

Ball-Indentation of Multi-layered Foils

This Measurement Note introduces a novel finite element (FE) solution to the elastic indentation of a multi-layered coated half-plane. It describes a method of predicting elastic indentation behaviour that can be used to check validity of experimental measurements. The problem was analysed using the commercial finite element software ANSYS, for which a macro was written in order to automatise the calculations, using the ANSYS Parametric Design Language (APDL). This macro was used to determine the effects of the number of layers and their relative thickness, of the friction between the indenter and the top layer of the coating, and of the residual stress state on the elastic response of the system. The inputs to the FE model were reduced so that the user needs only concentrate on the thermo-elastic properties of the layers and the substrate, the exact thickness of the layers, and the stress free temperature of the structure. All inputs normally used a database of referenced thermo-mechanical properties, based on values determined experimentally, although manual input in the macro remained an option. It was further shown that friction could be neglected, whereas residual stresses, introduced via the stress-free temperature, influenced to a great extent the elastic response of the coated system. An alternative approach to finite elements, based on recent semi-analytical solutions to the stress state and the penetration of the indenter, was further suggested for future developments.

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1. Introduction

This document describes the contents and the use of a macro for Finite Element solution of the axisymmetric nano-indentation of multi-layered systems. The macro is written in the ANSYS Parametric Design Language (APDL), and presents a fully automated alternative to compute the elastic response of a multi-layered system indented by a diamond sphere. The macro is called with a number of arguments that enable the definition of the substrate and coating materials, the number of bi-layers constituting the coating and their average individual thickness, as well as the indenter radius. The geometry of the problem and the mesh were optimised so that the user is required to provide a minimum of inputs. A range of materials are available for the substrate and the coating, and their mechanical properties are extracted from a database linked to the macro.

After a brief review of the state of the art, the geometry and the FE solution are described. Application examples, emphasizing the effects of the number of layers, of the friction between indenter and coating surfaces, as well as residual stresses, are further discussed.

2. Background

2.1. Models of elastic indentation

Nano-indentation has long been promoted as the ideal tool for the characterisation of materials on the nano-scale [1, 2]. The ability to load the material over very small areas, in the range of a few tens of square nanometers, together with its load control capabilities, down to the μN , is bound to enable the local determination of the mechanical properties of the material's surface. A thorough review was written recently [3], and the principles of nano-indentation will not be repeated here. However, the analysis of nano-indentation data remains far from being simple, and it is useful to complete the discussion of ref. [3] by arguments related to multi-layered geometries. As discussed in ref. [4], a number of unknowns clearly hamper the determination of elastic constants by nano-indentation in multi-layered systems. The extent of the effect of friction between the indenter and the surface, of residual stresses, and the experimentally unresolved contact area, to name only a few, are still issues of great importance, that pose the question: can elastic properties of coatings be determined by means of indentation? The measurement of an elastic property is based on the determination of the contact stiffness S , that relates to the area of contact A by:

$$S = \frac{2}{\sqrt{\pi}} E_r \sqrt{A} \quad (1)$$

where E_r is the reduced modulus of the tip and sample given by:

$$\frac{1}{E_r} = \frac{1 - \nu_t^2}{E_t} + \frac{1 - \nu_s^2}{E_s} \quad (2)$$

where E_t , ν_t and E_s , ν_s are the elastic modulus and Poisson's ratio of the tip and sample, respectively. It is important to note that the validity of Equation 1 has not been established on the basis of experimental results for nano-indentation, although other nano-mechanical contacts have been determined based on the use of, e.g., surface force apparatus. Most of the mathematical modelling of elastic contacts is based on the original works of Hertz [5] and Boussinesq [6]. Their models consider the contact of two axisymmetric curved surfaces, taken as axisymmetric half-spaces, where the contact region is small compared to the radius of curvature of the contacting surfaces. By letting one radius tend towards infinity, the geometry for the traditional indentation of a plane by a sphere is obtained, as shown in Fig. 1. Following the Hertz model, the radius of the contact region a relates to the radius R of the elastic sphere indenting the surface and the applied load P as:

$$a = \sqrt[3]{\frac{3PR}{4E_r}} \quad (3)$$

The displacement δ of the sphere into the surface being given by:

$$\delta = \sqrt[3]{\frac{9P^2}{16RE_r^2}} \quad (4)$$

Although the Hertz model is an exact solution for homogeneous materials, it has no equivalent for multi-layered systems, so that a FE approach was preferred.

