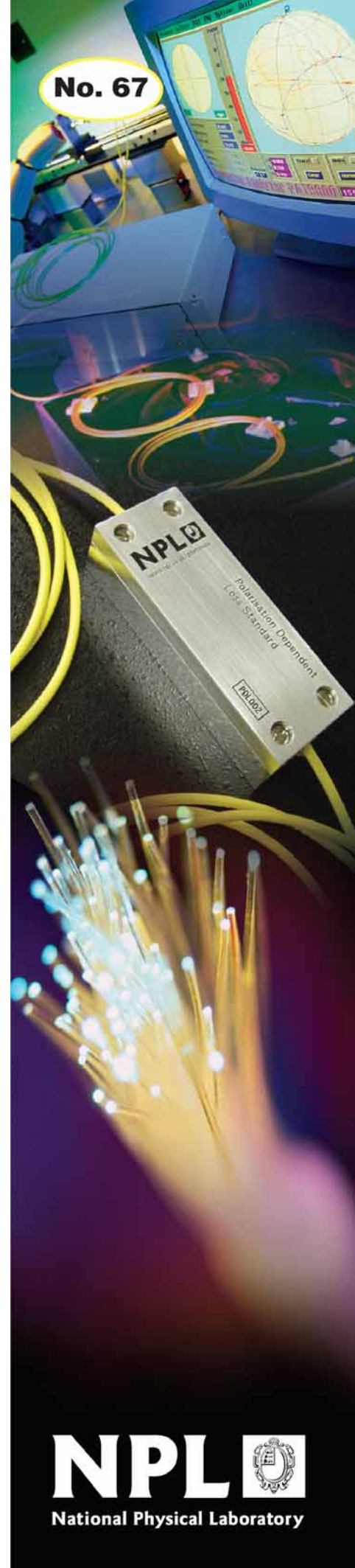


Measurement Good Practice Guide

Polarisation Effects and Measurements in Optical Fibre Systems

David Ives

No. 67



NPL 
National Physical Laboratory

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Abstract: This document has been written to give guidance and understanding to the array of polarisation properties and effects found within optical fibres, optical fibre components and optical fibre systems. The various polarisation properties and effects are described along with measurement and calibration techniques.

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Approved on behalf of Managing Director, NPL
by S Pollitt, Director of Enabling Metrology Division

Polarisation Effects and Measurements in Optical Fibre Systems

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1. Polarised Light

1.1 Polarised Light

The solution of the electro-magnetic wave equation shows light to be an oscillating electric and magnetic field. For a propagating wave where the electric field vector has a defined orientation the light is said to be polarised^{[1][2]}. The simplest case is where the electric field vector is confined to a plane containing the direction of propagation. This is known as linear polarised light, see Figure 1.

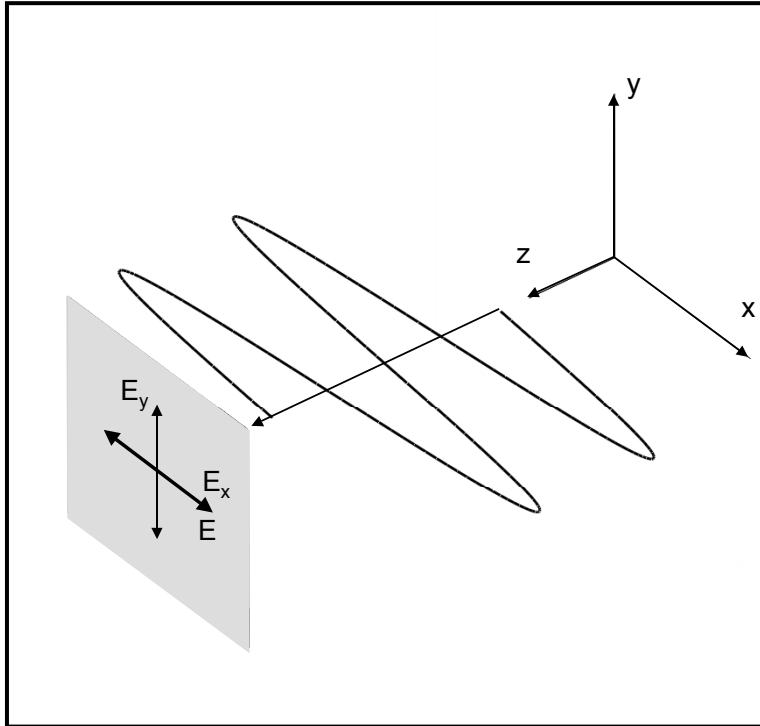


Figure 1 Electric Field Vector evolution for linear horizontal polarised light.

In contrast, natural light has no preferred orientation of the electric field and is said to be unpolarised. An example is light emitted from a hot thermal source where each photon has its electric field orientated differently. The degree of polarisation is defined as the polarised intensity divided by the total intensity and is 1 for polarised light and 0 for natural unpolarised light.

For the solution of the electro-magnetic wave equation in x, y and z space, where the wave is propagating in the z direction, we can consider the components of the electric field aligned with either the x axis or y axis. This describes two orthogonal states of polarisation, the linear horizontal and linear vertical states of polarisation. By combining two orthogonal polarised waves the resultant wave can form any state of polarisation depending on the exact amplitude and phase between the two waves. Thus by combining linear horizontal and linear vertical polarised light with the same amplitude and phase a linear polarised wave is produced with the electric field orientated at 45°, see Figure 2, and by combining a linear horizontal and linear vertical polarised light with the same amplitude and $\pi/2$ phase difference a circular polarised wave results, see Figure 3. In a circular polarised wave the electric field vector rotates about the z-axis over a wave period with constant amplitude where as in a linear polarised wave the electric field remains in the plane containing the propagation direction and

the amplitude varies sinusoidally during a wave period. For other combinations an elliptical polarised wave results where the amplitude and angle of the electric field vector evolve during the wave period.

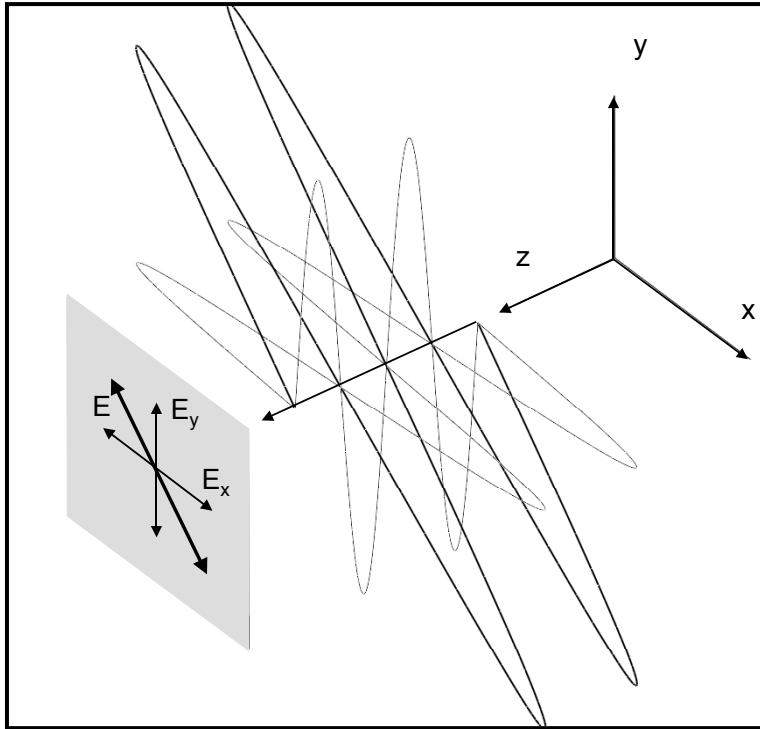


Figure 2 Electric field components E_x and E_y combine in phase to give a linear polarisation state at 45° .

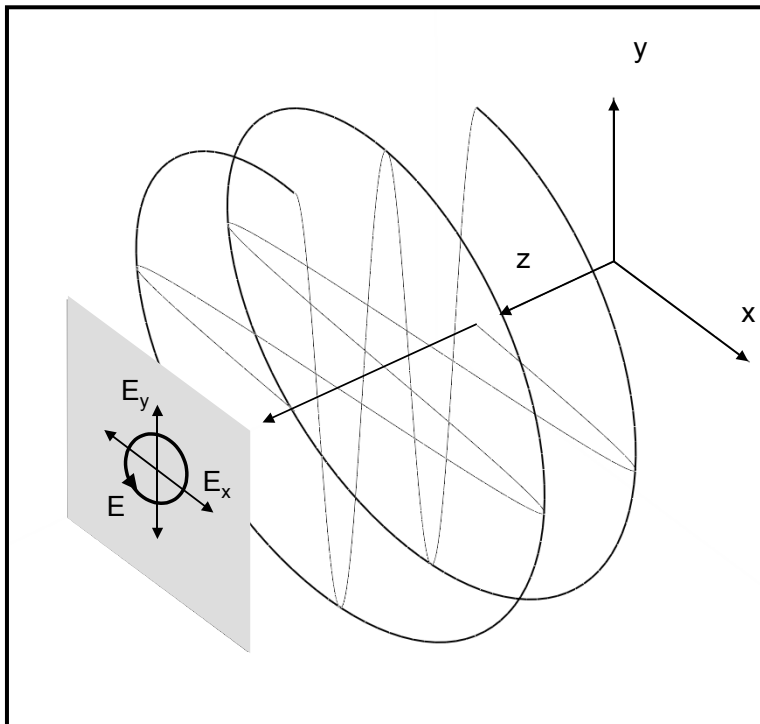


Figure 3 Electric field components E_x and E_y combine in quadrature to give a left hand circular polarisation state.

1.2 Polarisers

Unpolarised or natural light can be polarised by passing it through a polariser. This allows the transmission of one state of polarisation while absorbing or reflecting the other. For an unpolarised light wave this results in only half the power being transmitted. For the case of a linear polariser the resulting output wave will be linearly polarised. If this wave is incident on a second linear polariser the amount of transmitted light depends on the relative orientation of the two polarisation axes. The second polariser can be thought to pass the electric field component orientated parallel to its polarisation axis.

$$I(\theta) = [E_0 \cos(\theta)]^2 = I(0) \cos^2(\theta) \quad (1)$$

where $I(0)$ is the intensity of the input wave and θ the angle between the axis of the two linear polarisers, this assumes ideal lossless linear polarisers. A second polariser used to determine the component of a beam aligned with a state of polarisation is often referred to as a polarisation analyser.

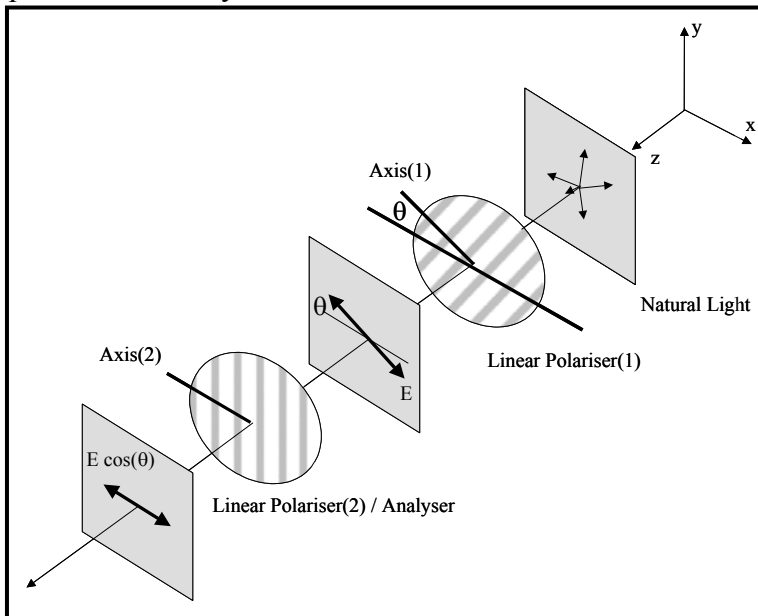


Figure 4 Effect of two linear polarisers crossed at an angle θ .

Polarisers can be made from dichroic materials that absorb one polarisation state in preference to its orthogonal state. This is true of the wire grid type polariser where a grid of fine wires allows the polarisation state with its electric field perpendicular to the wires to be transmitted while absorbing the polarisation state where the electric field is parallel to the wires. Wire grid polarisers can be made using a grid of fine wires for long wavelengths but for visible and near infrared optical radiation a similar effect exists in long conducting crystals and in Polaroid sheet.

Polarised light can also be formed from reflections from dielectric surfaces at Brewster's angle. At Brewster's angle the light polarised with its electric field vector in the plane of incidence (the plane that contains the direction of propagation and the normal to the surface) is not reflected. This leads the reflected beam to be fully polarised with the electric field vector perpendicular to the plane of incidence. In non-magnetic materials it can be considered that light polarised with its electric field in the plane of incidence is not reflected since the electric field of the transmitted beam would be in the direction of propagation, where no

electromagnetic propagation is supported. At boundaries where the magnetic permeability is constant Brewster's angle, θ_p , is given by

$$\tan(\theta_p) = \frac{n_t}{n_i} \quad (2)$$

where n_i is the refractive index of the incident medium and n_t the refractive index of the transmitting medium. This effect is usefully used in Polaroid sunglasses to block the sparkling reflection from water.

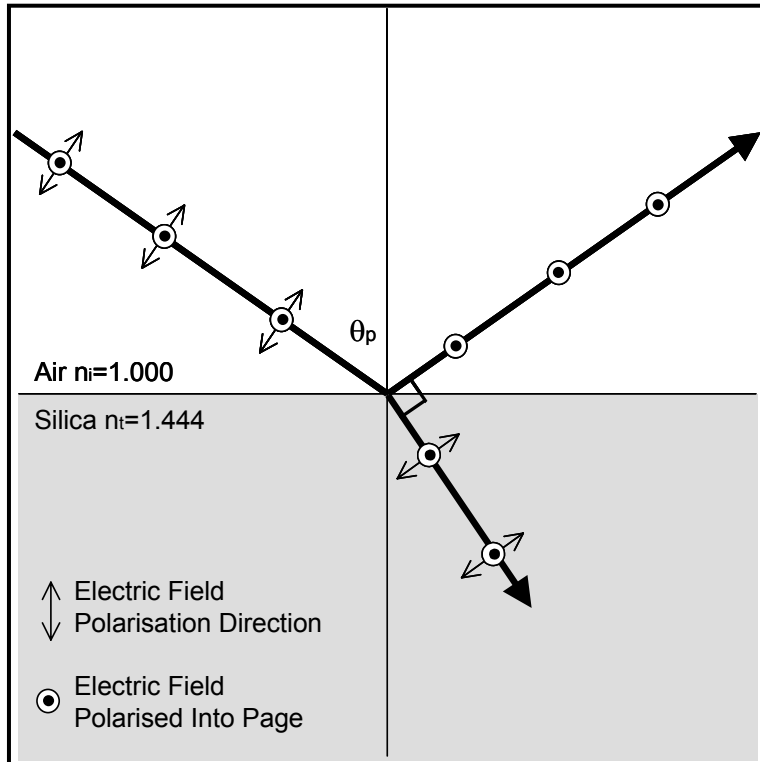


Figure 5 Light reflected at Brewster's angle from a surface is fully polarised with the electric field polarised perpendicular to the plane of incidence.

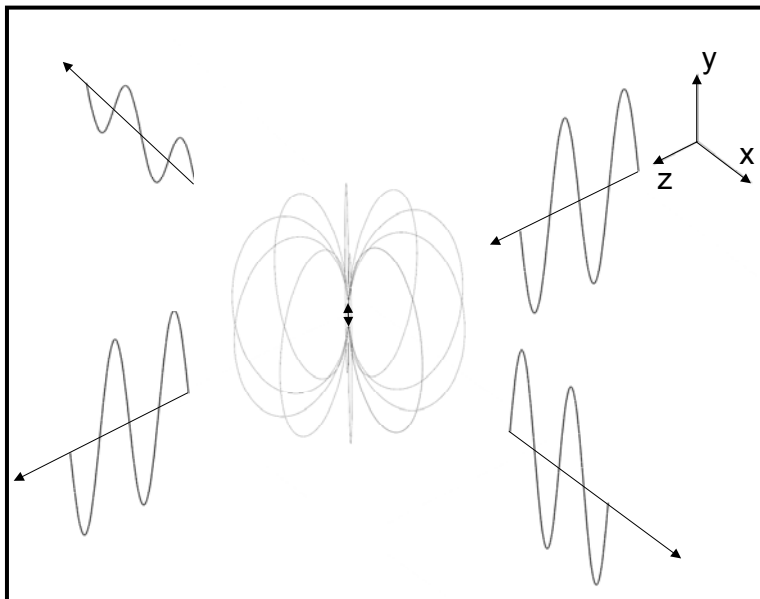


Figure 6 Scattering of polarised light by a small dipole.

Scattered light can also be strongly polarised. Scattering from small particles and molecules forms a dipole radiation pattern with zero scattering in the direction parallel to the incident electric field. Thus if we look at light scattering at right angles to the incident beam there will be partial polarisation with the preferred electric field perpendicular to the incident and scattered directions.

The highest quality linear polarisers are produced from birefringent crystals arranged to spatially separate the two polarisation states. This can be as a result of different refracting angles or by selectively reflecting one polarisation near Brewster's angle, see Figure 7.

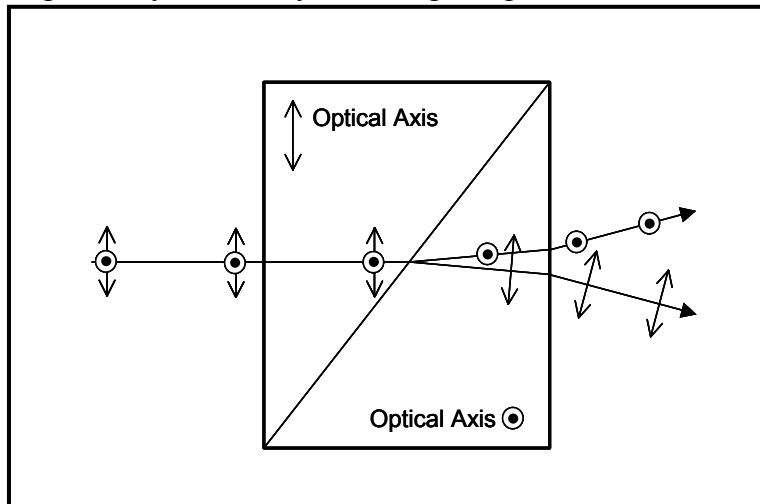


Figure 7 Wollaston prism acting as a polarising beam splitter.

1.3 Birefringence in Crystalline Materials

Many crystalline materials are not optically isotropic; this is due to an anisotropy in their crystal structure. The anisotropy in the crystal structure leads to an anisotropy in the dielectric polarisability that in turn leads to an anisotropy in the refractive index. A material that displays two different indices of refraction is said to be birefringent. The simplest form is a uniaxial crystal, for example Calcite and Quartz, where the crystalline structure has a single axis of symmetry. The electric polarisability of the crystal is different for electric fields parallel and electric fields perpendicular to this axis of symmetry, known as the optical axis. As a result the refractive index for electromagnetic waves with their electric field parallel and electromagnetic waves with their electric field perpendicular to this optical axis are different. In the simplest case where the optical axis is perpendicular to the propagating beam, the input optical beam is split into two, that with its electric field perpendicular to the optical axis and that with its electric field parallel to the optical axis, these two beams, known as the ordinary and extraordinary rays propagate at different velocities. This leads to a change in polarisation state of the beam by changing the relative optical phase between the two beam components.

