

VHF Impedance Measurement - A Review

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ABSTRACT

This report reviews the current status of top echelon impedance measurement in the UK over the frequency range 30 to 300 MHz (VHF) of the electromagnetic spectrum. The term impedance is used to describe one of the complex electrical properties of one-port passive terminations. Some fundamental concepts of VHF impedance measurement are given along with discussions on the measuring instruments, calibration standards and existing UK national standard capabilities.

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1 INTRODUCTION

The use of VHF signals dates back to the late 19th century and the experiments of Heinrich Hertz validating the theory of guided electromagnetic waves [1]. Today's users of this frequency region include; land, maritime and aeronautical mobile radio; astronomy, satellite and space operations; terrestrial broadcasting, cordless telephones, public radiophones, international distress signalling and many more [2]. VHF impedance measurements provide an essential support service for these users, and many others, in existing and emerging technologies.

The VHF (Very High Frequency) region of the electromagnetic spectrum is understood generally to include frequencies between 30 and 300 MHz, or equivalently, wavelengths from 10 to 1 m. It is an historical classification dating back to the early days of broadcasting when these frequencies were very high compared with other utilised frequencies. This is not so for contemporary microwave engineering where 'very high' might be more suited to describe frequencies above 100 GHz. The term VHF has remained however, and makes a convenient designation for this decade of the radio frequency (RF) spectrum.

The term impedance is used in this report to describe one-port passive terminations having complex characteristics, *i.e.*, both real and imaginary properties. These items are used commonly as transfer standards disseminating traceability to calibration laboratories through national measurement systems. They are used also for international measurement comparisons, promoting understanding and global harmonisation of impedance standards at VHF and other RF bands.

This report begins by outlining some fundamental concepts of VHF impedance measurement. This is followed by a discussion of the instrument types used for these measurements, including; (i) bridge circuits, representing well-established methods, and (ii) vector reflectometers and network analysers, representing more recent techniques. The standards used to calibrate these vector instruments and existing UK national standard measurement capabilities are also discussed. Finally, a list of references is included to allow points of interest to be investigated in more detail.

2 FUNDAMENTAL CONCEPTS

There are many textbooks dealing with the theory underlying RF and microwave impedance. The author recommends particularly Somlo and Hunter [3] and Collin [4], for RF and microwave technologists. The derivation of fundamental expressions will not be given here as they are readily available in these, and other, textbooks.

VHF signals occupy an awkward band of the RF spectrum. At higher frequencies, such as the microwave region, it is convenient to use electromagnetic field theory to describe circuit behaviour; the engineer thinking instinctively in terms of electric and magnetic fields and the concept of wave propagation. At lower frequencies, such as audio frequencies, it is convenient to describe circuit behaviour in terms of discrete circuit-elements. Examples of passive circuit-elements include resistors, capacitors and inductors. At VHF however, the engineer must be aware, and consider the appropriateness, of both concepts for any given circumstance.

An important consequence of electromagnetic theory is the use of transmission lines for the efficient transference of energy. The choice of transmission medium, type and size, is governed usually by the application; in particular, the operating frequency and power level. Power handling capacity is not usually a consideration for precision measurements, leaving the choice of transmission medium dependent only on the range of operating frequencies. At VHF the most suitable transmission medium in common use is coaxial line; hollow waveguide is too large at these frequencies and media such as microstrip and CPW (co-planar waveguide) are more difficult to characterise.

2.1 DISCRETE CIRCUIT-ELEMENT CONCEPTS

It is often convenient to represent an impedance using a series electrical circuit consisting of a resistor and an inductor as in Figure 1.

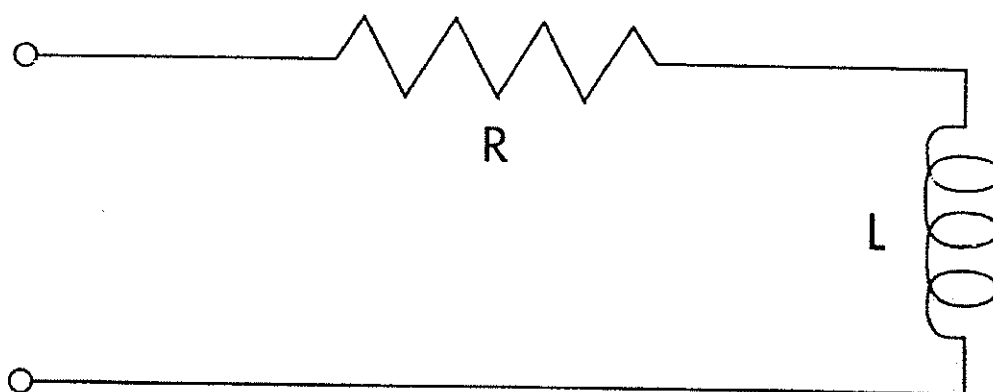


Figure 1: Series electrical circuit representing a terminating impedance.

The impedance Z , has both resistive and reactive properties and can be expressed as:

$$Z = R + jX \equiv R + j\omega L \text{ (Ohms)} \quad (1)$$

where R is the resistance, X the reactance, L the inductance and $j = \sqrt{-1}$. The angular frequency ω , indicates that the impedance of the termination is a function of frequency.

Alternatively, it is sometimes convenient to treat a termination at VHF in terms of admittance parameters. Admittance is the reciprocal of impedance. It is conventional to represent an admittance using a parallel electrical circuit consisting of a conductor and a capacitor as in Figure 2.

The admittance Y , has both conductive and susceptive properties and can be expressed as:

$$Y = G + jB \equiv G + j\omega C \text{ (Siemens)} \quad (2)$$

where G is the conductance, B the susceptance and C the capacitance. A term used commonly to indicate the equivalence between impedance and admittance parameters is immittance, which implies the use of either form of parameter representation.

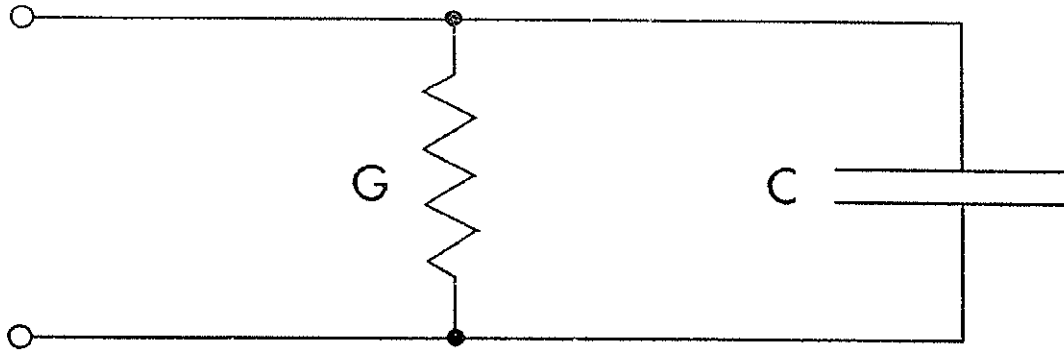


Figure 2: Parallel electrical circuit representing a terminating admittance.

An additional parameter used frequently in conjunction with immittance parameters is quality-factor, or Q-factor. It indicates the degree of purity for capacitors and inductors. The Q-factor for an impedance is defined as the ratio of reactive to resistive components:

$$Q = \frac{X}{R} \equiv \frac{\omega L}{R} \quad (3)$$

The Q-factor for an admittance is defined as the ratio of susceptive to conductive components. An alternative parameter used to indicate capacitor quality is the dissipation-factor, or loss tangent, $\tan \delta$. It is the reciprocal of Q-factor:

$$Q = \frac{1}{\tan \delta} = \frac{B}{G} \equiv \frac{\omega C}{G} \quad (4)$$

2.2 TRANSMISSION LINE CONCEPTS

The terminations modelled using discrete circuit-element concepts in the previous sub-section can be characterised in terms of their response to guided electromagnetic waves. Consider a coaxial transmission line of characteristic impedance Z_0 terminated by an impedance Z_T as in Figure 3.

A signal generator connected to the input of the line provides an electromagnetic signal varying sinusoidally with time, *i.e.*, $\mathcal{E} = \mathcal{E}_0 \sin \omega t$. The incident voltage and current at any point x along the line are described by V_i and I_i respectively. Similarly, the reflected voltage and current at any point along the line are described by V_r and I_r respectively.

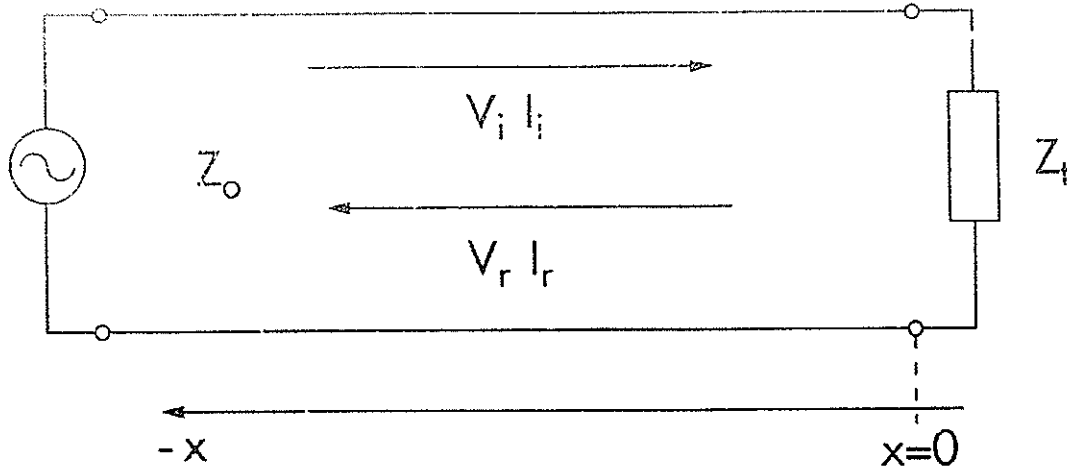


Figure 3: Transmission line of characteristic impedance Z_0 terminated by an impedance Z_T .

