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**Applications of
quantum entanglement
to metrology**

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Applications of Quantum Entanglement to Metrology

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ABSTRACT

Quantum entanglement, the inseparable entwinement of particles at the atomic scale, has emerged from the realms of theoretical debate and now has the potential to revolutionize many areas of science and technology. Entanglement-based technologies are truly disruptive: they rely on the quantum nature of matter to perform functions which are intrinsically inaccessible to technologies based on macroscopic devices ruled by the laws of classical physics. The use of entanglement in systems relevant to NPL, where there is already a technical capability such as for atomic particles, photons and superconducting qubit devices, is reviewed. Gains in precision metrology are assessed and recommendations made on where entanglement may benefit current and future projects in the NMS Quantum Metrology Programme.

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Approved on behalf of the Managing Director, NPL
by Dr. J. T. Janssen, Knowledge Leader, Time and Quantum Detection Team

Executive summary

Quantum entanglement, the inseparable entwinement of particles at the atomic scale, has the potential to revolutionize many areas of science and technology. Entanglement-based technologies are truly disruptive: they rely on the quantum nature of matter to perform functions which are intrinsically inaccessible to technologies based on macroscopic devices ruled by the laws of classical physics.

It is of paramount importance that a world-leading metrology laboratory such as NPL plays a leading role in the research and development of quantum-enhanced metrology applications. The field of QEM applications is vast and choices will have to be made where best NPL can make a contribution to these. In this respect NPL already has a considerable activity in optical clocks, solid state systems, and photonic applications. There are good reasons why we should focus on these technologies.

- Trapped ions are widely considered to be the only system which satisfies all the DiVincenzo criteria at present. The first real quantum computer is likely to be based on this technology. Entanglement enhanced trapped ion systems will directly lead to improved optical clocks and feed into timing and positioning applications.
- Solid state systems are considered to be the most scalable of all technologies. It offers the possibility for integration with existing classical computational hardware and is supported by mature background technology. In addition the critical mass of the semiconductor industry is enormous and fast progress can be expected now that the ultimate limit of Moore's Law for traditional CMOS technology comes into view.
- Photonic systems such as quantum cryptography are closest to market and the UK in particular has a very strong presence in this field.
- Cavity QED is the most promising vehicle for converting between static and flying qubits.

The principal recommendation of this report is to put these three topics together in one theme in the NMS 2007-2010 Quantum Metrology Programme with the title Quantum Information Processing and Communication (QIPC). There is a great deal of overlap between technologies in this field with similar underpinning technological requirements (nanotechnology, theory, cavity QED *etc.*). These topics have a similar customer focus and this grouping will be well-aligned with European Networks and UK initiatives. No winning technology has as yet been identified, so it will be important to maintain a strong presence in all these areas *via* strategic collaborations.

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Chapter 1

Introduction

Entanglement, the inseparable entwinement of particles at the atomic scale, is perhaps the most bizzare and counter-intuitive prediction of quantum mechanics. For much of the twentieth century, entangled states of matter were of interest only in the most abstruse discussions of the philosophical implications of quantum theory, yet now, at the beginning of the twenty-first century, a whole field of technologies based on quantum entanglement is emerging. These technologies are truly disruptive: they rely on the quantum nature of matter to perform functions which are intrinsically inaccessible to technologies based on macroscopic devices ruled by the laws of classical physics. Moreover, they encourage a new way of thinking about the physical world, in which the role of information is paramount.

Entanglement provides the route to quantum-enhanced measurement, pushing metrology to the fundamental Heisenberg limit. Already this can be seen in applications as diverse as nano-scale interferometry and lithography, atomic frequency standards, secure communications and clock synchronisation. On the other side, new technologies based on entanglement will require, as yet non-existent, metrology and the development of measurement standards and practices.

Seventy years ago, when Einstein, Podolsky, and Rosen (EPR) published their famous paper which discussed the strange implications of quantum mechanics for the accepted ideas of causality and physical reality [1], and Schrödinger coined the term *entanglement* to describe the lack of factorability of the states discussed by EPR [2], the concept of quantum entanglement lay in the realm of metaphysical discussion. Forty years ago, J. S. Bell showed how these concepts could be tested by devising real experiments with entangled states which could distinguish between the predictions of quantum mechanics and so-called hidden variable theories (Bell's inequalities) [3]. Twenty years ago, Aspect's experiment [4] provided the first conclusive laboratory test of Bell's inequalities, supporting the predictions of quantum mechanics.

Around the time of Aspect's experiment, Richard Feynman proposed that the best approach to the calculation of the behaviour of a complex quantum system would be to emulate it using another quantum system, giving birth to the concept of a *quantum simulator* [5]. This idea was extended a few years later by David Deutsch, who introduced the idea of the *quantum computer*, a device which would exploit quantum effects including entanglement to perform calculations which are unfeasible on a classical computer [6].

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Now we are at the point where laboratory experiments on entangled states of atoms, photons, and semiconductor devices are providing demonstrations of technologies. Figure 1.1 illustrates the evolution of some of the leading areas, showing how they have grown out of already-established quantum technologies, which have themselves given rise to quantum standards.

1.1 Entanglement for metrology

During the past few years, theorists have begun to work out ways to use entanglement and other extraordinary quirks of the quantum world to tackle important technological problems. Scientists have begun to experimentally test a few of those schemes, and some of them seem feasible. Although quantum computers are still considered to be a far-in-the-future goal, the first real applications of entanglement lie in the field of metrology.

The potential payoffs are many. Entanglement and other quantum weirdness may boost the accuracy of radar, Global Positioning System (GPS) receivers, and other navigation devices. It may improve manufacturers' ability to lay down tiny structures on microchips, expedite explorations for oil, improve the vision of telescopes, and increase the accuracy of atomic clocks and many other optical metrology techniques. Quantum technology might even enable us to see objects without actually looking at them and without being seen ourselves. Quantum entanglement in solid state environment (*i. e.* superconducting Josephson junctions, single electron transistors *etc.*) are still in their infancy, however, this field is making rapid progress. As scientists learn more about how to harness entanglement, new applications keep emerging all the time. The most recent of these is the teleportation. Teleportation is the name given by science fiction writers to the feat of making an object or person disintegrate in one place while a perfect replica appears somewhere else. Whilst transporting matter from one location to another without passing through intermediate locations is pure fantasy, quantum teleportation, the exact replication of a quantum state at a remote location, has been demonstrated by several groups and has important applications in quantum information processing.

Quantum entanglement can be exploited to circumvent the “fundamental” limits imposed on measurements by quantum projection noise. In any standard measurement, the signal-to-noise ratio (S/N) improves with the number of particles contributing to the measurement (N_p) as $1/\sqrt{N_p}$. However, in the instance where the “particles” are in a quantum-mechanically entangled state, the S/N scales as $1/N_p$, (an enhancement of $\sqrt{N_p}$) providing a route to realising the Heisenberg limit of the measurement. Therefore, the creation and control of entanglement can be exploited to achieve higher precision in a range of measurement problems. It must be noted that the “creation and control of entanglement” is extremely difficult due to the very fragile nature of such quantum states (this will be discussed in more detail later). Quantum effects have been used in metrology for many decades, however, the use of *quantum entanglement* is a completely new paradigm and the term *Quantum-Enhanced Metrology* has been coined to identify it.

1.2 Metrology of entanglement

Measurement takes on a different meaning in the quantum regime; some of the SI base units are no longer the appropriate units. Quantities such as current and flux are replaced by charge and phase. Other quantities become important to characterise quantum states, including entangled states, for instance concurrence, visibility, and fidelity. These are dimensionless measures of the quantum system; they will be used to define the specification of future quantum devices, replacing parameters such as phase noise, flux noise, *etc.*

Currently, the most advanced applications are in quantum communications. Systems are already commercially available which are claimed to be secure against eavesdropping by means of certain less-exotic features of quantum mechanics. Beyond this experiments have already shown that entanglement mechanisms improve these so-called quantum cryptography systems. The market for quantum cryptography systems is by some predicted to be as large as £10 bn/year in 10 years' time. A number of city banks have shown a keen interest in this technology; Toshiba Research Europe is running the first trials this year. At present there is no dedicated metrology which supports this technology. The characterisation and validation of these security-critical applications is of the utmost importance in order to ensure a successful exploitation and take-up. Metrology laboratories have to develop specific capabilities to characterise single-photon and entangled-photon sources and detectors which should be underpinned by fundamental research and development.

Quantum computation holds out tremendous promise for efficiently solving some of the most difficult problems in computational science, such as integer factorization, discrete logarithms, and quantum simulation and modeling that are intractable on any present or future conventional computer. New concepts for quantum computation implementations, algorithms, and advances in the theoretical understanding of the physics requirements for QC appear almost weekly in the scientific literature. This rapidly evolving field is one of the most active research areas of modern science, attracting substantial funding that supports research groups at internationally leading academic institutions, national laboratories, and major industrial-research centres. Start-up quantum-information companies are already in operation. A diverse range of experimental approaches from a variety of scientific disciplines are pursuing different routes to meet the fundamental challenges involved. Yet experimental achievements in QC, although of unprecedented complexity in basic quantum physics, are only at the proof-of-principle stage in terms of their abilities to perform quantum computation tasks. It will be necessary to develop significantly more complex quantum information processing (QIP) capabilities before quantum computer-science issues can begin to be experimentally studied. Realising this potential will require the engineering and control of quantum-mechanical systems on a scale far beyond anything yet achieved in any physics laboratory. This runs counter to the tendency of the essential quantum properties of quantum systems to degrade with time (*decoherence*). Yet, it is known that it should be possible to reach the *quantum computer-science test-bed regime* if challenging requirements for the precision of elementary quantum operations and physical scalability can be met. Although a considerable gap exists between these requirements and any of the experimental

Table 1.1: Estimates of the current worldwide annual investment in quantum information systems research. (Adapted from [7].)

Country	Annual investment		Euro equivalent	
USA	100 M	USD	75 M	EUR
Japan	2.5 b	YEN	20 M	EUR
Canada	20 M	CAD	12 M	EUR
Europe	8 M	EUR	8 M	EUR
Australia	10 M	AUD	6 M	EUR
Singapore	8 M	SGD	4 M	EUR
China	4 M	EUR	4 M	EUR
Total			130 M	EUR

implementations today, this gap continues to close.

1.3 International, European, and UK investment

International funding for research on applications of quantum entanglement, in particular to quantum information systems, has undergone a rapid increase over the last decade and is currently in the region of 130 million euros per year (see table 1.1). This level of funding is reflected in an exponential growth in the number of peer-reviewed scientific publications in this area, reaching approximately 900 in 2002 [7]. It is noteworthy that although European funding accounts for only 6% of the total, publications from European research groups accounted for nearly 40% of the number of publications.

Research on quantum information systems at the European level has been strongly supported by the EU. The 5th Framework funded 25 projects a total cost of 41 M euros, of which 31 M euros came from the EU and the remaining 10 M euros was matching funding from national sources. Within the current 6th Framework Programme, the Information Society Technologies activity provides support at a similar level through its Quantum Information Processing and Communications area.

Within the UK, EPSRC is investing £9 M over five years in an Interdisciplinary Research Collaboration on quantum information processing (QIP-IRC) [8]. This brings together the leading UK university research groups, NPL, and the leading industrial R&D groups:

- Tim Spiller at HP Labs(Bristol) [9],
- David Williams at Hitachi Laboratory (Cambridge) [10],
- John Jefferson at QinetiQ (Malvern) [11],
- Yuichi Harada at NTT Europe (Oxford) [12],
- Andrew Shields at Toshiba Research Europe (Cambridge) [13].

1.4 Current status of research at NPL

NPL has currently running projects in three critical technologies for the exploitation of quantum entanglement: trapped ions, photons, and superconducting devices.

Ions

NPL currently conducts experimental research in quantum state preparation and manipulation using trapped ions. This work has been funded to date by NPL's strategic research programme, at the level of one PhD scientist and one research student, for the past seven years. This research programme has established an experimental facility and demonstrated capabilities in several techniques relevant to entanglement and QIP with a single trapped ion:

- Optical Ramsey spectroscopy, showing quantum-mechanical coherence and the effects of quantum phase.
- Quantum state preparation for electronic and motional states.

This research activity has resulted in conference presentations, invited seminars, and five papers published in peer-reviewed journals [14, 15, 16, 17, 18].

In addition, QIP-relevant techniques developed in this research project are now available in-house for improving core frequency standards experiments. Recently, an additional Visiting Strategic Research Fellow has been appointed to work on the current project (active through June 2008), which seeks to demonstrate quantum entanglement using a micro-fabricated ion trap.

A further important aspect of this research is NPL's participation as a partner in the EU's quantum information research projects. QUBITS (Jan 99 – Dec 02) and QGATES (Jan 03 – Dec 05) were both three-year projects running in series, with a combined value to NPL of £200k. NPL is a partner in the SCALA integrated project (worth £120k to NPL over 4 years, due to start Nov 05), and also in the proposed MICROTRAP project (currently in negotiation, worth £270k to NPL over 3 years, anticipated start date of Mar 05).

Figure 1.2, not prepared specifically for this report, provides a pictorial roadmap for the development of QIP applications using trapped ions within DEM-TQD's Time, Frequency, and the Metre group. It illustrates the experimental milestones and metrological outputs along the road to quantum information processing with trapped ions, together with the underpinning technologies which are required.

Photons

NPL has three areas of research directly relevant to entangled photons; these have recently been brought together by the project entitled "Entangled photon metrology", within the current quantum metrology research programme. These three topics are:

- generation and detection of indistinguishable (transform-limited) single photons from a semiconductor quantum dot

Qubit :Entangled ions

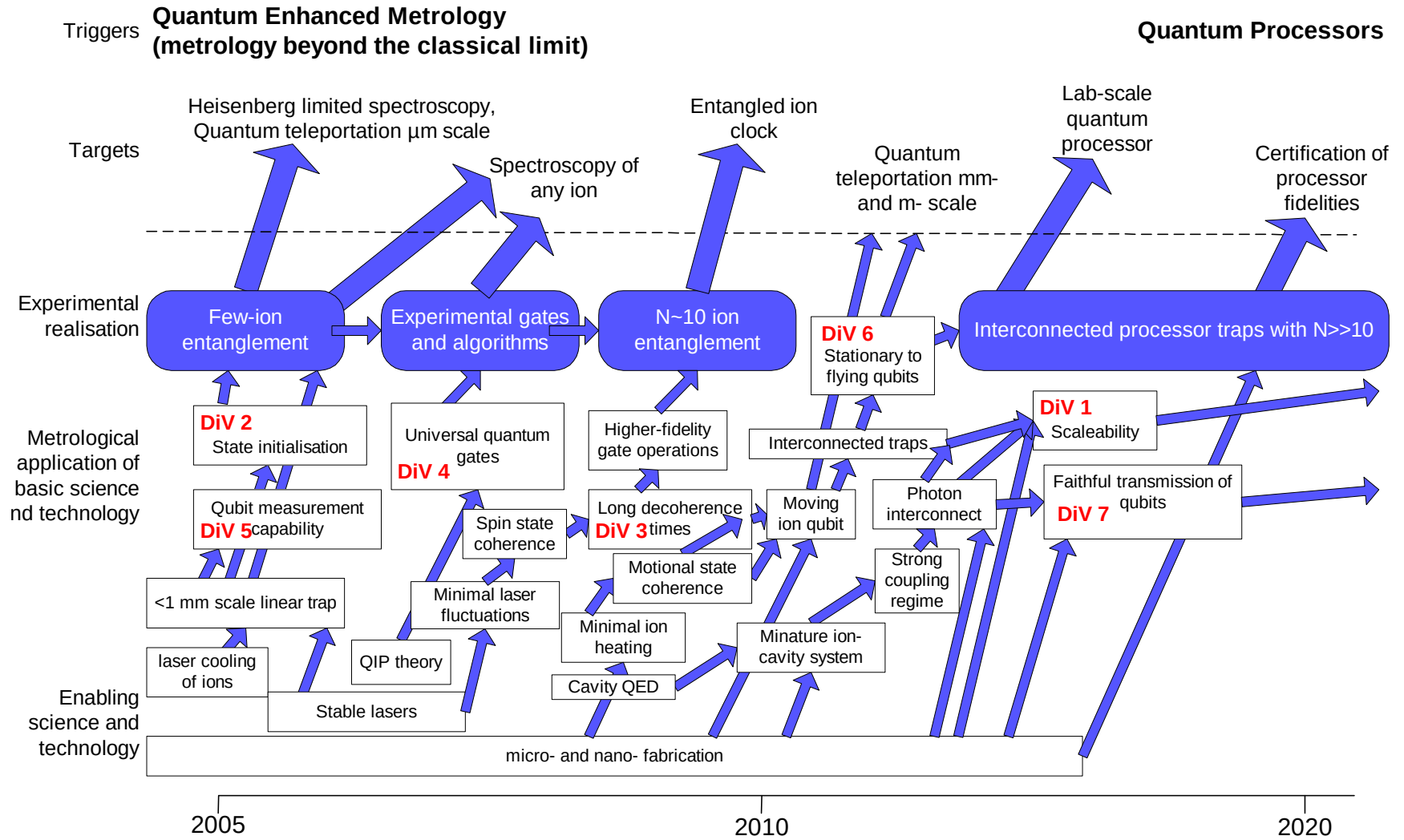


Figure 1.2: Roadmap for entangled ions.

- generation and detection of single photons from a SETSAW (single electron transistor surface acoustic wave) device
- generation of entangled photons *via* SPDC (spontaneous parametric down-conversion)

The first of these topics grew from a two-year Strategic Research Project (ended Aug 2004) that demonstrated single-photon emission from a single defect in room-temperature synthetic diamond. The second topic is very closely related to the research of the “SAW quantum transport metrology” Single electron current standard project, also within the current Quantum Metrology Programme. The third topic has emerged from research in the Optical Programme on SPDC for generating correlated photons to be used in radiometric applications. The “Entangled photon metrology” project is £800k in value, and is collaborating with Imperial College London, the Cavendish Laboratory (Cambridge), and Toshiba Research Europe Ltd., in order to gain access to appropriate source fabrication facilities. The expected outcome of this project is the capability and infrastructure to generate, detect and quantify the parameters associated with sources of single and entangled photons.

DTI Micro and nano technology project (MNT) “Narrow band-gap spintronic devices for information technology and standards” targeted towards single photon sources in the 2 - μm range. Collaboration between Qinetiq, Hitachi, and NPL, total value is £1.4M, with NPL in for £200k. This is leveraged by a much larger activity on InSb transistors between Qinetiq and Intel .

Additionally, a further project within the current Quantum Metrology Programme, entitled “Single particle spectroscopy: towards yoctocalorimetry” and based on superconducting devices, is relevant because of the possibility to detect single photons at frequencies not accessible by conventional silicon-based detectors.

This work has resulted in the publication of a review article on single photon sources in *Contemporary Physics* [19].

Superconducting devices

NPL currently conducts experimental research in quantum state preparation and readout using superconducting quantum devices. This work forms part of the Quantronics project which has been funded to date by NPL’s quantum metrology programme at the level of two PhD scientists for the past 18 months (total value £640k over 3 years: April 2004 to April 2007). Achievements include the development of an experimental facility with the following capabilities:

- Readout techniques based on detection of the phase escape from a Josephson junction.
- Spectroscopy techniques based on microwave enhancement of the phase escape.

This research activity has resulted in conference presentations and invited seminars. Recent publications include refs. [20, 21, 22, 23].

Another related activity at NPL is research into quantum noise thermometry using devices based on superconducting flux qubits [24]. Current activity in this area

Quantum Detection

Support of Sensor Semiconductor and IT industries : Continuation of Moore's Law

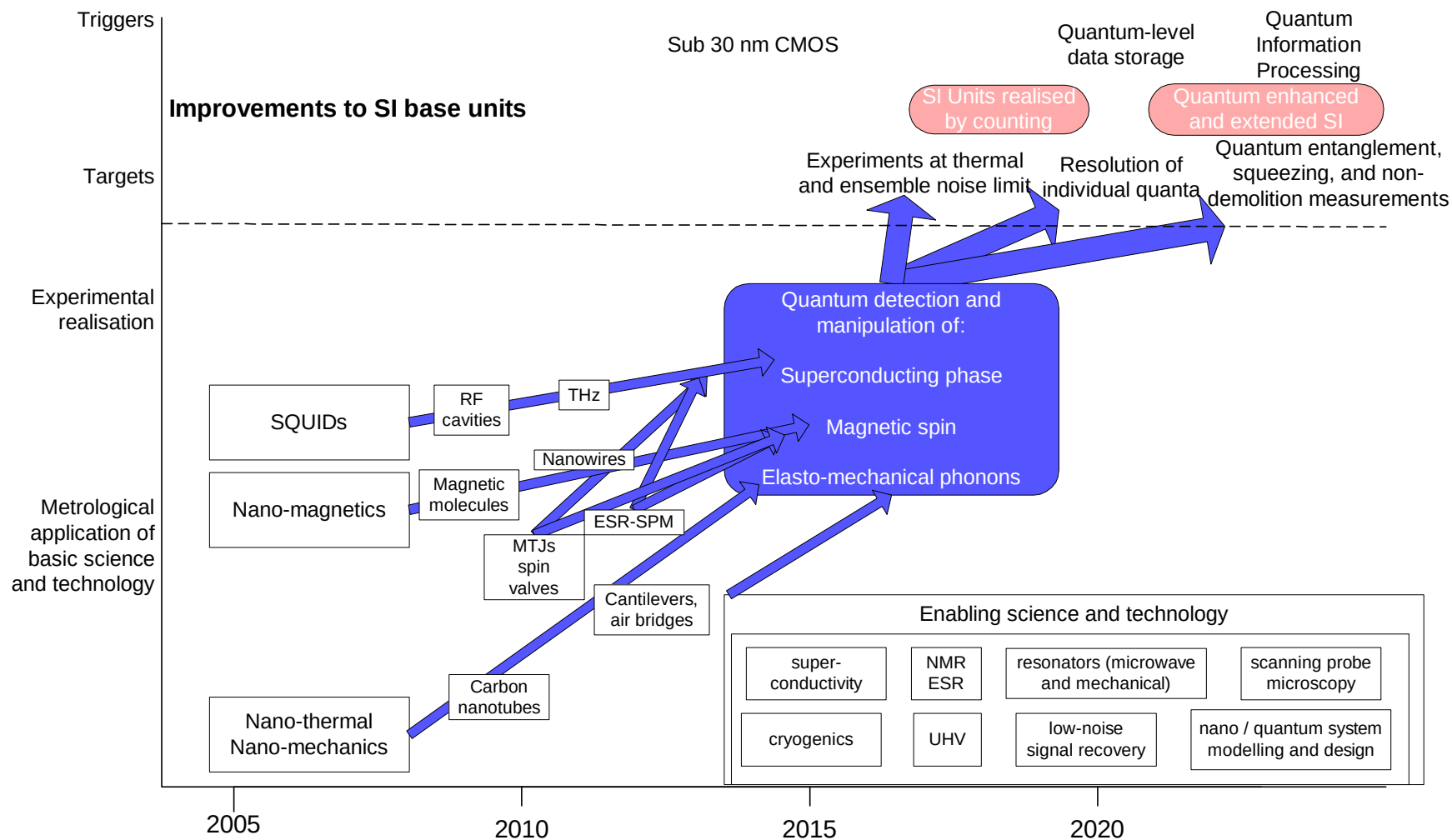


Figure 1.3: Roadmap for quantum detection.

is in the form of a study in the 2004-7 quantum metrology programme worth £35k. A feasibility project on superconducting qubits, in collaboration with NTT Basic Research Laboratories, Japan (worth £5k), is currently underway, with a view to applying to EPSRC or MNT for increased funding. Funding has also been obtained from NPL's strategic research programme for a study to design a superconducting cavity QED device which utilises entanglement of a superconducting qubit and resonant transmission line to perform quantum non-demolition measurements. This study is worth £13.5k and will run from February 2006 to April 2006. The results are expected to feed into a proposal to NPL's strategic research or quantum metrology programme for a future project on superconducting cavity QED devices.

Research on superconducting devices at NPL benefits from a solid background in superconducting microscopic and nanoscale devices, high- Q resonators and photonics, developed over several decades. Strong collaborations exist with research groups outside NPL who also perform research on superconducting quantum devices. These include groups at Chalmers University in Sweden, Jena in Germany, Naples in Italy, Royal Holloway University in London, NTT in Japan and the University of British Columbia in Canada.

Figure 1.3, prepared externally to this report, provides a pictorial roadmap for the development of QIP applications using superconducting devices within DEM-TQD's Quantum Detection group. It illustrates how the existing capabilities of this group complement each other and can be combined to provide not only improved quantum-based realisations and enhancement of the SI but also metrology in support of the sensor, semiconductor, and IT industries, as they move towards devices operating in the quantum régime.

1.5 Content of this report

The aim of this report is to emphasise areas in which research motivated by QIP has spin-offs which may benefit metrology and also areas in which QIP will itself require new metrology techniques.

Many of these examples discussed are drawn from QIP, which has been the dominant driver for the development of quantum-entanglement technologies; however, it is not the aim of this report to provide an overview of quantum computing *per se*.

Chapter 2 provides an introduction to the key concepts of quantum entanglement, introducing both the physics and the notation at the simplest level compatible with an understanding of the applications introduced in remainder of the report. These concepts are illustrated by a description of a *Bell experiment*, in which the validity of the quantum mechanical interpretation of the physical world is unambiguously demonstrated. The focus of this report is on quantum entanglement. Related quantum phenomena or processes such as squeezed number states (Fock states) or quantum non-demolition (QND) measurement schemes are only briefly touched upon. Aspects of these phenomena are discussed in a recent review paper by NPL authors (Mark Oxborrow and Alastair Sinclair) [19]. Quantum error correction, an important aspect of all quantum computing schemes and which uses entanglement as a resource, is also outside the scope of this report.

Subsequent chapters provide an illustrated review of techniques and applications

exploiting entanglement in a range of physical systems. No attempt has been made at an exhaustive review; the focus is on technologies in which NPL already has a capability (photonics, atomic systems, superconducting circuits). One or two key experiments are described in some detail as a means of introducing each technology. Experiments illustrating individual applications are only discussed in detail where they introduce distinctive techniques, the aim being to provide the reader with a broad overview of the possibilities opened up by entanglement and a flavour of the variety of experiments being performed.

Two reviews of QIP are freely-available on the web. The EU Information Society Technologies project on Quantum Information Processing and Communication (QIPC) has very recently produced a report “Strategic report on current status, visions and goals for research in Europe” [7], edited by Peter Zoller with contributions from the leading European researchers, including Rainer Blatt (Innsbruck), David Deutsch (Oxford), Artur Ekert (Cambridge), Nicolas Gisin (Geneva), Serge Haroche (Paris), Hans Mooij (Delft), Gerhardt Rempe (MPQ Garching), and Anton Zeilinger (Vienna). The same authors and others have contributed to a set of web review articles on the EU’s CORDIS (Community Research and Development Information Service) website [25]. The US Government’s Advanced Research and Development Activity (ARDA) Quantum Information Science and Technology Roadmapping Project has produced a similar report produced by a panel of, chiefly US, researchers, under the chairmanship of Richard Hughes (Los Alamos) [26]. Contributors to this report include David DiVincenzo (IBM), Paul Kwiat (University of Illinois), Seth Lloyd (MIT), and David Wineland (NIST). Appendix A, modified from similar material in the ARDA report [26], provides a summary of the leading research groups working with the technologies under discussion.

Chapter 7 draws together the material presented and sets out some recommendations for future activity within the NMS Quantum Metrology Programme.

Chapter 2

The quantum mechanics of entanglement

This chapter provides an overview of the quantum-mechanical framework of entangled systems, introducing concepts, terminology, and notation which will be used in subsequent chapters. These ideas are illustrated by a description of an EPR experiment which provides unambiguous confirmation of the standard interpretation of quantum mechanics. Quantum logic gates are an important tool in many applications of entanglement; much of chapters 3, 4 and 5 are concerned with practical implementations of these gates, the principles of which are outlined in section 2.3.

2.1 Quantum states and quantum bits

The state of a quantum-mechanical system is described by its *state vector* $|\psi\rangle$, which, like any ordinary vector, can be expressed as a linear combination of a set of orthogonal *basis states*. These basis states can be chosen arbitrarily, but are usually chosen in some convenient way that makes the system easy to understand. For instance, we often choose them to be *stationary states* (states that do not change with time, providing that the system is not disturbed by some external influence).

The form of the state vector depends on the nature of the system. In some cases the system can be described by a *discrete variable* like the spin angular momentum of an electron, which can take the values $+\frac{1}{2}\hbar$ (spin up) and $-\frac{1}{2}\hbar$ (spin down), or the polarisation state of a photon, which may be horizontal or vertical. In these cases the choice of basis states is straightforward. However, in other cases the system is described by a *continuous variable* like the position or momentum of a particle, and in such cases the system's state is more conveniently expressed as a *wavefunction*, e.g. $\psi(x)$ or $\psi(p)$, whose basis states consist of a set of orthogonal functions.

The thing that makes a quantum mechanical state different to a classical state is that we cannot determine it with certainty without making a measurement. However, the process of measurement disturbs the state itself. To explain this we mention two important postulates of quantum mechanics. Firstly, every observable is associated with a quantum mechanical operator. By an operator, we mean something that operates on the state vector to transform it in some way (since the state can be represented as a vector, an operator can be represented as a matrix), and by an ob-

servable we mean some real physical quantity, such as the spin of an electron. What this postulate really means is that if we choose the basis states of a system to be the eigenstates of an operator (for example, the eigenstates of the spin operator are the states spin up and spin down), then the eigenvalues will be the observable quantities (spin up has the eigenvalue $+\frac{1}{2}\hbar$ and spin down has the eigenvalue $-\frac{1}{2}\hbar$). Secondly, measurement of an observable collapses the state vector to one of its eigenstates, yielding the corresponding eigenvalue as the result of the measurement. What this postulate means is that, before we make a measurement of a physical quantity, we don't know whether the state $|\psi\rangle$ of the system is one eigenstate or another, or indeed, whether it is in a combination of several. However, after making a measurement, the system must be in a single eigenstate, so the measurement process destroys some of the information held in the quantum state.

An important operator that we will encounter a lot is the *Hamiltonian* $\hat{H}$, which determines how the system evolves with time. Often, the most important problem in studying a quantum mechanical system is working out how to write down its Hamiltonian. The effect of the Hamiltonian on the quantum state $|\psi\rangle$ is given by the Schrödinger equation

$$\hat{H}|\psi\rangle = i\hbar \frac{d}{dt}|\psi\rangle. \quad (2.1)$$

If the Hamiltonian itself is time independent, then its eigenvalues are the energy levels of the system and its eigenstates are stationary states.

2.1.1 Basis states and superposition states

The states of a system are defined relative to some external reference frame, described as the *quantisation axis*. For instance, spin is defined relative to the direction of an external magnetic field, with the spin up and spin down states corresponding to the spin lying parallel or antiparallel, respectively, to the field direction.

The basis states of a quantum-mechanical system are usually chosen to be the stationary states of the Hamiltonian in a particular external reference frame, the orientation of the reference frame being referred to as the *basis*. The basis states of a two-state system, such as electron spin or photon polarisation, can be labelled in a general (and suggestive) way as $|0\rangle$ and $|1\rangle$. If the external reference frame is changed the states $|0\rangle$ and $|1\rangle$ are no longer stationary states because the Hamiltonian has changed. In this new basis, the system can be described by a new set of stationary basis states which could be labelled $|-\rangle$ and $|+\rangle$.

These new basis states can be described in terms of the old basis states and *vice-versa*. Figure 2.1.1 illustrates the example of the polarisation states of a photon. The basis states are defined by the orientation of a polarising cube beam splitter (PBS), which forms part of a polarisation state analyser: reflected photons, measured by a single-photon detector D_R , are polarised in the interface plane of the PBS, whereas transmitted photons, measured by single-photon detector D_T , are polarised normal to the interface plane. Initially, the PBS is oriented such that its interface is vertical, defining basis states of vertical and horizontal polarisation $|0\rangle$ and $|1\rangle$. A photon which has been prepared with vertical polarisation will be detected at D_R , *i.e.* measured in the state $|0\rangle$ after the PBS. In this basis its state vector before the

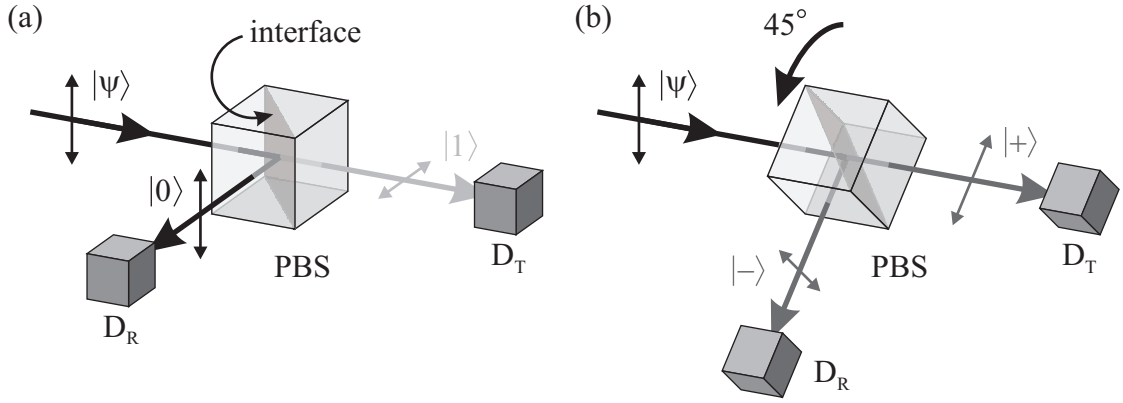


Figure 2.1: A polarisation state analyser consisting of a polarising beam splitter (PBS) and single-photon detectors D_R and D_T for reflected and transmitted photons respectively; the orientation of the PBS defines the basis states of polarisation. A single photon, prepared with vertical polarisation, is incident on the beam splitter. In basis (a), the PBS is oriented such that a vertically-polarised photon is reflected: it is in the state $|\psi\rangle = |0\rangle$. In (b), a change of basis has been made by rotating the PBS by 45° . In this new basis, a photon in state $|\psi\rangle$ is equally likely to be found in either of the basis states $|-\rangle$ or $|+\rangle$: it is in the superposition state $|\psi\rangle = \frac{1}{\sqrt{2}}(|-\rangle + |+\rangle)$.

analyser is $|\psi\rangle = |0\rangle$. If the PBS is rotated by 45° relative to this initial basis, there is an equal probability that the photon in polarisation state $|\psi\rangle$ will be detected by either D_R or D_T , *i.e.* that it will be measured either of the new basis states $|-\rangle$ or $|+\rangle$. The photon's state vector before the analyser is a *superposition* of the new basis states:

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|-\rangle + |+\rangle). \quad (2.2)$$

(The factor of $\frac{1}{\sqrt{2}}$ normalises the total probability of finding the photon in either state to unity, *cf.* equation (2.6) below.) Rotation of the analyser changes the basis of the measurement: the initial basis states $|0\rangle$, $|1\rangle$, are transformed into superpositions of the basis states in the rotated basis:

$$\begin{aligned} |0\rangle &\rightarrow \frac{1}{\sqrt{2}}(|-\rangle + |+\rangle), \\ |1\rangle &\rightarrow \frac{1}{\sqrt{2}}(|-\rangle - |+\rangle). \end{aligned} \quad (2.3)$$

Equally, the state vectors of photons measured in the states $|+\rangle$ or $|-\rangle$ in the rotated basis can be written

$$\begin{aligned} |+\rangle &\rightarrow \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle), \\ |-\rangle &\rightarrow \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle). \end{aligned} \quad (2.4)$$

The example of a change of basis by rotation of a photon polarisation analyser makes concrete the concept of a change of basis as corresponding to a rotation.

