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**EFFECTIVE WATER DEPTH
OF AN IONIZATION CHAMBER
IN A PROTON BEAM:
ANALYTICAL MODEL**

Hugo Palmans

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EFFECTIVE WATER DEPTH OF AN IONIZATION CHAMBER IN A PROTON BEAM: ANALYTICAL MODEL

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ABSTRACT

An analytical model is described to calculate the effective depth in water of cylindrical ionization chambers in clinical proton or heavy ion beams in the presence of modest and linear gradients. The model is compared with Monte Carlo simulations and recommendations in IAEA TRS-398. An extension of the model allows applying it for depth-dose curves even in the non-linear gradient region of the Bragg peak. Combined with information from the Monte Carlo simulations it also allows one to solve the inverse problem of deriving a depth-dose curve in homogeneous water from the depth-dose response obtained with a cylindrical ionization chamber.

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1. INTRODUCTION

Codes of practice for clinical proton dosimetry, such as AAPM report 16 (Lyman et al. 1986), the ECHED code of practice and its supplement (Vynckier et al. 1991, 1994), ICRU report 59 (ICRU 1998) and IAEA TRS-398 (Andreo et al. 2000) recommend cylindrical ionization chambers as reference instruments without taking into account any perturbation correction factors for the presence of the ionisation chamber in the proton field.

The centre of the cylindrical ionization chamber is normally taken as reference point for dosimetry. In general it is recommended to perform reference dosimetry in the absence of a depth-dose gradient, i.e. in the entrance region for non-modulated beams or in the middle of the spread-out Bragg peak (SOBP), such that volume averaging effects do not play a role, however, this is not always possible. Palmans (2000) showed that an analytical model gave a reasonable agreement with Monte Carlo simulations of the depth-dose response of an FWT IC-18 ionization chamber. But the statistics were not very good due to a lack of available CPU time for the Monte Carlo simulations to simulate this problem even for one ionization chamber.

Effects due to the presence of depth-dose gradients can be accounted for in two ways. A gradient (or displacement) perturbation correction factor can be applied or alternatively the concept of effective depth in water, $z_{w,tot}$, of the ionization chamber can be used. The effective depth in water includes gradient related perturbations of the cavity, the wall, the central electrode and the sleeve. It is thus different from the effective point of measurement, P_{eff} , which only applies to a theoretical air cavity surrounded by water. As was shown by Palmans and Verhaegen (2000) the meaning of P_{eff} and $z_{w,tot}$ is often confused.

Some authors have measured the effective depth in water for proton or carbon beams by comparing depth-dose curves obtained with cylindrical ionization chambers to those obtained using plane-parallel ionization chambers (Jäkel et al. 2000, Mobit et al. 2000 and Kanai et al. 2004). Hartmann et al. 1999 determined the shift that had to be applied to a cylindrical ionization chamber measurement such that the calibration of the monitor was consistent as a function of beam energy.

Palmans and Verhaegen (1998), Jäkel et al. (2000) and Kanai et al. (2004) used an analytical integration over the cavity geometry to obtain the shift of $z_{w,tot}$ from the centre of the ionization chamber related to the cavity and wall effect. Bichsel (1995) used an analytical model to calculate the convolution of the depth-dose curve with cylindrical and plane-parallel detectors with finite sizes (such as solid state detectors).

In this work, the analytical model from Palmans and Verhaegen (1998) and Palmans (2000) is extended to take the effect of the sleeve and central electrodes into account. The results will be compared with results from Monte Carlo simulations and with experimental data. In addition, it will be demonstrated that a slight modification to the model allows correcting depth-dose curves measured with a cylindrical ionization chamber even in the strongly non-linear gradient region of the Bragg peak, if the size of the ionization chamber is restricted to certain limits. It will be shown that the analytical model can be applied to the inverse problem as well, i.e. the reconstruction of the depth-dose curve from the depth-ionization function measured with a cylindrical ionization chamber.

2. METHOD

2.1. Analytical model

2.1.1. Analytical model for the effective depth in water of an ionization chamber

Palmans and Verhaegen (1998) showed an analytical model for the effective depth of a cylindrical or spherical cavity surrounded by a wall submersed in water, which is extended here. Four assumptions are made:

- (i) protons travel along straight lines,
- (ii) the distance a proton travels through a medium can be converted to a water equivalent distance by multiplying with the ratio of the csda ranges in water and the medium (see e.g. Palmans and Verhaegen 1997),
- (iii) processes like multiple scattering, energy straggling and nuclear absorption are not substantially altered by the presence of non-water materials over the distances relevant in this model (e.g. the wall thickness) such that the contributions from these processes to the exact shape of the depth-dose curve are inherently taken into account,
- (iv) the depth-dose gradient is approximately linear on the local scale of the ionization chamber dimensions.

Under these assumptions an unambiguous value of the effective depth can be calculated by integration over the projected area of the cavity perpendicular to the beam direction.

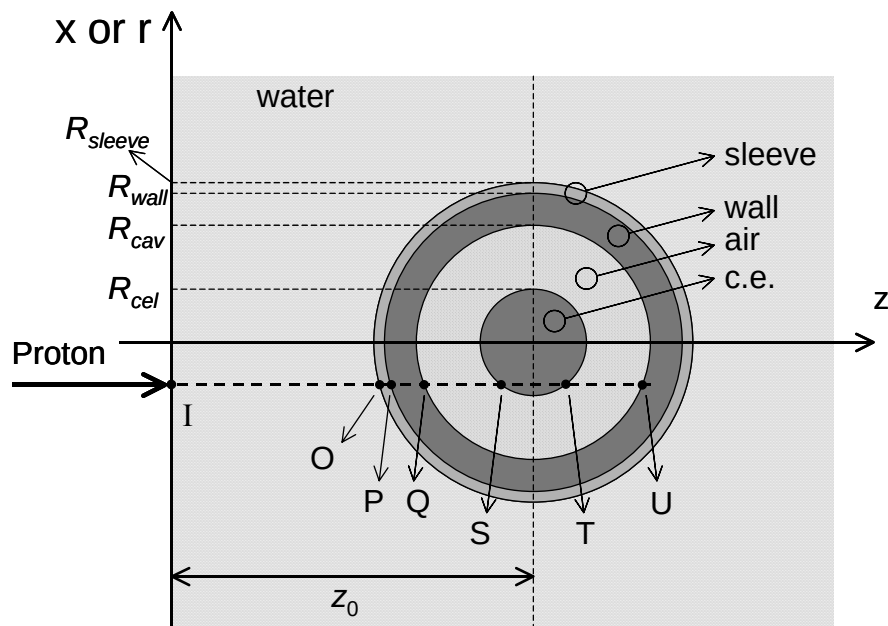


Figure 1. Transverse cross section through a cylindrical or spherical ionization chamber model for the analytical calculations of the effective depth in water.

Consider the transversal cross section through a cylindrical cavity in figure 1. The water equivalent depth the proton has travelled when entering the cavity at point Q is

$$z_{w,equiv}^{(a)} = |IO| + F_{sleeve} \cdot |OP| + F_{wall} \cdot |PQ| \quad (1)$$

and when it has travelled through the central electrode and exits at point T

$$z_{w,equiv}^{(b)} = |IO| + F_{sleeve} \cdot |OP| + F_{wall} \cdot |PQ| + F_{cel} \cdot |ST| \quad (2)$$

In these expressions, $F_{sleeve} = r_{0,water}/r_{0,sleeve}$, the ratio of the csda-range r_0 in water and that in the central electrode material and equally, $F_{wall} = r_{0,water}/r_{0,wall}$ and $F_{cel} = r_{0,water}/r_{0,cel}$. The superscript (a) refers to the front half of the cavity (towards the proton source) and (b) refers to the back half of the cavity (away from the proton source).

The weight of dose contributions from protons entering the cavity to the total dose is proportional to the track length in the cavity, hence

$$w^{(a)}(x) = |QS| \text{ and } w^{(b)}(x) = |TU| \quad (3)$$

If x is larger than R_{cel} , S and T become equal at a depth z_0 . The total dose in the cavity can thus be integrated as

$$D_{air}^*(z_0) = \frac{\int_{x=-R_{cav}}^{x=+R_{cav}} [w^{(a)}(x) \cdot D_{air}(z_{w,equiv}^{(a)}(x)) + w^{(b)}(x) \cdot D_{air}(z_{w,equiv}^{(b)}(x))] \cdot dx}{\int_{x=-R_{cav}}^{x=+R_{cav}} [w^{(a)}(x) + w^{(b)}(x)] \cdot dx} \quad (4)$$

where $D_{air}(z)$ is the dose to air in a point at depth z in homogeneous water and $D_{air}^*(z)$ is the average dose to air in the air cavity volume of the ionization chamber. If one depth $z_{w,tot}^c$ can be found where $D_{air}(z_{w,tot}^c) = D_{air}^*(z_0)$ then we call this the effective depth of the cylindrical ionization chamber geometry.

