $1.110 \times 10^1$ | 3.4                                                                                                 | 6.2       | 0.5          | 4.9        |
| $1.938 \times 10^1$ | 3.2                                                                                                 | 5.1       | 1.1          | -3.1       |
| $2.765 \times 10^1$ | 3.5                                                                                                 | 3.3       | -6.0         | -2.0       |
| /MPa                | Type A Uncertainty for $\Delta A_T$ ( $k=1$ )/(ppm of nominal area)                                 |           |              |            |
| $2.826 \times 10^0$ | 3.4                                                                                                 | 1.9       | 1.6          | 2.0        |
| $1.110 \times 10^1$ | 5.2                                                                                                 | 2.9       | 4.1          | 4.7        |
| $1.938 \times 10^1$ | 4.6                                                                                                 | 3.0       | 3.0          | 3.3        |
| $2.765 \times 10^1$ | 1.6                                                                                                 | 2.4       | 1.7          | 2.5        |
| /MPa                | Type B Uncertainty for $\Delta A_T$ ( $k=1$ )/(ppm of nominal area)                                 |           |              |            |
| $2.826 \times 10^0$ | 3.7                                                                                                 | 3.6       | 3.0          | 3.2        |
| $1.110 \times 10^1$ | 4.0                                                                                                 | 3.9       | 3.3          | 3.6        |
| $1.938 \times 10^1$ | 6.7                                                                                                 | 6.6       | 6.3          | 6.4        |
| $2.765 \times 10^1$ | 5.4                                                                                                 | 5.3       | 4.9          | 5.0        |
| /MPa                | Combined Uncertainty for $\Delta A_T$ ( $k=1$ )/(ppm of nominal area)                               |           |              |            |
| $2.826 \times 10^0$ | 5.0                                                                                                 | 4.1       | 3.4          | 3.8        |
| $1.110 \times 10^1$ | 6.6                                                                                                 | 4.9       | 5.3          | 5.9        |
| $1.938 \times 10^1$ | 8.1                                                                                                 | 7.3       | 7.0          | 7.2        |
| $2.765 \times 10^1$ | 5.6                                                                                                 | 5.8       | 5.1          | 5.6        |

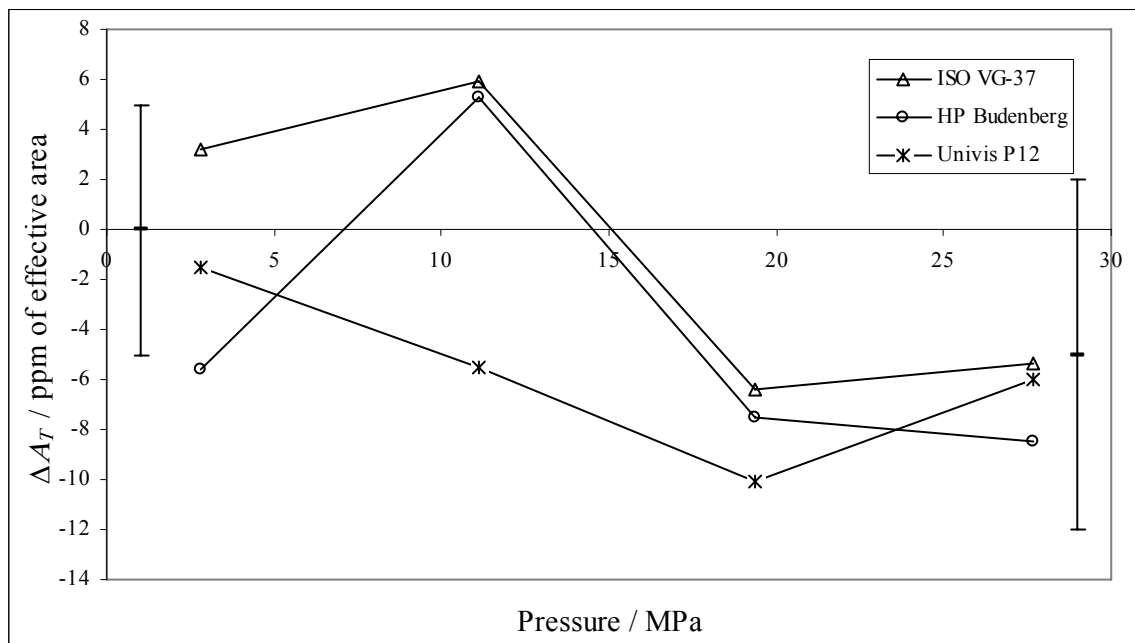
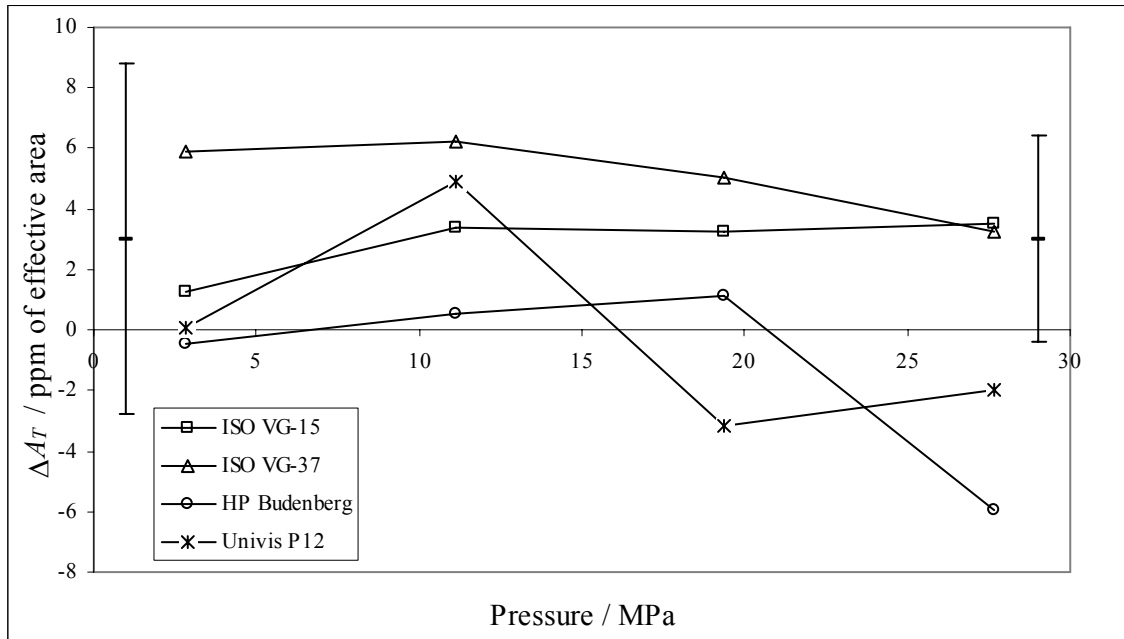
**Fig. 5** - Difference in  $B3_{T2}$  effective area w.r.t. CP Hall Sebacate (typical error bars shown at  $k = 1$ )

**Table 4** Results for  $B3_{TI} \nu B3_{St}$ 

| Pressure            | ISO VG-15                                                                                           | ISO VG-37 | HP Budenberg | Univis P12 |
|---------------------|-----------------------------------------------------------------------------------------------------|-----------|--------------|------------|
| /MPa                | Difference in effective area ( $\Delta A_T$ ) w.r.t. CP Hall Sebacate initial/(ppm of nominal area) |           |              |            |
| $2.826 \times 10^0$ | 14.6                                                                                                | 3.2       | -5.6         | -1.5       |
| $1.110 \times 10^1$ | 8.2                                                                                                 | 6.0       | 5.3          | -5.5       |
| $1.938 \times 10^1$ | 2.2                                                                                                 | -6.4      | -7.5         | -10.1      |
| $2.765 \times 10^1$ | 0.4                                                                                                 | -5.3      | -8.5         | -6.0       |
| /MPa                | Type A Uncertainty for $\Delta A_T$ ( $k=1$ )/(ppm of nominal area)                                 |           |              |            |
| $2.826 \times 10^0$ | 4.6                                                                                                 | 1.3       | 5.2          | 3.5        |
| $1.110 \times 10^1$ | 5.6                                                                                                 | 1.1       | 2.7          | 7.2        |
| $1.938 \times 10^1$ | 2.7                                                                                                 | 3.1       | 3.3          | 2.8        |
| $2.765 \times 10^1$ | 2.9                                                                                                 | 2.1       | 2.0          | 1.6        |
| /MPa                | Type B Uncertainty for $\Delta A_T$ ( $k=1$ )/(ppm of nominal area)                                 |           |              |            |
| $2.826 \times 10^0$ | 2.6                                                                                                 | 2.6       | 2.7          | 2.6        |
| $1.110 \times 10^1$ | 1.7                                                                                                 | 1.6       | 1.8          | 1.6        |
| $1.938 \times 10^1$ | 1.4                                                                                                 | 1.3       | 1.4          | 1.2        |
| $2.765 \times 10^1$ | 1.8                                                                                                 | 1.7       | 1.8          | 1.7        |
| /MPa                | Combined Uncertainty for $\Delta A_T$ ( $k=1$ )/(ppm of nominal area)                               |           |              |            |
| $2.826 \times 10^0$ | 5.3                                                                                                 | 2.9       | 5.8          | 4.3        |
| $1.110 \times 10^1$ | 5.9                                                                                                 | 2.0       | 3.2          | 7.4        |
| $1.938 \times 10^1$ | 3.0                                                                                                 | 3.4       | 3.6          | 3.0        |
| $2.765 \times 10^1$ | 3.4                                                                                                 | 2.7       | 2.7          | 2.3        |

**Fig. 6** - Difference in  $B3_{TI}$  effective area w.r.t. CP Hall Sebacate (typical error bars shown at  $k = 1$ )

## 4.2 RESULTS FOR B2 FAMILY

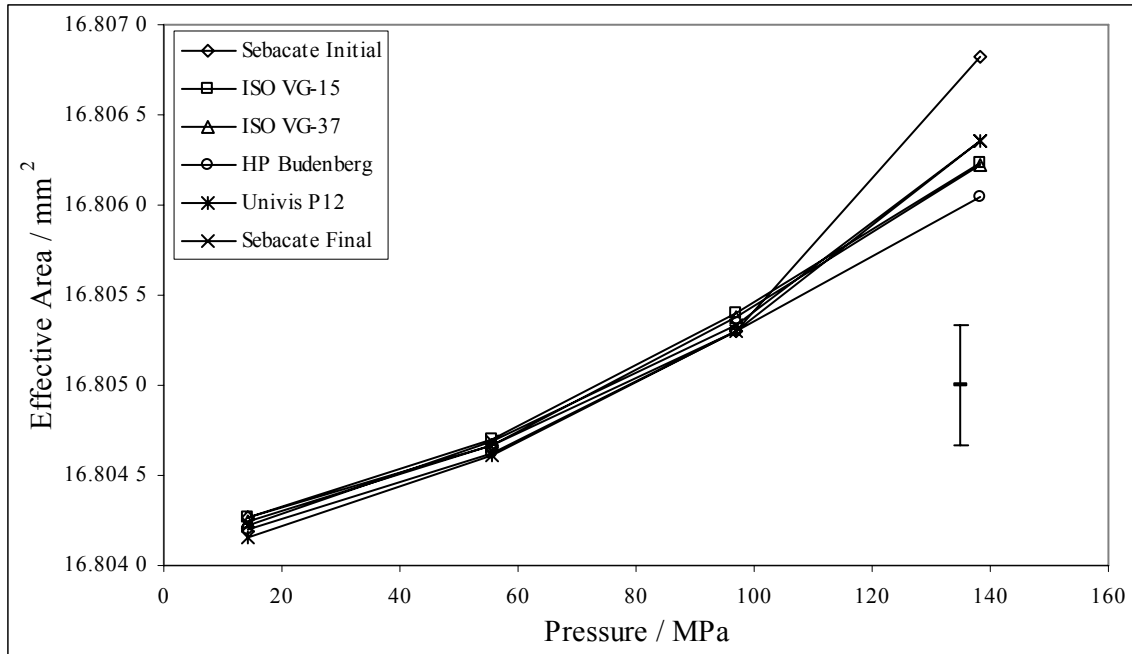


**Fig. 7** - *Effective area of  $B2_{T2}$  assembly when calibrated in different oils (typical error bar shown at  $k = 1$ )*