If the optical axis is not perpendicular to the direction of propagation then the electric field component that is parallel to the optical axis will experience a different polarisability to the electric field component perpendicular to the optical axis. This will cause the beam propagation direction to be deviated. The effect is observed in calcite as a double image where the ordinary and extraordinary images become separated, see Figure 8.

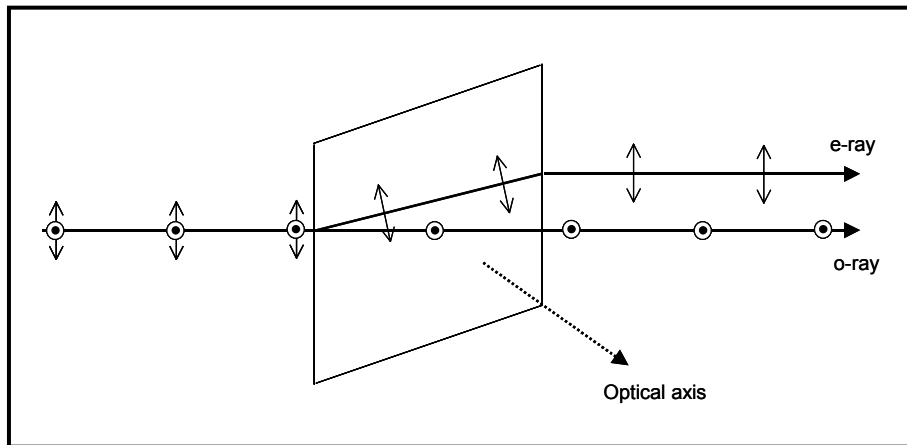


Figure 8 Double images in Calcite

In isotropic media, isotropic crystals and glasses there is no inherent birefringence as there is no asymmetry. External influences such as stress can induce a birefringence through the stress optic effect. This effect is important to polarisation effects in optical fibres.

1.4 Waveplates

Birefringent crystals cut such that their optical axis lies in the plane of the surface are used to form waveplates. An electromagnetic wave propagating perpendicular to the surface will be split into two components, one with its electric field parallel to the optical axis and one with the electric field perpendicular to the optical axis. These two components will propagate at different velocities giving an optical phase shift between them. This leads to a change in the state of polarisation of the emerging wave.

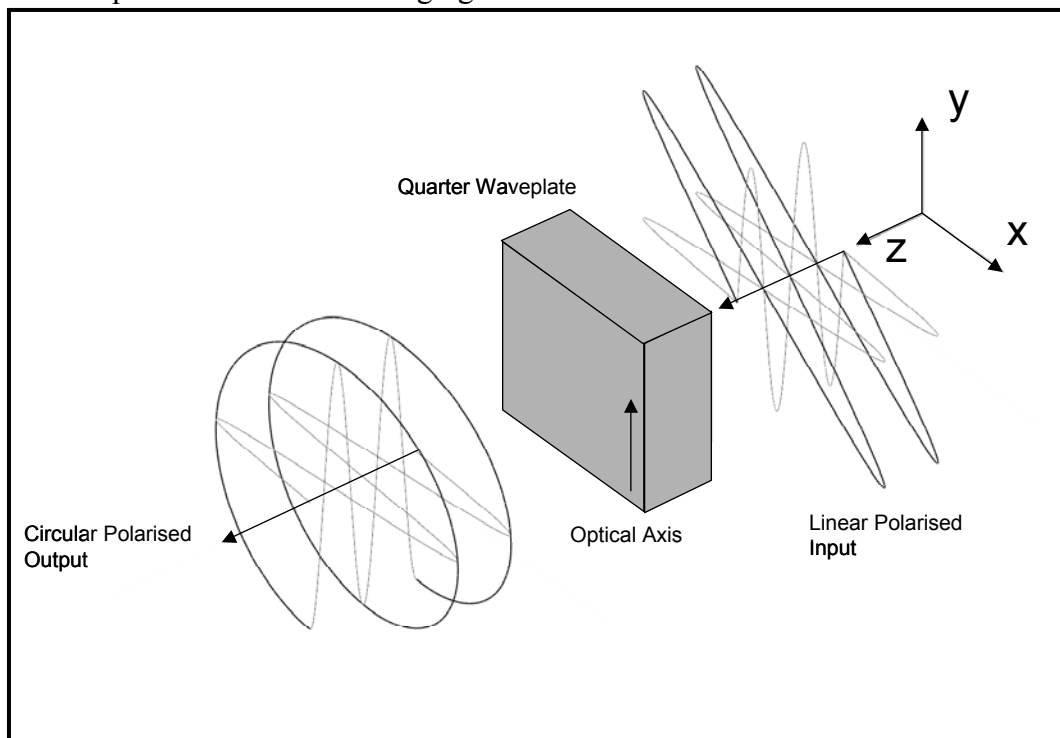


Figure 9 A quarter waveplate converts linear polarised light to circular polarised light when the input polarisation is at 45 degrees to the optical axis.

For example if a linear polarised wave is incident on a quarter wave retarder (that is a crystal plate where the phase retardance between the fast and slow axis is one quarter wave) at 45° to the birefringent axis then the emerging wave will have a circular state of polarisation, see Figure 9. For other retardances and angles of the retardance axis, the incident wave is decomposed into two orthogonal component waves that are aligned with the fast and slow axes of the crystal. The emerging waves are recombined taking into account their amplitudes and new phases to obtain the output state of polarisation. In general this will be an elliptical state of polarisation.

A quarter waveplate made from birefringent material is tuned to the operating wavelength. The thickness of the plate is adjusted so that the difference in phase velocity causes a phase shift of $\pi/2$ radians. In quartz the thickness required is extremely thin and so many quarter waveplates are either multi order that is the phase shift is equal to $(2n+1/2)\pi$ radians or are composed of two plates arranged with there fast and slow axis perpendicular such that the combined phase shift is only $\pi/2$ radians. Multi order waveplates show a stronger wavelength dependence. Some waveplates are produced from multiple layers of plastic film such that the various dispersions of each layer cancel giving a $\pi/2$ radians phase shift over a wide wavelength range.

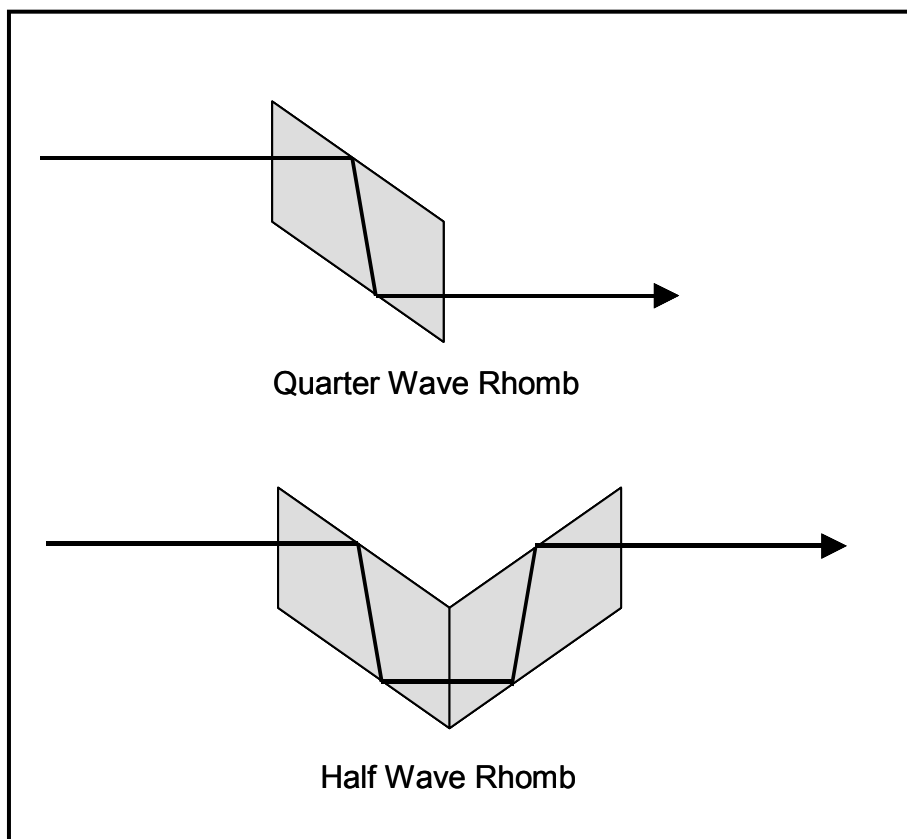


Figure 10 Fresnel Rhombs as achromatic retarders

It is also possible to create achromatic quarter waveplates based on the phase change on reflection in a Fresnel Rhomb. The angles are chosen such that each reflection gives a $\pi/4$ radian phase change between the two polarisation components. Such Rhombs are achromatic except for material dispersion. For a glass air interface the glass must have a refractive index greater than 1.497 to obtain this phase difference. This is possible for BK7 for an angle of incidence of 53.4° at 1550 nm.

1.5 Optical Activity and Circular Birefringence

Crystals that have a twist or spiral structure exhibit optical activity. The plane of polarisation of a linearly polarised optical input rotates as it passes through the material. Crystalline quartz is optically active for waves propagating along the optical axis. Also solutions of many organic substances show optical activity for example a solution of sugar. In such systems an input linearly polarised beam is split into its left and right hand circular polarised components and it is these states that propagate at differing velocities. When the left and right hand circular polarised components are recombined they form a linear polarised output and the phase shift between them determines azimuth of the linear polarisation. Since the difference in propagation velocities is between left and right handed circular states the phenomena is sometimes known as circular birefringence.

1.6 Electro-optic and Magneto-optic effects

In a number of crystalline materials an external static electric or magnetic field can modify the refractive index properties. There are a number of different effects.

The Kerr effect is observed in an isotropic medium where the electric field causes the medium to become birefringent with the optical axis aligned with the applied electric field. The birefringence $\delta\beta_1$ in rads.m^{-1} is given by

$$\delta\beta_1 = 2\pi B_K E^2 \quad (3)$$

where B_K is the Kerr constant and E the electric field strength. It should be noted that in the Kerr effect the birefringence is proportional to the square of the electric field.

The Pockels effect occurs in crystals that do not have a centre of symmetry. The electric field modifies the refractive index through the electro-optic tensor. With crystals such as KDP the effect is through a cross term in the tensor such that for the applied electric field along the crystals z -axis, the birefringence occurs along an axis at right angles in the x - y plane. This allows the applied electric field to be in the direction of propagation of the optical beam or perpendicular to the optical beam. However with crystals such as LiNbO_3 an applied electric field along the crystal z -axis directly affects the refractive index for optical beam with their electric field polarised parallel to the z -axis. The birefringence is changed with the optical axis parallel to the z -axis.

In the case of KDP and similar $\bar{4}2m$ crystal structures the change in birefringence is given by

$$\delta\beta_1 = \frac{2\pi}{\lambda} n_0^3 r_{63} E \quad (4)$$

where r_{63} is the element from the electro-optic tensor and E is the electric field applied along the crystals z -axis. The birefringence change has an optical axis in the x - y plane

In the case of LiNbO_3 and similar $3m$ crystal structures the change in birefringence is given by

$$\delta\beta_1 = \frac{2\pi}{\lambda} \frac{1}{2} E [r_{33} n_e^3 - r_{13} n_o^3] \quad (5)$$

where r_{ij} are the elements from the electro-optic tensor and E is the electric field applied along the crystals z -axis. The birefringence change has an optical axis along the crystal z – axis.

It should be noted that with these effects the change in birefringence is proportional to the electric field.

The Faraday effect occurs in materials when a strong magnetic field is applied in the direction of propagation of the optical beam. The magnetic field causes a circular birefringence, a velocity change between left and right circular polarised light. This causes the plane of polarisation of a linear polarised beam to rotate as it propagates. The direction of rotation of the plane of polarisation depends on whether the optical beam is propagating in the direction of the magnetic field or against the direction of the magnetic field. A circular birefringence $\delta\beta_C$ rads.m^{-1} is induced, given by

$$\delta\beta_C = 2 V_V H \quad (6)$$

where V_V is the Verdet constant.

The electro-optic effect is used to produce optical modulators and polarisation controllers. The Faraday effect is used to great effect in optical isolators that only allow light to propagate in the forward direction.

1.7 Effect on Reflection and Transmission

Light impinging on a dielectric surface is reflected from the surfaces and transmitted into the bulk. The reflection coefficient is a function of the angle of incidence, polarisation of the input light and the dielectric medium. For a lossless medium the total power remains unchanged.

The two polarisation states that are considered are those with the electric field vector in the plane of incidence, $E_{\parallel}$, and that with the electric field vector perpendicular to the plane of incidence, $E_{\perp}$. The plane of incidence is the plane containing the normal to the surface and the direction of propagation of the incident beam, see Figure 11.

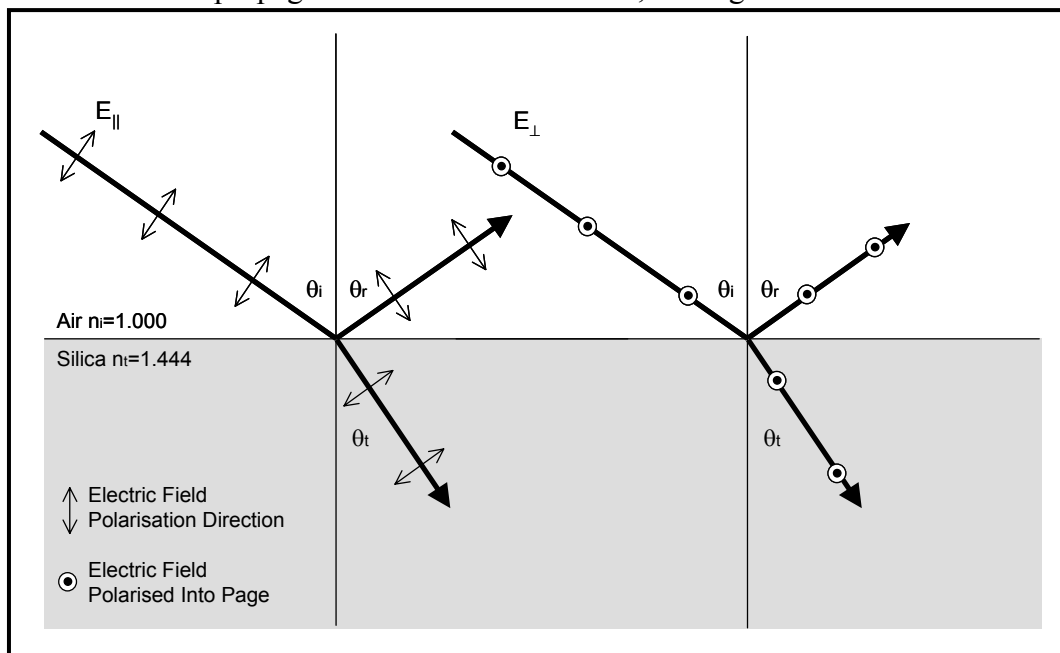


Figure 11 Definition of $E_{\parallel}$ and $E_{\perp}$. The plane of incidence is the plane of the paper.

For normal incidence on an isotropic medium the reflection coefficient for the two polarisation states is identical. However there is a difference in the optical phase shift between the two beams of π radians. This is because the perpendicular polarised beam, $E_{\perp}$, will experience an additional phase shift of π radians because the direction of the beam is reversed thus to the observer a field pointing to the left as looking into the beam will be pointing to the right after reflection. This is most significant when a circular polarised beam impinges on the surface as the reflected beam has the opposite handedness.

Solving the boundary conditions at the surface we can obtain the amplitude reflection coefficient for the electric field parallel and perpendicular to the plane of incidence.

$$r_{\parallel} = \left(\frac{E_r}{E_i} \right)_{\parallel} = \frac{\frac{n_t}{\mu_t} \cos(\theta_i) - \frac{n_i}{\mu_i} \cos(\theta_t)}{\frac{n_t}{\mu_t} \cos(\theta_i) + \frac{n_i}{\mu_i} \cos(\theta_t)} \quad (7)$$

$$r_{\perp} = \left(\frac{E_r}{E_i} \right)_{\perp} = \frac{\frac{n_i}{\mu_i} \cos(\theta_i) - \frac{n_t}{\mu_t} \cos(\theta_t)}{\frac{n_i}{\mu_i} \cos(\theta_i) + \frac{n_t}{\mu_t} \cos(\theta_t)} \quad (8)$$

where in most dielectric materials $\mu_i = \mu_t = 1$.

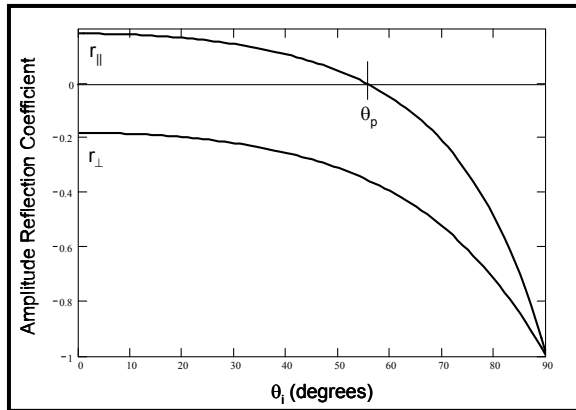


Figure 12 Reflection coefficient from an air – silica ($n=1.444$) surface.

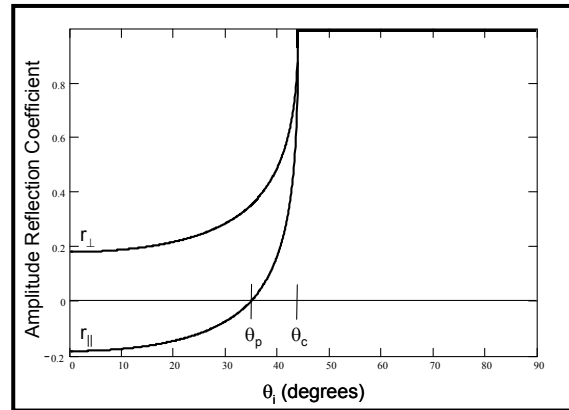


Figure 13 Reflection coefficient from a silica ($n=1.444$) - air surface.

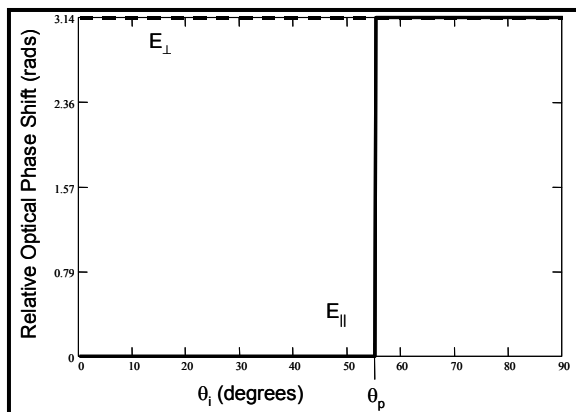


Figure 14 Reflection relative optical phase shift from an air - silica ($n=1.444$) surface.

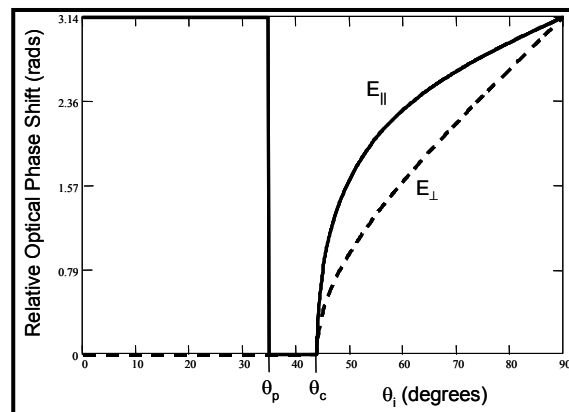


Figure 15 Reflection relative optical phase shift from a silica ($n=1.444$) - air surface.