If the depth dose gradient can be considered linear at the local scale of the ionization chamber dimensions, the effective depth can be calculated as

$$z_{w,tot}^c = \frac{\int_{x=-R_{cav}}^{x=+R_{cav}} [w^{(a)}(x) \cdot z_{w,equiv}^{(a)}(x) + w^{(b)}(x) \cdot z_{w,equiv}^{(b)}(x)] \cdot dx}{\int_{x=-R_{cav}}^{x=+R_{cav}} [w^{(a)}(x) + w^{(b)}(x)] \cdot dx} \quad (5)$$

In an analogous way an expression for a spherical cavity can be derived:

$$z_{w,tot}^s = \frac{\int_{r=0}^{r=R_{cav}} [w^{(a)}(r) \cdot z_{w,equiv}^{(a)}(r) + w^{(b)}(r) \cdot z_{w,equiv}^{(b)}(r)] \cdot r \cdot dr}{\int_{r=0}^{r=R_{cav}} [w^{(a)}(r) + w^{(b)}(r)] \cdot r \cdot dr} \quad (6)$$

By calculating the cord lengths in equations (1-3) as a function of the ionization chamber dimensions, equations (5) and (6) can be reduced to

$$\begin{aligned} z_{w,tot}^t = z_0 & - \frac{[I^t(R_{cav}, R_{sleeve}) - I^t(R_{cel}, R_{sleeve})]}{(1 - \gamma_{cel}^t) \cdot J^t(R_{cav})} \\ & + F_{sleeve} \cdot \frac{[I^t(R_{cav}, R_{sleeve}) - I^t(R_{cel}, R_{sleeve})] - [I^t(R_{cav}, R_{wall}) - I^t(R_{cel}, R_{wall})]}{(1 - \gamma_{cel}^t) \cdot J^t(R_{cav})} \\ & + F_{wall} \cdot \frac{[I^t(R_{cav}, R_{wall}) - I^t(R_{cel}, R_{wall})] - [K^t(R_{cav}) - I^t(R_{cel}, R_{cav})]}{(1 - \gamma_{cel}^t) \cdot J^t(R_{cav})} \\ & + F_{cel} \cdot \frac{[I^t(R_{cel}, R_{cav}) - K^t(R_{cel})]}{(1 - \gamma_{cel}^t) \cdot J^t(R_{cav})} \end{aligned} \quad (7)$$

where t stands for the type of cavity ($t = c$ for a cylindrical cavity and $t = s$ for a spherical cavity) and γ_{cel}^t is the ratio of the central electrode and the total cavity volume, i.e.

$$\gamma_{cel}^t = \frac{J^t(R_{cel})}{J^t(R_{cav})} \quad (8)$$

The integrals I, J and K are

$$\begin{aligned}
 I^c(R_a, R_b) &= \int_{x=-R_a}^{x=+R_a} \sqrt{(R_a^2 - x^2) \cdot (R_b^2 - x^2)} \cdot dx \\
 J^c(R_a) &= \int_{x=-R_a}^{x=+R_a} \sqrt{(R_a^2 - x^2)} \cdot dx = \frac{\pi}{2} \cdot R_a^2 \\
 K^c(R_a) &= \int_{x=-R_a}^{x=+R_a} (R_a^2 - x^2) \cdot dx = \frac{4}{3} \cdot R_a^3
 \end{aligned} \tag{9}$$

for a cylindrical cavity and

$$\begin{aligned}
 I^s(R_a, R_b) &= \int_{r=0}^{r=R_a} \sqrt{(R_a^2 - r^2) \cdot (R_b^2 - r^2)} \cdot r \cdot dr \\
 &= \frac{1}{8} \cdot R_a \cdot R_b \cdot (R_a^2 + R_b^2) + \frac{1}{16} \cdot (R_a^2 - R_b^2)^2 \cdot \ln\left(\frac{R_b - R_a}{R_b + R_a}\right) \\
 J^s(R_a) &= \int_{r=0}^{r=R_a} \sqrt{(R_a^2 - r^2)} \cdot r \cdot dr = \frac{1}{3} \cdot R_a^3 \\
 K^s(R_a) &= \int_{r=0}^{r=R_a} (R_a^2 - r^2) \cdot r \cdot dr = \frac{1}{4} \cdot R_a^4
 \end{aligned} \tag{10}$$

for a spherical cavity.

In the expressions for $I^t(R_a, R_b)$ R_a should always be smaller than R_b . The integral $I^c(R_a, R_b)$ can only be calculated numerically. A possible way to do this is shown in section 2.1.3.

The shift of the water equivalent depth relative to the centre of the cavity, $\Delta z_{w,tot}^t = z_{w,tot}^t - z_0$, can be regarded as the sum of four contributions by evaluating the additional shift that is caused by the individual contributions of cavity, wall, central electrode and sleeve.

For example, when in equation (7), $R_{cel}=0$, $R_{sleeve}=R_{wall}=R_{cav}$ and $F_{sleeve}=F_{wall}=1$ we obtain the equation for a (hypothetical) air cavity in water, which is by definition the depth z_{eff}^t of the effective point of measurement of the cavity:

$$z_{eff}^t = z_0 - \frac{K^t(R_{cav})}{J^t(R_{cav})} \quad (11)$$

For a cylindrical cavity this equation becomes:

$$z_{eff}^c = z_0 - \frac{8}{3 \cdot \pi} \cdot R_{cav} \quad (12)$$

and for a spherical cavity:

$$z_{eff}^s = z_0 - \frac{3}{4} \cdot R_{cav} \quad (13)$$

in agreement with the results of Palmans and Verhaegen (1998). The difference $\Delta z_{w,cav}^t = z_{eff}^t - z_0$ is then the contribution due to the cavity, or

$$\Delta z_{w,cav}^t = -\frac{K^t(R_{cav})}{J^t(R_{cav})} \quad (14)$$

In a similar way we can derive expressions for the contributions of the wall, $\Delta z_{w,wall}^t$:

$$\Delta z_{w,wall}^t = (F_{wall} - 1) \cdot \frac{I^t(R_{cav}, R_{wall}) - K^t(R_{cav})}{J^t(R_{cav})} \quad (15)$$

of the central electrode, $\Delta z_{w,cel}^t$:

$$\begin{aligned} \Delta z_{w,cel}^t = & \left[(F_{wall} - 1) \cdot \frac{I^t(R_{cav}, R_{wall})}{J^t(R_{cav})} - F_{wall} \cdot \frac{K^t(R_{cav})}{J^t(R_{cav})} \right] \cdot \frac{\gamma_{cel}^t}{1 - \gamma_{cel}^t} \\ & - \left[(F_{wall} - 1) \cdot \frac{I^t(R_{cel}, R_{wall})}{J^t(R_{cav})} - F_{wall} \cdot \frac{I^t(R_{cel}, R_{cav})}{J^t(R_{cav})} \right] \cdot \frac{1}{1 - \gamma_{cel}^t} \\ & + F_{cel} \cdot \frac{I^t(R_{cel}, R_{cav}) - K^t(R_{cel})}{J^t(R_{cav})} \cdot \frac{1}{1 - \gamma_{cel}^t} \end{aligned} \quad (16)$$

and of the sleeve, $\Delta z_{w,sleeve}^t$:

$$\begin{aligned} \Delta z_{w,sleeve}^t = & (F_{sleeve} - 1) \cdot \frac{I^t(R_{cav}, R_{sleeve}) - I^t(R_{cav}, R_{wall})}{J^t(R_{cav})} \cdot \frac{1}{1 - \gamma_{cel}^t} \\ & - (F_{sleeve} - 1) \cdot \frac{I^t(R_{cel}, R_{sleeve}) - I^t(R_{cel}, R_{wall})}{J^t(R_{cav})} \cdot \frac{1}{1 - \gamma_{cel}^t} \end{aligned} \quad (17)$$