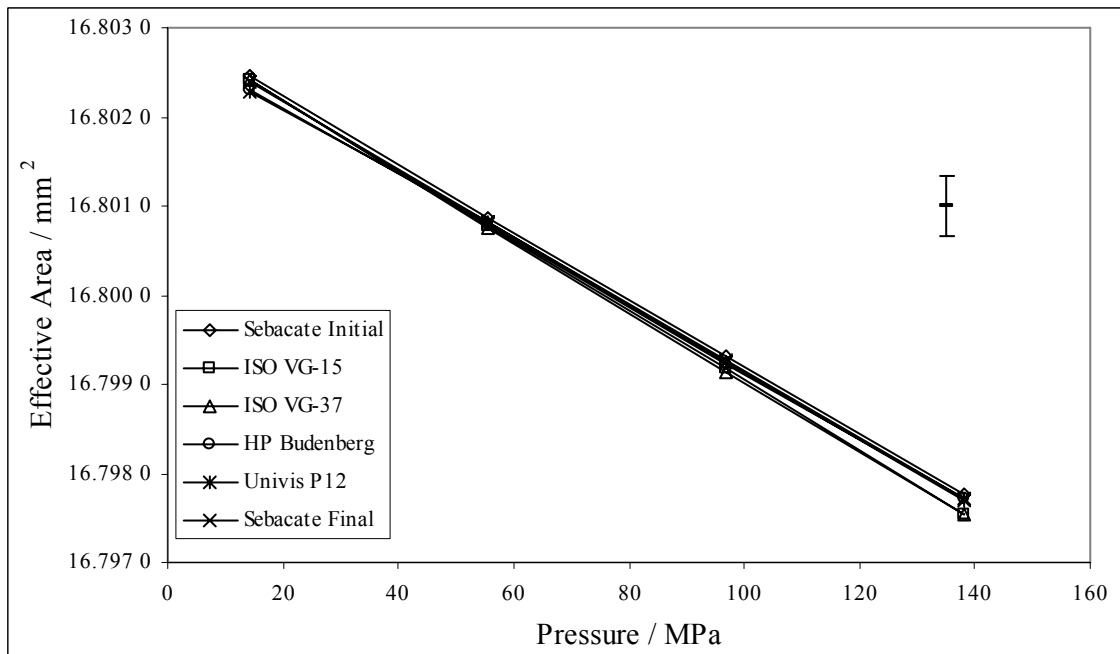


**Fig. 8** - *Effective area of  $B2_{T1}$  assembly when calibrated in different oils (typical error bar shown at  $k = 1$ )*

**Table 5 - Results for  $B2_{T2} \nu B2_{St}$** 

| Pressure            | ISO VG-15                                                                                           | ISO VG-37 | HP Budenberg | Univis P12 |
|---------------------|-----------------------------------------------------------------------------------------------------|-----------|--------------|------------|
| /MPa                | Difference in effective area ( $\Delta A_T$ ) w.r.t. CP Hall Sebacate initial/(ppm of nominal area) |           |              |            |
| $1.409 \times 10^1$ | 0.4                                                                                                 | -0.9      | -4.0         | -6.7       |
| $5.547 \times 10^1$ | 2.1                                                                                                 | -0.3      | -2.4         | -3.5       |
| $9.685 \times 10^1$ | 5.8                                                                                                 | 4.3       | -0.1         | -0.5       |
| $1.382 \times 10^2$ | -34.7                                                                                               | -35.8     | -46.0        | -27.7      |
| /MPa                | Type A Uncertainty for $\Delta A_T$ ( $k=1$ )/(ppm of nominal area)                                 |           |              |            |
| $1.409 \times 10^1$ | 5.3                                                                                                 | 3.6       | 5.3          | 4.1        |
| $5.547 \times 10^1$ | 2.1                                                                                                 | 1.6       | 2.1          | 2.9        |
| $9.685 \times 10^1$ | 1.9                                                                                                 | 1.9       | 1.9          | 2.5        |
| $1.382 \times 10^2$ | 7.5                                                                                                 | 7.5       | 7.5          | 7.6        |
| /MPa                | Type B Uncertainty for $\Delta A_T$ ( $k=1$ )/(ppm of nominal area)                                 |           |              |            |
| $1.409 \times 10^1$ | 1.9                                                                                                 | 1.9       | 1.7          | 1.7        |
| $5.547 \times 10^1$ | 1.5                                                                                                 | 1.5       | 1.3          | 1.3        |
| $9.685 \times 10^1$ | 1.6                                                                                                 | 1.6       | 1.4          | 1.5        |
| $1.382 \times 10^2$ | 16.1                                                                                                | 16.1      | 16.1         | 16.1       |
| /MPa                | Combined Uncertainty for $\Delta A_T$ ( $k=1$ )/(ppm of nominal area)                               |           |              |            |
| $1.409 \times 10^1$ | 5.6                                                                                                 | 4.1       | 5.6          | 4.5        |
| $5.547 \times 10^1$ | 2.6                                                                                                 | 2.2       | 2.5          | 3.2        |
| $9.685 \times 10^1$ | 2.5                                                                                                 | 2.5       | 2.4          | 2.9        |
| $1.382 \times 10^2$ | 17.8                                                                                                | 17.7      | 17.7         | 17.8       |

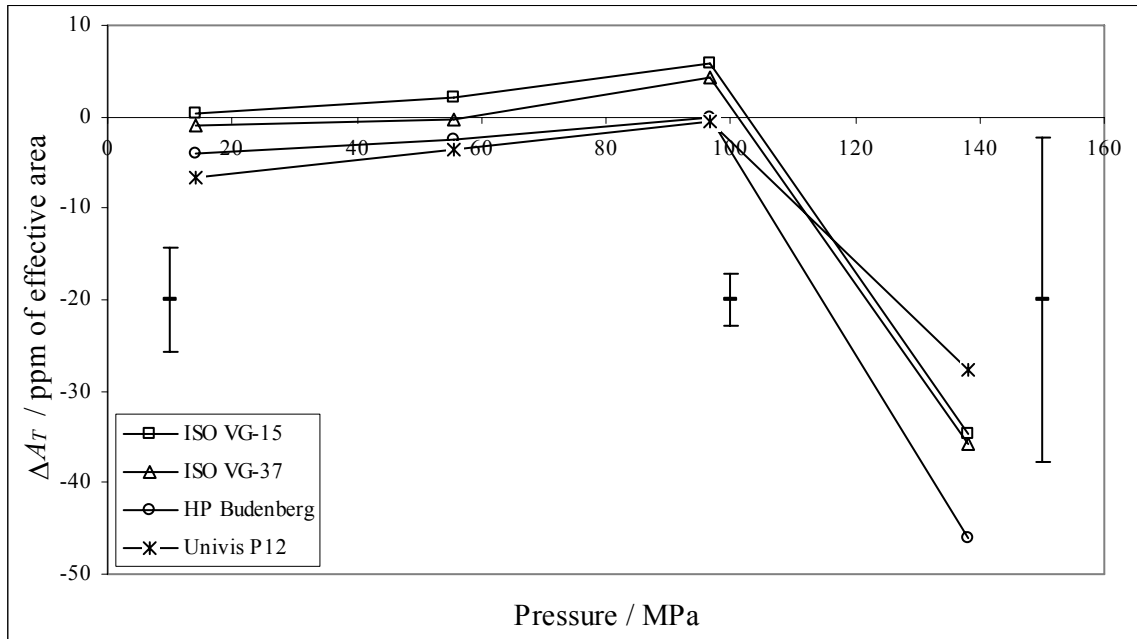
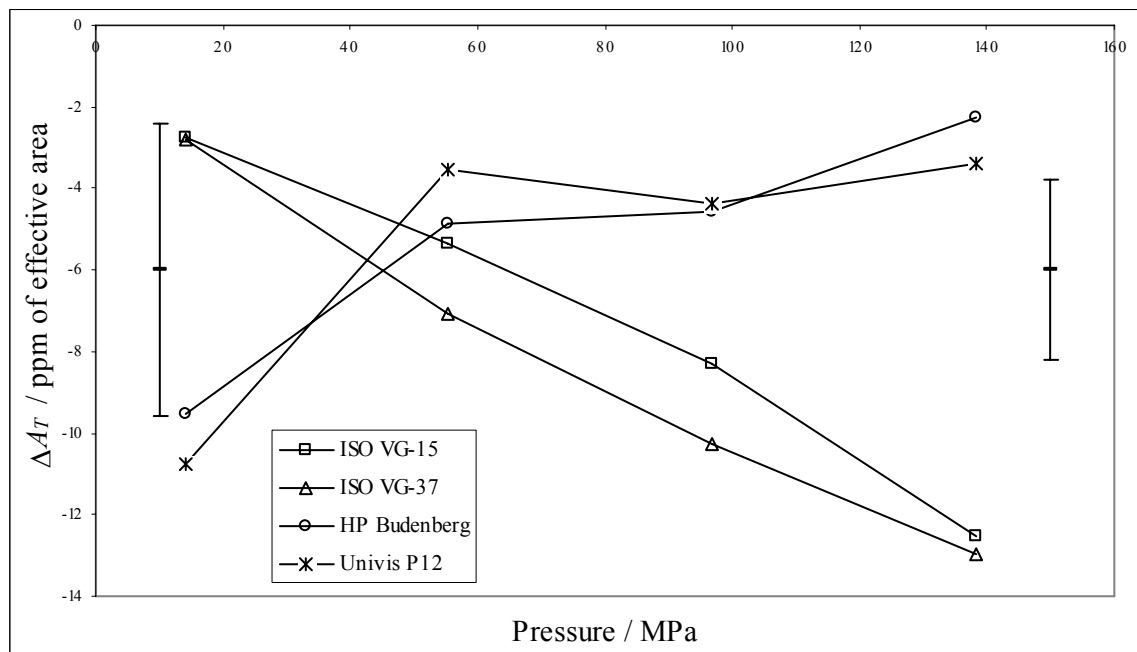
**Fig. 9 - Difference in  $B2_{T2}$  effective area w.r.t. CP Hall Sebacate (typical error bars shown at  $k = 1$ )**

**Table 6 - Results for  $B2_{TI} \nu B2_{St}$** 

| Pressure            | ISO VG-15                                                                                           | ISO VG-37 | HP Budenberg | Univis P12 |
|---------------------|-----------------------------------------------------------------------------------------------------|-----------|--------------|------------|
| /MPa                | Difference in effective area ( $\Delta A_T$ ) w.r.t. CP Hall Sebacate initial/(ppm of nominal area) |           |              |            |
| $1.409 \times 10^1$ | -2.8                                                                                                | -2.8      | -9.5         | -10.7      |
| $5.547 \times 10^1$ | -5.4                                                                                                | -7.1      | -4.9         | -3.6       |
| $9.685 \times 10^1$ | -8.3                                                                                                | -10.3     | -4.6         | -4.4       |
| $1.382 \times 10^2$ | -12.5                                                                                               | -13.0     | -2.3         | -3.4       |
| /MPa                | Type A Uncertainty for $\Delta A_T$ ( $k=1$ )/(ppm of nominal area)                                 |           |              |            |
| $1.409 \times 10^1$ | 2.5                                                                                                 | 1.5       | 1.6          | 2.9        |
| $5.547 \times 10^1$ | 1.3                                                                                                 | 0.9       | 1.2          | 1.9        |
| $9.685 \times 10^1$ | 1.1                                                                                                 | 1.3       | 1.2          | 1.2        |
| $1.382 \times 10^2$ | 1.1                                                                                                 | 1.1       | 1.4          | 1.6        |
| /MPa                | Type B Uncertainty for $\Delta A_T$ ( $k=1$ )/(ppm of nominal area)                                 |           |              |            |
| $1.409 \times 10^1$ | 2.3                                                                                                 | 2.5       | 2.3          | 2.2        |
| $5.547 \times 10^1$ | 2.0                                                                                                 | 2.3       | 2.0          | 1.9        |
| $9.685 \times 10^1$ | 1.9                                                                                                 | 2.3       | 1.9          | 1.8        |
| $1.382 \times 10^2$ | 1.6                                                                                                 | 2.0       | 1.6          | 1.5        |
| /MPa                | Combined Uncertainty for $\Delta A_T$ ( $k=1$ )/(ppm of nominal area)                               |           |              |            |
| $1.409 \times 10^1$ | 3.4                                                                                                 | 2.9       | 2.8          | 3.6        |
| $5.547 \times 10^1$ | 2.4                                                                                                 | 2.5       | 2.3          | 2.7        |
| $9.685 \times 10^1$ | 2.3                                                                                                 | 2.6       | 2.3          | 2.2        |
| $1.382 \times 10^2$ | 1.9                                                                                                 | 2.2       | 2.1          | 2.1        |

