

**NPL REPORT
DEM-TQD-007**

**The Future of Quantum
Roulette Noise Thermometry**

C. H. Webster

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JANUARY 2006



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The Future of Quantum Roulette Noise Thermometry

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ABSTRACT

The Quantum Roulette Noise Thermometer (QRNT) is a unique type of primary thermometer. Rather than probing a classical thermodynamic variable whose value depends on the average motion of an ensemble of microscopic particles, it makes a series of consecutive measurements of a quantum variable, the flux trapped within a superconducting ring. From the statistical distribution of flux measurements the temperature can be obtained. In principle, the device could operate over a wide range of temperatures, with the upper limit dependent on the superconducting material chosen for the ring. Its accuracy would be limited by statistical sample size. The QRNT directly addresses the primary objectives of the UK National Measurement System (NMS) Quantum Programme: an NMS based wholly on quantum processes, fundamental constants and the use of portable quantum standards with full primary accuracy. This report reviews preliminary research and assesses the future benefit of QRNT devices to thermal metrology.

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ISSN 1744-0491

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Approved on behalf of the Managing Director, NPL
by Dr. J. T. Janssen, Knowledge Leader, Time and Quantum Detection Team

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Chapter 1

Introduction

The Quantum Roulette Noise Thermometer (QRNT) is a novel type of primary thermometer with the potential to become a new primary standard for temperature, particularly in the sub 1 K range, where the temperature scale is currently least well defined.

Most thermometers rely on measurement of a macroscopic variable, such as pressure or resistance, whose temperature dependence is well understood and can be expressed in a relatively simple form. However, measurement of a macroscopic variable requires averaging over the motion of a vast ensemble ($\sim 10^{23}$) of interacting particles, e.g. molecules in a gas or electrons in a metal. In order to make the analysis of such systems tractable the interactions of the particles are often ignored. Inevitably, such assumptions limit the accuracy of a thermometer.

The QRNT offers an alternative approach, whereby the state of a single quantum system is sampled repeatedly over time to build up a temperature dependent stochastic distribution. Because the device behaves as a single entity its precision is limited only by statistical sample size (the number of sequential measurements of its state) and its accuracy is unaffected by interactions. We must not overlook the effects of time correlations, but in principle these can be avoided through modelling of the system dynamics and appropriate choice of the time interval between measurements.

The novelty of the QRNT lies in the method of sequential sampling of the quantized state of a single entity: the magnetic flux trapped within a superconducting ring. To the best of our knowledge, no other noise thermometer works on a similar principle. The most promising recent development in the field of low temperature noise thermometry is a device known as the Shot Noise Thermometer [1], which is a primary thermometer with 0.1% accuracy and 0.02% precision in the range near 1 K. However, this device, like all other noise thermometers, is based on a macroscopic variable, the spectral density of current noise through a tunnel junction, which is averaged over an ensemble of interacting electrons.

Since the QRNT is a superconducting device the upper limit of its temperature range is determined by the critical temperature T_c of the superconductor. For low temperature superconductors (LTS), such as Nb and Al, this limits the temperature range to below ~ 10 K, but for high temperature superconductors (HTS) such as $\text{YBa}_2\text{Cu}_3\text{O}_{7-\delta}$ and other cuprates, a device could in principle be made to operate

as high as 120 K.

Under the 2001-4 Quantum Programme of the UK National Measurement System (NMS), operation of an HTS QRNT device was demonstrated in the temperature range 15-40 K. Its precision was a few %, and was limited by statistics [2]. However, an unforeseen effect due to a combination of the fabrication technique and the *d*-wave symmetry of the superconducting wavefunction introduced an extra parameter with an unusual temperature dependence that severely hindered the accuracy of the device. This problem does not rule out the use of HTS devices in the future, as the effect can be eliminated by the use of different fabrication techniques, but at present it appears that the need for new thermometry techniques below 1 K would make an LTS device more advantageous.

An important recent development in temperature metrology, albeit on a provisional basis, has been the adoption of the first international scale for temperatures well below 1 K: the PLTS-2000. However, some discrepancies still remain between the scales of NIST and PTB on which the PLTS was founded. An LTS QRNT device would be a valuable tool to resolve these discrepancies, and would provide an effective means of realising the Kelvin at sub 1 K temperatures. The BCS theory of low temperature superconductivity is well established, so there is very little danger of an unforeseen exotic effect complicating the operation of an LTS device.

In the 2004-7 Quantum Programme the development of an LTS QRNT device was proposed, but co-funding from the thermal programme was not obtained due to higher priority of other projects. The recommendation of the working group was that a study should be undertaken to assess the future viability of HTS and LTS devices. This report summarizes the conclusions of that study.

Chapter 2

How the QRNT works

The QRNT consists of a ring of superconductor containing a superconducting switch (see Figure 2.1). The switch comprises one or more Josephson junctions, and its function is to switch the ring between the superconducting and the normal state.

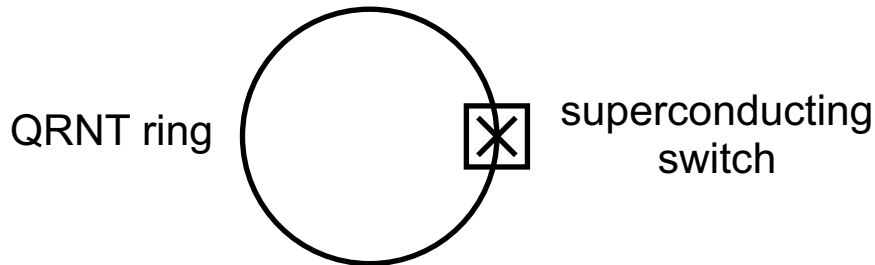


Figure 2.1: Schematic diagram of the QRNT device. The superconducting QRNT ring is interrupted by a superconducting switch comprising one or more Josephson junctions.

In the normal state the random thermal motion of electrons in the ring gives rise to fluctuating Nyquist noise currents, which in turn give rise to fluctuations in the magnetic flux enclosed within the ring. If the ring is mounted in thermal contact with a heat bath, then as the temperature of the bath rises the amplitude of the fluctuations increases. This is the basis of the thermometer.

The novelty of the QRNT lies in the detection scheme for measuring the fluctuations of the magnetic flux. Rather than detecting the fluctuations continuously, the property of flux quantisation in the superconducting state is exploited to freeze the flux before measurement.

The superconducting switch is employed to switch the ring repeatedly between the superconducting and normal states, trapping the flux in the superconducting state so that it can be measured, and releasing it again in the normal state so that it can undergo thermal fluctuation. By this method thermal fluctuations are recorded as a series of quantised flux values, each being an integer multiple of the flux quantum Φ_0 . The distribution of these values can be plotted as a histogram whose width is proportional to the temperature of the heat bath.