For a realistic thimble-shaped ionization chamber (figure 2) the effective water depth can be derived as a linear combination of the results for a spherical cavity and for a cylindrical cavity:

$$z_{w,tot} = \gamma^c \cdot z_{w,tot}^c + \gamma^s \cdot z_{w,tot}^s \quad (18)$$

where γ_c en γ_s are the fractions by volume of the cylindrical and the hemispherical part of the thimble-shaped cavity with total length L and radius R_{cav} :

$$\begin{aligned} \gamma^c &= \frac{1}{1 + \frac{2}{3} \cdot \frac{(R_{cav}^2 + R_{cav} \cdot R_{cel} + R_{cel}^2)}{(R_{cav} + R_{cel})(L - R_{cav})}} \\ \gamma^s &= \frac{1}{1 + \frac{3}{2} \cdot \frac{(R_{cav} + R_{cel}) \cdot (L - R_{cav})}{(R_{cav}^2 + R_{cav} \cdot R_{cel} + R_{cel}^2)}} \end{aligned} \quad (19)$$

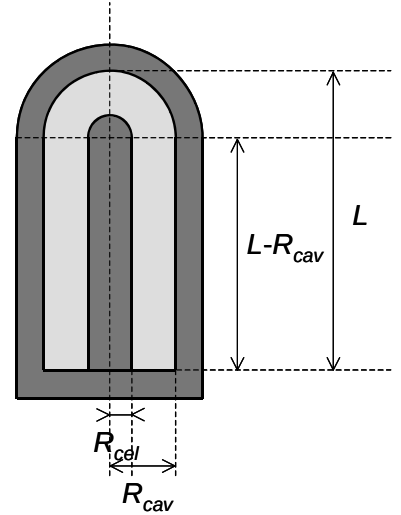


Figure 2. Geometry of a thimble ionization chamber with central electrode.

In this model, the centre of the spherical part of the electrode is assumed to coincide with the centre of the spherical part of the thimble cavity. This is not always correct since the electrode can be shorter or longer than given by this assumption, but it is difficult to adapt the analytical model for this. Since it is easier to implement this in a Monte Carlo simulation

(as e.g. done by Verhaegen and Palmans, 2001) the effect of taking a shorter or longer electrode into account is studied by simulations similar to the ones described in section 2.4.

2.1.2. Extension of the analytical model in the vicinity of the Bragg peak

In the presence of a substantial non-linearity of the gradient, such as in the vicinity of the Bragg peak, assumption (iv) in the previous section is not fulfilled and the integration in equation (4) should then in principle be performed with the function $D_{air}(z)$ explicitly in the integrand. In this case it is no longer always possible to assign a single and unambiguous effective depth to account for the gradient effect. The integration could be done numerically by dividing the integration interval in a number of subintervals. Figure 3 illustrates the integration over a subinterval and defines angles θ_1 , θ_2 , θ'_1 and θ'_2 .

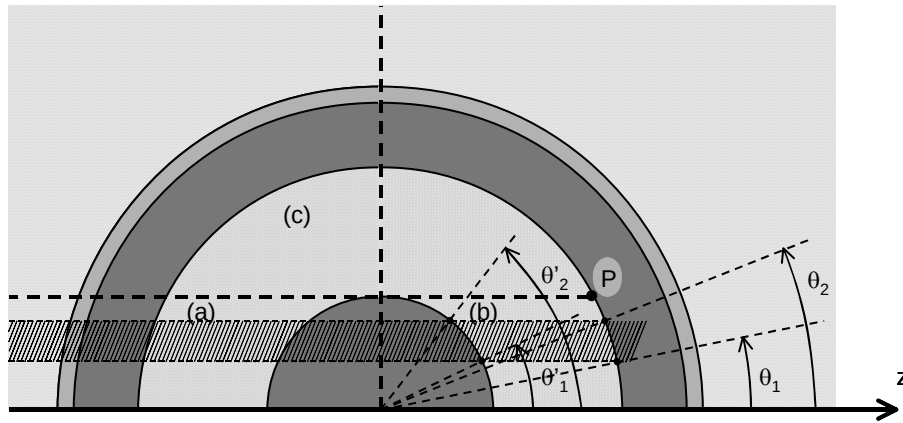


Figure 3. Indication of a subinterval for the integration in equation (4) according to the model of section 2.2. and definition of the angles θ_1 , θ_2 , θ'_1 and θ'_2 .

With θ_{cel} being the angle defined by the point P in figure 3 where the horizontal line tangent to the central electrode intersects the inner cavity wall. Three types of sectors can be indicated: (a) those with $\theta_2 < \theta_{cel}$ and upstream the central electrode, (b) those with $\theta_2 < \theta_{cel}$ and downstream the central electrode and (c) those with $\theta_1 > \theta_{cel}$. For simplicity, the case in which $\theta_1 < \theta_{cel} < \theta_2$ is not considered as it can be reduced to the situations (a), (b) and (c) by introducing an additional subinterval. For every sector the water equivalent depth $z_{w,tot}^t(\theta_1, \theta_2)$ can be calculated by the following expressions:

$$\begin{aligned}
 z_{w,i}^{t,(a)}(\theta_1, \theta_2) = & z_0 - \frac{IP^t(R_{cav}, R_{sleeve}, \theta_1, \theta_2) - IP^t(R_{cel}, R_{sleeve}, \theta'_1, \theta'_2)}{(1 - \gamma_{P,cel}^t) \cdot JP^t(R_{cav}, \theta_1, \theta_2)} \\
 & + F_{sleeve} \cdot \frac{IP^t(R_{cav}, R_{sleeve}, \theta_1, \theta_2) - IP^t(R_{cel}, R_{sleeve}, \theta'_1, \theta'_2)}{(1 - \gamma_{P,cel}^t) \cdot JP^t(R_{cav}, \theta_1, \theta_2)} \\
 & - F_{sleeve} \cdot \frac{IP^t(R_{cav}, R_{wall}, \theta_1, \theta_2) - IP^t(R_{cel}, R_{wall}, \theta'_1, \theta'_2)}{(1 - \gamma_{P,cel}^t) \cdot JP^t(R_{cav}, \theta_1, \theta_2)} \\
 & + F_{wall} \cdot \frac{IP^t(R_{cav}, R_{wall}, \theta_1, \theta_2) - IP^t(R_{cel}, R_{wall}, \theta'_1, \theta'_2)}{(1 - \gamma_{P,cel}^t) \cdot JP^t(R_{cav}, \theta_1, \theta_2)} \\
 & - F_{wall} \cdot \frac{KP^t(R_{cav}, \theta_1, \theta_2) - IP^t(R_{cel}, R_{cav}, \theta'_1, \theta'_2)}{(1 - \gamma_{P,cel}^t) \cdot JP^t(R_{cav}, \theta_1, \theta_2)}
 \end{aligned} \tag{20}$$

$$z_{w,i}^{t,(b)}(\theta_1, \theta_2) = z_{w,tot}^{t,(a)}(\theta_1, \theta_2) + 2 \cdot F_{cel} \cdot \frac{[IP^t(R_{cel}, R_{cav}, \theta'_1, \theta'_2) - KP^t(R_{cel}, \theta'_1, \theta'_2)]}{(1 - \gamma_{P,cel}^t) \cdot JP^t(R_{cav}, \theta_1, \theta_2)} \tag{21}$$

$$\begin{aligned}
 z_{w,i}^{t,(c)}(\theta_1, \theta_2) = & z_0 - \frac{IP^t(R_{cav}, R_{sleeve}, \theta_1, \theta_2)}{JP^t(R_{cav}, \theta_1, \theta_2)} \\
 & + F_{sleeve} \cdot \frac{IP^t(R_{cav}, R_{sleeve}, \theta_1, \theta_2) - IP^t(R_{cav}, R_{wall}, \theta_1, \theta_2)}{JP^t(R_{cav}, \theta_1, \theta_2)} \\
 & + F_{wall} \cdot \frac{IP^t(R_{cav}, R_{wall}, \theta_1, \theta_2) - KP^t(R_{cav}, \theta_1, \theta_2)}{JP^t(R_{cav}, \theta_1, \theta_2)}
 \end{aligned} \tag{22}$$

where

$$\gamma_{P,cel}^t = \frac{JP^t(R_{cel}, \theta'_1, \theta'_2)}{JP^t(R_{cav}, \theta_1, \theta_2)} \tag{23}$$

