

Calculation of the magnetic field strength in Helmholtz systems with square and polygonal coils

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ABSTRACT

Formulae are developed for the magnetic field strength of a Helmholtz system of two identical coils of square shape. The field of regular polygons of other than four sides is discussed, and it is shown that a simple change of co-ordinates leads to a general formula readily applicable to polygons of any number of sides.

The corners of the polygons cause departures of the magnetic field from the axial symmetry that circular coils display, but the irregularities are small, especially in the central region of the system. Examples of their amount for a square, an octagon and a dodecagon are given in tables. Departure from axial symmetry is negligible for the dodecagon.

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1 INTRODUCTION

It has sometimes been claimed that the pair of circular coils of a Helmholtz system designed to secure a calculable, uniform magnetic field in the central region of the assembly could advantageously be replaced by coils of square shape. The reason usually given is that the latter are easier to make. Although not everybody would agree that such is the case, it is instructive to compare the uniformity of field obtainable in the two systems. Naturally, triangles, square, and other polygons do not enjoy the beautiful axial symmetry that results from the circular shape, and calculation of the field at points near the centre is more complicated than for circles, for which spherical harmonics provide practical and elegant formulae. The study is nevertheless worth while, and is undertaken in what follows.

2 SQUARES AND OTHER REGULAR POLYGONS

In Figs 1 and 2, M is the mid-point of a side AB of a regular polygon of n sides, centred at the point O_1 on the Y axis. The angle AO_1M is π/n , and if the height O_1M be denoted by h ,

$$\text{Area of polygon} = \frac{n \cdot AM \cdot O_1M}{n \cdot h^2 \tan(\pi/n)} \quad (1)$$

To facilitate comparison, all polygons will be ascribed the same area, for which it is convenient to choose that of a square of side a . (1) then becomes

$$\begin{aligned} a^2 &= nh^2 \tan(\pi/n) \\ \text{and} \quad \frac{h}{a} &= \frac{1}{\sqrt{n \tan(\pi/n)}} \end{aligned} \quad (2)$$

It will be desirable in some cases to replace a by $\rho\sqrt{\pi}$, where ρ is the radius of a circle of area a^2 (see (34) and Table 1, page 7)