2.2. Indentation of multi-layers

The geometry of the problem is the indentation by a hard sphere (the indenter) of an elastic half-plane that is coated by a large number of elastic layers with thickness in the sub-micrometer range. The diameter of the indenter typically ranges between 5 μm and 100 μm , although in practice smaller radii can be used, and usually the largest diameter for testing nano-structured coatings is about 25 μm . The thickness of the individual coating layers is between a few nanometers and the micrometer. One of the difficulties of such simulation is the large variation of aspect ratio when the thickness of the layers reaches a few nanometers, together with the uncertainty of their mechanical properties at this scale. Invariably, the large aspect ratio of the coating layers leads to a large number of elements and, as a consequence, resource intensive models. Several meshing strategies can be used to reduce the number of elements, with relative success. In this study it was found that a good choice of dimensions enables the use of a number of elements consistent with the use of a standard PC, without particular need for an intricate mesh (although the mesh dimensions are only chosen to fit the purpose of most hard coatings on more compliant substrates). As such, the length of the substrate was chosen to be equal to twenty times the thickness of the thinnest coating layer times the ratio of the largest to the smallest modulus;

the thickness of the substrate was one half of this quantity. It was further checked that reaction forces on the boundaries were independent of changes to larger values, within computer error (10^{-12}). Simulations were performed for layers of thickness larger in all cases than the limit of 5-10 nm, where it is thought that molecular modelling should be used instead. Here, the thickness of layers was kept above 50 nm, where it has been demonstrated experimentally that traditional mechanics are valid [4]. Guidance on model requirements was obtained from earlier analytical and FE works at NPL of indentation of single-layered systems [7].

3. Presentation of the APDL macro

In order to deal properly with materials at the small scale, the macro was written for use with the set of consistent units [μm , g, nN, kPa, s, $^{\circ}\text{C}$]. The file begins with an automatic clearing of any previous results, making sure that the memory buffer is empty, followed by an instruction that toggles on the multi-processor capability. A number of definitions of parameters follow, including the load step history, and the number of entries in the materials database used. The substrate length and thickness are then defined, based on the smallest thickness of coating layer.

After these initialisations have been performed, the /PREP7 environment is accessed. The materials data array is created, and populated with the content of the file DATAMAT.DAT: contents of this file and references to the source of the data are found in the associated file DATAMAT.TXT. The density and elastic modulus are converted to the set of units in use. Poisson's ratio and coefficient of thermal expansion are non-dimensional quantities and therefore only need to be read. The indenter material properties are fixed at the moment, set to the properties of diamond. The friction coefficient between the indenter and the top layer of the coating is left as a parameter which can be tuned (it defaults to 0). The materials properties are shown in Table 1.

The next step involves building the finite element model. Here, the task is fully automated through the use of the macro. Upon introduction of the right parameters, one of the two configurations will be chosen, namely [substrate/AB...AB] or [substrate/AB...ABA]. The geometry is built by defining keypoints first, joining them