The optical phase change on reflection is given by

$$\tan\left(\frac{\delta_{\parallel}}{2}\right) = \frac{n_i \sqrt{n_i^2 \sin^2(\theta_i) - n_t^2}}{n_t^2 \cos(\theta_i)} \quad (9)$$

$$\tan\left(\frac{\delta_{\perp}}{2}\right) = \frac{\sqrt{n_i^2 \sin^2(\theta_i) - n_t^2}}{n_i \cos(\theta_i)} \quad (10)$$

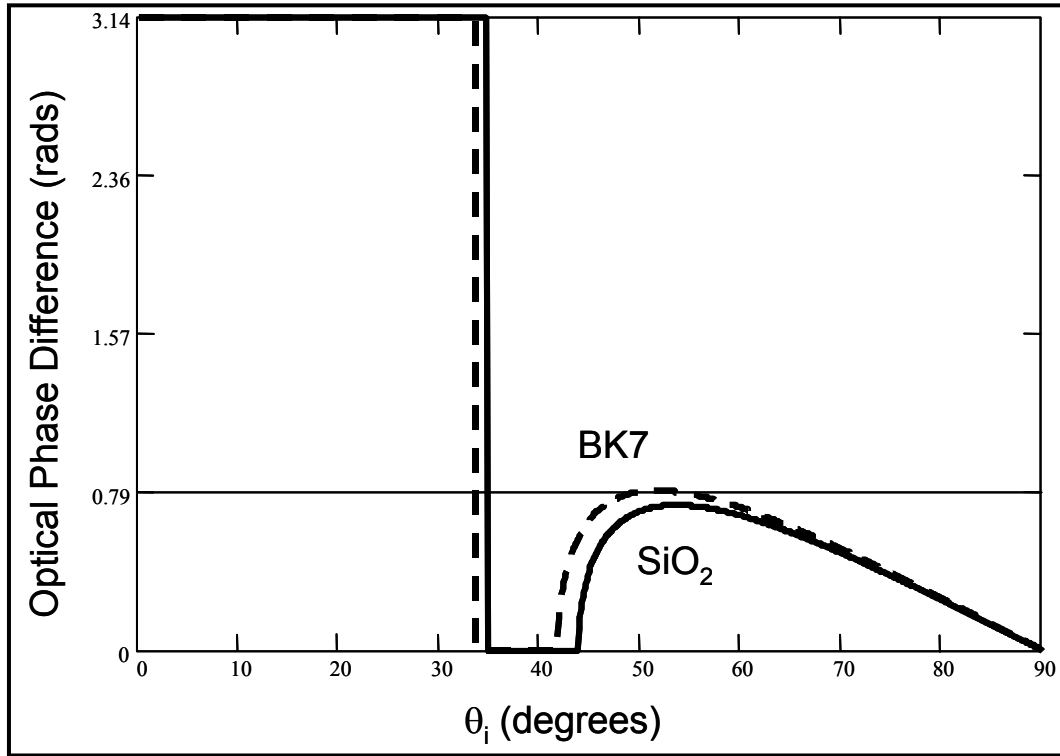


Figure 16 The optical phase difference between $\parallel$ and $\perp$ polarised beams on internal reflection.

Reflections from Metallic surfaces show similar curves to dielectric media. The reflection coefficient for light polarised perpendicular to the plane of incidence increases to one as the angle of incidence increases to 90° while the reflection coefficient for light polarised parallel to the plane of incidence decreases to a minima at the principle angle of incidence (equivalent to Brewster's angle) and then increases to one as the angle of incidence increases to 90° .

2. Mathematical Descriptions of Polarised Light

2.1 Jones Vector and Jones Matrixes

As stated previously the polarisation state of an optical wave can be described by the superposition of two orthogonally polarised waves with differing amplitudes and phases. Jones Vectors^[3] provide a two-element vector to describe the amplitude and phase of the electric field in the two basis polarisation states. The electric field is described by

$$\begin{pmatrix} a_1 \\ a_2 \end{pmatrix}_{12} e^{i\omega t} = [a_1|1\rangle + a_2|2\rangle] e^{i\omega t} \quad (11)$$

where $|1\rangle$ and $|2\rangle$ are the two basis polarisation states such that $\langle 1|1\rangle = \langle 2|2\rangle = 1$ and $\langle 1|2\rangle = \langle 2|1\rangle = 0$, the orthogonality condition and a and b are complex values giving the amplitude and phase of the component of the wave in each of the basis states. The vector is normalised such that $a_1 a_1^* + a_2 a_2^* = 1$, where the $*$ denotes the complex conjugate.

It is usual to take the basis states as linear polarised along x and y ie the linear horizontal and linear vertical polarised states. Thus a linear vertical polarised wave can be represented by

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad (12)$$

The most general elliptical polarised state and its orthogonal state are given by

$$\begin{pmatrix} \cos(v) e^{i\frac{\Delta}{2}} \\ \sin(v) e^{-i\frac{\Delta}{2}} \end{pmatrix} \text{ and } \begin{pmatrix} -\sin(v) e^{i\frac{\Delta}{2}} \\ \cos(v) e^{-i\frac{\Delta}{2}} \end{pmatrix} \quad (13)$$

where v is the azimuth angle of the major axis of the ellipse and Δ is the phase difference between the linear vertical and linear horizontal electric field components, $\Delta=0$ describes linear polarised states. For examples of common states of polarisation see Table 1.

The Jones Matrix describes the transmission of an optical element and the effect on the Jones Vector. The optical element is described by a 2×2 matrix, M , such that the output polarisation state E_O is related to the input polarisation state E_I by

$$E_O = M E_I \quad (14)$$

The matrix, M , can be used to describe polarisers and birefringent elements. The rotation matrix $R(\theta)$ can be used to rotate elements such that an optical element M at an azimuth angle θ is given by

$$M(\theta) = R(-\theta) M R(\theta) \quad (15)$$

where

$$R(\theta) = \begin{pmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{pmatrix} \quad (16)$$

For example the effect of a quarter waveplate at 45° to a linear polarised wave is given below

$$E_O = R(-45^\circ) M R(45^\circ) E_I = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} e^{i\frac{\pi}{4}} & 0 \\ 0 & e^{-i\frac{\pi}{4}} \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ i \end{pmatrix} \quad (17)$$

ie a right hand circular output polarisation. For a wave with polarisation state E_I incident on a number of optical elements $M_1, M_2 \dots M_N$ the output polarisation state E_O is given by

$$E_O = M_N \dots M_2 M_1 E_I \quad (18)$$

For examples of Jones Matrices for common optical elements see Table 2.

2.2 Stokes Vector and Mueller Matrixes

While Jones vectors describe the electric field of a polarised light wave the Stokes vector^[3] describes the intensity and can be directly measured. Stokes vectors are able to describe unpolarised light but cannot describe coherent combinations of optical waves. The Stokes vector is a four element vector where the elements describe the intensity of the beam, the difference in intensity for the beam passed through a horizontal linear polariser and a vertical linear polariser, the difference in intensity for the beam passed through a linear polariser at +45° and a linear polariser at -45° and the difference in intensity for the beam passed through a right hand circular linear polariser and a left hand circular polariser.

$$S = \begin{pmatrix} s_0 \\ s_1 \\ s_2 \\ s_3 \end{pmatrix} = \begin{pmatrix} I_0 \\ I_H - I_V \\ I_+ - I_- \\ I_R - I_L \end{pmatrix} \quad (19)$$

where I denotes the intensity of the wave passed through various analysers. H = linear horizontal polarisation state, V = linear vertical polarisation state, + = linear polarisation state at +45°, - = linear polarisation state at -45°, R = right hand circular polarisation state and L = left hand circular polarisation state. Thus a horizontal linear polarised state of unity intensity is described by

$$S = \begin{pmatrix} s_0 \\ s_1 \\ s_2 \\ s_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix} \quad (20)$$

The most general elliptical polarised state and its orthogonal state are given by

$$\begin{pmatrix} 1 \\ \cos(2v) \\ \sin(2v) \cos(\Delta) \\ \sin(2v) \sin(\Delta) \end{pmatrix} \text{ and } \begin{pmatrix} 1 \\ -\cos(2v) \\ -\sin(2v) \cos(\Delta) \\ -\sin(2v) \sin(\Delta) \end{pmatrix} \quad (21)$$

For examples of common states of polarisation see Table 1.

It is common to use the normalised Stokes vector where the s_0 term is dropped, and is taken to be 1, leaving a three element vector.

The degree of polarisation, DOP, is given by

$$DOP = \frac{I_{Pol}}{I_{Tot}} = \frac{\sqrt{s_1^2 + s_2^2 + s_3^2}}{s_0} \quad (22)$$

where I_{Pol} is the polarised intensity and I_{Tot} is the total intensity.

The Mueller Matrix describes the transmission of an optical element and the effect on the Stokes Vector. The optical element is described by a 4 x 4 matrix, M, such that the output polarisation state S_O is related to the input polarisation state S_I by

$$S_O = M S_I \quad (23)$$

The matrix, M can be used to describe polarisers and birefringent elements. The rotation matrix $R(\theta)$ can be used to rotate an elements azimuth such that an optical element M at an azimuth angle θ is given by

$$M(\theta) = R(-\theta) M R(\theta) \quad (24)$$

where

$$R(\theta) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(2\theta) & -\sin(2\theta) & 0 \\ 0 & \sin(2\theta) & \cos(2\theta) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (25)$$

For example the effect of a quarter waveplate at 45° on a linear polarised wave is given below
 $S_O = R(-45^\circ) M R(45^\circ) S_I$

$$= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos(\frac{\pi}{2}) & -\sin(\frac{\pi}{2}) \\ 0 & 0 & \sin(\frac{\pi}{2}) & \cos(\frac{\pi}{2}) \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix} \quad (26)$$

$$= \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

ie a right hand circular output polarisation. For a wave with polarisation state S_I incident on a number of optical elements $M_1, M_2 \dots M_N$ the output polarisation state S_O is given by

$$S_O = M_N \dots M_2 M_1 S_I \quad (27)$$

For examples of Mueller Matrices for common optical elements see Table 2.

2.3 Poincaré Sphere

As stated it is common to reduce the Stokes vector to just three elements by normalising to unit intensity such that $s_1 = S_1/S_0$, $s_2 = S_2/S_0$ and $s_3 = S_3/S_0$. If we consider only fully polarised light then $s_1^2 + s_2^2 + s_3^2 = 1$. Thus if we plot the Stokes vector onto a three dimensional graph we find that all the fully polarised states fall onto the surface of a sphere of radius one. This is the basis of the Poincaré sphere visualisation of polarised light. The plane containing s_1 and s_2 cuts the equator of the sphere and all linear states of polarisation fall onto the equator. The line $s_1=0, s_2=0, s_3$ pierces the poles of the sphere and circular states of polarisation fall onto the poles. The rest of the surface of the sphere describes a range of elliptical polarisation states. Polarisation states which are geometrically opposite on the surface of the Poincaré sphere are orthogonal states of polarisation.

The effect of a general retardance is to move the state of polarisation in an arc across the surface of the sphere. The arc will begin at the input state of polarisation, be centred on the fast axis of the retardance plate (the fast eigenstate of the retardance plate) and will describe an arc of angular length given by the retardance angle. This allows a quick visualisation of the effect of birefringent elements.

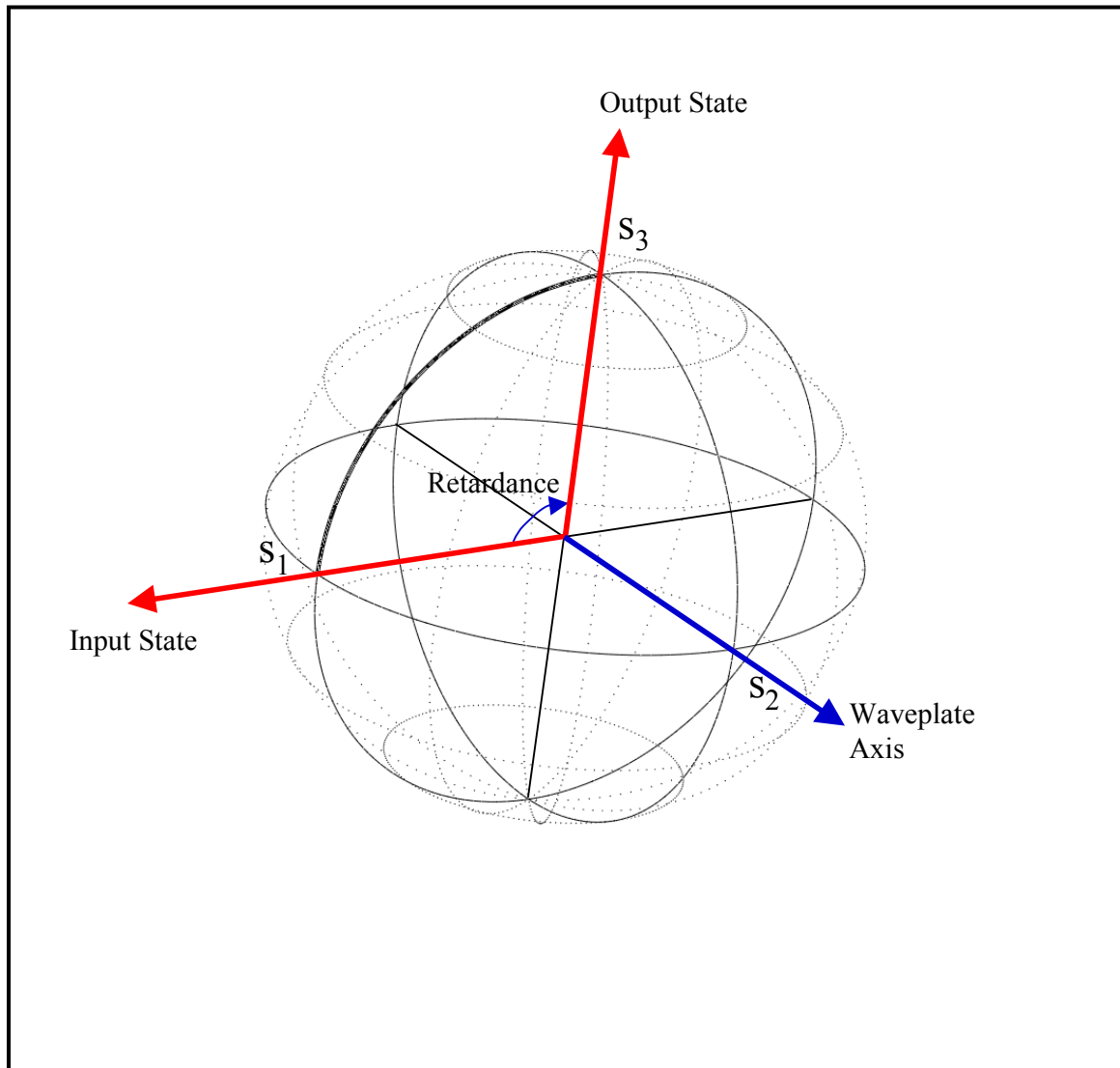


Figure 17 Poincaré sphere representation of a horizontal linear input state passing through a quarter waveplate at azimuth of 45° giving a right hand circular output.

2.4 Useful Matrices

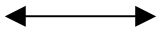
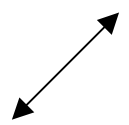
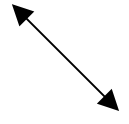
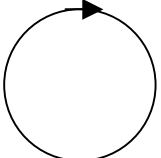
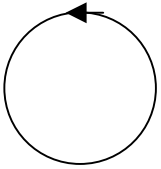
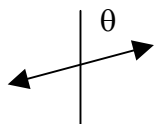
State		Jones Vector	Stokes Vector
H Linear Horizontal		$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}$
V Linear Vertical		$\begin{pmatrix} 0 \\ 1 \end{pmatrix}$	$\begin{pmatrix} 1 \\ -1 \\ 0 \\ 0 \end{pmatrix}$
+ Linear at +45°		$\frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$	$\begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}$
- Linear at -45°		$\frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix}$	$\begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix}$
R Right Hand Circular		$\frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ i \end{pmatrix}$	$\begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}$
L Left Hand Circular		$\frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -i \end{pmatrix}$	$\begin{pmatrix} 1 \\ 0 \\ 0 \\ -1 \end{pmatrix}$
Linear Polarization Azimuth θ		$\begin{pmatrix} \cos(\theta) \\ \sin(\theta) \end{pmatrix}$	$\begin{pmatrix} 1 \\ \cos(2\theta) \\ \sin(2\theta) \\ 0 \end{pmatrix}$
General Polarisation		$\begin{pmatrix} \cos(v) e^{i\frac{\Delta}{2}} \\ \sin(v) e^{-i\frac{\Delta}{2}} \end{pmatrix}$	$\begin{pmatrix} 1 \\ \cos(2v) \\ \sin(2v) \cos(\Delta) \\ \sin(2v) \sin(\Delta) \end{pmatrix}$

Table 1 Some common polarisation states in both Jones vector and Stokes vector format.

Element	Jones Matrix	Mueller Matrix
Ideal Horizontal linear polariser	$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$	$\frac{1}{2} \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$
Ideal Vertical linear polariser	$\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$	$\frac{1}{2} \begin{pmatrix} 1 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$
Quarter Waveplate at 45°	$\frac{1}{\sqrt{2}} \begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$
Azimuth Rotation, θ	$\begin{pmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{pmatrix}$	$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(2\theta) & -\sin(2\theta) & 0 \\ 0 & \sin(2\theta) & \cos(2\theta) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$
Linear Retarder, Azimuth 0°, retardance δ .	$\begin{pmatrix} e^{i\frac{\delta}{2}} & 0 \\ 0 & e^{-i\frac{\delta}{2}} \end{pmatrix}$	$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos(\delta) & -\sin(\delta) \\ 0 & 0 & \sin(\delta) & \cos(\delta) \end{pmatrix}$
Neutral Density Filter Transmission K		$\begin{pmatrix} K & 0 & 0 & 0 \\ 0 & K & 0 & 0 \\ 0 & 0 & K & 0 \\ 0 & 0 & 0 & K \end{pmatrix}$

Table 2 Some common optical elements in both Jones matrix and Mueller matrix format.

3. Polarisation Effects in Optical Fibres

3.1 Birefringence in Optical Fibres

The solution of the electromagnetic wave equation for single-spatial-mode optical fibre produces two guided polarisation modes. In ideal circular symmetric fibres these two modes are degenerate. However, any asymmetry in fibre structure or applied external stress leads to the fibre being weakly birefringent and causes polarisation mode dispersion, PMD. This PMD leads to pulse broadening and intersymbol interference and ultimately restricts the data capacity of an optical fibre system.

3.2 Causes of Linear Birefringence

There are a number of intrinsic and external effects that break the symmetry of single-mode optical fibre and give rise to linear birefringence. For example^[4]

- Asymmetrical external stress, e.g. bend, kink or squeezing
- Asymmetrical geometry of the guiding structure, e.g. an elliptical core.
- Asymmetrical internal stress associated with an elliptical core.
- Electric Field

Any external transverse stress across the fibre produces birefringence through the elasto-optic effect. Figure 18 shows possible arrangements to produce such stress ranging from pressing the fibre between parallel plates to bending around a mandrel under tension.