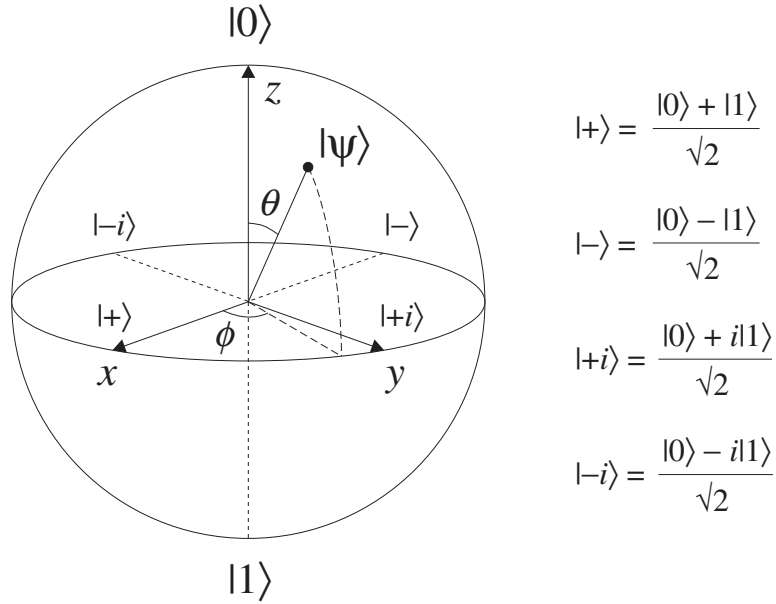


Figure 2.2: The state $|\psi\rangle = \cos \frac{\theta}{2} |0\rangle + e^{i\phi} \sin \frac{\theta}{2} |1\rangle$ can be visualised as a point on the surface of the Bloch Sphere. The basis states $|0\rangle$ and $|1\rangle$ are represented at the north and south poles, and at these points the phase ϕ has no measurable consequences. The states $|+\rangle$ and $|-\rangle$ lie in the equatorial plane, and here the phase is important because different values give rise to different measurement outcomes.

In this example, the analyser is rotated to a basis in which the horizontally-polarised photon has equal probability of being detected in state $|-\rangle$ or $|+\rangle$. More generally, the state vector $|\psi\rangle$ of a two-state quantum system is the superposition

$$|\psi\rangle = a |0\rangle + b |1\rangle \quad (2.5)$$

of the two basis states, where a and b are the *probability amplitudes* of the states: the probability of finding the system in state $|0\rangle$ is $|a|^2$ and the probability of finding it in state $|1\rangle$ is $|b|^2$. Since the total probability of finding the system in any state at all must be unity, the probability amplitudes must obey the normalisation condition

$$|a|^2 + |b|^2 = 1. \quad (2.6)$$

The probability amplitudes can be complex numbers. By writing these out explicitly, equation (2.5) can be expressed in the more general form

$$|\psi\rangle = \cos \frac{\theta}{2} |0\rangle + e^{i\phi} \sin \frac{\theta}{2} |1\rangle, \quad (2.7)$$

where ϕ is known as the *relative phase angle* and an overall phase, which is not usually physically significant, has been neglected. The form of this equation makes it natural to visualise the state in spherical polar coordinates as a point on the *Bloch sphere*, figure 2.2. This picture is well adapted for the visualisation of spin states, as the $|0\rangle$ and $|1\rangle$ states are at the poles of the Bloch sphere, separated by 180° . However, it can also be used to visualise photon states by observing that $\theta/2$ is equivalent to the angle of rotation of the polarisation analyser.

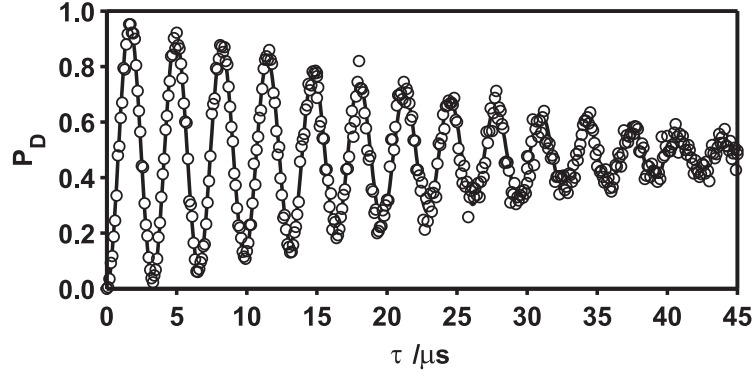


Figure 2.3: Rabi oscillations on the $^2S_{1/2} - ^2D_{5/2}$ optical transition in an NPL Sr^+ single ion spectroscopy experiment. The effect of decoherence is evident in the exponential damping of the oscillations.

In the language of quantum information processing, the quantum mechanical system is called a *qubit* (quantum bit), and the qubit state is represented by the vector $|\psi\rangle$. The result of a measurement on the qubit state will yield either a result assigned the value 0 (system in the state $|0\rangle$) or a result assigned the value 1 (system in the state $|1\rangle$). In this respect the qubit is analogous to a bit in classical information processing. However, the difference is that, between measurements, the qubit can be in a quantum superposition state, allowing computational operations to act on both basis states at once: something that is impossible in a classical computer.

2.1.2 Coherence and decoherence

A qubit can be placed in a superposition state either through a rotation of the basis, *i. e.* by changing the direction of the quantisation axis. As will be shown, rotation of the basis is particularly convenient for photons and for some solid-state spin systems. In atomic systems, the qubit states are often electronic energy levels of the atom; rotation of the basis is not a feasible process. The alternative method to create a superposition state is to drive a transition between the qubit states using electromagnetic radiation at the resonant frequency $\omega = \omega_1 - \omega_0$, where $\hbar\omega_{0(1)}$ is the energy of state $|0\rangle$ ($|1\rangle$). In the steady state, the qubit undergoes coherent *Rabi oscillations* between the basis states:

$$|\psi(t)\rangle = \cos(\tfrac{1}{2}\Omega t)|0\rangle + \sin(\tfrac{1}{2}\Omega t)|1\rangle, \quad (2.8)$$

where the *Rabi frequency* Ω is determined by the strength of the coupling of the basis states by the electromagnetic field. Rabi oscillations can be visualised as a cycling of the qubit on a great circle about the Bloch sphere; figure 2.1.2 illustrates a physical example: oscillations between the ground $^2S_{1/2}$ and metastable $^2D_{5/2}$ levels of a single trapped Sr^+ ion, driven by resonant laser radiation at 674 nm. Figure 2.1.2 is constructed by applying the laser radiation for a range of durations τ ; the probability of finding the ion qubit in the $^2D_{5/2}$ state after time τ is determined by averaging over a number of measurements. This measurement is destructive: it can only find the qubit in one or other of its basis states, thereby interrupting the evolution

of the superposition state. This illustration, using a single qubit, emphasises the distinction between a superposition state and a statistical mixture: it is not the case that half of one's ensemble is 'really' in state $|0\rangle$ whilst the other half is 'really' in state $|1\rangle$, a fact simply confirmed by the measurement; rather, each qubit is in a quantum superposition until the measurement is made.

In principle, the superposition state is *coherent*: there is an unchanging phase relationship between the two basis states. In practice, there will be *decoherence*: no quantum system is totally isolated from its environment. Sources of decoherence can be intrinsic, arising from *relaxation* of the system; for example, the metastable $^2D_{5/2}$ level of the Sr^+ ion undergoes spontaneous decay to the ground state with a natural lifetime of about 0.4 s. Alternatively, environmental fluctuations, such as the instability of the applied magnetic field in solid-state systems, can lead to fluctuations in the Rabi frequency. This effect is known as *dephasing*.

The term decoherence is often used rather loosely to refer to either relaxation or dephasing or both. However, to be really sure how the quantum information stored by a qubit decays with time we really need to know both the relaxation time and the dephasing time. The former can be measured by preparing the qubit in its excited state and then recording the occupancy of the excited state as a function of time. The latter can be measured by performing a Ramsey fringe experiment and fitting an exponential decay envelope to the oscillations. The review article by Zurek provides a comprehensive discussion of this subject [27].

2.1.3 Single qubit gates

In order to perform quantum computations, we need to be able to change the states of the qubits storing the information. In a classical computer there is only one transformation that can be performed on a single bit: the NOT gate. There is a quantum equivalent of the NOT gate, which transforms the state $|0\rangle$ into the state $|1\rangle$ and vice versa. In many physical systems, this transformation is produced by the interaction of the qubit with a pulse of electromagnetic radiation, for example, a microwave pulse in the case of a superconducting qubit or a pulse of laser light in the case of an atomic system. A pulse of radiation of duration $t = \Omega/\pi$ will swap the states in the superposition *cf.* equation (2.8), with an inversion of phase:

$$a|0\rangle + b|1\rangle \rightarrow -i(b|0\rangle + a|1\rangle). \quad (2.9)$$

This interaction is then called a π -pulse; it corresponds to a rotation in θ of 180° (π radians) on the Bloch sphere. However, an infinite number of other single qubit gates are also possible, because we can transform the state $|0\rangle$ into any superposition state $|\psi\rangle = a|0\rangle + b|1\rangle$.

A particularly useful single qubit gate is the Hadamard gate,

$$|0\rangle \rightarrow \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \quad \text{and} \quad |1\rangle \rightarrow \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle), \quad (2.10)$$

which is used to create the superposition states required by many quantum algorithms. The Hadamard gate corresponds to a rotation of θ by 90° on the Bloch sphere, and hence the physical implementation of the Hadamard gate is in many cases a $\pi/2$ -pulse of microwave or laser radiation, of duration $t = \Omega/2\pi$.

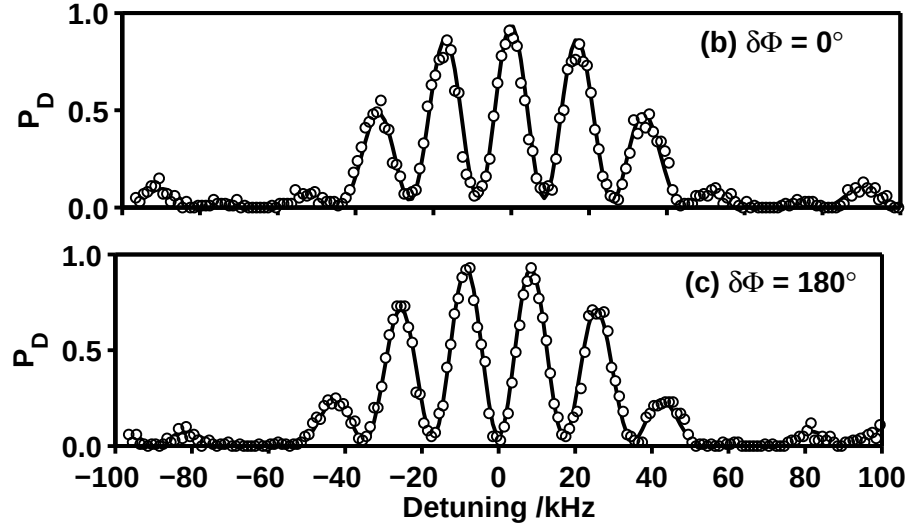


Figure 2.4: Ramsey spectroscopy on the $^2S_{1/2} - ^2D_{5/2}$ optical transition of a single Sr^+ ion at NPL. (a) Conventional Ramsey fringes. (b) Inverted fringes with 180° change of phase between the Ramsey pulses.

The use of the Hadamard gate can be illustrated by Ramsey's method of separated oscillatory fields, which forms the basis of the caesium atomic clock. The caesium ground state hyperfine transition is interrogated using two coherent microwave fields of equal duration τ and Rabi frequency Ω , which are separated by a period T of free precession. Each caesium atom can be thought of as a qubit, composed of the two hyperfine levels of the ground state, which for consistency will be labelled $|0\rangle$ and $|1\rangle$. The caesium atoms are prepared in one hyperfine level, $|\psi_0\rangle = |0\rangle$. In the first step of the Ramsey interaction, each atom is placed in the superposition state $|\psi_1\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ by a $\pi/2$ -pulse of microwave radiation. During the precession time T , the relative phase ϕ of the superposition state is free to evolve, until a second $\pi/2$ -pulse of microwave radiation completes the interaction.

In conventional Ramsey spectroscopy, the second $\pi/2$ -pulse is in phase with the first. The frequency of the driving field is stepped through resonance with the qubit transition frequency. The driving field will only be exactly a $\pi/2$ -pulse on resonance, in which case the superposition state does not evolve during the precession time and the second $\pi/2$ -pulse simply completes the inversion of the states: effectively, they combine to be equivalent to a single π -pulse. When the driving field is detuned from resonance, a relative phase will be accumulated during the precession time T , this can be pictured as precession around the equator of the Bloch sphere. The result is the familiar Ramsey fringe pattern seen in figure 2.1.3(a). Alternatively, the phase of the second $\pi/2$ -pulse can be stepped by 180° relative to the first, resulting in inverted Ramsey fringes, figure 2.1.3(b).

In a caesium atomic clock, the measured Ramsey fringes are obtained from an ensemble average over a large number of atoms. To emphasise the truly quantum character of the Ramsey interaction, the data shown here (figure 2.1.3) has been obtained on a single qubit, an optical transition in a single Sr^+ ion, in an experiment performed at NPL [16].

2.2 Entanglement

A quantum system can, of course, be made up of more than one particle. A classical many-body system can be broken down into its component parts; one might expect the same to be true of a quantum system: the wavefunction of the system could be expressed as the product of the wavefunctions of its component parts

$$|\psi\rangle_{\text{system}} = |\psi\rangle_1 |\psi\rangle_2 \dots |\psi\rangle_N. \quad (2.11)$$

Erwin Schrödinger realised that in quantum mechanics one could obtain states which could not be factorised in this way. He introduced the term ‘Verschränkung’ (*entangled*) to describe such states and called it “*the characteristic feature of quantum mechanics*” [2].

The simplest entangled states one can consider are an entangled states of a pair of two-state quantum systems (qubits), each of which has the basis states $|0\rangle$ and $|1\rangle$. Labelling the two systems A and B , one can easily envisage factorisable states in which each system is in a stationary state, *e.g.* $|\psi\rangle = |0\rangle_A |0\rangle_B$, or in which both systems are in superposition states, *e.g.*

$$|\psi\rangle = \frac{1}{2} (|0\rangle_A + |1\rangle_A) (|0\rangle_B + |1\rangle_B). \quad (2.12)$$

However, the state

$$|\psi\rangle = \frac{1}{\sqrt{2}} (|0\rangle_A |1\rangle_B + |1\rangle_A |0\rangle_B) \quad (2.13)$$

cannot be factorised as the product of two one-particle wavefunctions; it is entangled. The degree of entanglement can be quantified using the *concurrence*. The most general two-qubit wavefunction is

$$|\psi\rangle = \alpha|0\rangle_A|0\rangle_B + \beta|0\rangle_A|1\rangle_B + \gamma|1\rangle_A|0\rangle_B + \delta|1\rangle_A|1\rangle_B, \quad (2.14)$$

for which the concurrence is defined as $C = 2|\alpha\delta - \beta\gamma|$. C ranges from 0 for a completely unentangled state to 1 for a *maximally entangled* state [28]. An alternative measure is the *fidelity* of creation of an entangled state. This is defined as the overlap integral of the ideal entangled state $|\psi\rangle_{\text{ideal}}$ with the measured entangled state, represented by its density matrix ρ ,

$$F = {}_{\text{ideal}}\langle\psi|\rho|\psi\rangle_{\text{ideal}}. \quad (2.15)$$

A maximally-entangled two-particle entangled state, equation (2.13), is called a *Bell State*, after J. S. Bell, who used the properties of such states to distinguish between the predictions of quantum mechanics and of so-called hidden-variable theories (see section 2.2.2). The four Bell states

$$\begin{aligned} |\psi_{\pm}\rangle &= \frac{1}{\sqrt{2}} (|0\rangle_A |1\rangle_B \pm |1\rangle_A |0\rangle_B) \\ |\phi_{\pm}\rangle &= \frac{1}{\sqrt{2}} (|0\rangle_A |0\rangle_B \pm |1\rangle_A |1\rangle_B) \end{aligned} \quad (2.16)$$

form a complete orthogonal basis set of maximally-entangled states.

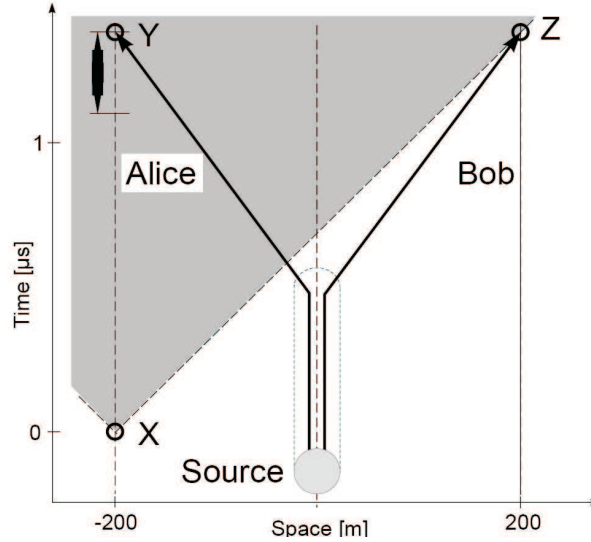


Figure 2.5: Spacetime diagram of a Bell experiment performed in Anton Zeilinger’s group at the University of Vienna. Entangled photons from the source are transmitted by optical fibres to Alice’s polarisation analyser at Y and Bob’s at Z . The shaded region, including the duration of Alice’s measurement process (indicated by the black bar) is invisible to Bob during his measurement. Reprinted figure with permission from G. Weihs *et al.*, Phys. Rev. Lett. **81**, 5039 (1998). Copyright (1998) by the American Physical Society.

2.2.1 Example: entangled photons

The consequences of this entanglement can be brought out by considering a concrete example, illustrated in figure 2.5. A pair of entangled photons can be generated in the Bell state of equation (2.13) by the process of parametric down-conversion. This process will be discussed in detail in chapter 3, here it is sufficient to view it as providing a source which emits pairs of entangled photons with (in this example) orthogonal polarisations. The photons are emitted simultaneously and with slightly different trajectories so that they become spatially separated. This spatial separation can be increased by coupling the photons into separate optical fibres which serve to transport the photons to observers (conventionally identified as Alice and Bob) equipped with polarisation state analysers such as that depicted in figure 2.1.1. Furthermore, the separation between Alice and Bob is sufficiently great that Alice cannot communicate the result of her measurement to Bob before he makes his measurement. (The photons travel at around two-thirds c in the fibres; Alice cannot send information to Bob any faster than c , the velocity of light in vacuum.)

Alice makes a measurement of the polarisation of her photon, A , in a basis determined by the orientation angle α of her analyser, finding it in either state $|-\alpha\rangle_A$ or $|+\alpha\rangle_A$, *cf.* equation (2.4). (The subscript α is introduced to make clear the choice of basis.) Measurement of photon A in state $|-\alpha\rangle_A$ implies that photon B is in the state $|+\alpha\rangle_B$ in this basis and, equally, measurement of photon A in state $|+\alpha\rangle_A$ implies that photon B is in the state $|-\alpha\rangle_B$. Now if Bob makes a measurement of the polarisation of photon B with his analyser oriented at a randomly-chosen angle

β the photon will be found in one of the polarisation states $|-\beta\rangle_B$ or $|+\beta\rangle_B$, with probability which is determined by the angle of rotation $(\beta - \alpha)$ of his basis relative to Alice's basis:

$$\begin{aligned} |\psi\rangle_A = |-\alpha\rangle &\implies |\psi\rangle_B = \sin(\beta - \alpha) |-\beta\rangle - \cos(\beta - \alpha) |+\beta\rangle, \\ |\psi\rangle_A = |+\alpha\rangle &\implies |\psi\rangle_B = \cos(\beta - \alpha) |-\beta\rangle + \sin(\beta - \alpha) |+\beta\rangle. \end{aligned} \quad (2.17)$$

According to this discussion, the probability of Bob finding photon B in either state is determined by the result of Alice's measurement. (For example, supposing Bob's randomly chosen basis happens to coincide with Alice's basis and Alice measures her photon to be in state $|-\alpha\rangle_A$, then Bob will find his photon in state $|+\beta\rangle_B$ with 100% probability.)

Thus it appears that the result of Bob's measurement is influenced by the result of Alice's measurement, even though their measurements are made in independent bases and faster-than-light communication would be required for Bob's apparatus to be influenced by the result of Alice's measurement.

Repeating the experiment for many pairs of entangled photons emitted by the source, the normalised coincidence rate for Alice to detect photon A in state $|-\alpha\rangle$ in her basis and for Bob to detect photon B in state $|+\beta\rangle$ in his basis is $C_{-+}(\alpha, \beta)$ (and similarly for the remaining three possible results). The expectation value of the correlation between Alice's and Bob's measurements is

$$E(\alpha, \beta) = [C_{++}(\alpha, \beta) + C_{--}(\alpha, \beta)] - [C_{+-}(\alpha, \beta) + C_{-+}(\alpha, \beta)]. \quad (2.18)$$

As can be seen from equation (2.17), quantum theory predicts that the coincidence rates have a sinusoidal dependence on the difference angle $(\beta - \alpha)$, $C_{-+}(\alpha, \beta) \propto \cos^2(\beta - \alpha)$, *etc*, as shown in figure 2.6. The correlation function is thus $E(\alpha, \beta) = -\cos[2(\beta - \alpha)]$.

Repeating the experiment for all combinations of two choices of basis for Alice, *i. e.* angles α, α' and two choices for Bob, β, β' , (sixteen coincidence rates) the expectation values can be combined to give a function

$$S(\alpha, \alpha', \beta, \beta') = |E(\alpha, \beta) - E(\alpha', \beta)| + |E(\alpha, \beta') - E(\alpha', \beta')| \quad (2.19)$$

which has a maximum value of $2\sqrt{2}$ for the set of angles $\alpha = 0^\circ$, $\beta = 45^\circ$, $\alpha' = 22.5^\circ$, $\beta' = 67.5^\circ$.

2.2.2 EPR and Bell's inequalities

The experiment described above is in all essentials the famous thought experiment of Einstein, Podolsky, and Rosen (EPR) [1]. They argued that, since no signal travelling at or below c can connect Alice and Bob's measurements and both Alice and Bob can choose the orientation of their polarisers at the very last moment before the photons A and B arrive at their respective analysers, one is forced to conclude that each photon is carrying some additional property, a *hidden variable*, determining the result of the polarisation measurement *in any basis*. This point of view is known as *local realism*. For EPR, the answer to the question posed in the title of their paper, "Can a quantum-mechanical description of physical reality be considered complete?", was, in contrast to Bohr's 'Copenhagen interpretation', no.

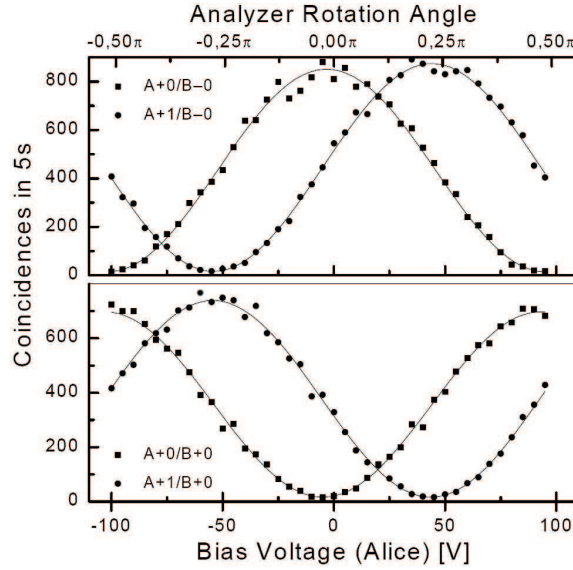


Figure 2.6: Results of the Vienna group’s Bell experiment. Four out of the sixteen possible coincidence rates between detection channels are shown. ‘Bias voltage’ corresponds to the rotation angle of Alice’s polarisation analyser. Reprinted figure with permission from G. Weihs *et al.*, Phys. Rev. Lett. **81**, 5039 (1998). Copyright (1998) by the American Physical Society.

This argument about the physical reality of quantum systems remained in the realms of philosophy until J. S. Bell demonstrated that local realism leads to predictions of the results of the EPR experiment which are in contradiction to the predictions of quantum mechanics [3]. Bell’s theorem, expressed mathematically by *Bell’s inequalities*, transformed the interpretation of the EPR experiment from a question of philosophical preference to one of quantitative, experimental, fact. It has been described as one of the most profound scientific discoveries of the twentieth century [29].

Bell’s inequalities were reformulated by Clauser, Horne, Shimony, and Holt (CHSH) in a form better suited to EPR experiments similar to that described above [30]. In the CHSH formulation, the assumption of local realism requires that the combination of expectation values S introduced in equation (2.19) must satisfy the inequality

$$S(\alpha, \alpha', \beta, \beta') \leq 2, \quad (2.20)$$

whereas quantum mechanics predicts a maximum value of $2\sqrt{2}$ (section 2.2.1).

The CHSH formulation of Bell’s inequalities led rapidly to the first experimental tests confirming the predictions of quantum mechanics [31]. A series of experiments using entangled photon pairs emitted in a cascade of atomic transitions culminated in the celebrated “Aspect’s experiment” which demonstrated deviation from the CHSH inequality at the 10 standard deviation level [4].

With the development of parametric down-conversion sources of entangled photons and of optical fibres as a means of ensuring that Alice’s and Bob’s measurements are truly well-separated, similar *Bell experiments* have been performed over 4 km in

Rarity’s group, then at Malvern [32], and over 10 km in Gisin’s group at Geneva [33].

A further refinement was introduced in an experiment in Zeilinger’s group at Vienna, which is essentially the experiment described in section 2.2.1 above [34]. In Aspect’s experiment, the relative orientations of the polarisers were fixed, or at best were rotated in a sinusoidal, *i. e.* predictable, fashion during the time-of-flight of the photons. It could be possible to reconcile the predictions of quantum mechanics with local realism by supposing that there is some kind of exchange of information between the polarisers (at a speed no greater than c). In the Vienna experiment, this loophole is closed by ensuring that Alice and Bob are sufficiently well-separated that the orientations of their polarisers can be chosen randomly during the photon’s time-of-flight and cannot be communicated between them. In figure 2.5, Alice’s measurement process (selecting a random analyser orientation, setting the analyser, and detecting a photon) must lie fully inside the shaded region which is invisible to Bob during his measurement, *i. e.* the decision about the setting has to be made after point X if the photons are detected at spacetime points Y and Z . The duration of the measurement process (indicated by the black bar in figure 2.5), including the choice of a random analyser setting, was less than one-tenth of the maximum allowed time. This experiment yielded violation of the CHSH inequality at the 30 standard deviation level, figure 2.6.

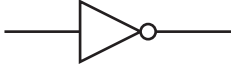
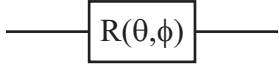
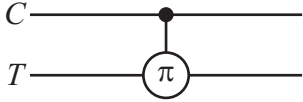
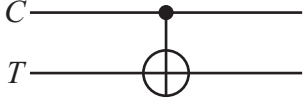
There remains a further loophole: the photon detection efficiency is not perfect. It has to be assumed that the detected photon pairs are a representative sample of all the emitted photon pairs. Bell commented succinctly on this point: “...it is hard for me to believe that quantum mechanics works so nicely for inefficient practical set-ups and is yet going to fail badly when sufficient refinements are made.” [35].

In conclusion, experiments with entangled photons have provided an unambiguous demonstration of the validity of the standard interpretation of quantum mechanics. These experiments have been discussed in some detail, since they have been the test-bed for concepts and techniques required in the application of entanglement in more technology-oriented applications.

2.3 Quantum logic

The Bell experiment described above uses entangled photons; because of the stochastic nature and low efficiency of spontaneous parametric down-conversion, the result has to be obtained by comparison with the statistical predictions of quantum mechanics. This is an example of *probabilistic* entanglement: the measurement of entanglement is performed on an ensemble of qubit pairs which cannot be individually prepared in a pre-determined state. In contrast, many applications of entanglement, particularly in quantum information processing, require *deterministic* entanglement: one wants to be able to carry out a sequence of operations involving the entanglement of individually-prepared qubits. Physical systems showing deterministic entanglement generally use well-localised entities such as trapped ions or atoms, or solid-state devices such as Josephson junctions. Although these systems do not provide such elegant and unambiguous demonstrations of the breach of Bell’s inequalities, they can exhibit some of the most intriguing consequences of quantum entanglement on an event-by-event level.

Table 2.1: Classical and quantum logic gates.

Classical	Quantum
1-bit NOT	Rotation (Hadamard gate)
	
$0 \rightarrow 1$	$ 0\rangle \rightarrow \cos(\theta/2) 0\rangle + e^{i\phi} \sin(\theta/2) 1\rangle$
$1 \rightarrow 0$	$ 1\rangle \rightarrow \cos(\theta/2) 1\rangle + e^{-i\phi} \sin(\theta/2) 0\rangle$
2-bit AND	π phase gate $U_{12}(\pi)$
	
$00 \rightarrow 0$	$ 0\rangle_C 0\rangle_T \rightarrow 0\rangle_C 0\rangle_T$
$01 \rightarrow 0$	$ 0\rangle_C 1\rangle_T \rightarrow 0\rangle_C 1\rangle_T$
$10 \rightarrow 0$	$ 1\rangle_C 0\rangle_T \rightarrow 1\rangle_C 0\rangle_T$
$11 \rightarrow 1$	$ 1\rangle_C 1\rangle_T \rightarrow - 1\rangle_C 1\rangle_T$
2-bit CNOT	Quantum CNOT
	
$00 \rightarrow 0$	$ 0\rangle_C 0\rangle_T \rightarrow 0\rangle_C 0\rangle_T$
$01 \rightarrow 1$	$ 0\rangle_C 1\rangle_T \rightarrow 0\rangle_C 1\rangle_T$
$10 \rightarrow 1$	$ 1\rangle_C 0\rangle_T \rightarrow 1\rangle_C 1\rangle_T$
$11 \rightarrow 0$	$ 1\rangle_C 1\rangle_T \rightarrow 1\rangle_C 0\rangle_T$

A powerful tool for manipulating deterministically-entangled qubits is the quantum gate. The manipulation of a single qubit by means of the generalisation of the classical NOT gate to a quantum superposition state, the rotation or Hadamard gate, has been discussed in section 2.1.3. Quantum equivalents of classical 2-bit logic gates can be constructed. Table 2.1 illustrates two fundamental examples, the phase gate and the quantum CNOT gate. A *control qubit*, C , acts on a *target qubit*, T , such that the final state of T depends both on its initial state and on the state of C . The phase gate is analogous to the classical AND gate: the final state of the system is only changed if both the target and control qubits are in state $|1\rangle$; the change being an inversion of the overall quantum phase. The phase gate can be combined with single-qubit rotations to build a quantum equivalent of the conditional-NOT or CNOT gate, in which the state of the target qubit is flipped conditionally on the state of the control qubit. The CNOT or phase gate, in combination with the rotation gate, provides a universal set of gate operations: any quantum information process can be performed using the appropriate sequence of these gates.

The truth tables presented in table 2.1 show the outcome of gate operations using qubits in the pure basis states $|0\rangle$, $|1\rangle$. Unlike a classical gate, a quantum gate can have a superposition state as its input and so can be used to generate entanglement. Consider the case of a CNOT gate where the target qubit is prepared in the state $|0\rangle_T$ but the control qubit is input in the superposition state $\frac{1}{\sqrt{2}}(|0\rangle_C + |1\rangle_C)$. Because the effect of the control qubit in $|1\rangle_C$ is to flip the state of the target qubit, an entangled Bell state $|\phi_+\rangle$ is generated at the output:

$$\frac{1}{\sqrt{2}}(|0\rangle_C + |1\rangle_C)|0\rangle_T \rightarrow \frac{1}{\sqrt{2}}(|0\rangle_C|0\rangle_T + |1\rangle_C|1\rangle_T).$$

It is this ability to carry out conditional operations on superposition states which underlies the potential power of a quantum computer, enabling it to implement algorithms impossible on a classical computer. The simplest such quantum algorithm is the Deutsch-Jozsa algorithm to determine the parity of a binary function [36]. This can be pictured as a way of identifying a trick coin (one which has ‘heads’ on both sides) by looking at only one side; classically, both sides must be examined. The ‘quantum advantage’ lies in the need to run the evaluating logic only once rather than twice: one could imagine a case in which evaluation of the function is a long and complicated process [37]. The Deutsch-Jozsa algorithm has been demonstrated in a number of physical implementations including trapped ions [38] and NMR; it provides the first step towards the implementation of more complex algorithms, such as Shor’s algorithm for the factorisation of large numbers [39].

2.4 Multi-particle entanglement

The entanglement a pair of qubits has been discussed in some detail, since it is the simplest example of entanglement and has also been realised across a wide range of physical systems, as will be discussed in subsequent chapters. However, quantum computation and quantum communication schemes have been proposed which call for the manipulation of entangled states of many particles. Multi-particle entangled states also offer the possibility of enhanced measurement techniques such as Heisenberg spectroscopy.

Some key features of multi-particle entanglement can be illustrated in the simplest case of the entanglement of three qubits. Two extreme types of entangled states of three qubits exist. Greenberger-Horne-Zeilinger (GHZ) states,

$$|\text{GHZ}\rangle = \frac{1}{\sqrt{2}}(|0\rangle_A|0\rangle_B|0\rangle_C + |1\rangle_A|1\rangle_B|1\rangle_C), \quad (2.21)$$

have the property that the measurement of the state of any one particle results in complete loss of entanglement: measurement of particle A in state $|0\rangle_A$ leaves particles B and C in the unentangled state $|0\rangle_B|0\rangle_C$. Alternatively, W -states,

$$|W\rangle = \frac{1}{\sqrt{3}}(|1\rangle_A|0\rangle_B|0\rangle_C + |0\rangle_A|1\rangle_B|0\rangle_C + |0\rangle_A|0\rangle_B|1\rangle_C), \quad (2.22)$$

have the property that measurement of the state of any one particle can still leave the other two particles entangled, *e. g.* measurement of particle A in state $|0\rangle_1$

leaves particles B and C in the Bell state $\frac{1}{\sqrt{2}}(|1\rangle_B|0\rangle_C + |0\rangle_B|1\rangle_C)$: the W state is maximally robust to retention of entanglement under loss of one qubit. The GHZ and W states can be readily generalised to N -particle systems. They represent two distinct classes of multi-qubit entanglement; it is not possible to translate one into the other through a series of quantum logic operations. In analysing an N -qubit system, it is not enough to ask whether the states are entangled; the measurement must determine how the qubits are entangled [40]. A quantum system in a GHZ state can exhibit violation of Bell's inequalities at the event-by-event level, rather than probabilistically as in two-particle Bell experiments. Situations have been demonstrated in which, on finding qubits A and B in state $|0\rangle$, local realism predicts that qubit C must also be found in state $|0\rangle$ whereas quantum mechanics predicts, and experiment confirms, that it is found in state $|1\rangle$ [41].

The entanglement of large numbers of particles offers a route to improved measurement. In a measurement made using a normal ensemble of particles, the signal-to-noise ratio (S/N) improves with the number of particles contributing to the measurement (N_p) as $1/\sqrt{N_p}$. However, for an ensemble of maximally-entangled particles, the S/N scales as $1/N_p$, (an enhancement of $\sqrt{N_p}$) providing a route to realising the Heisenberg limit of the measurement, as discussed in section 2.5.

Entanglement is not confined to two-state quantum systems; for example, entanglement of pairs of photons in superpositions of three orbital angular momentum states (*qutrits*) have recently been used to demonstrate a protocol to detect 'quantum cheating' in quantum cryptography (see section 3.2.3).