An important aspect of the QRNT is that it is a primary thermometer, i.e. it probes a variable which is related to temperature solely through fundamental

constants. Therefore it requires no calibration. We describe the theoretical basis of the QRNT below.

2.1 RF and DC SQUIDS

A SQUID (Superconducting Quantum Interference Device) is a device consisting of a superconducting ring interrupted by one or more Josephson junctions. Two types of SQUID are illustrated in Figure 2.2.

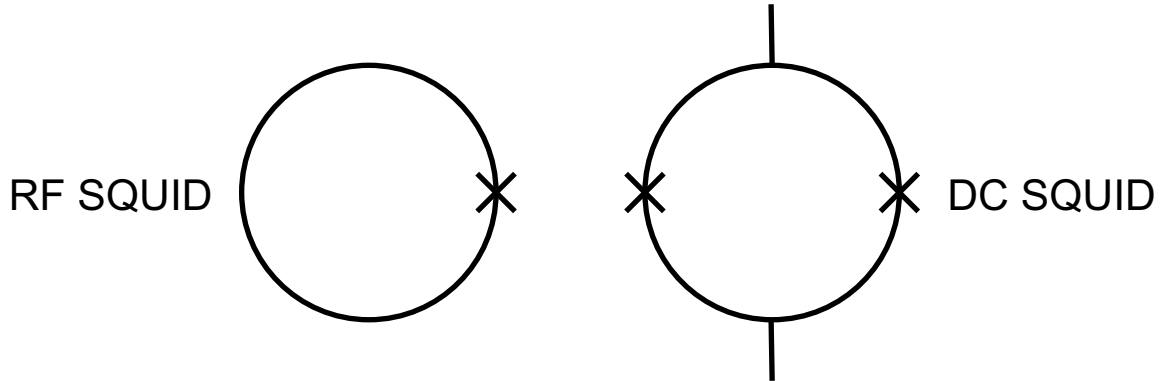


Figure 2.2: RF and DC SQUIDS. The RF SQUID contains a single Josephson junction and is isolated from its surroundings. The DC SQUID contains two parallel junctions and can be connected to an external circuit by its leads.

The RF SQUID is simply a loop containing one or more junctions. It possesses no electrical leads, so the only interaction it can have with the outside world is via magnetic induction. The QRNT falls into this category, and its electrical isolation affords it the advantage of being immune to self-heating.

The DC SQUID consists of a pair of Josephson junctions connected in parallel. Since this device possesses electrical leads it can be connected to a circuit, but since the two branches of the device form a superconducting loop it can also be affected by magnetic induction. Later on we will describe how a DC SQUID can be used to read out the flux trapped in a QRNT ring.

2.2 Flux trapping in an RF SQUID

The internal energy U of an RF SQUID is given by

$$U(\Phi_i) = \frac{(\Phi_x - \Phi_i)^2}{2L} - \frac{\Phi_0^2}{4\pi^2 L} \beta_L \cos\left(\frac{2\pi\Phi_i}{\Phi_0}\right), \quad (2.1)$$

where Φ_x is the external applied flux, Φ_i is the internal flux, Φ_0 is the flux quantum, L is the self-inductance of the ring and $\beta_L = 2\pi L I_c / \Phi_0$ is known as the screening parameter, where I_c is the critical current of the Josephson junction [3]. The first term in Equation (2.1) is the classical, parabolic flux dependence of the magnetic energy stored in the ring, while the second term arises from the presence of a Josephson

junction in the ring. The screening parameter β_L determines whether metastable quantised flux states exist within the ring (see Figure 2.3). For $\beta_L > 1$, metastable minima exist in $U(\Phi_i)$, and the flux enclosed within the ring is quantised at values corresponding to the minima. However, for $\beta_L < 1$, the barriers between the minima disappear so that metastable states no longer exist, and the flux enclosed within the ring ceases to be quantised. When $\beta_L = 0$ the Josephson junction no longer passes a supercurrent, and the ring behaves as if it were in the normal state.

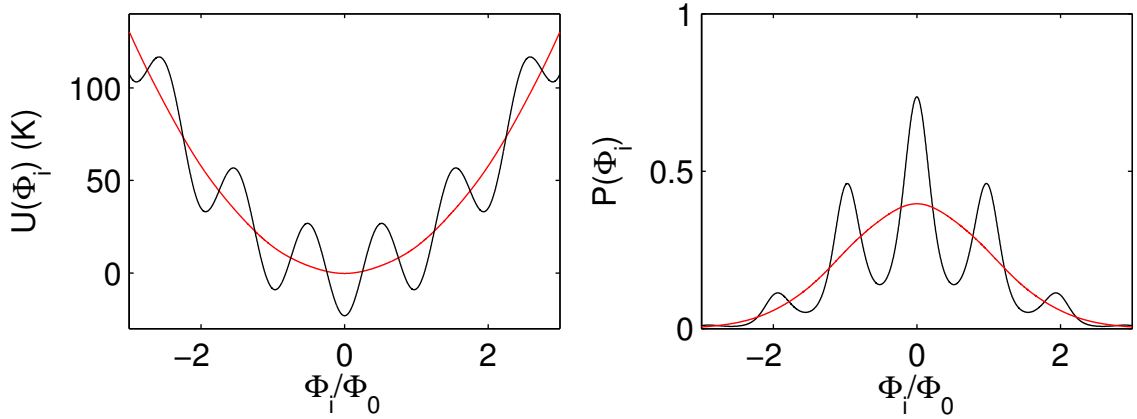


Figure 2.3: Left: Potential energy of an RF SQUID (the energy scale has been divided by k_B so that it appears in units of Kelvin). Right: internal flux distribution. Black line: $\beta_L > 1$ (quantised flux). Red line: $\beta_L < 1$ (continuous flux). The parameters used to make these plots were $L = 10$ nH, $\Phi_x = 0$, $T = 30$ K and $I_c = 1$ μ A (black)/ 0.01 μ A (red).

The probability distribution for magnetic flux enclosed within the ring is given by the Boltzmann distribution

$$P(\Phi_i, T) = P_0 \exp\left(-\frac{U(\Phi_i)}{k_B T}\right), \quad (2.2)$$

where P_0 is a normalisation factor, k_B is Boltzmann's constant, and T is the bath temperature. Figure 2.3 shows typical flux distributions for $\beta_L > 1$ (quantised flux) and $\beta_L < 1$ (continuous flux).

2.3 Flux noise thermometry

Noise thermometry using SQUID devices has been around for over thirty years. Many devices are still based on the original design by Webb, Giffard and Wheatley [4]. However, such devices detect fluctuations in magnetic flux continuously, in contrast to the QRNT process of sequential discrete measurements.