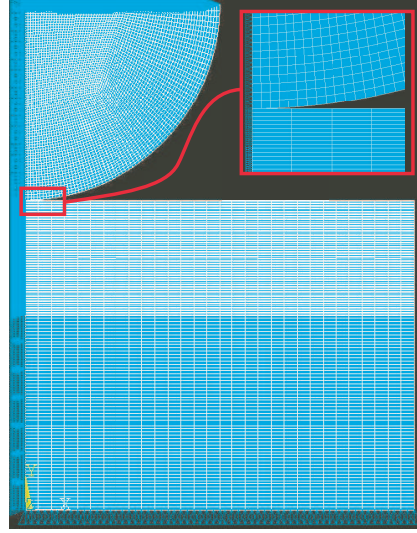


Figure 1: View of the finite element mesh

with lines, and then assigning an area number to the newly created layer. The material property is then attributed to the layer, and the layer is meshed. A free meshing was used, and all the constituents were meshed by PLANE183 elements, which are 2D axisymmetric 8-nodes quadrilateral solid elements (Figure 1). Once the discrete geometry has been obtained, the boundary conditions are fixed. In order to obtain an axisymmetric geometry, the y -axis is defined as a cylindrical symmetry axis, by having no displacements in the x -direction. The bottom of the substrate is clamped. A temperature boundary condition is further applied to all nodes of the model, in order to simulate the effect of a residual stress state. This temperature effectively corresponds to a stress free temperature, that is calculated for materials with constant thermo-elastic properties in the temperature range considered. While this is not true in general for any physical system, it has been shown to be a reasonable numerical approximation, for cases where it is possible to account for viscoelastic and other time dependent phenomena by adjusting ΔT accordingly [8].

Two options of ANSYS are used when solving this problem:

- gap closure: we did not use the fixed penalty stiffness to determine the area of contact, but fix the initial gap to zero, using the ICONT instruction. The contact stiffness is further calculated and used as a starting point for regression;
- non-linear geometry: although displacements are small, turning on the non-linear geometry option has been shown to help the convergence of the problem in similar situations.

Finally, for each load step we record the maximum indentation depth, and save the results database. The displacement δ versus load P tables are further exported to an ASCII file for easy representation.

Table 1: Thermo-mechanical properties of materials used in FE simulations

Material	E (GPa)	ν	α ($\mu\text{m}/\text{m}/\text{K}$)
Steel	200	0.29	17.28
Titanium	108	0.37	8.90
Titanium Nitride	600	0.20	9.40
Diamond	1200	0.10	1.18