2.5 Quantum-enhanced metrology

Through the Heisenberg uncertainty principle, quantum mechanics imposes a fundamental limit on the precision of measurement. Most conventional measurement techniques fail to reach this limit, and are bounded instead by non-fundamental limits such as the shot noise limit and the standard quantum limit. However, quantum strategies such as squeezing, quantum non-demolition, and the use of entangled states can be used to overcome the conventional limits and push measurement uncertainty to the fundamental Heisenberg limit. This field is known as *quantum-enhanced metrology*. A recent review is given in ref. [42].

2.5.1 The Heisenberg uncertainty principle

Near the beginning of this chapter we mentioned that it is one of the fundamental postulates of quantum mechanics that every real physical observable has an operator associated with it. It can be shown [43] that if two operators do not commute, the uncertainties on the physical observables associated with these operators are not independent, but are governed by an inequality known as an uncertainty relation.

If we have two operators $\hat{A}$ and $\hat{B}$ whose commutator can be expressed in the form

$$[\hat{A}, \hat{B}] = i\hat{C}, \quad (2.23)$$

where $\hat{C}$ is a third operator, then the uncertainties ΔA and ΔB on the physical observables A and B obey the uncertainty relation

$$\Delta A \Delta B \geq \frac{|\langle C \rangle|}{2}. \quad (2.24)$$

The position x and momentum p of a free particle are examples of two observables whose operators do not commute. The commutator is $[\hat{x}, \hat{p}] = i\hbar$, so the position-momentum uncertainty relation is

$$\Delta x \Delta p \geq \frac{\hbar}{2}. \quad (2.25)$$

This particular relation is known as the *Heisenberg uncertainty principle*. Other examples of non-commuting operators are the number and phase operators $\hat{n}$ and $\hat{\phi}$, referring to the number of quanta and the phase, respectively, of a harmonic oscillator, e.g. an electromagnetic cavity or a tuned circuit. As expected from the above discussion, the observables associated with these operators also obey an uncertainty relation.

The uncertainty relation defined in equation (2.24) imposes a fundamental limit on the measurement uncertainties of quantities whose operators are non-commuting (such pairs of quantities are often referred to as conjugate variables). If we make simultaneous measurements of the position and momentum of a free particle, and repeat this procedure many times (with identical preparation of the system each time) we will obtain a spread Δx in the measurements of the position, and a spread Δp in the measurements of the momentum, which are consistent with the Heisenberg uncertainty principle.

The Heisenberg limit is fundamental, and cannot be overcome, but most conventional measurements are subject to less fundamental limits, some of which we describe below.

2.5.2 The shot noise limit

The shot noise limit arises when we measure a quantity that depends on contributions from N uncorrelated sources. For example, if we want to measure the phase shift ϕ generated by an optical component, we can insert the component into one path of a Mach-Zehnder interferometer (see Figure 2.7). If we send a beam of light containing N photons into input A , the output through which it exits the interferometer depends on the phase ϕ . Due to the differences in phase acquired on reflection and transmission, if $\phi = 0$, all N photons will exit the interferometer at D . On the other hand, if $\phi = \pi$, all N photons will exit at C . If ϕ is somewhere between 0 and π , then the number of photons that exit at D will be equal to $N \sin^2(\phi/2)$ and the number that exit at C will be $N \cos^2(\phi/2)$. Therefore, if we measure the difference in intensities of the light at C and D , we can measure the phase ϕ .

The uncertainty on the measurement of ϕ depends on the uncertainty on the intensity. Since the beam contains N photons, by measuring the intensity at D we are essentially making N measurements of the probability that an individual photon will exit the interferometer at D , and taking the mean of the results. If the uncertainty on the result of an individual probability measurement is σ , then, according

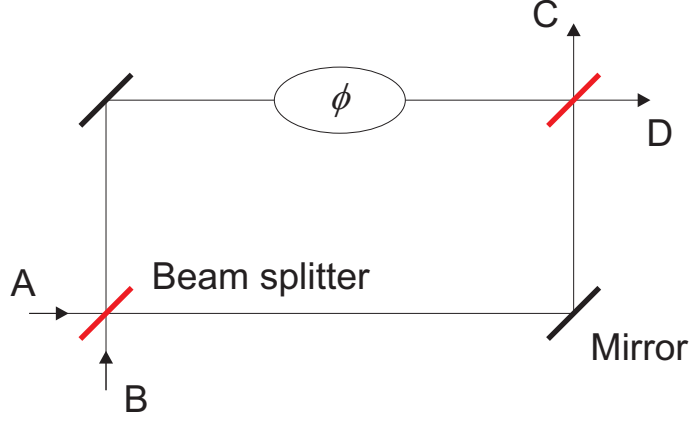


Figure 2.7: The Mach-Zehnder interferometer. Light can be input at A or B and output at C or D . The oval element introduces a phase shift ϕ .

to the central limit theorem, the uncertainty on the mean of N measurements will be equal to $\sigma/\sqrt{N}$. Thus, the uncertainty on the intensity scales as $1/\sqrt{N}$, and, by propagation of errors, the uncertainty on ϕ also scales as $1/\sqrt{N}$. This scaling is known as the *shot noise limit*. It is not a fundamental limit, so can be overcome by the use of quantum strategies such as squeezing or the use of entangled states.

2.5.3 Beyond the shot noise limit using entangled states

In this section we show that by using N entangled photons, rather than N uncorrelated photons, as the input to the interferometer, we can obtain an enhancement of $1/\sqrt{N}$ on the phase uncertainty.

Suppose the input beam is polarised. Photons in the beam can be vertically ($|0\rangle$) or horizontally ($|1\rangle$) polarised. If we use a polarising beam splitter at the input, then photons in the $|0\rangle$ state will pass through the empty arm of the interferometer and pick up no additional phase, whereas photons in the $|1\rangle$ state will pass through the phase shifter and pick up an additional phase ϕ . If we prepare each of the input photons in the superposition state $|\psi_{\text{in}}\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$, then each will exit the interferometer in the state $|\psi_{\text{out}}\rangle = (|0\rangle + e^{i\phi}|1\rangle)/\sqrt{2}$. If we now measure the probability that the output state is equal to the input state (using another polarising beam splitter at the exit), we obtain

$$p(\phi) = |\langle\psi_{\text{in}}|\psi_{\text{out}}\rangle|^2 = \cos^2(\phi/2). \quad (2.26)$$

The uncertainty $\Delta p(\phi)$ on this quantity is given by

$$\Delta^2 p(\phi) = \langle\psi_{\text{out}}|(|\psi_{\text{in}}\rangle\langle\psi_{\text{in}}|)^2|\psi_{\text{out}}\rangle - p^2(\phi) \quad (2.27)$$

(mean of the squares minus square of the mean), and since $(|\psi_{\text{in}}\rangle\langle\psi_{\text{in}}|)^2 = |\psi_{\text{in}}\rangle\langle\psi_{\text{in}}|$, we find that $\Delta p(\phi) = \sqrt{p(\phi) - p^2(\phi)}$. Now, to work out the uncertainty on ϕ we use the relation

$$\Delta\phi = \frac{\Delta p(\phi)}{|\partial p(\phi)/\partial\phi|}, \quad (2.28)$$

which gives us $\Delta\phi = 1$. By repeating the measurement N times (using N uncorrelated photons) we can reduce the uncertainty to $\Delta\phi = 1/\sqrt{N}$. However, if we use N entangled photons as the input we can improve upon this. We can write the entangled input state as

$$|\psi_{\text{in}}\rangle = \frac{1}{\sqrt{2}}(|0\rangle \cdots |0\rangle + |1\rangle \cdots |1\rangle). \quad (2.29)$$

Now the output state becomes

$$|\psi_{\text{out}}\rangle = \frac{1}{\sqrt{2}}(|0\rangle \cdots |0\rangle + e^{iN\phi}|1\rangle \cdots |1\rangle), \quad (2.30)$$

so the probability that the output state is equal to the input state is now $p(\phi) = |\langle\psi_{\text{in}}|\psi_{\text{out}}\rangle|^2 = \cos^2(N\phi/2)$, which means that when we evaluate $\Delta\phi$ using equation (2.28) we obtain

$$\Delta\phi = \frac{1}{N}, \quad (2.31)$$

an improvement on the unentangled uncertainty by a factor of $1/\sqrt{N}$. This is the essence of quantum-enhanced metrology. One must not forget, however, that the enhancement of $1/\sqrt{N}$ depends on the entangled state maintaining its coherence until it is measured at the exit of the interferometer. Any decoherence will degrade the enhancement.

The above arguments can be extended to other scenarios utilising phase shifts, such as frequency measurement.

2.5.4 The standard quantum limit

The standard quantum limit is a consequence of the phenomenon of measurement back action, where the outcome of a measurement is affected by the outcomes of previous measurements (and affects the outcome of subsequent measurements).

Consider a measurement of the position of a free mass at time $t = 0$, which has uncertainty $\Delta x(0)$. *Via* the Heisenberg uncertainty principle this corresponds to an uncertainty in momentum $\Delta p(0)$ at $t = 0$. However, the evolution of the wavefunction of a free mass is governed by the Hamiltonian $\hat{H} = \hat{p}^2/2m$, and causes the position at time t to be given by $x(t) = x(0) + p(0)t/m$. It appears that the momentum uncertainty generated by the initial position measurement has fed back into the position uncertainty at time t . This is known as measurement back action. If we work out the position uncertainty at time t by adding the position and momentum contributions in quadrature we obtain

$$\Delta^2 x(t) = \Delta^2 x(0) + \frac{\Delta^2 p(0)t^2}{m^2}, \quad (2.32)$$

and since $a^2 + b^2 = (a - b)^2 + 2ab$, we deduce that $a^2 + b^2 \geq 2ab$, which implies that $\Delta^2 x(t) \geq 2\Delta x(0)\Delta p(0)t/m$. Using the Heisenberg uncertainty principle we arrive at

$$\Delta x(t) \geq \sqrt{\frac{\hbar t}{m}}, \quad (2.33)$$

which is known as the *standard quantum limit*. This is another example of a non-fundamental limit that can be overcome using clever quantum strategies. For a start, by making N measurements of $x(t)$ (making sure to prepare the system in the same way before each measurement) we could reduce the uncertainty $\Delta x(t)$ by a factor of $1/\sqrt{N}$, and this could be further reduced by preparing N free masses in an entangled state. However, there are other ways to beat the standard quantum limit. One is squeezing, and the other is the use of quantum non-demolition measurements, both of which are described below.

2.5.5 Squeezing

Squeezing is the preparation of a quantum system in a state such that the uncertainty on one of a pair of complementary variables is reduced at the expense of the other. We can create a squeezed state simply by measuring the system. For example, if we use a scanning tunnelling microscope (STM) to measure the position x of a particle the uncertainty Δx will be determined by the diameter of the STM tip. If we reduce Δx by sharpening the tip, we increase the uncertainty Δp on the momentum. We have created a squeezed state. Similarly, we can create number-squeezed states of light (Fock states) using a single-photon source. If the uncertainty on the number of photons generated is small, the uncertainty on the phase of the light will be large.

We can use squeezed states to help overcome non-fundamental limits such as the shot noise limit and the standard quantum limit. If the input state is a squeezed state, the output state will benefit from the reduced uncertainty on the squeezed variable.

2.5.6 Quantum non-demolition measurements

Quantum non-demolition (QND) is a strategy used to get around the problem of measurement back action. The idea is to design the meter such that it monitors only one out of a pair of complementary observables, without perturbing it. The uncertainty on the other variable will adjust itself to satisfy the uncertainty relation, but since we aren't interested in this variable, this doesn't matter. A review is given in ref. [44].

QND measurements can only be performed under a certain condition. This is that the operator corresponding to the measured quantity must commute with the Hamiltonian:

$$[\hat{q}, \hat{H}] = 0, \quad (2.34)$$

where q is the measured quantity. If $\hat{q}$ commutes with the Hamiltonian, then $\hat{q}$ must share the same eigenstates as $\hat{H}$. The Hamiltonian governs the evolution of the state of the quantum system with time, but if the system is in an eigenstate of the Hamiltonian (as it will be, after a measurement of the quantity q), then it is in a stationary state and will not change with time. This means that the observable q is not affected by back action. The uncertainty relation for $\hat{q}$ and $\hat{H}$ is $\Delta q \Delta H \geq 0$, which means that there is no fundamental limit on the precision to which q can be measured. The downside to QND is that it is difficult to construct a meter and system whose combined Hamiltonian commutes with the operator to be observed.

The term “Quantum non-demolition” is a little misleading. To clarify, we point out that, as with any measurement of a quantum system, the state of the system following a QND measurement is an eigenstate of the operator corresponding to the measured quantity. Hence, if the system starts off in a superposition state, the QND measurement will collapse the wavefunction to one eigenstate. This “destroys” the initial superposition states, but doesn’t “demolish” it, where, by “demolition”, we mean complete obliteration of the quantum system such that it must be reset before another measurement can be made. An example of such demolition is the measurement technique used in a Stern-Gerlach device. The Stern-Gerlach device contains a magnetic field gradient which deflects incoming spin-1/2 particles in different directions, depending on their orientations (up or down). Detection is achieved by projecting the deflected spins onto a screen coated in some substance that is sensitive to the particles. Clearly, we cannot make a series of measurements on the spin state of a single particle. We demolish the quantum system when the particle hits the screen. QND allows us to make a series of measurements on a quantum system without demolishing it. In between measurements the system can be perturbed so that it changes its state, but every time a QND measurement is made, the state vector is collapsed back to an eigenstate of the observable.

How can we make a non-destructive measurement of the spin-state of a particle using a Stern-Gerlach device? The answer is to use entanglement. If we allow another spin-1/2 particle (the meter particle) to interact with the particle whose spin we want to measure (the target particle) the two spin states will become entangled, and a measurement of one will fully determine the state of the other. In this way we can make a destructive measurement of the meter particle using a Stern-Gerlach device, enabling us to read out the state of the target particle without demolishing it. To make a series of measurements on the target particle we just need a steady supply of meter particles. However, we must still make sure that the combined Hamiltonian of the meter and target particles, including the interaction Hamiltonian, commutes with the observable we want to measure.

Chapter 3

Entangled photons

In contrast to the atomic and solid-state systems to be reviewed in subsequent chapters, photons are essentially mobile, travelling with velocity c/n in a medium of refractive index n . Moreover, they interact only weakly with the environment, are thus relatively free from decoherence and can, therefore, exhibit entanglement over truly macroscopic distances. It is this property of long-distance entanglement which has enabled experiments using photons to provide the most rigorous demonstrations of the violation of Bell's inequalities to date and which makes the use of photons essential in applications such as quantum communication.

3.1 Sources of entangled photons

Early experiments with entangled photons, such as Aspect's landmark EPR experiment [4], used the correlated spontaneous emission of a pair of photons in an atomic cascade decay. In the first step of the cascade, a photon is emitted with random direction and polarisation. Conservation of linear and angular momentum requires that the direction and polarisation of the photon emitted in the second step of the cascade are fixed relative to the first photon: the pair of photons are in an entangled state. In Aspect's experiment, the correlations between the photons' polarisations are analysed; the randomness of the direction of emission serves only to decrease the efficiency with which photon pairs are detected, since the collimation optics associated with the polarisation analysers covers only a limited solid angle. A second limitation of an atomic cascade is that it uses a spontaneous decay process and is thus a source of only *probabilistic* entanglement.

The development of non-linear optics over the last quarter century has led to the development of parametric down-conversion and single-photon sources as practical sources of entangled photons which can be detected with high efficiency and, eventually, provide sources of deterministic entanglement of numerous photons.

3.1.1 Parametric down-conversion

Spontaneous parametric down-conversion (SPDC) is one of a group of non-linear optical processes which arise from the interaction of the electric field of the photon with the second-order susceptibility of the medium through which it propagates. It

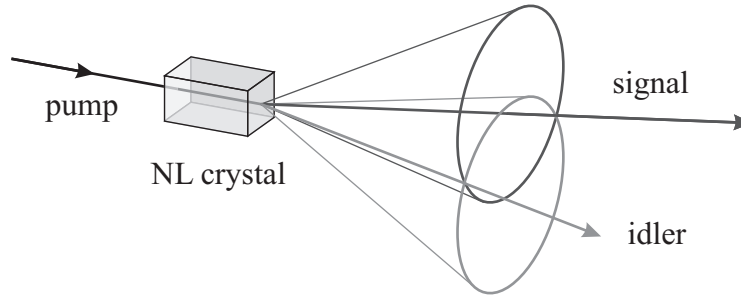


Figure 3.1: Generation of entangled photons by SPDC. The possible momentum vectors of signal (idler) photons describe cones of radius determined by the photon energy. (Dark grey denotes signal; light grey, idler.) Photon pairs emitted along the intersection of the two cones are in a Bell state of polarisation entanglement.

has become the most widely-used source for entangled photons, although it shares with the atomic cascade the drawback of providing only probabilistic entanglement.

In SPDC a single photon, referred to as the pump photon, is converted into two photons of lower energy. Of these two, the photon of higher frequency is referred to as the *signal* photon, and that of lower frequency, the *idler* photon. The sum of the signal and idler photon energy must equal the pump photon energy. The photon energies are further constrained by the phase-matching conditions in the non-linear medium, typically a birefringent crystal such as β -barium borate (BBO) or potassium niobate (KN). Depending on the spectral characteristics of the source of pump photons and on details of the experimental geometry (crystal length, pump focusing geometry) the phase-matching conditions will limit the signal and idler photon frequencies to within fairly narrow bandwidths. Depending on the crystal birefringence, the signal and idler photons can either have the same or orthogonal polarisation (type-I or type-II down-conversion).

Parametric down-conversion is the basis of the optical parametric oscillator: by constructing an optical cavity around the crystal the signal or idler radiation can be built up until the threshold for stimulated parametric down-conversion is reached, yielding high-efficiency down-conversion and hence a convenient source of tuneable laser light. In contrast, SPDC is an intrinsically low-efficiency process: fewer than one in 100 000 pump photons may be down-converted.

The phase-matching conditions constrain the possible momentum vectors of the signal (idler) photon of given frequency to describe a cone, figure 3.1. Conservation of momentum dictates that the momentum vector of the idler (signal) photon must lie at the opposite side of a similar cone. The signal and idler photons are entangled in both energy and momentum (which are both continuous variables). When the momenta of the two photons lie along the lines of intersection of the cones, Bell states of polarisation entanglement are obtained.

The geometry of the signal and idler momentum vectors makes it feasible to spatially separate the two photons by coupling them into separate optical fibres. This enables long-distance propagation of the entangled photons to be demonstrated in the laboratory, using fibre spools, or, alternatively, to transmit the photons over long distances using fibre networks.

An individual photon is in a state which can be described by continuous variables such as its momentum or spatial mode and by discrete variables such as its polarisation or angular momentum. Entangled photon pairs generated by SPDC can be entangled in several of these variables; the distinctive attributes of these variables give rise to different applications.

Polarisation and angular momentum entanglement

Polarisation-entangled photons have been discussed in chapter 2. Polarisation states are well-suited to experiments in the laboratory environment, providing robust decoherence-free qubits, but care has to be taken to eliminate or control sources of birefringence which may introduce rotation of the polarisation state. This is particularly significant for applications involving fibre transmission over long distances. Polarisation is an unambiguously two-state variable. In contrast, a photon can occupy an infinite number of angular momentum states, from the fundamental Gaussian mode through all orders of Laguerre-Gaussian modes. An experiment exploiting entangled qutrits of the Gaussian mode and the two lowest-order Laguerre-Gaussian modes is discussed in section 3.2.3. The Laguerre-Gaussian modes are not readily transmitted in conventional optical fibres.

Spatial entanglement

Conservation of momentum dictates that SPDC produces momentum-entangled photon pairs. Detection of one photon at a given position relative to the source fixes the momentum and hence path of the other photon of the pair. This can lead to unusual interference effects. Interference is fundamentally a quantum effect. A conventional interference pattern is created by the interference of each individual photon with itself (the photon is in a superposition state of its possible spatial paths, for example, transmission through the two slits in a double-slit experiment). An entangled photon pair can be thought of as a *biphoton*: the biphoton only interferes with itself and hence an interference experiment performed on one photon will be mirrored in measurements made on the other photon. In an elegant experiment performed in Zeilinger's group [45], this is shown to give rise to effects such as the loss of interference fringes when a double-slit experiment is performed on photons emitted in one direction when the position of the photons emitted in the other direction is measured. Spatial correlations between entangled photons have a variety of potential applications, some of which are discussed in section 3.2.1.

Time-bin entanglement

Both photons of an entangled pair are emitted simultaneously from the source. In other words, they have the same origin in space-time. This leads to a number of applications which rely on measurement of the difference in time of arrival of the photons, such as timing synchronisation (section 3.2.2).

Nicolas Gisin's group at the University of Geneva have developed the technique of *time-bin entanglement* [46]. A pulse of light from a femtosecond laser is passed through an interferometer with arms of unequal length. Macroscopically, the pulse is

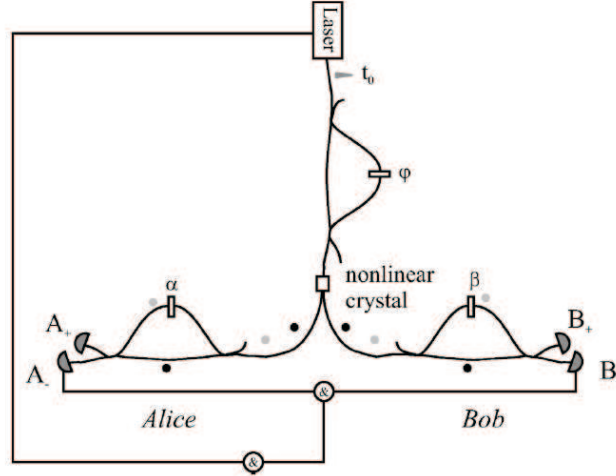


Figure 3.2: The Geneva group’s time-bin entanglement method. Time-bin qubits are prepared by passing a femtosecond pulse through the pump interferometer. Pairs of entangled photons are created by SPDC in the crystal. In the example shown, the amplitude of the first time-bin (grey) goes through the long arm and the second time-bin (black) goes through the short arm. In 50% of cases, this undoes the transformation introduced by the pump interferometer. Reprinted figure with permission from I. Marcikic *et al.*, Phys. Rev. A **66**, 062308 (2002). Copyright (2002) by the American Physical Society.

split into two, propagating with a time interval corresponding to the path difference in the interferometer, the path difference being significantly greater than the pulse length. At the quantum level, each photon in the pulse has an equal probability for traversing the long or the short arm of the interferometer; it is in a superposition state of time-of-arrival at a detector,

$$|\psi\rangle_p = \frac{1}{\sqrt{2}} (|1, 0\rangle - e^{i\phi}|0, 1\rangle), \quad (3.1)$$

where $|1, 0\rangle$ represents a photon detected in the first *time-bin* (going through the short arm of the interferometer) and $|0, 1\rangle$, a photon in the second time-bin (going through the long arm). Time-bin entangled photon pairs are generated by SPDC of this double pulse: each photon which undergoes SPDC gives rise to a photon pair which is in a Bell state of having been created in the first or second time-bin,

$$|\phi\rangle_p = \frac{1}{\sqrt{2}} (|1, 0\rangle_A |1, 0\rangle_B - e^{i\phi} |0, 1\rangle_A |0, 1\rangle_B). \quad (3.2)$$

Figure 3.2 is a sketch of the Geneva group’s Bell experiment performed using time-bin entangled photons. This experiment illustrates techniques which Gisin’s group have applied to quantum key distribution [47] and teleportation [48]. The sketch suggests an all-fibre implementation; in practise the pump pulses are produced by a femtosecond laser at 710 nm and consequently the first (pump) interferometer uses bulk optics. The SPDC produces signal and idler photons at 1310 nm and 1550 nm. A wavelength-division multiplexer (WDM) is used to separate the

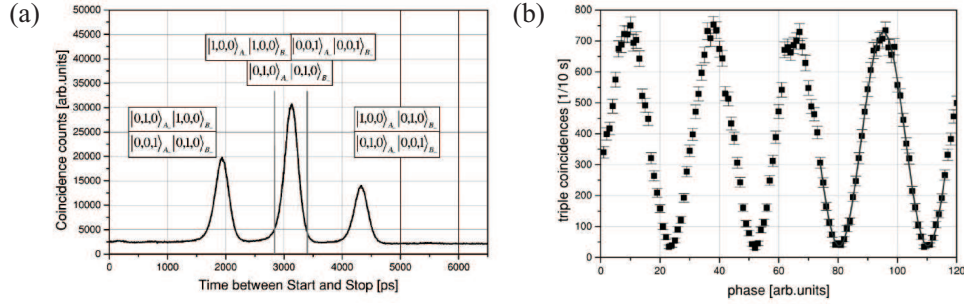


Figure 3.3: Results from the Geneva group’s Bell experiment. (a) Difference in arrival times between Alice’s and Bob’s detectors. (b) Interference fringes of the triple-coincidence detection of the post-selected state. Reprinted figure with permission from I. Marcikic *et al.*, Phys. Rev. A **66**, 062308 (2002). Copyright (2002) by the American Physical Society.

fibre-coupled signal and idler photons which are transmitted to Alice and Bob respectively. Alice and Bob use all-fibre interferometers to analyse the time-of-arrival of their photons. These interferometers have the same optical path difference as the pump interferometer but can introduce an adjustable phase α , β in the long arm. Alice or Bob can detect their photon in one of three time-bins: the photon can have passed through the short arm of both pump and analysis interferometer, state $|1, 0, 0\rangle$; through the long arm of both ($|0, 0, 1\rangle$); or through the short arm of one and the long arm of the other ($|0, 1, 0\rangle$). Measuring the difference in arrival time between Alice’s and Bob’s photons, three peaks are observed (figure 3.3(a)), corresponding to the different permutations of paths. Correlating events recorded in the central peak with the known emission time of the pump pulse, events in which both Alice’s and Bob’s photons are in states $|1, 0, 0\rangle$ or $|0, 0, 1\rangle$ can be rejected. The remaining post-selected photon pairs are in the state $|\psi\rangle = |0, 1, 0\rangle_A |0, 1, 0\rangle_B$. The probability of detecting this state is a function of the phases ϕ , α , β of the three interferometers: $P_\psi = 1 - \cos(\alpha + \beta - \phi)$. As the relative phases are changed, quantum interference fringes are observed in the rate at which this post-selected state is detected (figure 3.3(b)). Fringe visibility greater than 70.7% indicates violation of the CHSH inequality; in this experiment the visibility was 91.0(8)%.

3.1.2 Entanglement with linear optics

SPDC is a non-linear process and hence is inherently inefficient: it cannot provide a deterministic source of two-photon entanglement. In contrast, entangled states of photons can be achieved with good efficiency using linear optics, exploiting the phenomenon of quantum interference of indistinguishable photons [49]. Quantum interference is illustrated in figure 3.4. It can be used in conjunction with SPDC to generate multi-photon entanglement and to perform entanglement swapping (see section 3.1.3); it is thus an important tool in quantum teleportation.

Alternatively, quantum interference can potentially be used to provide a deterministic source of entangled photons, if it can be used with deterministic single-photon sources. A discussion of single-photon sources is outside the scope of this

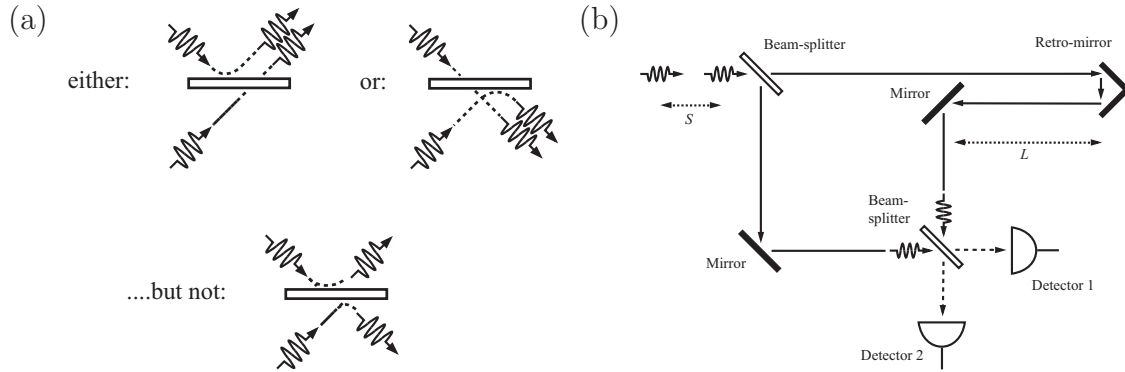


Figure 3.4: Interference of two indistinguishable photons arriving simultaneously at a beamsplitter. (a). The lower outcome, where the photons leave by different output ports, is forbidden by quantum mechanics. The upper two outcomes, photons leaving by the same port, are equally likely. (b). Two-photon interference using a train of single photons emitted at a constant rate. The Mach-Zender interferometer has short and long arms. The latter contains a delay line, adjusted such that a photon traversing the long arm will arrive at the second beamsplitter at the same time as the following photon, if the latter traverses the short arm. This will occur for 25% of pairs ($\frac{1}{4}$ of the single photon emission rate); quantum interference ensures that in these cases two photons will be measured at either detector 1 or detector 2, *i. e.* the probability of detecting two coincident photons at detector 1 is 1 in 8.

report (see the review article by Oxborrow and Sinclair [19]). Here, experiments using indistinguishable photon pairs generated by a quantum dot semiconductor single-photon source, performed in Yoshihisa Yamamoto's group at Stanford, are discussed [50, 51]. Their quantum dot source generates pairs of photons with a repetition rate of 1 pair/13 ns. The photons in each pair are emitted 2 ns apart. The photon pairs are sent into a Mach-Zender interferometer, as shown in figure 3.4, with the addition of a half-wave plate in the long arm. The photons are initially horizontally-polarised in the plane of the interferometer. At the second beamsplitter, each photon is in a superposition state of arriving *via* the short arm with horizontal polarisation and *via* the long arm with an (adjustable) delay and vertical polarisation. If the delay in the interferometer is adjusted to 2 ns, matching the delay between the pair of pulses emitted by the source, simultaneous detection of two photons after the second beamsplitter means that the joint polarisation state is

$$|\phi_i\rangle = |V\rangle_1 |H\rangle_2. \quad (3.3)$$

However, the second 50/50 beamsplitter places each photon in a superposition state of detection by either detector A or detector B, *i. e.* the final state is

$$|\phi_f\rangle = \frac{1}{2} (|H\rangle_A |V\rangle_A - |H\rangle_B |V\rangle_B - |H\rangle_A |V\rangle_B + |V\rangle_A |H\rangle_B), \quad (3.4)$$

with detectors A and B detecting 0, 1, or 2 photons. This state violates bell's inequality; however, poor detector efficiency prevents the detection of 1 or 2 photons at one detector from being reliably distinguished. consequently, in the Stanford

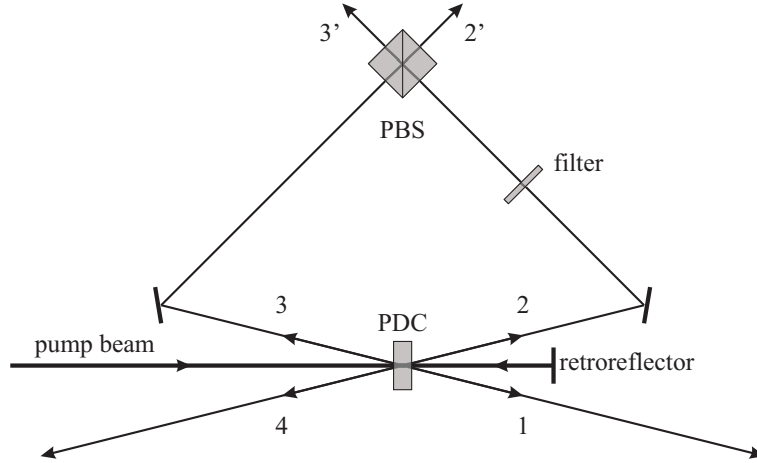


Figure 3.5: Scheme for observation of 4-photon entanglement in GHZ states. Polarisation-entangled pairs (1,2), (3,4) are generated by SPDC of the forward and retroreflected pump beam. Path lengths are adjusted such that photons 2 and 3 are simultaneously incident on a polarising beam-splitter (PBS). Simultaneously detected photons, one in each of the output modes 2' and 3', must be in the same polarisation state, projecting the 4-photon state into a GHZ state. To demonstrate efficient teleportation (section 3.2.3) [52], an addition filter is placed in the path of photon 2, with photon 1 being transmitted to Bob. Adapted figure with permission from J.-W. Pan *et al.*, Phys. Rev. Lett. **86**, 4435 (2001). Copyright (2001) by the American Physical Society.

experiment [50], post-selection was used: only events in which photons were simultaneously detected by both detectors were retained. The postselected state is thus the Bell state

$$|\phi_f'\rangle = \frac{1}{\sqrt{2}} (|H\rangle_A |V\rangle_B - |V\rangle_A |H\rangle_B). \quad (3.5)$$

Placing rotatable polarisation analysers before each detector, a Bell experiment analogous to Aspect's experiment (section 2.2.1) could be performed. The CHSH inequality was breached, with $S = 2.38(18)$, demonstrating that an entangled state of two independent photons had been generated.

In a further experiment [51], quantum teleportation was demonstrated. Using a double-interferometer configuration, the superposition state of one photon of the pair was transferred to the second photon with a fidelity of $F = 0.8$, demonstrating the feasibility of implementing a quantum gate using linear optics.

3.1.3 Multi-photon entanglement

Zeilinger's group have demonstrated the generation of 3 [54] and 4-photon [53] polarisation-entangled states. In the latter case, they have demonstrated the ability to create entanglement between pairs of photons which have never interacted. This process is called *entanglement swapping*. Figure 3.5 illustrates the principle of their experiment. A UV laser pulse is double-passed by retroreflection through a BBO crystal. With a small probability, two pairs of polarisation-entangled photons are

created by SPDC. Since the second pair are created by the retroreflected beam, their direction of propagation is opposite to that of the first pair. The time between the first and second passages of the pulse through the crystal is shorter than the coherence time of the down-converted photons. The entangled pairs are in Bell states of horizontal H and vertical V polarisation, yielding an initial 4-photon state

$$|\Psi_i\rangle_{1234} = \frac{1}{\sqrt{2}} (|H\rangle_1|V\rangle_2 - |V\rangle_1|H\rangle_2) \times \frac{1}{\sqrt{2}} (|H\rangle_3|V\rangle_4 - |V\rangle_3|H\rangle_4). \quad (3.6)$$

Photons 2 and 3 are simultaneously input onto a polarising beam-splitter (PBS). Simultaneous detection of a photon in each output mode $2'$, $3'$, indicates that either both photons are horizontally polarised (transmitted by the PBS) or vertically polarised (reflected at the PBS). The final 4-photon state is a GHZ state,

$$|\Psi_f\rangle_{12'3'4} = \frac{1}{\sqrt{2}} (|H\rangle_1|V\rangle_{2'}|V\rangle_{3'}|H\rangle_4 + |V\rangle_1|H\rangle_{2'}|H\rangle_{3'}|V\rangle_4). \quad (3.7)$$

If the detectors are not simply single-photon detectors but are also polarisation-state analysers, photons 2 and 3 can be projected into a Bell state. Equation (3.6) can be written in terms of the four orthogonal Bell states (equation 2.16),

$$|\Psi_i\rangle_{1234} = \frac{1}{2} (|\psi^+\rangle_{14}|\psi^+\rangle_{23} - |\psi^-\rangle_{14}|\psi^-\rangle_{23} - |\phi^+\rangle_{14}|\phi^+\rangle_{23} + |\phi^-\rangle_{14}|\phi^-\rangle_{23}). \quad (3.8)$$

Measurement of photons 2 and 3 in one of their four possible Bell states results in photons 1 and 4 being entangled in their corresponding Bell state, although they have never interacted.