The QRNT method uses a superconducting switch to set β_L to zero so that the flux can fluctuate as if the superconducting ring were in the normal state. In this régime the magnetic energy U_{th} associated with thermally driven noise currents is

given by

$$U_{\text{th}} = \frac{\langle (\Phi_x - \Phi_i)^2 \rangle}{2L}, \quad (2.3)$$

where the angular brackets represent a time average. Ideally, the external flux Φ_x should be zero, which can be achieved in practice with good magnetic shielding. In this case the only degree of freedom is the internal flux Φ_i of the ring. The equipartition theorem tells us that the thermal energy associated with each degree of freedom is $k_B T/2$, so, setting $\Phi_x = 0$ we obtain

$$\frac{\langle \Phi_i^2 \rangle}{2L} = \frac{k_B T}{2} \Rightarrow T = \frac{\langle \Phi_i^2 \rangle}{k_B L}. \quad (2.4)$$

We find that the temperature of the heat bath is directly proportional to the mean square flux enclosed within the ring, and the only constants of proportionality are a fundamental constant (k_B) and a purely geometrical quantity (L). The loop inductance L can be calculated to a high degree of accuracy using finite element modelling for a ring of arbitrary shape [5]. With precise engineering of the device dimensions, the QRNT is therefore a primary thermometer.

The primary nature of the QRNT is a key advantage over other flux noise thermometers. By utilising a series of discrete measurements, rather than a bandwidth-limited measurement of a continuous variable, unknown quantities associated with the amplifier chain are avoided.

2.4 The superconducting switch

The function of the superconducting switch is to control the screening parameter β_L . When $\beta_L > 1$ the flux enclosed within the ring is quantised, whereas when $\beta_L < 1$ it is continuous. Of the parameters on which β_L depends, the inductance L cannot be varied without changing the geometry of the ring, but the critical current I_c of the switch can be controlled by a number of methods. Hence, control of β_L relies on control of I_c . We discuss the various methods below.

2.4.1 Field modulation of I_c

The simplest type of superconducting switch comprises a single Josephson junction. In the presence of an external magnetic field the phase difference across the junction varies along its width, leading to quantum interference of the superconducting wavefunction. This phenomenon is analogous to the diffraction of light by a rectangular aperture, which is caused by interference of the electric field vector as it varies along the aperture. The interference of light results in a modulation of intensity known as the Fraunhofer diffraction pattern, whose nodes are determined by the width of the aperture and the wavelength of the light. Correspondingly, in a Josephson junction, quantum interference results in Fraunhofer modulation of the critical current, where the nodes are determined by the width of the junction. Since the magnetic flux passing through the junction is proportional to its width, we can express the

Fraunhofer modulation of the critical current as follows [6]:

$$I_c(\Phi_J) = I_c(0) \left| \frac{\sin(\pi\Phi_J/\Phi_0)}{\pi\Phi_J/\Phi_0} \right|, \quad (2.5)$$

where Φ_J is the flux threading the junction. The modulation pattern has nodes at $\Phi_J = n\Phi_0$, where n is an integer, as shown in Figure 2.4. Thus, we can tune the critical current of the junction from its maximum value to zero by applying an external flux of Φ_0 to the junction. The disadvantage of this technique is that the small area of the junction means that the magnetic field required to suppress I_c to zero is quite large. For example, to pass a flux Φ_0 through a junction of area $15 \mu\text{m}^2$ would require a magnetic field $B = 0.13 \text{ mT}$. It would be difficult to apply such a large field locally to the junction without affecting the state of the main QRNT ring.

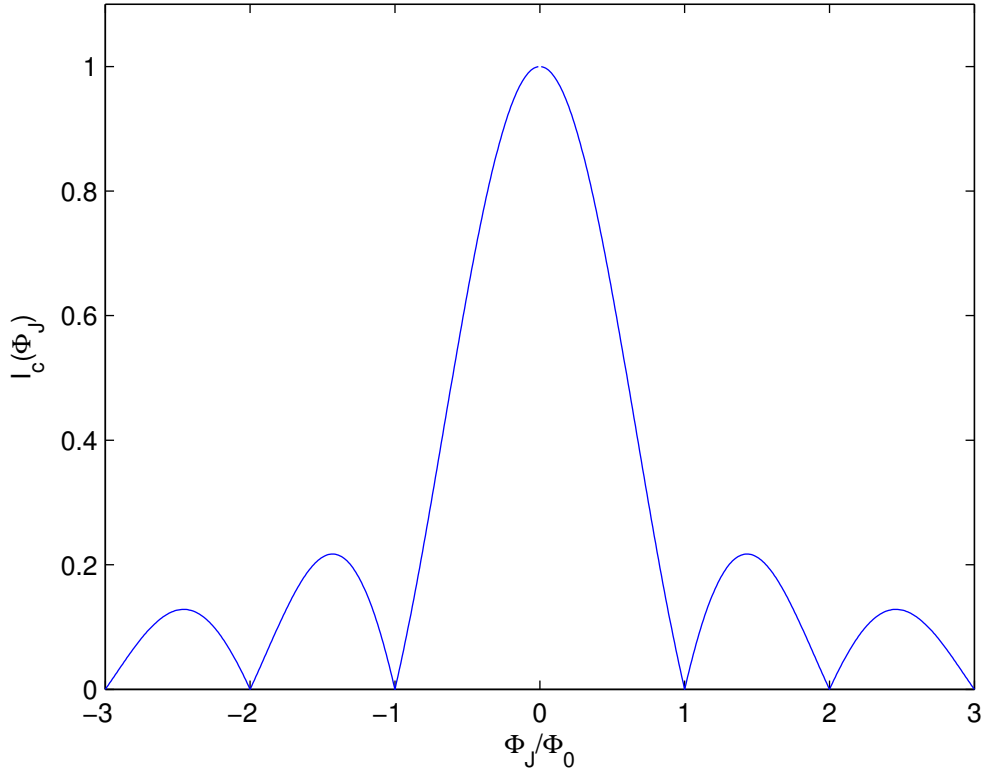


Figure 2.4: Fraunhofer modulation of the critical current of a Josephson junction under application of an external field (I_c is normalised to 1 at $\Phi_J = 0$).

In order to increase the effective area of the junction we can form a superconducting switch from a parallel combination of two junctions. The small superconducting loop enclosed by the two parallel arms forms a DC SQUID whose critical current is also modulated by quantum interference. However, the persistent current flowing in the loop changes the form of the modulation pattern such that its nodes depend on the flux threading the loop, rather than the flux threading the individual junctions.