4. APDL Macro

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=====
!== NANO-INDENTATION OF A MULTI-LAYERED COATING ON SUBSTRATE BY A SPHERICAL INDENTER ==
!== ALL UNITS IN kPa, um, g, s, nN WHERE NEEDED ==
!== ARGUMENT 1: SUBSTRATE MATERIAL (INTEGER) ==
!== ARGUMENT 2: COATING MATERIAL A (INTEGER) ==
!== ARGUMENT 3: COATING MATERIAL B (INTEGER) ==
!== ARGUMENT 4: NUMBER OF BI-LAYERS (INTEGER) ==
!== ARGUMENT 5: LAY-UP OF THE COATING (1) OR (2) ==
!== ARGUMENT 6: AVERAGE THICKNESS OF LAYERS A (REAL) ==
!== ARGUMENT 7: AVERAGE THICKNESS OF LAYERS B (REAL) ==
!== ARGUMENT 8: INDENTER RADIUS (REAL) ==
!== ARGUMENT 9: FRICTION COEFFICIENT BETWEEN INDENTER AND TOP FILM (REAL, VALUE BETWEEN [0;1]) ==
!== ARGUMENT 10: THERMAL GRADIENT (SIMULATION OF RESIDUAL STRESS) BETWEEN TOP LAYER AND SUBSTRATE ==
!=====
/CLEAR
/CONFIG,NPROC,2
/TITLE, AXISYMMETRIC NANO-INDENTATION
/COM, ANALYSIS PARAMETERS
NMATER=9 ! NUMBER OF MATERIALS IN DATABASE
PI=3.14159265359
RIND=ARG8 ! RADIUS OF SPHERICAL INDENTER
*IF, ARG6, LE, ARG7, THEN
    LX=20.*ARG7 ! LENGTH OF SUBSTRATE
*ELSE
    LX=20.*ARG6
*ENDIF
LY=0.5*LX ! THICKNESS OF SUBSTRATE
ELEM DIVY1=3 ! DEFAULT NUMBER OF ELEMENTS ACROSS SMALLEST THICKNESS
ARFILM=10 ! ASPECT RATIO OF ELEMENTS TO MESH FILM
TGAP=0. ! GAP CLOSURE FOR ICONT OPTION
NSTEPS=20
*DIM, FORCEVERT, ARRAY, 1+NSTEPS ! DEFINE ARRAY TO STORE APPLIED FORCE
*DIM, DISPLVERT, ARRAY, 1+NSTEPS ! DEFINE ARRAY TO STORE RESULTING PENETRATION
*DO, I, 2, 1+NSTEPS/2
    FORCEVERT(I) = 10000.*(I-1)
*ENDDO
*DO, I, 2+NSTEPS/2, 1+NSTEPS
    FORCEVERT(I) = 100000.*(I-NSTEPS/2)
*ENDDO
/PREP7
/COM, MATERIALS DATA
*DIM, MATDATA, , NMATER, 4
*VREAD, MATDATA(1,1), DATAMAT, DAT, , JIK, 4, NMATER
(4F7.1)
MP, DENS, 1, MATDATA(ARG1,3)*1E-15 ! MATERIAL 1 DENSITY
MP, EX, 1, MATDATA(ARG1,1)*1.E6 ! MATERIAL 1 YOUNG'S MODULUS
MP, PRXY, 1, MATDATA(ARG1,2) ! MATERIAL 1 POISSON'S RATIO
MP, ALPX, 1, MATDATA(ARG1,4)*1.E-6 ! MATERIAL 1 CTE
!!!... INSERT OTHER MATERIALS HERE .....
MP, MU, 9, ARG9 ! SURFACE CONTACT PROPERTIES WITH INDENTER
TREF, 0
/COM, FINITE ELEMENT MODEL
ANTYPE, STATIC ! STATIC ANALYSIS
MSHAPE, 0, 2D ! MESH USING QUAD ELEMENTS
MSHKEY, 1 ! USE FREE MESHING
ET, 1, PLANE183, , , 1 ! PLANE183 (UX,UY) AXISYMMETRIC 2D QUAD SOLID P-ELEMENT
ET, 2, 169 ! TARGET SURFACE ELEMENT
ET, 3, 172, 0, , , 1 ! CONTACT SURFACE ELEMENT
*IF, ARG6, LE, ARG7, THEN
    ELEM DIVY2=NINT(ARG7*ELEM DIVY1/ARG6)
*ELSE
    ELEM DIVY2=ELEM DIVY1
    ELEM DIVY1=NINT(ARG6*ELEM DIVY2/ARG7)
*ENDIF
ELEM DIVX1=NINT(LX*ELEM DIVY1/(ARFILM*ARG6))
ELEM DIVX2=ELEM DIVX1
RECTNG, 0, LX, 0, LY ! DEFINE RECTANGLE OF SUBSTRATE, CYLINDER AFTER ROTATION
LESIZE, 1, , , ELEM DIVX1
LESIZE, 3, , , ELEM DIVX1
LESIZE, 2, , , ELEM DIVX1*2
LESIZE, 4, , , ELEM DIVX1*2
TYPE, 1 ! SELECT ELEMENT TYPE
AATT, 1 ! ASSIGN TYPE TO AREA
AMESH, 1 ! MESH AREA
HTOT=LY
KLSTART=3
*DO, INOD, 1, ARG4 ! DEFINE FILM STACKING
    HTOT=HTOT+ARG6
    K, (1+4*INOD), LX, HTOT
    K, (2+4*INOD), 0, HTOT
    HTOT=HTOT+ARG7
    K, (3+4*INOD), LX, HTOT
    K, (4+4*INOD), 0, HTOT
    L, KLSTART, (1+4*INOD)
    L, (1+4*INOD), (2+4*INOD)