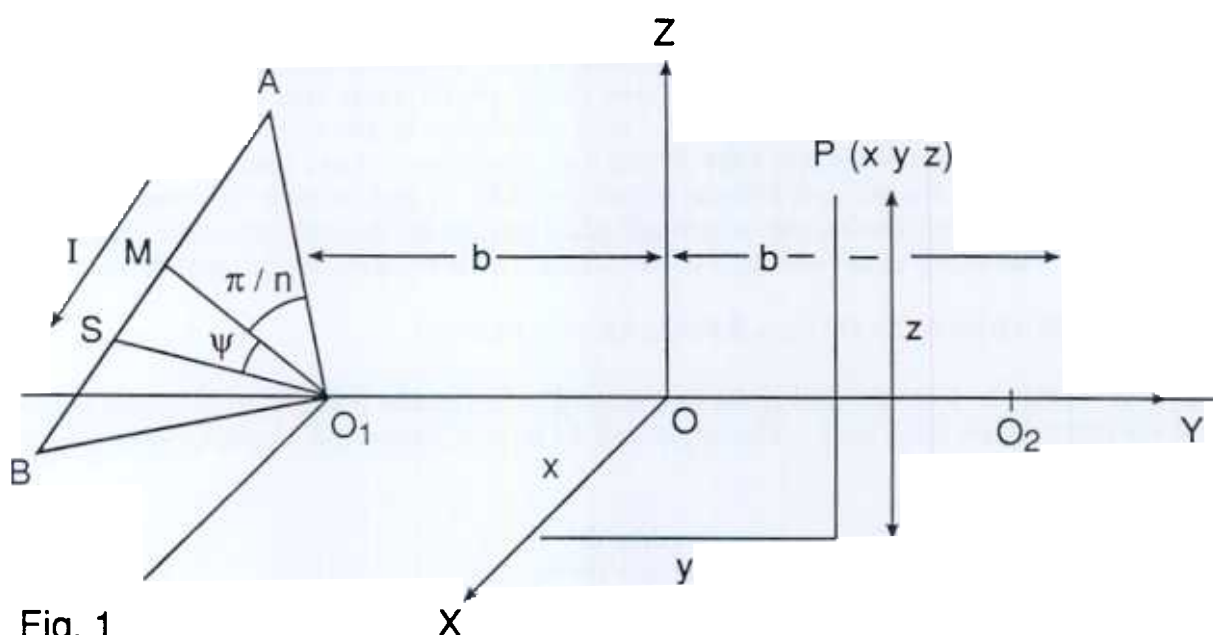


Fig. 1

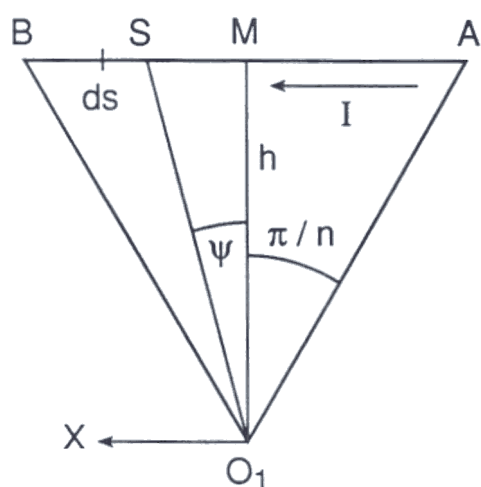


Fig. 2

Fig 3 for calculating field H_y , H_ϕ , H_r at point P (y , ϕ , r) due to current I in identical coils centred on the Y axis at distances b and $-b$ from the origin O.

Figs 1 and 2 for calculating field H_x, H_y, H_z at point P (x, y, z), due to current I in side AB of a regular polygon of n sides centred at $O_1 (0, -b, 0)$

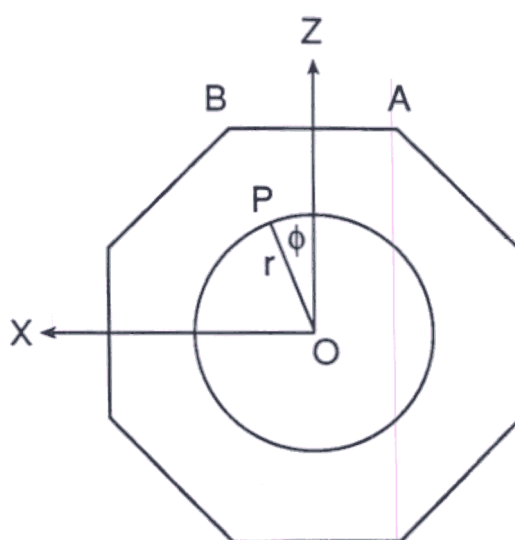


Fig. 3

3 MAGNETIC FIELD STRENGTH OF A STRAIGHT CONDUCTOR

In Figs 1 and 2, AB is the conductor forming one side of a regular polygon of n sides, S is a point on it, and P is the point at which the magnetic field strength due to a current I in AB is required. The co-ordinates of P are x, y, z ; those of S are $h \tan \phi, -b, h$. In vector notation

$$\mathbf{PS} = \mathbf{r} = \mathbf{PO} + \mathbf{OS} = -\mathbf{OP} + \mathbf{OS} \quad (3)$$

$$\mathbf{r} = i(-x + h \tan \psi) + j(-y-b) + k(-z+h) \quad (4)$$

$$r^2 = (x-h \tan \psi)^2 + (y+b)^2 + (z-h)^2 \quad (5)$$

The magnetic field strength dH due to the current I in the element ds , fig 2, is given by the vector formula

$$dH = \frac{\mathbf{r} \times d\mathbf{s}}{r^3} \cdot \frac{I}{4\pi} \quad (6)$$