The formula for an ideal DC SQUID with identical junctions is as follows [7]:

$$I_c(\Phi_S) = I_c(0) \left| \cos \left(\frac{\pi \Phi_S}{\Phi_0} \right) \right|, \quad (2.6)$$

where I_c is now the critical current of the whole SQUID (rather than a single junction) and Φ_S is the flux threading the DC SQUID loop. If the loop area is a good deal larger than the junction area, the magnetic field required to generate a flux $\Phi_S = \Phi_0$ is much lower than that required to generate $\Phi_J = \Phi_0$, making it easier to apply the flux locally to the junction without affecting the main QRNT ring. The disadvantage of this technique is that if the two junctions are not identical the modulation depth is reduced and the critical current cannot be suppressed completely to zero (see Figure 2.5). In this case the modulation pattern is given by

$$I_c(\Phi_S) = \left| \max \left[I_{c1} \sin \phi + I_{c2} \sin \left(\phi + \frac{2\pi \Phi_S}{\Phi_0} \right) \right] \right|, \quad (2.7)$$

where I_{c1} and I_{c2} are the critical currents of the two junctions and ϕ is the phase difference across one of the junctions. Fabrication of junctions that are similar enough to allow the condition $\beta_L < 1$ to be satisfied is non-trivial.

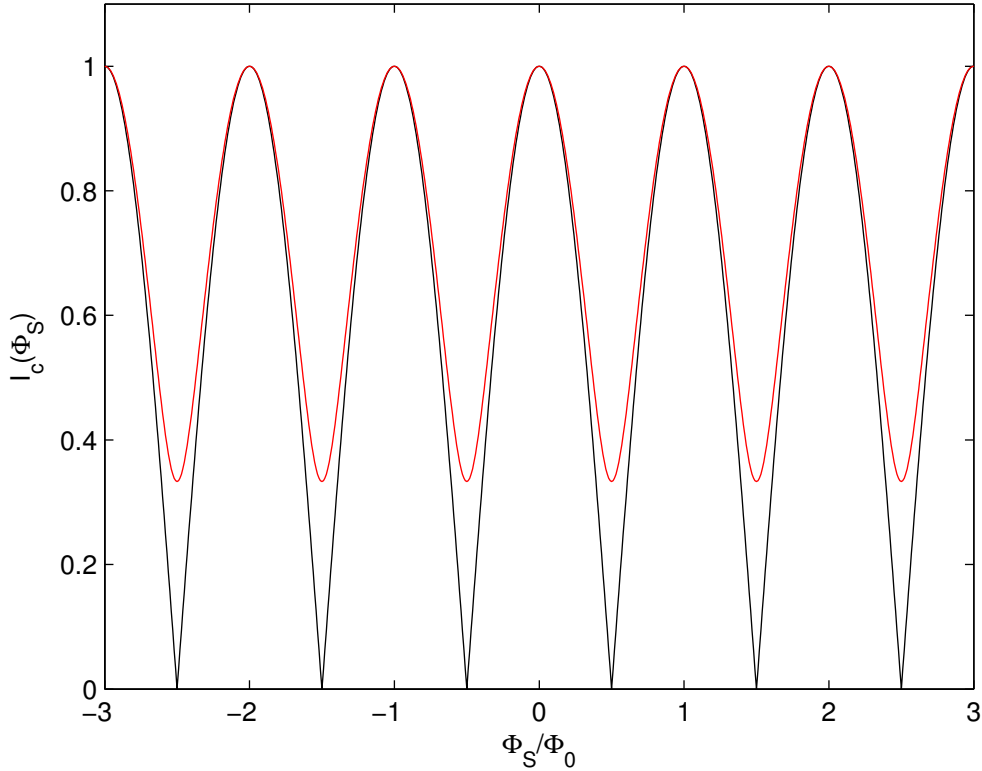


Figure 2.5: Modulation of the critical current of a DC SQUID under application of an external field (I_c is normalised to 1 at $\Phi_S = 0$). Black line: identical junctions. Red line: non-identical junctions.

2.4.2 RF modulation of I_c

An alternative way to control the critical current of a single junction is to irradiate it with radiofrequency (RF) electromagnetic radiation. This induces Shapiro steps [8] in the current voltage characteristic of the junction. The steps are equally spaced in voltage, but the currents at which they occur are dependent on the amplitude V_{RF} and frequency ν of the RF field. The zeroth order step occurs at the critical current of the junction, and its dependence on the RF field is given by

$$I_c(\eta) = I_c(0)J_0(\eta), \quad (2.8)$$

where J_0 is the zeroth order Bessel function and $\eta = V_{\text{RF}}/\nu\Phi_0$. A plot of I_c vs η is shown in Figure 2.6. The disadvantage of this technique is that it is difficult to predict the parameters V_{RF} and ν that will suppress I_c to zero, due to uncertainty in the coupling strength of the radiation to the junction. However, these parameters can be determined experimentally, as described in section 4.4. The advantage of the technique is that the values required are within the range of a standard function generator.

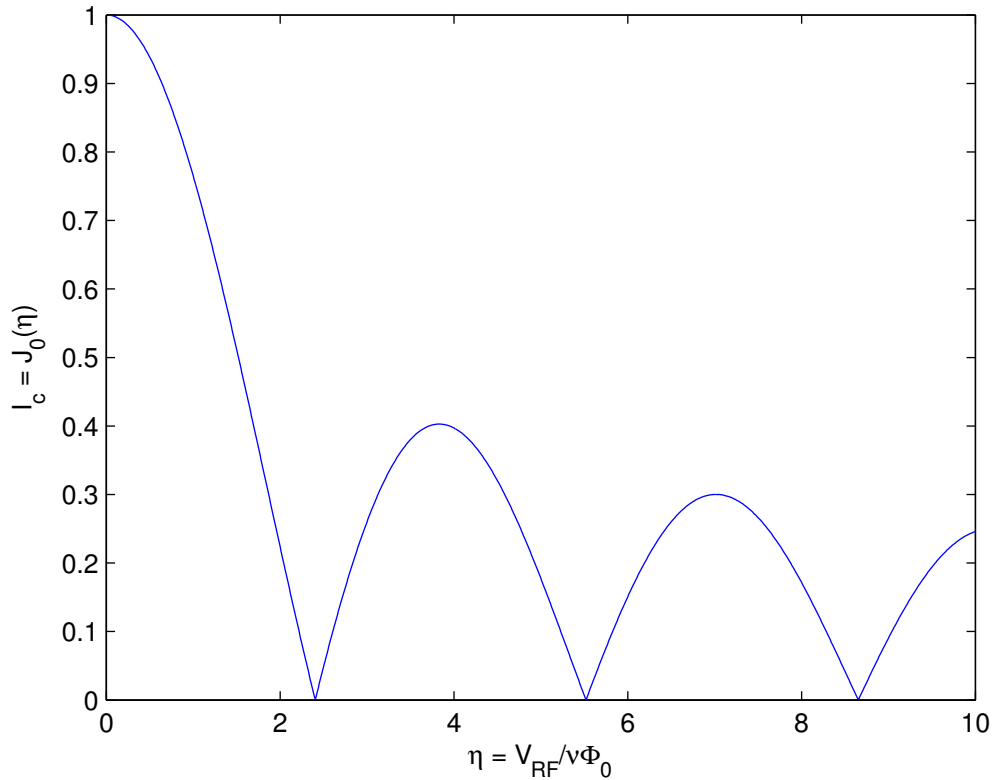


Figure 2.6: Modulation of the critical current of a DC SQUID under application of an RF field (I_c is normalised to 1 at $\eta = 0$).