**Fig. 10 - Difference in  $B2_{TI}$  effective area w.r.t. CP Hall Sebacate (typical error bars shown at  $k = 1$ )**

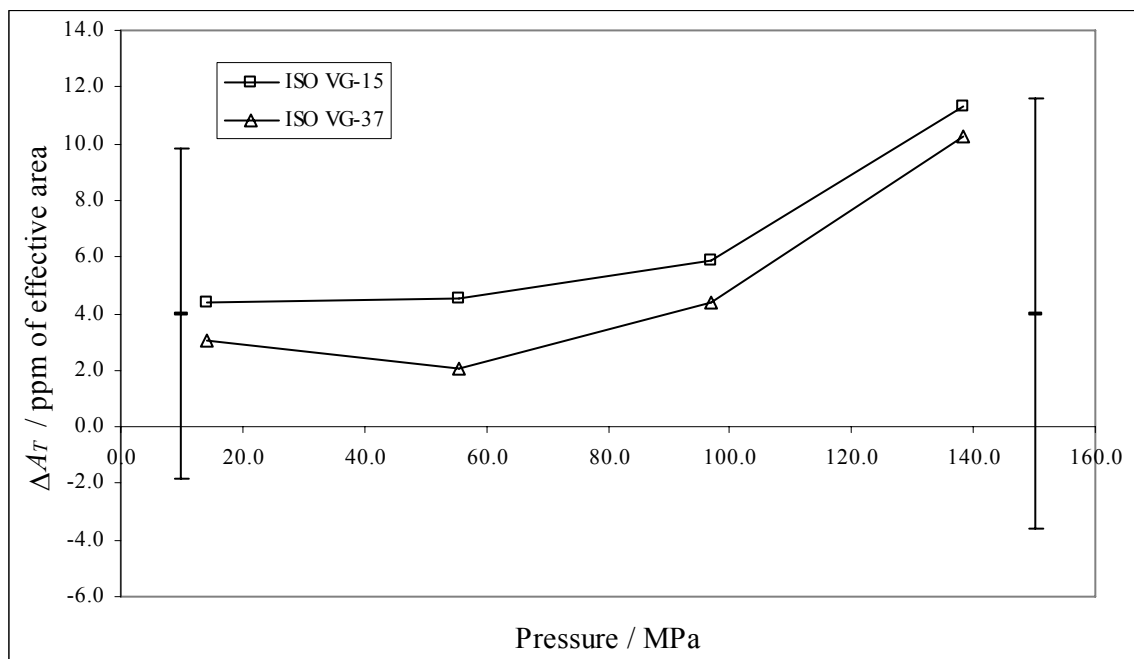


**Fig. 11** - Difference in  $B2_{T2}$  effective area w.r.t. HP-Budenberg

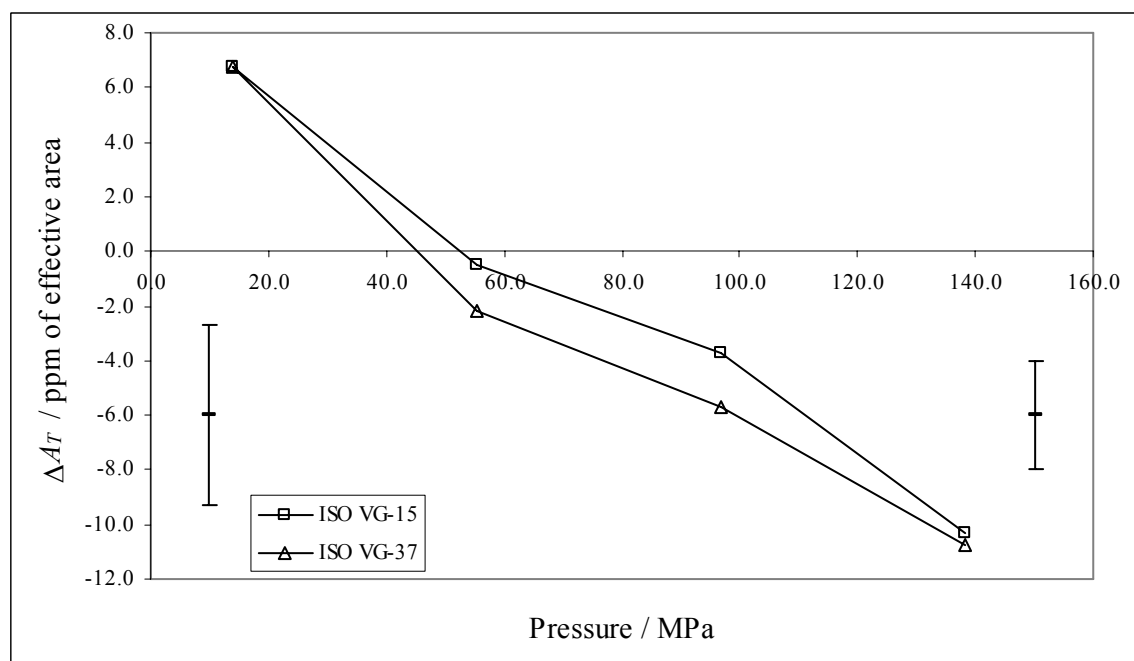


**Fig. 12** - Difference in  $B2_{T1}$  effective area w.r.t. HP-Budenberg

### 4.3 RESULTS FOR B1 FAMILY

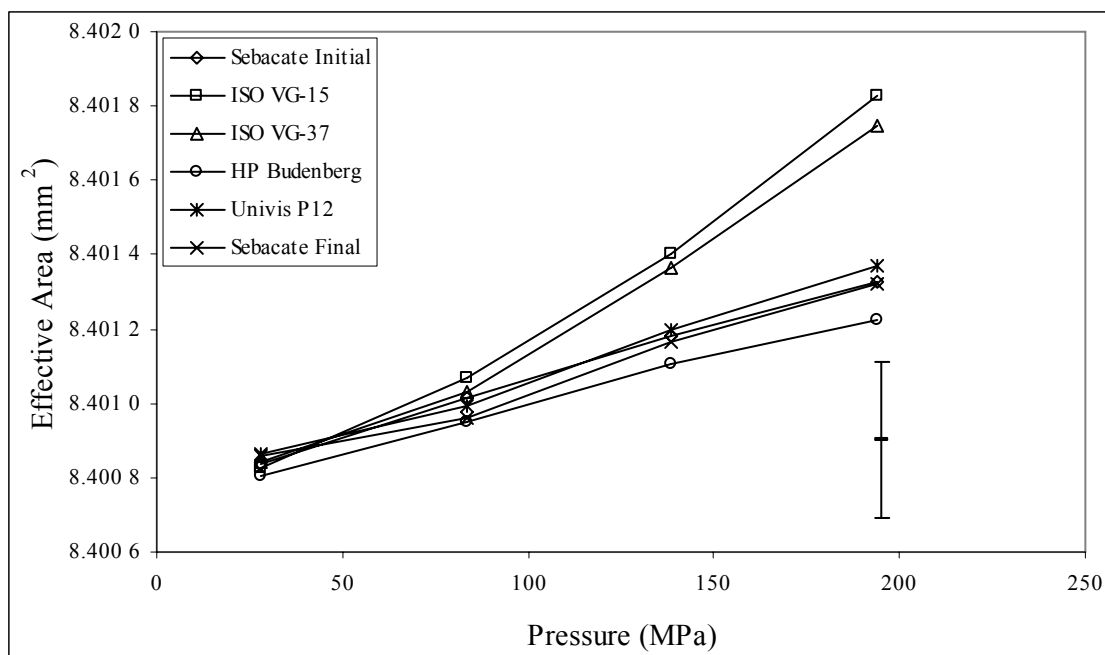


**Fig. 13** - *Effective area of  $B1_{T2}$  assembly when calibrated in different oils (typical error bar shown at  $k = 1$ )*

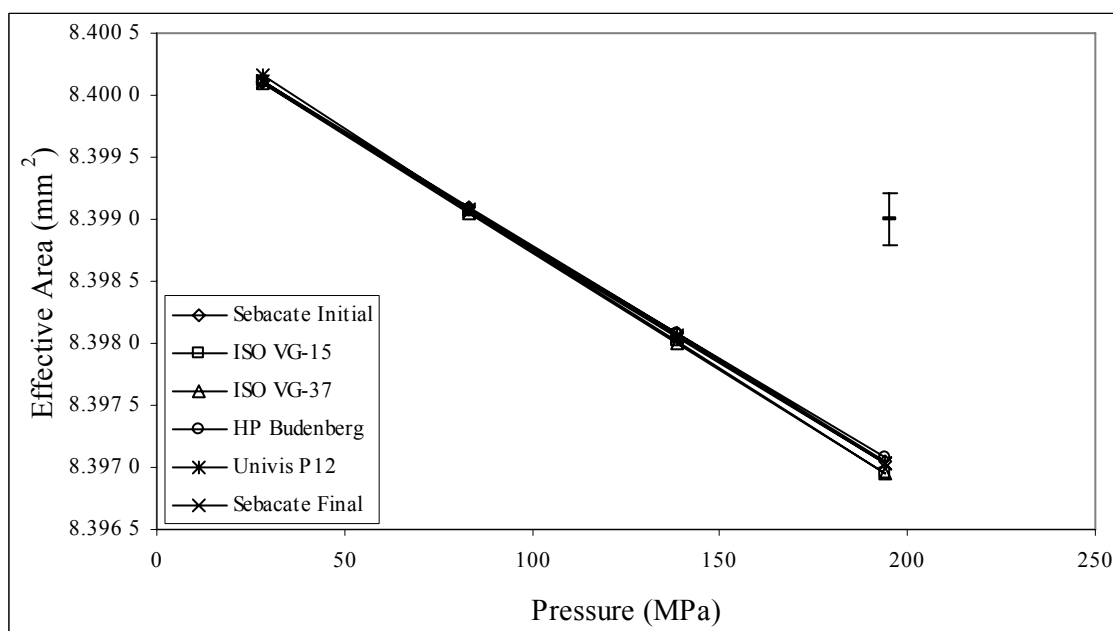


**Fig. 14** - *Effective area of  $B1_{T1}$  assembly when calibrated in different oils (typical error bar shown at  $k = 1$ )*

**Table 7 - Results for  $Bl_{T2} \nu Bl_{St}$** 

| Pressure            | ISO VG-15                                                                                           | ISO VG-37 | HP Budenberg | Univis P12 |
|---------------------|-----------------------------------------------------------------------------------------------------|-----------|--------------|------------|
| /MPa                | Difference in effective area ( $\Delta A_T$ ) w.r.t. CP Hall Sebacate initial/(ppm of nominal area) |           |              |            |
| $2.817 \times 10^1$ | -1.3                                                                                                | 0.4       | -3.9         | 2.8        |
| $8.336 \times 10^1$ | 6.3                                                                                                 | 1.9       | -7.5         | -2.4       |
| $1.386 \times 10^2$ | 26.3                                                                                                | 21.8      | -9.0         | 1.8        |
| $1.938 \times 10^2$ | 59.6                                                                                                | 49.6      | -12.1        | 4.9        |
| /MPa                | Type A Uncertainty for $\Delta A_T$ ( $k=1$ )/(ppm of nominal area)                                 |           |              |            |
| $2.817 \times 10^1$ | 4.9                                                                                                 | 5.0       | 4.8          | 4.4        |
| $8.336 \times 10^1$ | 2.8                                                                                                 | 3.2       | 3.0          | 2.9        |
| $1.386 \times 10^2$ | 1.2                                                                                                 | 1.1       | 1.2          | 2.4        |
| $1.938 \times 10^2$ | 1.8                                                                                                 | 1.8       | 1.7          | 1.8        |
| /MPa                | Type B Uncertainty for $\Delta A_T$ ( $k=1$ )/(ppm of nominal area)                                 |           |              |            |
| $2.817 \times 10^1$ | 1.8                                                                                                 | 1.8       | 1.9          | 1.9        |
| $8.336 \times 10^1$ | 4.0                                                                                                 | 4.0       | 4.0          | 4.0        |
| $1.386 \times 10^2$ | 1.6                                                                                                 | 1.7       | 1.8          | 1.8        |
| $1.938 \times 10^2$ | 1.2                                                                                                 | 1.3       | 1.4          | 1.4        |
| /MPa                | Combined Uncertainty for $\Delta A_T$ ( $k=1$ )/(ppm of nominal area)                               |           |              |            |
| $2.817 \times 10^1$ | 5.2                                                                                                 | 5.3       | 5.1          | 4.7        |
| $8.336 \times 10^1$ | 4.9                                                                                                 | 5.1       | 5.0          | 5.0        |
| $1.386 \times 10^2$ | 2.0                                                                                                 | 2.1       | 2.1          | 3.0        |
| $1.938 \times 10^2$ | 2.2                                                                                                 | 2.2       | 2.2          | 2.3        |