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L, (2+4*INOD), (KLSTART+1)
LESIZE, 6*INOD-1, , , ELEM DIVY1 ! ASSIGN DIVISIONS THROUGH THICKNESS
LESIZE, 6*INOD, , , ELEM DIVX1 ! ASSIGN DIVISIONS ACROSS LENGTH
LESIZE, 6*INOD+1, , , ELEM DIVY1 ! ASSIGN DIVISIONS THROUGH THICKNESS
A, KLSTART, (1+4*INOD), (2+4*INOD), (1+KLSTART) ! ASSIGN AREA NUMBER
AATT, 2
AMESH, (2*INOD)
KLSTART=1+4*INOD
L, KLSTART, (3+4*INOD)
L, (3+4*INOD), (4+4*INOD)
L, (4+4*INOD), (KLSTART+1)
LESIZE, 6*INOD+2, , , ELEM DIVY2 ! ASSIGN DIVISIONS THROUGH THICKNESS
LESIZE, 6*INOD+3, , , ELEM DIVX2 ! ASSIGN DIVISIONS ACROSS LENGTH
LESIZE, 6*INOD+4, , , ELEM DIVY2 ! ASSIGN DIVISIONS THROUGH THICKNESS
A, KLSTART, (3+4*INOD), (4+4*INOD), (1+KLSTART) ! ASSIGN AREA NUMBER
AATT, 3
AMESH, (2*INOD+1)
KLSTART=3+4*INOD
*ENDDO
NSEL, S, LOC, Y, HTOT
NSEL, R, LOC, X, 0
*GET, NODENO, NODE, 0, NUM, MAX
HTOT=HTOT+RIND+TGAP ! CALCULATE HTOT FOR TOP OF GEOMETRY
K, 4*ARG4+5, 0, HTOT ! KEYPOINT CORNER CIRCULAR INDENTER
CIRCLE, 4*ARG4+5, RIND, , 6, 90, 1 ! DEFINE QUARTER CIRCLE INDENTER
L, 4*ARG4+5, 4*ARG4+6 ! CLOSE QUARTER CIRCLE, LEFT
L, 4*ARG4+7, 4*ARG4+5 ! CLOSE QUARTER CIRCLE, TOP
ELEM DIVCIRC=3*NINT(ELEM DIVX1*PI*RIND/(2*LX))
*IF, MOD(ELEM DIVCIRC, 2), EQ, 1, THEN
ELEM DIVCIRC=ELEM DIVCIRC+1
*ENDIF
LESIZE, 6*ARG4+5, , , ELEM DIVCIRC, 2 ! ASSIGN DIVISIONS ON 1/4 CIRCLE
LESIZE, 6*ARG4+6, , , ELEM DIVCIRC ! ASSIGN DIVISIONS VERTICAL EDGE
LESIZE, 6*ARG4+7, , , ELEM DIVCIRC ! ASSIGN DIVISIONS TOP EDGE
AL, 6*ARG4+5, 6*ARG4+7, 6*ARG4+6 ! ASSIGN AREA 4 THROUGH LINES
AATT, 3 ! ASSIGN DIAMOND PROPERTIES TO INDENTER
AMESH, 2+2*ARG4 ! MESH AREA 4
TYPE, 2 ! --- CONTACT AREAS ---// TARGET SURFACE
MAT, 9 ! FRICTION COEFFICIENT
LSEL, S, , , 6*ARG4+3 ! TOP SURFACE, TOP ELECTRODE
NSLL, S, 1 ! SELECT ALL NODES
ESURF ! ASSIGN TARGET TO SURFACE
TYPE, 3 ! CONTACT SURFACE
LSEL, S, , , 6*ARG4+5 ! QUARTER CIRCLE (TOP)
NSLL, S, 1 ! SELECT ALL NODES
ESURF ! ASSIGN CONTACT TO SURFACE
ALLSEL, ALL
NLGEOM, ON
SAVE, ICONT, db1
RESUME, ICONT, db1
/COM, BOUNDARY CONDITIONS:
NSEL, S, LOC, X, 0 ! DEFINE OY AS SYMMETRY AXIS (OX = 0)
D, ALL, UX, 0 ! ZERO HORIZONTAL DISPLACEMENT
NSEL, S, LOC, Y, 0 ! SELECT BOTTOM SURFACE
D, ALL, UY, 0
D, ALL, UX, 0
NSEL, ALL ! SELECT ALL NODES
BFUNIF, TEMP, AR10 ! APPLY TEMPERATURE ON ALL NODES
/SOLUTION ! EXIT PREP7, GET INTO SOLVER
SOLVE ! INITIAL TEMPERATURE STEP SOLVE
/POST1 ! EXIT SOLVER< ENTER POST
*GET, DVERTINI, NODE, NODENO, U, Y ! RETRIEVE VERTICAL RESIDUAL DISPLACEMENT
DISPLVERT(1)=ABS(DVERTINI)
FORCEVERT(1)=0.
/SOLUTION ! RE-ENTER SOLVER
*DO, ISTEP, 2, 1+NSTEPS
NSEL, S, LOC, Y, HTOT ! SELECT TOP SURFACE
D, ALL, UX, 0 ! HORIZONTAL DISPLACEMENT CONSTRAINED
F, ALL, FY, -FORCEVERT(ISTEP)/ELEM DIVX1 ! APPLY FORCE BOUNDARY CONDITION
NSEL, ALL
SOLVE ! SOLVE STATIC PROBLEM
NSEL, S, LOC, Y, HTOT
*GET, NNDS, NODE, , COUNT
ND=0
UAVE=0
*DO, I, 1, NNDS
ND=NDNEXT(ND)
*GET, ULOC, NODE, ND, U, Y
UAVE=UAVE+ULOC
*ENDDO
UAVE=UAVE/NNDS
DISPLVERT(ISTEP)=ABS(UAVE)
*ENDDO
*CFOPEN, LOADDISPL, DAT ! OPEN FILE LOADDISPL.DAT FOR WRITING
*VWRITE, DISPLVERT(1), FORCEVERT(1) ! WRITE RESULTS