Single photon detection

It should be noted that all these experiments rely on the ability to detect single photons with high efficiency. A discussion of single photon detector technology is outwith the scope of this report; however, it is a key technology for the success of applications of quantum photonics.

3.2 Applications

The demonstration of violation of Bell's inequalities is one of the twentieth century's most significant experimental results. Zeilinger's experiment, showing a 30 standard deviation violation of the CHSH inequality [34], has been discussed in chapter 2, and Gisin's experiment [46] in section 3.1.1 above. The techniques and methodology which have been developed in their groups and elsewhere for these fundamental physics experiments provide a resource for many practical applications of entangled photons. Violation of Bell's inequalities is now frequently used as confirmation of the existence or preservation of entanglement; from being a result of profound philosophical importance, it has become a standard measurement technique.

In the rest of this section, some of these applications are discussed. Firstly, the use of entangled photons to provide an enhancement in the resolution of imaging or measurement techniques is reviewed. Next, applications in quantum communication, such as the detection of external interference or wilful lying are discussed. Finally, quantum computing with entangled photons is examined.

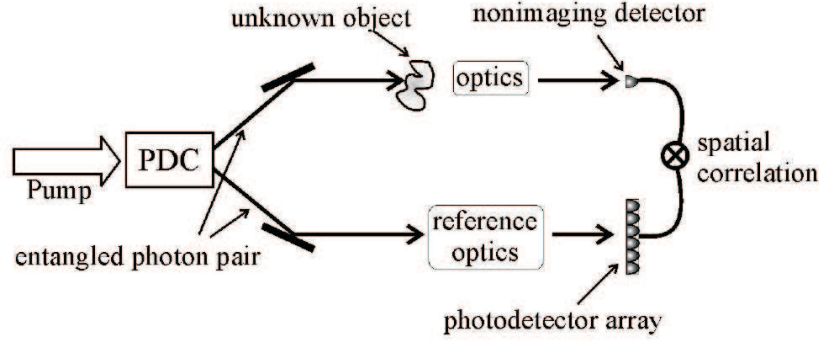


Figure 3.6: Schematic setup for two-photon imaging. (From Ref. [55].)

3.2.1 Quantum imaging

The photon pairs produced in SPDC can be used in novel two-photon imaging techniques, so-called **ghost imaging**, in which the transmission of an object placed in the beam of signal photons is measured by detecting coincidences between the signal and idler photons. Two-photon imaging may find applications in quantum communications and cryptography, providing a secure means for the transmission of complex images. figure 3.6 illustrates the concept: the signal photons are detected by a single-channel detector whereas an imaging detector is placed in the idler photon beam. The correlation between the photon pairs enables measurements to be made below the *standard quantum limit* (section 2.5.4). This technique was first demonstrated at the photon-counting level ten years ago by several groups including Yanhua Shih's group at the University of Maryland [56] and Zeilinger's Vienna group, in the double-slit experiment mentioned above [45]. The technique has been extended to the macroscopic intensity regime, where many photon pairs are detected and the photon-counting coincidences are generalised to the spatial correlation function of intensity fluctuations of the signal and idler beams [57]. However, it was recently demonstrated that all the features of these experiments can be reproduced using a classical incoherent light source [58], leading to an ongoing debate on the role of entanglement in ghost imaging [59]. It should be noted that the experiments discussed in this section are not explicitly interpreted in terms of violation of Bell's inequalities nor do they employ any measure of quality of entanglement.

An extension of ghost imaging is the technique of **quantum optical coherence tomography**, proposed by Sergienko's group at Boston University [60]. Classical optical coherence tomography is a 3-D imaging technique with a broad range of applications in biological and medical imaging. The sample forms the mirror in one arm of a Michelson interferometer; hence it is essentially an interferometric reflectance measurement. An image is formed of a slice of the sample at a depth determined by the position of the mirror in the other arm of the interferometer. The method uses the second-order coherence properties of the light source, hence its resolution is highest with sources of short coherence length such as superluminescent LEDs and femtosecond lasers. Imaging with photon pairs generated in SPDC has several advantages, including enhanced bandwidth (subject to the phase-matching conditions) and greater sensitivity for weakly reflecting samples.

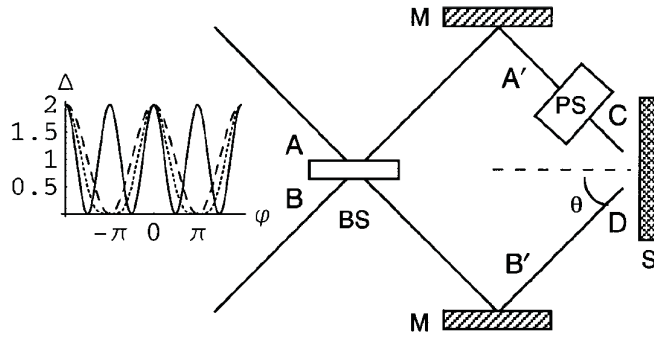


Figure 3.7: Two-photon interferometric lithography. Photons enter the interferometer at the beamsplitter BS. The photon probability amplitude in the upper path undergoes a phase shift of ϕ before interfering with the photon amplitude in the lower path at the substrate. The inset graph shows the deposition rate as a function of ϕ for uncorrelated single photon absorption (dashed), uncorrelated two-photon absorption (dotted), and entangled two-photon absorption (solid line). Reprinted figure with permission from A. N. Boto, Phys. Rev. Lett. **85**, 2733 (2000). Copyright (2000) by the American Physical Society.

A related application is **quantum lithography**. Optical lithography is a major technology in the semiconductor industry, being the principal method for fabricating circuit images on substrates. The resolution of conventional optical lithography cannot exceed the Rayleigh diffraction limit, $\lambda/2$; to achieve greater circuit density, it is necessary to use shorter wavelength lasers, to the point at which VUV or soft X-ray sources are contemplated. Schemes to beat the Rayleigh limit using classical two-photon interference techniques have been proposed. However, Dowling at JPL and Braunstein at the University of Wales at Bangor have proposed a technique using entangled photons which offers even greater resolution [61]. The principle of the technique is illustrated in figure 3.7, which shows an improvement of a factor of two in resolution over the Rayleigh limit. Using entangled states of N photons, the theoretical resolution limit is $\lambda/2N$.

Allied to spatial entanglement is *frequency entanglement*. Conservation of energy in SPDC requires that the frequency of an individual signal photon is correlated with that of the corresponding idler photon. Shih's group have exploited this to demonstrate **remote spectroscopy** [62]. Their experiment is analogous to ghost imaging: a spectral filter is placed in the signal beam whilst the spectral intensity of the idler photons is measured using a scanning monochromator. The transmission function of the filter as a function of signal wavelength is reproduced in the measured intensity profile as a function of idler wavelength. This technique could have applications in remote spectroscopy of the atmosphere or in space.

3.2.2 Clock synchronisation

Widely-separated entangled photons could be used in remote clock synchronisation (in other words, time transfer) and the related field of positioning.

In its simplest form, clock synchronisation using entangled photons depends only

on the simultaneity of creation of an entangled photon pair by SPDC. This procedure has been the subject of a proof-of-principle experiment by Shih's group [63]. In this experiment, which used fibre transmission, picosecond resolution was obtained over a distance of 3 km. As this is a one-way time transfer protocol, it suffers from the disadvantage that the dispersion of the transmitting medium has to be fully characterised and corrected for. Moreover, as in other experiments from Shih's group, it is not evident that the entanglement of the photon states is of particular significance, nor is any measure of entanglement used in the protocol.

The group of Seth Lloyd at MIT have suggested an alternative two-way time-transfer protocol which provides dispersion cancellation [64]. This *conveyor belt* protocol is purely classical; entanglement is used to enhance the accuracy of the scheme and to provide dispersion cancellation.

The MIT group have also proposed a more sophisticated scheme for both quantum-enhanced positioning and clock synchronisation [65]. This scheme provides an enhancement in the uncertainty δt with which the time of arrival of a pulse can be determined. For a classical light pulse, this is inversely proportional to the pulse bandwidth $\Delta\omega$ times the square root of the power in the pulse. Averaging over a pulse train of M coherent pulses

$$\Delta t = \frac{1}{\delta\omega\sqrt{MN}}, \quad (3.9)$$

where N is the number of photons in each pulse. Substituting the coherent pulse train for a train of frequency-entangled pulses, a gain in accuracy of $\sqrt{M}$ can be obtained using correlation measurements of the time-of-arrival. Additionally, using N -photon squeezed light pulses in place of classical pulses of average photon number N , an further gain of $\sqrt{N}$ is projected. (A discussion of squeezing and the generation of fixed photon number *Fock states* lies outside the scope of this report. For a recent review, see [19].) The enhancement factor of $\sqrt{MN}$ is shown to be the best physically possible. In principle, this scheme can be adapted to both time-transfer and positioning applications, characterised as QPS (Quantum Positioning System) in contrast to GPS. However, the technical challenges presented by the creation and distribution of the requisite entangled and squeezed photon states are formidable.

Neither of these proposals has to date been the subject of a demonstration experiment. These techniques have to be compared against a benchmark of the performance of current satellite time-transfer techniques, the best of which (two-way satellite time and frequency transfer, TWSTFT) currently achieves a precision of 1 ps and an instability of 100 ps at one day over site-to-site distances of up to 1000 km; improvements to an instability of 10 ps per day or better may be anticipated.

3.2.3 Quantum communication

Quantum communication techniques, in particular **quantum cryptography**, rely on the encoding of information in a stream of single qubits. No direct use of entanglement is made in the systems which are now commercially available. However, the use of entangled photons can enhance the security of these systems.

Table 3.1: Quantum coin tossing: Alice’s states and Bob’s bases. The labels identify these states in figure 3.8. Adapted figure with permission from G. Molina-Terriza *et al.*, Phys. Rev. Lett. **94**, 040501 (2005). Copyright (2005) by the American Physical Society.

Coin	Alice’s states	Label	Bob’s bases	Label
Heads	$\frac{1}{\sqrt{2}}(0\rangle + 1\rangle)$	A11	$\frac{1}{\sqrt{2}}(0\rangle + 1\rangle)$	B11
	$\frac{1}{\sqrt{2}}(0\rangle - 1\rangle)$	A12	$\frac{1}{\sqrt{2}}(0\rangle - 1\rangle)$	B12
			$ 2\rangle$	B13
Tails	$\frac{1}{\sqrt{2}}(0\rangle + 2\rangle)$	A21	$\frac{1}{\sqrt{2}}(0\rangle + 2\rangle)$	B21
	$\frac{1}{\sqrt{2}}(0\rangle - 2\rangle)$	A22	$\frac{1}{\sqrt{2}}(0\rangle - 2\rangle)$	B22
			$ 1\rangle$	B23

Quantum cryptography

Quantum cryptography, also known as **quantum key distribution**, provides a secure method for the exchange of a secret encryption key between two parties. In contrast to classical key distribution schemes, which depend for their security on mathematical complexity, quantum key distribution depends for its security on the fact that it is impossible to measure a quantum system without modifying it. Without full knowledge of the original quantum state prepared by the transmitter, Alice, an eavesdropper (conventionally named Eve) cannot cover her tracks by replicating the qubit she has measured: this is known as the *no-cloning theorem*. Consequently, her presence is signalled by an increased error rate in the transmitted data.

Most quantum cryptography systems, including those now commercially-available, transmit data using trains of faint laser pulses, in which most pulses contain zero photons. Schemes using entangled photon pairs offer various advantages; in particular they are more robust against eavesdropping strategies. Gisin’s group have demonstrated one such scheme using time-bin photon Bell states [47]. Coupled with their demonstration of distribution of entanglement by fibre over distances of up to 10 km [33] and similar experiments by Zeilinger’s group demonstrating free-space distribution of entanglement [66], the feasibility of this technique has been established. However, for truly long-range quantum communication, the teleportation techniques discussed below need to be brought into play.

Quantum cheating

Much of the work on the application of entanglement to quantum cryptography and quantum communication is directed towards methods for detecting the interference of an eavesdropper, Eve, in communication between two parties who trust each other. What happens when the two parties do not trust each other? Suppose Alice and Bob agree to reach a decision by Alice tossing a coin, but Bob cannot see the coin and does not trust Alice to honestly report the outcome. Can Bob detect Alice’s dishonesty? This coin-tossing protocol is a paradigm for cryptographic problems such

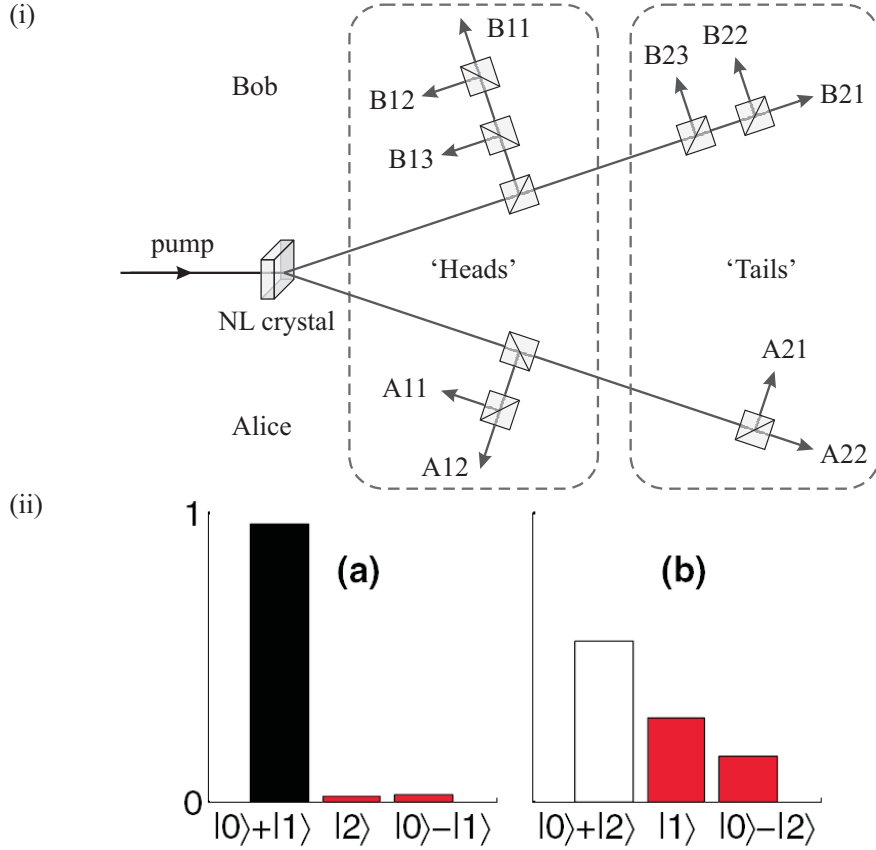


Figure 3.8: (i). The Vienna group's quantum coin tossing experiment. Adapted figure with permission from G. Molina-Terriza *et al.*, Phys. Rev. Lett. **94**, 040501 (2005). Copyright (2005) by the American Physical Society. (ii). Statistics of Bob's quantum coin tossing measurements. Black represents 'heads'; white 'tails'; grey, failures of the protocol. (a). Alice is honest, having sent the state $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ (heads). (b). Alice cheats, sending a mixture of two states. Here she claims to have sent the state $\frac{1}{\sqrt{2}}(|0\rangle + |2\rangle)$. Reprinted figure with permission from G. Molina-Terriza *et al.*, Phys. Rev. Lett. **94**, 040501 (2005). Copyright (2005) by the American Physical Society.

as mail certification or contract signing and is also applicable to the computation of a function with distributed outputs. There is no classical protocol which gives security against cheating on Alice's part. The Vienna group have demonstrated a quantum coin tossing protocol [67], in which the box is replaced by a quantum state. The protocol is a success if the state Bob measures is that which Alice claims she sent, otherwise it is counted as a failure. Cheating on Alice's part can be detected if the number of failures increases over the number expected from statistical errors.

Alice has a Type-I SPDC source of photon pairs, which are entangled in orbital angular momentum. The photons retained by Alice and sent to Bob can be in three possible states (*qutrits* rather than qubits): a Gaussian mode ($|0_A\rangle$ and $|0_B\rangle$), or Laguerre-Gaussian modes $m = \pm 1$ ($|1_A\rangle$ and $|2_B\rangle$ or $|2_A\rangle$ and $|1_B\rangle$). The entangled state used in the experiment is $|\Psi\rangle = \frac{1}{\sqrt{2}}|0_A 0_B\rangle + \frac{1}{2}|1_A 1_B\rangle + \frac{1}{2}|2_A 2_B\rangle$. Using a series

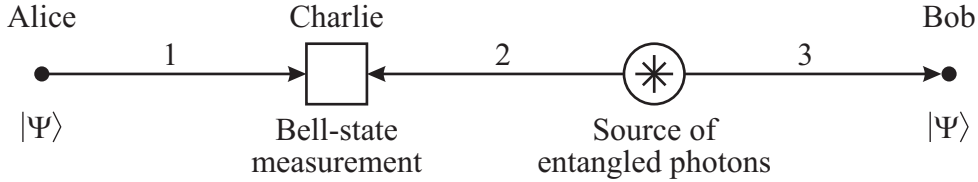


Figure 3.9: Quantum teleportation used as a quantum relay. A photon sent by Alice is teleported to Bob using the entangled photon pair. Two photons are travelling towards Bob and one towards Alice. The time-ordering of Charlie’s Bell-state measurement and Bob’s qubit state measurement is irrelevant: the logical qubit can be considered as propagating all the way from Alice to Bob. (Adapted from Ref. [48].)

of beamsplitters and photon mode analysers (figure 3.8i), Alice projects her photon onto one of four states, one pair of which correspond to ‘heads’ and the other, ‘tails’. Each pair of states are orthogonal, but the ‘heads’ states have a non-zero projection onto the ‘tails’ states (table 3.1). Bob’s photon, entangled with Alice’s, is projected into a corresponding state. Alice’s detection of a photon confirms that a photon qutrit has been sent to Bob; she indicates this via a classical channel and Bob makes his guess ‘heads/tails’. Alice then tells Bob what state her photon was in. Bob now measures the angular momentum state of his photon in this basis; he needs to be able to measure in two bases, corresponding to ‘heads’ and ‘tails’, each of which contains not only Alice’s states but also a third, orthogonal, state. These additional states are crucial to the detection of cheating.

Figure 3.8ii(a) shows the outcome of Bob’s measurements when Alice is honest. She sends the state $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ (heads). Bob makes his guess, which is correct in 50% of cases; Alice tells him which state she sent and Bob measures in the appropriate basis, confirming Alice’s report. In the experiment, the protocol fails in about 6% of cases due to small misalignments of the apparatus.

Figure 3.8ii(b) shows what happens if Alice cheats. In this case, she always sends a random mixture of the states $\frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$ (heads) and $\frac{1}{\sqrt{2}}(|0\rangle + |2\rangle)$ (tails). When Bob makes his guess, Alice always tells him he has lost: *e.g.* if Bob calls ‘tails’, Alice tells him the state was $\frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$ and Bob measures in this basis. Cheating in this way, Alice will win in 62.5% of cases, the remaining 37.5% of cases appearing to Bob as ‘failures’. (In the experiment, the failure rate rises to 46% because Alice is not sending a perfect mixture of the two states.) For the majority of ‘failures’, Bob finds the photon in the third orthogonal state, providing clear confirmation of Alice’s cheating, quite apart from the fact that Alice wins all the successful cases.

Teleportation

In the context of quantum systems, *teleportation* denotes the reconstruction of the quantum state of a system at a remote location, the necessary quantum information being transferred by a qubit. Quantum teleportation was first proposed by Bennett in 1993 [68] and first demonstrated with photons [69, 70]. The main application envisioned for quantum teleportation using entangled photons is the *quantum relay* 3.9, which would be used to extend the range of quantum communication networks.

In quantum cryptography, the signal decreases exponentially with distance due to attenuation, whereas the dominant noise source, the dark count rate of the detector, is independent of the distance. This imposes an effective range of 100 km or less. Gisin's group have demonstrated teleportation of qubits in fibre optics at telecommunications wavelengths over distances of up to 2 km [48], whilst Zeilinger's group have demonstrated free-space teleportation over 600 m in an urban environment [71]. An important feature of these Vienna experiments is that the teleported photon remains available as a resource for further processing [52]. The experimental scheme is similar to the entanglement-swapping scheme described in section 3.1.3, except that an additional attenuator is placed in one arm of Alice's two-photon interferometer (see figure 3.5). Consequently, an equivalent attenuation is introduced in the frequency with which an entangled photon is transmitted to Bob. A successful teleportation event can be detected with high probability using the auxiliary entangled photon pair, without performing a measurement at Bob's location. A fidelity of 85% is obtained, well above the classical limit and hence at a level which is adequate for the implementation of a quantum repeater.

3.2.4 Quantum computing

The goal of constructing a quantum computer is the principal driver for the development of techniques for the manipulation of entangled states of photons, no less than of other technologies. A scheme for quantum computation using linear optics (LOQC) has been proposed by Knill, Laflamme, and Milburn [72], based on sequences of unitary quantum logic gates constructed from simple components such as beam-splitters, waveplates, and phase-shifters. Techniques such as entanglement swapping and teleportation are the building-blocks for such a computer; a key requirement is thus the ability to generate multi-photon entanglement.

An alternative approach is the **one-way quantum computer** proposed by Raussendorf and Briegel [73]. This scheme, which has been demonstrated by Zeilinger's group [74], requires the qubits to be initialised in a *cluster state*, a highly-entangled state related to the GHZ state. Quantum computation proceeds by a series of measurements on single qubits; the algorithm to be implemented corresponds to a certain order and choice of measurements. The key role played by measurement in the progress of the computation makes it irreversible, hence the description *one-way*. In the Vienna experiment, a four-photon cluster state was used to implement Grover's search algorithm [39].

Because of the low efficiency of non-linear processes such as SPDC and of single-photon detectors, processes relying on entangled photon states are probabilistic, in contrast to the deterministic entanglement which will be encountered in the following chapters. The principal role of photons in quantum computing may be to transfer entanglement from one deterministic qubit system to another, *i. e.* as *flying qubits*, using techniques such as those discussed in section 4.1.3 and chapter 6.

3.3 Summary conclusions

The field of quantum cryptography has demonstrated over the last twenty years how a concept based on quantum mechanics and single photons can be transformed into innovative products now available in the marketplace. Currently there are four companies worldwide that are marketing fibre or free-space based quantum cryptography systems: QinetiQ (UK), Toshiba Research Europe (UK), idQuantique (Switzerland), and MagiQ Technologies (US). This is clear evidence of how a photon-based quantum technology has developed from ideas through laboratory demonstrations to products. Metrology at the single-photon level has not really kept pace with these developments, so there is now a requirement for companies and NMIs to work towards this.

The example of quantum cryptography illustrates the future possibilities for technologies based on entangled photons. While many ideas are at the laboratory demonstration level (or even yet to be demonstrated), it is possible to envisage varieties of quantum information processors using entangled photons on a photonic technology platform. In addition, the possibilities for enhanced precision in specific measurements will progress as sources of entangled photons are developed further, driven by the QIP incentive. We anticipate that it is highly desirable (and likely) that optical photons will feature in transmission of quantum information.

In order to determine the utility of any future source of entangled photons, there are specific parameters that need to be quantified. We anticipate that as this technology progresses, the need for more precise measures of these relevant parameters. NPL is currently developing a measurement infrastructure for single and entangled photons, through its “Entangled Photon Metrology” project in the current quantum programme. This project will allow NPL to be an active player in developing current measurement techniques, and be ideally positioned to innovate in future measurement techniques as and when next generation technology demands greater precision.

Currently there exists no visible route to achieving large-scale deterministic entanglement with photons. This status may change in the future if and when efficient and repeatable deterministic sources of single photons be developed. Until such times, entangled-photon sources will be limited to N of a few. It is therefore unclear that technologies such as clock synchronisation based on entangled photons will in fact be capable of improving the precision beyond more conventional means. In order to determine this (or alternatively to determine the level of N at which it becomes viable), a detailed analysis of the achievable signal-to-noise is required.

Chapter 4

Entangled atomic particles

Research on entangled states of ensembles of trapped atomic particles has principally been driven by the 1995 proposal of J. I. Cirac and P. Zoller, then at the University of Innsbruck, that such systems could be used to implement the basic elements of a quantum computer [75]. Each individual atom constitutes a qubit, entanglement being achieved through coupling of the atoms by their collective quantised motion within the trapping potential. In contrast to photons, entangled states of atoms are intrinsically deterministic: because the atoms are confined and individually addressable, their entangled states can be manipulated and interrogated at will and can be available as a resource for further quantum information processing.

Although neutral atoms confined in optical lattices are also being studied (see section 4.3 below), the field is led by research groups working with singly-ionised atoms confined in electrodynamic traps, to the point at which it is possible to sketch out a design for a 300 qubit, 1 Gop quantum computer using proven or realistic technology [76]. The development of quantum information processing using neutral atoms or ions is closely related to the development of optical frequency standards based on such systems. The apparatus required, and the technical challenges to

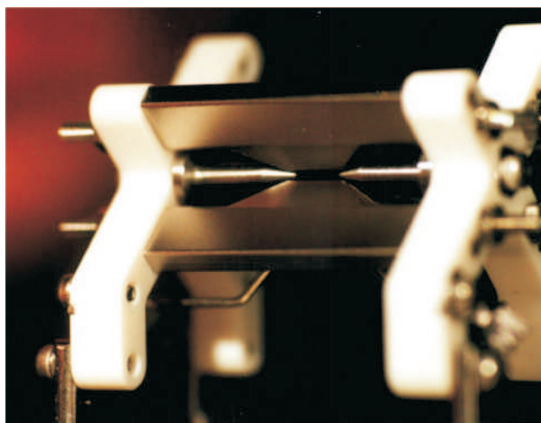


Figure 4.1: Linear quadrupole ion trap used by Rainer Blatt's group at the University of Innsbruck. The endcap electrodes (pointed) are separated by 6 mm. Image ©Inst. f. Experimentalphysik, Univ. Innsbruck. Reproduced with permission.

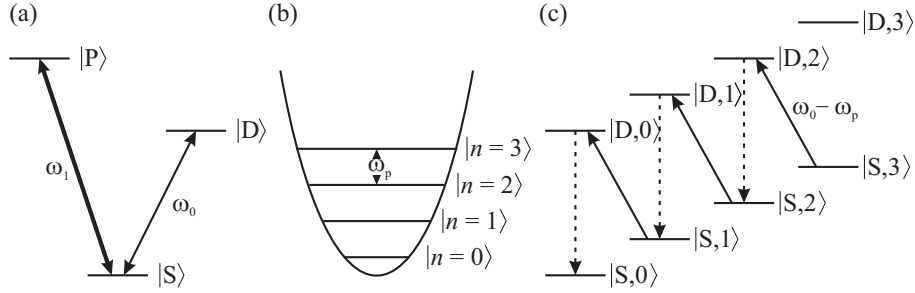


Figure 4.2: (a). A three-level atom as a qubit. The ground state $|S\rangle$ is coupled to the metastable state $|D\rangle$ by the clock transition at ω_0 and to the short-lived state $|P\rangle$ by the resonance transition at ω_1 . (b). The harmonic oscillator potential of the lattice (atoms) or Paul trap (ions). The vibrational energy levels are separated by the phonon frequency ω_p . (c). Dressed-state energy levels of the trapped atom, illustrating the preparation of the atom in its internal ground state and the lowest vibrational state $n = 0$ by sideband cooling.

be faced, are very similar. The prior development of techniques for the trapping, cooling, and interrogation of small numbers of ions for optical frequency standards enabled several research groups to move quickly into the field; now, techniques developed for the manipulation of trapped ion quantum logic gates are beginning to feed back into improvements in the performance of optical frequency standards. This symbiotic relationship is best illustrated by the work of the Ion Storage Group, led by Dave Wineland and Jim Bergquist, in the Time and Frequency Division of NIST, the highlights of which will be discussed below, but is also evident in the Time, Frequency, and the Metre Group at NPL.

In this chapter, entanglement of atomic qubits is principally discussed using the example of the experiments with ions performed in the groups of Rainer Blatt at the University of Innsbruck and Dave Wineland at NIST, Boulder. The technologies now emerging to scale the current generation of experiments to larger numbers of qubits are discussed along with techniques for sharing entanglement between remotely-located atomic qubits (so-called *flying qubits*). Applications to quantum information processing and optical frequency standards using ions are surveyed, along with some promising techniques in the less well-developed field of neutral atom entanglement.

4.1 Quantum gates with atomic qubits

Figure 4.2(a) illustrates a generic three-level atom as a qubit. The qubit states are the ground state, $|S\rangle$ and a metastable state, $|D\rangle$. The metastable state decays only to the ground state *via* a weak optical transition; its natural lifetime constitutes the limiting source of decoherence in the system. The qubit can be manipulated using laser radiation tuned to this transition (frequency ω_0). Readout of the qubit state is performed using laser radiation tuned to the resonance transition to the state $|P\rangle$ (frequency ω_1): the presence or absence of fluorescence on the resonance transition indicates whether the atom is in the ground state $|S\rangle$ or the ‘dark’ state $|D\rangle$.

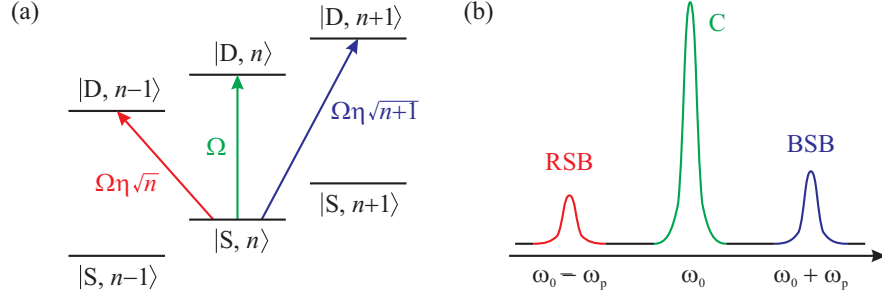


Figure 4.3: Sidebands observed in spectroscopy of an ion cooled to low- n vibrational state of the trap. (a). Dressed-state energy levels of the trapped atom, showing transitions between states of the same n ($\Delta n = 0$), Rabi frequency Ω , and $\Delta n = \pm 1$ transitions, with Rabi frequencies $\Omega\eta\sqrt{n+1}$ and $\Omega\eta\sqrt{n}$. (b). The observed spectrum, consisting of a carrier C at ω_0 ($\Delta n = 0$ transitions) and blue (BSB) and red (RSB) sidebands at $\omega_0 \pm \omega_p$ ($\Delta n = \pm 1$ transitions).

This is essentially the pulsed-probe spectroscopy using Dehmelt’s electron shelving technique (so-called *quantum jump spectroscopy*) which is used in trapped ion optical frequency standards, where the $|S\rangle \rightarrow |D\rangle$ transition is the clock transition.

The atomic qubits are confined within a trapping potential. For neutral atoms, this is the potential well formed by each optical lattice site. For ions, it is the quadrupole field of a Paul trap; this is typically a linear trap, in which a string of ions can be stored with a spacing of a few μm between each ion. To a good approximation, the motional modes of the atom in the trap are uniformly-spaced harmonic oscillator states, characterised by vibrational quantum number n , figure 4.2(b). These vibrational states are separated by the phonon frequency ω_p . The atom is thus fully described by a ladder of *dressed states* $|i, n\rangle$, where i represents the internal electronic state, figure 4.2(c). Spectroscopy of an ion in an external state of low n on the $|S\rangle \rightarrow |D\rangle$ transition reveals not only a strong peak at ω_0 , corresponding to transitions between pairs of dressed states with no change in vibrational quantum number but also weaker *blue* and *red* sidebands at frequencies $\omega_0 \pm \omega_p$, figure 4.3. These sidebands arise from transitions $|S, n\rangle \rightarrow |D, n \pm 1\rangle$, *i. e.* transitions in which the vibrational state is changed. Denoting the Rabi frequency of the $\Delta n = 0$ transition by Ω , the Rabi frequencies of the blue and red sidebands are $\Omega\eta\sqrt{n+1}$ and $\Omega\eta\sqrt{n}$ respectively, where the Lamb-Dicke parameter $\eta \ll 1$.

By tuning a laser to the red sideband, the ions can be cooled to their lowest collective vibrational state, since the $\Delta n = 0$ transition is the dominant channel for spontaneous decay. This technique is called *sideband cooling*. All the ions are then in the state $|S, 0\rangle$; a single ion can now be excited to the $|D\rangle$ state by a π -pulse of laser radiation tuned to the blue sideband. Since the ions’ vibrational motion is strongly coupled through their Coulomb repulsion, all the ions are excited into the common vibrational mode $n = 1$.

In Cirac and Zoller’s original proposal for a trapped ion quantum computer [75], entanglement between neighbouring ions in a linear trap is achieved through the excitation of collective vibrational modes of the ion string. These vibrational states are therefore referred to as *bus modes*.

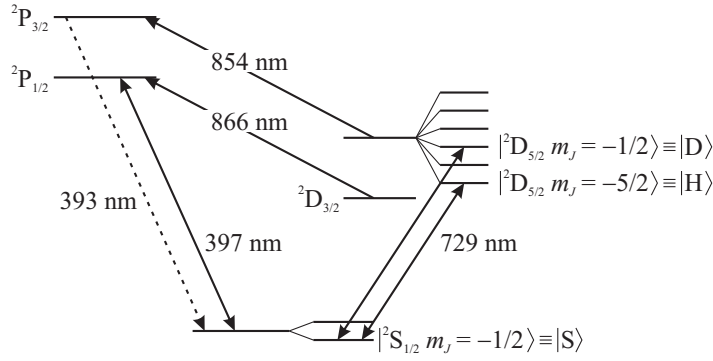


Figure 4.4: Implementation of a qubit using the $^2\text{S}_{1/2}(m_J = -1/2) - ^2\text{D}_{5/2}(m_J = -1/2)$ electric quadrupole transition in $^{40}\text{Ca}^+$.

4.1.1 The Cirac-Zoller scheme implemented at Innsbruck

The group of Rainer Blatt at the University of Innsbruck employs the Cirac-Zoller scheme in $^{40}\text{Ca}^+$ ions. Figure 4.4 illustrates in more detail the internal level structure of the $^{40}\text{Ca}^+$ ion. The qubit state $|S\rangle$ is the $m_J = -1/2$ magnetic substate of the $^2\text{S}_{1/2}$ ground state, whilst qubit state $|D\rangle$ is the $m_J = -1/2$ magnetic substate of the $^2\text{D}_{5/2}$ state. The D-state is metastable, with a natural lifetime of 1.0 s, decaying to the ground state by an electric quadrupole transition at 729 nm. State detection is performed on the $^2\text{S}_{1/2} - ^2\text{P}_{1/2}$ resonance transition at 397 nm. This resonance transition does not form a closed two-level system. The $^2\text{P}_{1/2}$ state can also decay to the $^2\text{D}_{3/2}$ state; a repumper laser at 866 nm is required during the state detection. If, after state detection, the ion is in the metastable state, it can be desirable rapidly to de-excite it from this state. A clearout laser at 854 nm is used to return the ion to the ground state *via* the $^2\text{P}_{3/2}$ state.

In order to implement quantum logic gates, a number of ions are held in the trap. Figure 4.1 shows the design of ion trap used in their recent work. The ions are separated by $5\text{ }\mu\text{m}$ and can be individually interrogated. The laser light, focused to $2.5\text{ }\mu\text{m}$, can be directed onto each ion by steering the beam using an electro-optic deflector. The intensity incident on the neighbouring ion is a factor of 400 less than at the target ion; additionally the neighbouring ions can be protected from accidental excitation by transfer to a hidden state $|H\rangle$, which does not couple to the laser light. The $m_J = -5/2$ magnetic substate of the metastable $^2\text{D}_{5/2}$ state is used; its 1 s lifetime enables the ion to be stored while a complex series of operations is carried out on neighbouring ions.