The voltage reflection coefficient (VRC) of the terminating impedance is defined as the ratio of reflected to incident voltage. It is specified usually at the reference plane of the termination, *i.e.*, $x = 0$, although it can be given at any point along the line. Voltage reflection coefficient is denoted by Γ :

$$\Gamma = \frac{V_r}{V_i} \quad (5)$$

The current reflection coefficient is defined, for completeness, as the ratio of reflected to incident current. Current reflection coefficients are seldom used in RF and microwave engineering practice.

Two additional parameters relating to VRC are voltage standing wave ratio (VSWR) and return loss. They can be defined in terms of the magnitude of the VRC:

$$\text{VSWR} = \frac{1 + |\Gamma|}{1 - |\Gamma|} \quad (6)$$

$$\text{Return loss} = 20 \log_{10} \left[\frac{1}{|\Gamma|} \right] \text{ (dB)} \quad (7)$$

2.3 DISCRETE CIRCUIT-ELEMENT AND TRANSMISSION LINE CONCEPT EQUIVALENCE

It can be shown that the VRC for an impedance Z_T terminating a line of characteristic impedance Z_0 , as in Figure 3, can be expressed as:

$$\Gamma = \frac{Z_T - Z_0}{Z_T + Z_0} \quad (8)$$

Similarly, for an admittance Y_T terminating a line of characteristic admittance Y_0 , the VRC is given by

$$\Gamma = \frac{Y_0 - Y_T}{Y_0 + Y_T} \quad (9)$$

The terminating impedance Z_T and admittance Y_T in equations (8) and (9) are equivalent to Z and Y in equations (1) and (2), respectively. These four equations allow VRCs to be transformed to immittance parameters and vice versa, illustrating alternative parameters expressing the same physical property.

Parameter equivalence is also apparent when charts are used to represent VRC and immittance properties. It is conventional to represent VRCs as points in the complex plane. The magnitude of VRC for a passive termination has a value between zero and one; zero when the termination absorbs all incident energy, and one when it reflects all incident energy. The phase of VRC varies between $\pm 180^\circ$ depending on the termination's complex characteristics. VRC of a termination at any given frequency can be represented as a point within, or on the boundary of, a unit circle centred on the origin of the complex plane. Any impedance, or admittance, can be represented in the VRC plane by superimposing an appropriate grid on the unit circle. These grids, or charts, were used by Smith [5,6] in the 1930s to represent impedance characteristics, and Smith charts are still used as a convenient means to represent device immittance characteristics.

Parameter equivalence can be extended easily to transform from immittance and VRC to Q-factor, dissipation-factor, VSWR and return loss.

3 MEASURING INSTRUMENTS

The instruments used to measure impedance at VHF can be divided into two broad categories; bridges and reflectometers. Bridge techniques, in general, are understood best in terms of discrete circuit-element concepts, and reflectometer techniques in terms of transmission line concepts. However, the equivalence relationships discussed in the previous section should be considered when instrument performance is compared.

It is informative to examine trends of instrument usage given in reviews of top echelon VHF impedance measuring capabilities carried out over the past 30 years [7-15]. Bridge techniques, in general, represented the dominant technology throughout the 1960s and 1970s, and although still used in the 1980s and 1990s, they appear less common. Contemporary reflectometer techniques, represented by automatic network analysers based on either four-port or six-port principles, represent an emerging technology developing throughout the 1980s and 1990s.

This section reviews firstly RF bridge techniques; in particular, the Twin-T dual admittance bridge which is still used in some standards laboratories. This is followed by an examination of the six-port technique; in particular, the current UK national standard measuring instrument for VHF impedance measurement. Finally, commercially available vector network analysers based on four-port techniques are discussed. Although emphasis will be given to top echelon measurements performed by standards laboratories, some commercially available network analysers are rivalling these capabilities, as observed by Jones [16].

3.1 BRIDGE CIRCUITS

3.1.1 Historical perspective. The bridge circuit was first introduced by S H Christie in 1833 and later made famous by Wheatstone in 1843. Maxwell, in 1873, described several forms of bridge which represent the basis of the modern impedance bridge. Max Wein was the first to report applying AC signals to Maxwell's bridges in 1891, representing the development of the modern AC bridge. The general theory of the bridge circuit will not be given here as it is described in a large number of textbooks on elementary AC theory.

Several reviews of bridge networks can be found in the literature [8,9,17]. These reviews hint at the bewildering variety of networks which can be classified as bridges. In the most part, they are based on the four-arm impedance bridge in which, the components in two of the arms remain constant, the impedance to be determined is placed in the third arm, and the network restored to balance by adjustment of a variable component in the fourth arm (the numbering of the bridge arms is arbitrary). The bridge is energised by an RF source and uses a null detector to establish when the balance condition is reached. A generalised four-arm impedance bridge is illustrated in Figure 4.

Some of the bridge circuits used during the middle part of this century are discussed in references [18-25]. The Twin-T circuit proposed by Tuttle [18] and Sinclair [19] in 1940 formed the basis of a modern high-precision bridge - the Woods Twin-T dual admittance bridge [26]. Woods developed the Twin-T circuit and extended its frequency range to cover the complete VHF region. The instrument was used during the 1970s and 1980s for national standard immittance measurements at VHF in the UK. A description of this instrument type is given because of its importance at VHF for impedance measurement.

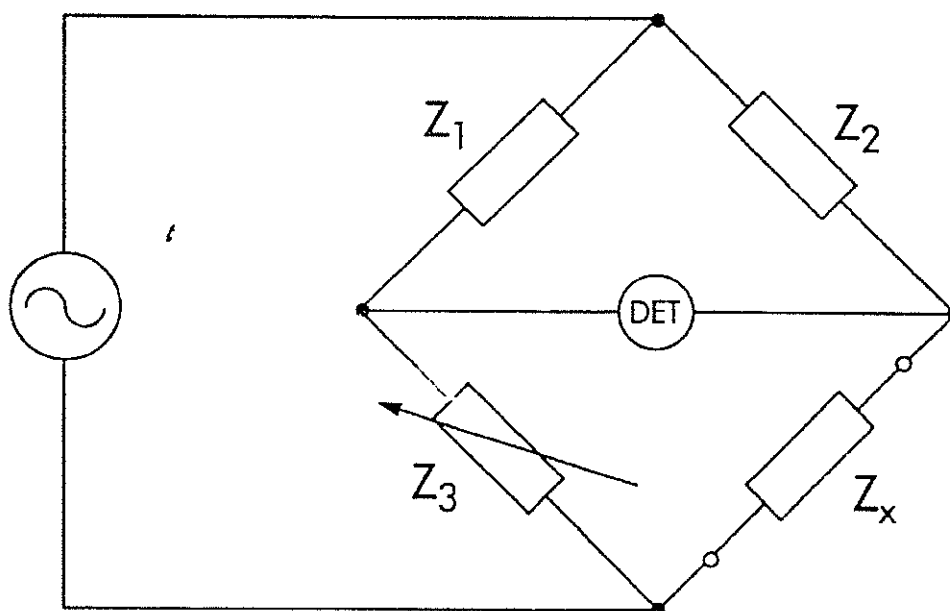


Figure 4: A four-arm impedance bridge containing two fixed components Z_1 and Z_2 , a variable component Z_3 , the device under test Z_x , and a RF source and null detector.