(E12.5, ' ', E12.5)
*CFCLOSE ! CLOSE FILE LOADDISPL.DAT

```

5. Results

5.1. Number of layers and relative thickness

The first set of results that need to be examined is the influence of the number of layers and their thickness. Two sets of layer thickness were used: 500 nm/100 nm and 250 nm/50 nm. Load versus displacement curves for such $[\text{Ti}, \text{TiN}]_N$ bi-layers on steel, with N equal to 5 or 10, are shown in Figure 2. Firstly, all curves follow the trend given by Equation (4) upon adjustment of E_r , i.e. they are all approximated within less than 0.01% error by a power law of exponent $3/2$. This is not surprising, as a purely elastic response is calculated, which corresponds in all boundaries to the Hertzian case. No easily identifiable quantity, such as the composite modulus in bending or tension, could be extracted from the contact stiffness determined by fitting these curves to Eq. (4), though. It was found that the problem cannot be inverted easily, and resolving individual layer properties from a load-displacement curve need additional numerical regression to be coupled to the FE solution.

Secondly, the four plots displayed show a dominant influence of the bending stiffness of the hardest layer, TiN in this case: the closer to the substrate and the thicker the top layer, the stiffer the contact. This is shown in Fig. 3 as well, where the large variation in vertical stress σ_z occurs in the first Ti layer: the stiffer TiN layer above it buffers the indenter's vertical stress by spreading it away from the contact zone. In Fig. 3, this is illustrated by the zone of maximum stress extending to about 5 times the area of contact.

Other specific features of this model, not discussed here, include the stress transfer singularity at the interfaces between layers: above three elements in the thickness of the layers, it can be seen on, e.g. a plot of shear stress against the vertical axis, which shows large discontinuities between layers.

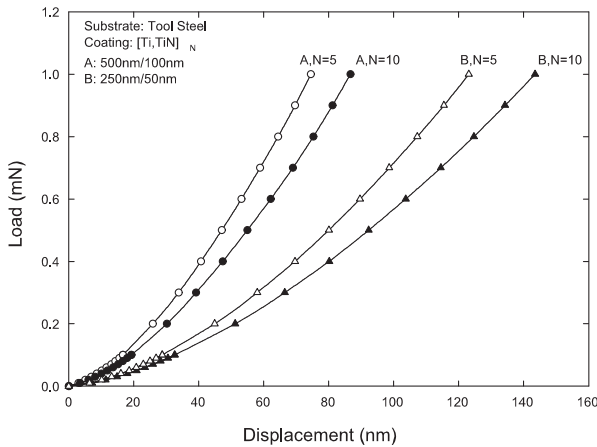


Figure 2: Load-displacement curves for various number of layers of Ti,TiN bi-layers, as indicated on the Figure

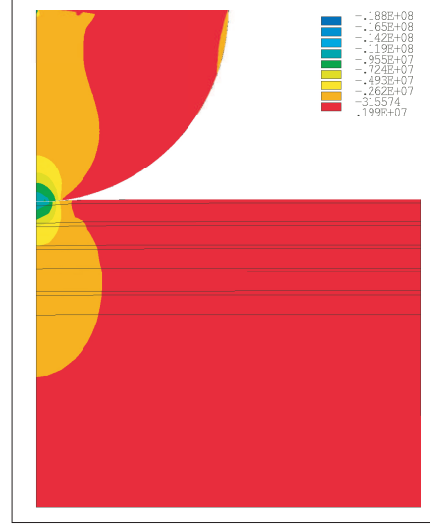


Figure 3: Vertical stress distribution in a $[\text{Ti}, \text{TiN}]_5$ system on a steel substrate

This type of problem, involving more than one million elements at least, was deemed too resource intensive for this study, where the main objective remains the characterisation of layers and composite behaviour, rather than their interfaces.

5.2. Effects of friction

Next, it was decided to examine the effects of friction between the indenter and the top layer on the stress response. The reason is associated with the experimental uncertainty arising when the indenter is pushed inside a surface: up to now, it has not been possible to measure the contact area while the indenter is in contact, and therefore contact models need to be used in order to approximate the contact behaviour. Figure 4 displays

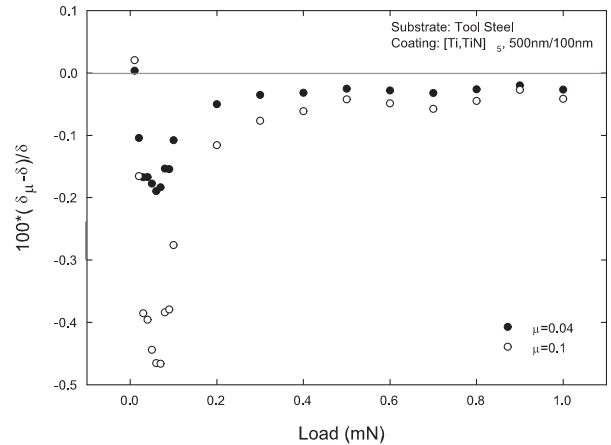