The quantities entering (6) are tabulated in (7) below.

	i	j	k
$\mathbf{r}$	$h \tan \psi - x$	$-b - y$	$h - z$
$d\mathbf{s}$	$h \sec^2 \psi d\psi$	0	0
$\mathbf{r} \times d\mathbf{s}$	0	$(h-z)h \sec^2 \psi d\psi$	$(b+y)h \sec^2 \psi d\psi$

(7)

From (6) and (7), and $\mathbf{r}$ as in (5),

$$H_x = 0 \quad (8)$$

$$H_y = \frac{I}{4\pi} \int_{-\pi/n}^{\pi/n} \frac{h(h-z) \sec^2 \psi}{r^3} d\psi \quad (9)$$

$$H_z = \frac{I}{4\pi} \int_{-\pi/n}^{\pi/n} \frac{h(b+y) \sec^2 \psi}{r^3} d\psi \quad (10)$$

On integration these expressions yield

$$H_x = 0$$

$$H_y = \frac{I}{4\pi} \left[\frac{(z-h)(x-h \tan \psi)}{(y+b)^2 + (z-h)^2} \cdot \frac{1}{r} \right]_{-\pi/n}^{\pi/n}$$

$$H_z = \frac{I}{4\pi} \left[-\frac{(y+b)(x-h \tan \psi)}{(y+b)^2 + (z-h)^2} \cdot \frac{1}{r} \right]_{-\pi/n}^{\pi/n}$$

When the limits in (11), (12), (13) are inserted, the expressions can be put into the form

$$H_x = 0, \quad H_y = \frac{I}{4\pi} (z-h) F, \quad H_z = -\frac{I}{4\pi} (y+b) F$$

where

$$F = \frac{x-h \tan(\pi/n)}{(y+b)^2 + (z-h)^2} \{ (x-h \tan(\pi/n))^2 + (y+b)^2 + (z-h)^2 \}^{-1/2} \\ + \frac{x+h \tan(\pi/n)}{(y+b)^2 + (z-h)^2} \{ (x+h \tan(\pi/n))^2 + (y+b)^2 + (z-h)^2 \}^{-1/2}$$

4 CHOICE OF CO-ORDINATES

Expressions (14) and (15) are simple enough when the field of only one side AB, figs 1 and 2, is required. But they assume different values for other sides of the polygon, and complicate evaluation by computer. With a square, for instance, the side BC, at right angles to AB, would lead to expressions similar to (14) and (15), but with x and z interchanged.

It is possible, however, to arrange for (14) and (15) to be identical for all the sides of a regular polygon, except for a single variable ϕ , the inclination of O_1M to OZ , by choosing new co-ordinates r, ϕ , such that

$$z = r \cos \phi \tag{16}$$

$$x = r \sin \phi$$

The magnetic field strength at the point x, y, z is then given by two components, H_y parallel to the axis OY , and H_r perpendicular to it, and at an angle ϕ with OZ .

To effect the change it is sufficient to ignore H_x , to write H_r instead of H_z in (14), and to replace z and x by $r \cos \phi$ and $r \sin \phi$ wherever they occur in (14) and (15).

The symbol r used in (16) and (17) above, also for OP in Fig 3, has no relation to the same symbol given to PS in (3), page 3 and in Fig 1.