2.4.3 Local heating of the switch

An early idea for a superconducting switch was to focus a He-Ne laser beam on a thin superconducting ring to provide local heating that would drive it normal [9]. However, heat input is undesirable for accurate thermometry.

2.5 Readout techniques

The flux enclosed by the main QRNT ring can be read out by means of an inductively coupled DC readout SQUID. Two readout schemes are presented below.

2.5.1 Flux-locked loop

In the first scheme the readout SQUID is used as a magnetometer and is operated in a circuit known as a flux-locked loop [6]. In this configuration the SQUID is biased above its critical current so that it operates in the resistive régime. A modulation coil couples a low amplitude ($< \Phi_0/2$) AC flux into the SQUID loop, causing the voltage across the SQUID to oscillate. A feedback coil maintains the flux threading the SQUID loop at a value close to $\Phi_0/2$ (where the voltage-flux gradient is highest). Any change in the external flux coupled into the loop causes a change in the amplitude of the voltage oscillation, which is fed back to the feedback coil to oppose the change. The feedback current is therefore a measure of the external flux.

2.5.2 “Time-of-flight” technique

The second readout scheme involves a “time-of-flight” technique which is used to determine the critical current of the readout SQUID. The critical current of a DC SQUID is dependent on the external flux as described in Eq. (2.6), so a measurement of the critical current of the readout SQUID constitutes a measurement of the flux state of the QRNT. The time-of-flight technique works as follows. The bias current of the readout SQUID is ramped from zero to just above the critical current. When the bias exceeds the critical current the SQUID switches from the superconducting to the resistive régime and a finite voltage appears across its terminals. A time interval counter is used to measure the time between the start of the ramp (triggered by the ramp generator) and the switching of the SQUID to the resistive régime (triggered by the appearance of a voltage across the SQUID). If the ramp rate is known, the critical current of the SQUID (and hence the flux state of the QRNT) can be obtained from the time interval between the start of the ramp and the switching event.

Chapter 3

Limitations

In this chapter we discuss fundamental theoretical and practical limits to the ultimate operation of the QRNT.

3.1 Switching speed

The time taken for the flux state of the QRNT to change is determined by the time constant $\tau = L/R$, where L is the inductance of the QRNT ring and R is normal state resistance of the Josephson junction. For an HTS junction, a realistic upper limit to R would be $\sim 100 \, \Omega$, but for an LTS junction it could be more like $10 \, \text{k}\Omega$. The inductance L is determined by the intended temperature range of the device, as described in section 3.3. For an HTS device with $L = 10 \, \text{nH}$, operating at $T > 25 \, \text{K}$, or for an LTS device with $L = 1 \, \mu\text{H}$, operating at $T < 1 \, \text{K}$, the time constant would have a lower limit of $\tau \sim 0.1 \, \text{ns}$.

The time taken to change the state of the superconducting switch depends on the switching method used. The method of field modulation is limited by the time taken to ramp up the magnetic field applied to the switch. A rapidly changing flux induces a voltage across a Josephson junction through the AC Josephson effect [3]. In order to avoid excessive pair-breaking, which would lead to dissipation, we want to ensure that the induced voltage does not exceed the superconducting gap voltage by a very large amount. For conventional superconductors the gap is of order a few mV, so the flux ramp rate is limited to $d\Phi_x/dt < 10^{-3} \, \text{Wb s}^{-1}$. For a flux change of $\Phi_0/2 = 10^{-15} \, \text{Wb}$ (required for a parallel combination of two junctions) we obtain a switching time $\tau_{\text{sw}} > 1 \, \text{ps}$. The method of RF modulation has a rather longer switching time, as it is limited by the frequency of the applied radiation, which is limited by the range of the function generator and the bandwidth of the leads. Laboratory microwave sources can typically provide frequencies up to $40 \, \text{GHz}$, a value which is within the bandwidth of semi-rigid coaxial cable. This gives us a switching time of $\tau_{\text{sw}} > 25 \, \text{ps}$.

From the above discussion it appears that the time constant of the QRNT ring is the limiting factor. This would result in an upper limit to the sampling speed of 10^{10} samples per second.

3.2 Readout speed

A magnetometer in the flux-locked loop configuration is capable of measuring changes in flux from a fraction of Φ_0 to several Φ_0 . However, its bandwidth is limited by the AC modulation frequency, which is typically ~ 100 kHz. This would ultimately limit the sampling speed of the QRNT to $\sim 10^5$ samples per second. This is a good deal slower than the limit imposed by the time constant of the QRNT ring. We also note that the state of the art in the SQUID noise is of order $1 \mu\Phi_0/\sqrt{\text{Hz}}$. A bandwidth of 100 kHz would therefore put the noise floor at around $0.3 \text{ m}\Phi_0$. In order to resolve quantised flux states in the QRNT ring we would need a noise floor well below Φ_0 . Assuming a realistically weak coupling between the QRNT and readout SQUID, a change of Φ_0 in the QRNT ring is likely to produce a change of $0.1\Phi_0$ at the readout SQUID, so we are likely to require a noise floor of $< 10 \text{ m}\Phi_0$. This is well within the scope of a modern magnetometer.

The time-of-flight readout technique is potentially much faster than the flux-locked loop technique, as the readout speed is limited only by the ramp rate of the bias current, which is determined by the ramp generator and the bandwidth of the leads.