Figure 4.5 illustrates the sequence used to implement the CNOT gate in the Innsbruck experiment [77]. The two ion qubits are prepared in the desired superposition states: for the data shown in figure 4.6(a), ion 2 (the target qubit) is prepared in state $|S\rangle$ and ion 1 (the control qubit) in the superposition $\frac{1}{\sqrt{2}}(|S\rangle + |D\rangle)$. The first step is to transfer this superposition state to the vibrational motion, by means of a π -pulse on the blue sideband of the ion 1. This SWAP operation generates the vibrational state $\frac{1}{\sqrt{2}}(|1\rangle + |0\rangle)$. A sequence of pulses on the blue sideband of ion 2 then perform a phase gate operation. This sequence is framed by $\pi/2$ -pulses on

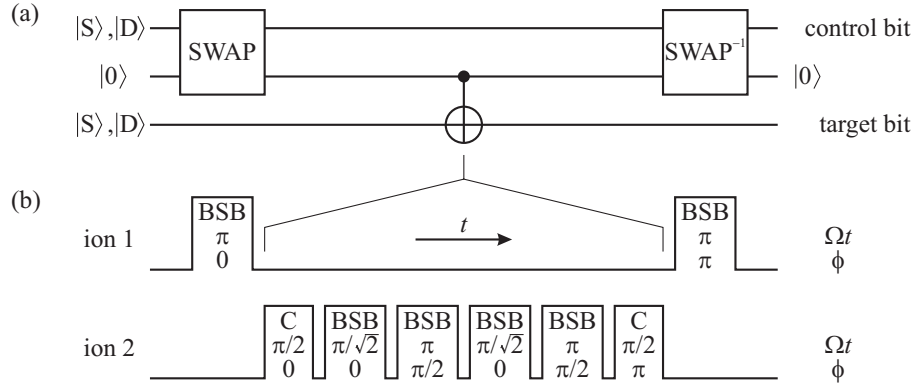


Figure 4.5: Sequence for the quantum CNOT gate. (a) Block diagram of the gate operations: the SWAP operation transfers quantum information from ion 1, the control qubit, to the common vibrational mode; then, the internal state of ion 2, the target qubit, is flipped, conditional on the state of the vibrational mode; finally, the SWAP $^{-1}$ operation transfers the quantum information back from the vibrational mode to the control qubit. (b) The corresponding laser pulse sequence. For each pulse, the laser frequency (carrier C or blue sideband BSB) is indicated (top), along with the value of Ωt (middle) and the phase ϕ (bottom) in radians.

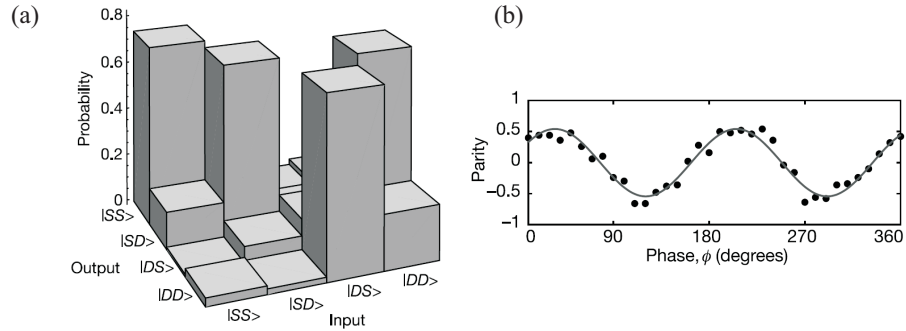


Figure 4.6: The Innsbruck group's Cirac-Zoller CNOT gate. (a) Representation of the experimentally-observed truth table. (b) Demonstration of the quantum nature of the gate operation. Inputting the superposition $1/\sqrt{2}(|S\rangle + |D\rangle)|S\rangle$ to the gate results in an entangled output state $1/\sqrt{2}(|SS\rangle + e^{i\alpha}|DD\rangle)$. After the gate operation, a $\pi/2$ -pulse is applied to each ion, with relative phase ϕ . The parity $P = (P_{SS} + P_{DD}) - (P_{SD} + P_{DS})$ (P_{ij} denotes the probability of finding the ions in the state $|ij\rangle$) is measured as a function of ϕ . P is observed to oscillate at $\cos(2\phi)$, whereas a non-entangled state P would vary with $\cos(\phi)$. From the visibility of the oscillation, the fidelity of creation of an entangled state is $F = 0.71(3)$. Reprinted by permission from Macmillan Publishers Ltd: Nature. F. Schmidt-Kaler *et al.*, Nature **422**, 408 (2003), copyright (2003).

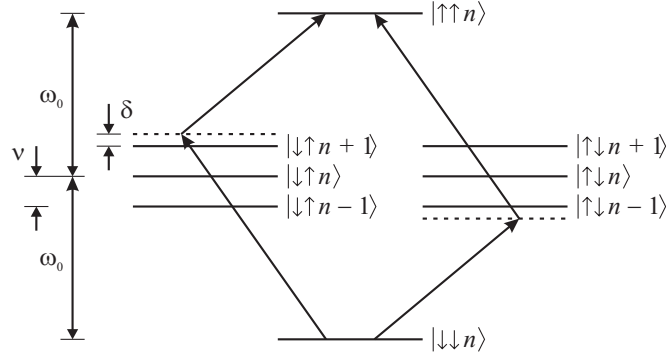


Figure 4.7: The Mølmer-Sørensen scheme for creation of entanglement. The qubit states of each ion, separated in frequency by ω_0 , are labelled $|\downarrow\rangle$ and $|\uparrow\rangle$. The collective vibrational states, containing n quanta, are separated in frequency by ν . The ions are prepared in the state $|\downarrow\downarrow n\rangle$. Laser light at frequencies $\omega_0 + \nu + \delta$ and $\omega_0 - \nu - \delta$, close to the blue and red motional sidebands, is applied to both ions with equal intensity, driving the transition $|\downarrow\downarrow n\rangle \rightarrow |\uparrow\uparrow n\rangle$ through the intermediate states $\frac{1}{\sqrt{2}}(|\downarrow\uparrow n+1\rangle + e^{i\phi_-}|\downarrow\uparrow n-1\rangle)$; ϕ_- is the phase difference between the laser fields at the two ions. The detuning δ is large enough that these intermediate states are negligibly occupied, so that the motional state is unaltered. Applying the laser fields for a time corresponding to a $\pi/2$ -pulse, the entangled state $\frac{1}{\sqrt{2}}(|\uparrow\uparrow\rangle + e^{i\phi_+}|\downarrow\downarrow\rangle)$ is created, where ϕ_+ is the sum of the field phase at the two ion positions.

the carrier, effectively performing a Ramsey interaction with 180° change of phase; the full sequence of six pulses constitutes the CNOT operation between the vibrational and target qubits. Finally, an inverse SWAP operation on ion 1 transfers the outcome of the CNOT operation from the vibrational qubit to the control qubit. In this case, the result is entanglement of the two ions in a Bell state. Applying an additional $\pi/2$ -pulse to each ion, with a phase difference ϕ , measurement of the parity confirms the entangled nature of the final state to be confirmed and provides a measurement of the fidelity (see figure 4.6(b)).

This CNOT operation takes around 0.6 ms. The fidelity is limited by technical issues such as laser frequency noise and control of the laser beam position; to achieve $F \sim 1$, more complex pulse sequences along with quantum control techniques may be required, together with a very low decoherence rate (see section 4.1.4).

4.1.2 The Mølmer-Sørensen scheme implemented at NIST

The group of Dave Wineland at NIST, Boulder, employ the Mølmer-Sørensen scheme in which, rather than entangling each ion sequentially as in the Cirac-Zoller scheme, a N -ion entangled state can be created in a single step, offering the possibility of faster gate speed. [78]. Figure 4.7 illustrates the principle of the Mølmer-Sørensen scheme for creating entanglement between two ions. This scheme has the advantage that the ions do not need to be in the motional ground state of the trap; however, the need to be detuned from resonance with the motional sidebands means that the entangling operation is much slower than a single sideband operation. To achieve *fast*

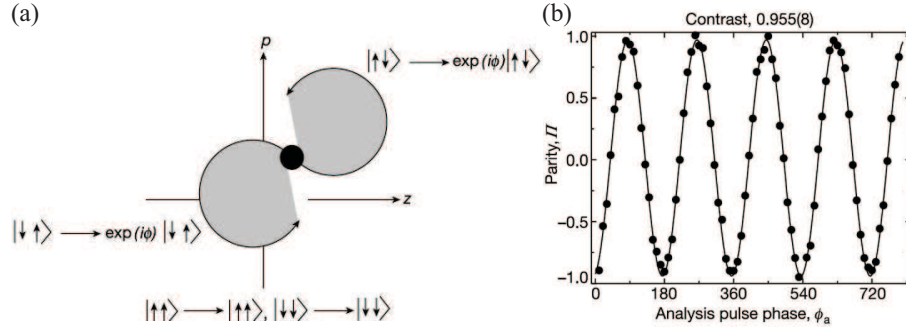


Figure 4.8: The NIST group’s phase gate. (a) Phase-space trajectories of the intermediate vibrational states of the ion pair. (b) Demonstration of the quantum nature of the gate operation, obtained in a similar manner to figure 4.6(b). The parity $P = (P_{\downarrow\downarrow} + P_{\uparrow\uparrow}) - (P_{\downarrow\uparrow} + P_{\uparrow\downarrow})$ is observed to oscillate at $\cos(2\psi)$, with fidelity $F = 0.97(2)$. Reprinted by permission from Macmillan Publishers Ltd: Nature. D. Leibfried *et al.*, Nature **422**, 412 (2003), copyright (2003).

entanglement, the NIST group use a modification of the Mølmer-Sørensen scheme, in which the laser fields are resonant with the sidebands. In this case, the intermediate vibrational states are occupied. Figure 4.8(a) illustrates the circular trajectories of the states in phase space. Both intermediate vibrational states accumulate the same phase since the enclosed phase-space areas are equal and the trajectories rotate in the same sense. At the moment when the entangled state is created, the occupation of the intermediate states vanishes. This process is only a factor of two slower than a single sideband operation and has the additional advantage that it is less sensitive to magnetic field fluctuations.

The Mølmer-Sørensen scheme provides a route to *scaleable entanglement* (section 4.1.5): the same operation can be used to entangle any even number of ions. For more than two ions, care has to be taken in choosing modes which avoid unwanted heating and decoherence effects. Suitable ‘stretch’ modes exist, in which ions in the string oscillate out of phase with their neighbours but with the same amplitude; alternatively, the laser intensity at each ion can be adjusted to balance the difference in motion, ensuring that all ions experience the same sideband coupling.

Most of the NIST group’s work has been with $^9\text{Be}^+$ ions. The qubit states are the hyperfine sublevels $|F = 2, m_F = -2\rangle$ and $|F = 1, m_F = -1\rangle$ of the $^9\text{Be}^+$ ground state ($|\downarrow\rangle$ and $|\uparrow\rangle$ in their preferred notation), transitions between which are driven by a two-photon Raman transition detuned by ~ 82 GHz from the $^2\text{S}_{1/2} - ^2\text{P}_{1/2}$ resonance transition at 313 nm. The difference frequency between the two lasers used to drive the Raman transition (ω_0 in figure 4.7) is the hyperfine splitting frequency, 1.25 GHz. This scheme provides the possibility of very long coherence times. The excited hyperfine states have long natural lifetimes, ensuring that spontaneous decay is not the limiting source of decoherence (see section 4.1.4).

Using these techniques, the NIST group have demonstrated a two-ion phase gate, corresponding to the central four-pulse sequence of the Innsbruck CNOT gate [79], flanked by state preparation pulses. The speed of this gate was $78 \mu\text{s}$, limited by the laser intensity. Figure 4.8(b) illustrates the high fidelity obtained in this experiment.

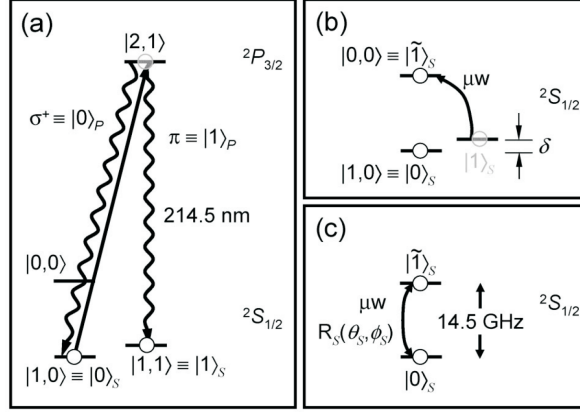


Figure 4.9: Entangled ion-photon Bell experiment at the University of Michigan. (a). The ion is excited to the ${}^2P_{3/2} |F=2, m_F=1\rangle$ state from which it can decay to either the $|F=1, m_F=0\rangle (\equiv |0\rangle_S)$ or $|F=1, m_F=1\rangle (\equiv |1\rangle_S)$ ground state hyperfine levels by spontaneous emission, creating a superposition of these two ion qubit states. The emitted photon is σ -polarised ($|0\rangle_P$) if the decay is to $|0\rangle_P$, or π -polarised ($|1\rangle_P$) if to $|1\rangle_S$; thus its polarisation state is entangled with the ion's hyperfine state. (b). A microwave pulse transfers the qubit state $|1\rangle_S$ to an effective qubit state $|\tilde{1}\rangle_S \equiv |F=0, m_F=0\rangle$ in preparation for state detection. (c). A microwave pulse at the hyperfine splitting frequency, 14.5 GHz, performs a rotation of the hyperfine qubit to a randomly-chosen basis. Correlations between the photon polarisation state and the ion's hyperfine state are then measured. Reprinted figure with permission from D. L. Moehring *et al.*, Phys. Rev. Lett. **93**, 090410 (2004). Copyright (2004) by the American Physical Society.

4.1.3 Flying qubits

The two-ion gates discussed above make use of three qubits: the internal state qubits of the ions and the common vibrational mode qubit, which is the ‘bus’ for transfer of information between the ions. The distance over which this quantum information can be transferred is limited by the physical dimensions of the trap; even with the microfabricated traps under development by the NIST group, this distance will remain small. In a real quantum computer, it will be necessary to have a means of transferring quantum information over larger distances. As discussed in chapter 3, quantum information can be best carried over large distances by photons: the vibrational bus mode of the ion trap could be replaced by the exchange of photons between remote trapped ions: so-called *flying qubits*. The group of Chris Monroe at the University of Michigan have demonstrated entanglement of a trapped ${}^{111}\text{Cd}^+$ ion and emitted photon [80, 81]. Figure 4.9 illustrates a Bell experiment on the entangled state of the the polarisation of a spontaneously-emitted photon and the ion's internal hyperfine state [82]. In this experiment, analysis of the photon's polarisation state was performed at a distance of 1.1 m from the trapped ion, demonstrating the non-local nature of the entanglement. The Bell state $|\psi\rangle = \frac{1}{\sqrt{2}}(|0\rangle_S|0\rangle_P + |1\rangle_S|1\rangle_P)$ was created with a fidelity of $F = 0.87$.

Although the entanglement is probabilistic (the photon is emitted spontaneously), it should be possible to perform deterministic quantum gate operations between remote ions *via* entanglement of the photons emitted by remote ions [83]. If the ions are prepared in the same internal superposition state, the superposition states of the emitted photons can be entangled using the entanglement swapping techniques discussed in 3.1.3: if the photons are in an entangled state, then the ions with which they are entangled must also be entangled with each other. Deterministic entanglement of atoms and photons can be achieved using the cavity QED techniques discussed in chapter 6.

4.1.4 Robust entanglement

It is widely held that quantum entanglement is fragile: any interaction with the environment which distinguishes between the entangled particles will bring about collapse of the entangled state, *i. e.* entangled states must decohere rapidly. This argument is frequently advanced against the feasibility of a quantum computer. To rebut this, the Innsbruck group have recently demonstrated the survival of an entangled state of two $^{40}\text{Ca}^+$ ions for more than 20 s: so-called robust entanglement [84].

In the $^{40}\text{Ca}^+$ experiments discussed above, the 1 s lifetime of the $|D\rangle$ state limits the coherence time of the qubit. Recall that the qubit states $|S\rangle$ and $|D\rangle$ are the $m_J = -\frac{1}{2}$ magnetic substates of the ground and metastable states. Just as an ion in the state $|S\rangle$ can be hidden from the readout laser tuned to the resonance transition by transfer to the ‘hidden’ $m_J = -\frac{5}{2}$ substate of the $^2D_{5/2}$ state, so an ion in the state $|D\rangle$ can be stored beyond the natural lifetime of that state by transfer to the $m_J = +\frac{1}{2}$ substate of the ground state: the ion is transferred to a *decoherence-free sub-space*. In both cases the transfer is accomplished by a π -pulse of laser radiation tuned to the appropriate Zeeman component of the clock transition. The effective qubit states are now both substates of the ground state, prevented from decoherence by an applied magnetic field which lifts their degeneracy; the main source of decoherence is heating of the vibrational modes of the trap, as in the NIST scheme (which should in principle show similarly long coherence times). After the required storage time, the ion is restored to the qubit state $|D\rangle$ by a second π -pulse.

This operation is performed on a pair of ions prepared in the Bell state $|\psi\rangle = \frac{1}{\sqrt{2}}(|S\rangle|D\rangle + |D\rangle|S\rangle)$. This entangled state is relatively immune to decoherence due to external perturbations since its two component two-ion states $|S\rangle|D\rangle$, $|D\rangle|S\rangle$ have the same energy; it shows a decoherence time of 1 s corresponding to the metastable state lifetime. This Bell state is recovered with a fidelity F significantly greater than 0.5 (indicating the presence of entanglement) for storage times of up to 20 s; the inferred decoherence time is 34(3) s.

4.1.5 Scaleable entanglement

One of the principal challenges to all quantum information processing technologies is to find ways of scaling up single gate demonstration experiments to many-gate devices capable of running complex algorithms. The NIST group are using techniques for storing quantum information held by trapped ions in decoherence-free

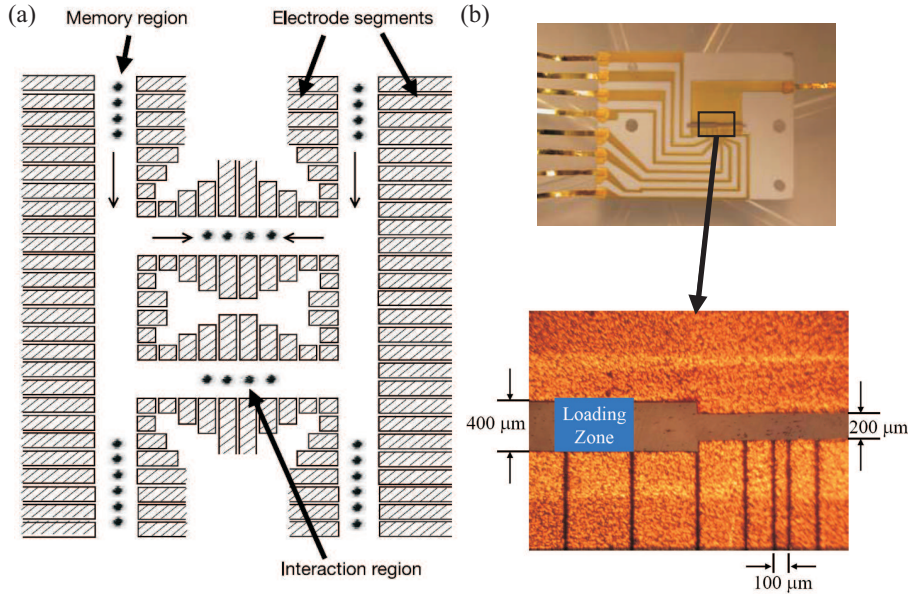


Figure 4.10: The quantum charge-coupled device: a prototype trapped ion quantum computer. (a) Concept of a microfabricated segmented ion trap. Reprinted by permission from Macmillan Publishers Ltd: Nature. D. Kielpinski *et al.*, Nature **417**, 709 (2002), copyright (2002). (b) Microfabricated segmented trap for Be^+ and Mg^+ ions constructed at NIST. (Images cty. NIST. Reproduced with permission.)

sub-spaces along with other manipulation techniques and microfabrication technologies demonstrate the feasibility of a large-scale trapped-ion quantum computer. Figure 4.10(a) illustrates their concept of a *quantum charge-coupled device* [85]. Physically, the device is a microfabricated segmented trap, with regions for cooling, state preparation, interaction, and storage of ion qubits, with the possibility of performing a number of logic gate operations in parallel in different parts of the device. The scale of the device is illustrated by the prototype six-zone trap of figure 4.10: the electrode separations of the order of $100\ \mu\text{m}$ but the device must not suffer from RF loss or breakdown with 500 MHz fields at 500 V. MEMS technology is used to manufacture gold-leaf electrode patterns on alumina or silicon-based substrates.

4.2 Achievements with trapped ion logic gates

The NIST and Innsbruck groups have led the field in the development of trapped ion quantum logic gates, with rapid progress over the last few years, often resulting in back-to-back publication of their results in *Nature* or *Science*. Both groups demonstrated two-ion logic gates as described above during 2002 [86, 77, 79], then rapidly moved on to demonstrate entanglement of three ions [87]. The NIST group characterised this experiment as Heisenberg-limited spectroscopy (see section 4.2.2) [88] whilst the Innsbruck group demonstrated the creation of GHZ states [89]. At the 2005 International Conference on Laser Spectroscopy (ICOLS), the NIST and Innsbruck groups reported the entanglement of six and seven ions respectively.

Before turning to techniques for quantum information processing and applica-

tions to precision spectroscopy and optical frequency standards, fundamental tests of quantum mechanics with trapped ion should be noted. The NIST group have demonstrated violation of a Bell's inequality in a two-ion system [90]. The significance of this experiment is that, since the ion entanglement is deterministic, it closes the 'detection loophole' (section 2.2.2): the entangled state is detected with near 100% probability, as opposed to the experiments with photons in which only a small fraction of the entangled pairs are detected.

4.2.1 Quantum teleportation

The achievement of three-ion entanglement made possible the demonstration of the deterministic teleportation of atomic qubits, *i. e.* the complete transfer of information from one qubit to another by reconstruction of the internal state of one ion at a second ion, by entanglement with a third, auxiliary ion (ancilla qubit) [91, 92]. The significance of these experiments is greater than simply the demonstration of teleportation: both the NIST and Innsbruck schemes use many of the complex procedures which would be needed for scaling quantum computing using trapped ions. Together with 'flying qubits' and the quantum teleportation of photon states, deterministic teleportation will become an essential tool in quantum information science, providing a means whereby distant qubits could interact without the need for physical proximity (quantum wiring) [93].

4.2.2 Heisenberg Spectroscopy

The NIST group's three-ion entanglement experiment demonstrates the $\sqrt{N}$ improvement in spectral resolution expected through the use of an entangled system (see section 2.5.2) [88]. The ions are prepared in a GHZ state, with a fidelity of $F = 0.89(3)$. A series of phase-gate operations used to perform a Ramsey interaction on the entangled ensemble. For three unentangled particles, the shot noise limit to the visibility of the Ramsey fringes is $1/\sqrt{3} = 0.577$; using entangled particles, a visibility of 1 is theoretically possible. In the experiment, the fringe visibility is 0.84, exceeding the shot-noise limit by 45%.

This gain in visibility using GHZ states would be offset by faster decoherence, if the Ramsey free precession time is greater than the dephasing time. However, a relatively short Ramsey time is often desirable from other considerations, such as the need to stabilise a local oscillator to the Ramsey fringe. Sørensen, Lukin and co-workers at Harvard have analysed this issue [94], along with the effect of other sources of technical noise. If Heisenberg spectroscopy is used as the basis of an atomic clock, they show that the optimum enhancement in stability, compared to the quantum projection noise limit, scales as $N^{1/6}$. This rather moderate improvement can be contrasted with previous studies, which indicated no improvement at all using entangled states [95]. However, the NIST group have demonstrated that entanglement techniques can have a significant impact on atomic clocks, as the following section shows.

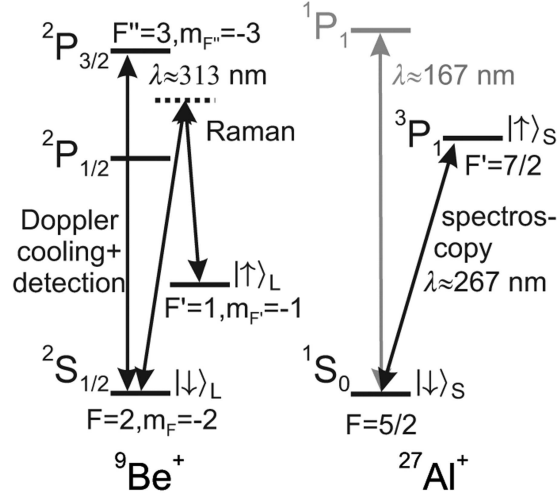


Figure 4.11: Energy level diagrams for the ${}^9\text{Be}^+$ ‘logic’ ion and the ${}^{27}\text{Al}^+$ ‘spectroscopy’ ion used in the NIST group’s quantum logic clock. A conventional trapped ion clock using ${}^{27}\text{Al}^+$ would require laser radiation at 167 nm for Doppler cooling on the ${}^1\text{S}_0 - {}^1\text{P}_1$ transition. Reprinted from Science. P. O. Schmidt *et al.*, Science **309**, 749 (2005).

4.2.3 Quantum logic clock

The achievements discussed above demonstrate techniques which are chiefly required for quantum information processing. However, the NIST group have recently demonstrated that trapped ion quantum logic gates can provide a route to improved optical frequency standards [96]. Conventional ion trap frequency standards make use of a single ion of a species having the energy level scheme of figure 4.2(b). The resonance transition, used for Doppler cooling and state detection, is necessarily at a shorter wavelength than the clock transition. For the ion species used for optical frequency standards at NPL, the resonance transition wavelengths are at 422 nm (Sr^+) and 369 nm (Yb^+), both of which can be generated with relative ease by frequency doubling of diode or solid-state lasers in the near infrared. Other ion species, in particular ions of elements in group IIIB of the periodic table, have clock transitions which have potentially superior metrological features (immunity to systematic frequency shifts due to external perturbations) but resonance transitions at technically challenging wavelengths: 167 nm in the case of Al^+ .

In the NIST group’s *quantum logic clock*, the functions of state preparation (spectroscopy of the clock transition) and interrogation (readout *via* the resonance transition) are split between two ions of different species, ${}^{27}\text{Al}^+$ and ${}^9\text{Be}^+$ respectively, figure 4.11. Doppler cooling is performed on the ${}^2\text{S}_{1/2} - {}^2\text{P}_{3/2}$ resonance transition in the Be^+ ion at 313 nm; the Al^+ ion is sympathetically cooled by the strong coupling of the ions’ collective motion in the trap. The sequence of operations required to detect excitation of the Al^+ ion’s clock transition is outlined in figure 4.12.

Compared to a conventional trapped ion clock, this sequence is more clearly more complex and may take more time, which, all other factors being equal, will

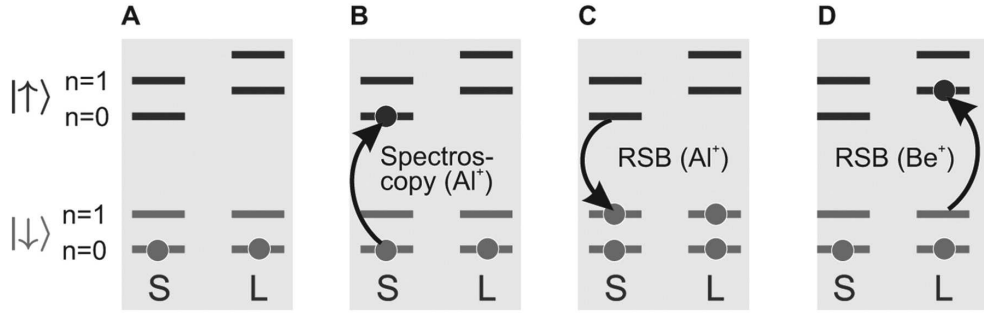


Figure 4.12: Sequence of operations in the NIST quantum logic clock. S: spectroscopy or clock ion, $^{27}\text{Al}^+$; L: logic or readout ion, $^9\text{Be}^+$; n : quantum number of the common vibrational mode. (a). The ions are prepared in their internal ground state and the ground vibrational state. (b). Interrogation of the spectroscopy (clock) transition. (c). Transfer of the internal superposition state of the spectroscopy ion into a superposition state of vibration by the application of a π -pulse of radiation tuned to the red sideband of the clock transition. (d). Transfer of the motional superposition state into an internal superposition state of the logic ion by a π -pulse on the red sideband of the Raman transition in the logic ion. Reprinted from Science. P. O. Schmidt *et al.*, Science **309**, 749 (2005).

degrade the short-term stability of the standard. For the quantum logic clock to show a real improvement over a conventional clock, the spectroscopy ion's clock transition must have significant advantages. For a given natural or interrogating laser linewidth, a shorter-wavelength clock transition will yield a proportionately better short-term stability. A frequently-cited advantage of group IIIB ions is their immunity to the quadrupole shift which affects the metastable states in ions such as those studied at NPL; however, a powerful technique to average out this shift has been demonstrated in $^{88}\text{Sr}^+$ at NPL [97]. A limiting source of systematic uncertainty for all the ion species currently being studied arises from the Stark shift due to the blackbody radiation field. Evaluation of this shift depends upon complex many-body calculations of the atomic polarisability and also requires accurate knowledge of temperature of the apparatus, which may be non-uniform. The quantum logic clock is not immune to this blackbody shift, which can only be eliminated by operating the ion trap at near to zero kelvin. NIST operate their $^{199}\text{Hg}^+$ trap in this way; the added complexity of the cryogenic apparatus is probably more of a burden on the experiment than the quantum logic scheme.

A particularly interesting candidate for the quantum logic scheme is He^+ . In this hydrogen-like ion, a number of two-photon transitions could be used as frequency references which can be directly linked to fundamental constants. A conventional ion trapping scheme calls for Doppler cooling on the Lyman- α transition at 60 nm.

4.2.4 Quantum computation

In addition to the demonstration of two-ion quantum gates, a number of simple quantum computations have been implemented using trapped ions.

The Innsbruck group have demonstrated the Deutsch-Jozsa algorithm to deter-

Table 4.1: Truth table for the Deutsch-Jozsa algorithm. ID: identity, CNOT: controlled NOT, Z-CNOT: zero-controlled NOT. (Adapted from [38]).

	constant functions		balanced functions	
a	$f_1(a)$	$f_2(a)$	$f_3(a)$	$f_4(a)$
heads?	heads	tails	heads	tails
tails?	heads	tails	tails	heads
gate	ID	NOT	CNOT	Z-CNOT

mine the parity of a binary function (section 2.3) [38, 98], using the entanglement of the internal states of a single ion with its vibrational states. Recalling the fake coin example, the Deutsch-Jozsa algorithm takes an input bit, having values $a = 0$ or 1 , standing for the questions “is it ‘heads’?” or “is it ‘tails’?”, and maps it to an output bit *via* a function $f(a) = 0$ or 1 , standing for the answer “it is ‘heads’ ” or “it is ‘tails’ ”. There are four possible functions: two constant functions, $f_1(a) = 0$ and $f_2(a) = 1$, standing for the fake coins (always answering “heads” or always answering “tails”) and two balanced functions $f_3(a) = a$ and $f_4(a) = \text{NOT } a$ (answering “your call was right” or “your call was not right”). Table 4.1 shows the truth table for these functions. Since the input bit is a qubit and can be in a superposition state, it can, in a sense, call both ‘heads’ and ‘tails’ at once.

The internal states of the ion are the ‘input’ qubit $|a\rangle$, representing the ‘which side of the coin’ information, and the vibrational states are the ‘working’ qubit $|w\rangle$, which reads out the answer ‘heads’ or ‘tails’.

Similar demonstrations of simple quantum computations have been performed by the NIST group, including a semi-classical implementation of Shor’s algorithm [99].

4.3 Entanglement with neutral atoms

The discussion above has focused on experiments with trapped ions, in which many of the basic building-blocks for QIP have been demonstrated. With neutral atoms, techniques based on Bose-Einstein condensates (BECs) and optical lattices are now being used to prepare and manipulate entangled states of neutral atoms. An advantage of such systems is ability to prepare very large number of qubits in ground state. Entanglement and quantum logic operations can be achieved through the manipulation of lattice sites [100].

A key experiment, by the group of Ted Häensch at the Ludwig-Maximilian University in Munich, is the demonstration of the *Mott Insulator* phase, in which atom number fluctuations at each lattice site are suppressed [101, 102]. The use of *atom chip* technology in QIP applications is being pursued in the groups of Jörg Schmiedmayer at the University of Heidelberg, Ed Hinds at Imperial College London and is the subject of a number of EU projects [103].

4.4 Summary conclusions

This is a rapidly developing research field with several significant “proof-of-principle” experimental demonstrations made in the last few years. The two leading ion trap groups (*i. e.* NIST Boulder and Innsbruck), are now being followed by a number of others. NPL (*via* its strategic research on this topic) is recognised as being a credible part of this community, as evidenced by invitations received to contribute to focused international workshops on this topic.

There are several reasons why NPL should at least maintain or accelerate its current level of activity on this R & D topic.

- There is now direct experimental evidence of how QIP techniques with trapped ions can now be used to perform metrology in a fundamentally different manner than before. NIST Boulder have demonstrated precision measurements on a specific optical clock transition which would not have been possible using conventional techniques. This opens up possibilities for using more exotic atomic species with more desirable clock transitions.
- Entanglement has also been shown to be a vehicle for making precision measurements below the classical noise limit, thereby improving the signal-to noise of the measurement. This has been demonstrated with a 3-ion entangled state and is in principle scalable. This represents a real prospect of significantly enhancing the measurement precision in an optical clock.
- Trapped ions are currently the most advanced technology platform for QIP demonstrations. While there are going to be significant technical demands to scaling up this technology (*cf* semiconductor-based systems), there are no fundamental reasons why trapped ions cannot be envisaged at the heart of a quantum processor. If progress with trapped ions continues, then it is anticipated that there will be an increasing need for appropriate metrological expertise in assessing the true and measured viability of this technology platform, prior to further significant investment.
- As a consequence of progress and prospects with trapped ions, there is significant world-wide investment in R & D on this topic. One spin-off from this is that technical developments are a natural part of progress. Frequently, these same developments are of significant value for core frequency standards research, and would otherwise not have taken place on the same timescale. Therefore, developments such as microfabricated ion traps, and integrated photonic systems, are two examples of developments that are advantageous in the miniaturisation of trapped ion optical frequency standards.
- History has shown that NPL’s trapped ion research is successful in partnering with European institutions in EU projects in order to leverage its strategic research funds. Opportunities for future EU projects on this topic are to be expected.

Given that NPL is only the second NMI (alongside NIST) that is active in trapped ion QIP, it is very well placed to capitalise on the opportunities ahead of other NMIs.

Chapter 5

Entanglement of superconducting quantum circuits

In recent years there has been an explosion in the field of superconducting quantum information processing. Until the late 1990s the only viable candidates for qubit realisation seemed to be stable systems well isolated from their environments, such as trapped atoms or ions or nuclear spins. It was thought that solid state systems, including superconducting circuits, would interact so strongly with their surroundings (through unavoidable capacitive and inductive coupling to electromagnetic noise signals) that they would be unable to maintain coherence over sufficiently long times to allow measurements and manipulations of the quantum state.

However, not all researchers were put off, and around the end of the 1990s the first demonstrations of superconducting qubit operation appeared, indicating short, but just about workable coherence times. The dawn of the new millenium saw a proliferation of ideas to lengthen coherence times through improved qubit isolation, and the field has now reached the point where it is possible to create entangled states of coupled qubits.