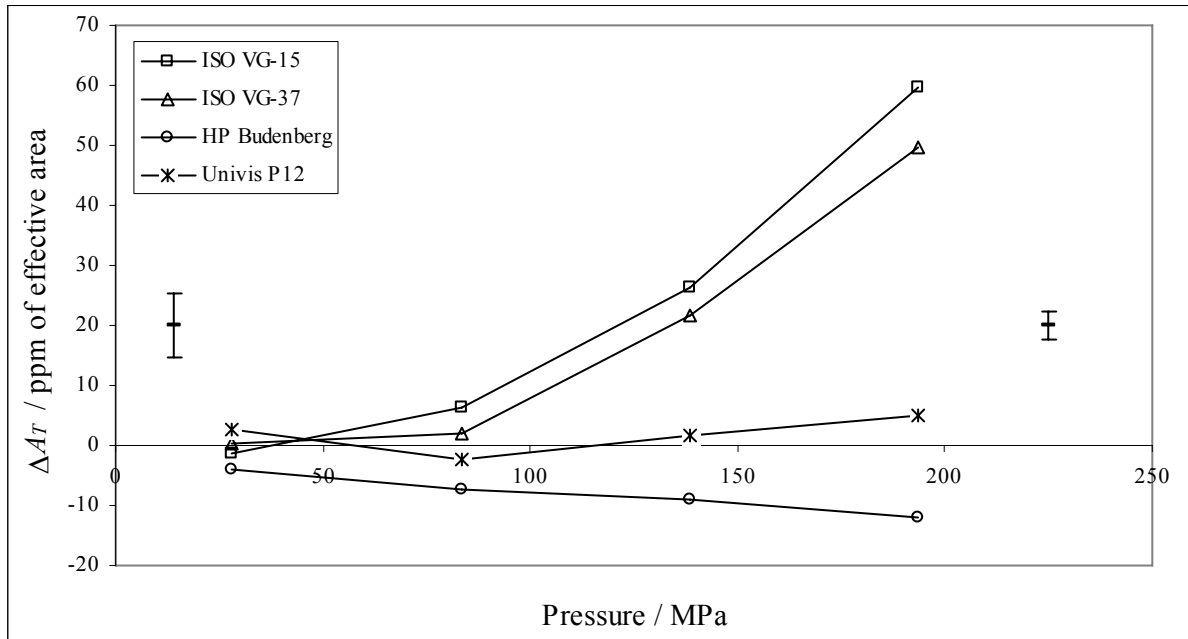
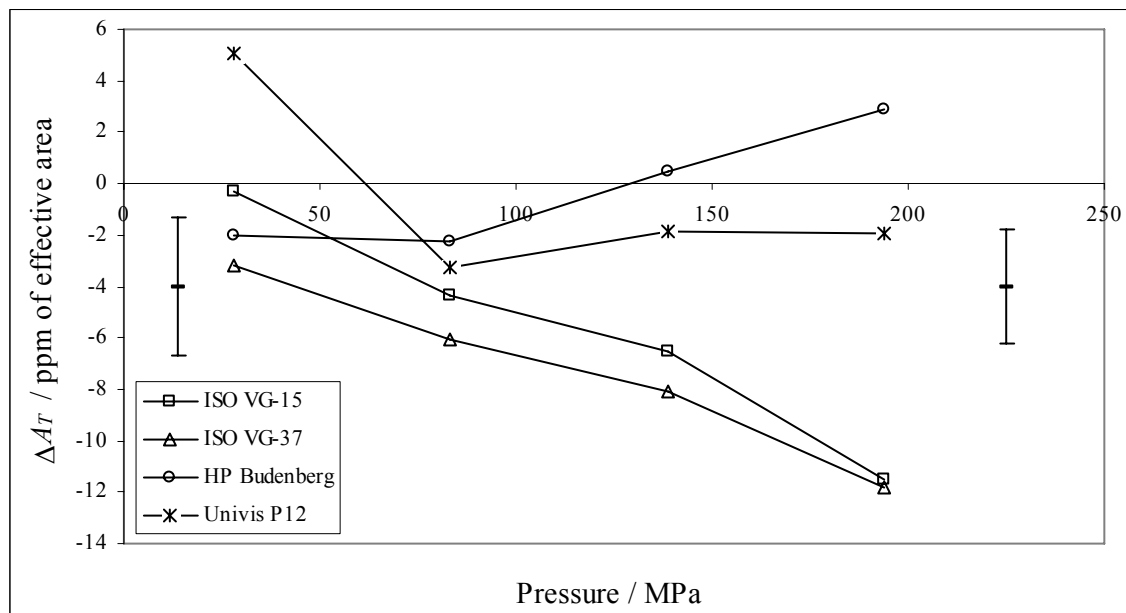
**Fig. 15 - Difference in  $Bl_{T2}$  effective area w.r.t. CP Hall Sebacate (typical error bars shown at  $k = 1$ )**

**Table 8** - Results for  $Bl_{TI} \nu Bl_{St}$ 

| Pressure            | ISO VG-15                                                                                           | ISO VG-37 | HP Budenberg | Univis P12 |
|---------------------|-----------------------------------------------------------------------------------------------------|-----------|--------------|------------|
| /MPa                | Difference in effective area ( $\Delta A_T$ ) w.r.t. CP Hall Sebacate initial/(ppm of nominal area) |           |              |            |
| $2.817 \times 10^1$ | -0.3                                                                                                | -3.2      | -2.0         | 5.1        |
| $8.336 \times 10^1$ | -4.4                                                                                                | -6.1      | -2.3         | -3.2       |
| $1.386 \times 10^2$ | -6.6                                                                                                | -8.1      | 0.5          | -1.8       |
| $1.938 \times 10^2$ | -11.5                                                                                               | -11.8     | 2.9          | -1.9       |
| /MPa                | Type A Uncertainty for $\Delta A_T$ ( $k=1$ )/(ppm of nominal area)                                 |           |              |            |
| $2.817 \times 10^1$ | 2.3                                                                                                 | 2.1       | 2.5          | 2.4        |
| $8.336 \times 10^1$ | 2.7                                                                                                 | 2.5       | 2.7          | 2.5        |
| $1.386 \times 10^2$ | 1.3                                                                                                 | 1.1       | 1.0          | 0.8        |
| $1.938 \times 10^2$ | 0.9                                                                                                 | 0.8       | 0.8          | 0.7        |
| /MPa                | Type B Uncertainty for $\Delta A_T$ ( $k=1$ )/(ppm of nominal area)                                 |           |              |            |
| $2.817 \times 10^1$ | 1.7                                                                                                 | 1.7       | 1.1          | 1.0        |
| $8.336 \times 10^1$ | 3.4                                                                                                 | 3.4       | 3.1          | 3.1        |
| $1.386 \times 10^2$ | 2.3                                                                                                 | 2.3       | 1.8          | 1.8        |
| $1.938 \times 10^2$ | 2.1                                                                                                 | 2.1       | 1.5          | 1.5        |
| /MPa                | Combined Uncertainty for $\Delta A_T$ ( $k=1$ )/(ppm of nominal area)                               |           |              |            |
| $2.817 \times 10^1$ | 2.9                                                                                                 | 2.7       | 2.7          | 2.6        |
| $8.336 \times 10^1$ | 4.3                                                                                                 | 4.2       | 4.1          | 4.0        |
| $1.386 \times 10^2$ | 2.6                                                                                                 | 2.5       | 2.1          | 2.0        |
| $1.938 \times 10^2$ | 2.2                                                                                                 | 2.2       | 1.7          | 1.7        |

**Fig. 16** - Difference in  $Bl_{TI}$  effective area w.r.t. CP Hall Sebacate (typical error bars shown at  $k = 1$ )

## 4.4 DISCUSSIONS OF RESULTS

The results of the calibrations performed in different oils for B1 family are plotted in the graphs presented in Fig. 13 and Fig. 14. The graphs show the change in effective area with pressure for the B1<sub>T2</sub> and B1<sub>T1</sub> assemblies, respectively. The pressure ranges from 0 MPa to 198 MPa for both cases.

The change in effective area with respect to the initial CP Hall Sebacate calibration, together with the associated uncertainties (see Appendix) is presented in Fig. 15 and Fig. 16.

The results for the B1<sub>T2</sub> assembly (operated in free-deformation mode) show:

- the largest change in effective area for both ISO VG15 and ISO VG37 oil;
- the change seems to be positive for oils with lower densities than CP Hall Sebacate and negative for oils with bigger densities than CP Hall Sebacate, therefore change  $\Delta A_T$  is anti-correlated with oil density;
- the effect is consistent over most of pressure range;
- at highest pressures (138.6 MPa and 193.8 MPa) the differences are significant when compared with the uncertainties .

The results for B1<sub>T1</sub> assembly show:

- the largest change in effective area still occurs for ISO VG15 and ISO VG37 oil;
- the effect is not as large in re-entrant mode (by a factor of 4);
- the effect is positively correlated with density but its magnitude is smaller when compared to the free-deformation;
- at highest pressures (138.6 MPa and 193.8 MPa) the differences are significant when compared with the uncertainties .

For the B2 family, the pattern of behaviour is similar to that of the B1 family. The effective areas when using different oils, for the range 0 MPa to 140 MPa are displayed in Fig. 7 and Fig. 8. The differences in effective area for the two assemblies of the B2 family are presented in Fig. 9 and Fig. 10.

The results for B2<sub>T2</sub> assembly when calibrated in CP Hall Sebacate and Univis P12 at pressures above 100 MPa show significant discrepancies relative to the expected calibration curve. This was most likely due to the large fall rate of the PCA as a result of a large gap combined with the low viscosity of the CP Hall Sebacate and Univis P12.

Despite the complications at high pressure the remainder of data (after excluding highest pressure 138 MPa) still show the same behaviour as B1<sub>T2</sub>, i.e:

- $\Delta A_T$  is anti-correlated with oil density;
- at 97 MPa the effect is still significant when compared to uncertainties. (The situation at 140 MPa can be clarified if we compare data for the ISO VG oils against same data for the HP Budenberg oil. The results are presented in Fig. 11 and show clearly the anti-correlation between  $\Delta A_T$  and oil density.)

The results for B2<sub>T1</sub> assembly show similar behaviour to the B1<sub>T1</sub> assembly, i.e.  $\Delta A_T$  is positively correlated with the oil density, apart from the lowest pressure where uncertainty is particularly large.

Finally, the results for the B3 family, calibrated in the range 0 MPa to 28 MPa are presented in Fig. 3 and Fig. 4. The changes in effective area relative to CP Hall Sebacate are shown in Fig. 5 and Fig. 6.

With the exception of the Univis P12 data, the results for the B3<sub>T2</sub> assembly also appear to show the same anti-correlation between  $\Delta A_T$  and oil density especially at 28 MPa.

The results for B3<sub>T1</sub> assembly are not particularly informative as the variation of effective area is generally smaller than the expanded uncertainties. The fluid head was not recorded properly when ISO VG15 oil was tested therefore the results have been excluded in the graph presented in Fig. 6.

## 5 SUMMARY AND CONCLUSIONS

The project was designed to study differences in the performance of hydraulic pressure balances when the pressure transmitting fluid is changed. The experiment involved testing five oils with different physical characteristics using piston cylinder assemblies (PCA) emulating the hydraulic pressure balances (HPB) available on the market.

Three PCAs, of nominal sizes  $84 \text{ mm}^2$ ,  $16.8 \text{ mm}^2$  and  $8.4 \text{ mm}^2$ , were tested over their corresponding pressure ranges 0 MPa to 28 MPa, 0 MPa to 140 MPa and 0 MPa to 280 MPa, respectively. A high-pressure sensor with ppm resolution was used to assist the measurements.

Oil selection was carried out with the help of representatives of organisations working in the field. Valuable information was collected which provided a useful insight into the multitude of liquids used in pressure balances, as well as the pressure range covered by the hydraulic pressure balances. Four oils were chosen for comparing with CP Hall Sebacate, which is the oil used by NPL in routine calibrations. The oil selection was intended to cover the wide range of densities and viscosities of the lubricating liquids used in practice in HPB: oil density ranged from  $862 \text{ kg m}^{-3}$  to  $988 \text{ kg m}^{-3}$  and the viscosity from 21.5 mPa s to 131 mPa s. The maximum pressure was limited to 200 MPa, as the vast majority of instruments do not exceed this limit.

The experimental set up was designed so that the '*Test*' PCA was not contaminated by CP Hall Sebacate when tested in the selected oils listed in Table 1. This was accomplished by creating a special arrangement consisting of two constant volume valves and a high-pressure sensor which confined the '*Standard*' and '*Test*' oil mixing to only a very small volume at the interface.

A mathematical model was established and extensive uncertainty budgets were developed for the measurement results. For some of the results, the magnitude of the uncertainty was larger than the change in the metrological characteristics produced by a different type of oil. This was due to a variety of factors, some of them controllable and some impossible to overcome. For example, maintaining the environmental conditions within specific limits could easily reduce the maximum influence of the temperature in the type B uncertainty from  $\pm 3.5 \text{ ppm}$  to as low as  $\pm 0.5 \text{ ppm}$ . Furthermore, increasing the number of calibrations and using additional instruments could probably improve the figure for the type A uncertainty. In contrast, the

significant effect of the uncertainty in the fluid head, especially when the PCA with large nominal area was used, could not easily be addressed.

The main conclusion is that significant systematic errors are found in the effective area when using different fluids as pressure transmitting media in a pressure balance. The change in effective area relative to CP Hall Sebacate is dependent on the pressure and therefore on gap size although at this time we cannot make any quantitative predictions relating the size of the effect to the initial gap. Although a difference related to the liquid viscosity (due to its contribution to the pressure distribution in the piston-cylinder inner space) was intuitively expected [1], the experiment instead appeared to show the effective area dependency is more closely correlated to liquid density.