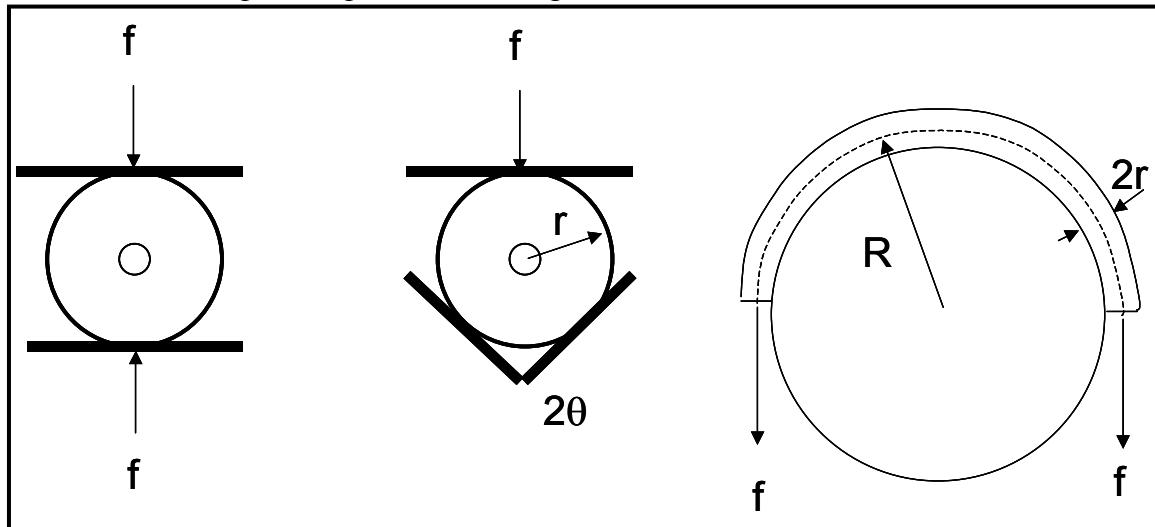


Figure 18 External influences on fibre birefringence.

All such external stresses give rise to a linear birefringence given by the form

$$\delta\beta_1 = F(f, r, R, \theta) \frac{\omega_0}{c} \frac{C_s}{k_0} \quad (28)$$

where $F(f, r, R, \theta)$ is a function of the external force, radius etc. $\frac{C_s}{k_0}$ is known as the strain-optical coefficient and is given by:

$$\frac{C_s}{k_0} = 0.5 n_0^3 (\rho_{11} - \rho_{12}) (1 + \nu_p) \sim 0.27 \quad (29)$$

where n_0 is the average refractive index of the fibre, ρ_{11} and ρ_{12} are the components of the strain-optic tensor and ν_p is Poissons ratio.

For a fibre pressed into a V groove as shown in Figure 18 then

$$F(f, r, R, \theta) = 2(1 - \cos(2\theta)\sin(\theta)) \frac{f}{\pi r E} \quad (30)$$

where E is Young's modulus of the fibre, this takes into account friction between the fibre and V groove but assumes the forces act as point forces on the fibre. In the case of two flat plates this simplifies to

$$F(f, r, R, \theta) = 4 \frac{f}{\pi r E}. \quad (31)$$

For a fibre bent around a radius under tension as shown in Figure 18 then

$$F(f, r, R, \theta) = 0.5 \frac{r^2}{R^2} + \left[\frac{2 - 3\nu_p}{1 - \nu_p} \right] \frac{r}{R} \left[\frac{f}{\pi r^2 E} \right]. \quad (32)$$

For a free bend then the second term in Equation 32 is zero. For a standard single-mode fibre with cladding diameter 125 μm loosely bent around a 5 cm diameter then the birefringence is approximately 3.4 rad.m^{-1} such that 3 full loops gives approximately $\frac{1}{2}\pi$ rad birefringence. This therefore forms is a quarter waveplate.

If a fibre has an elliptical core then there will be differing propagation constants for modes where the electric field is aligned with the major or minor axis of the ellipse. This leads to a linear birefringence component with the fast axis along the minor axis of the ellipse. Several papers give the functional form of this linear birefringence as:

$$\delta\beta_1 = G(V) \frac{\omega}{c} \left(\frac{a}{b} - 1 \right) \Delta n^2. \quad (33)$$

where Δn is the refractive index difference between the core and cladding, a and b are the major and minor axes of the elliptic core, and $G(V)$ is a function of the normalised frequency parameter, V . For single mode fibre near cut off where $V \sim 2.4$ then $G(V) \sim 0.2$.

Fibres that have an elliptical core will inevitable also have internal stress. The stress results from the differential expansion and cooling rates between the core and cladding during manufacture. In circular symmetric fibre this has no net effect but in elliptical core fibre this will cause a linear birefringence. This stress birefringence is given by:

$$\delta\beta_1 = \left(1 - \frac{U^2}{V^2} \right) \frac{\omega}{c} \frac{C_s}{k_0} \frac{1}{(1 - \nu_p)} \Delta\alpha \Delta T \left(\frac{a - b}{a + b} \right) \quad (34)$$

where $\Delta\alpha$ is the expansion coefficient difference between the core and cladding, ΔT is the temperature difference between room temperature and the glass softening temperature, a and b are the major and minor axes of the elliptical core. For a core doped with germanium the expansion coefficient will be greater then the cladding leading to the fast axis aligned with the minor axis of the elliptical core. The factor $\left(1 - \frac{U^2}{V^2} \right)$ is related to the overlap of the optical mode with the stress field and is approximately 0.5 near the cut off wavelength.

A transverse electric field will introduce a linear birefringence through the Kerr effect given by:

$$\delta\beta_1 = 2\pi B_K E^2 \quad (35)$$

where B_K is the Kerr Electro-optic constant and E the electric field strength. In silica $B_K^{[5][6]}$ is approximately 2 to $5 \times 10^{-16} \text{ m.V}^{-2}$ such that a DC electric field of 10^6 V.m^{-1} will give a

birefringence of just 3 mrad.m^{-1} . For AC electric fields electrostriction effects must be taken into account particularly when mechanical resonance may enhance any effect.

3.3 Causes of Circular Birefringence

There are a number of external effects that break the symmetry of single-mode optical fibre and give rise to circular birefringence. For example

- Twist
- Axial Magnetic Field

For a fibre that is twisted the resulting stress gives rise to circular birefringence. That is the left and right circular modes experience differential phase shift as they propagate through the twisted fibre. The circular birefringence is given by

$$\delta\beta_C = g t \quad (36)$$

where t is the twist rate (rad.m^{-1}) and

$$g = -0.5 n_0^2 (\rho_{11} - \rho_{12}) . \quad (37)$$

g is reported to be ~ 0.14 .

An axial magnetic field will introduce a circular birefringence via the Faraday effect given by

$$\delta\beta_C = 2 V_V H \quad (38)$$

where V_V is the Verdet constant of the fibre material and H is the axial magnetic field. For silica fibre the effect is small since V_V is approximately $4.7 \times 10^{-6} \text{ rad.A}^{-1}$. Thus for a large axial magnetic field of say 1 Tesla = $8 \times 10^5 \text{ A.m}^{-1}$, there is approximately 7.5 rad.m^{-1} of circular birefringence. The Faraday effect is very important in the development of non-reciprocal components such as optical isolators and for this special doped crystals have been developed with an enhanced V_V .

3.4 Local Effects of Fibre Birefringence

As light propagates down a length of fibre the polarisation state will evolve due to the local birefringence. The local birefringence can be expressed as a vector in the Poincaré sphere such that the magnitude of the vector gives the total birefringence and the vector intercepts the Poincaré sphere at the fast polarisation. As with bulk birefringence as the light propagates through the birefringent media the polarisation state rotates around the birefringence vector, the rate of rotation being one full revolution for a total birefringence to π radians.

The birefringence vector, W , is given by

$$W = \begin{pmatrix} \delta\beta_l \cos(2\theta) \\ \delta\beta_l \sin(2\theta) \\ \delta\beta_C \end{pmatrix} \quad (39)$$

where $\delta\beta_l$ is the linear birefringence component at azimuth θ and $\delta\beta_C$ is the circular birefringence component. The change of the polarisation state of light propagating through this birefringence is given by

$$\frac{\partial s}{\partial z} = W \times s \quad (40)$$

The solution to this differential equation is a circle on the surface of the Poincaré sphere that is centred on the point where the vector b pierces the surface of the sphere. The polarisation state makes one revolution of the circle for a distance, L_b , known as the beat length and given by

$$L_b = \frac{2\pi}{\delta\beta} \quad (41)$$

where $\delta\beta$ is the magnitude of the birefringence vector W .

3.5 Global Effects in Fibre, Polarisation Mode Dispersion

If the birefringence of each fibre section is identical then the global birefringence will equal the local birefringence and the fibre is said to show no-mode coupling. If the local birefringence varies along the length of the optical fibre, due to internal imperfections and external perturbations, then these perturbations cause coupling between the two polarisation modes. This coupling is described by a coupling parameter, h , that is equal to the fractional power transferred per meter. $1/h$ is often referred to as a coupling length, L_C .

For a medium where there is no polarisation dependent loss the Mueller matrix of a small section of fibre is a simple rotation matrix, $R(W, \delta z)$. The rotation matrix is a function of the birefringence vector and the section length, the birefringence vector depends on its position within the fibre, z , and the optical frequency, ω . The Rotation matrix for the whole fibre span is the multiplication of the rotation matrix for each element.

$$\begin{aligned} R(\omega) &= R(W_Z(\omega), \delta z) R(W_{Z-1}(\omega), \delta z) \dots R(W_1(\omega), \delta z) R(W_0(\omega), \delta z) \\ &= \prod_Z^0 R(W_Z(\omega), \delta z) \end{aligned} \quad (42)$$

Such that the output polarisation state, S_O , is given by

$$S_O = R(\omega) S_I. \quad (43)$$

As there is no polarisation dependent loss and this is a simple rotation of the polarisation state on the Poincaré sphere any two orthogonal input polarisation states will emerge as two orthogonal output polarisation states. There exist two orthogonal input states of polarisation that emerge unchanged. These are the polarisation eigenstates.

The transmission of a polychromatic source through a high polarisation mode coupled fibre leads to a range of output states and, in general, a depolarising of the light. It has been shown that over a limited source bandwidth there exist two orthogonal input polarisation states which lead to two orthogonal output states which are independent of the source frequency (to first order). These are known as the principle states of polarisation (PSP)^[7], they are not the same as the polarisation eigenstates. To first order there is a phase difference between the states that is proportional to the source frequency. This is equivalent to a time delay between signals propagating in the two PSPs. This time delay is the differential group delay (DGD).

For monochromatic light as the optical frequency is changed the output state of polarisation will describe a circle on the surface of the Poincaré sphere centre on the principle state of polarisation and making one full revolution for an optical frequency change of $1/(\text{DGD})$.

We define a polarisation dispersion vector, Ω , aligned with the principle state of polarisation, p , and with magnitude equal to the differential group delay, $\delta\tau$, such that

$$\Omega = \begin{pmatrix} \Omega_1 \\ \Omega_2 \\ \Omega_3 \end{pmatrix} = \delta\tau \begin{pmatrix} p_1 \\ p_2 \\ p_3 \end{pmatrix} \quad (44)$$

The change of the output state of polarisation with optical frequency is given by

$$\frac{\partial s}{\partial \omega} = \Omega \times s = \frac{\partial R(\omega)}{\partial \omega} R(\omega)^{-1} s \quad (45)$$

Similar to the birefringence equation, the solution to this differential equation is a circle on the surface of the Poincaré sphere that is centred on the point where the vector Ω pierces the surface of the sphere. The polarisation state makes one revolution of the circle for a change in optical frequency, $\delta\omega$, given by

$$\delta\omega = \frac{2\pi}{|\Omega|} = \frac{2\pi}{\delta\tau}. \quad (46)$$

This can be written in terms of optical wavelength such that the DGD, $\delta\tau$, is given by

$$\delta\tau = \frac{\theta}{2\pi} c \frac{\lambda_1 \lambda_2}{(\lambda_2 - \lambda_1)^2} \quad (47)$$

Where the wavelength evolves from λ_1 to λ_2 and the polarisation state rotates through an angle, θ in radians around the principle state of polarisation on the Poincaré sphere.

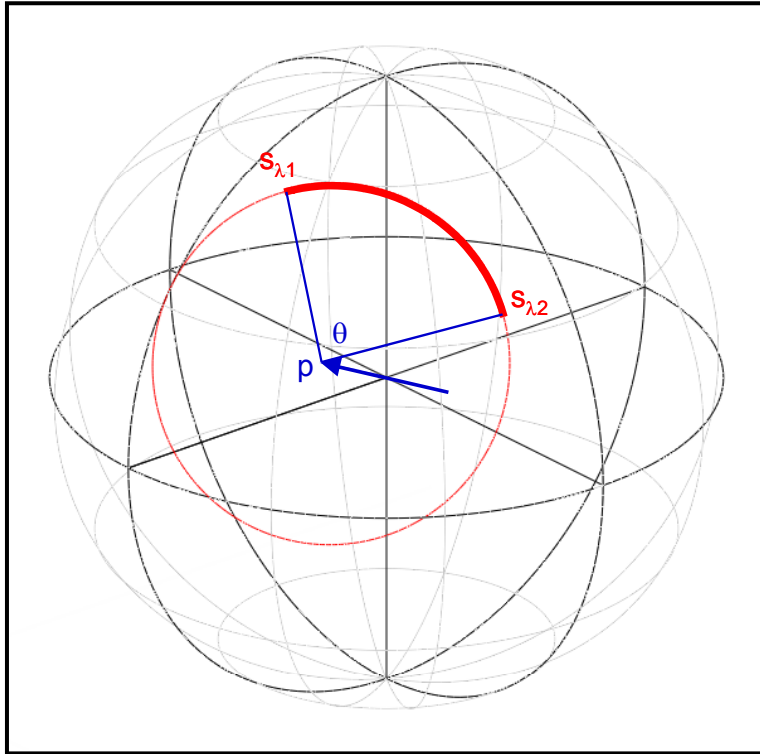


Figure 19 Loci of polarisation state on surface of sphere as wavelength is changed from λ_1 to λ_2 .

To first order where the polarisation vector, Ω , is not a function of the optical frequency the rotation matrix can be written as^[8]

$$\begin{aligned}
R(\delta\omega) = & \cos(\delta\tau\delta\omega) \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + [1 - \cos(\delta\tau\delta\omega)] \begin{bmatrix} p_1p_1 & p_2p_1 & p_3p_1 \\ p_1p_2 & p_2p_2 & p_3p_2 \\ p_1p_3 & p_2p_3 & p_3p_3 \end{bmatrix} \\
& + \sin(\delta\tau\delta\omega) \begin{bmatrix} 0 & -p_3 & p_2 \\ p_3 & 0 & -p_1 \\ -p_2 & p_1 & 0 \end{bmatrix}
\end{aligned} \tag{48}$$

where $\delta\omega$, is the optical frequency difference from the central optical frequency to which the polarisation vector corresponds.

For fibres with no polarisation mode coupling the DGD is a constant with optical frequency and the principle state of polarisation is a constant. If a pulse is propagated through such a fibre the polarisation components aligned and orthogonal to the principle axis propagate at different group velocities leading to two pulses at the exit of the fibre.

For highly mode coupled fibres the DGD and PSP are strong functions of optical frequency. An optical impulse input into such a fibre will be broadened and in the absence of chromatic dispersion the output pulse will be a Gaussian pulse. This is the phenomenon known as polarisation mode dispersion. For an ideal optical impulse (a theoretical pulse with infinite spectral width) the output pulse will have an RMS width equal to the PMD. The DGD and PSP are also strong functions of the environmental conditions. However, while the DGD and PSP will change at a specific optical frequency the PMD is constant. The PMD can be found from the RMS average of the DGD such that $\text{PMD}, \langle \Delta\tau^2 \rangle^{1/2}$,

$$\langle \Delta\tau^2 \rangle^{1/2} = \sqrt{\frac{\sum_{i=1}^N \delta\tau_i^2}{N}} \tag{49}$$

where the DGD, $\delta\tau$, is averaged over optical frequency or environmental conditions.

The DGD is the magnitude of the polarisation dispersion vector, Ω . Perturbation theory has shown that the elements of the dispersion vector are taken from a normal distribution with zero mean and standard deviation α , where $\alpha^2 = \langle \Delta\tau^2 \rangle / 3$. The statistical distribution of the magnitude of such a three-element vector is described by the Maxwellian distribution^[9],

$$P(\delta\tau) = \sqrt{\frac{2}{\pi}} \frac{\delta\tau^2}{\alpha^3} e^{-\frac{\delta\tau^2}{2\alpha^2}}. \tag{50}$$

This describes the probability of a single measurement or a single sample of DGD having the value $\delta\tau$. The long tail indicates that a DGD greater than 3 times the PMD has a probability of 6×10^{-5} of occurring (approximately 1/2 hour per year). It is normal to consider that the DGD may take on values as high as 5 times the PMD.

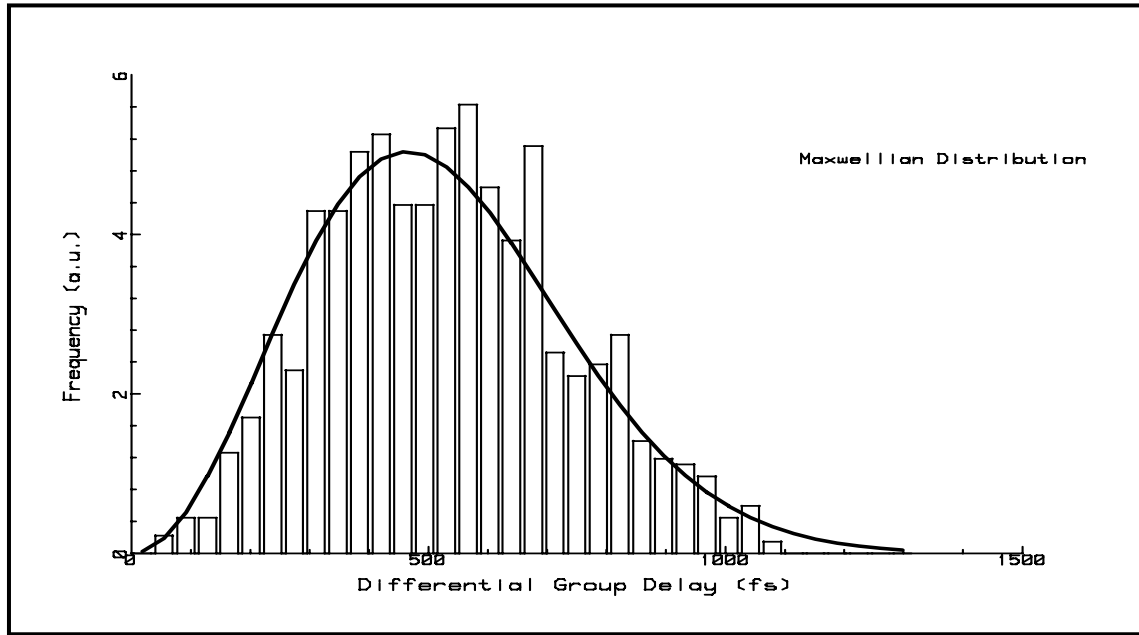


Figure 20 Distribution of measured DGD values for a highly polarisation mode coupled fibre.