5 NORMALISATION

In addition to the change of co-ordinates mentioned in the previous section, we "normalise" by dividing all lengths by a , the side of the square of the same area as the polygon under consideration. For the sake of distinction denote the resulting dimensionless quantities by capital letters, for example B for b/a , R for r/a , etc. The expressions for H_y and H_r then give place to (18) to (22), in which F , M , N are now dimensionless, although for simplicity these same letters have been retained.

$$H_y = \frac{I}{4\pi a} \{R \cos \phi - (n \tan(\pi/n))^{-0.5}\} F$$

$$H_r = \frac{-I}{4\pi a} (Y+B) F$$

$$F = \frac{M-N}{(Y+B)^2 + \{R \cos \phi - (n \tan(\pi/n))^{-0.5}\}^2}$$

$$M = R \sin \phi - \left(\frac{1}{n} \tan \left(\frac{\pi}{n} \right) \right)^{0.5} \left\{ R \sin \phi - \left(\frac{1}{n} \tan \frac{\pi}{n} \right)^{0.5} \right\}^2 \\ + (Y+B)^2 + \left\{ R \cos \phi - \left(n \tan \left(\frac{\pi}{n} \right) \right)^{-0.5} \right\}^2 \right]^{-0.5}$$

$$N = \left\{ R \sin \phi + \left(\frac{1}{n} \tan \left(\frac{\pi}{n} \right) \right)^{0.5} \right\}^2 \left\{ R \sin \phi + \left(\frac{1}{n} \tan \left(\frac{\pi}{n} \right) \right)^{0.5} \right\}^2 \\ + (Y+B)^2 + \left\{ R \cos \phi - \left(n \tan \left(\frac{\pi}{n} \right) \right)^{-0.5} \right\}^2 \right]^{-0.5}$$

Apart from the current I , the only dimensional quantity in the right-hand terms of the above expressions is the side a of the square of area equal to that of the polygon, confirming the well-known result that the magnetic field strength of similar circuits varies inversely as their linear dimensions.

If it is desired to express the magnetic field strength by its cartesian components H_y , H_z and H_x , the first requires no change, whereas the last two are obtained by multiplying H_r by $\cos \phi$ and $\sin \phi$ respectively.

The field strengths H_y and H_z of (18) and (19) are those due to the side AB of the polygon centred at O_1 . AB is parallel to X, M is its midpoint, figures 1 and 2, and O_1M is parallel to Z. The variable ϕ of (18) and (19) is chosen according to the direction in which H_z is required.

For the second side BC of the polygon nothing is changed in (18) and (19) except that ϕ is replaced by $\phi + 2\pi/n$; and for the I th side it is $\phi + (I-1) 2\pi/n$. To account for all the sides, I is given values from 1 to n, and the results added.

Likewise, to account for the identical coil centred at O_2 , B is replaced by - B, and the results added.

6 OPTIMUM SEPARATION OF COILS

In order to calculate the field strength at any point O (y, ϕ, r), which is the object of this note, the optimum separation 2B of the coils at O_1 and O_2 must be determined. For the purpose we put R equal to zero in (18) to (22), and thus obtain H_y and its derivatives on the axis; and since R and ϕ do not appear in H of (18) to (22), it is sufficient to multiply by n to obtain H for the whole of one polygon. Hence

$$H_y = \frac{2I}{4\pi a} \cdot \left[(Y+B)^2 + \frac{1}{n \tan(\pi/n)} \right]^{-1} \times \left[\frac{1}{n} \tan(\pi/n) + (Y+B)^2 + \frac{1}{n \tan(\pi/n)} \right]^{-0.5}$$

Put

$$U = (Y+B)^2 + \frac{1}{n \tan(\pi/n)}$$

$$W = U + \frac{1}{n} \tan(\pi/n)$$

Then

$$\frac{dU}{dY} = \frac{dW}{dY} = 2(Y+B)$$

$$H_y = \frac{2I}{4\pi a} U^{-1} W^{-0.5}$$

$$\frac{dH_y}{dY} = -\frac{2I}{4\pi a} \cdot 2(Y+B)(U^{-2} W^{-0.5} + 0.5 U^{-1} W^{-1.5})$$

$$\frac{d^2 H_y}{dY^2} = -\frac{4I}{4\pi a} \left[U^{-2} W^{-0.5} + 0.5 U^{-1} W^{-1.5} \right]$$