Existing theories do not provide a mechanism for a density dependent effect. Efforts to validate any new theoretical model (that attempts to describe the behaviour seen in this work) are likely to require a wider selection of oils having a range of density and viscosity. Additionally, the experimental work will need to be extended to assemblies having better metrological characteristics than those used here (e.g. better repeatability and higher pressures ranges) and a wider range of PCA gap sizes.

It is predicted on the basis of the current results that the variation in the effective area will increase for PCAs with smaller effective area and bigger distortion coefficient than the ones used in this study. For instance, if we calibrate an instrument similar to the B1<sub>T2</sub> assembly, using CP Hall Sebacate when it is designed to use ISO VG37, we have a possible systematic error of 60 ppm in effective area at 200 MPa. In the work reported here the uncertainty would be 68 ppm ( $k = 2$ ) of effective area, which may be unacceptable. It follows that calibration providers should either calibrate instruments using the prescribed oil or take extra account of the different oil when evaluating uncertainties.

Using piston cylinder assemblies with smaller gaps reduces the magnitude of the effect. When cross-floating two pressure balances with the *same* fluid many of the systematic effects, such

as a change to effective area, are largely cancelled out - it is possible that larger effects would be seen if different oils were used to calibrate high precision electronic instruments.

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## APPENDIX: UNCERTAINTY ESTIMATION

This section describes the evaluation of the combined standard uncertainty associated with the measurement result  $\Delta A_T$ . The uncertainty analysis is performed in line with the recommendations of the ISO-GUM [11].

### A 1 TYPE A UNCERTAINTY

The value for  $\Delta A_T$  is derived using equation (7) with  $A_T^f$  and  $A_T^i$  representing the mean of the ‘Test’ effective area obtained, by cross-float at the same nominal pressure, for the *final* and the *initial* calibrations.

The type A standard uncertainty associated with each mean value of the effective area is estimated using the experimental standard variance of the corresponding mean:

$$\sigma_i = \sqrt{\frac{\sum_{j=1}^n (A_{Tj}^i - \overline{A_T^i})^2}{n(n-1)}} \quad \dots(12)$$

$$\sigma_f = \sqrt{\frac{\sum_{j=1}^n (A_{Tj}^f - \overline{A_T^f})^2}{n(n-1)}} \quad \dots(13)$$

For  $n = 4$  repeat measurements.

As  $\Delta A_T$  represents the difference between  $A_T^f$  and  $A_T^i$ , the associated experimental standard deviation is given by:

$$U_A = 3.18 \times \sqrt{\sigma_i^2 + \sigma_f^2} \quad \dots(14)$$

Where 3.18 is the coverage factor obtained for 95% confidence level and  $\nu = 3$  degrees of freedom.

## A 2 TYPE B UNCERTAINTY

When evaluating the type B uncertainty associated with the  $\Delta A_T$  value, the input quantities  $X_j$  of Eq. (10) are considered.

The type B standard variance is given by:

$$u_B^2(\Delta A_T) = \sum_j \left( \frac{\partial \Delta A_T}{\partial x_j} \right)^2 u^2(x_j) = \sum_j c_j^2 u^2(x_j) = \sum_j u_{x_j}^2(\Delta A_T) \quad \dots(15)$$

Where  $u_B(\Delta A_T)$  is type B standard uncertainty and  $u(x_j)$  is the standard uncertainty associated with the measured estimates,  $x_j$  of the input quantities  $X_j$ . The term  $c_j$  is known as the sensitivity coefficient and  $u_x(\Delta A_T)$  is the standard uncertainty associated with the output estimate  $\Delta A_T$ .

$u_x(\Delta A_T)$  can be written as:

$$u_{x_j}(\Delta A_T) = \left| \frac{\partial \Delta A_T}{\partial x_j} \right| u(x_j) = |c_j| u(x_j) \quad \dots(16)$$

The sensitivity coefficients for an equation such as (10) are quite complicated, however when evaluating the sensitivity coefficients a number of simple approximations can be made, as summarised in Table 9.

Once the sensitivity coefficient expressions are known the standard uncertainty associated with the  $\Delta A_T$  value generated by any input quantity can be estimated. It is also useful to estimate the maximum value of the uncertainty due to each input quantity, as this may allow further simplification of the final calculation of the type B standard uncertainty.

The remainder of this section will discuss and estimate the effect each input quantity has on the type B uncertainty, first by calculating the sensitivity coefficient and then estimating the maximum value of  $u_x(\Delta A_T)$ . It will be shown that many of the terms do not make significant contribution to the overall uncertainty. Those terms that *are* identified as significant have been evaluated on a point-by-point basis for evaluation of the uncertainties reported in the main body of this document.

**Table 9** - *Approximations used for evaluation of sensitivity coefficients and estimate of closeness to correct value. The relative error is evaluated by comparing the effective area calculated using the full equation against the area calculated when the approximation is applied.*

| Approximation used for evaluation of sensitivity coefficients.                 | Relative error approximation /ppm of value |
|--------------------------------------------------------------------------------|--------------------------------------------|
| $1 + \alpha_{St}(t_{St}^f - 20) = 1$                                           | 30 ppm                                     |
| $1 + \alpha_{St}(t_{St}^i - 20) = 1$                                           | 30 ppm                                     |
| $1 + \alpha_T(t_T^f - 20) = 1$                                                 | 30 ppm                                     |
| $1 + \alpha_T(t_T^i - 20) = 1$                                                 | 30 ppm                                     |
| $b_{Tj}^f = 1$                                                                 | 156 ppm                                    |
| $b_{Stj}^f = 1$                                                                | 156 ppm                                    |
| $b_{Tj}^i = 1$                                                                 | 156 ppm                                    |
| $b_{Stj}^i = 1$                                                                | 156 ppm                                    |
| $\Delta m_T b_T = 0$                                                           | 60 ppm                                     |
| $\Delta m_{St} b_{St} = 0$                                                     | 60 ppm                                     |
| $(\rho_{IT} - \rho_a^f) V_T = 0$                                               | 30 ppm                                     |
| $(\rho_{ISt} - \rho_a^f) V_{St} = 0$                                           | 30 ppm                                     |
| $(\rho_{IT} - \rho_a^i) V_T = 0$                                               | 30 ppm                                     |
| $(\rho_{ISt} - \rho_a^i) V_{St} = 0$                                           | 30 ppm                                     |
| $[(\rho_{ISt} - \rho_a^f) h_{St}^f - (\rho_{IT} - \rho_a^f) h_t^f] A_{St} = 0$ | 70 ppm                                     |
| $[(\rho_{ISt} - \rho_a^i) h_{St}^i - (\rho_{IT} - \rho_a^i) h_t^i] A_{St} = 0$ | 70 ppm                                     |
| $\frac{\gamma_T C_T}{g} = 0$                                                   | 10 ppm                                     |
| $\frac{\gamma_{St} C_{St}}{g} = 0$                                             | 10 ppm                                     |
| $\frac{\gamma_{St} C_T}{g} = 0$                                                | 10 ppm                                     |

## A 2.1 Mass Contribution

### *Mass loaded on ‘Standard’*

The simplified sensitivity coefficient describing the variation of  $\Delta A_T$  with one mass artefact  $m_{Stj}$  loaded on ‘Standard’ is given by:

$$c_{m_{Stj}} = \frac{\partial \Delta A_T}{\partial m_{Stj}} \cong - \frac{\Delta A_T}{\sum_j m_{Stj}} \quad \dots(17)$$

The standard uncertainties associated with the mass are mainly systematic in nature therefore the uncertainty associated with  $\Delta A_T$  generated by the total mass loaded on ‘Standard’ is obtained from the arithmetic summation of the contributions of each mass loaded on ‘Standard’ rather than using a quadrature summation [12].

From (16) and (17):

$$u_{m_{St}}(\Delta A_T) = \sum_j \left( \frac{\Delta A_T}{\sum_j m_{Stj}} u(m_{Stj}) \right) = \frac{\Delta A_T}{\sum_j m_{Stj}} \sum_j u(m_{Stj}) \quad \dots(18)$$

| Input                | Maximum or Minimum Value        | Source of Value                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |
|----------------------|---------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| $u(m_{Stj}) _{\max}$ | $\pm 5 \text{ ppm of } m_{Stj}$ | The same mass was loaded on ‘Standard’ to generate an identical nominal pressure when establishing $A_T^i$ and $A_T^f$ , respectively. As a result, the drift is the only component affecting the standard uncertainty associated with the mass (measurements were up to 4 weeks apart). The historical data represents a good source of information for the drift estimation that was found not to exceed $\pm 2 \text{ ppm}$ of mass per year [12]. We consider that a value of $\pm 5 \text{ ppm}$ is a better estimate for the mass drift, due to of the relatively harsh treatment of the mass artefacts in this experiment. A rectangular distribution was assumed for the mass drift when calculating the standard |

| Input                | Maximum or Minimum Value       | Source of Value                                                                                                                                         |
|----------------------|--------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------|
|                      |                                | uncertainty.                                                                                                                                            |
| $\Delta A_T _{\max}$ | 60 ppm of $A_{\text{nominal}}$ | The maximum difference of the 'Test' effective area was 60 ppm of nominal effective area when B1 <sub>T2</sub> assembly was calibrated in ISO VG37 oil. |

Using Eq. (18), the maximum contribution of mass loaded on 'Standard' will then be given by:

$$\begin{aligned}
 u_{m_{St}}(\Delta A_T)|_{\max} &= \frac{\Delta A_T|_{\max}}{\sum_j m_{Stj}} \sum_j u(m_{Stj}) \Big|_{\max} = \frac{1}{\sqrt{3}} \frac{0.000\,06\, A_{\text{nominal}}}{\sum_j m_{Stj}} 0.000\,005 \sum_j m_{Stj} \\
 &= 1.73 \times 10^{-10} A_{\text{nominal}}
 \end{aligned} \quad \dots(19)$$

This indicates that the estimated uncertainty associated with  $\Delta A_T$  generated by the mass loaded on 'Standard' is not greater than  $\pm 0.000\,2$  ppm of effective area ( $k = 1$ ).

#### Mass loaded on 'Test'

The sensitivity coefficient describing the variation of  $\Delta A_T$  with one mass artefact  $m_{Tj}$  loaded on 'Test' is given by:

$$c_{m_{Tj}} = \frac{\partial \Delta A_T}{\partial m_{Tj}} \cong \frac{\Delta A_T}{\sum_j m_{Tj}} \quad \dots(20)$$

Hence:

$$u_{m_T}(\Delta A_T) = \sum_j \left( \frac{\Delta A_T}{\sum_j m_{Tj}} u(m_{Tj}) \right) = \frac{\Delta A_T}{\sum_j m_{Tj}} \sum_j u(m_{Tj}) \quad \dots(21)$$

The same considerations used for the mass loaded on 'Standard' apply to the 'Test' load.

$$u_{m_T}(\Delta A_T)|_{\max} = 1.73 \times 10^{-10} A_{\text{nominal}} \quad \dots(22)$$

This shows the estimated uncertainty associated with  $\Delta A_T$  generated by the mass loaded on 'Test' to be less than  $\pm 0.000\,2$  ppm of effective area, at  $k = 1$ .

*Effective trim mass*

$$c_{\Delta m} = \frac{\partial \Delta A_T}{\partial \Delta m} \cong \frac{A_{St}}{\sum_j m_{Stj}} \quad \dots(23)$$

Therefore:

$$u_{\Delta m}(\Delta A_T) = \frac{A_{St}}{\sum_j m_{Stj}} u(\Delta m) \quad \dots(24)$$

| Input                 | Maximum or Minimum Value                          | Source of Value                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |
|-----------------------|---------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| $u(\Delta m) _{\max}$ | $\pm 6 \times 10^{-3}$ ppm<br>of $\sum_j m_{Stj}$ | <p>Using data supplied with the calibration certificates of the mass set, the uncertainty associated with the trim mass <math>u(\Delta m)</math> is not bigger than <math>\pm 100</math> ppm. A normal distribution was assumed for the trim mass when used in calculating the standard uncertainty.</p> <p>The estimated uncertainty associated with <math>\Delta A_T</math> generated by the trim mass reaches its maximum when the <math>\Delta A_T</math> is a maximum. As the maximum difference of the effective area obtained during the experiment was found to be less than 60 ppm of effective area, the trim mass difference did not exceed 60 ppm of total mass loaded on one instrument.</p> |

Using Eq. (24), the maximum contribution of the difference in trim mass is:

$$\begin{aligned}
 u_{\Delta m}(\Delta A_T)|_{\max} &= \frac{A_{St}}{\sum_j m_{Stj}} u(\Delta m)|_{\max} = \frac{A_{St}}{\sum_j m_{Stj}} \frac{0.000\ 06 \sum_j m_{Stj} 0.000\ 1}{2} \\
 &= 3 \times 10^{-9} A_{\text{nominal}}
 \end{aligned} \quad \dots(25)$$

This shows that the estimated uncertainty associated with  $\Delta A_T$  generated by the trim mass is less than  $\pm 0.003$  ppm of effective area,  $k = 1$ .