The alternative definition of PMD based on the linear mean is often used, $\langle \Delta\tau \rangle$ where

$$\langle \Delta\tau \rangle = \frac{\sum_{i=1}^N \delta\tau_i}{N}. \quad (51)$$

The difference between these definitions can be calculated from the probability distribution and is given by

$$\langle \Delta\tau^2 \rangle^{1/2} = \sqrt{\frac{3\pi}{8}} \langle \Delta\tau \rangle = 1.085 \langle \Delta\tau \rangle. \quad (52)$$

This factor is only valid in the limit of high polarisation mode coupling.

Figure 21 shows the evolution of pulse arrival times as the amount of polarisation mode coupling is increased, while maintaining a constant modal dispersion and system length^[10]. It can be seen that the pulse broadening is reduced if there is a large amount of polarisation mode coupling.

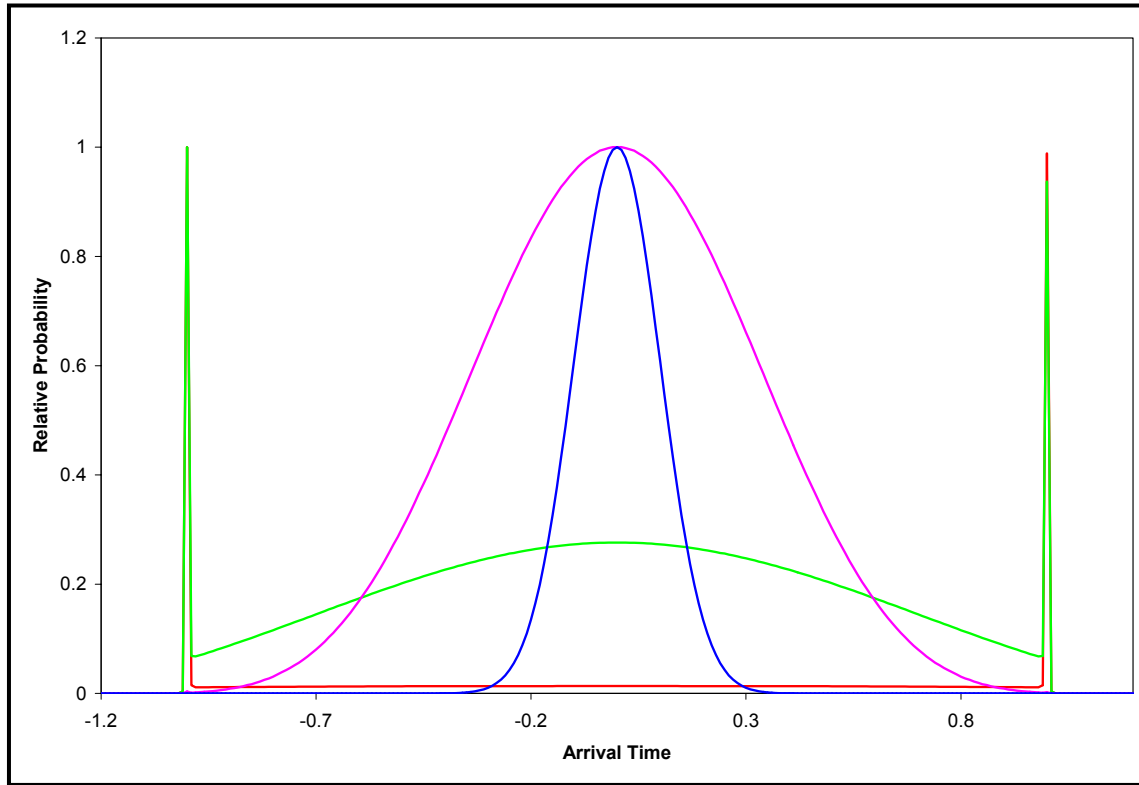


Figure 21 Effect of polarisation mode coupling on pulse arrival times. Red $hz=1$, Green $hz=3$, Blue $hz=10$ and Pink $hz=100$. The curves have been normalised to a maximum of 1.

The length dependence of the PMD is given by^[11]:

$$\langle \Delta\tau^2 \rangle = \frac{\delta\beta'^2}{2h^2} [e^{-2hz} + 2hz - 1] \quad (53)$$

where h is the coupling parameter and $\delta\beta'$ is the modal dispersion given by

$$\delta\beta' = \frac{\partial\delta\beta}{\partial\omega} = c \frac{\partial\delta\beta}{\partial k} \quad (54)$$

The length dependence given in equation (53) can be simplified to:

$$\langle \Delta\tau^2 \rangle^{1/2} = \delta\beta' z = \langle \Delta\tau \rangle \quad (55)$$

in the short length regime where $z \ll 1/h$ and

$$\langle \Delta\tau^2 \rangle^{1/2} = \delta\beta' \sqrt{\frac{z}{h}} = 1.085 \langle \Delta\tau \rangle \quad (56)$$

in the long length regime $z \gg 1/h$.

The description of polarisation mode dispersion based on principle states of polarisation and differential group delay is suitable to first order only. Higher order polarisation mode dispersion, in particular second order PMD, is described in terms of the optical frequency dependence of the polarisation vector, $\Omega(\omega)$,

$$\Omega(\omega) = \begin{pmatrix} \Omega_1(\omega_0) + (\omega - \omega_0) \frac{\partial \Omega_1(\omega_0)}{\partial \omega} + \frac{1}{2} (\omega - \omega_0)^2 \frac{\partial^2 \Omega_1(\omega_0)}{\partial \omega^2} + \dots \\ \Omega_2(\omega_0) + (\omega - \omega_0) \frac{\partial \Omega_2(\omega_0)}{\partial \omega} + \frac{1}{2} (\omega - \omega_0)^2 \frac{\partial^2 \Omega_2(\omega_0)}{\partial \omega^2} + \dots \\ \Omega_3(\omega_0) + (\omega - \omega_0) \frac{\partial \Omega_3(\omega_0)}{\partial \omega} + \frac{1}{2} (\omega - \omega_0)^2 \frac{\partial^2 \Omega_3(\omega_0)}{\partial \omega^2} + \dots \end{pmatrix} \quad (57)$$

$$= \Omega(\omega_0) + (\omega - \omega_0) \Omega_\omega(\omega_0) + \dots$$

where Ω_ω is known as the second order PMD vector.

This second order PMD vector can be broken into polarisation dependent chromatic dispersion and a change in the principle state of polarisation. These are the components of the second order PMD vector that are aligned and orthogonal to the first order PMD vector.

$$\Omega_\omega = \frac{\partial \Omega}{\partial \omega} = \frac{\partial \Delta \tau}{\partial \omega} \mathbf{p} + \Delta \tau \frac{\partial \mathbf{p}}{\partial \omega} \quad (58)$$

where $\mathbf{p}$ is the principle state. The length dependence of the second order PMD magnitude is given by^[12]

$$\langle \Omega_\omega^2 \rangle = \frac{\delta \beta'^4}{3 h^4} \left[\frac{17}{8} - \frac{5}{2} h z + h^2 z^2 - \frac{1}{8} e^{-4 h z} - 2 e^{-2 h z} - 2 h z e^{-2 h z} \right]. \quad (59)$$

For high polarisation mode coupled fibres the average second order PMD magnitude is given by^[13]

$$\langle \Omega_\omega^2 \rangle = \frac{1}{3} \langle \Delta \tau^2 \rangle^2 = \left\langle \frac{\partial \Delta \tau}{\partial \omega} \mathbf{p}^2 \right\rangle + \left\langle \Delta \tau^2 \frac{\partial \mathbf{p}}{\partial \omega}^2 \right\rangle = \frac{1}{27} \langle \Delta \tau^2 \rangle^2 + \frac{8}{27} \langle \Delta \tau^2 \rangle^2, \quad (60)$$

from which it can be seen that the first term, the polarisation induced chromatic dispersion, is considerably smaller than the change in the direction of the PSP with wavelength.

The probability distribution of the second order PMD components, $\Omega_{\omega,i}$, are taken from a ‘‘Soliton’’ distribution given by^[12]

$$P(\Omega_{\omega,i}) = \frac{1}{2\alpha^2 \cosh\left(\frac{\pi \Omega_{\omega,i}}{2\alpha^2}\right)} \quad (61)$$

where, as before, $\alpha = \frac{1}{3} \delta \beta'^2 h z$. From this the probability distribution of the second order PMD magnitude, $|\Omega_\omega|$, is given by^[12]

$$P(|\Omega_\omega|) = \frac{\pi |\Omega_\omega| \sinh\left(\frac{\pi |\Omega_\omega|}{2\alpha^2}\right)}{2\alpha^4 \cosh^2\left(\frac{\pi |\Omega_\omega|}{2\alpha^2}\right)}. \quad (62)$$

Figures 22 and 23 illustrate the probability distributions of the second order PMD components and the second order PMD magnitude for highly mode coupled fibres.

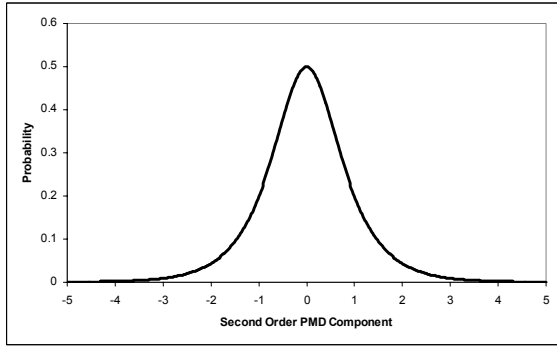


Figure 22 Probability distribution of second order PMD components for highly polarisation mode coupled fibre.

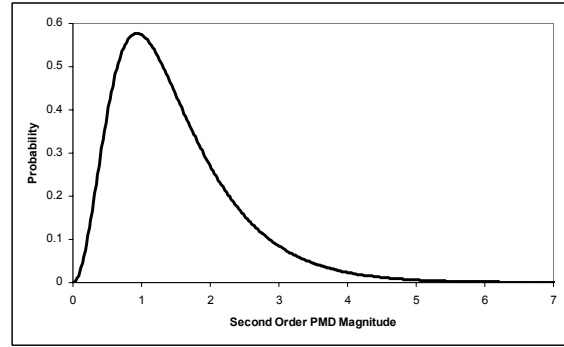


Figure 23 Probability distribution of second order PMD magnitude for highly polarisation mode coupled fibre.

A full analysis of the statistics of second order PMD in the case of high polarisation mode coupling leads to the following average results shown in table 3^{[14][12]}.

Quantity	Value
α^2	$\frac{\delta\beta'^2 z}{3h}$
$\langle\Omega_i\rangle$	0
$\langle\Omega_i^2\rangle$	α^2
$\langle\Omega_i \Omega_j\rangle \ i \neq j$	0
$\langle\Delta\tau^2\rangle = \langle\Omega^2\rangle$	$3\alpha^2$
$\langle\Delta\tau\rangle^2 = \langle \Omega \rangle^2$	$\frac{8}{\pi}\alpha^2$
$\sigma_{ \Omega }^2$	$(3 - \frac{8}{\pi})\alpha^2$
$\sigma_{\sqrt{\Omega^2}}^2$	$\frac{1}{2}\alpha^2$
$\langle\Omega_{\omega i}\rangle$	0
$\langle\Omega_{\omega i}^2\rangle$	α^4
$\langle\Omega_{\omega i} \Omega_{\omega j}\rangle \ i \neq j$	0
$\langle\Omega_{\omega}^2\rangle$	$3\alpha^4$
$\langle(\Omega \cdot \Omega_{\omega}/ \Omega)^2\rangle$	$\frac{1}{3}\alpha^4$
$\langle(\Omega \times \Omega_{\omega}/ \Omega)^2\rangle$	$\frac{8}{3}\alpha^4$

Table 3 Useful relationships for high polarisation mode coupled systems.

Since the detail of the polarisation mode coupling depends on a large number of small local mode couplings each of which are determined by internal and external influences on the fibre the differential group delay changes considerably as the external influences on the fibre change. Thus if the DGD of a fibre is measured at a given wavelength then, as the external influences change, the measured DGDs will be samples from the Maxwellian distribution. However, given sufficient samples the RMS mean will be a constant and equal to the polarisation mode dispersion.

3.6 Polarisation Maintaining Fibre

These fibres are better described by their alternative name of highly birefringent, HiBi, fibre. The idea is to produce a fibre that will maintain the state of polarisation on transmission and hold the polarisation under moderate external influence. This is done by giving the fibre a high intrinsic birefringence such that any external influence does not produce any significant change. The greater the intrinsic birefringence the greater the fibres polarisation holding ability under external perturbations. It is typical for such fibres to have beat lengths of order mm. The polarisation is only “maintained” if the input state of polarisation is aligned with the birefringence axis. If the input state of polarisation is not aligned with the birefringence axis then the fibre acts as a long waveplate. The input light is split into the two axes and propagates at differing velocities, at the output of the fibre the two beams recombine with a different phase giving the final state of polarisation. Since the fibre is highly birefringent the polarisation state can undergo a significant change. This change will also depend on external perturbations on the fibre.

There are several types of HiBi fibre using internal stress and elliptical cores to give the intrinsic birefringence, see Figure 24.

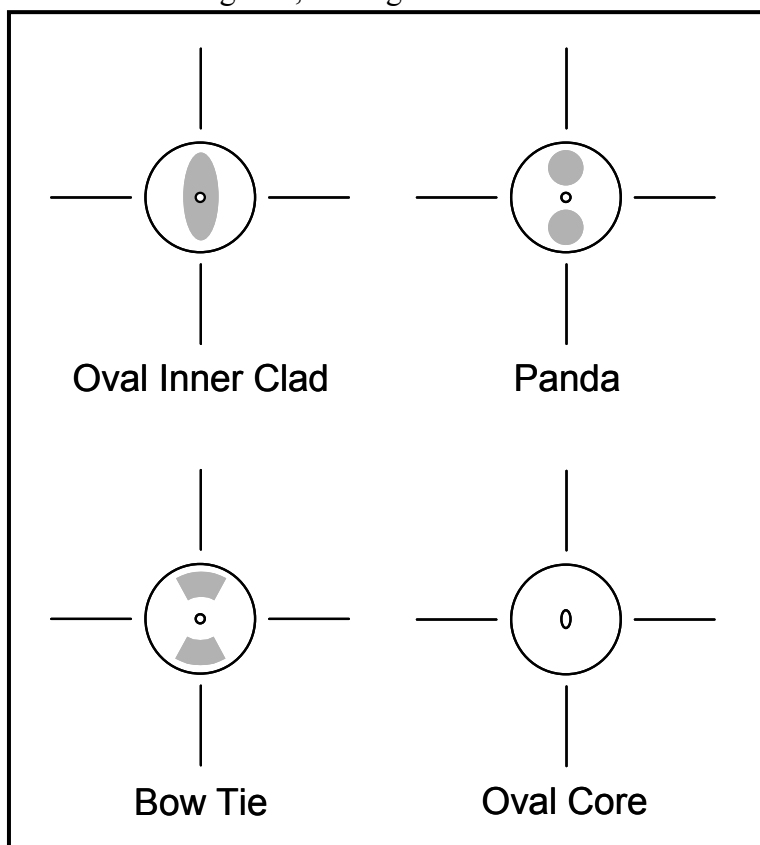


Figure 24 Various designs of HiBi fibre.

HiBi fibre is used in relatively short pieces, due to its increased cost, in specialist applications for example joining a laser source to an electro-optic modulator. In this situation the laser source is linearly polarised and the electro-optic modulator is polarisation sensitive. If ordinary fibre were used then as the external perturbations changed the polarisation state of the light entering the electro-optic modulator would shift leading to a variation in the modulation depth. The important parameter here is how much light enters the electro-optic

modulator in the correct polarisation state. This is described by the polarisation extinction ratio defined as

$$ER = 10 \log \left(\frac{P_S}{P_F} \right) \quad (63)$$

where P_F is the power propagating in the fast axis of the HiBi fibre and P_S that propagating in the slow axis. The extinction ratio parameter can be applied to a HiBi coupled laser source or electro-optic modulator or a HiBi patch cord. In systems with connectors then P_S and P_F should be replaced with $P_{\parallel}$ and $P_{\perp}$ where $P_{\parallel}$ is the power linear polarised with azimuth aligned to the connector keyway and $P_{\perp}$. The extinction ratio is determined by the mechanical alignment of the sources and components with the axis of the HiBi fibre and also by any external perturbation (notably during connectorisation).

3.7 Polarisation Dependent Loss

The polarisation effects of the previous sections have been limited to phase and time based phenomena. In optical components an equally significant phenomena is polarisation dependent loss. This describes the differences in loss experienced by orthogonal polarisation states. Polarisation dependent loss (PDL) is defined as the difference between the maximum and minimum attenuation of the device or component under test. PDL is described by

$$PDL_{dB} = A_{max} - A_{min} = 10 \text{ LOG} \left[\frac{P_{max}}{P_{min}} \right] \quad (64)$$

where A_{max} and A_{min} are the maximum and minimum attenuation of the device or component under test and P_{max} and P_{min} are the maximum and minimum transmitted power as the polarisation state of the input light is changed over all possible states.

A more detailed description of polarisation loss is in terms of the polarisation loss vector, Γ . The polarisation loss vector describes both the magnitude and polarisation state for which the loss is minimised. The linear transmission factor for an input state, s_{in} , is given by

$$T = T_{depol} (1 + s_{in} \cdot \Gamma) \quad (65)$$

where T_{depol} is the transmission for depolarised light and is equal to the average transmission over all states or the average of the maximum and minimum transmission. From this it can be seen that

$$PDL_{dB} = 10 \log \left(\frac{T_{max}}{T_{min}} \right) = 10 \log \left(\frac{1 + \Gamma}{1 - \Gamma} \right) \quad (66)$$

where T_{max} and T_{min} are the maximum and minimum linear transmission.

In the absence of birefringence an input polarisation state, s_{in} , is transformed to an output polarisation state, s_{out} , since the components of the input polarisation state aligned to the polarisation loss vector and those counter aligned will experience different transmission. The output polarisation state is given by s_{out} , where^[15]

$$s_{out} = \frac{\sqrt{1 - \Gamma^2}}{1 + s_{in} \cdot \Gamma} s_{in} + \frac{1 + s_{in} \cdot \Gamma (1 - \sqrt{1 - \Gamma^2}) / \Gamma^2}{1 + s_{in} \cdot \Gamma} \Gamma \quad (67)$$

Thus for a depolarised input polarisation state in the absence of local birefringence the output polarisation state will be equal to the polarisation loss vector, Γ .

3.8 Polarisation Dependent Response

Polarisation dependent response is the equivalent parameter to polarisation dependent loss applied to optical to electrical converters, for example photodiodes, power meters and optical spectrum analysers. The polarisation dependent response in (dB) is defined by

$$\text{PDR}_{\text{dB}} = 10 \text{ LOG} \left[\frac{R_{\text{max}}}{R_{\text{min}}} \right] = 10 \text{ LOG} \left[\frac{P_{\text{max}}}{P_{\text{min}}} \right] \quad (68)$$

where R_{max} and R_{min} are the maximum and minimum responsivity and P_{max} and P_{min} are the maximum and minimum displayed power or voltage.