$$-2(Y+B)^2 (2U^{-3} W^{-0.5} + U^{-2} W^{-1.5} + \frac{3}{4} U^{-1} W^{-2.5})$$

As the object is to use (29) to determine the value of B at zero Y, the coil at O_2 can be

accounted for by simply doubling (29). Also, for convenience, (29) may be multiplied on divided by any common factor. Multiplication by $U^3 W^{2.5}$ gives the equation to be solved as

$$B^2 (8W^2 + 4UW + 3U^2) - 2UW^2 - U^2W = 0$$

A simple computer programme helps to determine by trial and error, B or b/a for various values of n, the number of sides of the polygon. When n tends to infinity it is not necessary to solve (30) by this method, because if Y is made zero in (24), and n infinite in (24) and (25), the resulting values of U and W inserted in (30) yield (32), (33) and (34)

$$U = W = B^2 + \frac{1}{\pi}$$

$$5B^2 = U = B^2 + \frac{1}{\pi}$$

$$B = \frac{b}{a} = \frac{1}{2\sqrt{n}} = 0.282\ 095$$

and, since the radius ρ of the circle of area a^2 is $a/\sqrt{\pi}$,

$$b = \frac{\rho}{2} \quad (34)$$

a well-known result.

Table 1 shows the dependence of B, referred to ρ , on the number n of sides of polygons all of the same area a^2 .

Table 1 Optimum Separation of Coils

Polygon	Number of sides n	$B = b/\rho$
Triangle	3	.4401
Square	4	.48255
Octagon	8	.49895
Dodecagon	12	.49983
Circle	∞	.5

Square coils should be approximately 3.5 per cent closer than circular ones. If the number of sides is doubled, the difference shrinks to 0.21 per cent, whereas it is negligible for a dodecagon. The values of B, still referred to ρ , plotted against the number of sides n of the polygons, starting with the triangle (n=3) and ending with the circle (n= ∞), fit roughly the curve

$$B = 1/2 - 2.52\sqrt{\pi}/n^4 \quad (35)$$

To simplify the computer programmes, this formula has been adopted for them rather than

the values given in Table 1. The effect on the results is negligible.

7 MAGNETIC FIELD STRENGTH ON THE AXIS

The optimum separation for various polygons having been determined and tabulated in Table 1, it is instructive to compare their field strengths at points on the axis on each side of the origin. Steps of $a/20$ have been used. Table 2 gives the figures that have to be multiplied by I/a to yield the magnetic field strength H if I is the current in each of the two identical coils, and if the area of each is that of a square of side a .

Table 2

Table 2. Field strength on the axis of pairs of polygonal coils Columns 3 and 8 give $(aH_y)/I$ at the origin and at points y on the axis. The area of each coil is a^2 , the current in it is I , and the range of y covered in this table is $a/4$.

N	$B=b/\rho$	$y/a = 0$	.05	.10	.15	.20	.25
3	.444 857	1.365 487	1.365 589	1.364 123	1.356 327	1.336 015	1.297 804
4	.482 552	1.296 110	1.296 006	1.294 496	1.288 309	1.273 032	1.244 439
8	.498 910	1.269 934	1.269 840	1.268 514	1.263 096	1.249 679	1.224 402
12	.499 785	1.268 593	1.268 503	1.267 194	1.261 826	1.248 511	1.223 406
∞ (1000)	$1/2$.500 000	1.268 265	1.268 175	1.266 872	1.261 516	1.248 227	1.223 165

The table shows that, as might have been expected, triangular coils differ most from circular ones. At the origin the field strength of the triangle exceeds that of the circle by 7.7%, whereas at a point still on the axis but distant $a/4$ from the origin, the difference has decreased to about 6.1%. For the dodecagon the relative difference has shrunk to about 0.026% at the origin, and, $a/4$ away, to less than 0.02%.