and

$$\begin{aligned}
IP^c(R_a, R_b, \theta_1, \theta_2) &= 2 \cdot \int_{x=+R_a \cdot \sin \theta_1}^{x=+R_a \cdot \sin \theta_2} \sqrt{(R_a^2 - x^2) \cdot (R_b^2 - x^2)} \cdot dx \\
JP^c(R_a, \theta_1, \theta_2) &= 2 \cdot \int_{x=+R_a \cdot \sin \theta_1}^{x=+R_a \cdot \sin \theta_2} \sqrt{(R_a^2 - x^2)} \cdot dx \\
&= \frac{1}{2} \cdot R_a^2 \cdot [2 \cdot (\theta_2 - \theta_1) + (\sin 2\theta_2 - \sin 2\theta_1)] \\
KP^c(R_a, \theta_1, \theta_2) &= 2 \cdot \int_{x=+R_a \cdot \sin \theta_1}^{x=+R_a \cdot \sin \theta_2} (R_a^2 - x^2) \cdot dx \\
&= \frac{2}{3} \cdot R_a^3 \cdot (\sin \theta_2 - \sin \theta_1) \cdot (1 + \cos^2 \theta_2 + \cos^2 \theta_1 - \sin \theta_2 \cdot \sin \theta_1)
\end{aligned} \tag{24}$$

for a cylindrical cavity and

$$\begin{aligned}
IP^s(R_a, R_b, \theta_1, \theta_2) &= \int_{r=R_a \cdot \sin \theta_1}^{r=R_a \cdot \sin \theta_2} \sqrt{(R_a^2 - r^2) \cdot (R_b^2 - r^2)} \cdot r \cdot dr \\
&= \frac{1}{8} \cdot R_a \cdot R_b \cdot \cos \theta_1 \cdot \sqrt{1 - \varepsilon^2 \cdot \sin^2 \theta_1} \cdot [R_a^2 \cdot \cos^2 \theta_1 + R_b^2 \cdot (1 - \varepsilon^2 \cdot \sin^2 \theta_1)] \\
&\quad - \frac{1}{8} \cdot R_a \cdot R_b \cdot \cos \theta_2 \cdot \sqrt{1 - \varepsilon^2 \cdot \sin^2 \theta_2} \cdot [R_a^2 \cdot \cos^2 \theta_2 + R_b^2 \cdot (1 - \varepsilon^2 \cdot \sin^2 \theta_2)] \\
&\quad + \frac{1}{16} \cdot (R_a^2 - R_b^2)^2 \cdot \ln \left[\frac{(R_b \cdot \sqrt{1 - \varepsilon^2 \cdot \sin^2 \theta_1} - R_a \cdot \cos \theta_1)^2}{(R_b \cdot \sqrt{1 - \varepsilon^2 \cdot \sin^2 \theta_2} - R_a \cdot \cos \theta_2)^2} \right] \\
JP^s(R_a, \theta_1, \theta_2) &= \int_{r=R_a \cdot \sin \theta_1}^{r=R_a \cdot \sin \theta_2} \sqrt{(R_a^2 - r^2)} \cdot r \cdot dr \\
&= \frac{1}{3} \cdot R_a^3 \cdot (\cos^3 \theta_1 - \cos^3 \theta_2) \\
KP^s(R_a, \theta_1, \theta_2) &= \int_{r=R_a \cdot \sin \theta_1}^{r=R_a \cdot \sin \theta_2} (R_a^2 - r^2) \cdot r \cdot dr \\
&= \frac{1}{4} \cdot R_a^4 \cdot (\sin^2 \theta_2 - \sin^2 \theta_1) \cdot (\cos^2 \theta_2 + \cos^2 \theta_1)
\end{aligned} \tag{25}$$

for a spherical cavity, with $\varepsilon = R_a/R_b$.

The dose measured by the cavity should than be derived by combining the dose for the different sections with the appropriate weight of the sector, which equals the volume fraction of that sector:

$$D_{w,ch}^t = \sum_i \gamma_i^t \cdot D(z_{w,i}^t) \quad (26)$$

where

$$\begin{aligned} \gamma_i^{t,(a)} = \gamma_i^{t,(b)} &= \frac{1}{2} \cdot \frac{JP^t(R_{cav}, \theta_1, \theta_2) - JP^t(R_{cel}, \theta'_1, \theta'_2)}{J^t(R_{cav}) - J^t(R_{cel})} \\ \gamma_i^{t,(c)} &= \frac{JP^t(R_1, \theta_1, \theta_2)}{J^t(R_{cav}) - J^t(R_{cel})} \end{aligned} \quad (27)$$

2.1.3. Calculation of the integrals $I^c(R_a, R_b)$ and $IP^c(R_a, R_b, \theta_1, \theta_2)$

The elliptic integral $I^c(R_a, R_b)$ cannot be solved analytically and must therefore be calculated numerically. A series expansion is the most straightforward solution and can be done in the following way. By performing the substitution $\frac{x}{R_a} = \sin \theta$ in the expression for

$I^c(R_a, R_b)$ and defining $\varepsilon = \frac{R_a}{R_b}$ we obtain:

$$\begin{aligned} I^c(R_a, R_b) &= R_a^2 \cdot R_b \cdot \int_{-\pi/2}^{\pi/2} (1 - \sin^2 \theta) \cdot \sqrt{1 - \varepsilon^2 \cdot \sin^2 \theta} \cdot d\theta \\ &= R_a^2 \cdot R_b \cdot \int_{-\pi/2}^{\pi/2} I^*(\theta, \varepsilon) \cdot d\theta \end{aligned} \quad (28)$$

Given the general expression for the series expansion of $(1+x)^s$ with $|x| < 1$:

$$(1+x)^s = 1 + \sum_{k=1}^{\infty} \frac{s \cdot (s-1) \cdot \dots \cdot (s-k+1)}{k!} \cdot x^k \quad (29)$$

we obtain

$$\begin{aligned}\sqrt{1 - \varepsilon^2 \cdot \sin^2 \theta} &= 1 + \sum_{k=1}^{\infty} \frac{\frac{1}{2} \cdot \left(-\frac{1}{2}\right) \cdot \dots \cdot \left(\frac{1}{2} - k + 1\right)}{k!} \cdot \left(-\varepsilon^2 \cdot \sin^2 \theta\right)^k \\ &= 1 + \sum_{k=1}^{\infty} (-1)^k \cdot \frac{1 \cdot 3 \cdot \dots \cdot (2 \cdot k - 3)}{2^k \cdot k!} \cdot \varepsilon^{2 \cdot k} \cdot \sin^{2 \cdot k} \theta\end{aligned}\tag{30}$$

and

$$\begin{aligned}I^*(\theta, \varepsilon) &= 1 - \sin^2 \theta + \sum_{k=1}^{\infty} (-1)^k \cdot \frac{1 \cdot 3 \cdot \dots \cdot (2 \cdot k - 3)}{2^k \cdot k!} \cdot \varepsilon^{2 \cdot k} \cdot \sin^{2 \cdot k} \theta \\ &\quad - \sum_{l=1}^{\infty} (-1)^l \cdot \frac{1 \cdot 3 \cdot \dots \cdot (2 \cdot l - 3)}{2^l \cdot l!} \cdot \varepsilon^{2 \cdot l} \cdot \sin^{2 \cdot l + 2} \theta\end{aligned}\tag{31}$$

where in the 4th term at the right hand side the index k was replaced with l in order to clarify the substitution $l = k - 1$, such that:

$$I^*(\theta, \varepsilon) = 1 - \left(1 + \frac{1}{2} \cdot \varepsilon^2\right) \cdot \sin^2 \theta + \sum_{k=2}^{\infty} \frac{1 \cdot 3 \cdot \dots \cdot (2 \cdot k - 5)}{2^{k-1} \cdot (k-1)!} \cdot \varepsilon^{2 \cdot (k-1)} \cdot \sin^{2 \cdot k} \theta \cdot \left(1 - \frac{2 \cdot k - 3}{2 \cdot k} \cdot \varepsilon^2\right)\tag{32}$$

for the integration of this expression, integrals of the following type are required:

$$B_k = \int_{-\pi/2}^{\pi/2} \sin^{2 \cdot k} \theta \cdot d\theta\tag{33}$$

for which a recursive expression can be derived by partial integration:

$$\begin{aligned}
B_k &= \int_{-\pi/2}^{\pi/2} \sin^{2k-1} \theta \cdot \sin \theta \cdot d\theta \\
&= \sin^{2k-1} \theta \cdot (-\cos \theta) \Big|_{-\pi/2}^{\pi/2} - \int_{-\pi/2}^{\pi/2} (2 \cdot k - 1) \cdot \sin^{2k-2} \theta \cdot \cos \theta \cdot (-\cos \theta) \cdot d\theta \\
&= 0 + \int_{-\pi/2}^{\pi/2} (2 \cdot k - 1) \cdot \sin^{2(k-1)} \theta \cdot d\theta - \int_{-\pi/2}^{\pi/2} (2 \cdot k - 1) \cdot \sin^{2(k-1)} \theta \cdot \sin^2 \theta \cdot d\theta \\
&= (2 \cdot k - 1) \cdot B_{k-1} - (2 \cdot k - 1) \cdot B_k
\end{aligned} \tag{34}$$

such that

$$B_k = \frac{2 \cdot k - 1}{2 \cdot k} \cdot B_{k-1} = \frac{(2 \cdot k - 1) \cdot (2 \cdot k - 3) \cdot \dots \cdot 1}{(2 \cdot k) \cdot (2 \cdot k - 2) \cdot \dots \cdot 2} \cdot B_0 = \frac{(2 \cdot k - 1) \cdot (2 \cdot k - 3) \cdot \dots \cdot 1}{2^k \cdot k!} \cdot \pi \tag{35}$$