Superconducting circuits have a number of advantages over the earlier developed atomic and nuclear qubits. To begin with, the superconducting state itself has intrinsic benefits. The wavefunction is coherent over nm to μm length scales (depending on the material), and the ground state is separated from excited states by an energy gap¹. The latter provides a certain degree of isolation from external noise. On a practical note, the nanofabrication technology required to manufacture superconducting qubit circuits is well developed from the silicon electronics industry, allowing new qubit designs to be produced, tested and customised with relative ease. Scalability is also an advantage. An entire quantum circuit, including the qubit, its manipulation, readout and bias elements, can be fabricated on a microscopic scale, allowing large numbers of qubits to be accommodated on a single chip. Trapping an atom or ion, on the other hand, requires a vast array of bulky equipment which typically occupies most of a room. Superconducting qubits can easily be coupled

¹In ordinary metallic superconductors the energy gap is always non-zero, but this is not always the case for unconventional superconductors. For instance, in the cuprate compounds that exhibit *d*-wave symmetry the gap goes to zero along certain crystal symmetry directions, and Cooper-pair states are no longer separated from single quasiparticle states. The implications of this are discussed in Section 5.1.4.

together for the very reason they were first thought to be unsuitable for quantum information processing: they interact with their surroundings. Coupling atomic qubits is much more tricky because their interactions are so weak. Finally, the fact that circuits can be tuned means that superconducting qubits can be made distinct, and can thus be addressed individually with much greater ease than atoms or ions.

Although superconducting qubit technology is not yet as advanced as that of atomic or ionic qubits, it is rapidly catching up and looks very promising.

5.1 Introduction to superconducting qubits

Superconducting qubits fall into three main categories defined by their control variables: charge, phase and flux. However, all superconducting qubits share a common element: the Josephson junction. The most progress to date has been made on devices fabricated from low- T_c elements such as Al or Nb, using well established techniques such as optical and e-beam lithography and shadow evaporation [104]. However, there has also been theoretical and some preliminary experimental work on devices fabricated from high- T_c materials such as $\text{YBa}_2\text{Cu}_3\text{O}_{7-\delta}$.

The first superconducting qubits were proposed between 1997 and 1999, and many different types were successfully demonstrated within the following few years. More recently, coupling of two or more qubits has been achieved and entangled states have been observed.

In the following section we review the most promising low- T_c qubits in each of the categories *charge*, *phase* and *flux*, and discuss the current state of high- T_c qubit research.

5.1.1 Charge qubits

A typical charge qubit consists of a superconducting island (often referred to as a Cooper pair box) coupled on one side to a superconducting reservoir (a superconducting electrode) through a Josephson junction and on the other side to a voltage source through a capacitor (figure 5.1). The capacitor is used as a *gate* through which to transmit signals to the island, and the Josephson junction acts like a tunnel barrier through which Cooper pairs can tunnel on and off the island.

To understand how the Cooper pair box works, let's forget about the capacitor and the Josephson junction for a moment, and consider an isolated superconducting island. Even a nanoscale island holds a large number of Cooper pairs N , and since its potential energy depends on the number of pairs present, it forms an N -level quantum system. We can use this as a qubit if we limit its operation to just two states: those with N_0 and $N_0 + 1$ Cooper pairs. It now becomes more convenient to define $n = N - N_0$, where n is the number of excess pairs above the number in the ground state N_0 . An isolated island can therefore act as a two-level system with ground state $|n = 0\rangle$ and excited state $|n = 1\rangle$. However, we can't use an isolated island as a qubit, as there must be a way for the Cooper pairs to tunnel on and off, and for this we need the Josephson junction. We also need a way to manipulate the qubit state, and for this we need the capacitor. The Hamiltonian for the whole

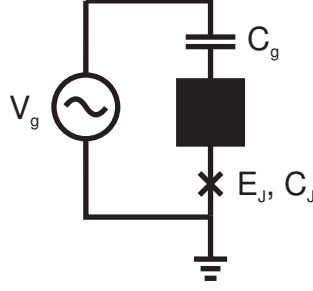


Figure 5.1: A simple charge qubit consisting of a superconducting island (Cooper pair box), gate capacitor and Josephson tunnel junction. A gate charge n_g can be induced on the island by applying a voltage V_g to the capacitor. The island is connected to a superconducting reservoir through the Josephson junction.

charge qubit is

$$\hat{H} = E_C(\hat{n} - n_g)^2 - E_J \cos \hat{\theta}, \quad (5.1)$$

where $E_C = (2e)^2/2(C_g + C_J)$ is the Cooper pair charging energy of the island², e is the charge on an electron, C_g and C_J are the capacitances of the gate capacitor and the Josephson junction respectively, $n_g = V_g C_g / 2e$ is the dimensionless *gate charge* induced on the island by applying voltage V_g to the gate capacitor, E_J is the Josephson coupling energy (a parameter that depends on the size of the Josephson junction) and θ is the phase difference of the superconducting wavefunction across the Josephson junction, commonly referred to simply as the *phase*.

This Hamiltonian is not in a particularly convenient form, as it contains both the operators $\hat{n}$ and $\hat{\theta}$. These are known to be quantum conjugate variables which do not commute, and do not share common eigenstates. Because both operators are present, the eigenstates of the system are not pure charge states, but rather, superpositions of charge states. In order to work out what the eigenstates (basis states) of the charge qubit are, we make the substitutions $\cos \hat{\theta} = \frac{1}{2}(|n\rangle\langle n+1| + |n+1\rangle\langle n|)$ ³ and $\hat{n} = |n\rangle\langle n|$, where we have used the inner product notation [43] to represent the operators. The Hamiltonian becomes

$$\hat{H} = \sum_{n=0,1} \left[E_C(n - n_g)^2 |n\rangle\langle n| - \frac{E_J}{2} (|n\rangle\langle n+1| + |n+1\rangle\langle n|) \right], \quad (5.2)$$

which can be written in matrix form as

$$\hat{H} = \begin{pmatrix} E_C n_g^2 & -E_J/2 \\ -E_J/2 & E_C(1 - n_g)^2 \end{pmatrix}. \quad (5.3)$$

To find the eigenstates and eigenvalues of this Hamiltonian we just diagonalise it [106]. We find that, at $n_g = 1/2$, the eigenstates are $|0\rangle = \frac{1}{\sqrt{2}}(|n=0\rangle + |n=1\rangle)$ and

² $E_C = 4E_e$, where E_e is the single electron charging energy. Some authors use E_C and others E_e .

³The conversion from $\hat{\theta}$ to $\hat{n}$ comes from the relation $\cos \hat{\theta} = \frac{1}{2}(e^{i\hat{\theta}} + e^{-i\hat{\theta}})$ and the fact that the terms $e^{i\hat{\theta}}$ and $e^{-i\hat{\theta}}$ are associated with changes of $+1$ and -1 respectively in the number of Cooper pairs transferred across a Josephson junction [105].

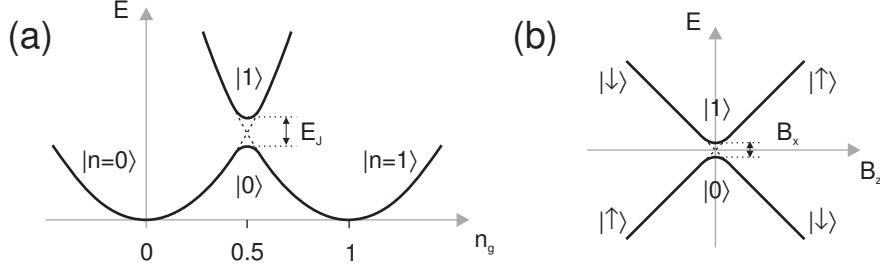


Figure 5.2: (a) Anticrossing of charge qubit levels at degeneracy point $n_g = 1/2$. (b) Anticrossing of spin-1/2 levels at $B_z = 0$. The thick lines are the eigenstates of the system and the thin dashed lines are the pure charge/spin levels that would be obtained for zero coupling ($E_J = 0$ or $B_x = 0$).

$|1\rangle = \frac{1}{\sqrt{2}}(|n=0\rangle - |n=1\rangle)$, i.e. the symmetric and antisymmetric superpositions of the two pure charge states, and the eigenvalues (energy levels) are $E_C \pm E_J/2$. As n_g gets further away from the value $1/2$, the eigenstates tend more towards the pure charge states $|n=0\rangle$ and $|n=1\rangle$, as shown in Figure 5.2(a).

For n_g close to $1/2$, the charge qubit is analogous to a spin-1/2 system in a magnetic field $\mathbf{B}$ whose Hamiltonian can be written

$$\hat{H} = -\frac{1}{2}B_z\hat{\sigma}_z - \frac{1}{2}B_x\hat{\sigma}_x, \quad (5.4)$$

where $\hat{\sigma}_x$ and $\hat{\sigma}_z$ are the Pauli spin matrices

$$\hat{\sigma}_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \hat{\sigma}_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \quad (5.5)$$

The transverse field B_x perturbs the system from its pure spin up $|\uparrow\rangle$ and spin down $|\downarrow\rangle$ states with respect to B_z . If $B_z \gg B_x$, the eigenstates will be close to $|\uparrow\rangle$ and $|\downarrow\rangle$, but at $B_z = 0$, a finite value of B_x leads to an avoided level crossing (anticrossing) where the eigenstates are $|0\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle)$ and $|1\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle - |\downarrow\rangle)$, as shown in Figure 5.2(b). In an analogous way, the finite Josephson coupling E_J leads to an anticrossing of the charge qubit states. At $n_g = 1/2$, the level splitting is at its minimum value of $\Delta E = E_J$, and because the splitting is at a minimum, the qubit is insensitive to fluctuations in the gate charge (to first order) at this value of n_g .

Because the eigenstates of the charge qubit can be tuned by the gate charge n_g , we can view a change in n_g as effecting a rotation of the charge qubit's basis, in the same way that a change of B_z rotates the spin system's basis. We can therefore use the gate capacitor to manipulate the charge qubit. By rapidly (i.e. adiabatically) changing the DC voltage applied to the gate we can rotate the basis, causing the position of the qubit state on the Bloch sphere to change. Alternatively, by applying an AC voltage we can perform NMR-type manipulations.

We have shown that the minimum splitting of the qubit levels is $\Delta E = E_J$. We therefore note that, in order for the qubit to be adequately protected from thermal excitation we require $k_B T \ll E_J$. We also require the Josephson coupling to be

small compared to the charging energy of the island, such that we work in the charge, rather than phase basis. This requires $E_J \ll E_C$. Finally, we must ensure that the energy required for a Cooper pair to tunnel onto the island is much less than the energy required to split the pair, otherwise the island will be able to accommodate single quasiparticle states, which are much more noisy than Cooper pair states. This requires $E_C \ll \Delta$, where Δ is the superconducting energy gap. For a charge qubit fabricated from a low T_c superconductor it is therefore very important to cool it to the lowest possible temperature. A temperature around 25 mK is ideal, and can be achieved with a dilution refrigerator.

In recent years a number of research groups have implemented charge qubits [107, 108, 109]. Using a variety of techniques to manipulate the qubit and read out its state, these groups have observed coherent oscillations (Rabi oscillations, Ramsey fringes and similar oscillations of the qubit state vector) and have demonstrated coherence times compatible with the di Vincenzo criteria. We review two of the most studied systems to date [107, 108].

In 1999, Nakamura *et al.* demonstrated coherent oscillations in an Al qubit they named the *single-Cooper-pair box*, shown in figure 5.3. This qubit differs slightly from the simple charge qubit described above in two respects. Firstly, the island (box) is coupled to the superconducting reservoir through two Josephson junctions in parallel (forming a DC SQUID) rather than through a single junction. This allows the coupling energy $E_J \cos(\pi\Phi/\Phi_0)$ to be tuned using an external magnetic field ($\Phi_0 = h/2e$ is the flux quantum). By setting the coupling energy to zero at $n_g = 1/2$ it is possible to freeze the state of the qubit [110]. Secondly, there is a probe electrode coupled to the island via a highly resistive tunnel junction, whose function is to read out the qubit state. The idea is that, when a very tiny voltage bias V_b is applied to the probe junction, we expect a small current $I = dq/dt$ to pass through it. This means that, if we wait for a time Δt , a charge Δq is likely to tunnel across the junction. If the qubit is in the state $|1\rangle$, we will see a pulse of current equal to $2e/\Delta t$ if we wait for time Δt . However, if it is in the state $|0\rangle$ we will see nothing. Of course, we must make sure that we don't read out the state of the qubit accidentally before we have finished manipulating it, but this only requires us to choose Δt to be much longer than the time period of the coherent oscillations. Since Δt is inversely proportional to I we can lengthen Δt by decreasing I , and this means we must decrease V_b . A large resistance R_b of the probe junction makes this easier⁴.

The disadvantage of the probe junction is that it is in continuous contact with the island and is continually voltage biased. This means that the qubit can undergo premature relaxation if a Cooper pair spontaneously tunnels through the probe junction. In addition, fluctuations of the probe's bias voltage are coupled to the island, causing dephasing. These problems limit the coherence time of the qubit⁵ and also detract from the visibility of the coherent oscillations.

A more recent charge qubit which overcomes these disadvantages has been demonstrated by Vion *et al.* [108]. Named the *quantronium*, it is an Al circuit based on

⁴Nakamura *et al.* used $R_b \approx 30 \text{ M}\Omega$ and $\Delta t \approx 14 \text{ ns}$

⁵Nakamura *et al.* achieved a coherence time of order 2 ns in 1999, but they didn't measure this properly by doing a Ramsey fringe experiment.

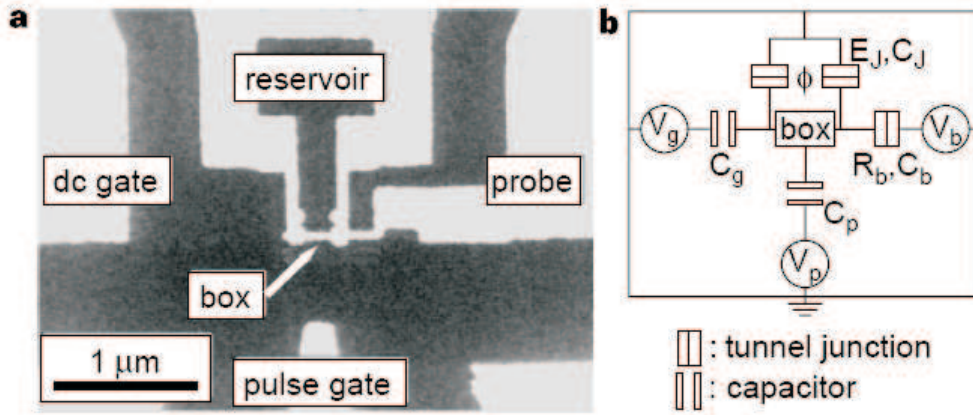


Figure 5.3: Single-Cooper-pair-box qubit. Reprinted by permission from Macmillan Publishers Ltd: Nature. Y. Nakamura *et al.*, Nature **398**, 786 (1999), copyright (1999).

the Cooper pair box, and is shown in figure 5.4. This qubit is very similar to that of Nakamura *et al.*, but differs in its readout mechanism. The Cooper-pair box again comprises two small Josephson junctions with a superconducting island in between (Cooper-pair transistor geometry). But the box is shunted by a much larger junction. Instead of reading out the qubit state by waiting for the excess charge to tunnel off the island, the quantronium exploits the entanglement of the qubit charge state with the phase state of the large junction⁶. The excited qubit state gives rise to a larger phase difference across the two small Josephson junctions than the ground state, and this causes a small additional current to flow through the large junction, which is already biased close to its critical current by a current source. The idea is that if the qubit is in the state $|1\rangle$ the extra bias current will cause the large junction to switch to the finite voltage régime, whereas if the qubit is in the state $|0\rangle$ the large junction will remain in the superconducting, zero voltage régime. The state $|1\rangle$ can therefore be detected as a voltage pulse across the large junction.

The reason why this readout scheme is better than that of Nakamura *et al.* is that the large junction does not need to be continually current biased. While the qubit is being manipulated via signals to the gate capacitor the large junction can remain dormant with zero current bias, a régime in which it does not influence the qubit state. When it is time to read out the qubit state the large junction can be activated by ramping up the bias current to a value just below its critical current. The quantronium is insensitive to first order charge noise at $n_g = 1/2$, and it is also immune to first order flux noise when the flux passing through the loop enclosed by the three Josephson junctions is zero. Vion *et al.* have measured a coherence time of $\approx 0.50 \mu\text{s}$ in a Ramsey fringe experiment, which is at least a factor of 100 better than that observed by Nakamura *et al.*, and is compatible with the di Vincenzo criteria. The visibility of the oscillations was poorer than they had hoped, but this was possibly due to relaxation of the qubit caused by flux noise, which could be

⁶This is analogous to the entanglement of the spin state with the position state in a Stern Gerlach experiment.

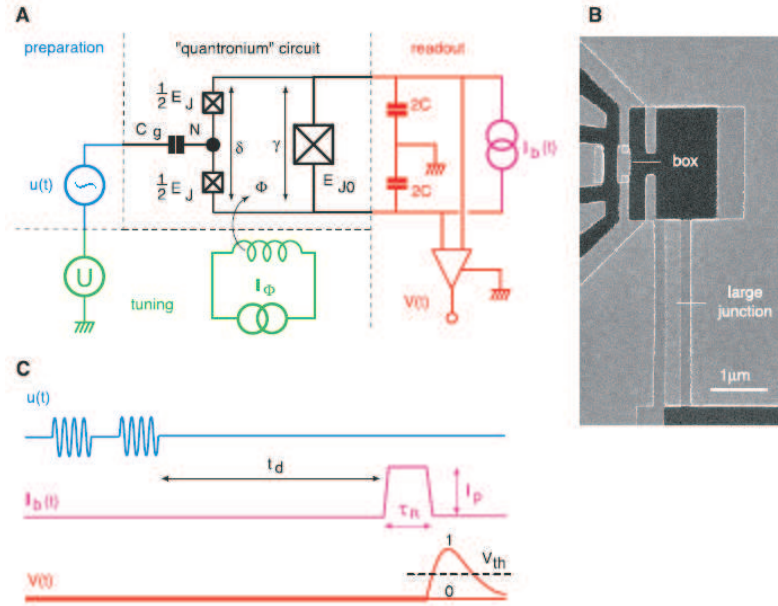


Figure 5.4: The quantronium qubit. (A) circuit diagram, (B) scanning electron micrograph, (C) pulse sequence used for preparation and readout. Reprinted with permission, D. Vion *et al.*, Science **296**, 886 (2002). Copyright (2002) American Association for the Advancement of Science.

reduced by improving the magnetic shielding.

The dominant source of decoherence in charge qubits is thought to be the fluctuating charge in the substrate and the insulating material of the junction. As much as the Cooper-pair box is sensitive to the gate potential, it is also sensitive to any background charge through stray capacitance. This problem could perhaps be alleviated by using alternative materials. However, there is no such problem with flux qubits, described below, whose magnetic environment can easily be made relatively stable.

5.1.2 Flux qubits

Charge qubits are so called because qubit manipulation is achieved by controlling the gate charge n_g . Flux qubits, therefore, are manipulated by applying an external magnetic flux Φ_{ext} . A typical flux qubit consists of a superconducting ring interrupted by one or more Josephson junctions.

Early flux qubit designs were based on the single junction r.f. SQUID, shown in figure 5.5. The Hamiltonian of this system is

$$\hat{H} = \frac{\hat{q}^2}{2C_J} + \frac{\hat{\Phi}^2}{2L} - E_J \cos \hat{\theta}, \quad (5.6)$$

where q is the charge stored capacitively by the Josephson junction, C_J is the capacitance of the junction, $\Phi = \Phi_{\text{int}} - \Phi_{\text{ext}}$ is the flux generated by the persistent current flowing in the ring (Φ_{int} is the internal flux passing through the qubit loop and Φ_{ext} is the external applied flux), L is the inductance of the ring, and E_J and θ

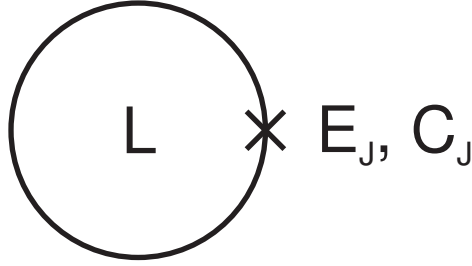


Figure 5.5: Early flux qubit designs were based on the single junction r.f. SQUID, which consists of a superconducting ring of inductance L interrupted by a single Josephson junction.

are, respectively, the Josephson coupling energy and the phase difference across the junction.

The phase difference around a superconducting ring containing a Josephson junction is directly related to the flux passing through the ring by the equation

$$2r\pi = \theta + 2\pi \frac{\Phi_{\text{int}}}{\Phi_0}, \quad (5.7)$$

where r is an integer. This equation describes quantisation of the fluxoid $\theta + 2\pi\Phi_{\text{int}}/\Phi_0$ [111]. We can arbitrarily choose $r = 0$ to obtain $\theta = -2\pi\Phi_{\text{int}}/\Phi_0$. If we now substitute this into equation (5.6) we obtain

$$\hat{H} = \frac{\hat{q}^2}{2C_J} + \frac{\hat{\Phi}^2}{2L} - E_J \cos \left(2\pi \frac{\hat{\Phi} + \Phi_{\text{ext}}}{\Phi_0} \right), \quad (5.8)$$

where the charge q and the flux Φ are quantum conjugate variables in direct analogy with the Cooper pair number n and the phase θ . Now, the flux Φ is a continuous variable, so the best way to find the eigenstates of a Hamiltonian like the one above is to use wave mechanics, where the basis states $|\psi\rangle$ of the qubit are represented by wavefunctions $\psi(\Phi)$. Then, all we have to do to find the eigenfunctions (basis states) and eigenvalues (energy levels) of the flux qubit is to solve the Schrödinger equation $\hat{H}\psi(\Phi) = E\psi(\Phi)$.

This must be done numerically⁷, but we can visualise the solution by making the following analogy. We notice that equation (5.8) is analogous to the Hamiltonian for a particle of mass m moving in a gravitational potential $U(x)$, where C_J corresponds to m , q corresponds to the momentum p and Φ corresponds to the position x of the particle. The first term corresponds to the kinetic energy of the particle and the other terms $U(\Phi) = \Phi^2/2L - E_J \cos[(2\pi\Phi + \Phi_{\text{ext}})/\Phi_0]$ correspond to the gravitational potential $U(x)$. Therefore, if we plot the potential landscape $U(\Phi)$ we can get a qualitative idea of what the eigenfunctions will look like. From figure 5.6 we can see that the shape of the potential well, and hence the form of the eigenfunctions and eigenvalues, depends on the external flux Φ_{ext} . When $\Phi_{\text{ext}} = 0.5\Phi_0$ the potential

⁷Numerical solution of the Schrödinger equation is done by making the substitution $\hat{q} = -i\hbar\partial/\partial\hat{\Phi}$ in equation (5.8) and using the Runge-Kutta method or a similar numerical technique to solve the resulting differential equation $\hat{H}\psi(\Phi) = E\psi(\Phi)$.

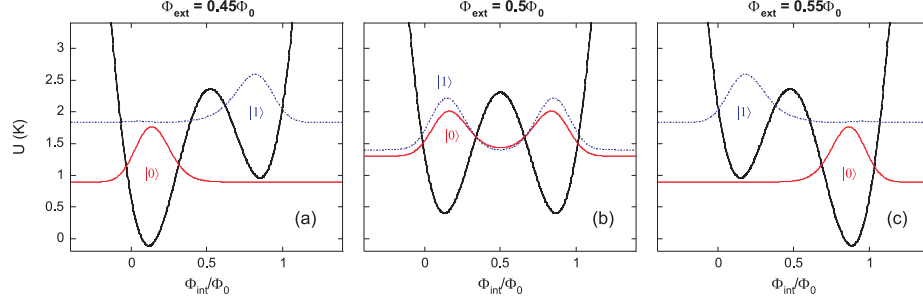


Figure 5.6: Potential energy and eigenfunctions of the single junction r.f. SQUID flux qubit (we have plotted the square of the wavefunction here to show the probability of the system being in each state). The red solid line is the ground state and the blue dotted line is the excited state. The thick black line is the potential $U(\Phi_{\text{int}})$. When $\Phi_{\text{ext}} = 0.5\Phi_0$ (centre plot) the eigenfunctions are the symmetric $|0\rangle$ and antisymmetric $|1\rangle$ superpositions of the two localised flux states that would exist in the absence of tunnelling. Both $|0\rangle$ and $|1\rangle$ have expectation values $\langle\Phi\rangle = \langle\Phi_{\text{int}} - \Phi_{\text{ext}}\rangle = 0$. However, for other values of Φ_{ext} (outer plots) the eigenfunctions are localised flux states with expectation values $\langle\Phi\rangle$ approaching $\pm 0.5\Phi_0$.

is symmetric and the eigenfunctions would be degenerate, except that tunnelling through the barrier causes the degeneracy to be lifted and gives rise to an anticrossing of the energy levels (see figure 5.7). For $\Phi_{\text{ext}} \neq 0.5\Phi_0$ the two lowest eigenfunctions, labelled $|0\rangle$ and $|1\rangle$, each give rise to a non-zero expectation value of the total flux Φ . For Φ_{ext} far away from $0.5\Phi_0$, the expectation values are $\langle 0|\Phi|0\rangle, \langle 1|\Phi|1\rangle \approx \pm 0.5\Phi_0$ and correspond to the flux being localised in one well or the other. However, at $\Phi_{\text{ext}} = 0.5\Phi_0$ the eigenfunctions are the symmetric and antisymmetric superpositions of these localised flux states, and the expectation value of Φ is zero for both.

A difficulty with a flux qubit based on a single-junction r.f. SQUID is that it requires a large loop to ensure quantised flux states. The reason for this is that the existence of quantised flux states depends on the inductance L of the loop and the critical current I_c of the junction⁸. L determines the slope of the parabolic component of the potential, and I_c determines the amplitude of the oscillatory component that gives rise to the quantised flux states. Therefore, if L is too small compared to I_c the steepness of the parabolic component will wash out the oscillatory component and there will be no metastable minima—hence no quantised flux states. We therefore require a large value of L , and since L is directly proportional to the area of the loop, we require a large loop. However, this causes a problem because the larger the loop is, the more susceptible it is to external flux noise which will cause decoherence. Although the anticrossing of the energy levels at $\Phi_{\text{ext}} = 0.5\Phi_0$ makes the flux qubit insensitive to first order fluctuations in Φ_{ext} , this is not enough to provide it with a substantial coherence time when the loop is large.

This difficulty can be overcome by including more than one Josephson junction in the superconducting ring. This introduces extra degrees of freedom into the Hamiltonian and allows localised flux states to exist in nanoscale loops. Two designs proposed in 1999 by Mooij *et al.* [112] are shown in figure 5.8. These have

⁸The criterion for the existence of quantised flux states is $\beta_L > 1$, where $\beta_L = 2\pi LI_c/\Phi_0$ [111].

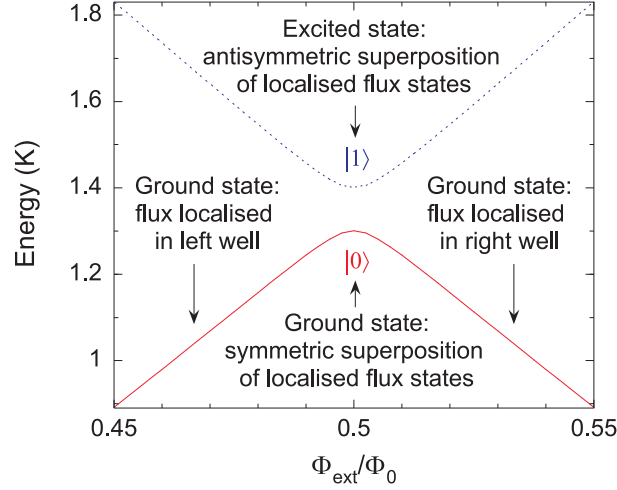


Figure 5.7: Energy levels of the single junction r.f. SQUID flux qubit as a function of external flux Φ_{ext} . The red solid line is the ground state and the blue dotted line is the excited state. An anticrossing occurs at the point $\Phi_{\text{ext}} = 0.5\Phi_0$.

been dubbed *persistent current qubits* because the qubit states are characterised by persistent currents flowing in opposite directions around the loop, giving rise to localised flux states.

We first discuss the 3-junction qubit shown in Figure 5.8(A). The Hamiltonian of this qubit is

$$\hat{H} = \frac{\hat{q}^2}{2C} + \frac{\hat{\Phi}^2}{2L} - \sum_{i=1}^3 E_{Ji} \cos \hat{\theta}_i, \quad (5.9)$$

where q is the total charge stored on all the junctions, C is the total capacitance of all the junctions, and E_{Ji} and θ_i are, respectively, the Josephson coupling energy and the phase difference across the i th junction. In the persistent current qubit the localised flux states have such small expectation values $\langle \Phi \rangle \approx 0$ that the parabolic component of the potential (the second term in equation (5.9) can be ignored. We then notice that, due to quantisation of the fluxoid in a superconducting ring, as expressed in equation (5.7), the phases of the three junctions cannot all be independent. We therefore make the substitution $\theta_3 = -2\pi f - \theta_1 - \theta_2$, where the filling factor $f = \Phi_{\text{int}}/\Phi_0 \approx \Phi_{\text{ext}}/\Phi_0$. Finally, the proposal of Mooij *et al.* requires that two of the junctions have identical coupling energies E_J , while the third has a lower value αE_J , where $0.5 < \alpha < 1$. Putting all these modifications together, we arrive at the Hamiltonian

$$\hat{H} = \frac{\hat{q}^2}{2C} - E_J \cos \hat{\theta}_1 - E_J \cos \hat{\theta}_2 - \alpha E_J \cos(2\pi f + \hat{\theta}_1 + \hat{\theta}_2). \quad (5.10)$$

We find that, since the new degrees of freedom are the phases across the two identical junctions, we are now working with the variable *phase*, rather than flux. However, since phase is a continuous variable we can once again use wave mechanics to determine the eigenstates and eigenvalues. This time we visualise the results by plotting the potential landscape $U(\theta) = -E_J \cos \theta_1 - E_J \cos \theta_2 - \alpha E_J \cos(2\pi f + \theta_1 + \theta_2)$, as shown in Figure 5.9.

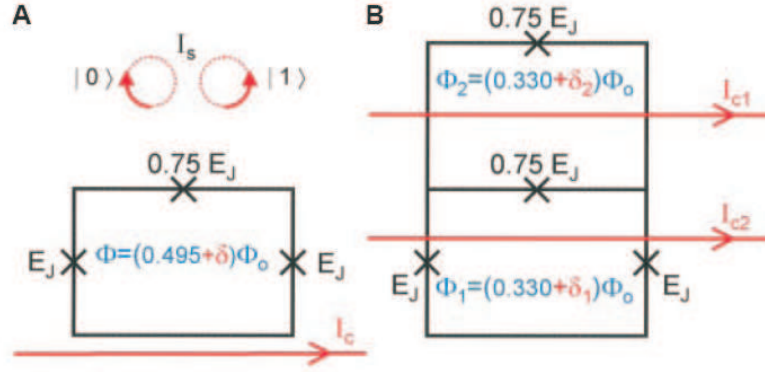


Figure 5.8: The persistent current qubit. (A) 3-junction qubit, (B) 4-junction qubit. The red lines represent wires used to bias the device with DC magnetic flux or to manipulate it with microwave signals. Reprinted with permission, J. E. Mooij *et al.*, *Science* **285**, 1036 (1999). Copyright (1999) American Association for the Advancement of Science.

As with the simple flux qubit described earlier, a filling factor of $f = 0.5$ causes the potential to be symmetric, with eigenstates whose degeneracy is lifted by tunnelling between the wells. As there are now two degrees of freedom, θ_1 and θ_2 , the potential landscape is two-dimensional. Consequently, the phase can tunnel in many directions between different wells. However, the parameter α can be used to tune the different tunnelling probabilities. A value of $\alpha \sim 0.75$ allows strong intra-pair tunnelling while suppressing tunnelling to other adjacent pairs. For values of f far away from 0.5, the qubit states are characterised by well-defined values of the phases θ_1 and θ_2 and consist of localised flux states ($\langle \Phi \rangle \sim \pm 10^{-3} \Phi_0$) arising from persistent currents circulating in opposite directions. At $f = 0.5$, the qubit states are symmetric and antisymmetric superpositions of these persistent current states, their energy separation depending on the size of the potential barrier between the wells. In the 3-junction persistent current qubit the parameter α is fixed by the size of the smaller junction. However, in the 4-junction qubit shown in figure 5.8(B) the small junction is replaced with a parallel combination of two junctions (DC SQUID geometry) allowing α to be tuned by an applied magnetic field.

Readout of the persistent current qubit can be achieved through weak inductive coupling to an underdamped DC SQUID (a DC SQUID without shunting resistors). This is usually realised by enclosing the qubit within the DC SQUID loop [113, 114]. The small difference in the expectation value of the flux Φ for the two qubit states can be detected as a difference in the switching current of the DC SQUID (the current at which the SQUID switches from the superconducting, zero voltage state to the resistive, finite voltage state).

There is one slight disadvantage of this readout scheme. At $f = 0.5$ the expectation value of the flux Φ through the qubit loop is zero, because the qubit states are symmetric and antisymmetric superpositions of the persistent current states. This means that if we want to be able to read out the qubit state we must operate it at a slightly smaller value of f , like 0.495. However, the qubit is least sensitive to external flux noise at the centre of the anticrossing, $f = 0.5$. So the DC SQUID readout

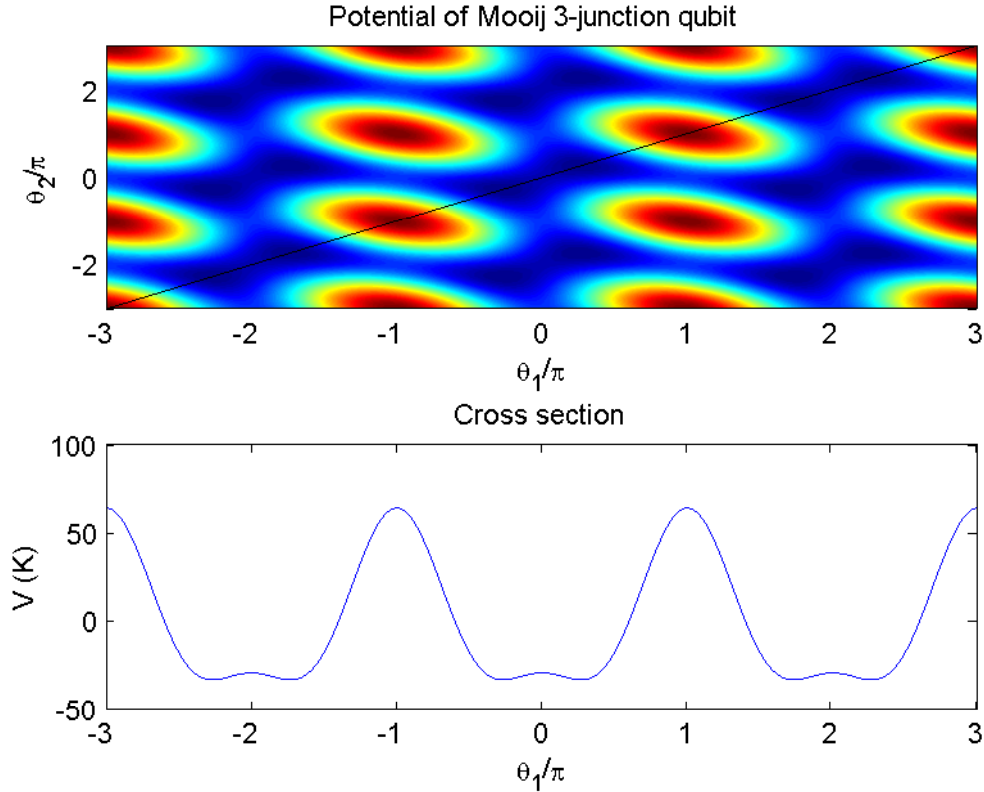


Figure 5.9: Potential landscape of the 3-junction persistent current qubit ($\alpha = 0.75$, $f = 0.5$). The cross section along the line $\theta_1 = \theta_2$ shows that the potential is a symmetric double well. In the colour plot, pairs of wells appear in dark blue. Each pair is equivalent because the phases θ_1 and θ_2 are 2π periodic.

scheme exposes the qubit to a small source of decoherence that might be avoided with a better readout method⁹. However, the small size of the qubit is enough to screen the qubit from major external flux noise, and the fact that the qubit states are formed from persistent current states rather than charge states makes the qubit virtually insensitive to charge noise—a phenomenon abundant in nanoscale circuits.