4.1.2 The Twin-T bridge. A generic circuit diagram of a Twin-T bridge is shown in Figure 5.

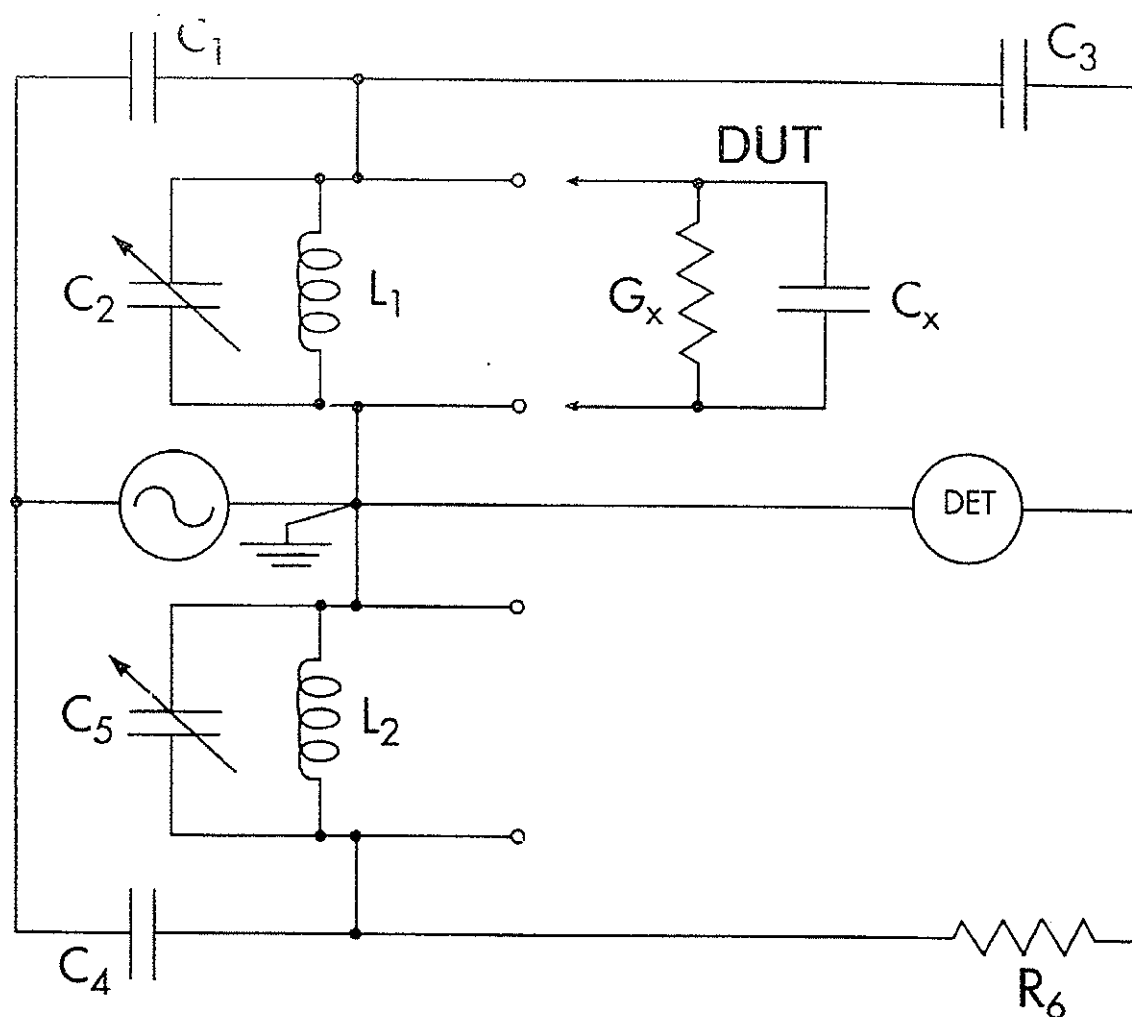


Figure 5: A generic circuit diagram of a Twin-T bridge

The circuit consists of two T-networks connected in parallel. The series elements in each T-network are C_1 , C_3 and C_4 , R_6 respectively. The parallel elements consist of variable susceptance standards (capacitors C_2 , C_5), inductors L_1 , L_2 and the measurement ports. One arm of both T-networks is connected to an ultra-stable RF source while the other arm of the networks is connected to a narrow bandwidth null detector. An advantage with this configuration is that both the parallel elements, including test ports, RF source and detector, share a common ground - aiding shielding and reducing potential earth-loop errors.

Initial balance is achieved against a shielded open-circuit on one of the test ports. The device under test (DUT) is connected to the other test port and balance restored by adjusting the variable capacitors C_2 , C_5 . The DUT characteristics (admittance parameters) are determined from the balance equations, overleaf:

$$G_X = \omega^2 \left[R_c \frac{C_1 C_3}{C_4} \right] \Delta C_1 \quad (10)$$

$$C_X = \Delta C_2 \quad (11)$$

Conductance and capacitance for the DUT are both given directly in terms of capacitance changes. Hence, measurements can be traceable directly to national capacitance standards and therefore to the fundamental physical quantities of length and time.

The Woods version of this bridge [26] was used until recently as a national standard measuring instrument in the UK. Slack and Graham [27] evaluated the measurement uncertainty for the bridge establishing the useful operating frequency range from approximately 3 to 300 MHz. Measurement uncertainties were found to be unacceptably large above this frequency. There have been several notable contributions to the further development of the Woods Twin-T bridge; (i) Clarke [28] described a method whereby initial balance could be achieved with test port adaptors in place, facilitating measurements in different coaxial line sizes and characteristic impedances, (ii) Williams [29] extended the capabilities of the network and automated parts of the instrument's operation; this version is now available commercially, and (iii) the network's upper frequency limit has been extended to 1 GHz by Grno [30] using quarter-wave line techniques.

The Twin-T bridge is a high precision instrument and remains firmly in the domain of standards laboratories. It is laborious and tedious to use, and requires a high level of operator skill and judgement. These reasons, and the availability of fully automated technologies, led to the instrument's replacement as the UK national standard in 1991.

3.2 SIX-PORT REFLECTOMETERS

3.2.1 General description. The six-port technique has attracted a great deal of attention since its introduction by Hoer and Engen [31,32] in 1972. Since then, six-port instruments have been reported, or proposed, at operating frequencies from RF [33], microwave and millimetre wave [34,35], submillimetre wave [36] and even optical frequencies [37]. Circuit fabrication appears unaffected by the choice of transmission medium with designs appearing in most waveguide types in common use [38-42]. An overview of the six-port principle is given here which is sufficient to appreciate its use in a VHF impedance measuring instrument.

A block diagram of a six-port is given in Figure 6. A source is connected to one port, the DUT to another and scalar detectors to the remaining four ports. One detector monitors primarily the energy incident on the DUT whereas the remaining three detectors monitor a combination of incident and reflected energies, separated by different path lengths. The ratios of the incident wave detector to each of the three other detectors are used to determine the characteristics, *e.g.*, reflection coefficient, of the DUT. A computer is used to generate and apply correction factors to simulate perfect performance from the imperfect circuit components. A major advantage with the six-port technique is that vector information (magnitude and phase) can be obtained from scalar observations.

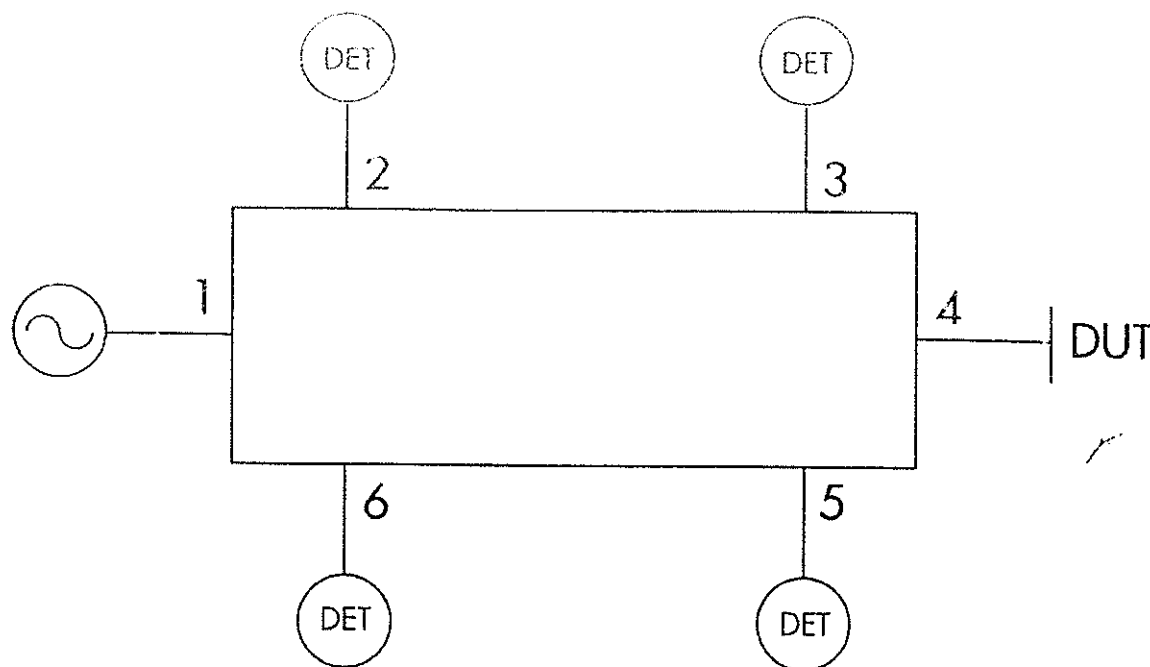


Figure 6: Block diagram of a six-port reflectometer.

Six-port junctions based on a wide range of circuits have been constructed. Designs depend upon required performance and available components. A large number of junctions have been constructed using 3 dB quadrature couplers, power dividers and directional couplers [32,43-46]. Several recent configurations use a single device, such as a five-port or six-port coupler, in conjunction with conventional directional couplers [47-49]. These devices have the advantage that circuit coupling occurs in the same physical space, minimising effects due to thermal expansion caused by temperature gradients existing throughout the circuit.

3.2.2 VHF six-port reflectometer. This sub-section describes briefly the six-port reflectometer which is the current UK national standard instrument for VHF impedance measurement. It superseded the Woods Twin-T bridge in 1991.

The RF circuit comprising the six-port junction is given in Figure 7. It is based on a circuit reported by Engen [43] and has been evaluated theoretically by Griffin and Hodgetts [50]. It uses five broadband 3dB quadrature hybrid couplers whose bandwidth is the limiting factor to the bandwidth of the six-port junction. The resistor symbols in Figure 7 represent nominally matched 50 ohm loads. The signal path lengths, θ_i (for $i = 1$ to 6), were adjusted during the instrument's development to ensure that only one of the detectors P_1, P_2, P_3 can ever approach zero signal at any time at all the frequencies of operation, which in this case is any frequency in the range 50 to 250 MHz. These path lengths remain fixed when this criterion has been achieved.

The hermetically sealed couplers used for the six-port junction are temperature stabilised using a temperature-controlled plate in contact with all of the couplers [51]. The RF power sensors are also temperature controlled to minimise drift caused by external thermal fluctuations. Both temperature-controlled plates maintain a nominal temperature of 301K to within 50mK. The couplers and detectors are housed within an aluminium casing with connections for the RF input, the measurement test port and the four detectors. The entire

apparatus is kept in a temperature controlled laboratory maintained at 296K to within 1K. This degree of temperature stabilisation produces a very mechanically and electrically stable instrument, enabling calibrations up to a month old to be used without appreciable performance degradation.