The integral $I^c(R_a, R_b)$ eventually becomes:

$$I^c(R_a, R_b) = \pi \cdot R_a^2 \cdot R_b \cdot \left(1 - \frac{1}{2} \cdot \left(1 + \frac{1}{2} \cdot \varepsilon^2 \right) + \sum_{k=2}^{\infty} C_k \right) \tag{36}$$

with

$$C_k = \frac{1 \cdot 3 \cdot \dots \cdot (2 \cdot k - 5)}{2^{k-1} \cdot (k-1)!} \cdot \frac{1 \cdot 3 \cdot \dots \cdot (2 \cdot k - 1)}{2^k \cdot k!} \cdot \varepsilon^{2(k-1)} \cdot \left(1 - \frac{2 \cdot k - 3}{2 \cdot k} \cdot \varepsilon^2 \right) \tag{37}$$

The convergence of this series is assured since:

$$\lim_{k \rightarrow \infty} \left| \frac{C_{k+1}}{C_k} \right| = \lim_{k \rightarrow \infty} \frac{(2 \cdot k - 3) \cdot (2 \cdot k + 1)}{(2 \cdot k) \cdot (2 \cdot (k + 1))} \cdot \varepsilon^2 \cdot \frac{\left(1 - \frac{2 \cdot k - 1}{2 \cdot (k + 1)} \cdot \varepsilon^2\right)}{\left(1 - \frac{2 \cdot k - 3}{2 \cdot k} \cdot \varepsilon^2\right)} = \varepsilon^2 < 1 \quad (38)$$

For typical ionization chamber geometries, 5 to 20 terms are sufficient to obtain an accuracy of 1 μm on z_{wd} . Since $I^c(R_a, R_b)$ could be reduced to an elliptic integral, it could be remarked that more elegant solutions in terms of Bessel functions might be found, but they are unlikely to reduce the amount of calculation effort.

For the integrals $IP^c(R_a, R_b, \theta_1, \theta_2)$, the only difference is that in the calculation of the factors B_k , the integration interval is reduced:

$$B_k = 2 \cdot \int_{\theta_1}^{\theta_2} \sin^{2 \cdot k} \theta \cdot d\theta \quad (39)$$

and the recursive expression for B_k becomes

$$B_k = \frac{\sin^{2 \cdot k - 1} \theta_1 \cdot \cos \theta_1 - \sin^{2 \cdot k - 1} \theta_2 \cdot \cos \theta_2}{k} + \frac{2 \cdot k - 1}{2 \cdot k} \cdot B_{k-1} \quad (40)$$

with

$$B_0 = 2 \cdot (\theta_2 - \theta_1) \quad (41)$$

2.2. Monte Carlo simulations

Monte Carlo simulations of the gradient effects were performed with McPTRAN.MEDIA and McPTRAN.CAVITY, which have both been described by Palmans (2004).

McPTRAN.MEDIA was used for calculating dose to air in homogeneous water as a function of depth by calculating the air to water stopping power ratios in-line calculation in the simulation. McPTRAN.CAVITY was used to simulate dose in the air cavity as a function of depth for all 52 cylindrical ionization chamber types tabulated in IAEA TRS-398 for proton energies of 60 MeV, 80 MeV, 100 MeV, 150 MeV and 200 MeV. Per incident energy, the code allows one to simulate all 52 ionization chambers and all depths in one run. The implementation of geometry interrogation, history splitting and lateral range rejection reduced the calculation time per energy to 5600 CPU hours on a P4 2GHz PC, which took about 1.5 days per energy when run on the NPL distributed computing grid.

All ionization chambers were modelled as thimble shaped. For Farmer type chambers that have a conical instead of a hemispherical top end, it is assumed that the error due to this approximation is small since the cylindrical part is the dominant contribution to the volume. The length of the cavity (cylinder + hemisphere), the cavity radius and the wall thickness were taken from IAEA TRS-398. Central electrode diameters were taken as 1.0 mm, except for Exradin A2, Exradin T2, FWT IC-18, NE2581, PTW23331 and PTW23332 for which they were respectively taken as 4.6 mm, 4.6 mm, 2.0 mm, 3.0 mm, 1.5 mm and 2.0 mm.

For ionization chambers where the manufacturers data show substantial deviations from the dimensions given in IAEA TRS-398 it was verified whether or not these differences have a significant influence on the results. This includes the hollow electrode of a NE2561 and NE2611 ionization chamber, but also a number of PMMA walled ionization chambers that have a conducting graphite layer or graphite/epoxy paste on the inner surface of the cavity. For all non-waterproof ionization chambers, a 0.5 mm PMMA sleeve was assumed and included in the simulations as was done for generating the data in IAEA TRS-398.

3. RESULTS

3.1. Comparison of the analytical model results with Monte Carlo calculations

3.1.1. Effective water depth in entrance and distal regions

Figure 4 shows for two situations the dose in air, D_{air} , as a function of depth in water, the same curve shifted in depth by the analytical $\Delta z_{\text{w,tot}}$ and D_{air} in the ionization chamber's air cavity from the Monte Carlo simulation. The type-A uncertainty on the Monte Carlo results was smaller than 0.05% on all data points except those in the tail of the distal edge of the Bragg peak.

It is obvious that up to a certain depth a single shift is adequate and for sufficiently high energies it can be even sufficient at any depth (except that it leads to a small underestimation of the dose maximum) as illustrated in figure 4b. A criterion for this can be derived from the simulations of all ionization chambers: for those with a ratio of central electrode diameter and cavity diameter smaller than 0.25 and a radius smaller than 4 mm the differences will be smaller than 3% of the maximum dose above 150 MeV (the difference being mainly in the Bragg peak). At low energies, on the other hand, the Bragg peak in the Monte Carlo results is heavily distorted for all ionization chambers due to volume averaging in the ionization chamber geometry and thus a simple shift is too crude in that region. Figure 4a gives an impression of the range of depths where the more detailed model of section 2.1.2 will be required.

Values for $\Delta z_{\text{w,tot}}$ can be derived from the Monte Carlo simulations by calculating the distance from the centre of the ionization chamber to the depth where D_{air} in homogeneous water (in a point) equals the average D_{air} in the air cavity. This can of course not be done in the distorted depth dependence of dose to air in the Bragg peak region, but it is equally difficult to get an accurate value if the gradient is very small such as at shallow depths in the entrance region. For these reasons, the following rather arbitrary criterion was used: only depths where the local gradient of the ionization chamber depth dose is between 3% per mm and 10% per mm were considered. This interval has been indicated in the plots of figure 4. Only for one ionization chamber, the Exradin A2, an exception was made; since the distortion of the Bragg peak by this ionization chamber extends to a very wide range of depths, overlapping considerably with the range defined by the criterion, the upper limit was set to 8% per mm. The results are averaged over all calculated points that pass the criterion.

Since there was no obvious trend of the result as a function of energy, an average over the five energies was taken. The standard deviation of the results was smaller than 0.03% for all ionization chambers.