8 MAGNETIC FIELD STRENGTH OFF THE AXIS

Equations (18) and (22) can now serve to calculate the magnetic field strengths H_y and H_r at any point $P_{(y,\phi,r)}$ of a system of two identical polygonal coils placed at equal distances b on each side of the origin, each of area a^2 and carrying a current I .

In fig 3 the origin is at O in the plane of the paper, and the axis of Y is through O , perpendicular to the paper, with the positive direction upwards. The coils shown, in this case octagons, are centred on the Y axis at distances b above and below the plane of the paper.

If the co-ordinates y and r of P are kept constant whilst ϕ , the inclination of OP to OZ , is varied, P moves on the circumference of a circle whose plane is perpendicular to the axis, and whose centre is on the axis at a distance y from O . If the coils, instead of being octagons, had been circles, H_y and H_r would have had the same values all round the circle, whereas here the corners of the polygons cause departures from this uniformity. Therein lies one of the advantages of coils of circular shape.

The departures are not however very large. Their magnitude and sign depend on the co-ordinates y , ϕ , r of the point P , fig 3, at which the components H_y , H_r of the magnetic field strength are being calculated.

Table 3 gives two examples of this calculation for a square coil ($N=4$). In the first the point P is at a distance from the origin of approximately a third the distance of the coil. At that point the corner of the square is more effective than the central region of the side, so that $H_y(\phi)$ increases gradually from $\phi=0$ to reach a maximum at $\phi=\pi/4$, for which the relative difference between $H_y(\phi)$ and $H_y(0)$ is 1.45×10^{-4} , as can be seen from the penultimate column of Table 3. In that table the results have been shown to $\phi=\pi/2$ only as a reminder of the symmetry at the corner; and the same figures are repeated at each of the four corners of the square. Thus, if steps of $\pi/12$ for ϕ are considered sufficient, only four numbers need be calculated. With a polygon of N sides the steps would be $\pi/3N$, and for the whole polygon at one point P just four calculations would suffice.

Table 3

Magnetic field of a Helmholtz system of two identical co-axial coils distant $2b$ apart, each of area a^2 , and carrying a current I . The magnetic field strengths H_y and H_r refer to a point P of co-ordinates y, ϕ, r (see Section 4, page 4).

y/a	ϕ	r/a	N	b/ρ	$aH_y(\phi)/I$	$aH_y(0)/I$	$\frac{H_y(\phi)}{H_y(0)} - 1$	H_r/H_y
0	0	0	4	.482 552	1.296 110	1.296 110	0	0
.1	0	.1			1.298 661	1.298 661	0	(-2)809 019
.1	$\pi/12$	.1			1.298 708		(-5)363 164*	(-2)809 809
	$\pi/6$				1.298 802		(-4)108 923	(-2)811 388
	$\pi/4$				1.298 849		(-4)145 212	(-2)812 177
	$\pi/3$				1.298 802		(-4)108 923	(-2)811 388
	$5\pi/12$				1.298 708		(-5)363 164	(-2)809 809
	$\pi/2$				1.298 661		0	(-2)809 019
.2	0	.1	4	.482 552	1.287 076	1.287 076	0	(-1)187 848
	$\pi/12$				1.286 988		-(-5)685 473	(-1)187 937
	$\pi/6$				1.286 812		-(-4)205 633	(-1)188 116
	$\pi/4$				1.286 723		-(-4)274 172	(-1)188 205
	$\pi/3$				1.286 812		-(-4)205 633	(-1)188 116
	$5\pi/12$				1.286 988		-(-5)685 473	(-1)187 937
	$\pi/2$				1.287 076		0	(-1)187 848

*Exponential notation: (-5)363 164 stands for 3.63164×10^{-5} ; the decimal sign always follows the first digit, and is omitted for simplicity.

For the second example in Table 3 the y co-ordinate of P has been doubled, which has brought the point to approximately two thirds of b, the distance of the coil from the origin. The figures show that the effect of the corners is now to decrease H_y as ϕ is increased from 0 to $\pi/4$, but again the changes are small.