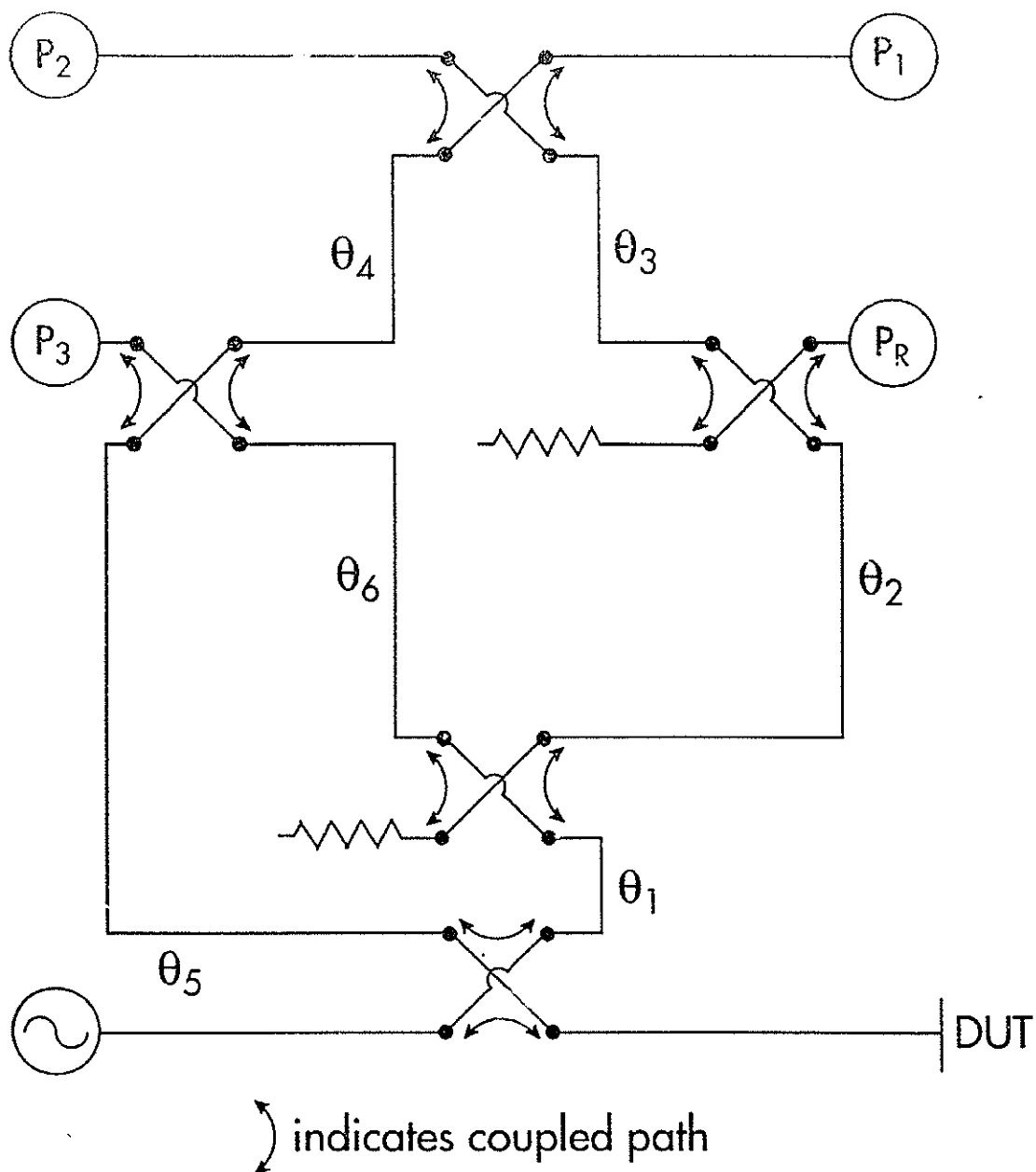


Figure 7: Diagram of the VHF six-port RF circuit.

A block diagram of the complete six-port system is given in Figure 8. The computer controls all the instrumentation via the GPIB. It also provides screen instructions for the operator when required to connect specific items (e.g., during calibration) or items for measurement.

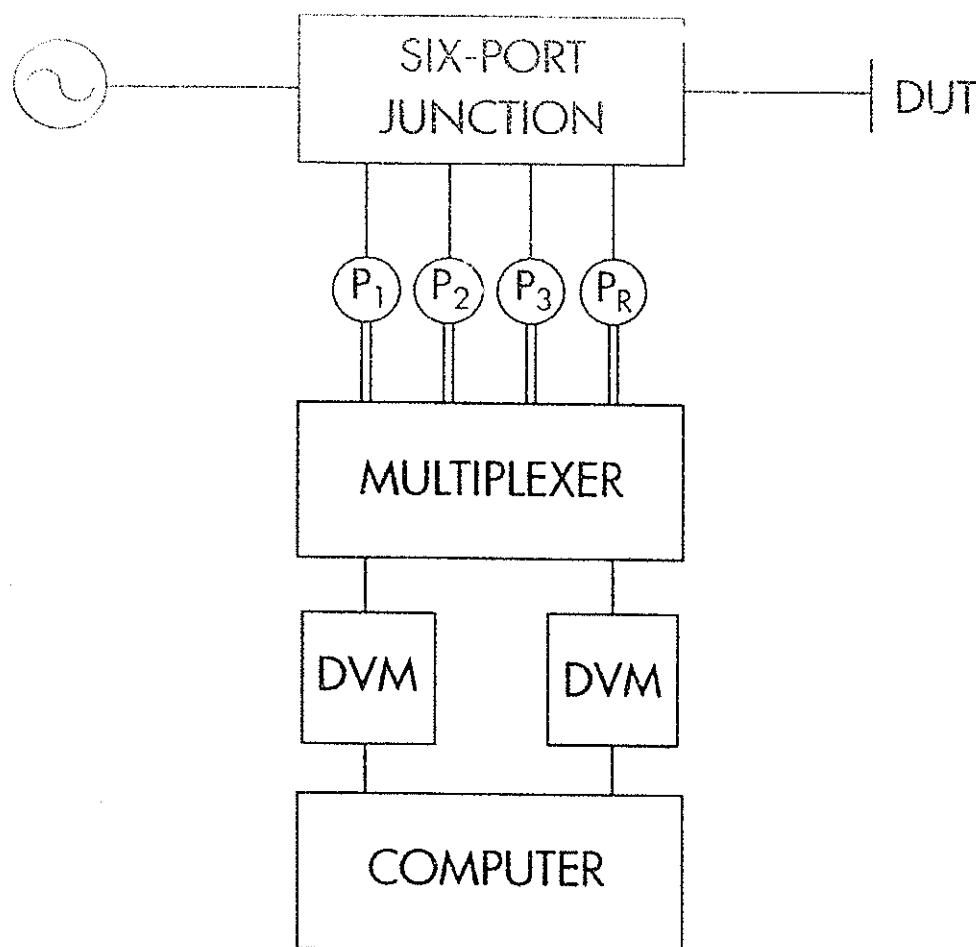


Figure 8: Block diagram of the VHF six-port instrumentation.

One of the six-port junction's ports is energised by an RF source, phase locked to a microwave frequency counter which uses a 10 MHz external frequency standard as a reference. The instrument test port consists of a 50 Ω 14mm precision coaxial connector, since this is the highest quality readily available connector at VHF. Measurements in other coaxial line sizes and other transmission line types are achieved using adaptors.

Thermistors are used as the detectors because of their proven linearity [52]. P_R is the incident, or reference, detector and P_1 , P_2 and P_3 sample the combined incident and reflected signals. Two digital voltmeters (DVMs) are used to convert the analogue indications of the self-balancing bridges in the power meters into digital readings capable of being read by the computer via the GPIB. The switching of signal paths from the various power meters to the DVMs is performed by a multiplexer.

3.2.3 Calibration. A wide variety of schemes have been proposed to calibrate the six-port reflectometer [53-60]. The calibration process can be divided into two processes: (i) the mathematical reduction of the six-port network to an equivalent vector indicating four-port network, and (ii) the calibration of the resultant four-port. The six-to-four port reduction process usually requires a range of terminations (usually between nine and fifteen), known only to have different voltage reflection coefficients, to be measured by the six-port. Five constants which characterise the reduction process are then determined at each frequency. The process used for the VHF six-port is that given in reference [55].

The second of these two processes, the four-port calibration, has the same requirements as the calibration of any arbitrary vector-indicating four-port reflectometer. Since a one-port calibration on a network analyser falls into this category, this process will be discussed in the following sub-section which is concerned with vector network analysers.

3.3 VECTOR NETWORK ANALYSERS

3.3.1 General description. The term "reflectometer" is used in this report to describe a one-port measuring instrument which determines the complex characteristics of terminations. The term "network analyser" is used in this report to describe a two-port measuring instrument determining all four scattering coefficients (all complex) for a two-port device. It is clear from these definitions that two compatible six-port reflectometers used in conjunction with each other constitute a network analyser [61,62]. Similarly, a network analyser measuring one-port terminations is equivalent to a reflectometer.

A block diagram of a network analyser used as a one-port measuring instrument is given in Figure 9.