3.3 Temperature range

The temperature range of the QRNT is mainly determined by the inductance L of the QRNT ring. In order to obtain a good fit to the observed flux distribution it is desirable that the distribution spans at least $2\Phi_0$. A sensible criterion for the low temperature limit of operation is that the state containing $2\Phi_0$ has a population no lower than 10% of the population of the global minimum (zero flux quanta in the ring). This requires

$$\exp\left(-\frac{(2\Phi_0)^2/2L}{k_B T}\right) > 0.1. \quad (3.1)$$

Therefore, if the QRNT is to be operated at low temperatures, a high inductance is required. The inductance of a thin film washer with a square hole of side d , where d is much smaller than the outer diameter of the washer, has been shown [10] to be

$$L = 1.25\mu_0 d. \quad (3.2)$$

For a washer of side $d = 6$ mm, we obtain $L \sim 10$ nH. Substituting this in Eq. (3.1) we find that such a device would operate best at $T > 25$ K. For a device to operate at 1 K, we would require $L \sim 250$ nH, which requires $d = 150$ mm. To be reduced to a practical size, such a device would require of order 15 turns. This would be difficult to achieve with current HTS fabrication technology, but would be much easier with LTS.

In order to operate the QRNT over a range of temperature it is necessary to demonstrate that the flux distribution is not significantly affected by the temperature dependence of the critical current of the superconducting switch. The temperature dependence of I_c for a conventional superconductor is given by the Ambegaokar-

Baratoff formula [6]:

$$I_c = \frac{\pi\Delta}{2eR_n} \tanh\left(\frac{\Delta}{2k_B T}\right), \quad (3.3)$$

where Δ is the superconducting gap energy and R_n is the normal state resistance of the junction. This function is relatively flat for $T \ll T_c$, where T_c is the critical temperature of the superconductor, but the formula changes for dirty junctions, acquiring a steeper gradient. Nevertheless, provided that $\beta_L \gg 1$ in the quantised state of the QRNT, the trapped flux distribution should be relatively insensitive to variations in I_c , owing to the steepness of the potential gradient.

3.4 Fabrication techniques

In order to achieve high precision with a QRNT device its dimensions must be known to a high precision in order to calculate the inductance. Fabrication techniques such as e-beam lithography currently have a positional uncertainty of order ~ 10 nm, but the difficulty would lie in achieving this precision on a macroscopic length scale. Current techniques would struggle to achieve this, but one can envisage methods for improvement: for instance, a translation stage whose travel is measured interferometrically. Alternatively, a 3D inductor could be envisaged, in which the dimensions would be controlled by the former, which could be made from a material with high crystallinity and few defects, such as sapphire.

3.5 Back action

Operation of the QRNT device could potentially be limited by the phenomenon of back-action, in which flux generated by currents flowing in the readout SQUID could couple to the QRNT ring and disturb the flux distribution. However, this can be minimised by avoiding strong coupling between the QRNT and readout SQUID.

3.6 Relaxation

Since the QRNT relies upon the population of metastable flux states there is a danger that it could relax to the ground state before readout. At low temperatures the probability of thermal excitation over the barrier between metastable states is low, provided that β_L is large. Quantum tunnelling can occur at any temperature, but again, provided that β_L is large, the barrier width will be large enough to render the probability of quantum tunnelling minimal.

Relaxation is also a danger if the switching speed is comparable to or slower than the time constant of the QRNT ring. However, for each of the switching methods outlined above, the switching time τ_{sw} is much faster than the time constant τ .

Chapter 4

Research to date

The QRNT project was first conceived in 1996 [11]. An initial feasibility study was followed by experimental work, the bulk of which was done between 1999 and 2002 [12, 2]. Unusual results were obtained, and much of the research from 2003 to 2004 consisted of modelling in an effort to understand the system [13].

4.1 Fabrication

All QRNT devices tested to date have been HTS devices designed for operation in the temperature range 15–70 K. Funding has not been obtained for the development of LTS devices. The HTS devices were fabricated from thin films of $\text{YBa}_2\text{Cu}_3\text{O}_7$ (~ 200 nm thick), grown by pulsed laser deposition on SrTiO_3 bi-crystal substrates. Bi-crystal substrates consist of two single crystals, misoriented with respect to one another. The grain boundary propagates through to the $\text{YBa}_2\text{Cu}_3\text{O}_7$ layer, so that when microbridges are patterned across the join by etching the superconductor, grain boundary Josephson junctions are formed.

The design used for the QRNT is shown in Figure 4.1. The main QRNT ring consists of a square washer containing a rectangular hole of area $6 \times 8 \text{ mm}^2$. The superconducting switch consists of a parallel combination of two grain boundary Josephson junctions formed from microbridges of area $5 \times 3 \text{ }\mu\text{m}^2$. The area of the small DC SQUID loop enclosed by the two microbridges is $5 \times 5 \text{ }\mu\text{m}^2$. The other Josephson junction formed by the grain boundary on the wide part of the washer loop has a very large critical current, and can therefore be ignored. A control loop is patterned close to the superconducting switch, for the purpose of supplying a DC or RF magnetic field. The switch is located far from the main QRNT ring in an effort to localise the control field.

Substrates with two different misorientation angles were tested: 24° and 30° . The latter produced junctions with smaller critical currents than the former, according to the formula

$$I = I_c \cos 2\alpha \cos 2\beta \sin \phi, \quad (4.1)$$

where α and β are the angles the bicrystals make with the grain boundary [14], and ϕ is the phase difference across the junction. Both the substrates tried were symmetric, meaning that α and β were equal (for a total misorientation angle of 24° , $\alpha = \beta = 12^\circ$).

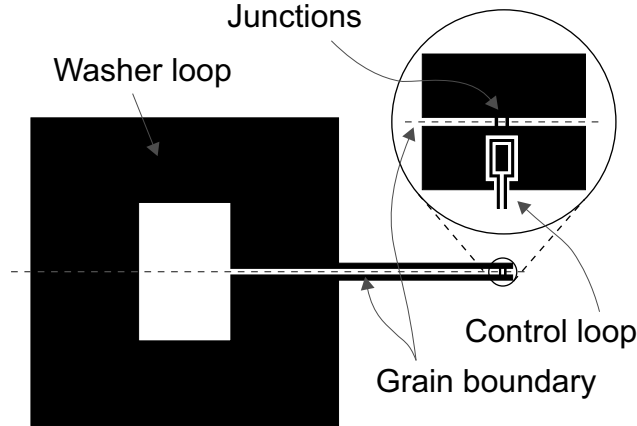


Figure 4.1: Schematic diagram of washer and Josephson junctions (not to scale). The dashed line represents the grain boundary. The control loop is used to provide external magnetic flux to the junctions comprising the superconducting switch.

4.2 Proof of principle

Using HTS devices, most of the techniques described in Chapter 2 have been demonstrated at a basic level. In particular, operation of the superconducting switch has been verified using the RF modulation technique (the field modulation technique was found to influence the flux state of the QRNT ring [12]), and it has been shown that flux can be trapped and released by means of the superconducting switch [2]. Flux distributions have been measured at a series of temperatures and the results modelled [13].