*Combined mass influence*

Taking account of the high degree of correlation in the mass uncertainty we can write:

$$u_m(\Delta A_T) = u_{m_{Si}}(\Delta A_T) + u_{m_T}(\Delta A_T) + u_{\Delta m}(\Delta A_T) \quad \dots(26)$$

Considering the calculated maximum values obtained in Eqs. (19), (22), (25) for  $u_{m_{Si}}(\Delta A_T)$ ,  $u_{m_T}(\Delta A_T)$  and  $u_{\Delta m}(\Delta A_T)$  respectively, the maximum value thus obtained for  $u_m(\Delta A_T)$  does not exceed  $\pm 0.0034$  ppm of effective area ( $k = 1$ ).

## A 2.2 Density of the applied mass

### *Density of the mass loaded on 'Standard'*

For each mass artefact of density  $\rho_{Stj}$ :

$$c_{\rho_{Stj}} = \frac{\partial \Delta A_T}{\partial \rho_{Stj}} \cong - \frac{A_{St}}{\sum_j m_{Stj}} m_{Stj} \frac{\rho_a^f - \rho_a^i}{\rho_{Stj}^2} \quad \dots(27)$$

Therefore, for the assembly of masses:

$$u_{\rho_{St}}(\Delta A_T) = \sum_j \left( \frac{A_{St}}{\sum_j m_{Stj}} m_{Stj} \frac{|\rho_a^f - \rho_a^i|}{\rho_{Stj}^2} u(\rho_{Stj}) \right) = \frac{A_{St} |\rho_a^f - \rho_a^i|}{\sum_j m_{Stj}} \sum_j \frac{m_{Stj}}{\rho_{Stj}^2} u(\rho_{Stj}) \quad \dots(28)$$

| Input                          | Maximum or Minimum Value    | Source of Value                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |
|--------------------------------|-----------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| $u(\rho_{Stj}) _{\max}$        | $\pm 1.2 \text{ kg m}^{-3}$ | The value of the mass density was assumed to be constant for the duration of the experimental work thus the associated uncertainty was only dependent upon the thermal stability of the environment. For typical thermal coefficient of $45 \text{ ppm } ^\circ\text{C}^{-1}$ for stainless steel of density $7900 \text{ kg m}^{-3}$ and a maximum difference recorded in the ambient temperature during the experiment of $3.5^\circ\text{C}$ , the uncertainty associated with the mass density did not exceed $\pm 1.2 \text{ kg m}^{-3}$ . A rectangular distribution was assumed for the mass density when calculating the standard uncertainty. |
| $ \rho_a^f - \rho_a^i _{\max}$ | $0.055 \text{ kg m}^{-3}$   | The maximum difference in the air density between the <i>final</i> and <i>initial</i> calibration and for a minimum value of the mass density was less than $0.055 \text{ kg m}^{-3}$                                                                                                                                                                                                                                                                                                                                                                                                                                                                  |
| $\rho_{Stj} _{\min}$           | $7900 \text{ kg m}^{-3}$    | The minimum value of the mass density was $7900 \text{ kg m}^{-3}$ .                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |

Using Eq. (28), the maximum contribution due to the mass density loaded on ‘Standard’ will then be given by:

$$\begin{aligned}
 u_{\rho_{St}}(\Delta A_T) \Big|_{\max} &= \frac{A_{St} |\rho_a^f - \rho_a^i|_{\max}}{\sum_j m_{Stj}} \sum_j \frac{m_{Stj}}{\rho_{Stj}^2 \Big|_{\min}} u(\rho_{Stj}) \Big|_{\max} \\
 &= \frac{A_{St} 0.055}{\sum_j m_{Stj}} \sum_j \frac{m_{Stj}}{7900^2} \frac{2}{\sqrt{3}} \\
 &= 10^{-9} A_{\text{nominal}}
 \end{aligned} \tag{29}$$

This indicates that the estimated uncertainty associated with  $\Delta A_T$  generated by the mass density loaded on ‘Standard’ is not greater than  $\pm 0.001$  ppm of effective area ( $k = 1$ ).

#### *Density of the mass loaded on ‘Test’*

For mass artefacts of density  $\rho_{Tj}$  loaded on ‘Test’:

$$c_{\rho_{Tj}} = \frac{\partial \Delta A_T}{\partial \rho_{Tj}} \cong \frac{A_{St}}{\sum_j m_{Tj}} m_{Tj} \frac{\rho_a^f - \rho_a^i}{\rho_{Tj}^2} \tag{30}$$

Hence:

$$u_{\rho_T}(\Delta A_T) = \sum_j \left( \frac{A_{St}}{\sum_j m_{Tj}} m_{Tj} \frac{|\rho_a^f - \rho_a^i|}{\rho_T^2} u(\rho_{Tj}) \right) = \frac{A_{St} |\rho_a^f - \rho_a^i|}{\sum_j m_{Tj}} \sum_j \frac{m_{Tj}}{\rho_{Tj}^2} u(\rho_{Tj}) \tag{31}$$

The same considerations used for the density of the mass loaded on ‘Standard’ apply to the density of the mass loaded on ‘Test’, therefore:

$$u_{\rho_T}(\Delta A_T) \Big|_{\max} = 10^{-9} A_{\text{nominal}} \tag{32}$$

#### *Combined influence of the density of the mass loaded on ‘Standard’ and ‘Test’*

The estimated uncertainty associated with  $\Delta A_T$  generated by the density of the mass loaded on ‘Test’ and on ‘Standard’, respectively is given by their arithmetical summation:

$$u_{\rho_m}(\Delta A_T) = u_{\rho_{St}}(\Delta A_T) + u_{\rho_T}(\Delta A_T) \tag{33}$$

The maximum value thus obtained for  $u_{\rho_m}(\Delta A_T)$  does not exceed  $\pm 0.002$  ppm of effective area ( $k = 1$ ).

### A 2.3 Standard Effective area

For the effective area of ‘*Standard*’:

$$c_{A_{St}} = \frac{\partial \Delta A_T}{\partial A_{St}} \cong \frac{\Delta A_T}{A_{St}} \quad \dots(34)$$

Hence:

$$u_{A_{St}}(\Delta A_T) = \frac{\Delta A_T}{A_{St}} u(A_{St}) \quad \dots(35)$$

| Input                      | Maximum or Minimum Value            | Source of Value                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |
|----------------------------|-------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| $u(A_{St})$                | $\pm 2$ ppm of $A_{\text{nominal}}$ | The drift in ‘ <i>Standard</i> ’ effective area during the experiment is the only contribution affecting the uncertainty in $\Delta A_T$ . The effective area of a pressure balance such as the one used in our experiments does not drift more than a few ppm per year, thus a $\pm 2$ ppm drift is not unreasonable for the month long period of our measurements [12]. A rectangular distribution was assumed for the ‘ <i>Standard</i> ’ effective area when calculating the standard uncertainty. |
| $\Delta A_T _{\text{max}}$ | 60 ppm of $A_{\text{nominal}}$      | The estimated uncertainty associated with $\Delta A_T$ generated by the ‘ <i>Standard</i> ’ effective area is again obtained when the $\Delta A_T$ is maximum.                                                                                                                                                                                                                                                                                                                                         |

Using Eq. (35), the maximum will be given by:

$$\begin{aligned}
 u_{A_{St}}(\Delta A_T)|_{\text{max}} &= \frac{\Delta A_T|_{\text{max}}}{A_{St}} u(A_{St}) = \frac{0.000\ 06 A_{St}}{A_{St}} \frac{0.000\ 002 A_{St}}{\sqrt{3}} \\
 &= 7 \times 10^{-11} A_{\text{nominal}}
 \end{aligned} \quad \dots(36)$$

This shows that the uncertainty associated with  $\Delta A_T$  generated by the ‘*Standard*’ effective area is less than  $\pm 0.000\ 07$  ppm of effective area ( $k = 1$ ).

## A 2.4 Piston circumference

*‘Standard’ piston circumference*

$$c_{C_{St}} = \frac{\partial \Delta A_T}{\partial C_{St}} \cong 0 \quad \dots(37)$$

Hence:

$$u_{C_{St}}(\Delta A_T) \cong 0 \quad \dots(38)$$

*‘Test’ piston circumference*

$$c_{C_T} = \frac{\partial \Delta A_T}{\partial C_T} \cong \frac{A_{St}}{g \sum_j m_{Tj}} (\gamma_T - \gamma_{St}) \quad \dots(39)$$

Hence:

$$u_{C_T}(\Delta A_T) = \frac{A_{St}}{g \sum_j m_{Tj}} |\gamma_T - \gamma_{St}| u(C_T) \quad \dots(40)$$

| Input    | Maximum or Minimum Value | Source of Value                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |
|----------|--------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| $u(C_T)$ | $\pm 0.000 \text{ 1 m}$  | The piston circumference is derived from the PCA nominal area. Historical data confirms that the effective area of any PCA from the B3 family does not exceed its nominal value by more than $\pm 0.5 \text{ mm}^2$ , therefore the standard uncertainty associated with the piston circumference is not greater than 0.1 mm. This estimate is an upper limit for any piston used in the currently described experimental work (as the nominal area of the two other families is smaller than the effective area of B3 family). A rectangular distribution was assumed for the circumference when calculating the standard uncertainty. |

| Input                           | Maximum or Minimum Value | Source of Value                                                                                                                 |
|---------------------------------|--------------------------|---------------------------------------------------------------------------------------------------------------------------------|
| $ \gamma_{St}-\gamma_T _{\max}$ | 0.006 N m <sup>-1</sup>  | The difference in the ‘ <i>Test</i> ’ and ‘ <i>Standard</i> ’ oil surface tension was maximum when HP Budenberg oil was tested. |
| $\sum_j m_{Tj}  _{\min}$        | 23 kg                    | The minimum value of the mass applied on ‘ <i>Test</i> ’ was when HPBs were generating the lowest pressure                      |

Using Eq (40), the maximum contribution of the ‘*Test*’ circumference will be:

$$\begin{aligned}
 u_{C_T}(\Delta A_T) \Big|_{\max} &= \frac{A_{St}}{g \sum_j m_{Tj} \Big|_{\min}} |\gamma_T - \gamma_{St}|_{\max} \quad u(C_T) = \frac{A_{St}}{9.8 \times 23} \frac{0.006 \times 0.0001}{\sqrt{3}} \\
 &= 1.5 \times 10^{-9} A_{\text{nominal}}
 \end{aligned}
 \tag{41}$$

This shows that the estimated uncertainty associated with  $\Delta A_T$  generated by the ‘*Test*’ circumference does not exceed  $\pm 0.0015$  ppm of effective area ( $k = 1$ ).

## A 2.5 Buoyancy volume

*‘Standard’ buoyancy volume*

$$c_{V_{St}} = \frac{\partial \Delta A_T}{\partial V_{St}} \cong 0 \quad \dots(42)$$

Hence:

$$u_{V_{St}}(\Delta A_T) \cong 0 \quad \dots(43)$$

*‘Test’ buoyancy volume*

$$c_{V_T} = \frac{\partial \Delta A_T}{\partial V_T} \cong - \frac{A_{St}}{\sum_j m_{Tj}} (\rho_{IT} - \rho_{ISt}) \quad \dots(44)$$

The uncertainty due to the test buoyancy volume is:

$$u_{V_T}(\Delta A_T) = \frac{A_{St}}{\sum_j m_{Tj}} |\rho_{IT} - \rho_{ISt}| u(V_T) \quad \dots(45)$$

| Input                             | Maximum or Minimum Value             | Source of Value                                                                                                                                                                                                                                                                                                                                                                                                                                                |
|-----------------------------------|--------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| $u(V_T) _{\max}$                  | $\pm 12 \times 10^{-11} \text{ m}^3$ | As temperature variation during experiment was no bigger than 3.5 °C the standard uncertainty associated with the buoyancy volume does not exceed 150 ppm (stainless steel thermal coefficient is 45 ppm). A rectangular distribution was assumed for the buoyancy volume when calculating the standard uncertainty.<br><br>The estimated uncertainty associated with buoyancy volume reaches its maximum for a maximum buoyancy volume (800 mm <sup>3</sup> ) |
| $ \rho_{StI} - \rho_{TI} _{\max}$ | $75 \text{ kg m}^{-3}$               | The difference in the ‘Test’ and ‘Standard’ oil density was maximum when HP Budenberg oil was tested.                                                                                                                                                                                                                                                                                                                                                          |

| Input                    | Maximum or Minimum Value | Source of Value                                                                                        |
|--------------------------|--------------------------|--------------------------------------------------------------------------------------------------------|
| $\sum_j m_{Tj}  _{\min}$ | 23 kg                    | The minimum value of the mass applied on 'Test' occurred when HPBs were generating the lowest pressure |

The maximum influence of the buoyancy volume will thus be:

$$\begin{aligned}
 u_{V_T}(\Delta A_T) \Big|_{\max} &= \frac{A_{St}}{\sum_j m_{Tj} \Big|_{\min}} |\rho_{IT} - \rho_{ISt}|_{\max} u(V_T) \Big|_{\max} = \frac{A_{St}}{23} 75 \times \frac{15 \times 10^{-5} \times 8 \times 10^{-7}}{\sqrt{3}} \\
 &= 2 \times 10^{-10} A_{\text{nominal}}
 \end{aligned} \quad \dots(46)$$

The estimated uncertainty associated with  $\Delta A_T$  generated by the 'Test' buoyancy volume does not therefore exceed  $\pm 0.000\,2$  ppm of effective area ( $k = 1$ ).