Table 4 shows examples for an octagon and a dodecagon. The latter is almost indistinguishable from a circle, for $H_y(\phi)$ equals $H_y(0)$ to the seventh figure as ϕ is varied from 0 to the corner ($\pi/12$). There is however a real difference, as the penultimate column shows, but it is only -9.04×10^{-8} in relative value.

In Tables 3 and 4 the last column has been included to yield the ratio of the radial to the axial flux densities at the point P(y, ϕ , r).

Table 4

Magnetic field of a Helmholtz system of two identical co-axial coils distant $2b$ apart, each of area a^2 , and carrying a current I . The magnetic field strengths H_y and H_r refer to a point P of co-ordinates y, ϕ, r (see Section 4, page 4).

y/a	ϕ	r/a	N	b/ρ	$aH_y(\phi)/I$	$aH_y(0)/I$	$\frac{H_y(\phi)}{H_y(0)} - 1$	H_r/H_y
0	0	0	8	.498 910	1.269 934	1.269 934	0	0
.1	0	.2			1.279 326	1.279 326	0	(-2)518 994
	$\pi/24$				1.279 332		(-6)494 089*	(-2)519 075
	$\pi/12$				1.279 345		(-5)148 235	(-2)519 237
	$\pi/8$				1.279 381		(-5)197 653	(-2)519 318
	$\pi/6$				1.279 345		(-5)148 235	(-2)519 237
0	0	0	12	.499 785	1.268 593	1.268 593	0	0
.2	0	.2			1.301 623	1.301 623	0	(-1)141 551
	$\pi/36$						(-8)226 110	
	$\pi/18$						(-8)678 390	
	$\pi/12$						(-8)904 490	
	$\pi/9$						(-8)678 390	

* Exponential notation: (-6)494 089 stands for 4.94089×10^{-6} ; the decimal sign always follows the first digit, and is omitted for simplicity.

9 CONCLUSION

A Helmholtz system is frequently used to cancel the Earth's magnetic field in a small region in which to study a property of some material in zero field, or in a known applied field. A measurement of the gyromagnetic ratio of the proton, for instance, might entail the use of a small sphere, say 3 or 4 cm in diameter, filled with water or some other liquid, subjected to a known field, but unaffected by the terrestrial field or its variations.

If the Helmholtz system has square coils of side 1 m, about 0.5 m apart, the small variations in axial symmetry due to the corners, studied in Section 8 of this report, would be negligible to perhaps a few parts in 10^8 in that small region round in origin. On the other hand, if the system is applied to the study of materials or instruments requiring a larger region, say about a quarter of a metre from the origin, the departures from axial symmetry caused by the corners may be unacceptably large. A computer programme like that used for the examples given in Tables 3 and 4, pages 22 and 24, will enable the experimenter to decide.

The results obtained from (18) to (22), page (10), were applied to Table 3 and 4, on the assumption of a single turn and sharp corners. Whilst it is possible to produce a formula of the sort given in Vigoureux (1993), the bending of the wire at the corners is likely to cause difficulties both as regards calculation and permanence of the instrument constant.

Why not then secure the desired axial symmetry by employing circular coils? A E Drake and D A Rollett have recently wound a triaxial Helmholtz coil system of approximately 3 metres in diameter to calibrate instruments, other coil systems, and determine the effect of low magnetic permeability materials on the ambient magnetic field. The coils were wound on circular formers of Fibrelam, produced from resin impregnated paper in the form of a honeycomb sandwiched between layers of resin bonded paper. The material had the desired properties of being non-magnetic, non-conducting and was lightweight yet rigid and mechanically stable.

In 1959 Mr K Burley, of NPL's wood workshop, made and wound two pairs of circular frames about 2 m in diameter to cancel the vertical and the horizontal components of the Earth's field. Non-magnetic wood is available, and if the construction is performed in rooms where no work on metals is allowed, contamination by dust from magnetic materials is avoided. Mr Burley's coils secured the success of the measurement of the gyromagnetic moment of the proton by Vigoureux and Dupuy (1980).