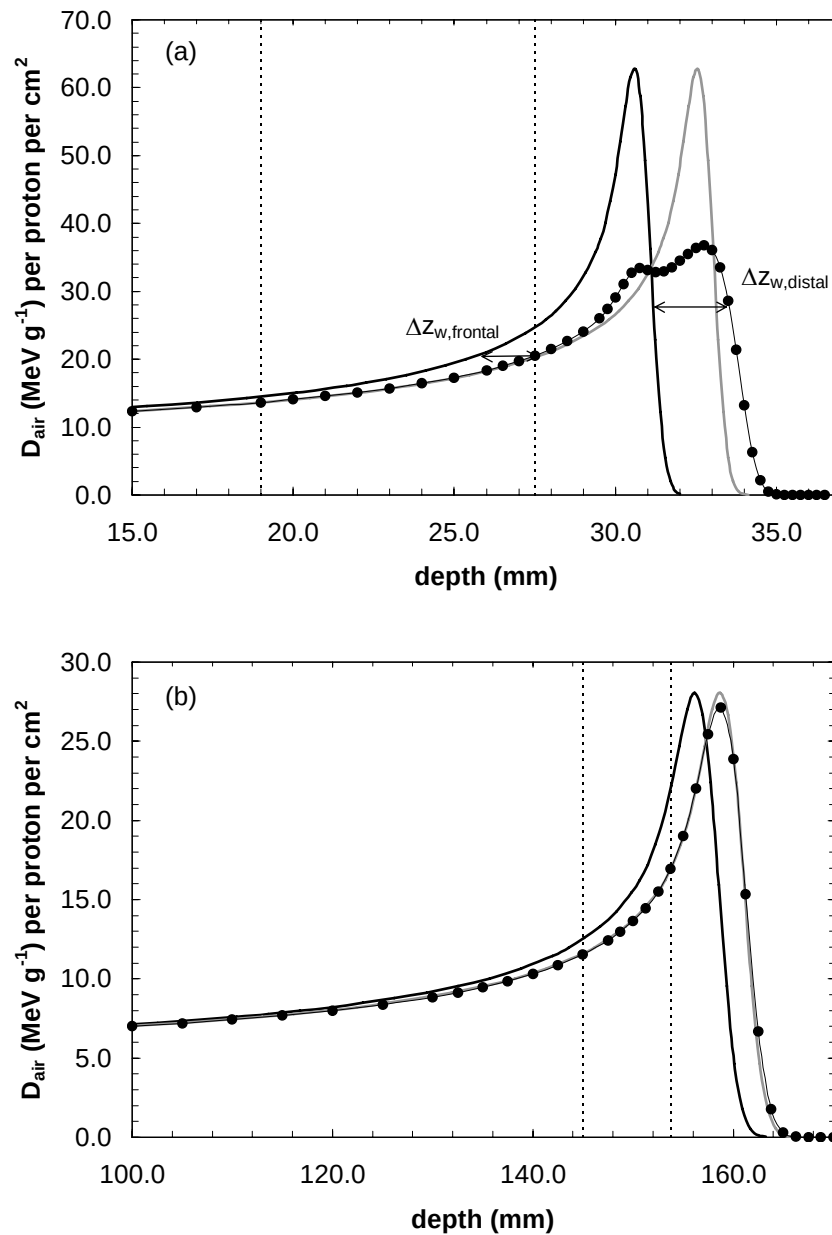


Figure 4. Depth dose to air curve calculated by Monte Carlo in homogeneous water (black curve), depth dose to air curve shifted over $\Delta z_{w,tot}$ (grey curve) and depth dose to air as a function of depth for a thimble ionization geometry (symbols connected by thin lines), (a) for an NE2581 in a 60 MeV mono-energetic beam and (b) for an NE2561 in a 150 MeV mono-energetic beam. The vertical dashed lines indicate the interval where the local gradient is between 3% and 10% of the local dose per mm.

Another possibility is to derive $\Delta z_{w,tot}$ from the shift of the distal edge of the Bragg peak. This requires a criterion for the dose level at which this is evaluated. Since usually the depth where the dose has dropped to 80% of the dose maximum, z_{80} , is used as a reference to determine the csda range we have chosen to evaluate the shift at this dose level. In this case there was an obvious dependence on energy with the values being larger for lower energies and reaching an asymptotic value at higher energies. Only values above this asymptotic value, which depends on the ionization chamber type, were taken into account.

Table 1 shows the values of (-) $\Delta z_{w,tot}$ obtained from the analytical model and from the Monte Carlo simulations using both methods explained in the two previous paragraphs. The selection of ionization chamber types listed in table 1 have all been reported in the literature as being used as reference instruments. The results are compared with the general recommendation in IAEA TRS-398 to use 0.75 times the radius of the cavity.

Table 1. $-\Delta z_{w,tot}$ for a selection of ionization chambers (ICs) listed in IAEA TRS-398, according the analytical model, the Monte Carlo simulations (frontal: MC_f and distal: MC_d) and the general recommendation of 0.75 times the cavity radius given in IAEA TRS-398. The ionization chambers for which the difference with the IAEA model is more than 0.5 mm are highlighted in italic, those for which the difference is more than 1 mm in bold.

Type of IC	$-\Delta z_{w,tot}$ (mm)			
	This work			IAEA
	MC _f	MC _d	Analyt.	
Capintec PR06C	2.4	2.4	2.4	2.4
<i>Exradin A2</i>	2.0	2.1	2.1	3.6
<i>Exradin T2</i>	2.8	2.9	2.8	3.6
<i>Exradin A1</i>	1.0	0.9	0.9	1.5
Exradin T1	1.5	1.4	1.4	1.5
Exradin A12	2.2	2.2	2.2	2.3
<i>FWT IC18</i>	1.3	1.2	1.2	1.7
NE2505	2.2	2.2	2.2	2.3
NE2505/A	2.3	2.2	2.3	2.3
NE2505/3	2.3	2.3	2.3	2.4
NE2571	2.3	2.3	2.3	2.4
<i>NE2581</i>	1.9	2.0	2.0	2.4
NE2561	2.6	2.5	2.5	2.8
PTW30001	2.4	2.4	2.4	2.3
PTW30002/11	2.2	2.1	2.1	2.3
PTW30004/12	2.1	2.1	2.1	2.3
PTW30006/10/13	2.4	2.3	2.4	2.3
Wellhöfer IC69/FC65-P	2.3	2.3	2.3	2.3
Wellhöfer IC70/FC65-G	2.2	2.2	2.2	2.3

Both the frontal and distal results are in general in good agreement with the analytical results. For those ionization chambers of which the cylindrical section constitutes the largest part of the volume and the central electrode has a small diameter compared to the cavity diameter, the agreement is excellent. For other ionization chambers it can be worse, in particular when the cylindrical section is not much longer or even shorter than the diameter of the cavity as well as when the central electrode diameter is large compared to the cavity dimensions. The deviation from the IAEA recommendation is small for most ionization chambers but can be considerable (up to 1.5 mm) for some chambers commonly used in

proton dosimetry such as the FWT IC-18. For Farmer type ionization chambers as well as for most ionization chambers with a thin wall and a central electrode diameter that is small compared to the cavity diameter, the agreement is good in general, consistent with the observations of Jäkel et al. (2000) and Palmans and Verhaegen (2000).

Table 2 shows similar results for some ionization chambers of which the geometry is different from what is specified in IAEA TRS-398 (the results in the IAEA column were also based on the slightly different values of the cavity radius, to indicate the difference this gives compared to table 1). In particular, the hollow central electrode of the NE2561 ionization chamber and the mixed wall of a number of PTW ionization chambers could potentially result in different results. However, comparing the results for these ionization chambers in tables 1 and 2 it is clear that the differences are minor and that the data as specified in IAEA TRS-398 are sufficiently accurate for calculating $\Delta z_{w,tot}$.

Table 2. $-\Delta z_{w,tot}$ for some ionization chambers that have characteristics different from the ones specified in IAEA TRS-398.

Type of IC	R_{cav} (mm)	R_{cel} (mm)	Wall	$-\Delta z_{w,tot}$ (mm)			
				This work			IAEA
				MC _f	MC _d	Analyt.	
NE2561	3.675	0.875 ^(*)	0.5 mm graphite	2.4	2.5	2.5	2.8
PTW30006 /10/13	3.05	0.55	0.09 mm graphite + 0.34 mm PMMA	2.2	2.2	2.3	2.3
PTW30001	3.05	0.50	0.15 mm graphite/epoxy-paste + 0.28 mm PMMA	2.4	2.4	2.2	2.3
PTW23331	3.95	0.75	0.15 mm graphite/epoxy-paste + 0.40 mm PMMA	2.9	3.0	2.9	3.0
PTW23332	2.50	1.00	0.15 mm graphite/epoxy-paste + 0.35 mm PMMA	1.3	1.6	1.3	1.9

^(*)Hollow with inner radius of 0.7mm

3.1.2. Depth dose distributions

Figure 5 shows the depth distribution of D_{air} in the air cavity of an NE2581 ionisation chamber calculated with McPTRAN.CAVITY and the distribution by integrating the depth dose in homogeneous water according to the model of section 2.1.2. The integration was done by dividing the central electrode sections (a) and (b) (see figure 3) into three sub-intervals and section (c) into four sub-intervals, which is still a quite crude approach. As can be seen the model from section 2.1.2 gives excellent agreement even for an ionization chamber with a thick electrode such as the NE2581 in a low-energy 60 MeV beam.