## A 2.6 Fluid head

$$c_{h_{St}^i} = \frac{\partial \Delta A_T}{\partial h_{St}^i} \cong - \frac{A_{St}^2}{\sum_j m_{Stj}} \rho_{lSt} \quad \dots(47)$$

$$c_{h_{St}^f} = \frac{\partial \Delta A_T}{\partial h_{St}^f} \cong \frac{A_{St}^2}{\sum_j m_{Stj}} \rho_{lSt} \quad \dots(48)$$

$$c_{h_T^i} = \frac{\partial \Delta A_T}{\partial h_{St}^i} \cong \frac{A_{St}^2}{\sum_j m_{Stj}} \rho_{lSt} \quad \dots(49)$$

$$c_{h_T^f} = \frac{\partial \Delta A_T}{\partial h_T^f} \cong - \frac{A_{St}^2}{\sum_j m_{Stj}} \rho_{lT} \quad \dots(50)$$

The uncertainty due to the fluid head is:

$$\begin{aligned} u_h(\Delta A_T) &= \sqrt{u_{h_{St}^i}^2(\Delta A_T) + u_{h_{St}^f}^2(\Delta A_T) + u_{h_T^i}^2(\Delta A_T) + u_{h_T^f}^2(\Delta A_T)} \\ &\cong \frac{A_{St}^2 \sqrt{3\rho_{lSt}^2 + \rho_{lT}^2}}{\sum_j m_{Stj}} u(h) \end{aligned} \quad \dots(51)$$

| Input                    | Maximum or Minimum Value | Source of Value                                                                                                                                                                                                                                                                                                                                                            |
|--------------------------|--------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| $u(h)$                   | $\pm 0.0005 \text{ m}$   | The fluid head was measured using a cathetometer. The pistons' vertical positions during the experiments were monitored using capacitance sensors. In these conditions the standard uncertainty associated with the fluid head does not exceed $\pm 0.5 \text{ mm}$ . A rectangular distribution was assumed for the fluid head when calculating the standard uncertainty. |
| $\rho_{lT} _{\max}$      | $989 \text{ kg m}^{-3}$  | The 'Test' oil density was maximum when HP Budenberg oil was tested.                                                                                                                                                                                                                                                                                                       |
| $\sum_j m_{Stj} _{\min}$ | $23 \text{ kg}$          | The minimum value of the mass applied on 'Standard' occurred when HPBs were generating the lowest pressure                                                                                                                                                                                                                                                                 |

| Input            | Maximum or Minimum Value | Source of Value                                                      |
|------------------|--------------------------|----------------------------------------------------------------------|
| $A_{St} _{\max}$ | 83 mm <sup>2</sup>       | The 'Standard' effective area was maximum when B3 family was tested. |

Using Eq (51), the maximum influence of fluid head is given by:

$$\begin{aligned}
 u_h(\Delta A_T)|_{\max} &= \frac{A_{St}^2|_{\max} \sqrt{3\rho_{lSt}^2 + \rho_{IT}^2}|_{\max}}{\sum_j m_{Stj}|_{\min}} u(h) \\
 &= \frac{0.000083 A_{St} \sqrt{3 \times 914^2 + 989^2}}{23} \frac{0.0005}{\sqrt{3}} \\
 &= 2 \times 10^{-6} A_{\text{nominal}}
 \end{aligned} \tag{52}$$

This indicates that the estimated uncertainty associated with  $\Delta A_T$  generated by the fluid head does not exceed  $\pm 2$  ppm of effective area ( $k = 1$ ).

## A 2.7 Temperature coefficient

*‘Standard’ temperature coefficient*

$$c_{\alpha_{St}} = \frac{\partial \Delta A_T}{\partial \alpha_{St}} \cong A_{St} (t_{St}^f - t_{St}^i) \quad \dots(53)$$

Hence:

$$u_{\alpha_{St}}(\Delta A_T) = A_{St} |t_{St}^f - t_{St}^i| u(\alpha_{St}) \quad \dots(54)$$

| Input                          | Maximum or Minimum Value                   | Source of Value                                                                                                                                                                                                                                                                                                                                                                        |
|--------------------------------|--------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| $u(\alpha_{St})$               | $\pm 0.9 \text{ ppm } ^\circ\text{C}^{-1}$ | The uncertainty of the piston-cylinder temperature coefficient is estimated to be about $\pm 10 \%$ of its nominal value, 9 ppm, as is suggested in previous studies [12]. A rectangular distribution was assumed for the piston-cylinder temperature coefficient when calculating the standard uncertainty.                                                                           |
| $ t_{St}^f - t_{St}^i _{\max}$ | $\pm 3.5 \text{ } ^\circ\text{C}$          | The estimated uncertainty associated with $\Delta A_T$ due to the <i>‘Standard’</i> temperature coefficient is maximum when the difference in the <i>‘Standard’</i> temperature between the <i>initial</i> and <i>final</i> calibration is a maximum. The maximum difference in the <i>‘Standard’</i> temperature registered during the experiment was $3.5 \text{ } ^\circ\text{C}$ . |

Using Eq. (54), the maximum influence of the *‘Standard’* temperature coefficient will be:

$$\begin{aligned} u_{\alpha_{St}}(\Delta A_T) \Big|_{\max} &= A_{St} |t_{St}^f - t_{St}^i|_{\max} u(\alpha_{St}) = 3.5 \frac{9 \times 10^{-7}}{\sqrt{3}} A_{St} \\ &= 1.8 \times 10^{-6} A_{\text{nominal}} \end{aligned} \quad \dots(55)$$

Eq. (55) indicates that the uncertainty associated with  $\Delta A_T$  generated by the *‘Standard’* temperature coefficient does not exceed  $\pm 1.8 \text{ ppm}$  of effective area ( $k = 1$ ).

*'Test' temperature coefficient*

$$c_{\alpha_T} = \frac{\partial \Delta A_T}{\partial \alpha_T} \cong -A_{St} \times (t_T^f - t_T^i) \quad \dots(56)$$

Hence:

$$u_{\alpha_T}(\Delta A_T) = A_{St} |t_T^f - t_T^i| u(\alpha_T) \quad \dots(57)$$

The same considerations used for the *'Standard'* temperature coefficient apply to the *'Test'* temperature coefficient. Therefore:

$$u_{\alpha_T}(\Delta A_T) \Big|_{\max} = 1.8 \times 10^{-6} A_{\text{nominal}} \quad \dots(58)$$

The uncertainty associated with  $\Delta A_T$  generated by the *'Test'* temperature coefficient does not exceed  $\pm 1.8$  ppm of effective area ( $k = 1$ ) as can be seen from Eq (58).

*Combined 'Standard' and 'Test' temperature coefficient*

The uncertainty associated with  $\Delta A_T$  generated by the uncertainty due to the *'Test'* and *'Standard'* temperature coefficient is given by their quadratical summation:

$$u_{\alpha}(\Delta A_T) = \sqrt{u_{\alpha_{St}}(\Delta A_T) + u_{\alpha_T}(\Delta A_T)} \quad \dots(59)$$

Considering the calculated maximum values obtained in Eqs. (55), (58) the maximum value thus obtained for combined *'Standard'* and *'Test'* temperature coefficient uncertainty does not exceed  $\pm 2.5$  ppm of effective area ( $k = 1$ ).

## A 2.8 Temperature

*‘Standard’ temperature*

$$c_{t_{St}} = \frac{\partial \Delta A_T}{\partial t_{St}} \cong \Delta A_T \alpha_{St} \quad \dots(60)$$

Hence:

$$u_{t_{St}}(\Delta A_T) = |\Delta A_T| \alpha_{St} u(t_{St}) \quad \dots(61)$$

| Input                | Maximum or Minimum Value       | Source of Value                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |
|----------------------|--------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| $u(t_{St})$          | $\pm 0.05^\circ\text{C}$       | The temperature probe was kept at the same position on the outside wall of the base column throughout the experiment. With the temperature measured at the same position of the PCA only the linearity and resolution of the temperature indicator contributes to the standard uncertainty due to the measured temperature. Thus the uncertainty associated with the <i>‘Standard’</i> temperature was estimated to be no bigger than $\pm 0.05^\circ\text{C}$ . A rectangular distribution was assumed for the temperature when calculating the standard uncertainty. |
| $\Delta A_T _{\max}$ | 60 ppm of $A_{\text{nominal}}$ | The estimated uncertainty associated with $\Delta A_T$ generated by the <i>‘Standard’</i> temperature is obtained when the $\Delta A_T$ is maximum.                                                                                                                                                                                                                                                                                                                                                                                                                    |

Using Eq. (61), the maximum influence of the *‘Standard’* temperature is given by:

$$\begin{aligned} u_{t_{St}}(\Delta A_T)|_{\max} &= \Delta A_T|_{\max} \alpha_{St} u(t_{St}) = 0.00006 A_{St} 0.000009 \frac{0.05}{\sqrt{3}} \\ &= 1.5 \times 10^{-11} A_{\text{nominal}} \end{aligned} \quad \dots(62)$$

This shows that the estimated uncertainty associated with  $\Delta A_T$  generated by the ‘*Standard*’ temperature does not thus exceed  $\pm 0.000\ 15$  ppm of effective area ( $k = 1$ ).

‘*Test*’ temperature

$$c_{t_T} = \frac{\partial \Delta A_T}{\partial t_T} \cong -\Delta A_T \times \alpha_T \quad \dots(63)$$

Hence:

$$u_{t_T}(\Delta A_T) = |\Delta A_T| \alpha_T u(t_T) \quad \dots(64)$$

The same considerations used for the ‘*Standard*’ temperature apply to the ‘*Test*’ temperature, therefore:

$$u_{t_T}(\Delta A_T)_{\max} = 1.5 \times 10^{-11} A_{\text{nominal}} \quad \dots(65)$$

Combined ‘*Standard*’ and ‘*Test*’ temperature

The uncertainty associated with  $\Delta A_T$  generated by the uncertainty due to the ‘*Test*’ and ‘*Standard*’ temperature is given by their quadratical summation:

$$u_t(\Delta A_T) = \sqrt{u_{t_{St}}(\Delta A_T) + u_{t_T}(\Delta A_T)} \quad \dots(66)$$

Considering the calculated maximum values obtained in Eqs. (62), (65) the maximum value thus obtained for combined ‘*Standard*’ and ‘*Test*’ temperature uncertainty does not exceed  $\pm 0.000\ 2$  ppm of effective area ( $k = 1$ ).

## A 2.9 Air density

$$c_{\rho_a^i} = \frac{\partial \Delta A_T}{\partial \rho_a^i} \cong - \frac{A_{St}^2}{\sum_j m_{Stj}} \times (h_{St}^i - h_T^i) \quad \dots(67)$$

$$c_{\rho_a^f} = \frac{\partial \Delta A_T}{\partial \rho_a^f} \cong - \frac{A_{St}^2}{\sum_j m_{Stj}} \times (h_{St}^f - h_T^f) \quad \dots(68)$$

Hence:

$$u_{\rho_a}(\Delta A_T) = \sqrt{u_{\rho_a^i}^2(\Delta A_T) + u_{\rho_a^f}^2(\Delta A_T)} = \frac{A_{St}^2}{\sum_j m_{Stj}} \sqrt{(h_{St}^i - h_T^i)^2 + (h_{St}^f - h_T^f)^2} u(\rho_a) \quad \dots(69)$$

| Input                    | Maximum or Minimum Value     | Source of Value                                                                                                                                                                                                                            |
|--------------------------|------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| $u(\rho_a)$              | $\pm 0.05 \text{ kg m}^{-3}$ | The air density was calculated using the equation of Davies [13] with an estimated uncertainty of $\pm 0.05 \text{ kg m}^{-3}$ [12]. A rectangular distribution was assumed for the air density when calculating the standard uncertainty. |
| $h_{St} - h_T _{\max}$   | 0.01 m                       | The maximum difference in height between 'Test' reference level and 'Standard' reference level is 10 mm.                                                                                                                                   |
| $\sum_j m_{Stj} _{\min}$ | 23 kg                        | The minimum value of the mass applied on 'Standard' occurred when HPBs were generating the lowest pressure                                                                                                                                 |
| $A_{St} _{\max}$         | $83 \text{ mm}^2$            | The 'Standard' effective area was maximum when the B3 family was tested.                                                                                                                                                                   |

Using Eq. (69), the maximum influence of the air density will be give by:

$$\begin{aligned}
u_{\rho_a}(\Delta A_T) \Big|_{\max} &= \frac{A_{St}^2 \Big|_{\max}}{\sum_j m_{Stj} \Big|_{\min}} \sqrt{\left(h_{St}^i - h_T^i\right)^2 \Big|_{\max} + \left(h_{St}^f - h_T^f\right)^2 \Big|_{\max}} u(\rho_a) \\
&= \frac{0.000083 A_{St}}{23} 0.01 \sqrt{2} \frac{0.05}{\sqrt{3}} \\
&= 1.5 \times 10^{-9} A_{\text{nominal}}
\end{aligned} \tag{70}$$

The uncertainty associated with  $\Delta A_T$  due to the air density does not exceed  $\pm 0.0015$  ppm of effective area ( $k = 1$ ).