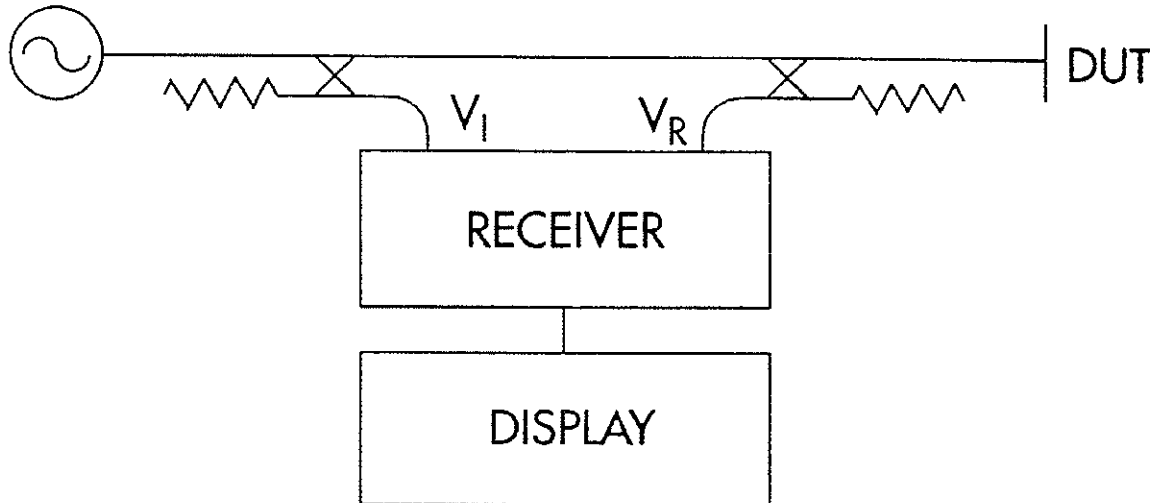


Figure 9: Block diagram of a network analyser used as a one-port measuring instrument.

The network analyser's RF source energises the DUT. Samples of the incident and reflected signals, V_I and V_R respectively, are taken by the two couplers or power splitters. The receiver converts the RF signals into intermediate frequency (IF) signals where amplitude and phase information can be measured directly and presented on a display.

3.3.2 Calibration techniques. For a generalised reflectometer, the complex indication w , produced by the reflectometer is related to the DUT reflection coefficient Γ , by the following bilinear transformation:

$$w = \frac{A\Gamma + B}{C\Gamma + 1} \quad (12)$$

where A, B and C are complex constants characterising the reflectometer at each frequency. Calibration is the process by which these three constants are determined. The conventional

method of determination is to connect three items with known characteristics to provide three values of Γ to solve three complex simultaneous equations in A , B and C . The types of item assumed to have known characteristics will be discussed in the next section. This calibration principle can be applied to both six-port reflectometers and to one-port measurements on a network analyser.

An alternative method for an instrument with two test ports (*e.g.*, a dual six-port, or a vector four-port, network analyser) is to perform a two-port calibration. Either test port of the calibrated instrument can then be used for impedance measurements. The most popular of these two-port calibration techniques for high precision measurements is the TRL (Thru-Reflect-Line) technique [63] and its generalisation, the LRL (Line-Reflect-Line) technique [64]. Generalised approaches to network analyser calibration techniques are given in references [65,66], and [15] reviews the suitability of routines in current practice.

4 CALIBRATION STANDARDS

This section describes the types of item suitable for reflectometer and network analyser calibration, particularly at VHF. The item types can be divided into two categories; (i) two-port items, and (ii) one-port items, or terminations.

The most common two-port calibration item is the precision air-dielectric coaxial transmission line, or air line, although some calibration techniques have been described using an attenuator as a calibration item [65,66]. Air lines are used in conjunction with terminations as calibration items for reflectometers; one end of the air line connected to the reflectometer test port and the other end to the termination. They are used also for network analyser two-port calibrations, such as TRL and LRL, where they constitute the line standard placed between the two test ports. In both cases, the inner conductor is unsupported (no dielectric support beads) and is held in place by the inner conductors of the test port and termination for a reflectometer, or the two test ports for a network analyser two-port calibration.

Terminations are used for reflectometer and network analyser calibrations. They can be connected directly to the instrument test port or separated by a length of air line called an "offset". The three most common terminations are short-circuits, open-circuits and matched loads (including sliding loads). Mismatched loads and capacitance standards can also be used, particularly at lower frequencies.

Less information, in general, is required about the air line standard for a two-port than a one-port calibration. Full characterisation of air line and termination is usually required for a one-port calibration. The modelling of these standards suitable for reflectometer calibration will be given here.

4.1 AIR LINES

Air lines have been used as impedance standards for many years [67,68]. Measurement accuracy achieved using modern network analysers and six-ports requires the electrical characteristics of air lines to be defined very precisely [69].

A coaxial line can be characterised using the distributed circuit model given in Figure 10, where R , L , G and C are the series resistance and inductance, and the shunt conductance and capacitance, respectively, all per unit length of line.

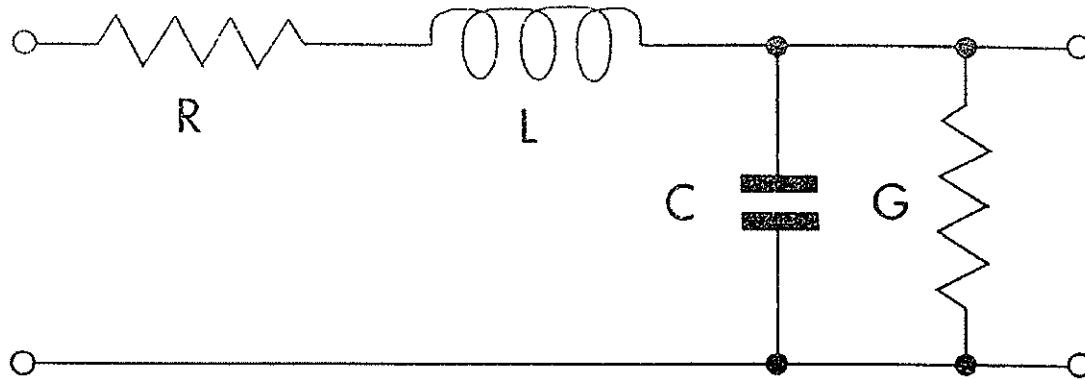


Figure 10: Distributed circuit model for a coaxial line.

Expressions for the four line elements R , L , G and C can be used to obtain further expressions for the two fundamental line parameters; the characteristic impedance Z , and the propagation constant γ , defined as follows:

$$Z = \sqrt{\frac{(R + j\omega L)}{(G + j\omega C)}} \quad (13)$$

$$\gamma = \alpha + j\beta = \sqrt{(R + j\omega L) \cdot (G + j\omega C)} \quad (14)$$

where: $\omega = 2\pi f$ (15)

and:

α = attenuation constant (Nepers.m⁻¹)

β = phase constant (Rads.m⁻¹)

f = frequency (Hz).

The next sub-section reviews lossless line models in order to introduce lossy line models used currently for high precision impedance metrology.

4.1.1 Lossless line models. The series resistance and the shunt conductance for a lossless line are both zero. The series inductance and the shunt capacitance have fixed values independent of frequency. They are represented by L_o and C_o respectively, and are given by:

$$L_o = \frac{\mu \ln(b/a)}{2\pi} \quad (16)$$

$$C_o = \frac{2\pi\epsilon}{\ln(b/a)} \quad (17)$$

where

$$\epsilon = \epsilon_0 \epsilon_r \quad (18)$$

$$\mu = \mu_0 \mu_r \quad (19)$$

and:

μ_0 = permeability of a vacuum ($H.m^{-1}$)

ϵ_0 = permittivity of a vacuum ($F.m^{-1}$)

μ_r = relative permeability of the dielectric

ϵ_r = relative permittivity of the dielectric

a = radius of the inner conductor (m)

b = inner radius of the outer conductor (m).

The relative permeability μ_r is equal to unity for non-magnetic materials (ignoring diamagnetic and paramagnetic effects). The relative permittivity ϵ_r for the air dielectric is a function of temperature, relative humidity (RH) and atmospheric pressure of the air. It can be calculated, as in [70, Appendix B], and has the value 1.000649 for 'standard air' at 296.2K, 50% RH and 101.325 kPa.

Values for the two physical constants μ_0 and ϵ_0 can be found in the literature [71] and are given as:

$$\mu_0 = 4\pi \times 10^{-7} H.m^{-1}$$

$$\epsilon_0 = 8.854187817... \times 10^{-12} F.m^{-1}$$

The characteristic impedance Z of the line, from equations (13), (16) and (17), is given by:

$$Z = \sqrt{\frac{L_0}{C_0}} = \frac{\ln(b/a)}{2\pi} \sqrt{\frac{\mu}{\epsilon}} \quad (20)$$

Equation (20) shows that the line's characteristic impedance is a *real* quantity (*i.e.*, no imaginary part), independent of frequency and determined by the ratio (b/a) . For example, to achieve a characteristic impedance of 50Ω the ratio is approximately 2.3.

The propagation constant γ of the line, from equations (14), (16) and (17), is given by:

$$\begin{aligned} \gamma = j\beta &= j\omega \sqrt{L_0 C_0} \\ &= j\omega \sqrt{\mu \epsilon} = j \frac{\omega}{v} = j \frac{2\pi}{\lambda} \quad Rad.m^{-1} \end{aligned} \quad (21)$$

where v and λ are the speed and wavelength of the propagating wave, respectively. Equation (21) shows that the line's propagation constant is purely *imaginary* (*i.e.*, no real part) and is determined only by the wavelength (or equivalent) of the propagating wave. The attenuation constant α is zero, consistent with a lossless line. The phase constant β is a linear function of frequency, indicating a non-dispersive line.