These preliminary demonstrations utilised slow sampling rates (~ 10 samples per second) and low measurement bandwidths (~ 5 kHz), leaving plenty of scope for improvement. However, the results were far from ideal, with the devices exhibiting confusing behaviour that was ultimately traced to the d -wave symmetry of the superconducting wavefunction. Below we describe some of the key results in more detail.

4.3 Experimental setup

A schematic diagram of the experimental setup is shown in Figure 4.2. Inside an evacuated glass canister the QRNT device was mounted on the end of a sapphire rod in thermal contact with the cold finger of a continuous flow helium siphon. Temperature control of the device was achieved by passing high frequency AC current¹ through a wire wound heater. The DC readout SQUID was positioned outside the glass, approximately 6 mm from the QRNT ring, and well inductively coupled. The whole system was immersed in a bath of liquid nitrogen.

¹A 5 kHz low pass filter on the readout SQUID was used to filter out the AC field generated the heater coil.

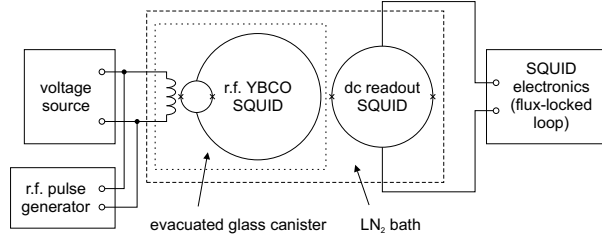
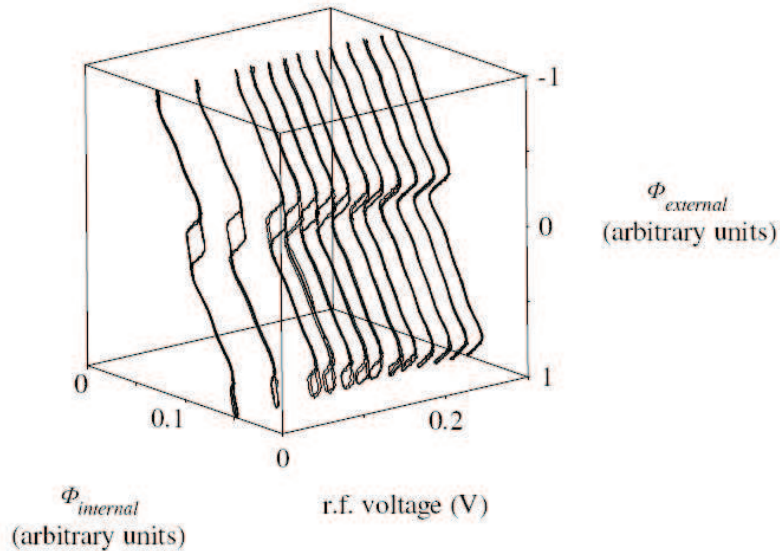


Figure 4.2: Schematic diagram of the experimental setup.

4.4 RF operation of the superconducting switch

As mentioned in Chapter 2, it is difficult to predict the amplitude V_{RF} and frequency ν of the RF field required to suppress the critical current I_c of the junction(s) to zero. However, these parameters can be determined experimentally using a technique dubbed “hysteresis loop suppression” [2]. The idea is to measure the flux hysteresis of the QRNT ring by ramping the external flux up and down while measuring the flux enclosed by the QRNT ring with the readout SQUID. When no RF radiation is applied to the superconducting switch, the flux enclosed by the QRNT ring should be quantised ($\beta_L > 1$), and should exhibit hysteretic step-like behaviour as the external magnetic flux is ramped up and down. However, when the RF radiation is switched on and tuned to the correct amplitude and frequency, the flux enclosed by the QRNT should be continuous and should not exhibit hysteresis on ramping. By making repeated measurements of the flux hysteresis while varying V_{RF} and ν , one can determine the RF field parameters required to suppress I_c to zero and thus operate the superconducting switch. Figure 4.3 shows the experimental results. As V_{RF} is increased (while ν is held constant) the flux hysteresis is gradually suppressed until $\beta_L < 1$.

Figure 4.3: Suppression of flux hysteresis as the amplitude V_{RF} of the applied RF field is increased.

4.5 Flux distribution measurement

Once we have determined the RF amplitude and frequency required to suppress I_c to zero, the flux distribution of the QRNT ring can be sampled by applying a series of RF pulses and recording the flux measured by the readout SQUID after each pulse. From the flux hysteresis curves measured earlier, the response of the readout SQUID to a flux change of Φ_0 in the QRNT ring is known. Hence, measurements of the readout SQUID signal can be binned into a histogram of QRNT flux states. Figure 4.4 shows the experimental results as a function of temperature. The device is clearly non-ideal. At low temperatures no metastable flux states are visible. The flux only gets trapped in the ground state. As T increases the distribution broadens, suggesting a strong change in I_c . Then, at 35 K the distribution suddenly bifurcates into two Gaussian peaks separated by $\Phi_0/2$. This behaviour cannot be explained by conventional Josephson dynamics. The only condition under which two peaks should be seen is when the external flux is equal to $\Phi_0/2$, but in this case the separation of the peaks ought to be Φ_0 , i.e. twice the separation observed.

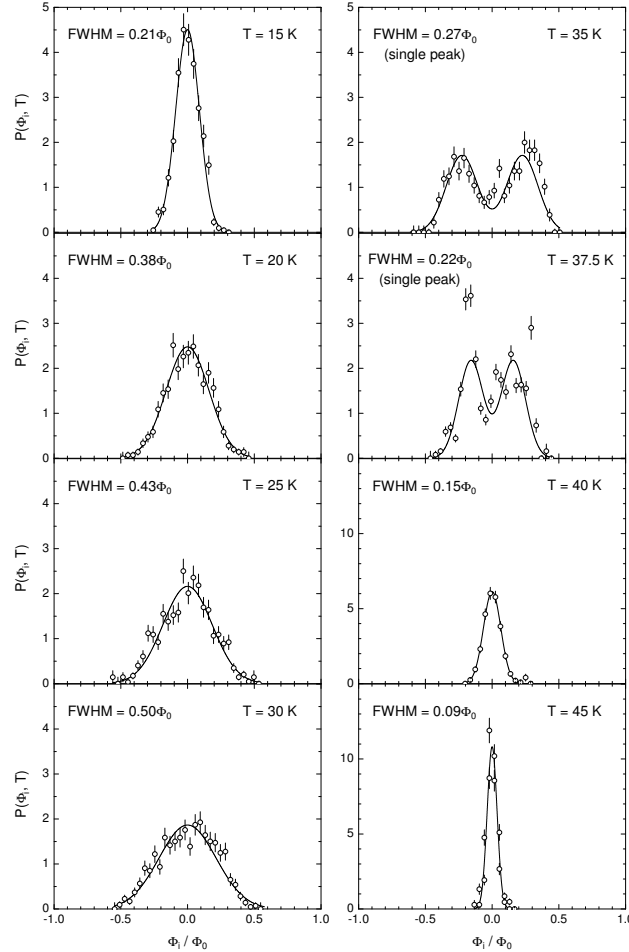


Figure 4.4: Temperature dependence of flux distribution. The open circles are the normalised histogram data and the lines are gaussian fits using one or two gaussians. The distribution bifurcates at $T = 35$ K.