## A 2.10 Oil surface tension

$$c_{\gamma_{St}} = \frac{\partial \Delta A_T}{\partial \gamma_{St}} \cong - \frac{A_{St}}{\sum_j m_{Stj}} \frac{C_T}{g} \quad \dots(71)$$

$$c_{\gamma_T} = \frac{\partial \Delta A_T}{\partial \gamma_T} \cong \frac{A_{St}}{\sum_j m_{Stj}} \frac{C_T}{g} \quad \dots(72)$$

Hence:

$$u_{\gamma}(\Delta A_T) = \sqrt{u_{\gamma_T}^2(\Delta A_T) + u_{\gamma_{St}}^2(\Delta A_T)} = \frac{A_{St}}{\sum_j m_{Stj}} \frac{C_T}{g} \sqrt{u(\gamma_T)^2 + u(\gamma_{St})^2} \quad \dots(73)$$

| Input                     | Maximum or Minimum Value       | Source of Value                                                                                                                                                                                                                                                                                                 |
|---------------------------|--------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| $u(\gamma)$               | $\pm 10 \% \text{ of } \gamma$ | The uncertainties for the surface tension value are not provided with the manufacturers' oil data, therefore the standard uncertainty of the oil surface tension is estimated to be $\pm 10 \%$ . A rectangular distribution was assumed for the oil surface tension when calculating the standard uncertainty. |
| $\sum_j m_{Stj}  _{\min}$ | 23 kg                          | The minimum value of the mass applied on 'Standard' occurred when HPBs were generating the lowest pressure                                                                                                                                                                                                      |
| $C _{\max}$               | 0.032 m                        | The 'Standard' and 'Test' effective area was maximum when B3 family was tested.                                                                                                                                                                                                                                 |

Using Eq. (73), the maximum influence of the air density will be give by:

$$\begin{aligned}
 u_{\gamma}(\Delta A_T)|_{\max} &= \frac{A_{St}|_{\max}}{\sum_j m_{Stj}|_{\min}} \frac{C_T|_{\max}}{g} \sqrt{u(\gamma_{St})^2 + u(\gamma_T)^2} \\
 &= \frac{A_{St}}{23} \frac{0.032}{9.8} \frac{0.003 \times \sqrt{2}}{\sqrt{3}} \\
 &= 3.5 \times 10^{-7} A_{\text{nominal}}
 \end{aligned} \quad \dots(74)$$

The uncertainty associated with  $\Delta A_T$  due to the fluid surface tension does not exceed  $\pm 0.35$  ppm of effective area ( $k = 1$ ).

## A 2.11 Oil density

## CP Hall Sebacate density

$$c_{\rho_{lSt}} = \frac{\partial \Delta A_T}{\partial \rho_{lSt}} \cong \frac{A_{St}}{\sum_j m_{Stj}} \left[ V_T + (h_{St}^i - h_{St}^f - h_T^i) A_{St} \right] \quad \dots(75)$$

Hence:

$$u_{\rho_{lSt}}(\Delta A_T) = \frac{A_{St}}{\sum_j m_{Stj}} \left| V_T + (h_{St}^i - h_{St}^f - h_T^i) A_{St} \right| u(\rho_{lSt}) \quad \dots(76)$$

| Input                     | Maximum or Minimum Value  | Source of Value                                                                                                                                                                                                                                                                                                    |
|---------------------------|---------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| $u(\rho_{lSt})$           | $\pm 3 \text{ kg m}^{-3}$ | When the temperature dependency is not corrected out, the standard uncertainty associated with CP Hall Sebacate density does not exceed $\pm 3 \text{ kg m}^{-3}$ of its value (see section 2). A rectangular distribution was assumed for the CP Hall Sebacate density when calculating the standard uncertainty. |
| $\sum_j m_{Stj}  _{\min}$ | 23 kg                     | The minimum value of the mass applied on 'Standard' occurred when HPBs were generating the lowest pressure                                                                                                                                                                                                         |
| $h _{\max}$               | 0.015 m                   | The maximum 'Test' or 'Standard' fluid head is 15 mm.                                                                                                                                                                                                                                                              |
| $V _{\max}$               | 0.032 m                   | The 'Standard' and 'Test' buoyancy volume was maximum when the B3 family was tested.                                                                                                                                                                                                                               |

Using (76):

$$\begin{aligned}
 u_{\rho_{lSt}}(\Delta A_T) \Big|_{\max} &= \frac{A_{St}}{\sum_j m_{Stj} \Big|_{\min}} \left| V_T \Big|_{\max} + (h_{St}^i - h_{St}^f - h_T^i) A_{St} \Big|_{\max} \right| u(\rho_{lSt}) \\
 &= \frac{A_{St}}{23} \left( 800 \times 10^{-9} + 15 \times 10^{-3} \times 83 \times 10^{-6} \right) \frac{3}{\sqrt{3}} \\
 &= 1.5 \times 10^{-7} A_{\text{nominal}}
 \end{aligned} \quad \dots(77)$$

The uncertainty associated with  $\Delta A_T$  due to the fluid surface tension does not exceed  $\pm 0.15$  ppm of effective area ( $k = 1$ ).

*'Test' oil density*

$$c_{\rho_{IT}} = \frac{\partial \Delta A_T}{\partial \rho_{IT}} \cong - \frac{A_{St}}{\sum_j m_{Stj}} (V_T - h_T^f A_{St}) \quad \dots(78)$$

Hence:

$$u_{\rho_{IT}}(\Delta A_T) = \frac{A_{St}}{\sum_j m_{Stj}} |V_T - h_T^f A_{St}| u(\rho_{IT}) \quad \dots(79)$$

| Input       | Maximum or Minimum Value | Source of Value                                                                                                                                                                                                                                                                                                                           |
|-------------|--------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| $u(\rho_T)$ | $\pm 5 \%$ of $\rho_T$   | The densities for the oils tested in this experiment were provided by the manufacturer's specifications. In the absence of a declared value the estimated uncertainty for the 'Test' oil density is taken to be $\pm 5 \%$ . A rectangular distribution was assumed for the 'Test' oil density when calculating the standard uncertainty. |

The same considerations used for the CP Hall Sebacate oil apply to the 'Test' oil, therefore:

$$\begin{aligned}
 u_{\rho_{IT}}(\Delta A_T) \Big|_{\max} &= \frac{A_{St}}{\sum_j m_{Stj} \Big|_{\min}} |V_T - h_T^f A_{St}| u(\rho_{IT}) \\
 &= \frac{A_{St}}{23} (800 \times 10^{-9} - 15 \times 10^{-3} \times 83 \times 10^{-6}) \frac{5}{\sqrt{3}} \\
 &= 6 \times 10^{-8} A_{\text{nominal}}
 \end{aligned} \quad \dots(80)$$

*Combined CP Hall Sebacate and 'Test' oil density*

$$u_{\rho_{oil}}(\Delta A_T) = \sqrt{u_{\rho_{IT}}^2(\Delta A_T) + u_{\rho_{Is}}^2(\Delta A_T)} \quad \dots(81)$$

Considering the calculated maximum values obtained in Eqs. (77) and (80) the maximum value thus obtained for  $u_{\rho_{oil}}(\Delta A_T)$  does not exceed  $\pm 0.16$  ppm of effective area ( $k = 1$ ).

$$\begin{aligned} u_{\rho_{oil}}(\Delta A_T)_{\max} &= \sqrt{u_{\rho_{IT}}^2(\Delta A_T)_{\max} + u_{\rho_{ISi}}^2(\Delta A_T)_{\max}} \\ &= 1.6 \times 10^{-7} A_{\text{nominal}} \end{aligned} \quad \dots(82)$$

## A 2.12 Acceleration due to gravity

$$c_g = \frac{\partial \Delta A_T}{\partial g} \cong -\frac{A_{St}}{g^2 \sum_j m_{Stj}} (\gamma_T - \gamma_{St}) C_{St} \quad \dots(83)$$

Hence:

$$u_g(\Delta A_T) \cong \frac{A_{St}}{g^2 \sum_j m_{Stj}} |\gamma_T - \gamma_{St}| C_{St} u(g) \quad \dots(84)$$

| Input                             | Maximum or Minimum Value             | Source of Value                                                                                                                                                                                                                          |
|-----------------------------------|--------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| $u(g)$                            | $\pm 0.000\,000\,2 \text{ m s}^{-2}$ | The acceleration due to gravity is known at this location with an uncertainty of $\pm 0.000\,000\,2 \text{ m s}^{-2}$ . A normal distribution was assumed for the acceleration due to gravity when calculating the standard uncertainty. |
| $\sum_j m_{Stj}  _{\min}$         | 23 kg                                | The minimum value of the mass applied on 'Standard' occurred when HPBs were generating the lowest pressure                                                                                                                               |
| $ \gamma_{St} - \gamma_T _{\max}$ | $0.006 \text{ N m}^{-1}$             | The difference in the 'Test' and 'Standard' oil surface tension was maximum when HP Budenberg oil was tested.                                                                                                                            |
| $C _{\max}$                       | 0.032 m                              | The 'Standard' and 'Test' circumference was maximum when B3 family was tested.                                                                                                                                                           |

Using (84):

$$\begin{aligned}
 u_g(\Delta A_T) |_{\max} &= \frac{A_{St}}{g^2 \sum_j m_{Stj} |_{\min}} |\gamma_T - \gamma_{St}|_{\max} C_{St} |_{\max} u(g) \\
 &= \frac{A_{St}}{9.8^2 \times 23} \times 0.006 \times 0.032 \times \frac{0.000\,002}{2} \\
 &= 9 \times 10^{-14} A_{\text{nominal}}
 \end{aligned} \quad \dots(85)$$

The maximum uncertainty associated with  $\Delta A_T$  generated by the acceleration due to gravity does not exceed  $\pm 10^{-7}$  ppm pm of effective area ( $k = 1$ ).

### A 2.13 Reproducibility of cross float

At the end of the experiment the ‘*Test*’ assembly was again calibrated in CP Hall Sebacate. The difference,  $\Delta A_{T-CP \text{ Hall Sebacate}}$  between the final calibration in CP Hall Sebacate with respect to the initial calibration is used to estimate the reproducibility. Assuming a rectangular distribution and a sensitivity coefficient equal to one, the uncertainty due to the cross-float repeatability at  $k = 1$  is given by:

$$u_R(\Delta A_T) = \frac{\Delta A_{T-Sebacate}}{\sqrt{3}} \quad \dots(86)$$

### A 2.14 Tilt and piston taper

The combined uncertainty due to ‘*Standard*’ and ‘*Test*’ tilt and taper does not exceed  $\pm 1.6$  ppm of effective area at  $k = 1$  [12].

## A 2.15 Summary of type B uncertainty, maximum values

The components of type B uncertainty, including their estimated maximum values are listed in Table 10.

**Table 10** – *Summary of maximum type B contributions.*

| Source of uncertainty       | Symbol    | $u_{i-\max}$ /ppm of effective area |
|-----------------------------|-----------|-------------------------------------|
| Mass contribution           | $m$       | 0.003 4                             |
| Density of the applied mass | $\rho_m$  | 0.002                               |
| ‘Standard’ effective area   | $A_{St0}$ | 0.000 07                            |
| Pistons circumference       | $C$       | 0.001 5                             |
| Buoyancy volume             | $V$       | 0.000 2                             |
| Fluid head                  | $H$       | 2                                   |
| Temperature coefficient     | $\alpha$  | 2.5                                 |
| Temperature                 | $t$       | 0.000 2                             |
| Air density                 | $\rho_a$  | 0.001 5                             |
| Fluid surface tension       | $\gamma$  | 0.35                                |
| Oils density                | $\rho_l$  | 0.16                                |
| Acceleration due to gravity | $G$       | $10^{-7}$                           |
| Tilt and piston taper       | $N/A$     | 1.6                                 |

Table 10 is used to identify those terms that are insignificant and those that should be evaluated on a point-by-point basis for evaluation of the overall uncertainty. Hence:

$$u_B = \sqrt{u_h^2(\Delta A_T) + u_R^2(\Delta A_T) + u_t^2(\Delta A_T) + u_{Tilt\&Taper}^2(\Delta A_T)} \quad \dots(87)$$

and

$$u_c = \sqrt{u_A^2 + u_B^2} \quad \dots(88)$$