4.1.2 Lossy line models Metallic air-filled coaxial lines are not lossless. An important part of line characterisation at RF is the evaluation of loss. The first serious attempt to calculate coaxial line losses was made by Russell [72], in 1909. Expressions were derived for the series RF resistance and inductance using discrete circuit-element methods. The problem was re-evaluated by Stratton [73], in 1941, obtaining approximate expressions for these terms using electromagnetic field theory. In 1966, Nelson and Coryell [74] used these expressions to evaluate effects on electrical parameters of air lines for RF immittance standardisation. The problem has been re-addressed in the 1990s by Daywitt [75,76], obtaining a rigorous solution using Stratton's approach. The expressions relevant to the VHF region [75] were found to contain an additional frequency dependant term affecting all four line elements; R , L , C and G . Daywitt also solved the problem for lower frequency regions, obtaining exact field equations for a lossy coaxial line [76].

The expressions derived by Daywitt [75] for the four line elements at VHF are as follows:

$$R = 2\omega L_o d_o \left[1 - \frac{k^2 a^2 F_o}{2} \right] \quad (22)$$

$$L = L_o \left[1 + 2d_o \left[1 - \frac{k^2 a^2 F_o}{2} \right] \right] \quad (23)$$

$$G = \omega C_o d_o k^2 a^2 F_o \quad (24)$$

$$C = C_o (1 + d_o k^2 a^2 F_o) \quad (25)$$

where:
$$F_o = \frac{(b^2/a^2) - 1}{2 \ln(b/a)} - \frac{(b/a) \ln(b/a)}{(b/a) + 1} - \frac{1}{2} \left[\frac{b}{a} + 1 \right] \quad (26)$$

$$d_o = \frac{\delta_s (1 + (b/a))}{4b \ln(b/a)} \quad (27)$$

$$k = \omega \sqrt{\mu \epsilon} \quad (28)$$

and:
$$\delta_s = \sqrt{\frac{2\rho}{\omega \mu}} \quad (29)$$

where ρ is the resistivity of the conductors (ohm.m). Other symbols are as used for the lossless line expressions. The resistivity is dependent on the material of the conductors.

The characteristic impedance can be calculated from equations (13), (22), (23), (24) and (25). Clearly, the characteristic impedance of a line with non-zero resistivity is a complex quantity, material dependent and a function of frequency. Figures 11 and 12 illustrate the effect on the characteristic impedance, Z , for a nominal 50 ohm 7 mm air-dielectric line made of beryllium copper with an assumed resistivity of 8×10^{-8} ohm.m.

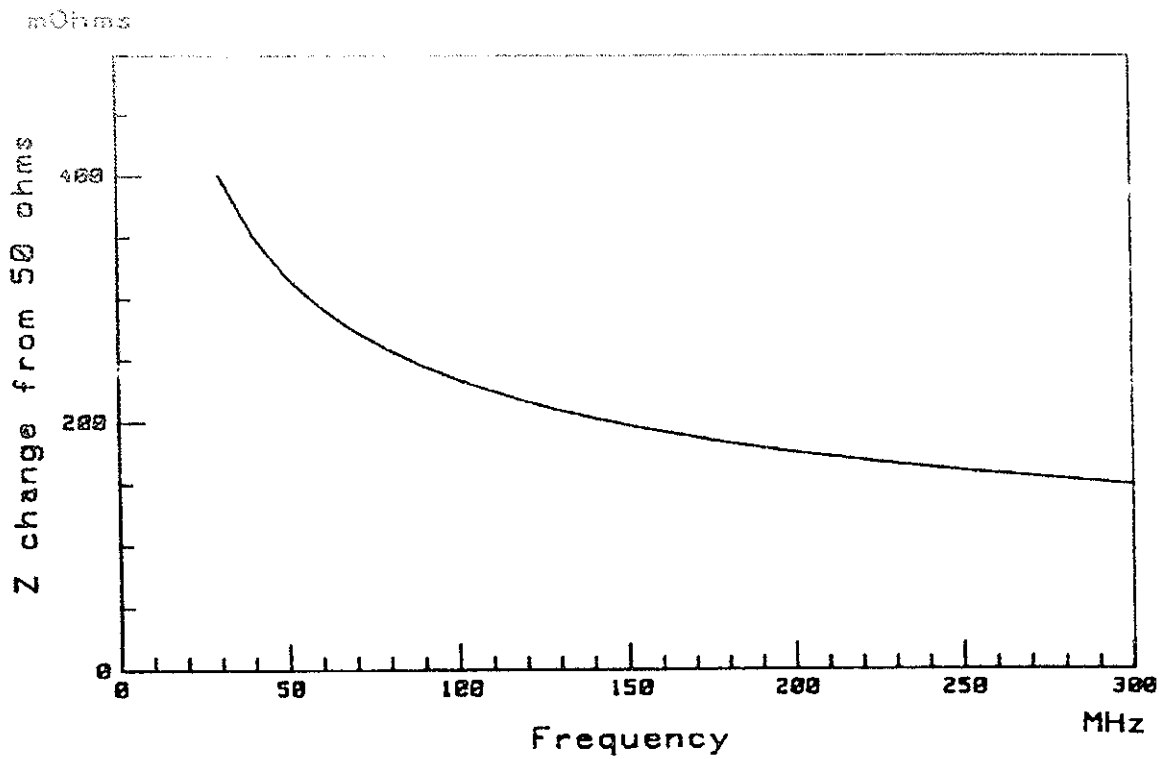


Figure 11: Change in characteristic impedance magnitude as a function of frequency for a nominal 50 ohm 7 mm line made of beryllium copper with assumed resistivity of 8×10^{-8} ohm.m.

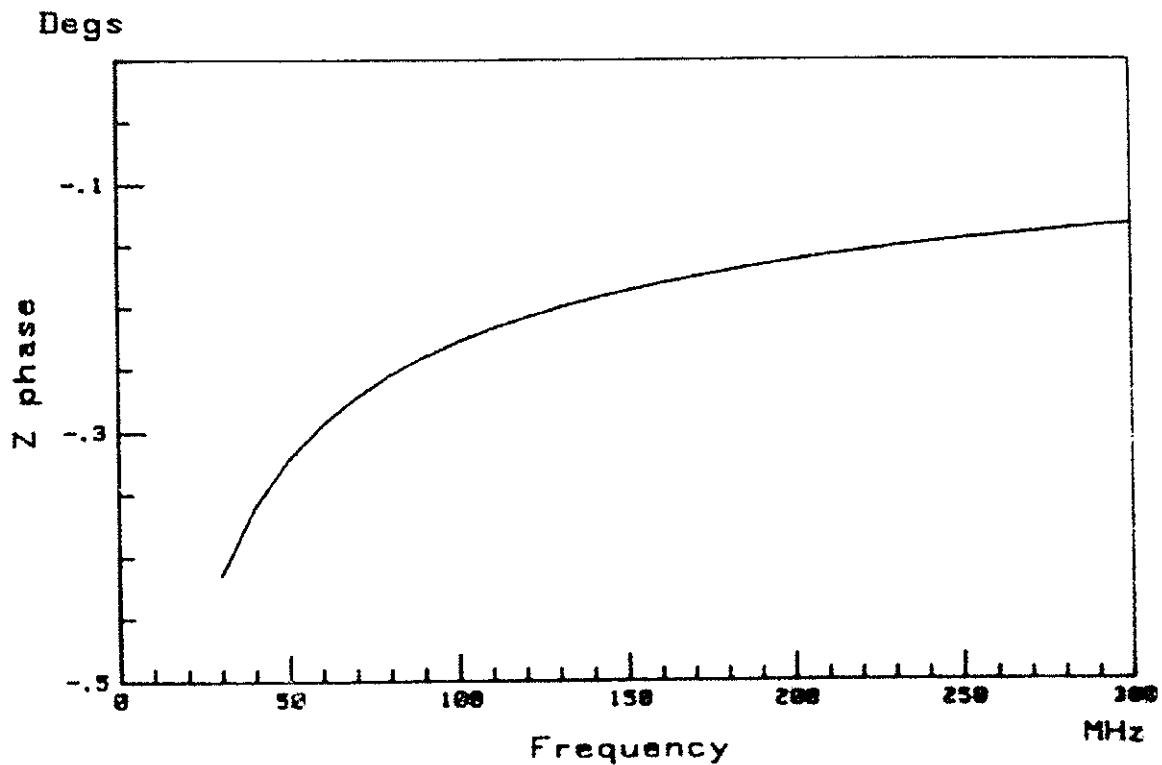


Figure 12: Characteristic impedance phase angle as a function of frequency for a nominal 50 ohm 7 mm line made of beryllium copper with assumed resistivity of 8×10^{-8} ohm.m.

The deviation in the characteristic impedance causes a problem for reflection coefficient measurements as they are usually specified with respect to the lossless line value. Measurements made on instruments calibrated with lines of different material will vary systematically since the measured reflection coefficients will be with respect to different characteristic impedances. This problem is overcome by transforming from actual to lossless line impedance. The transformation is given by:

$$\Gamma_o = \frac{\left\{ \left[\frac{Z_1}{Z_o} \right] \cdot \left[\frac{1 + \Gamma_1}{1 - \Gamma_1} \right] - 1 \right\}}{\left\{ \left[\frac{Z_1}{Z_o} \right] \cdot \left[\frac{1 + \Gamma_1}{1 - \Gamma_1} \right] + 1 \right\}} \quad (30)$$

where Γ_o is the reflection coefficient transformed to the lossless line impedance Z_o (e.g., 50 ohms), and Γ_1 is reflection coefficient measured with respect to Z_1 , the actual line impedance. Further discussion on the effects caused by this transformation are given in the literature [77].

The propagation constant can be calculated from equations (14), (22), (23), (24) and (25). Clearly, the propagation constant of a line with non-zero resistivity has both real and imaginary parts and is non-linear with frequency. Figures 13 and 14 illustrate the effect on the propagation constant for a nominal 50 ohm 7 mm air-dielectric line made of beryllium copper. The attenuation constant is non-zero, consistent with a lossy line. The increase in the phase constant from its lossless value indicates that the line's electrical length is longer than its physical length; the discrepancy increasing as the frequency is lowered. The line is dispersive and imparts group delay to broadband signals.