10 REFERENCES

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11 NOTE ON APPENDICES

The formulae given in the text of this paper suffice for the calculation of the components of magnetic field strength at any point of the system. The arithmetic is considerable, and even for a single point a computer is of great help. The one used was a Casio PB-100 without a printer. The three programmes in BASIC, written for Tables 1 to 4, are reproduced merely to inform readers of what was done. Should they need to apply the formulae to their own requirements, they might well wish to secure shorter and more efficient programmes from a more powerful computer and a simpler language.

12 APPENDIX 1

Optimum distance of coil from the origin. By a simple and quick trial and error method the programme determines $B(=b/\rho)$ for a coil of N sides to cancel expression (30), page 7. The last column of Table 1, page 7, shows the results.

```

10  INPUT N, B
15  SET F5
20   $U = ((B \uparrow 2) / \pi + (N * (\text{TAN}(\pi/N))) \uparrow -1$ 
30   $W = U + (\text{TAN}(\pi/N)) / N$ 
40   $G = (B * B)(\pi) * (3 * U * U + 4 * U * W + 8 * W * W)$ 
50   $H = 2 * U * W * W + U * U * W$ 
60   $I = G - H$ 
70  PRINT I
80  GOTO 10

```

13 APPENDIX 2

Magnetic field strength on the axis.

This programme was used for a few points on the axis, see Table 2. It is easy to modify for other points, also on the axis

```

10  INPUT N
20  SET F6
30  FOR I = 0 TO 5
40  Y = .05*I
50  B = 0.5-2.52*(SQR π)/N↑4
60  IF I = 0; PRINT B
70  U = (Y+B)↑2+(N*(TAN(Ti/N)))↑-1
80  W = U+(TAN(π/N))/N
90  S = 0
100 S = ((1/(2*π))*(U↑-1)*(W↑-0.5)+S
110 U = (Y-B)↑2+(N*(TAN(π/N)))↑-1
120 W = U+(TAN(π/N))/N
130 S = ((1/2*π))*(U↑-1)*(W↑-0.5)+S
140 PRINT S
150 NEXT I
160 GOTO 10

```

14 APPENDIX 3

Magnetic field strength off the axis.

The programme was used to illustrate in Tables 3 and 4 the departures from circular symmetry that occur when polygonal coils replace circular ones.

```

10  INPUT E, W, R, N
20  SET F6
30  B = .5-2.52*(SQRπ)/N↑4
40  PRINT B
50  S = 0
60  P = 0
70  FOR I = 0 to N-1
80  G = W+I*2*π/N
90  T = (N*(TAN(π/N)))↑.5
100 X = -1/T+R*(COS G)
120 J = (E+B)↑2+X↑2
130 K = X/J
140 L = -(E+B)/J
150 M = (R*(SIN G)-T/N)*((R*(SIN G)
      -T/N)↑2+J)↑-.5
      O = (R*(SIN G)+T/N)*((R*(SIN G)
      +T/N)↑2+J)↑-.5
170 Q = MK-O
180 S = (K+Q)/(4*π)+S
190 P = (L*Q)/(4*π)+P
220 J = (E-B)↑2+X↑2
230 K = X/J
240 L = -(E-B)/J
250 M = (R*(SIN G)-T/N)*((R*(SIN G)
      -T/N)↑2+J)↑-.5
      O = (R*(SIN G)+T/N)*((R*(SIN G)
      +T/N)↑2+J)↑-.5
270 Q = M-O
280 S = (K*Q)/(4*π)+S
290 P = (L*Q)/(4*π)+P
300 NEXT I
310 PRINT I
320 IF I<N-1 THEN 80
330 PRINT S
340 IF W = 0; H = S
350 PRINT H
360 U = S/H - 1
370 SET E6
380 PRINT U
390 V = P/S
400 PRINT V
410 GOTO 10
420 END

```