Figure 4: Relative difference of displacement obtained for various friction coefficients μ

the relative difference between displacements obtained with and without friction, that is $(\delta_\mu - \delta)/\delta$. The friction coefficients of 0.04 and 0.1 were chosen because the former was experimentally determined as static friction in a bending experiment [4] with similar surfaces, and the latter is close to values obtained for dynamic friction from nano-scratch tests. Surprisingly, it was not possible to distinguish between load versus displacement curves, so that the type of representation of Fig. 4 was preferred instead. The maximum difference, for $\mu = 0.04$, is never greater than 0.2%, and never exceeds 0.6% for $\mu = 0.1$. These simulations therefore indicate that friction between the indenter and the top layer has a negligible effect on the load-displacement curve, well within the experimental error of such measurement. Consequently, when the top layer is a smooth metal or ceramic, friction can be neglected.

Apart from that important fact, it is interesting to notice that the relative difference increases sharply (in an absolute value sense) during the early loading stages, which correspond to the largest increase in contact area, and then stabilises when the contact area becomes almost constant.

5.3. Effects of residual stresses

Last, but not least, the effects of residual stresses were examined. It is well known, in polymer-based multi-layered systems, that residual stresses can change not only the shape of the composite, but also the internal stress state, to the point that internal damage can be sustained during demoulding of structures. During the past ten years, conclusive measurements of residual stresses, and in particular the internal stress level in thin layers, have been obtained for various material combinations, and shown to reach up to several GPa for diamond-like coatings. The point is, even though the internal stress state is essentially tensile within the layers, it is enough to have a large influence on the through-thickness component of stress.

It is not surprising, therefore, to witness a large shift in the load-displacement curves when a temperature difference is applied first, prior to loading of the structure by the spherical indenter (Fig. 5). This effectively generates a change of the structural response of the coating, which is in effect pre-loaded. In the present case, because the coefficient of thermal expansion of the coating layers is smaller than that of the substrate, when the material is cooled-down the coating is put in tension, with a stiffness smaller than would be obtained by indentation only. The effect of residual stresses is a decrease of the secant stiffness, particularly in the early contact stages. Notice as well that the tangent stiffness, changed by almost a factor ten at low loads, tends to be similar when high enough loads, in the order of 0.5 mN, are reached.

To conclude, the effect of internal stresses, which changes the coated material's response enormously, cannot be neglected when analysing indentation results. Sim-