Although an arbitrary choice such as equal volume subintervals (per section (a), (b) and (c)) gives a good agreement, the choice of the intervals was optimised in order to give the best (least squares) fit to the Monte Carlo simulations for assessing the inverse problem in the next section.

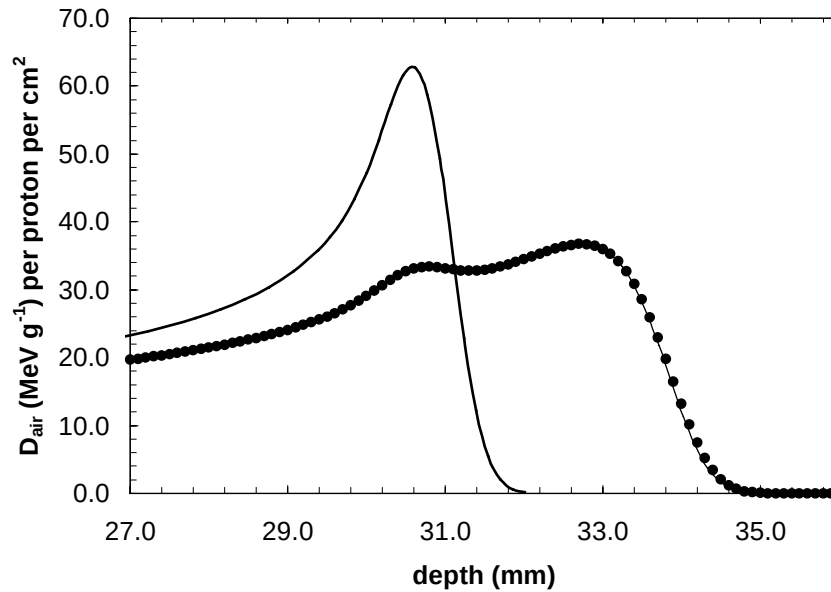


Figure 5. Depth dose to air curve calculated by Monte Carlo in homogeneous water (black curve), depth dose to air as a function of depth for a thimble ionization geometry (symbols) and predicted depth dose to air curve calculated with the analytical model of section 2.1.2 (the black curve which almost coincides with the symbols) for an NE2581 in a 60 MeV mono-energetic beam.

3.1.3. Inverse problem of estimating the depth dose curve from the measured depth dose response using a thimble ionization chamber

The possibility of applying the analytical model to the more challenging problem of converting a depth dose curve obtained with an ionization chamber to the depth dose in homogeneous water is demonstrated in figure 6 for a few ionization chambers and energies.

A function consisting of an arbitrary set of depth-dose values connected by a cubic spline was constructed. This function was convolved with the analytical model and a least square optimisation of the set of depth-dose values was performed in order to minimise the difference with the Monte Carlo simulated depth-dose function for the ionization chamber. In this work, this optimisation was performed with Microsoft Excel's solver.

This method is illustrated for the NE2581 in figure 6, which shows the optimised data points and the difference of the cubic spline with the depth dose curve in homogeneous water. This results in differences to the real depth dose curve smaller than 1% of the dose maximum in the region frontal to the Bragg peak and 5% in the distal edge, where these differences are less important since they correspond to minor shifts in depth. Even when deliberately adding Gaussian noise to the Monte Carlo data points (we could as well have simulated less histories) the outcome of the inverse optimisation is still reasonable. This indicates that the method will be applicable to measured depth dose curves, which will always exhibit a certain level of fluctuations, as well. It is again for ionization chambers with very thick, non-water equivalent walls or thick electrodes that the technique is difficult to apply with satisfying results.

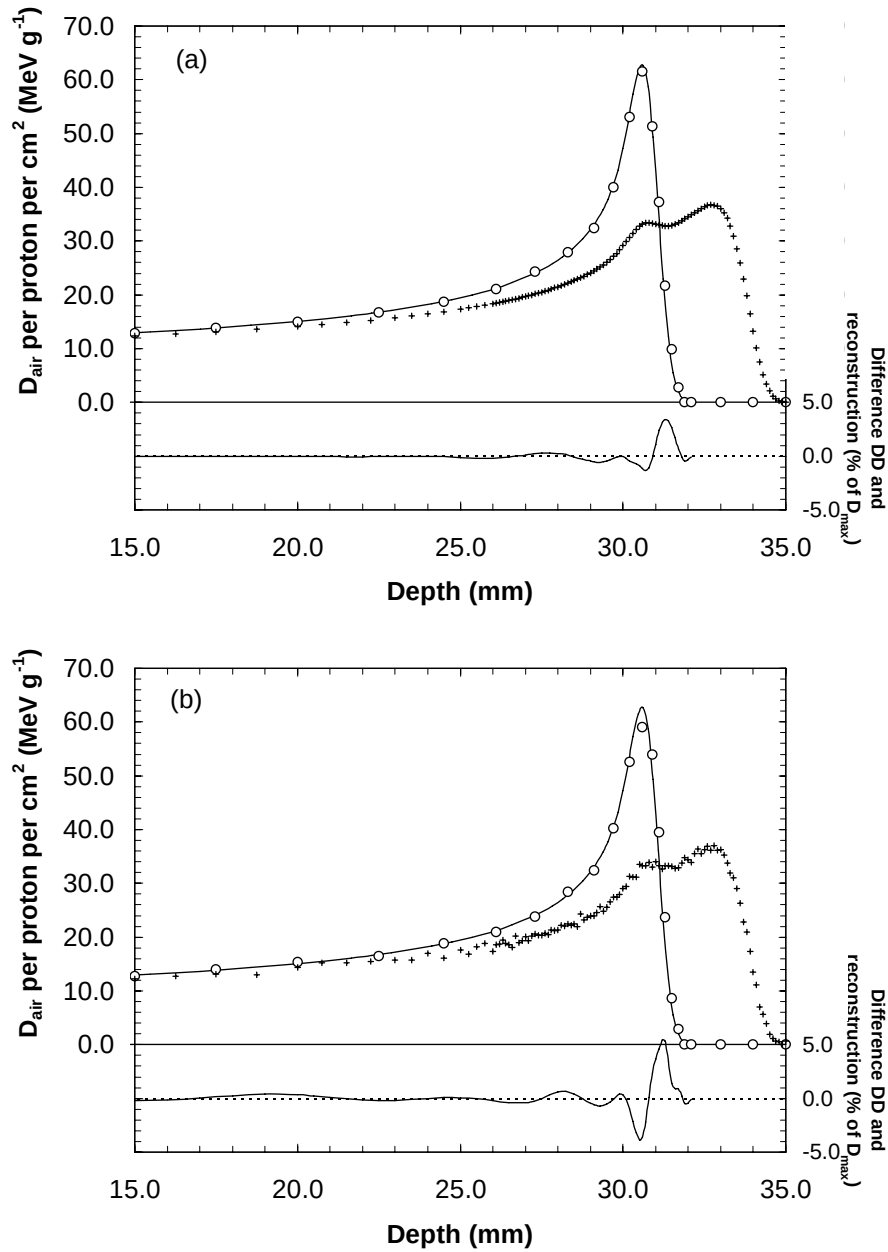


Figure 6. Reconstruction of depth-dose curves from an ionization chamber response for an NE2581 in a 60 MeV mono-energetic beam (a) without and (b) with deliberately added noise on the Monte Carlo simulations ($1\sigma = 2\%$). The + symbols are the result from a Monte Carlo simulation of the ionization chamber response, the full curve is the Monte Carlo calculated depth-dose curve in homogeneous water and the circles are the points left as free parameters for the reconstructed depth-dose curve. The lower panel gives the difference, expressed as a percentage of the dose maximum, between the cubic spline through the reconstructed data points and the curve in homogeneous water.

3.2. Comparison with theoretical and experimental data from the literature

Bichsel (1995) developed an analytical model for the simpler geometry of a pure cylindrical air cavity in homogeneous water and modelled the beam angle and energy spread widening

analytically as well. The analytical model of section 2.1.2. can be applied for a cavity without wall and central electrode to compare with Bichsel's results using his calculated depth dose curve to calculate the integrals. The cavity was divided in five strips with equidistant angles for this purpose. The results are shown in figure 7a and as can be seen the agreement is good even though the analytical model presented here is conceptually simpler and cruder.

Table 3 compares experimental determinations of the effective depth in water from the literature with the values calculated in this work showing that in all cases the agreement is within the experimental uncertainties. For the PTW-30001 and PTW-30006 type ionization chambers the values from table 2, incorporating the effect of the mixed wall composition, were taken except for the IAEA values that are just the values which would be derived from the IAEA data and recommendation (listed in table 1).