3.9 PMD and PDL Combined

The previous paragraphs have considered PMD in the absence of PDL and PDL in the absence of PMD. In real systems it is likely that both PMD and PDL will be present. The full effect of combining PMD with PDL requires a detailed analysis, see for example [16],[17] and [18]. The physical effect of PDL on PMD is to move the principle states of polarisation such that the two principle states are no longer orthogonal. We maintain our definitions of principle state of polarisation and differential group delay as the two output states of polarisation that are independent of optical frequency to first order and the differential delay between modes propagating in these two principle states. However, the principle states of polarisation no longer hold the fastest and slowest mode. There is also the possibility of pulse broadening in excess of the combined DGD of a fibre system in the presence of PDL. This is because the non-orthogonal PSPs lead to interference between the polarisation components propagating in the two PSPs. This can lead to cancellation of the centre of the pulse where the output from the two PSPs overlap effectively widening the pulse. An alternative view is to consider that the PDL combined with the PMD leads to a frequency dependent loss that can suppress the wings of the pulse spectrum. This will effectively broaden the pulse in the time domain. It is also possible for this filtering process to suppress the centre of the optical spectrum leading to an effective increase in the spectral width and a pulse compression in the time domain^[19].

4. The Effects of Polarisation on Data Transmission

4.1 Impact of first order PMD

The effect of polarisation mode dispersion on a transmission system is to cause eye closure which reduces the signal to noise ratio and increases the likelihood of transmission errors. The effect of first order DGD alone is to split the signal into two, delaying one part with respect to the other. The effect is minimised when the input polarisation state is aligned with a principle state of polarisation and maximised when the two principle states of polarisation are equally excited.

A simulation was calculated based on; a pseudo random bit pattern, a first order DGD transfer function, and ideal intensity detection followed by a low pass 5th order Bessel filter with a 3 dB frequency set at 0.75 of the bit rate. The eye patterns are shown in Figure 25.

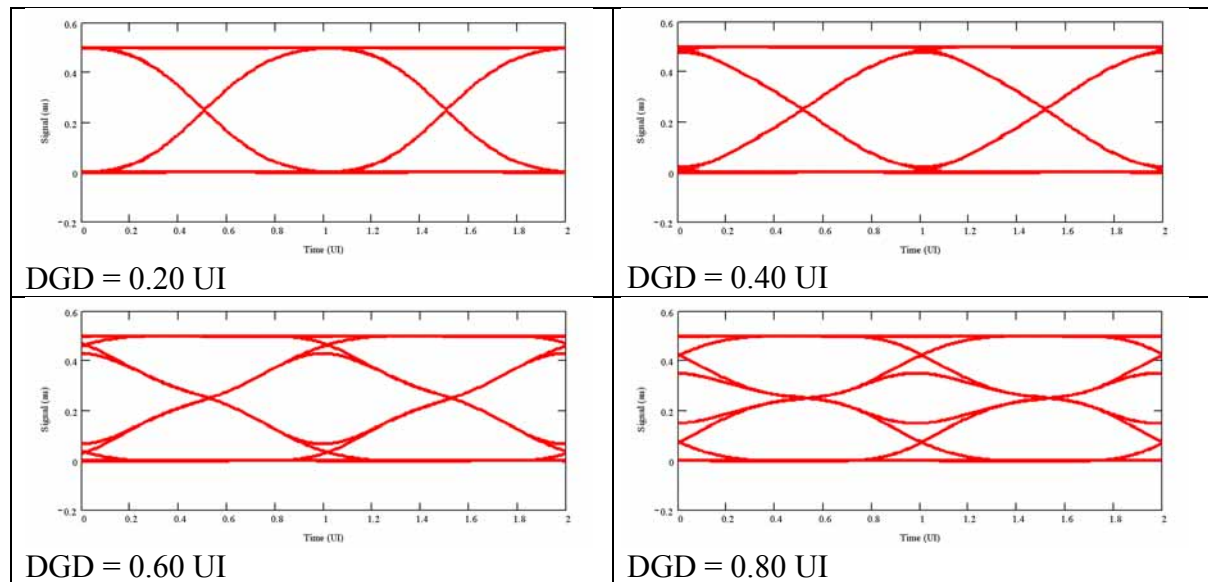


Figure 25 Eye diagrams for simulated bit pattern and various 1st order DGD. UI = unit interval = 1 bit period.

The parameter of interest to system designers is the power penalty of any signal degrading effect. This turns any signal degradation into an effective requirement to increase the signal to noise. From the eye diagrams the signal quality, Q , can be calculated as

$$Q = \frac{\mu_1 - \mu_0}{\sigma_1 + \sigma_0} \quad (69)$$

where μ_1 is the mean one level, μ_0 the mean zero level, σ_1 is the standard deviation of the one level and σ_0 the standard deviation of the zero level all taken at the decision point. The Q can be related to the expected bit error rate based on an assumed Gaussian statistical probability of the noise such that the BER is approximated by

$$\text{BER} \approx \frac{1}{2} \operatorname{erfc}\left(\frac{Q}{\sqrt{2}}\right). \quad (70)$$

For a BER of 10^{-9} a Q of 6 is required. The question is what is the power penalty due to the first order DGD degradation. Considering noise free simulations we calculate the basic s/n that is required to maintain the Q at 6. Assuming the noise adds in quadrature this gives

$$Q_{\text{Resulting}} = \frac{1}{\sqrt{\frac{1}{Q^2} + \frac{1}{\frac{s}{n}^2}}} \quad (71)$$

Thus the signal to noise that can be accepted for a BER of 10^{-9} is given by

$$\frac{s}{n}^2 = \frac{36Q^2}{Q^2 - 36} \quad (72)$$

and the power penalty is

$$20 \log\left(\frac{s}{n}_{b-b}\right) - 20 \log\left(\frac{s}{n}\right) = 10 \log\left(\frac{Q_{b-b}^2}{Q_{b-b}^2 - 36}\right) - 10 \log\left(\frac{Q^2}{Q^2 - 36}\right) \quad (73)$$

where Q_{b-b} is the back to back Q without the degrading effect under consideration.

Using this method to analyse the simulated data of Figure 25 the power penalties shown in Figure 26 were obtained. These are worst case penalties since the PSP were equally excited.

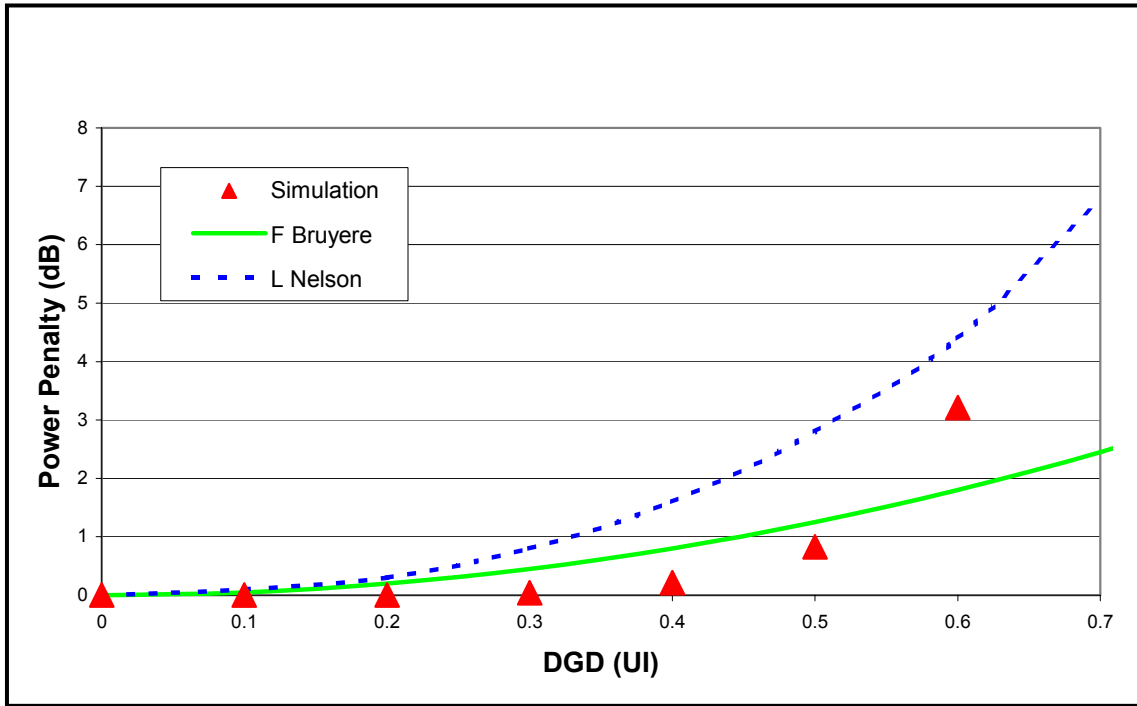


Figure 26 Power penalties for first order DGD, measured, simulated and published F Bruyère^[20], L Nelson^[21].

While the power penalties depend on the exact system arrangement; modulation format, bandwidth and filtering there appears to be a significant degradation for first order DGD greater than 0.5 UI.

4.2 Impact of Second Order PMD

For any optical system containing real fibre there will be significant polarisation mode coupling that will lead to second-order PMD. The second-order PMD consists of two parts, the change of DGD with optical frequency and the change in direction of the PSP with optical frequency as described in equation (58). A full analysis of the effect of second order PMD on pulse propagation has shown that there are two dominating pulse broadening components^[12],

an effect equivalent to chromatic dispersion that involves the magnitude of the second order PMD and an effect which depends on the product of second order PMD and chromatic dispersion. This second component has a sign that depends on the alignment of the input polarisation state and the PSPs.

The degrading effects of second order PMD particularly in the presence of chromatic dispersion are more significant than first order PMD for the highly polarisation mode coupled regime.

4.3 PMD of Installed Links

The PMD of installed systems can vary considerably depending on the quality of the fibre, cable design etc. Reported values vary from $0.1 \text{ ps.km}^{-1/2}$ to $10 \text{ ps.km}^{-1/2}$. There is also the question of how fast the PMD vector evolves, as this will determine the speed of any tracking that may be required for active compensation. A number of studies show differing results depending on the cable route. For buried cables the evolution is slow with periodic variations over timescale of 24 hours, while suspended cables show fast variations due to wind.

4.4 Optical PMD Compensation

Unlike chromatic dispersion, PMD cannot be simply compensated with a passive device since any changes in environmental conditions cause the PMD vector orientation and magnitude to change. There are a number of optical PMD compensation schemes employing increasing complexity^[22]. Such a compensation system is independent of the optical data format.

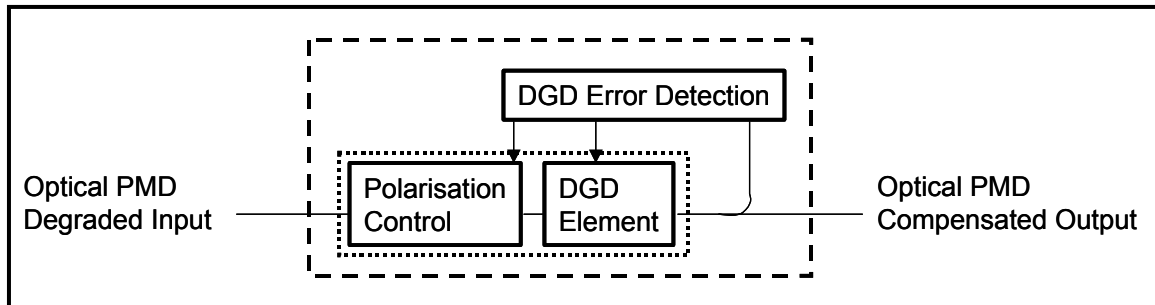


Figure 27 Block diagram of an Optical PMD compensation system

There are three basic parts to an optical adaptive PMD compensator, see Figure 27;

- 1) A PMD detector giving a PMD error signal,
- 2) A DGD or PMD compensation element,
- 3) An adaptive polarisation controller to align the compensation element with the system.

Associated with each scheme is the number of degrees of freedom (DOF) that the scheme can compensate. The higher the number of degrees of freedom the greater the complexity of the PMD element and the greater the orders of PMD that are compensated. Three degrees of freedom will fully compensate first order PMD while six degrees of freedom are required to compensate first and second order PMD.

4.5 Compensation Advantage

Simulations of fibre were performed using the random rotation of the birefringence model^[23]. Each element of the fibre is assumed to be a linear birefringent element where the angle, θ , of the birefringence axis follows a random process

$$\frac{\partial \theta}{\partial z} = \sqrt{\partial z} \sigma_{\theta} v \quad (74)$$

where v is an independent random variable taken from a Gaussian distribution with zero mean and unit standard deviation. The coupling length, L_c , is determined by, σ_{θ} , as $L_c \sim 4/\sigma_{\theta}^2$. The simulation took 200 elements each 50 m long with modal dispersion of 1 fs.m⁻¹, $\sigma_{\theta} = 0.2$ giving a coupling length of 100 m. This gave an uncompensated PMD of 1043 fs with a second order PMD magnitude of 6.301×10^5 fs². The ratio of the RMS PSP rotation to the RMS DGD change was $\sqrt{8.5}$ compared with the statistical accepted value of $\sqrt{8}$.

Using this simulated fibre and providing compensation at the centre optical frequency, the PMD either side of the compensation point was calculated. The compensation was based on either a variable DGD element (3DOF) or compensation with 6DOF. Figure 18 and 19 shows the PMD and 2nd order PMD magnitude as a function of frequency offset. For the PMD it was found that, for compensation with 3DOF and with 6DOF, the compensated PMD within 386 GHz was lower than the original uncompensated PMD but further away there is degradation as the PMD compensation acts to increase the total system PMD. The compensated PMD with 6DOF was lower than that with 3DOF within this bandwidth. A similar simulation for an uncompensated system PMD of 0.5 ps gave a crossing at 687 GHz suggesting a bandwidth for improved PMD of approx $\sim 0.35/\text{PMD}$. Thus the compensated PMD bandwidth appears to be related to the uncompensated PMD. The 2nd order PMD magnitude shows immediate degradation for 3 DOF compensation but shows improvement for 6 DOF compensation within a bandwidth of $\sim 0.15/\text{PMD}$. Higher order PMD effects may show narrower bandwidths.

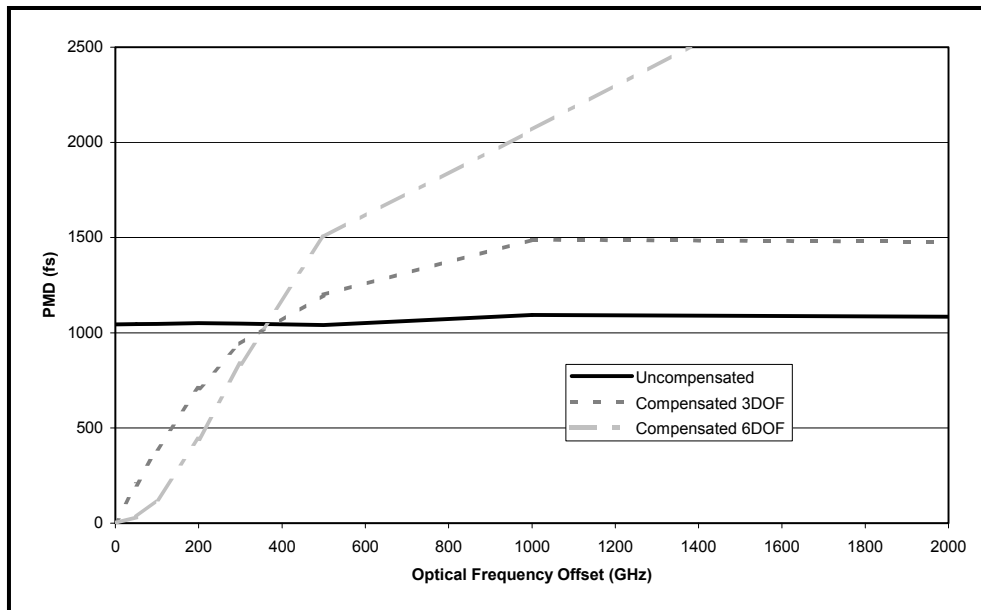


Figure 28 Effect of PMD compensation on first order PMD.

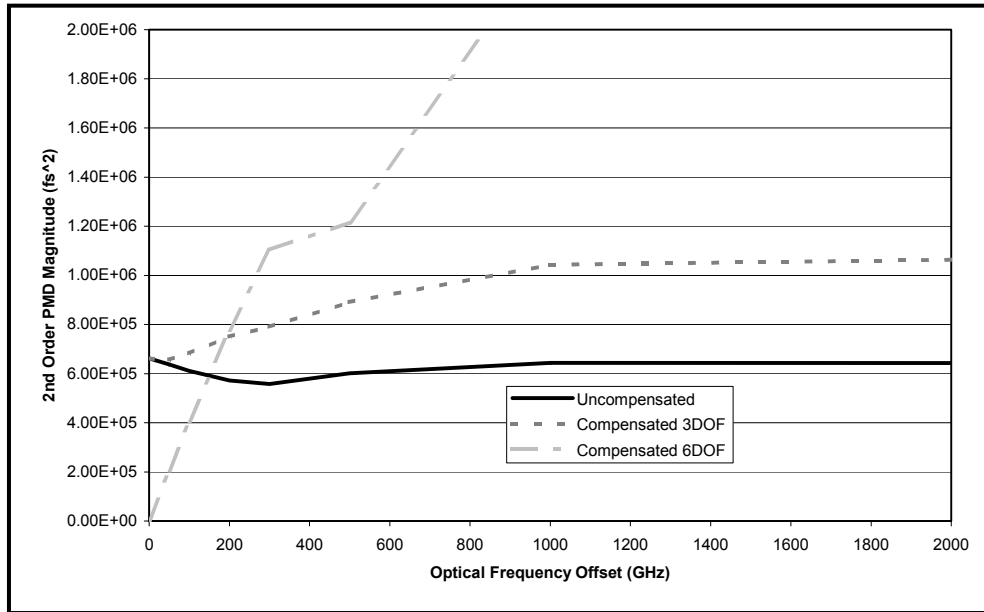


Figure 29 Effect of PMD compensation on 2nd order PMD magnitude.

Following the work by Frank Bruyère^[20] the worst case penalty due to first and second order PMD is given by

$$P(\text{dB}) = 10 \frac{|\Omega|^2}{T_{\text{bit}}^2} + 408 C_0 \frac{|\Omega_\omega|}{T_{\text{bit}}^2} \quad (75)$$

where C_0 is the normalised chromatic dispersion given by

$$C_0 = \frac{1}{2} \left(\frac{\beta''}{T_{\text{bit}}^2} \right) \quad (76)$$

where β'' is the second differential of the propagation constant with optical frequency, ω .

For an arbitrary outage probability of 1 second / day (10^{-5}) we have the maximum DGD equal to 2.9 times the PMD and the maximum 2nd order PMD magnitude equal to 5.3 time the RMS 2nd order PMD magnitude. Taking the worst case that both these maxima occur together we can find the PMD that will give a 1 dB penalty for the various compensation schemes, for a normalised chromatic dispersion of 0.08, see Table 4.

PMD Compensation	Maximum Uncompensated PMD (UI)
None	0.09
Fixed DGD, 2DOF	0.10
Variable DGD, 3DOF	0.13
Two fixed DGD, 4 DOF	0.16

Table 4

While the exact PMD values rely on worst case statistics and the work of F Bruyère^[20] the trend suggests that in order for PMD compensation to provide any real improvement high order compensation will be required and that even then the original system PMD allowable is only doubled.

5. Polarisation Measurement

5.1 Polarisation State

In order to measure polarisation properties it is necessary to know which polarisation state is incident upon the device under test. It is therefore necessary to either set or measure the polarisation state of the optical input. An instrument that measures the polarisation state of an incident optical input is known as a polarisation state analyser or polarimeter.