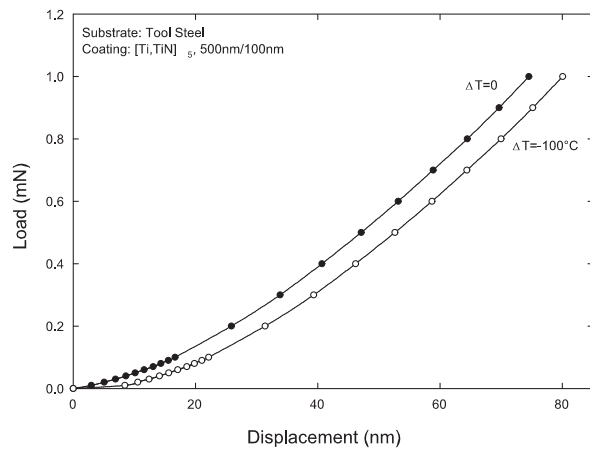


Figure 5: Load-displacement curves with or without residual stresses, as indicated by the stress free temperature on the Figure

ilarly, it is recommended that residual stresses are measured independently, e.g. by means of transmission electron microscopy of ion-milled sections.

6. Future Work

To summarise the arguments above, it was concluded that large variations in the elastic indentation behaviour were obtained when the number and relative thickness of the coating layers were changed, and also when the residual stress state was accounted for. The effects of friction between the indenter and the top layer, within a reasonable range restricted to hard coatings and diamond indenter, were found to be negligible.

As a result, we claim that the behaviour of the composite under indentation can be determined by carefully specifying the dimensions of the problem, the thermo-elastic properties, and the stress free temperature. Stemming from these assumptions, finding a closed-form solution to the problem is easier, and although no such working has been proposed comprehensively, the following approach is relevant. Starting from the works of Shield and Boggy, Green's tensor describing continuity of stress and displacement between layers is determined [9]. This approach makes use of Hankel transforms and is followed by a cascade elimination, so that all applied boundary conditions (top layer and substrate) are linked in the same equation. Shield and Boggy's approach was for a flat, cylindrical indenter. A similar argument has been used to solve the spherical indentation problem by Aizikovich et al. [10], who obtain an analytical solution for a graded half space, that can be extended to a coated half-space. This approach, leading to an integral equation, could not be solved, thus being of limited use for solving experimental contact problems.

Very recently, however, Perriot and Barthel have re-formulated the analysis to enable the problem to be solved at low numerical cost [11]. It is recommended that this latter approach is followed, after being completed by including the residual stress term.

7. Conclusions

In this work, finite element solutions for the elastic indentation of a multi-layered coated half-plane have been devised and used to determine the effects of the number of layers, of the friction between the indenter and the top layer of the coating, and of the residual stress state. The inputs to the FE model were reduced so that the user needs only concentrate on the thermo-elastic properties of the layers and the substrate, the exact thickness of the layers, and the stress free temperature. It was further shown that friction could be neglected, whereas residual stresses influence to a great extent the elastic response of the coated system. An alternative approach, based on recent semi-analytical solutions to the stress state and the penetration of the indenter, was suggested.

8. Acknowledgements

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