Manipulation of the qubit can be achieved by applying AC current pulses to a control line inductively coupled to the qubit loop. Thus, NMR-type manipulation of the qubit can take place, and such techniques have recently been used to observe coherent oscillations of the qubit state [113]. The measured dephasing time was 20 ns, which is better than that observed in the single-Cooper-pair-box charge qubit, but not as good as that observed in the quantonium. There is, however, scope

⁹An alternative readout scheme called the *impedance measurement technique* has been implemented by Il'ichev *et al.* [115]. A small AC signal is coupled to the qubit through an inductor to enable the complex impedance to be measured. The two qubit states are distinguished by different values of the current-voltage phase angle α (the phase difference between the current signal supplied to the inductor and the voltage signal measured across it). However, it is not clear whether this readout scheme is any better than the DC SQUID method, as an anticrossing in α as a function of external flux has not been demonstrated.

for improvement through increased magnetic shielding, optimisation of the device parameters to avoid accidental resonances¹⁰ and improved readout schemes.

As with the charge qubit, for flux qubits to be protected from thermal noise, $k_B T \ll E_J \ll \Delta$, where Δ is the superconducting energy gap. The Josephson coupling energy E_J must be greater than the Cooper pair charging energy E_C , so that we work in the phase basis rather than the charge basis.

5.1.3 Phase qubits

Having discussed charge qubits and flux qubits in the preceding sections we now come to the final category of superconducting qubits: phase qubits. The simplest form of phase qubit is a single Josephson junction, and it can be manipulated by controlling the bias current, which is directly related to the phase of the junction through the first Josephson relation $I = I_c \sin \theta$.

A current-biased Josephson tunnel junction, operating in the superconducting régime ($I < I_c$) behaves like an inductor¹¹ with inductance $L_J = \hbar/2eI_c \cos \theta$. It also has capacitance C_J , so it behaves like an anharmonic LC resonator [116]. This means that, in the superconducting régime, the energy of the junction oscillates between the capacitive and inductive components in the same way that the energy of a particle in a gravitational potential well oscillates between its kinetic and potential components. We have met this analogy already in the description of the flux qubit. The phase θ is analogous to the position x of the particle. However, the Hamiltonian of the phase qubit is slightly different to that of the flux qubit due to the presence of a current source and the absence of a superconducting ring (see ref. [105] for a derivation). It is as follows:

$$\hat{H} = \frac{\hat{q}^2}{2C_J} - E_J \cos \hat{\theta} - E_J \eta \hat{\theta}, \quad (5.11)$$

where $E_J = I_c \Phi_0/2\pi$ and $\eta = I/I_c$. As with the flux qubit, the first term is analogous to the kinetic energy of the particle, and the other terms to its potential energy. The potential landscape $U(\theta) = -E_J \cos \hat{\theta} - E_J \eta \hat{\theta}$ is known as the *washboard potential*, because it looks like a corrugated washboard. The eigenfunctions are dependent on the current bias, which corresponds to the slope η of the washboard, and are shown in Figure 5.10. When η is close to 1, the potential well contains only a few eigenfunctions. We can use the lowest two as qubit states.

The qubit can be manipulated by altering the bias current I . This changes the slope η of the washboard and alters the eigenfunctions, effecting a change of basis. NMR-type manipulations can be performed by applying pulses of AC bias current in addition to the DC bias that determines the separation of the qubit's energy levels.

Readout can be achieved through the phenomenon of *phase escape*. When $\eta < 1$ the junction is in the superconducting, zero voltage state. This corresponds to the

¹⁰The energy separation of the qubit states at $f = 0.4$ was rather close to the plasma frequency of the junctions [113].

¹¹If we differentiate the first Josephson relation with respect to time t we obtain $dI/dt = (I_c \cos \theta)d\theta/dt$. Combining this with the second Josephson relation $V = (\hbar/2e)d\theta/dt$ we obtain $V = (\hbar/2eI_c \cos \theta)dI/dt$. The voltage across an inductor is $V = L dI/dt$.

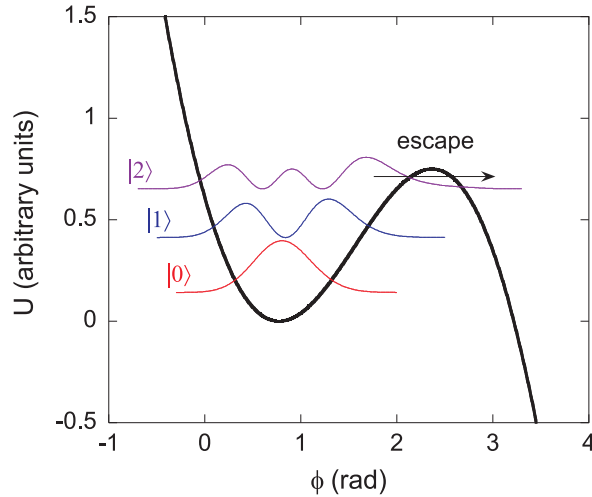


Figure 5.10: Energy levels of the phase qubit in the washboard potential. The uppermost level has a much higher escape probability than the lower levels. In this plot, η is very close to 1, so there are only three energy levels in the well. As η is increased the potential well becomes shallower, and the uppermost levels disappear one by one into the continuum.

particle being trapped in one of the metastable wells of the washboard potential. In this régime the phase of the junction can only oscillate over a small range of values and has a constant expectation value. However, when $\eta > 1$ the junction enters the resistive state and develops a voltage across its terminals. In this régime the depth of the potential well goes to zero and the particle is no longer trapped, running away down the washboard with velocity dx/dt . The velocity of the particle is analogous to the rate of change of the phase $d\theta/dt$. The appearance of a finite voltage across the junction is known as phase escape. For $\eta < 1$ it can occur via thermal excitation over the barrier or quantum tunnelling through the barrier.

In order to read out the qubit state we bias the junction to a value of η where only three energy levels remain in the well. The qubit is cooled to a very low temperature, so there is not enough thermal energy to cause thermal phase escape from any of the levels. However, the probability of tunnelling through the barrier increases by a factor of $\sim 10^3$ for each successively higher energy level. This means that, although the probability of tunnelling is very low for the two qubit levels, it is quite high for the topmost level. Therefore, we can read out the qubit state by using a microwave bias signal to excite the transition $|1\rangle \rightarrow |2\rangle$. If the qubit is in the state $|0\rangle$ it will not be excited, so it will remain in the well. However, if it is in the state $|1\rangle$ it will be excited to the state $|2\rangle$, where it will rapidly tunnel out of the well and escape, causing a voltage to appear across the junction.

Two groups have recently observed Rabi oscillations (see Figure 5.11) in phase qubits [117, 116, 118], demonstrating coherence times of $5 \mu\text{s}$ in a NbN/AlN/NbN junction and [117] and 41 ns in an Al junction with an oxide barrier [118].

Because phase qubits are relatively large in size ($\sim 10 \mu\text{m} \times 10 \mu\text{m}$), they are easy to fabricate reproducibly with optical lithography. A disadvantage is that they do not share the first order insensitivity to charge/flux noise that charge/flux

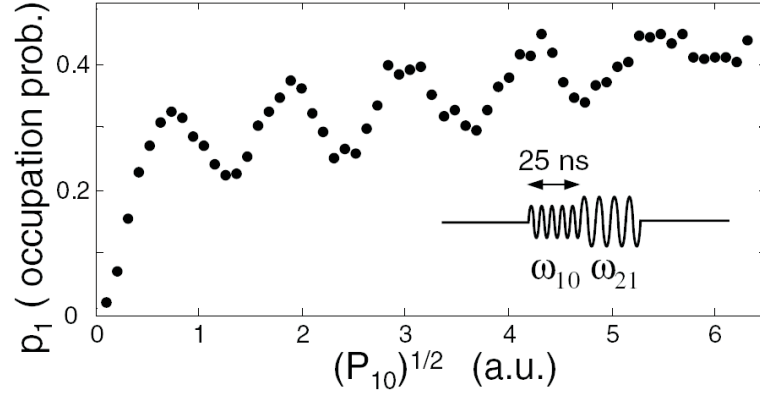


Figure 5.11: Rabi oscillations in a Josephson junction phase qubit. Reprinted figure with permission from J. M. Martinis *et al.*, Phys. Rev. Lett. **89**, 117901 (2002). Copyright (2002) by the American Physical Society.

qubits exhibit. However, at present their coherence times are competitive.

A major cause of decoherence in phase qubits is the existence of *two-level resonators* in the oxide tunnel barrier of the Josephson junction [118]. These are thought to consist of small current channels which switch spontaneously between *on* and *off* states with a resonant frequency comparable to the qubit level separation at certain values of the bias current¹². The switching of these current channels couples to the qubit state through their effect on the critical current I_c , creating unwanted entanglement of the qubit and resonator states. The decoherence caused by such resonators can be minimised by choosing the bias current of the qubit carefully, but this is not an easy task, as the resonator frequencies and strengths are unique to every junction and change with thermal cycling or even spontaneously. Some materials seem to be more susceptible than others to resonator-induced decoherence, so there is scope for improvement through materials research.

5.1.4 High T_c qubits

In 1999, Ioffe *et al.* [122] proposed a qubit device made at least partially with high- T_c superconductor materials such as $\text{YBa}_2\text{Cu}_3\text{O}_{7-\delta}$. This would have the advantage that the *d*-wave symmetry of the superconducting wavefunction could be exploited to design a qubit with a doubly degenerate ground state at zero applied magnetic field.

In this respect, high- T_c qubit designs are rather similar to the persistent current qubit [112], consisting of a superconducting loop interrupted by more than one Josephson junction. The difference between low- T_c and high- T_c devices is that high- T_c devices need only two junctions to obtain a doubly degenerate ground state, and the degeneracy occurs at zero applied field. Such devices make use of the unconventional current-phase relationship of certain types of high- T_c junction (e.g. *s*-wave-*d*-wave-*s*-wave junctions or *d*-wave-*d*-wave junctions). Rather than obeying

¹²An alternative suggestion is that of the *two-level fluctuator*, which consists of an electron trapped in the tunnel barrier, hopping back and forth between two potential wells [119, 120, 121].

the conventional first Josephson relation $I = I_c \sin \theta$, such junctions are described by a current-phase relationship involving higher harmonics. In practice those beyond the second harmonic are negligible, so the current-phase relationship is given by

$$I = I_c \sin \theta - I'_c \sin 2\theta. \quad (5.12)$$

The second harmonic component is negative and temperature-dependent, as shown in ref. [123]. If two such junctions are incorporated in a nanoscale superconducting loop of low inductance the Hamiltonian is

$$\hat{H} = \frac{\hat{q}^2}{2C} + E_{J1} \cos \hat{\theta}_1 - E'_{J1} \cos 2\hat{\theta}_1 + E_{J2} \cos \hat{\theta}_2 - E'_{J2} \cos 2\hat{\theta}_2, \quad (5.13)$$

where the first term is the kinetic energy and the rest represent the potential landscape. The numbers 1 and 2 denote the two junctions, and we note that their phases are not independent, but are related by $\theta_1 + \theta_2 + f = 2r\pi$, where $f = 2\pi\Phi/\Phi_0$, giving, for $r = 0$, $\theta_2 = -\theta_1 - f$. Making this substitution, the Hamiltonian becomes

$$\hat{H} = \frac{\hat{q}^2}{2C} + E_{J1} \cos \hat{\theta} - E'_{J1} \cos 2\hat{\theta} + E_{J2} \cos(\hat{\theta} + f) - E'_{J2} \cos(2\hat{\theta} + 2f), \quad (5.14)$$

The advantage of a doubly degenerate ground state at zero applied field is that the qubit can be screened from low frequency external flux noise. However, there is a caveat: due to the d -wave symmetry of the superconducting wavefunction, high- T_c materials have a superconducting gap that vanishes along certain symmetry directions. This means that there are always some quasiparticles present (referred to as nodal quasiparticles), which introduce unwanted noise. One way to get round this problem is to create a size-induced energy gap by restricting the junction size [124]. Calculations suggest that a d -wave grain boundary junction must be less than 100 nm wide to create a gap of 1 K [20]. However, it is difficult to fabricate junctions this small. In fact, it has been found that in the very smallest junctions the second harmonic is suppressed. A sizeable second harmonic has only been observed in micron-sized junctions [20].

Nevertheless, research into high- T_c qubits continues, with new designs being proposed [23], awaiting improved fabrication technology. Recent theoretical work [125, 126] suggests that decoherence from nodal quasiparticles and other sources may not be as bad as was once suspected.

5.2 Coupled systems and entanglement

The coupling of two or more qubits to create entangled states causes the energy levels of the combined system to be altered. For example, spin-1/2 particles influence each other through the magnetic dipole fields they generate. If the spins are very far apart the mutual effects of their dipole fields will be negligible due to $1/r^2$ scaling. In this régime the spins are said to be uncoupled. However, if they are brought close together, each will interact with the fields of the others. Consider a pair of spins. When brought close together their energy levels will undergo a shift $\Delta E = -\boldsymbol{\mu}^A \cdot \mathbf{B}^B$, where $\boldsymbol{\mu}^A$ is the magnetic moment due to one spin (A) and $\mathbf{B}^B$ is the magnetic field

generated by the other (B). If the spins are sitting in an applied field along the z -direction they are likely to be in stationary eigenstates of the $\hat{\sigma}_z$ operator, so $\mu_z^A \propto \hat{\sigma}_z^A$ and $B_z^B \propto \mu_z^B \propto \hat{\sigma}_z^B$. Thus, $\boldsymbol{\mu}^A \cdot \mathbf{B}^B \propto \mu_z^A \mu_z^B \propto \hat{\sigma}_z^A \hat{\sigma}_z^B$, and we obtain $\Delta E = J \hat{\sigma}_z^A \hat{\sigma}_z^B$, where J is known as the *exchange constant* or *exchange coupling*. When $J = 0$, the spins are uncoupled and there is no shift in their energy levels.

The interaction Hamiltonian in the qubit eigenbasis can contain diagonal or off-diagonal terms. The spin-spin interaction described above is an example of an interaction that gives rise to diagonal terms in the Hamiltonian. This is because it involves only $\hat{\sigma}_z$ operators, and the eigenstates of a spin in a magnetic field along the z -direction are eigenstates of $\hat{\sigma}_z$. This sort of interaction does not mix the eigenstates of the coupled system, so does not give rise to entangled stationary states (the effect of the coupling is merely to shift the energy levels with respect to one another). However, it can cause a system in a non-stationary state to evolve through entangled states.

Other interactions exist in which the interaction Hamiltonian contains off-diagonal terms. This sort of interaction *does* mix the states of the coupled system, giving rise to a new set of eigenstates that include superpositions, e.g. of $|01\rangle$ and $|10\rangle$. Such a system possesses entangled stationary states.

In recent years significant progress has been made in the coupling of superconducting qubits and the creation of entangled states. Coupled pairs of low- T_c charge, flux and phase qubits have been demonstrated by a number of research groups. Coherent oscillations between states of the coupled system have been observed in charge [127] and phase [128] qubits. During such oscillations both systems evolve through maximally entangled Bell states. Entangled states have also been prepared in coupled charge qubits using microwave pulses corresponding to the Hadamard and CNOT gates [129]. Coupled charge [127] and phase [130] qubits have been probed spectroscopically to demonstrate anticrossings of coupled states. A pair of coupled flux qubits has been shown to exhibit entangled eigenstates by a technique similar to spectroscopy, based on the impedance measurement technique [131]. We discuss these results in more detail below.

The coupled single-Cooper-pair-box qubits of Yamamoto *et al.* are shown in figure 5.12. One of the qubits has a fixed Josephson energy E_J , whereas the other has a SQUID geometry that allows E_J to be varied. The two qubits are coupled through a capacitor, giving rise to an interaction Hamiltonian

$$\hat{H} = \sum_{n_1=0}^1 \sum_{n_2=0}^1 E_m (n_{g1} - n_1)(n_{g2} - n_2) |n_1 n_2\rangle \langle n_1 n_2|, \quad (5.15)$$

where n_1, n_2 are the number of excess Cooper pairs on qubits 1 and 2 and n_{g1}, n_{g2} are the gate charges induced by DC voltages applied to the DC gates. E_m is the coupling energy and depends on the capacitance of the coupling capacitor. Since we work in the charge basis in the analysis of the single-Cooper-pair box, the interaction Hamiltonian is diagonal, like spin-spin coupling. Far from the charge degeneracy points $n_{g1}, n_{g2} = 1/2$ the eigenstates of the coupled system are the pure charge states $|00\rangle$, $|01\rangle$, $|10\rangle$ and $|11\rangle$. However, by applying short DC pulses to the pulse gates the individual qubits can be manipulated into superposition states as described in

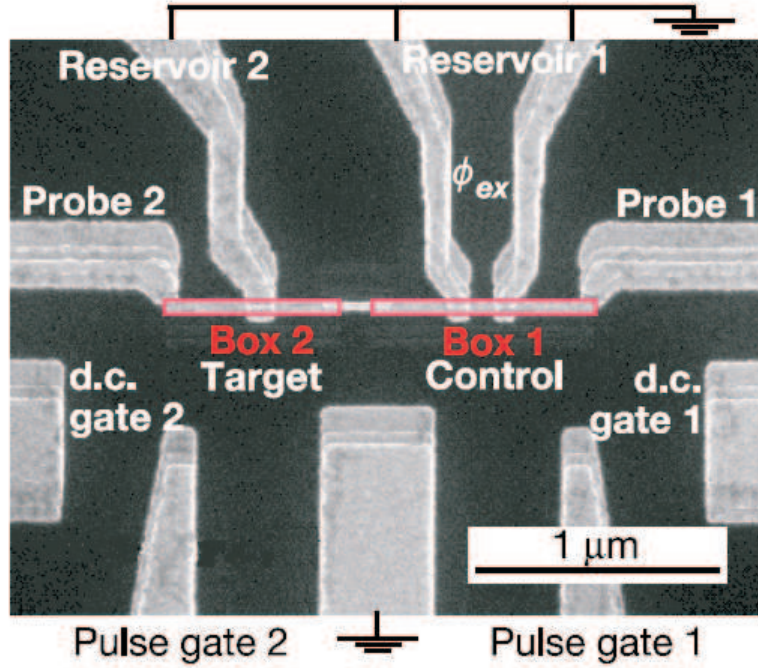


Figure 5.12: Coupled single-Cooper-pair-box qubits. Reprinted by permission from Macmillan Publishers Ltd: Nature. T. Yamamoto *et al.*, Nature **425**, 941 (2003), copyright (2003).

section 5.1.1. The final states are read out by probe junctions of the same type used by Nakamura *et al.* [107].

The crucial thing about this coupled system is that, for $n_{g1} \ll 1/2$ (qubit 1 far from degeneracy point), the gate charge n_{g2} required to bring qubit 2 to the degeneracy point is different for different values of qubit 1. So for qubit 1 in the ground state $|0\rangle$, $n_{g2} = n_{g2}^L$, whereas for qubit 1 in the excited state $|1\rangle$, $n_{g2} = n_{g2}^U$, where $n_{g2}^U > n_{g2}^L$. This means that, if we start from the state $|00\rangle$ and apply a pulse to gate 2 of amplitude n_{g2}^L , the state of qubit 2 will evolve with time between the $|00\rangle$ and $|01\rangle$ states. For a π -pulse, it will end up in the state $|01\rangle$. However, if we start from $|10\rangle$ and apply the same pulse, qubit 2 will not be brought to the degeneracy point, and hence will not evolve during the pulse, ending up in the same state as it started in: $|10\rangle$. Similarly, if we start from the state $|01\rangle$ and apply a π -pulse of amplitude n_{g2}^L the state will evolve to $|00\rangle$, but if we start from $|11\rangle$ and apply the same pulse the state will not change.

The operation we have just described (π -pulse of amplitude n_{g2}^L , when $n_{g1} \ll 1/2$) constitutes a zero-controlled NOT gate (Z-CNOT: switching of the target qubit occurs when the control qubit is $|0\rangle$, rather than $|1\rangle$). Yamamoto *et al.* have used a combination of a $\pi/2$ -pulse (Hadamard gate) to qubit 1 and a π -pulse (CNOT gate) to qubit 2 to prepare entangled states. However, they were not able to achieve maximally entangled Bell states with high fidelity due to the finite rise time of their pulses, which made the CNOT transition $|00\rangle \leftrightarrow |01\rangle$ fairly unreliable (although the $|10\rangle \leftrightarrow |11\rangle$ had good fidelity).

Flux qubits have also been coupled. Izmailkov *et al.* [131] have inductively cou-

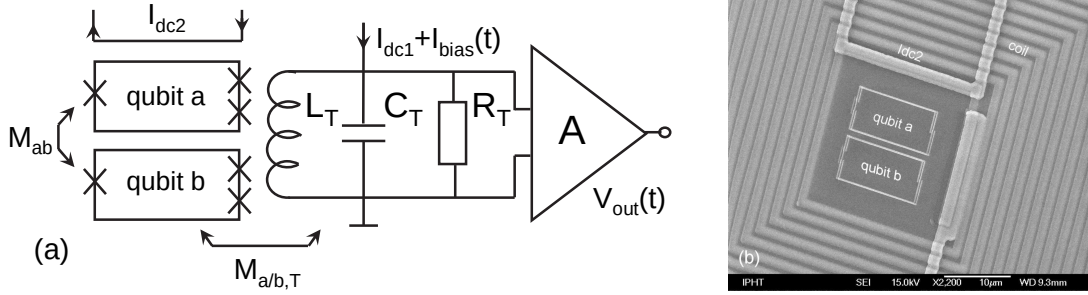


Figure 5.13: Coupled flux qubits enclosed within the loop of a large inductor. (a) Circuit diagram, (b) micrograph. Reprinted figure with permission from A. Izmailkov *et al.*, Phys. Rev. Lett. **93**, 037003 (2004). Copyright (2004) by the American Physical Society.

pled two persistent current qubits of the type proposed by Mooij *et al.* [112] by enclosing them within the loop of a large inductor (see figure 5.13). The qubits are both manipulated and read out via this inductor, with high frequency microwave pulses being used for the manipulation, and low frequency r.f. pulses being used for the readout (which is accomplished by measuring the phase difference between the current and voltage across the inductor). Using some fairly complicated theoretical analysis, the eigenstates of the coupled system were shown to be entangled at a certain value of the externally applied DC magnetic flux. The amount of entanglement (the concurrence C) in each eigenstate was calculated from the qubit parameters determined from earlier measurements. Two of the eigenstates had $C = 0.39$ while the other two had $C = 0.97$. A concurrence of $C = 1$ corresponds to a maximally entangled Bell state, so two of the eigenstates of the system appeared to be close to maximal entanglement. The disadvantage of this system is that the two qubits are not individually addressable.

Considerable progress has been made recently with coupled phase qubits. Berkley *et al.* [130] have performed spectroscopy on a system of two simple Josephson junction phase qubits coupled through a capacitor (see figure 5.14), and have demonstrated an anticrossing of the $|01\rangle$ and $|10\rangle$ states. Here, the capacitive coupling gives rise to an off-diagonal interaction Hamiltonian because we are working in the phase basis. In contrast to the coupled single-Cooper-pair-box qubits of Pashkin *et al.* and Yamamoto *et al.*, this causes two of the eigenstates of the coupled phase qubit system to become entangled when the qubits are in resonance with each other. Off resonance, the eigenstates are $|00\rangle$, $|01\rangle$, $|10\rangle$ and $|11\rangle$, but on resonance they are $|00\rangle$, $\frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle)$ and $|11\rangle$. The qubits are brought in and out of resonance by varying their bias currents. Spectroscopy is performed by applying an additional microwave signal to the bias current of one of them to excite transitions between the qubit states. Readout is achieved by monitoring the voltage across each junction. Each qubit is operated at a bias current so close to its switching current that there are only two levels in the washboard potential. If the qubit is in the excited state $|1\rangle$ it is very likely to escape and cause a voltage pulse, whereas if it is in the ground state $|0\rangle$ it is likely to remain trapped in the superconducting zero-voltage state.

McDermott *et al.* [128] have very recently observed coherent oscillations between

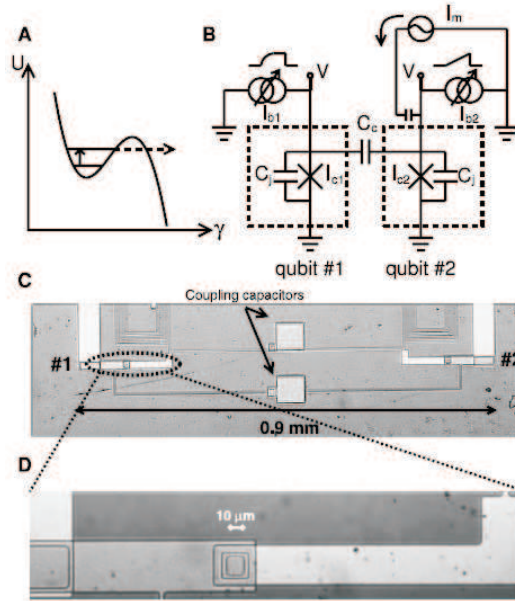


Figure 5.14: Phase qubits coupled through a capacitor. (A) Washboard potential, (B) circuit diagram, (C) photograph of device, showing coupling capacitors, (D) closeup of one junction. Reprinted with permission, A. J. Berkley *et al.*, Science **300**, 1548 (2003). Copyright (2003) American Association for the Advancement of Science.

the $|01\rangle$ and $|10\rangle$ states in a pair of capacitively coupled phase qubits. These phase qubits are more advanced than those of Berkley *et al.*, consisting of single Josephson junctions enclosed in superconducting rings. The qubit states are still eigenstates of the washboard potential, but the potential is altered slightly by the presence of the ring. There are several advantages to this configuration. Firstly, the qubit can be biased inductively by supplying a bias current to an inductor coupled to the qubit loop, causing a current to flow around the ring and tilting the washboard potential. This inductive bias gets rid of any low frequency noise that could cause decoherence. Secondly, the readout scheme is improved because, when the phase escapes, it tunnels to a global minimum of the potential, rather than running away down the washboard. The tunnelling event changes the flux trapped within the qubit loop by Φ_0 , which can easily be read out at a later time with a DC SQUID. In contrast to the readout scheme of Berkley *et al.*, McDermott *et al.* bias their qubits such that the washboard potential contains 3 or 4 levels, ensuring that the phase cannot escape prematurely from the excited state of the qubit. To read out the qubit state, the bias current is suddenly increase so that the $|1\rangle$ state is very likely to escape but the $|0\rangle$ state remains trapped. This can be done simultaneously to both qubits. The flux states of the rings (which are stable over long periods of time) can be read out independently.

Simultaneous fast readout is a great improvement over readout schemes such as the probe junction used for the single-Cooper-pair-box, which rely on the excited state tunnelling after some time period much longer than the gate operation time. With the probe junction there is always the possibility that the qubit will tunnel

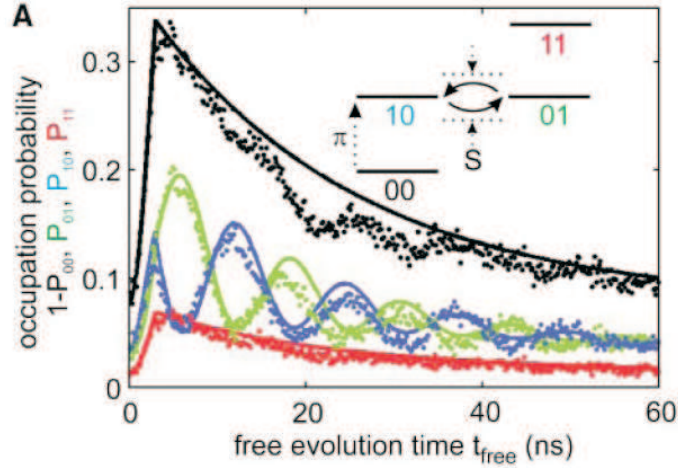


Figure 5.15: Coherent oscillations in coupled phase qubits. The green and blue data show the occupation probabilities of the $|01\rangle$ and $|10\rangle$ states respectively. The oscillations are in antiphase, indicating that the system evolves through maximally entangled Bell states. Reprinted with permission, R. McDermott *et al.*, Science **307**, 1299 (2005). Copyright (2005) American Association for the Advancement of Science.

too early, while the gate operation is still underway. The scheme of McDermott *et al.* allows the moment of readout to be controlled very precisely so that both qubits can be read out with sub-ns simultaneity. This allows measurement crosstalk to be minimised. They have worked out that, when there is a significant delay between the measurement pulses to the two qubits, a measurement of $|1\rangle$ in the first qubit can lead to an enhanced probability of measurement of $|1\rangle$ in the second qubit. This occurs because oscillations of the phase in the global potential minimum after tunnelling are transmitted through the coupling capacitor to the other qubit, and, as they decay in frequency with time, come into resonance with the $|0\rangle \rightarrow |1\rangle$ transition of the second qubit. The easiest way to avoid this is to make sure that the measurement pulses are as close to simultaneous as possible (< 2 ns delay between measurement pulses), so that the phase oscillations do not have time to come into resonance with the other qubit.

Using this readout scheme, McDermott *et al.* have been able to observe coherent oscillations of the coupled qubit state between the $|01\rangle$ and $|10\rangle$ states (see figure 5.15): the qubits are tuned into resonance and the system is then allowed to relax to the ground state $|00\rangle$. A microwave π -pulse is applied to one of the qubits, resulting in the state $|10\rangle$. Because this is not an eigenstate of the coupled system at resonance, the state evolves with time, oscillating between $|10\rangle$ and $|01\rangle$. This sequence is performed many times and the two qubits are read out after varying lengths of time evolution. The occupation probabilities of the excited states of the two qubits are found to oscillate in antiphase, with high fidelity, indicating that the system evolves through the maximally-entangled Bell states $\frac{1}{\sqrt{2}}(|10\rangle \pm |01\rangle)$.

Above we have discussed the most significant recent advances in superconducting quantum devices. Three main types of qubit have been implemented, each resem-

bling an artificial atom, and all have exceeded expectations regarding coherence time, ease of manipulation and readout. The field is evolving fast and is rapidly gaining critical mass.

Research efforts on single qubits are currently focused on ironing out problems with decoherence (to do with shielding, fabrication and circuit design) and on improving readout fidelity. Coupled qubit research is also gathering momentum, with a number of recent demonstrations of entangled states.

A decade ago, it was believed that strong coupling to the environment would render superconducting devices incapable of storing quantum information over times long enough to perform manipulation and readout. However, it has now been demonstrated unequivocally that this view was too pessimistic. In fact, strong coupling to the environment provides superconducting qubits with an advantage over atomic qubits in that they are easy to control and easy to couple together.

5.3 Applications of superconducting qubits

Superconducting quantum devices are now sufficiently mature that they can begin to be exploited for practical applications. Many of these applications are metrological, but there is also much scope for industrial spin off. The fabrication technology required to produce superconducting quantum devices has already been well developed by the semiconductor industry, so mass production is already possible.

5.3.1 SET current and capacitance standards

Superconducting quantum devices are already being developed for metrological application in the area of single electron tunnelling (SET) current standards [132, 133, 134]. Such devices utilise the same quantum circuitry as charge qubits. The idea is to transport Cooper pairs along a chain of superconducting islands by alternately raising and lowering the potential barriers between the islands. The current can be controlled by changing the frequency of the barrier suppression signal. Single Cooper pair current standards offer the potential to close the metrological electrical triangle of voltage, current and resistance. This field is actively pursued by the DC and low frequency team at NPL. A capacitance standard based on counting electrons with an SET device was demonstrated in 1999 [135].

5.3.2 Sub-Kelvin, sub-Poisson noise standards

Another area of active research at NPL is quantum thermal metrology. This offers an alternative to the canonical ensemble of interacting particles in the form of a single quantum entity sampled repeatedly in time. Such a device could in principle achieve high accuracy thermometry in the sub-Kelvin region of the temperature scale, limited only by sample size and sampling speed. In the current 2004-7 Quantum Metrology programme the Quantum Roulette Noise Thermometer (QRNT) project explores this possibility [24]. The QRNT utilises the same superconducting circuitry as a flux qubit, but other devices based on charge and phase are possible.

5.3.3 Quantum computing and cryptography

The most obvious industrial application of superconducting quantum devices is quantum computing. It is well known that quantum computers have the potential to perform certain computational tasks at a much higher efficiency than classical computers. Typical examples are Shor's algorithms for factoring and discrete logarithms [39] and Grover's quantum searching algorithm [136]. Such algorithms rely on the entanglement of multiple qubits, and superconducting devices offer good prospects for the scalability required.

Quantum information processing, in particular quantum cryptography is a burgeoning field that is already attracting heavy investment. Quantum cryptography requires error correction [137] to allow quantum states to be transmitted faithfully over long distances, and this requires large numbers of qubits. Superconducting circuits currently offer the best prospects for the scalability required to implement error correction.

5.3.4 Quantum simulation

A future metrological and industrial application of entanglement is *quantum simulation*. This was first proposed by Richard Feynmann in the 1980s [5], to address the problem of simulating complex quantum mechanical systems, and is now becoming important for nanotechnology. The task of modelling systems on the atomic scale, such as how molecules bind to surfaces and how self assembly of nanostructures occurs, currently requires vast classical computational power that scales exponentially with the number of elements involved. And to model quantum mechanical effects requires extraordinary computational power, far beyond the capabilities of classical computers. The idea behind quantum simulation is to use quantum systems themselves to perform the simulations. The resources of superposition and entanglement could be utilised to process many calculations simultaneously, thus allowing efficient modelling of systems that are too complex for classical simulation. Quantum simulation is a long way off at present, but would greatly facilitate the production of new materials, nanoscale devices and biological systems. Once again, superconducting qubits currently offer the best prospects for the scalability required.

5.3.5 Frequency standards

Ramsey interferometry is currently used to probe the frequency of atomic transitions to provide time and frequency standards. This is an area where quantum-enhancement of the uncertainty can be obtained through the use of entangled states. By entangling many atoms the uncertainty on the frequency measurement can be reduced by a factor of $1/\sqrt{N}$ [42]. The same sort of measurement can be performed on superconducting qubits, and in fact these offer better prospects for quantum enhancement because of their increased scalability. The caveat is that long coherence times are needed, and improvement is needed in this area before superconducting systems can compete with atoms and ions in this area.

5.3.6 Single microwave sources and detectors

Entanglement of superconducting devices with resonant transmission lines offers the potential to create deterministic r.f./microwave single-photon sources and detectors. These ideas utilise the strong coupling régime of cavity QED and will be discussed in more detail in chapter 6.