If we use the Stokes vector method of specifying polarisation state then this can be achieved by measuring the intensity of the light passed through a number of polarisation analysers. A minimum of four analysers is required but this can be extended to six analysers. The analysers must form a Mueller set for example, linear vertical (V), linear horizontal (H), linear at +45° (+) and right hand circular (R). The intensity of the light transmitted by each analyser in turn is measured making sure that the polarisation state of the input light remains constant. Alternatively the input light can be split and the intensity of the light transmitted by each analyser measured simultaneously in which case the splitting device must not alter the polarisation state of each part of the light. The insertion loss of each analyser must be equal or known and a correction applied. Thence the normalised Stokes vector is given by

$$s = \begin{pmatrix} s_1 \\ s_2 \\ s_3 \end{pmatrix} = \begin{pmatrix} \frac{I_V - I_m}{I_m} \\ \frac{I_+ - I_m}{I_m} \\ \frac{I_R - I_m}{I_m} \end{pmatrix} = \begin{pmatrix} \frac{I_V - I_H}{I_V + I_H} \\ \frac{I_+ - I_-}{I_+ + I_-} \\ \frac{I_R - I_L}{I_R + I_L} \end{pmatrix} \quad (77)$$

where I_m is the mean Intensity $\frac{1}{2}(I_V + I_H)$, and I_j is the intensity transmitted through analyser j . For fully polarised inputs it is possible to avoid using the circular polarisation analyser and calculate s_3 assuming $|s|^2 = s_1^2 + s_2^2 + s_3^2 = 1$.

An alternative technique is to measure the transmission through a rotating quarter waveplate and fixed linear polarisation analyser, see Figure 30.

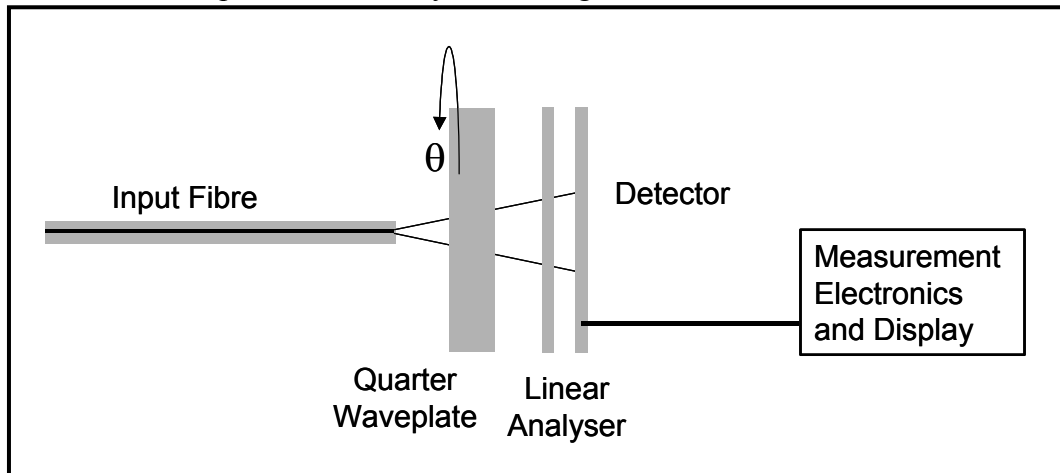


Figure 30 Layout of a rotating quarter waveplate polarimeter.

Taking the Mueller matrixes from section 2.4 and setting the azimuth of the linear polarisation analyser to zero, the detected intensity is given by

$$I = \left\{ \frac{1}{2} \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(2\theta) & -\sin(2\theta) & 0 \\ 0 & \sin(2\theta) & \cos(2\theta) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(2\theta) & \sin(2\theta) & 0 \\ 0 & -\sin(2\theta) & \cos(2\theta) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} s_0 \\ s_1 \\ s_2 \\ s_3 \end{pmatrix} \right\}_0$$

$$= s_0 + \frac{1}{4}s_1 + \frac{1}{4}\cos(4\theta)s_1 + \frac{1}{4}\sin(4\theta)s_2 + \frac{1}{2}\sin(2\theta)s_3$$

(78)

By rotating the quarter waveplate at a frequency f (revolutions per second) and detecting the electrical signal at frequency $2f$, $4f$ and DC it is possible to calculate s_0 , s_1 , s_2 and s_3 . A small correction is required if the quarter waveplate is not exactly a quarter wave and this correction is likely to be wavelength dependent.

It is possible to simplify this if the input polarisation state is known to be fully polarised. The quarter waveplate is no longer required and the linear polariser is rotated, the detected intensity is then given by

$$I = \frac{1}{2}s_0 + \frac{1}{2}\cos(2\theta)s_1 + \frac{1}{2}\sin(2\theta)s_2. \quad (79)$$

Since it is assumed that the input is fully polarised then s_3 can be calculated from $s_0^2 = s_1^2 + s_2^2 + s_3^2$, except for the ambiguity of the sign.

In both these polarisation state measurement schemes it is necessary to ensure that there is no intensity variation due to wobble in the mechanical alignment of the components.

5.2 Birefringence and Beat Length

It is possible to measure the linear retardance of an optical element by rotating that element and observing the output polarisation state on a polarisation state analyser. For a linear polarised input state the locus of the output state on the Poincaré sphere will form a figure of eight with the cross occurring when the linear retardance axis are aligned with the linear polarised input. The extrema of the loops are related to the retardance. Such a method can measure a linear retardance up to a half waveplate. Beyond this ambiguities of π in the retardance occur.

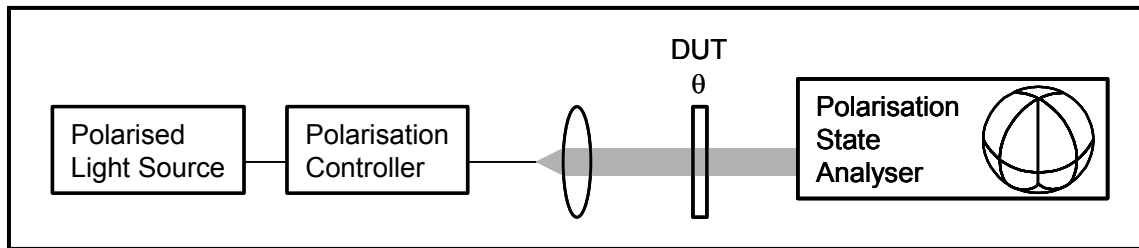


Figure 31 Set up to measure retardance of DUT

Figure 31 shows a system to measure the retardance of a bulk waveplate. Figure 32 shows the loci of the output state of polarisation for a linear polarised input for three different retardances. It is possible to calculate the effect for a general input polarisation state but the technique is most sensitive for near linear polarised inputs.

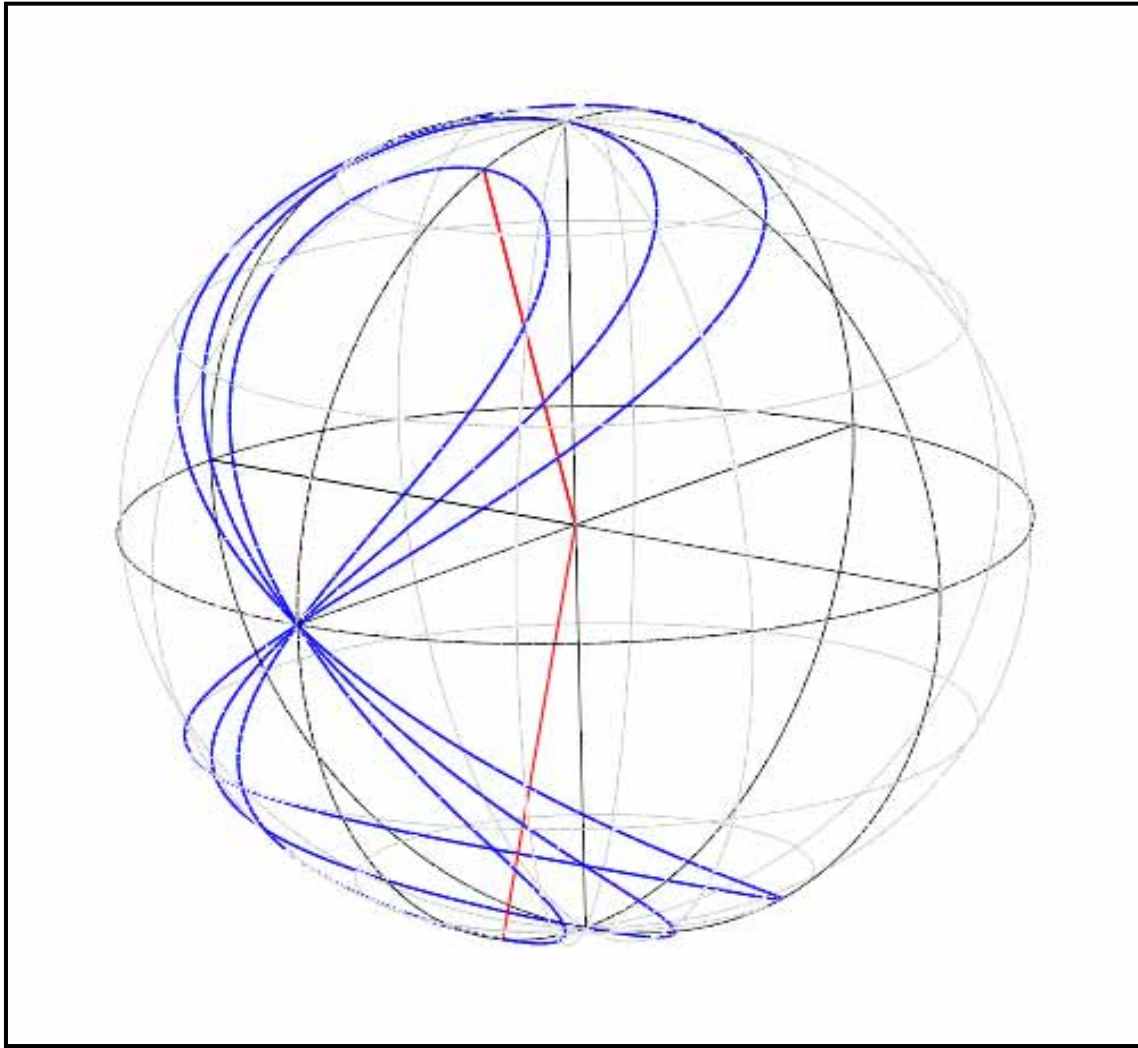


Figure 32 Showing the loci of the output polarisation state for a linear input polarisation state and three different retardances (1.27 rad, 1.57 rad and 1.87 rad) as they are rotated. The red vectors illustrate the extremes of the 1.27 rad retardance.

Figure 33 shows the loci of the output state of polarisation for an all fibre quarter waveplate formed of 4 turns of fibre around a 50mm diameter. The locus is plotted as the azimuth of the loops is rotated through 180° for three wavelengths of operation. The linear retardance was found to be near $\pi/4$. The loci do not form a closed figure of eight because the connecting fibre is twisted as the azimuth of the loop is changed and this twist introduces a circular retardance that effectively reduces the change in azimuth. The pattern is not centred on the equator since the input state to the fibre loop was not a linear polarised state and the output state undergoes a random polarisation transformation in the connecting fibre to the polarisation state analyser.

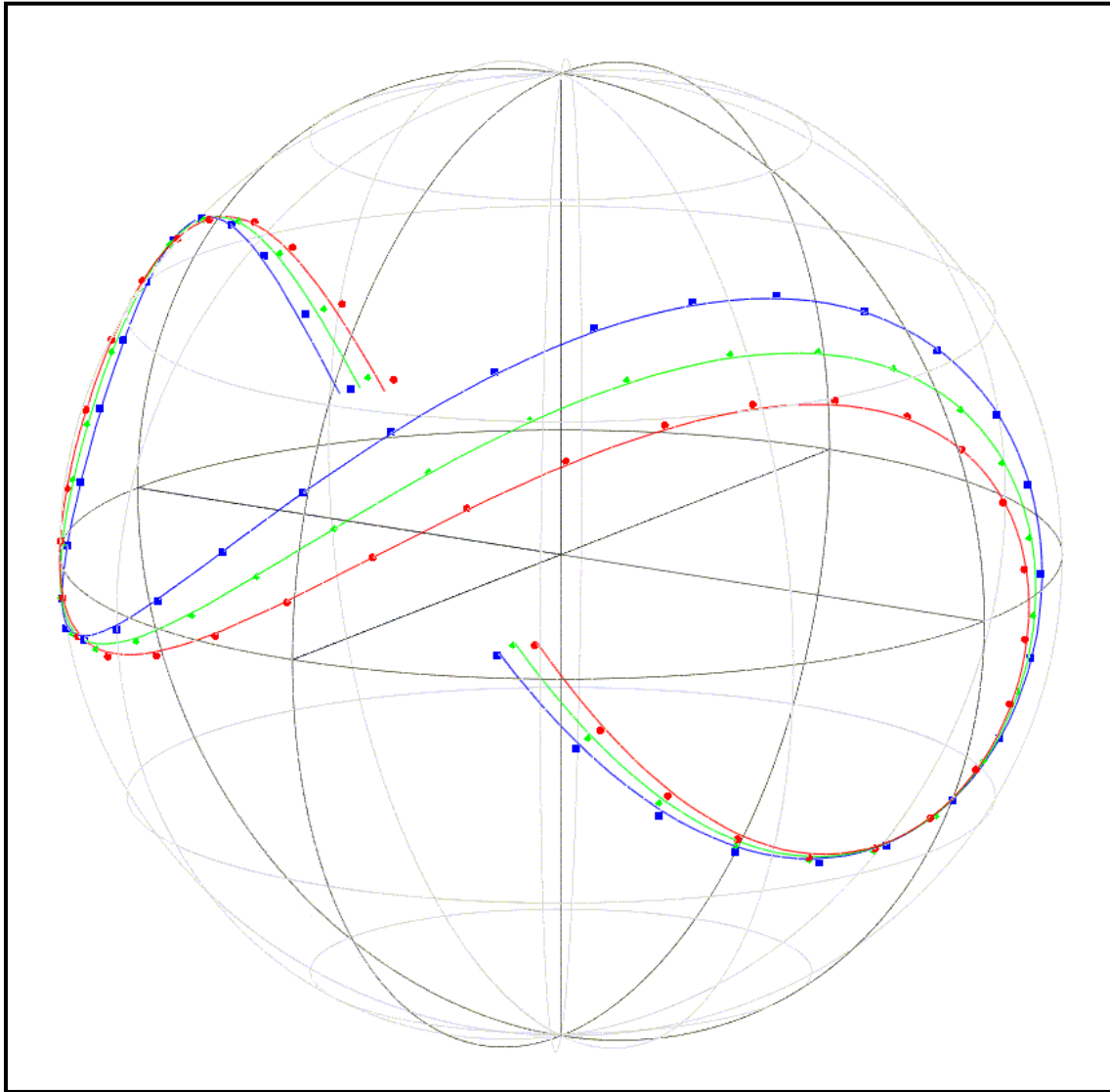


Figure 33 Loci of output polarisation state as loop azimuth is changed from 0° to 180°. Red Circles = 1540 nm, Green Diamonds = 1500 nm and Blue Squares = 1460 nm. Points are measured, lines are fitted.

The beat length is the period over which the polarisation state of the light propagating in an optical fibre evolves through one complete cycle. For example with a HiBi fibre if an optical beam is launched at 45° to the birefringence axis then as the optical beam propagates along the fibre the polarisation state will evolve through circular to linear at -45°, circular again and finally back to linear at 45°. Since there is no Rayleigh scatter in the direction of the electric field if the fibre is viewed perpendicular to the direction of propagation at 45° to the birefringent axis, the Rayleigh scatter will appear as bands with dark bands where the electric field of the propagating beam is pointing at the observer^[24].

The beat length can also be measured using POTDR, a polarised version of normal optical time domain reflectometry. The Rayleigh scattered light maintains the polarisation of the scattering signal such that the backreflected signal evolves at twice the rate of the forward propagating beam. By passing the backscatter signal through an analysing polariser the periodic rotation of the polarisation state is converted into an amplitude variation from which the beat length can be calculated^[25]. The OTDR resolution must be greater than the beat length. In practice for practical HiBi fibre the OTDR technique has insufficient resolution.

The optical frequency domain reflectometer technique, with its short distance resolution, has been shown to give good results^[26].

5.3 Differential Group Delay and Polarisation Mode Dispersion

Numerous methods for measuring polarisation mode dispersion have been developed over the years. These methods fall into two categories, those based on frequency domain measurements and those based on time domain measurements. Frequency domain methods include the polarisation state analyser based methods and fixed analyser method. Time domain methods include; interferometric methods, phase shift methods and pulse time of flight methods. No one method is ideal for all measurement situations since there is a broad range of system and components types requiring PMD measurement.

These components can be grouped, for example

- Installed Fibres (large bandwidth, no noise, unstable)
- Laboratory Fibres (large bandwidth, no noise, stable)
- DWDM component (narrow bandwidth, no noise, stable)
- Optical Amplifiers (mid bandwidth, ase noise, stable)

Table 5 below shows these component groups and the PMD measurement methods, marking with a tick the suitable methods and a cross the least suitable method.

	Installed Fibre	Laboratory Fibre	DWDM Component	Optical Amplifier
Polarimeter Methods	×	✓		✓
Fixed Analyser	✓	✓	×	
Interferometric	✓	✓	×	
Phase Shift			✓	

Table 5 Methods and their suitability.

5.3.1 Polarisation State Analyser based measurement of DGD and PMD

As stated above the polarisation state analyser based methods used to measure DGD and PMD are frequency domain techniques. The state of polarisation exiting the fibre or component under test is measured as a function of optical frequency. Measurements are made for a number of known input states of polarisation as the optical frequency is scanned. The measurements are analysed to calculate DGD and PMD. A possible experimental arrangement is shown in Figure 34.

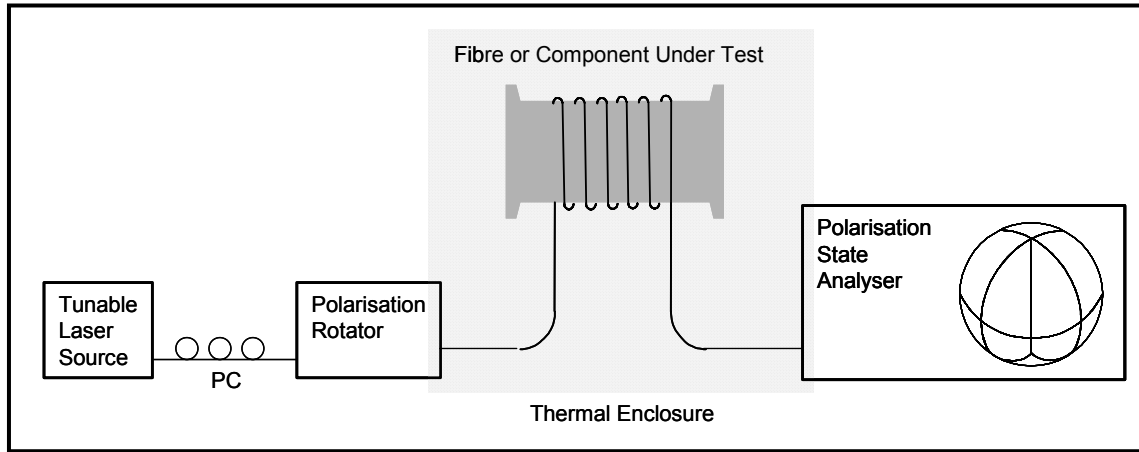


Figure 34 Possible experimental arrangement for polarisation state analyser based PMD measurement.