4.1.3 Skin depth and resistivity. A comparison of parameters characterising lossless and lossy lines reveals that only one parameter is added to allow for loss effects; the conductors' resistivity. Setting $\rho = 0$ causes equations (29) and (27) to be zero, and equations (22), (23), (24) and (25) to revert to their lossless expressions, *i.e.*:

$$R = 0 : L = L_o : G = 0 : C = C_o \quad (31)$$

Non-zero resistivity is responsible therefore for the departure from the lossless condition. Equation (29) also contains an ω^{-2} term indicating that δ_s increases as ω (or frequency) is decreased, causing a subsequent increase in the four line element values.

As frequency decreases, the non-zero resistivity of a line causes the propagating wave to penetrate the walls of the conductors to a greater extent. The attenuation constant associated with the wave propagating into the walls of the conductors is considerably higher than for the wave propagating in the dielectric between the conductors, and therefore the wave attenuates rapidly as it penetrates the walls of the conductors. The reciprocal of this attenuation constant is δ_s , called the skin depth, and represents the distance the wave travels into the walls of the conductors before being attenuated by one Neper (≈ 8.686 dB). The wave decays exponentially as it penetrates the walls of the conductors.

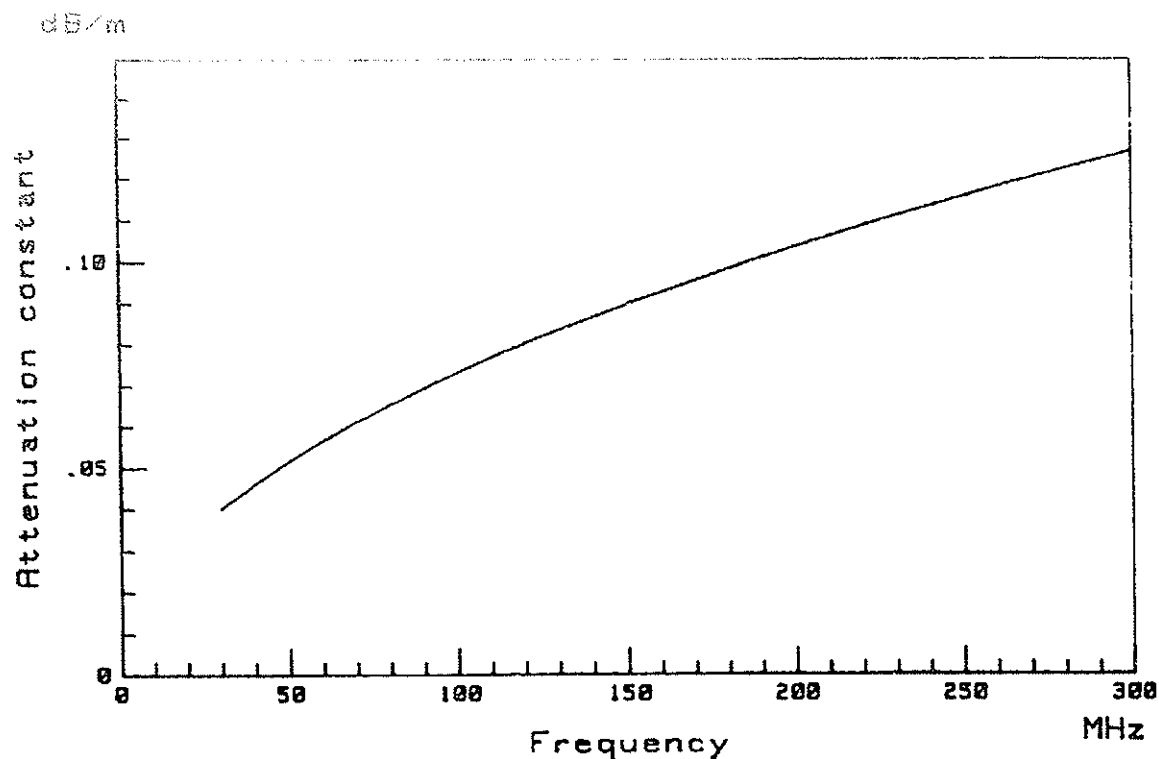


Figure 13: Attenuation constant as a function of frequency for a nominal 50 ohm 7 mm line made of beryllium copper with assumed resistivity of 8×10^{-8} ohm.m.

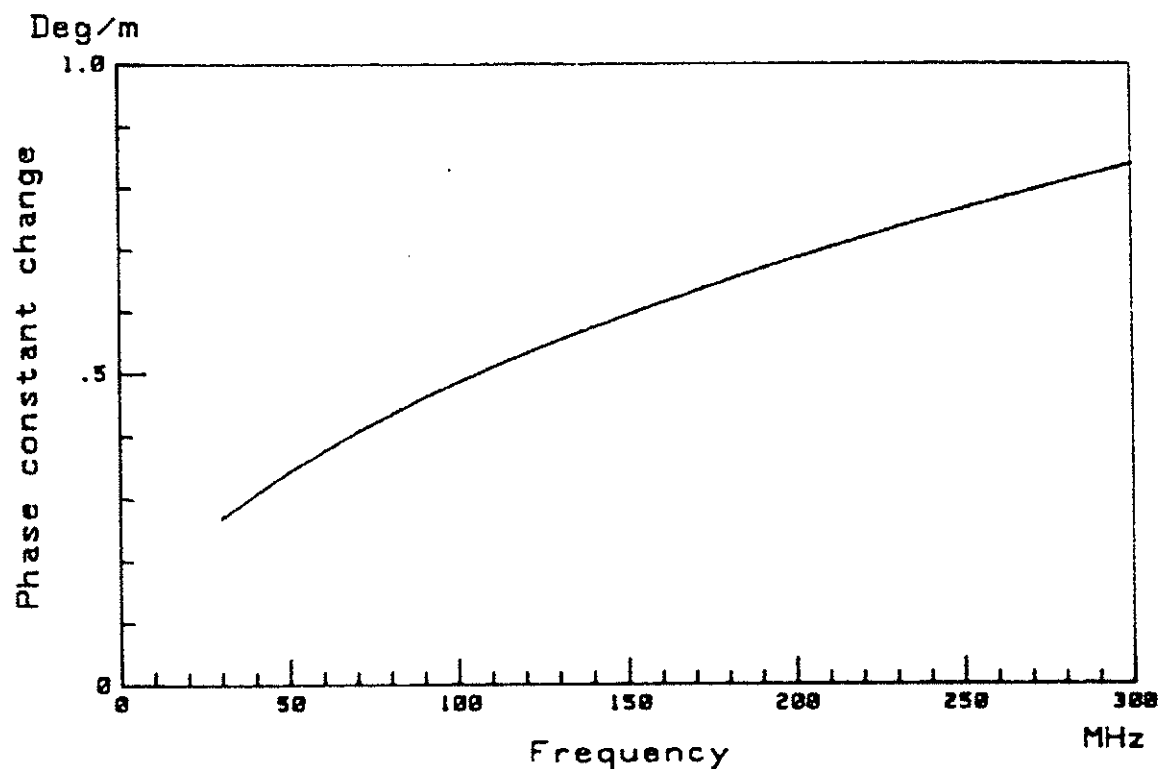


Figure 14: Change in phase constant as a function of frequency for a nominal 50 ohm 7 mm line made of beryllium copper with assumed resistivity of 8×10^{-8} ohm.m.

this phenomenon imparts physical meaning to equation (29), indicating that skin depth increases as frequency decreases, and as resistivity increases. Values for skin depth are given in Table 1, illustrating the effect on conductors made of copper, brass and beryllium copper, with assumed resistivities 1.7×10^{-8} , 6.3×10^{-8} and 8×10^{-8} ohm.m, respectively. Further discussions on skin depth effects can be found in references [78-80].

Frequency (MHz)	Skin depth (μm)		
	Copper ($\rho = 1.7 \times 10^{-8} \Omega\text{m}$)	Brass ($\rho = 6.3 \times 10^{-8} \Omega\text{m}$)	Beryllium copper ($\rho = 8 \times 10^{-8} \Omega\text{m}$)
3	37.9	72.9	82.2
30	12.0	23.1	26.0
300	3.8	7.3	8.2
3000	1.2	2.3	2.6

Table 1: Table showing the skin depth of copper, brass and beryllium copper as a function of frequency.

It is generally only necessary to specify accurately the resistivity of the conductors at VHF in order to characterise adequately the line's characteristics, because lines are used rarely as calibration standards below VHF, and skin depth becomes less of a problem to their use at higher frequencies. This requirement, however, is not trivial. If the line's constitution is known then a value may be obtained from tables of physical data, *e.g.*, Kaye and Laby [81]. Values specified in tables refer usually to bulk material samples. These values are often different from actual values for the same material subjected to machining, as is the case for air lines, as observed by Gray [82] and Weinschel [83].

An additional problem in determining a value for the resistivity of a line is caused by plating layers applied by manufacturers either to increase conductivity (silver plating) or increase longevity (gold flashing). Several studies have been carried out evaluating effects of plating on the effective resistivity of conductors [84-86] but these assume prior knowledge of the material of each layer and ignore additional complications caused by impurities which will doubtlessly be present. A recent validation of theoretical predictions based on assumed resistivity values has been performed by comparison with precision attenuation measurements [87].

Before leaving the subject of lossy line characterisation another contributing factor to this problem should be mentioned: the non-uniformity of conducting surfaces caused either by changes in the longitudinal dimensions of the line [88,89] or conductor surface roughness [90]. The latter of these factors leads to the anomalous skin effect observed at microwave frequencies [91].