Figures 7b-d apply the method of section 2.1.2. to the experimental depth dose curves from Mobit et al. (2000) for a 78 MeV proton beam, Jäkel et al. (2000) for a 250 MeV $u^{-1} {}^{12}C$ beam and Kanai et al. (2004) in a 290 MeV $u^{-1} {}^{12}C$ beam (the experimental data from these papers were digitised). For the data of Mobit et al. (2000) a small re-normalisation of the optimised curve of 1.6% was needed similar to the normalisation they did when shifting the curve. This is consistent with what one expects since, given that there must be a shift of the effective depth of the cylindrical ionization chamber, one makes an error by normalising this to the curve for the plane-parallel ionization chamber. For the data of Jäkel et al. (2000) a more substantial re-normalisation of 6% was needed. This is probably similar to what they have done, but no information is provided on the normalisations that were used. Even though the number of measured data points by Mobit et al and Jäkel et al. was rather limited (consequently the number of free data points that could be allowed in the optimisation was limited as well), the method still works well and given the fluctuations on the experimental data no significant differences can be observed between the reconstructed curves and the measurements using plane-parallel ionization chambers. For the data of Kanai et al. (2004), which were presented by them as non-normalised ionization curves, no additional normalisation was required. This is an indication that the assumption made in IAEA TRS-398 that the ionization chamber perturbation correction factors are unity is valid. Again the method works well, but the fine resolution of their measurements allows resolving the small shift of 0.2 mm between the reconstructed curve and the plane-parallel ionization chamber measurement. Such small differences are of course within common levels of positioning uncertainty, but are also influenced by small differences in the stopping power ratios for protons and heavier ions.

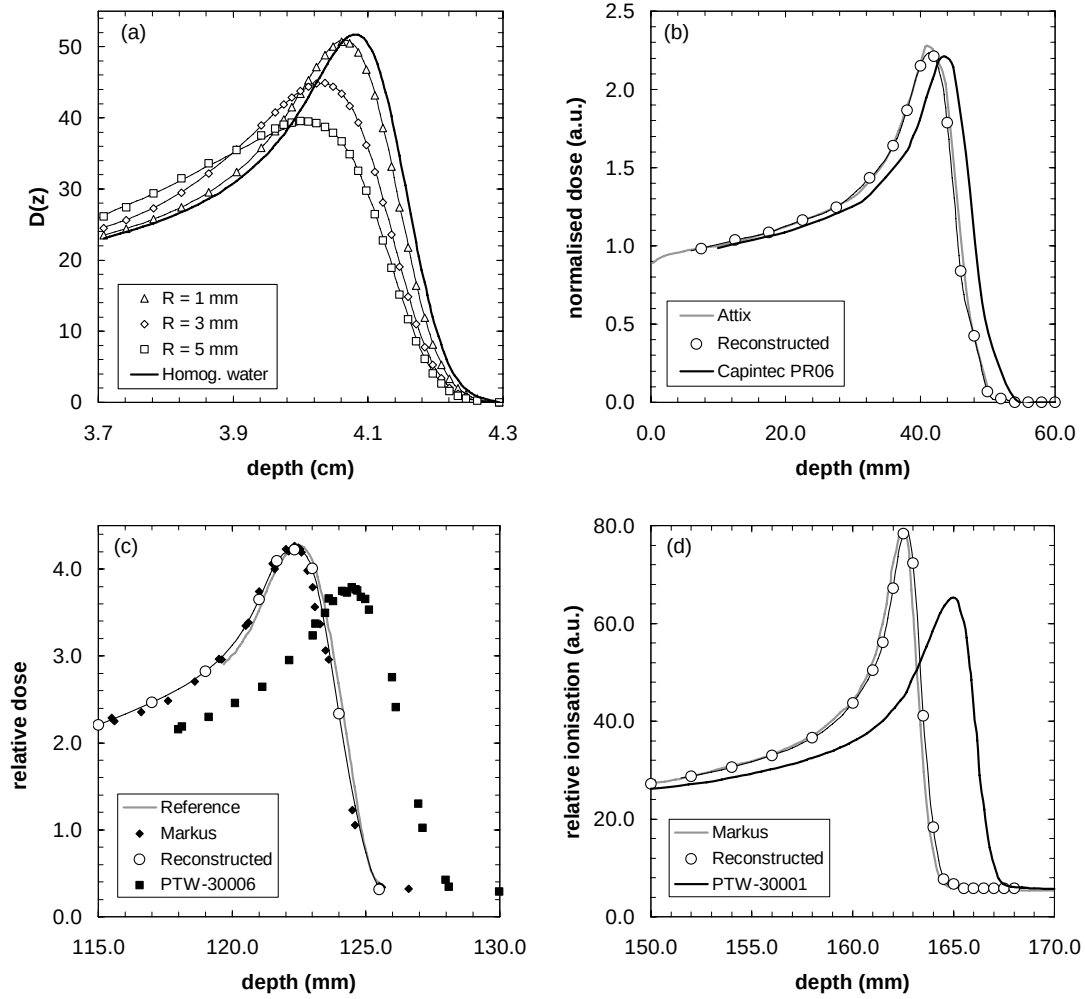


Figure 7. (a) Comparison with the analytical model of Bichsel (1995); the lines are the results from Bichsel, the symbols the results according to the model of section 6.2. (b) Comparison of the inverse dose reconstruction method with experimental results using a cylindrical and a plane-parallel chamber by Mobit et al. (2000), (c) idem with results by Jäkel et al. (2000) and (d) idem with results by Kanai et al. (2004).

Table 3. Experimental data from the literature on the shift (towards the beam source) of the effective depth of some cylindrical ionization chambers in mm.

Reference	Type of IC	Beam	$-\Delta z_{w,tot}$ (mm)			
			Experiment	This work		IAEA
				MC	Analytical	
(Mobit et al. 2000)	Capintec-PR06C	^1H	2.0 ± 0.4	2.4	2.4	2.4
(Hartmann et al. 1999)	PTW-30001	^{12}C	3 ± 1	2.4	2.2	2.3
(Jäkel et al. 2000)	PTW-30006	^{12}C	2.2 ± 0.2	2.2	2.3	2.3
(Kanai et al. 2004)	PTW-30001	^{12}C	2.5 ± 0.1	2.4	2.2	2.3

Some additional experimental evidence was obtained by Palmans et al. (2001) who performed an experiment in which graphite build-up caps were used to increase the wall thickness of a NE2571 ionization chamber. The agreement of their experimental results with the theoretical calculations was good. In the same paper, it was shown that the application of the gradient correction factors to a number of ionization chambers brings the mutual differences between ionization chambers in better agreement with the mutual differences on the SOBP at the same depth (where it is assumed that no gradient corrections are needed). The remainder of the mutual differences was then attributed to secondary electron perturbations calculated by Verhaegen and Palmans (2001). The models used by Palmans et al. (2001) were simpler but consistent with the models in this paper.

4. CONCLUSIONS

An analytical model was presented to calculate the effective depth in water, $z_{w,tot}$, of cylindrical (thimble) ionization chambers in clinical proton or a heavy ion beams. The shift of this quantity from the centre of the cavity was then calculated according to this model as well as with Monte Carlo simulations for all commercial cylindrical ionization chambers listed in IAEA TRS-398. The agreement between the analytical results and the Monte Carlo results was good, whereas the agreement with the recommended value of 0.75 times the cavity radius was good for most ionization chamber but can be different by up to 1.5 mm for some ionization chamber types. In general the recommendation of the IAEA is in good agreement for Farmer type chambers and for other chamber types with a thin wall and a central electrode diameter that is small compared to the cavity diameter. Furthermore, it was found that for ionization chamber with a ratio of central electrode diameter and cavity diameter smaller than 0.25 and a radius smaller than 4 mm and for proton energies above 150 MeV, a simple shift of the entire depth dose curve is sufficient in order for the difference with the correct curve to be smaller than 3% (the difference being mainly in the Bragg peak).

A modification to the analytical model, dividing the integrations in a number of sub steps, allowed correcting depth dose curves measured with a cylindrical ionization chamber even in the strongly non-linear gradient region of the Bragg peak. Agreement with the Monte Carlo simulations was again good. This model also enabled to solve the inverse problem of deriving a depth dose curve in homogeneous water from the depth dose response obtained with a cylindrical ionization chamber, even from rather noisy data. This inverse optimisation was applied to experimental results from a few papers from the literature and gave satisfactory results although more (and more detailed) data would be welcomed to support this.

5. ACKNOWLEDGEMENTS

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