4.6 Modelling

At the beginning of the QRNT project the behaviour of HTS grain boundary junctions was not well understood. It was assumed they behaved in a similar way to other types of Josephson junction. However, during the course of the project new research on the current-phase relationship (CPR) of grain boundary junctions has improved our understanding a great deal [15, 16, 17, 18].

It has been shown that the CPR in HTS junctions is not purely sinusoidal with period ϕ , as it is for conventional junctions. In fact, the CPR of an HTS junction possesses a second harmonic component with period 2ϕ :

$$I = I_c^I \sin \phi - I_c^{II} \sin 2\phi, \quad (4.2)$$

where I_c^I and I_c^{II} are the magnitudes of the first and second harmonic components respectively. These depend on the relative orientation of the $d_{x^2-y^2}$ -wave superconductors on either side of the junction [15]. In addition, the magnitudes of I_c^I and I_c^{II} are temperature dependent [16], and can possibly even change sign.

In an asymmetric 45° grain boundary junction between two $d_{x^2-y^2}$ -wave superconductors, the first harmonic I_c^I is expected to disappear by symmetry [14] (although in practice it does not disappear completely, due to microscopic or nanoscale faceting of the boundary [19]). The second harmonic I_c^{II} then becomes prominent, and has been observed in recent studies [17, 18].

Following these discoveries, we began to understand the strange behaviour exhibited by our HTS QRNT devices. We were able to model the distribution of trapped flux, including the bifurcation at 35 K, using a CPR with a temperature-dependent second harmonic term [13].

4.7 Implications for future devices

The discovery of a strong temperature dependence of the first and second harmonics of the CPR is bad news for thermometry with HTS QRNT devices. As mentioned in Chapter 3, the QRNT relies on having a superconducting switch whose critical current is relatively independent of temperature so that the flux distribution is not affected by changes in I_c . This means that future QRNT devices must not contain high angle HTS Josephson junctions. HTS grain boundary junctions are ruled out. Even low angle grain boundary junctions can exhibit unusual CPRs due to microscopic faceting along the boundary, which causes some facets to have a high misorientation angle.

The HTS research has not been entirely misguided, however. The essential operational techniques of the QRNT have been demonstrated, and there is no reason to believe they cannot be transferred to an LTS device with a conventional sinusoidal CPR.

Chapter 5

Future Directions

There are two possible future directions for developing a working QRNT:

- Develop an LTS device for use at sub 1 K temperatures. Proof of principle has already been demonstrated with HTS devices. An LTS device could be used to help resolve discrepancies between the low temperature scales of PTB and NIST.
- Develop a charge conjugate device in which the charge state of a metallic island is sampled repeatedly by a sensitive electrometer such as an RF single electron transistor (SET). This type of device would be much smaller than the flux QRNT, and could in principle operate at much higher temperatures. A feasibility study would be a sensible first step.

Further development of HTS devices seems unwise for the moment, due to ongoing uncertainties arising from the lack of a microscopic theory. A MgB₂ device is not currently viable due to lack of Josephson fabrication technology.

5.1 The LTS QRNT

An LTS QRNT device would most likely be fabricated from Nb ($T_c = 9$ K) and would operate at temperatures in the range 0.1–1 K. An Al device would be an alternative option, but the lower T_c (1 K) would restrict its temperature range to < 0.1 K due to the requirement that the critical current of the superconducting switch should vary slowly with temperature. A Nb device operating in the range 0.1–1 K would require an inductance of ~ 1 μ H. This could be realised with a multitrans coil a few mm in diameter with around 100 turns.

5.2 The charge QRNT

The quantum conjugate variable to the magnetic flux Φ is the electric charge Q . Therefore, an analogous device to the flux QRNT may be envisaged in which the discrete charge on a metallic island is sampled repeatedly using a sensitive detector such as an RF SET. The charge version of the QRNT would have to satisfy the

condition

$$T_{\max} \ll \frac{e^2}{2k_{\text{B}}C}, \quad (5.1)$$

where $T_{\max}$ is the maximum operating temperature and C is the island capacitance. To enable $T_{\max} \sim 300$ K requires $C \ll 10^{-18}$ F. This is the capacitance of a nanoscale sphere of diameter 50 nm, well within the fabrication range of e-beam lithography. The absolute temperature of the device would be determined from the expression

$$\frac{1}{2}C\langle V(t)^2 \rangle = \frac{1}{2}k_{\text{B}}T. \quad (5.2)$$

Unlike the flux QRNT, the charge QRNT would be of nanometre scale and would not rely on cryogenic operation, so, in principle, its use at room temperature and above would be possible. In practice this would rely on a room temperature readout device such as a carbon-nanotube-based SET [20].

Chapter 6

Conclusions

The QRNT is a unique idea for a primary, quantum thermometer. Preliminary research has demonstrated its feasibility, although the initial direction taken in pursuing HTS devices has proved erroneous. There is still plenty of scope for the development of an LTS device for operation at temperatures below 1 K, or a charge-based device for operation at higher temperatures. The novelty of the QRNT can be summarised as follows:

- It behaves as a single, isolated quantum entity, sampled repeatedly over time. This distinguishes it from the majority of thermometers which rely on variables averaged over a vast ensemble of interacting particles.
- It is dissipationless. Because there is no heat input during measurement, the QRNT is immune to self-heating.
- It is a primary thermometer whose parameters can be optimised for operation over a range of temperatures.
- Its discrete nature means that its precision is limited only by statistics and not by bandwidth.

The QRNT is an excellent example of portable, quantum technology based on fundamental constants, thus satisfying the primary objectives of the NMS Quantum Programme. It is unique to NPL and occupies a niche in the field of noise thermometry. Therefore, its further development would enhance NPL's international standing amongst other NMIs and academic institutions. At present it is not considered a priority for the Thermal Programme, but it is a promising idea and should be kept in mind for the future.

Acknowledgements

The author would like to acknowledge the detailed experimental work performed by R. A. M. Lee and many illuminating discussions with J. C. Gallop and A. Ya. Tzalenchuk.

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