5.3.7 Quantum measurement

In the quantum regime the concept of measurement is radically altered from that in the classical world. When we make a classical measurement we expect the measured quantity to remain unperturbed by the measurement. However, in many quantum systems the measurement process destroys the quantum state of the system, requiring it to be reset before another measurement can be made. One way to avoid the problem of measurement-induced state destruction is to develop so called *quantum non-demolition* (QND) readout techniques (see section 2.5.6) [44]. These require the use of a meter that is a quantum system itself, and is entangled with the target system. Readout of the target system can be achieved by performing a destructive measurement of the meter system, which simultaneously determines the state of the target system via entanglement. Superconducting systems are ideal for the construction of QND readout schemes (some such schemes are described in chapter 6), as they are highly customisable and tuneable. In principle there is no limit to the precision with which a QND observable can be measured, so QND represents the ultimate goal in precision quantum metrology.

5.4 Summary conclusions

The field of superconducting qubit research is rapidly gaining critical mass. Entanglement of coupled qubits has been demonstrated, and the advantages of scalability, tunability and individual addressability are being recognised both by industry and by other NMIs. Great potential exists for metrological applications as outlined above. The main reasons why NPL should maintain or accelerate its current level of activity in this field are as follows:

- NPL is already strong in areas of traditional quantum metrology such as SET-based electron-counting current/capacitance standards, but these capabilities could be extended further by capitalising on entanglement-based advances in qubit technology such as quantum non-demolition readout methods. Such techniques would build on NPL's solid background in quantum metrology and allow the frontiers of measurement to be pushed towards the fundamental Heisenberg limit.
- The sensor, semiconductor, and IT industries are already moving toward devices operating in the quantum régime, and will require metrological capabilities in the quantum régime to support their development. Quantum devices of the future will be characterised by quantities such as visibility, fidelity and concurrence, and it is vital that we develop the capability to measure such quantities precisely.

Chapter 6

Cavity QED

The vacuum radiation field can be made to interact with a stationary qubit system, however it is necessary to enhance the strength of the vacuum field using a build-up resonator (or cavity). This resonator modifies the vacuum field that interacts with the stationary qubit; this field of research is known as cavity quantum electrodynamics (QED). The basic physics of the cavity QED systems relevant to this report is illustrated by way of an example in figure 6.1.

The high-finesse cavity is resonant with an optical transition in the qubit, *e. g.* an atomic particle. The rate g characterises the strength of the coupling between the atomic transition and the cavity mode. The rate κ , at which photons decay from the cavity mode, is determined by the cavity finesse. The excited state of the atomic particle has a spontaneous decay rate γ . The most interesting regime of this system is when $g \gg \kappa, \gamma$; this is known as the *strong-coupling regime*: the interaction between the vacuum field of the cavity and the atomic transition dominates over spontaneous emission and cavity decay. The physical consequence is that oscillatory exchanges of energy between the atom and the cavity can be observed (in the form of a photon), before any irreversible process can take place, enabling superposition states of the atomic energy levels and the cavity mode photon number to be created. Should the photon leaves the cavity (rate κ), or be emitted spontaneously from the atom (rate γ), then it is irreversibly lost from the system.

The same basic principles apply to any system consisting of a stationary qubit interacting with the vacuum field: *i. e.* the aim is to ensure that reversible processes dominate over irreversible ones. Cavity QED systems in the strong coupling regime are a prerequisite to achieve deterministic entanglement of static and flying qubits.

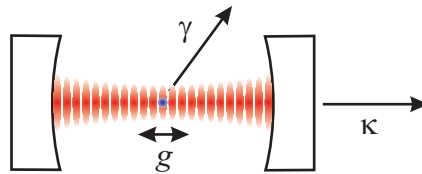


Figure 6.1: A single atomic particle interacting with the TEM₀₀ mode of a high-finesse optical cavity. The parameters g , κ , and γ are explained in the text.

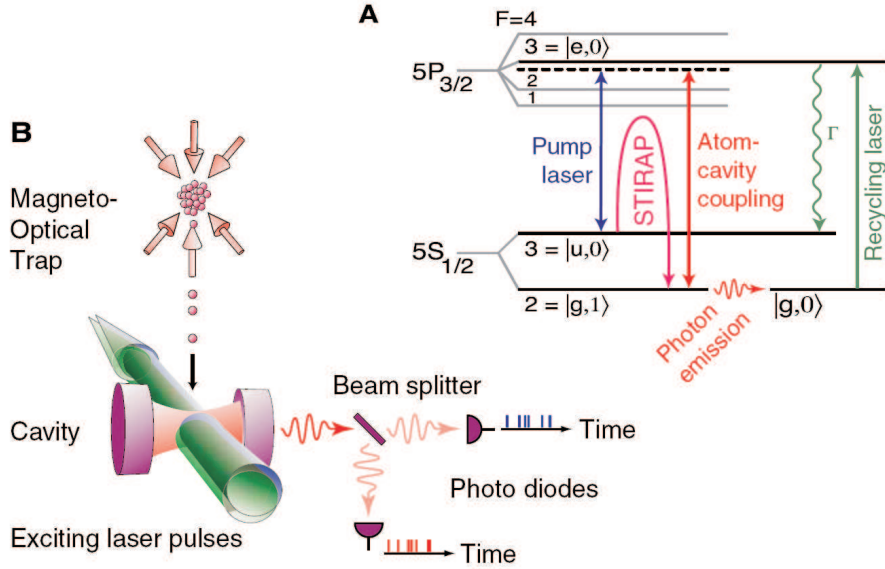


Figure 6.2: The Munich group's experimental scheme for ^{85}Rb . (a) energy level diagram showing the pump laser and cavity mode frequencies for driving the stimulated Raman transition *via* adiabatic passage. Reprinted figure with permission from A. Kuhn *et al.*, Phys. Rev. Lett. **89**, 067901 (2002). Copyright (2002) by the American Physical Society.

6.1 Atomic systems and optical cavities

In terms of achieving strong coupling, the most successful experiments to date in the optical regime have been with neutral atoms (of Rb or Cs) and high finesse cavities in the near infrared. This field is led by the groups of Gerhard Rempe at the Max Planck Institute for Quantum Optics, Munich and Jeff Kimble at Caltech, who use very similar experimental approaches. The basis of these experiments is the technique of stimulated Raman scattering *via* adiabatic passage (STIRAP); this uses a three-level atomic system, an external laser field, and the vacuum field of the optical cavity [138]. This scheme is illustrated in figure 6.2. The adiabatic passage occurs between two ground-state hyperfine levels, with the external laser field and the cavity vacuum field driving the transition.

In practice these are technically demanding experiments. They rely on dropping single atoms from a magneto-optic trap, into the centre of the cavity. Various means exist to cool and trap the atom inside the cavity. Trapping times were limited to about 1 s, however very recently the Munich group have extended this to 17 s [139]. Perhaps the biggest challenge is to hold a single neutral atom inside the optical cavity for much greater periods of time.

For several years the major goal of these neutral atom experiments was to demonstrate a deterministic source of single photons. Both the Munich and Caltech groups achieved this [138, 140], as well as other significant results such as the realisation of a one-atom laser [141], the observation of the vacuum Rabi splitting [142, 143], and the interference of two single photons [144]. Other research groups active in this field

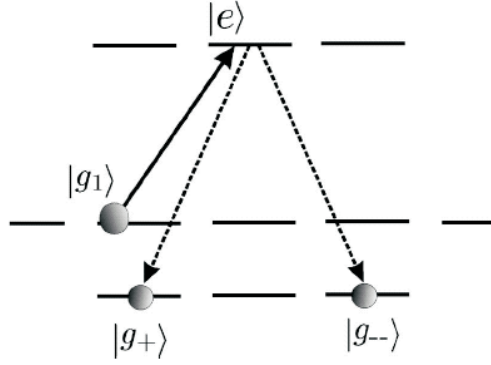


Figure 6.3: Atom-photon entanglement for a multi-level atom. The STIRAP scheme uses an external laser field ($g_1 \rightarrow e$, solid line), and the vacuum cavity field resonant with the $e \rightarrow g_+/g_-$ degenerate transitions.

of cavity QED with neutral atoms are those of Michael Chapman at Georgia Tech and Dieter Meschede at Bonn. The latter group has demonstrated very impressive control of single atoms in an optical conveyor belt [145].

Trapping atomic particles for extended periods is a routine task for the ion trapping community, however these systems present their own separate technical challenges. One successful experiment on cavity QED with a trapped ion has been reported [146]. Walther's group in Munich demonstrated controlled single-photon emission from a single trapped ion into a well-defined optical mode. However, it should be noted that this experiment was not conducted in the strong coupling regime of cavity QED, unlike its neutral atom competitors. Some stiff technical challenges need to be overcome before ion trap experiments will be able to realise the strong coupling regime. Nevertheless, the groups of Chris Monroe at Michigan and Michael Chapman at Georgia Tech, in addition to that of Wolfgang Lange at Sussex, are all pursuing this goal (see table A.5 in appendix A).

Research in atomic particle cavity QED is strongly driven by the prospects for QIP and related topics. One example is the prospect of a quantum network capable of demonstrating quantum state transfer between remote nodes [147]. Such a network could be envisaged as consisting of several atom-cavity nodes, interconnected by optical fibers. The interaction of a high-finesse cavity with stationary atomic qubits (*i. e.* trapped atoms or ions) is the mechanism for coupling the qubits state to an optical mode, for transmission to another qubit processing node.

A further example of the relevance of cavity QED to QIP is the possibility of deterministic atom-photon entangled states. While beyond the capability of current experiments for purely technical reasons, an example of atom-photon entangled states has been proposed by Chapman's group [148]. In this example, the simple three-level atomic system and STIRAP scheme discussed above has been extended to a multi-level atom, as illustrated in figure 6.3.

By careful choice of atomic energy levels addressed by the pump laser field, together with selection rules for atomic transitions, the photon emitted into the cavity mode could be in a superposition of right and left circular polarisations.

Since the polarisation state of the photon determines the final atomic energy level, the atom is also in a superposition. The system is therefore in an entangled state, denoted by

$$|\psi\rangle = \frac{1}{\sqrt{2}} (|g_{-}\rangle |n_L = 1, n_R = 0\rangle + |g_{+}\rangle |n_L = 0, n_R = 1\rangle), \quad (6.1)$$

where n_R and n_L denote the number of right and left circular photons respectively.

A semiconductor quantum dot is often viewed as an *artificial atom*. The quantum dot research community has also been pursuing cavity QED experiments. In the first instance there have been successful demonstrations of single-photon sources based on quantum dots within a microcavity [149, 150]. However, since the Q of these cavities was limited to $\sim 10^3$, they were not able to realise the strong coupling regime. The utility of such devices is evidenced by the entangled photon states [50] and quantum teleportation [51] demonstrated by the Yamamoto group, as mentioned in section 3.1.2 of this report. More recently, the exciton-photon strong coupling regime has been demonstrated for a single quantum dot embedded in a microdisk microcavity [151]. This therefore opens up possibilities for generation of the exciton-photon entangled states, similar to the atom-photon entangled states outlined above. Therefore, this system is a highly relevant tool for QIP based on semiconductor quantum dots.

6.2 Superconducting qubits and resonators

In superconducting circuits the strong coupling regime is reached when a qubit can be made to interact strongly with a resonant circuit such as a transmission line or a DC SQUID. The qubit and the resonator exchange energy through absorption and reemission of a microwave photon, and strong coupling is achieved when many cycles of vacuum Rabi oscillation occur before either the photon is lost from the resonator or the qubit relaxes.

In the following discussion we write coupled qubit and resonator states as $|\psi\rangle = |nm\rangle$, where $|n\rangle$ is the state of the qubit ($n = 0, 1$) and $|m\rangle$ is the state of the resonator ($m = 0, 1, \dots$).

Cavity QED has recently been demonstrated in all three types of superconducting qubit: charge, flux and phase. Spectroscopy of a Cooper-pair-box charge qubit coupled capacitively to a superconducting transmission line has revealed a splitting of the $|01\rangle$ and $|10\rangle$ states at resonance, indicating entangled eigenstates of the combined system of resonator and qubit [152]. In the case of a persistent current flux qubit coupled to a DC SQUID, vacuum Rabi oscillations were observed between the $|01\rangle$ and $|10\rangle$ states, indicating a cycle of energy exchange between the resonator and the qubit, and showing that the system evolves through maximally entangled Bell states with time [153]. Even greater progress has been achieved with phase qubits, with coupling of two qubits to an LC resonator being demonstrated. Spectroscopic measurements have revealed anticrossings of the qubit and resonator states at resonance and indicate the existence of entangled three-particle eigenstates [154]. We discuss each of these results in more detail below.

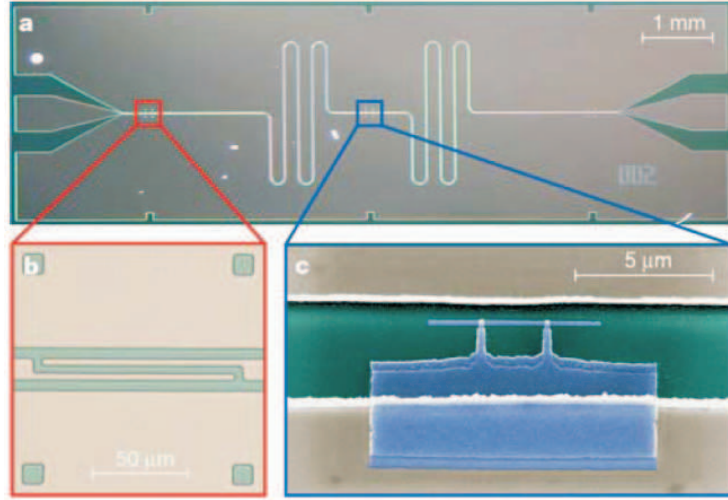


Figure 6.4: Cooper-pair-box qubit (bottom right) coupled to a superconducting transmission line (top). Reprinted by permission from Macmillan Publishers Ltd: Nature. A. Wallraff *et al.*, Nature **431**, 162 (2004), copyright (2004).

The coupled Cooper-pair-box qubit and transmission line of Wallraff *et al.* [152] are shown in Figure 6.4. The transmission line is coupled capacitively to the qubit, acting as the qubit's gate electrode. The resonator frequency is fixed at $\omega_r = 1/\sqrt{LC}$, where L is the inductance and C the capacitance of the thin film superconducting transmission line. The qubit transition frequency can be varied by altering the DC gate voltage supplied *via* the transmission line. Spectroscopy of the energy levels of the coupled system is performed by supplying a microwave signal to the transmission line and observing the transmitted power as a function of signal frequency. Resonant enhancement of the transmitted microwave power is observed whenever the signal frequency comes into resonance with the transition frequency of the coupled system. By varying the gate voltage the qubit levels can be tuned in and out of resonance with the transmission line resonator. When out of resonance only a single peak is observed in the transmitted microwave power, corresponding to the resonant frequency of the transmission line ($|00\rangle \rightarrow |01\rangle$). However, at resonance, two peaks are observed, corresponding to the entangled eigenstates $\frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle)$ of the qubit and the transmission line.

Wallraff *et al.* also note that they can utilise the non-resonant (dispersive) régime of the circuit to perform a quantum non-demolition readout of the qubit state. This exploits the entanglement of the qubit and resonator to ensure that measurement of one fully determines the state of the other. The transition frequency of the resonator is slightly modified by the qubit state because of the strong coupling between them, so a measurement of the resonator frequency (using the same technique as described above, where the transmitted microwave power or phase angle with respect to the input signal is measured as a function of frequency) allows them to distinguish between the two qubit states. The same authors have used the same technique to detect Rabi oscillation of the qubit with almost unit visibility (see Figure 6.5). Using a Ramsey fringe experiment they demonstrated a coherence time of 500 ns [155].

Chiorescu *et al.* have demonstrated Rabi oscillations of a persistent current flux

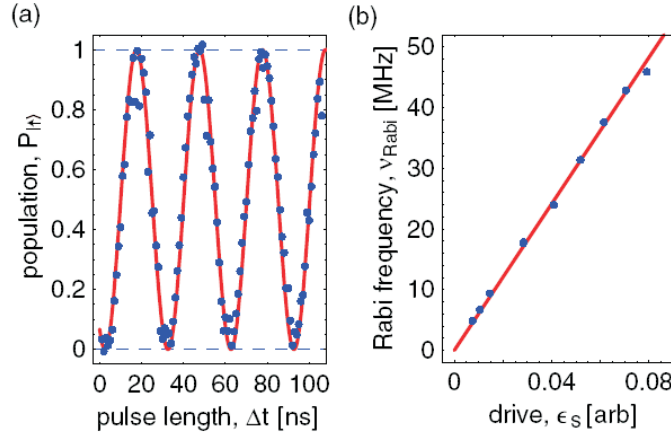


Figure 6.5: Left: Rabi oscillations with close to unit visibility, observed using a QND readout of a charge qubit. Right: Rabi frequency vs pulse amplitude. Reprinted figure with permission from A. Wallraff *et al.*, Phys. Rev. Lett. **95**, 060501 (2005). Copyright (2005) by the American Physical Society.

qubit coupled to a DC SQUID resonator [153]. This system differs from that of Wallraff *et al.* by the tunability of the resonator frequency through the bias current supplied to the SQUID (a DC SQUID has inductance $L_{\text{loop}} + L_J/2$, where L_{loop} is the loop inductance of the SQUID and $L_J = \hbar/2eI_c\sqrt{1 - (I/I_c)^2}$ is the Josephson inductance of each of its junctions, which depends on the bias current I). The qubit transition frequency can also be tuned by varying the DC external magnetic flux (this has a negligible effect on the SQUID's critical current due to the much larger loop area of the SQUID). The qubit of Chiorescu *et al.* was designed to have a minimum transition frequency higher than the maximum resonant frequency of the DC SQUID, so the two were never brought into resonance. But they were able to observe Rabi oscillations between the $|00\rangle$ and $|10\rangle$ states (the qubit transition), the $|01\rangle$ and $|10\rangle$ states (the red sideband) and the $|00\rangle$ and $|11\rangle$ states (the blue sideband) by driving the coupled system strongly with external microwave power (supplied *via* a control line coupled inductively to the SQUID-qubit system). Rabi oscillations between the $|00\rangle$ and $|01\rangle$ states (the resonator transition) could not be observed because the oscillations were detected *via* measurement of the changing occupancy of the excited qubit state. The qubit state was read out by the SQUID switching method described in section 5.1.2, and the parameters of the circuit were chosen such that the SQUID would be much more likely to switch at a low bias current if the qubit were in its excited state than if the resonator were in its excited state. The coherence time of the red sideband was 2.9 ns, the shortness of which was attributed to resonator relaxation. It is likely that this can be improved.

Finally, the coupled phase qubits and LC resonator of Xu *et al.* [154] are shown in figure 6.6. We note that this is actually the same chip used for the coupled phase qubit experiments described in section 5.2, but operated in a different regime (for the non-cavity-QED régime they operated the device at frequencies much less than the frequency of the resonator, so the resonator provided purely capacitive coupling between the qubits). This system is distinct from an atomic cavity QED

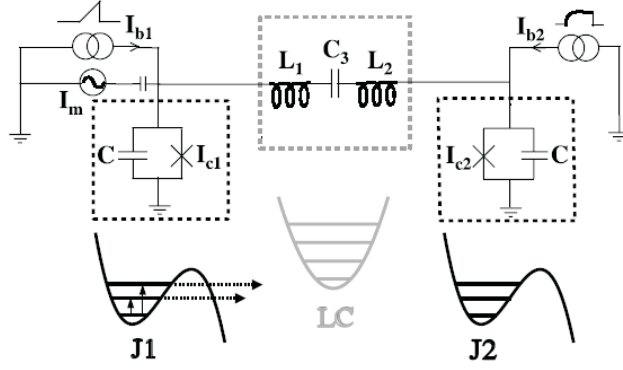


Figure 6.6: Schematic of two Josephson junction phase qubits coupled to an LC resonator. Reprinted figure with permission from H. Xu *et al.*, Phys. Rev. Lett. **94**, 027003 (2005). Copyright (2005) by the American Physical Society.

system in that the transition frequencies of the Josephson junction phase qubits are distinguishable and independently tunable. In contrast to the setup of Chiorescu *et al.*, but in common with Wallraff *et al.*, the frequency of the LC resonator is fixed.

Xu *et al.* made spectroscopic measurements of the energy levels of the coupled system, with one or both qubits tuned into resonance. Transitions between the eigenstates of the coupled system were excited by applying a microwave signal to one of the junctions in addition to the DC bias current. Excitations were detected as an enhancement of the phase escape rate from the junction. Anticrossings of the $|001\rangle$ and $|100\rangle$ states and the $|001\rangle$ and $|010\rangle$ states were observed when each qubit was tuned separately into resonance with the resonator. The last number represents the resonator state. When both qubits were tuned into resonance with the resonator, anticrossings between the $|001\rangle$, $|010\rangle$ and $|100\rangle$ states were observed, giving rise to the entangled eigenstates $\frac{1}{2}(|100\rangle - |010\rangle - \sqrt{2}|001\rangle)$, $\frac{1}{\sqrt{2}}(|100\rangle + |010\rangle)$ and $\frac{1}{2}(|100\rangle - |010\rangle + \sqrt{2}|001\rangle)$ at the triple resonance frequency.

Although the qubits are separated by almost 1 mm, strong coupling between them can be achieved by tuning them into resonance with the resonator. This bodes well for future efforts in quantum communication and for the building of quantum logic gates. It is hoped that by improving the isolation of the qubits from their environment, coherence times long enough to observe coherent oscillations will be achieved. The observation of coherent oscillations in phase qubits coupled in the non-cavity-QED regime by McDermott *et al.* [128] suggests that incorporating the Josephson junctions in superconducting loops and biasing them inductively would be a good way to improve isolation.

The entanglement of cavity (resonator) modes with superconducting qubit states offers the possibility of applications such as r.f./microwave single-photon sources and detectors. Such applications have potential both for metrology (r.f./microwave radiometry) and industry (deterministic generation of single r.f./microwave photons). Entanglement is crucial to these applications, as both generation and detection of photons rely on control of the resonator state *via* control of the qubit. The qubit state must be entangled with the resonator state in order to do this.

Superconducting devices are ideal for photon generation and detection because they are tuneable in frequency. When the resonator and qubit are in resonance we obtain cavity QED behaviour, involving coherent energy exchange between them, but the tunability of superconducting devices allows us to use the same devices to access the non-resonant régime in which quantum non-demolition measurements of the qubit state can be made. This is done *via* measurement of the phase of driven oscillations of the coupled system.

6.3 Summary conclusions

Technology in atomic-optical photon cavity-QED systems is currently led by neutral atoms. However, trapped ions do offer significant advantages once specific technical demands are met. Since (1) cavity QED is the most promising vehicle for converting between static and flying qubits, (2) a highly-developed technology platform exists for transmitting photons, and (3) currently trapped ions are the most developed system for processing quantum information, then optical cavity QED with trapped ions should be viewed as a significant prospect for processing and transmitting quantum information. It is anticipated that if and when this technology emerges as the front-runner for transmitting quantum information, then for example, quantifying the fidelity of quantum information transmission (together with associated decoherence) will be crucial in determining the viability of the technology in the long term. An integral part of optical cavity-QED is the capability to perform appropriate measurements on photons. NPL is already developing its capability in this area through the “Entangled Photon Metrology” project; indeed, all of the measurement capabilities under development are considered essential for any advanced optical cavity-QED experimental facility. Consequently, current activities in trapped ions and single photons are an appropriate platform for progressing in optical cavity-QED, and NPL should consider future possibilities in this area.

- NPL is currently pursuing research in trapped ions and single-photon measurements that are an integral part of any R&D capability in optical cavity QED. Current expertise in this research base is a sound foundation from which to pursue optical cavity QED with trapped ions. A significant achievement for this research field would be to demonstrate an ion-cavity system in the strong-coupling regime; this would have strong impact in QIP research with trapped ions.
- In addition to QIP applications, superconducting cavity QED devices could lead to single photon microwave sources and detectors which would find applications in fields such as radio astronomy.
- Superconducting devices are particularly flexible because the qubit can be tuned in and out of resonance with the transmission line. On resonance the device operates in the cavity QED regime, whereas off resonance the transmission line can be used to perform quantum non-demolition measurements of the qubit state. A study to design a device whose parameters are optimised for both regimes has recently been awarded funding by NPL’s strategic research programme, and will commence in February 2006.

Chapter 7

Conclusions

This report emphasises areas in which research motivated by QIP has spin-offs which may benefit metrology — **entanglement for metrology**, such as Heisenberg spectroscopy and the quantum logic clock, and areas in which QIP will itself require new metrology techniques — **metrology of entanglement**, such as the quantification of entanglement through measurement of the fidelity.

7.1 Comparison of technologies

The state-of-the art in the technologies for quantum entanglement applications in which NPL has a capability has been reviewed in the preceding chapters. In concluding, we briefly draw compare the potential of these technologies before making some recommendations for the future direction of NMS-funded research.

In order to compare and map the progress in the many different fields of quantum computation, David DiVincenzo from IBM, devised a set of necessary criteria which any viable quantum computation technology should meet. These *DiVincenzo criteria* are:

1. a scalable physical system of well-characterized qubits
2. the ability to initialize the state of the qubits to a simple state
3. long (relative) decoherence times
4. a universal set of quantum gates
5. a qubit-specific measurement capability

There are, additionally, two necessary conditions for QIP *networkability*:

7. the ability to interconvert stationary and flying qubits
8. the ability to faithfully transmit flying qubits between specified locations

Although the DiVincenzo criteria are specifically designed with reference to quantum information processing, they provide a reliable indication of the state of development and potential for each technology in related applications. Table 7.1 scores

Table 7.1: Scoring of technologies reviewed in this report against the DiVincenzo criteria. **Y**: A potentially viable approach has been demonstrated at at least a proof-of-principle level. **?**: A potentially viable approach has been proposed but not yet adequately demonstrated. **N**: No viable approach is known (Adapted from [26].)

Technology	DiVincenzo criteria						
	quantum computation					networkability	
	1	2	3	4	5	6	7
Photons	?	?	Y	?	?	?	Y
Trapped ions	?	Y	?	Y	Y	?	?
Neutral atoms	?	Y	?	?	?	?	?
Superconducting	?	Y	?	?	?	N	N
Cavity QED	?	Y	?	?	Y	?	?

the technologies reviewed in this report against each criterion. Technologies in which NPL has no activity, such as NMR or semiconductor quantum devices, score worse, or at least no better than, the technologies under review. Scoring each technology individually has the weakness that it does not reflect the potential power of combining technologies to make best use of their individual strengths, for example, using photons as flying qubits to transmit quantum information between registers in a trapped-ion quantum computer.

The ability to meet the DiVincenzo criteria depends crucially on the availability of high precision measurement science related to quantum properties, *i. e.* quantum enhanced metrology.

7.2 Investment

As discussed in chapter 1, the annual global expenditure on R&D in quantum information systems and related applications of quantum entanglement is in well in excess of 100 M euros, with well over half of this sum being invested in the United States, largely through defence and security-related programmes. Although European funding, both through the EU and through national programmes, is at a lower level, it has yielded well above par in both quantity and quality of output. NPL shares in this rich field of research through its participation in EU projects such as SCALA and UK collaborations such as QIP-IRC. NPL’s activity is currently at the level of approximately five man-years per year of staff scientists, together with two full-time PhD students.

7.3 Recommendations for the NMS QMP

This report has investigated the impact of quantum entanglement on metrology and future industry applications. Entanglement, the inseparable entwining of

particles at the atomic scale, is a unique resource in science today and has the potential to revolutionize many areas of science and technology. These technologies are truly disruptive: they rely on the quantum nature of matter to perform functions which are intrinsically inaccessible to technologies based on macroscopic devices ruled by the laws of classical physics.

The report has discussed in detail the benefits to be gained from harnessing quantum entanglement for metrology. Entanglement provides the route to quantum-enhanced measurement, pushing metrology to the fundamental Heisenberg limit. The precision of atomic frequency standards can be improved which will benefit the most demanding timing and positioning applications such as deep space navigation and tests of the fundamental laws of physics.

A firm understanding of quantum entanglement is a prerequisite for advancing the development of quantum information processing and communication applications. Manipulation and quantification of entanglement is one of the most challenging measurement problems. Quantum states are extremely fragile and require the ultimate control of the measurement environment in every possible aspect (both on a macroscopic and microscopic scale). Therefore, quantum enhanced metrology must be seen as an enabling or underpinning technology for future QIPC applications.

On the basis of the above considerations it is of paramount importance that a major metrology laboratory such as NPL plays a leading role in the research and development of quantum-enhanced metrology applications. However, the field of QEM applications is vast and choices will have to be made where best NPL can make a contribution to these. In this respect NPL already has a considerable activity in trapped-ion systems, solid state systems, and photonic applications. The choice of these technology areas is motivated by the following considerations.

- Trapped ions are widely considered to be the only system which satisfies all the DiVincenzo criteria at present. The first real quantum computer is likely to be based on this technology. Entanglement enhanced trapped ion systems will directly lead to improved optical clocks and feed into timing and positioning applications.
- Solid state systems are considered to be the most scalable of all technologies. They offer the possibility for integration with existing classical computational hardware and are supported by mature background technology. In addition the critical mass of the semiconductor industry is enormous and fast progress can be expected now that the ultimate limit of Moore's Law for traditional CMOS technology comes into view.
- Photonic systems such as quantum cryptography are closest to market and the UK in particular has a very strong presence in this field.
- Cavity QED is the most promising vehicle for converting between static and flying qubits.

This report recommends that these technology areas are amalgamated into one theme in the next NMS 2007-2010 Quantum Metrology Programme with the title **Quantum Information Processing and Communication (QIPC)**. There are several benefits to this approach. QIPC is fast becoming an accepted term within the

science community (academia, industry and large research organisations), as such a clearly identifiable theme within the Quantum Programme would align well with this trend. At present there is no single winning technology within QIPC and many research organisations have activities in several technology fields. A single theme within the Quantum Programme would benefit the establishment of collaborations between NPL and these outside research organisations.

Another important factor is the collaborative research effort supported by a number of large European Framework 6 initiatives (such as SCALA, EuroSQIP and QAP) and the UK QIP-IRC. NPL is a partner in the SCALE project and several collaborations with partners of the EuroSQIP and QIP-IRC projects.

As can be seen from this report, the technologies involved in the separate topics are very different, however, the underlying principles are identical in each of them. There is a great deal of overlap between technologies in this field with similar underpinning technological requirements (nanotechnology, theory, cavity QED *etc.*). These topics have a similar customer focus and this grouping will be well-aligned with European Networks and UK initiatives. No winning technology has as yet been identified, so it will be important to maintain a strong presence in all these areas *via* strategic collaborations.

Appendix A

Leading research groups

The following tables are updated from those in the roadmap document on the ARDA website [26] but are not to be taken as exhaustive lists.

Table A.1: Photon research groups.

Research Leader(s)	Institution	Country
Bouwmeester	UC	US
DeMartini	Rome U.	It
Dowling	Louisiana State U.	US
Franson	Johns Hopkins U.	US
Gisin	U. Geneva	CH
Howell	U. Rochester	US
Imamoglu	UC	US
Kwiat	U. Illinois	US
Milburn & Ralph	U. Queensland	Au
Nakamura	NEC, Tskuba	Jp
Rarity	U. Bristol	UK
Sergienko	Boston U.	US
Shih	U. Maryland	US
Steinberg	U. Toronto	Ca
Takeuchi	Hokkaido U.	Jp
Walmsley	U. Oxford	UK
Weinfurter	U. Munich	De
White	U. Queensland	Au
Yamamoto	Stanford U.	US
Zeilinger	U. Vienna	At

Table A.2: Ion trap research groups.

Research Leader(s)	Institution	Country	System
Berkeland	LANL	US	Sr^+
Blatt	U. Innsbruck	At	Ca^+
Drewsen	Aarhus		Ca^+
Gill	NPL	UK	Sr^+
King	McMaster U.	Ca	Mg^+
Monroe	U. Michigan	US	Cd^+
Schmidt-Kaler	U. Ulm	De	
Steane	U. Oxford	UK	Ca^+
Wunderlich	U. Siegen	De	Mg^+, In^+
Wineland	NIST, Boulder	US	$^9\text{Be}^+, \text{Mg}^+$

Table A.3: Neutral atom research groups.

Research Leader(s)	Institution	Country	System
Bloch	U. Mainz	De	
Chapman	Georgia Tech.	US	magnetic & optical trapping
Ertmer & Birkel	U. Hannover	De	optical trapping with micro-optics
Gould	U. Connecticut	US	optical trapping of Rydberg atoms
Grainger	Institute d'Optique	Fr	single-atom trapping
Haensch	MPQ	De	BEC/optical trapping
Hinds	Imperial College	UK	magnetic microtraps
Jessen	U. Arizona	US	optical lattices
Meschede	U. Bonn	De	single-atom trapping
Phillips & Rolston	NIST, Gaithersburg	US	optical lattices
Reichel	MPQ	De	magnetic microtraps
Saffman & Walker	U. Wisconsin	US	optical trapping of Rydberg atoms
Schiedmayer	U. Heidelberg	De	magnetic microtraps

Table A.4: Superconducting research groups.

Research Leader(s)	Institution	Country	System
Berggren	MIT	US	flux qubits
Buisson	Grenoble	CH	charge qubits
Clarke	Berkeley	US	flux qubits
Cosmelli	Rome	It	flux qubits
Delsing	Chalmers	Se	charge qubits
Devoret	Yale	US	charge qubits
Echternach	JPL	US	charge qubits
Esteve	Saclay	Fr	charge qubits
Feldman & Bocko	Rochester	US	flux qubits
Han	Kansas	US	flux and single-junction phase qubits
Kock	IBM	US	flux qubits
Kouwenhoven	Delft	NL	charge qubits
Ladizinsky	TRW	Pl	flux qubits
Likharev	Stonybrook	US	charge and flux qubits
Manheimer	LPS	?	charge qubits
Martinis	UC	US	single-junction phase qubits
Mooij	Delft	NL	flux qubits
Nakamura	NEC	Jp	charge qubits
Oliver & Gouker	Lincoln Lab	US	flux qubits
Orlando	MIT	US	flux qubits
Schoelkopf	Yale	US	charge qubits
Silvestri	Naples	It	flux qubits
Simmonds	NIST	US	phase qubits
Tanaka	NTT	Jp	flux qubits
Ustinov	Erlangen	De	flux qubits
van Harlingen	Illinois	US	flux qubits
Wellstood, Anderson & Lobb	Maryland	US	flux and single-junction phase qubits

Table A.5: Atomic cavity QED research groups.

Research Leader	Institution	Country	System
Blatt	U. Innsbruck	At	Ca ⁺
Chapman	Georgia Tech.	US	Rb, Ba ⁺
Esslinger	ETH	CH	Rb
Feld	MIT	US	Ba
Haroche	ENS	Fr	Rb (Rydberg)
Kimble	Caltech	US	Cs
Kuga	U. Tokyo	Jp	Rb
Lange	U. Sussex	UK	
Mabuchi	Caltech	US	Cs
Meschede	U. Bonn	De	Cs
Orozco	U. Maryland	US	Rb
Rempe	MPQ	De	Rb
Stampur-Kurn	UC	US	Rb

Table A.6: Superconducting cavity QED research groups.

Research Leader(s)	Institution	Country	System
Delsing	Chalmers	Se	charge qubits
Han	Kansas	US	flux and single-junction phase qubits
Mooij	Delft	NL	flux qubits
Schoelkopf	Yale	US	charge qubits
Wellstood & Anderson	Maryland	US	flux and single-junction phase qubits

Table A.7: Acronyms and abbreviations used in the above tables.

Caltech	California Institute of Technology
ENS	Ecolé Normale Supérieure, Paris
ETH	Swiss Federal Institute of Technology, Zurich
Georgia Tech.	Georgia Institute of Technology
JPL	Jet Propulsion Laboratory, Caltech
MIT	Massachusetts Institute of Technology
MPQ	Max Planck Institute for Quantum Optics, Garching
NIST	National Institute of Standards and Technology
UC	University of California

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