4.1.4 Broadband considerations. Air lines are used primarily to introduce phase changes from a given reference position. Modern networks analysers and reflectometers often have operating frequency ranges of several decades or more. The phase change produced by a given length of line will vary greatly over this range of frequencies. This can cause problems when the phase change coincides, at a given frequency, with reference positions produced

by other standards used in the calibration process. Examples are given to illustrate this potential problem for typical one-port and two-port calibration routines.

(i) One-port calibration techniques usually require three known and different items to be connected to the reflectometer to solve the bilinear transformation given in equation (12) characteristic of the instrument. If lines are used to fabricate standards, care must be taken to ensure the VRCs produced by any given standard are removed sufficiently from other known VRCs in the complex plane. If this is not the case, the inevitable error caused by system noise will produce unreliable calibrations. For example, if several nominal unity reflection standards are used, *e.g.*, offset short-circuits (S/Cs) or offset open-circuits (O/Cs), the length of each offset must be chosen so that, for the required frequency range, the phase angles produced by the standards must differ significantly. As a rule of thumb, it is unsuitable to allow phase separations between offset S/Cs and O/Cs to be less than 20° for satisfactory calibrations. Suitable choices of line length combinations for offset S/Cs and O/Cs as calibration standards at VHF have been given in [70].

A similar precaution is required when a sliding load is used as a standard. Phase changes with respect to other sliding load positions approaching multiples of 360° should be avoided as the positions are indistinguishable in the complex plane. A reasonable minimum phase separation limit should be applied to allow for system noise effects.

(ii) Two-port calibration techniques employ line standards usually to introduce phase changes with respect to a reference position; for the TRL calibration technique the reference is the 'thru' connection (both test ports together), for the LRL technique the reference is the length of one line standard. Phase differences approaching multiples of 180° should be avoided between line and reference position as these produce problems due to lack of resolution between line and reference position. The same rule of thumb given for one-port calibration techniques can be used for minimum acceptable phase separation between line and reference position for calibration at any frequency, *i.e.*, not less than 20° . A suitable line length selection process for the TRL technique is given in [92] and this can be extended easily to the LRL technique.

A problem common to one-port and two-port calibration techniques employing line standards is that the phase change produced by any given length of line reduces as frequency decreases. The minimum acceptable phase separation (20°) between standards establishes a low frequency limit to the techniques. For example, using the above rule of thumb, a 300 mm air line has a low frequency limit of 56 and 28 MHz for two-port and one-port calibrations, respectively (one-port calibrations utilise both forward and return signal paths). This indicates that only one-port calibration methods employing line standards are suitable for the complete VHF region; two-port calibrations are likely to be susceptible to error in the lowest octave of this frequency region.

The lower frequency limit established above using the minimum phase difference criterion coincides approximately with problems caused by the line's resistivity discussed in the previous sub-section. The VHF region represents therefore the lowest RF band where lines can be used, under normal circumstances, as standards for vector reflectometers and network analysers. Alternative standards are required below these frequencies.

4.2 TERMINATIONS

Terminations can be connected directly to an instrument test port or via a length of line; the line can be an integral part of the item or connected separately. Terminations used in both one-port and two-port calibration techniques can be divided into four categories; (i) short-circuits, (ii) open-circuits, (iii) 'matched' loads, and (iv) mismatched loads and capacitors.

Mismatched loads and capacitors are used rarely at VHF; their characteristics vary significantly with frequency making them difficult to characterise. They are not discussed here since they are more suited as standards at lower RF. The other classes of termination are discussed briefly.

4.2.1 Short-circuits. A coaxial line S/C is simply a flat metallic disc connected normally to the line's inner and outer conductors. Its radius must exceed the inner radius of the outer conductor and be of sufficient thickness to form an effective shield for the electromagnetic wave propagating in the line. The disc is made usually of a similar material as the line's conductors. S/Cs can be connected directly to an instrument test port or via a length of line producing an offset S/C.

S/Cs approximate to the lossless condition at VHF ($R = X_L = 0$) producing a reflection coefficient with real and imaginary parts -1, 0 respectively. Loss due to skin depth and surface finish of the disc can be considered for high precision metrology applications. Skin depth loss has been evaluated by Collin [4] and Stratton [73] by the analysis of a TEM wave incident normally on a conduction plane. An expression for the reflection coefficient of a lossy S/C has been given in [70].

4.2.2 Open-circuits. In principle, a coaxial O/C is produced by having nothing connected to the instrument test port. This produces a poorly defined standard, however, for two reasons; (i) it will radiate energy producing a reflected signal dependent on obstacles in the vicinity of the test port, and (ii) the connector's mating mechanism affects the established measurement reference plane limiting accurate characterisation.

The first of these problems can be overcome by extending the line's outer conductor sufficiently beyond the open-circuited inner so that the evanescent radiating field decays to zero within the shield: the extended outer conductor acting as an effectively infinite length of circular waveguide below cut-off.

The second problem can be overcome either by depressing the mating mechanism using a dielectric plug or attaching a length of line to the inner terminated in an abrupt truncation. The dielectric plug technique is used as a standard with numerous network analyser calibration kits. The abruptly truncated line technique has been used to realise primary national impedance standards [70]. The O/C behaves as a frequency dependent terminating capacitance in both cases. Calculations for the capacitance of an abruptly truncated coaxial line can be found in the literature [93-97]. Values produced by Hodgetts [97] have been verified recently at VHF using a computer-intensive equivalent circuit technique [98].

O/Cs have a reflection coefficient magnitude of nominally unity with a phase angle dependent on the capacitance. They constitute a useful standard for VHF applications, especially when used in conjunction with S/Cs, as they can offer large spatial separation in the complex plane.

Tractorial loads - A low reflection coefficient load can be produced using a cylindrical non-film resistor mounted in the inner line with a tractorially-shaped outer. A parabolic transition between the conventional coaxial line and the tractorial section transforms incident plane wave-fronts to spherical wave-fronts required to propagate in the tractrix, producing near-uniform energy absorption along the length of the resistor with minimal frequency dependence. This design of resistor has been discussed in [10,99-101]. Other designs have also been used [102,103].

Low reflection loads are assumed usually to have zero reflection coefficient during instrument calibration. Loads with measurable mismatch can be characterised using sliding load techniques; the response of the load is measured at several positions along a variable length of line allowing the characteristics of a matched load to be synthesised. However, problems due to line quality and inadequate phase differences produced by realisable lengths of sliding sections make this technique of limited use at VHF.

5 EXISTING CAPABILITIES

This section gives an overview of the current national standard VHF impedance measurement capabilities in the UK. A comparatively up-to-date global perspective of impedance capabilities, and other electrical parameters, can be found in reference [104].

Specifying the capabilities of a measurement system requires a detailed assessment of errors inherent in the system. The identified sources of error should be corrected, where possible, leaving only residual errors which are allowed for in an uncertainty budget for the system. The uncertainty budget establishes the performance capabilities of the system.

A number of error, accuracy and uncertainty assessments have been reported in the literature for six-port reflectometers [105-108], dual six-port network analysers [64,109-113] and commercial network analysers [114-116]. An uncertainty budget applicable to six-ports and network analysers is used currently in the UK for top echelon impedance measurements at VHF [70]. This budget indicates that accuracies comparable with existing UK national standards are obtainable using a commercial network analyser [117]. Tables 2 and 3 illustrate this capability using measurements of a termination containing a nominal 100 ohm resistor fitted with a coaxial connector. Table 2 presents the results in terms of VRC magnitude and phase, and Table 3 presents the same measurements in terms of equivalent impedance parameters; resistance and inductance. The estimated uncertainties given in the Tables are specified at a confidence level of not less than 95%. Measurements performed on a range of different value terminations indicate typical performance of 0.0008 uncertainty in VRC magnitude corresponding to an equivalent return loss of 62 dB. High quality components fitted with precision connectors are required to achieve this level of accuracy.

Frequency (MHz)	Magnitude		Phase (Degrees)	
	Mean	Uncertainty	Mean	Uncertainty
50	0.33302	0.00066	-4.69	0.12
100	0.33295	0.00067	-9.42	0.12
150	0.33289	0.00067	-14.16	0.12
200	0.33275	0.00067	-18.90	0.12
250	0.33272	0.00068	-23.62	0.13
300	0.33276	0.00068	-28.34	0.13

Table 2: Measurements of a nominal 100 ohm resistor in terms of complex voltage reflection coefficient.

Frequency (MHz)	Resistance (ohms)		Inductance (nH)	
	Mean	Uncertainty	Mean	Uncertainty
50	99.43	0.16	-19.38	0.48
100	97.94	0.16	-19.12	0.24
150	95.56	0.15	-18.58	0.16
200	92.42	0.15	-17.83	0.12
250	88.75	0.14	-16.938	0.085
300	84.70	0.14	-15.964	0.069

Table 3: Measurements of a nominal 100 ohm resistor in terms of impedance; resistance and inductance.

6 CONCLUSIONS

This report has outlined some topics relevant to VHF impedance measurement. A review of capabilities past and present has been given. It has been shown that, although a relatively mature technology in the context of microwave engineering, VHF impedance is experiencing a renaissance with the advent of new measurement methods (six-port techniques) and new measuring instruments (network analysers). This has led to a need to characterise the artefacts used during calibration to new levels of accuracy. The resulting performance is impressive: multi-octave frequency coverage, measurement periods reduced by several orders of magnitude compared with earlier techniques, and typical discrimination of 62 dB return loss. This provides the customer for these measurements with more information at a reduced cost, representing better value for money. This will benefit directly the technologies relying on these capabilities in the years to come.

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