A considerable number of data analysis methods for polarisation state analyser based measurements of this type have been developed. These include Jones matrix eigenanalysis, Poincaré sphere analysis, Poincaré sphere arc method, Poole method and Mueller matrix method. All of these methods give essentially the same result. However since the calculation of DGD involves a numerical differentiation with optical frequency some of the methods are better suited for larger steps of optical frequency. Other methods are able to calculate the full polarisation dispersion vector, Ω , and not just the DGD magnitude.

For Jones matrix Eigenanalysis^{[27][28]} the input states of polarisation are taken to be the linear states of polarisation with azimuth 0° , 45° and 90° . These can be easily set using a linear polariser giving set states that will be independent of optical frequency. The measured output polarisation states are S_H , S_+ and S_V respectively. The measured output states are converted to Jones vectors and these are converted to complex ratios using

$$\begin{aligned} k_1 &= \frac{a_{Hx}}{a_{Hy}} = \frac{1}{S_{H2}^2 + S_{H3}^2} [S_{H2}(1 - S_{H1}) + i S_{H3}(1 - S_{H1})] \\ k_2 &= \frac{a_{Vx}}{a_{Vy}} = \frac{1}{S_{V2}^2 + S_{V3}^2} [S_{V2}(1 - S_{V1}) + i S_{V3}(1 - S_{V1})] \\ k_3 &= \frac{a_{+x}}{a_{+y}} = \frac{1}{S_{+2}^2 + S_{+3}^2} [S_{+2}(1 - S_{+1}) + i S_{+3}(1 - S_{+1})] \\ k_4 &= \frac{k_3 - k_2}{k_1 - k_3} \end{aligned} \quad (80)$$

where a_{Hx} and a_{Hy} are the two complex elements of the Jones matrix for the output polarisation state corresponding to the linear horizontal input polarisation state. Following the Jones algorithm the transmission matrix, T , for the system under test is given, to within a complex constant, by

$$T(\omega) = \begin{pmatrix} k_1 k_4 & k_2 \\ k_4 & 1 \end{pmatrix} \quad (81)$$

The DGD is then approximated by

$$\Delta\tau = \left| \frac{\text{Arg}\left(\frac{\rho_1}{\rho_2}\right)}{\delta\omega} \right| \quad (82)$$

where ρ_1 and ρ_2 are the eigenvalues of $T(\omega+\delta\omega)T(\omega)^{-1}$ and the Arg function is defined by $\text{Arg}(\alpha e^{i\theta}) = \theta$.

For Poincaré sphere analysis the output polarisation states S_H and S_+ are measured for the two linear input polarisation states at azimuth 0° and 45° . The vector S_C is calculated as the cross product of S_H and S_+ . These three vectors define a rectangular coordinate system that rotates around the polarisation dispersion vector, Ω , as the optical frequency is changed. By geometry the magnitude of the polarisation dispersion vector, equal to the DGD is given by

$$|\Omega| = \sqrt{\frac{1}{2} \left[\frac{\partial S_H^2}{\partial \omega} + \frac{\partial S_+^2}{\partial \omega} + \frac{\partial S_C^2}{\partial \omega} \right]}. \quad (83)$$

For practical measurements only finite differences are known and a more accurate expression can be obtained given by

$$|\Omega| = \frac{2}{\delta\omega} \arcsin \left(\frac{1}{2} \sqrt{\frac{1}{2} [\Delta S_H^2 + \Delta S_+^2 + \Delta S_C^2]} \right). \quad (84)$$

where ΔS_H is the magnitude of the change of the vector S_H as the optical frequency is changed from ω to $\omega+\delta\omega$.

For Poole^[29] analysis the output polarisation states are measured for three input polarisation states S_H , S_+ and S_L , the two linear input polarisation states at azimuth 0° and 45° and the left hand circular polarised input. There is no requirement for these exact input states of polarisation, however the input states must remain fixed as the optical frequency is changed. The polarisation vector, Ω , is given by

$$\Omega = \frac{\frac{\partial S_i}{\partial \omega} \times \frac{\partial S_j}{\partial \omega}}{\frac{\partial S_i}{\partial \omega} \cdot S_j}. \quad (85)$$

where S_i and S_j are any pair taken from S_H , S_+ and S_L . For output polarisation states that are close to the principle state of polarisation large uncertainties can arise from noise in the calculation of the derivatives. Poole suggests the following criteria for selecting the pair of states used in calculating the polarisation vector

$$\begin{aligned} \frac{\frac{\partial S_i}{\partial \omega} \times \frac{\partial S_j}{\partial \omega}}{\frac{\partial S_i}{\partial \omega} \cdot \frac{\partial S_j}{\partial \omega}} &> \frac{1}{\sqrt{2}} \\ \left| \frac{\Omega}{|\Omega|} \times S_{i,j} \right| &> \frac{1}{\sqrt{2}} \end{aligned} \quad (86)$$

The DGD can then be calculated from the magnitude of the polarisation vector.

For Mueller matrix analysis^[30] the output polarisation states S_1 and S_a are measured for two linear input polarisation states where $S_1 \times S_a \neq 0$. These could be linear polarisation states at azimuth 0° and 45° . The Mueller rotation matrix is determined from S_1 , $S_3 = S_1 \times S_a / |S_1 \times S_a|$ and $S_2 = S_3 \times S_1$ using

$$R = \begin{pmatrix} (S_1)_1 & (S_2)_1 & (S_3)_1 \\ (S_1)_2 & (S_2)_2 & (S_3)_2 \\ (S_1)_3 & (S_2)_3 & (S_3)_3 \end{pmatrix} R_0. \quad (87)$$

where R_0 is an arbitrary rotation connecting the actual input polarisation state to the output states for output states S_1 and S_a . Such that

$$S_1 = \begin{pmatrix} (S_1)_1 & (S_2)_1 & (S_3)_1 \\ (S_1)_2 & (S_2)_2 & (S_3)_2 \\ (S_1)_3 & (S_2)_3 & (S_3)_3 \end{pmatrix} R_0 S_{1\_in}$$

$$S_a = \begin{pmatrix} (S_1)_1 & (S_2)_1 & (S_3)_1 \\ (S_1)_2 & (S_2)_2 & (S_3)_2 \\ (S_1)_3 & (S_2)_3 & (S_3)_3 \end{pmatrix} R_0 S_{a\_in} \quad (88)$$

where S_{1_in} and S_{a_in} are the input polarisation states which give output polarisation states S_1 and S_a . The change in the rotation matrix as the optical frequency is changed from ω to $\omega + \delta\omega$ is given by

$$R_\delta = R(\omega + \delta\omega) R(\omega)^T$$

$$= \cos(\phi) \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + [1 - \cos(\phi)] \begin{bmatrix} p_1 p_1 & p_2 p_1 & p_3 p_1 \\ p_1 p_2 & p_2 p_2 & p_3 p_2 \\ p_1 p_3 & p_2 p_3 & p_3 p_3 \end{bmatrix} + \sin(\phi) \begin{bmatrix} 0 & -p_3 & p_2 \\ p_3 & 0 & -p_1 \\ -p_2 & p_1 & 0 \end{bmatrix}. \quad (89)$$

This can be solved to obtain

$$\cos(\phi) = \frac{1}{2} [(R_\delta)_{11} + (R_\delta)_{22} + (R_\delta)_{33} - 1] \quad (90)$$

and

$$p \sin(\phi) = \frac{1}{2} \begin{bmatrix} (R_\delta)_{32} - (R_\delta)_{23} \\ (R_\delta)_{13} - (R_\delta)_{31} \\ (R_\delta)_{21} - (R_\delta)_{12} \end{bmatrix} \quad (91)$$

The polarisation vector $\Omega = \delta\tau p$, where

$$\delta\tau = \frac{\phi}{\delta\omega} \quad (92)$$

Since all of these measurements involve the change of polarisation state with optical frequency and measurements will be made at a sequence of optical frequencies the size of the step will affect the accuracy of the calculated result. The table 6 shows the upper limit of the product of optical frequency step, $\delta\omega$, and RMS DGD, $\langle \delta\tau^2 \rangle^{1/2}$ required to give various uncertainties for each data analysis method. The Jones matrix method, PSA method and Mueller method show considerably better performance for large optical steps with no-mode coupled devices as they make assumptions that the differential of the output state of polarisation will follow an arc on the Poincaré sphere. These methods give good performance when measuring the PMD, as an average or RMS DGD with an optical frequency step that is in effect limited by the likely maximum DGD. In the case where the local DGD is required then all the methods require a much smaller optical frequency step since the higher order PMD effects mean that the output state no longer follows a simple arc on the Poincaré sphere. This will be particularly important if the data is being analysed to calculate the PMD vector with the intention to differentiate this to then calculate the second order PMD.

Polarisation Mode Coupling of Device Under Test	Parameter	Max value of $\delta\omega < \Delta\tau^2 >^{1/2}$ vs Method			
		Jones	PSA	Mueller	Poole
None	Error DGD $< 1\%$	3.14	3.14	3.14	0.5
High	Error PMD, $< \Delta\tau^2 >^{1/2} < 1\%$	0.85	0.85	0.85	0.4
High	RMS error in DGD $< 1\%$	0.065	0.065	0.065	0.065
High	Max error in DGD $< 1\%$	0.007	0.007	0.007	0.007

Figure 6 Optical frequency step requirement.

The method is relatively slow and requires that the fibres polarisation properties do not change during the measurement process. As such the techniques are best suited to laboratory conditions where the fibre or component under test can be maintained in stable environmental conditions. The technique is less suitable for installed cables and unsuitable for aerial cables where the polarisation properties change in seconds. The technique is able to measure the full PMD vector, DGD and the principle state of polarisation, PSP, that allows the subsequent calculation of second and higher order PMD effects.

5.3.2 Fixed Analyser Method to measure PMD

The fixed analyser method used to measure PMD can provide either frequency domain or time domain measurements depending on the data analysis techniques^[31]. A polarised broadband source is transmitted through the fibre or component under test and the output signal passed through an analyser to an optical spectrum analyser. The optical intensity as a function of optical frequency is recorded, this is normalised to the intensity obtained with the analyser removed or the sum of intensities with the analyser rotated through 90° . A possible experimental arrangement is shown in Figure 35. As an alternative the source can be tunable and the optical spectrum analyser replaced by a power meter. The polarisation controller is only required when the fibre or component under test has low mode coupling and is used to maximize the intensity variation with optical frequency. However with high polarisation-mode coupled fibres the measurement uncertainty can be reduced by averaging over the 9 state Mueller set, that is for all combinations of three input states orthogonal on the Poincaré sphere and three analyser states orthogonal on the Poincaré sphere.

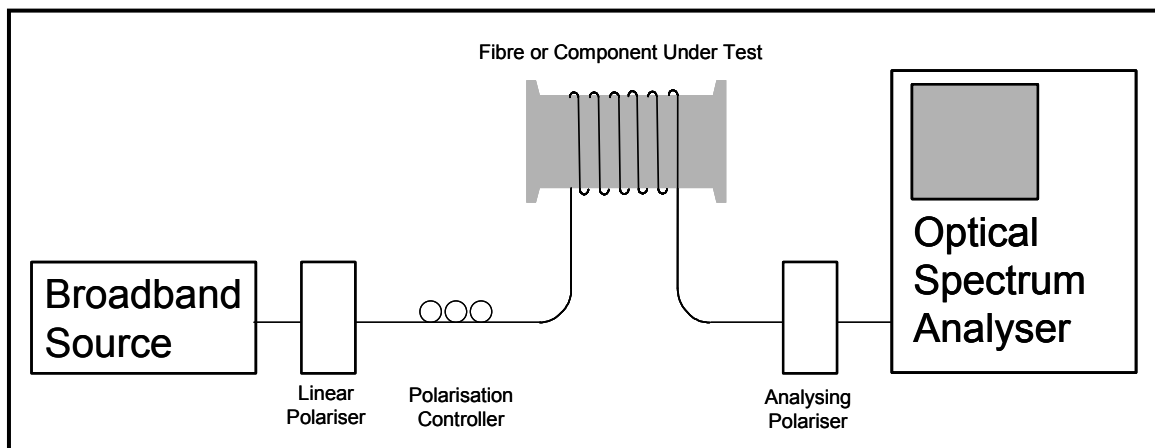


Figure 35 Possible experimental arrangement for Fixed analyser PMD measurement.

Typical measurement scans for a fixed low mode coupled device and a high mode coupled device are shown in Figures 36 and 37.

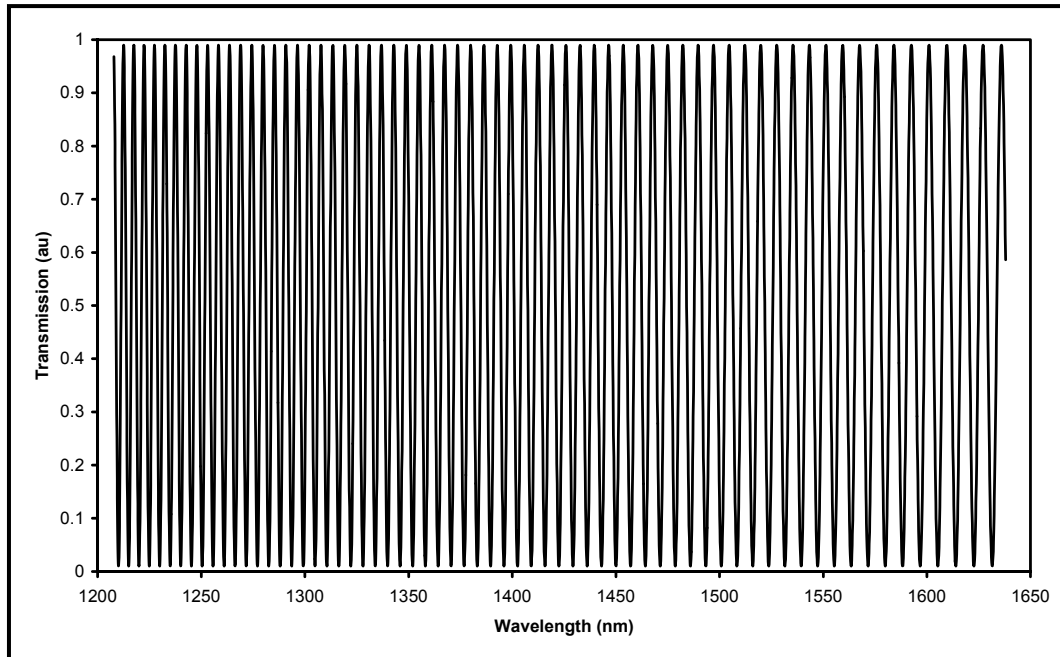


Figure 36 Fixed analyser spectrum of a low polarisation mode coupled fibre with 1000 fs of DGD

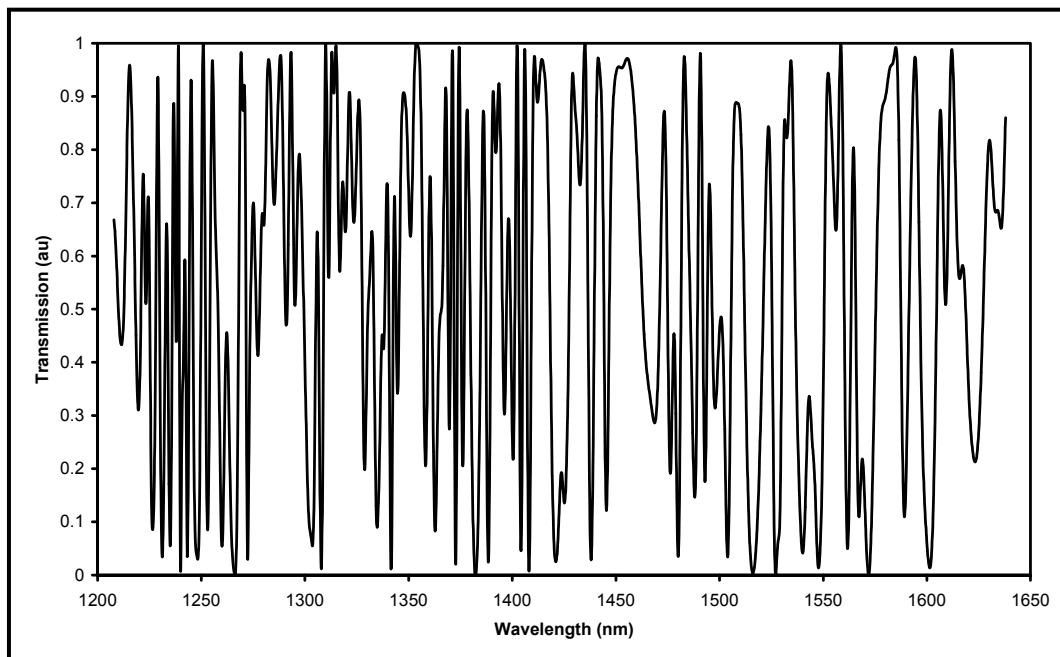


Figure 37 Fixed analyser spectrum of a high polarisation mode coupled fibre with ~1000 fs of PMD

The data can be analysed through cycle counting or by Fourier transform.

With cycle counting the number of extrema, N_e , (peaks and troughs) or the number of mean level crossings, N_m , are counted within the measurement window from λ_1 to λ_2 . If extrema or mean level crossings form the endpoints of the window then the number is reduced by 1.

For low polarisation mode coupling the mean DGD is given by^[32]

$$\Delta\tau = \frac{\pi N_e}{\Delta\omega} = \frac{\pi N_m}{\Delta\omega} = \frac{N_e \lambda_1 \lambda_2}{2c(\lambda_2 - \lambda_1)} = \frac{N_m \lambda_1 \lambda_2}{2c(\lambda_2 - \lambda_1)} , \quad (93)$$

where $\Delta\omega$ is the width of the optical frequency window. Figure 36 shows $N_e = N_m = 130$ for a wavelength window from 1208 nm to 1638 nm giving a DGD = 997 fs.

For high polarisation mode coupling the mean DGD is given by^[32]

$$\langle \Delta\tau \rangle = 0.824 \frac{\pi N_e}{\Delta\omega} = \frac{4}{\pi} \frac{\pi N_m}{\Delta\omega} = 0.824 \frac{N_e \lambda_1 \lambda_2}{2c(\lambda_2 - \lambda_1)} = \frac{4}{\pi} \frac{N_m \lambda_1 \lambda_2}{2c(\lambda_2 - \lambda_1)} , \quad (94)$$

and thus the RMS DGD is

$$\langle \Delta\tau^2 \rangle^{1/2} = 0.894 \frac{N_e \lambda_1 \lambda_2}{2c(\lambda_2 - \lambda_1)} = 1.382 \frac{N_m \lambda_1 \lambda_2}{2c(\lambda_2 - \lambda_1)} . \quad (95)$$

Thus in the high polarisation mode coupling regime $\langle N_e \rangle = 1.54 \langle N_m \rangle$. Figure 37 shows $N_e = 149$, $N_m = 94$ for a wavelength window from 1208 nm to 1638 nm giving a RMS DGD of 1021 fs and 996 fs respectively.

An alternative method of analysis for a fixed analyser scan is to use a Fourier transform. For this the data set must be in equal intervals of optical frequency and not wavelength. Scans that are measured with equal wavelength steps must first be transformed to equal frequency steps by interpolation. The DC component of the fixed analyser vs frequency scan is usually removed by subtracting the mean signal level and the data is often multiplied by a Hamming window to reduce the noise in the Fourier transform due to the non-repetitive nature of the scan. The optical power transmission of the fixed analyser scan in the frequency domain, $R(\omega)$ is Fourier transformed and the magnitude of the amplitude in the time domain calculated as $P(\delta\tau)$. The Fourier transform of the low polarisation mode coupled fibre shown in Figure 36 is shown in Figure 38.

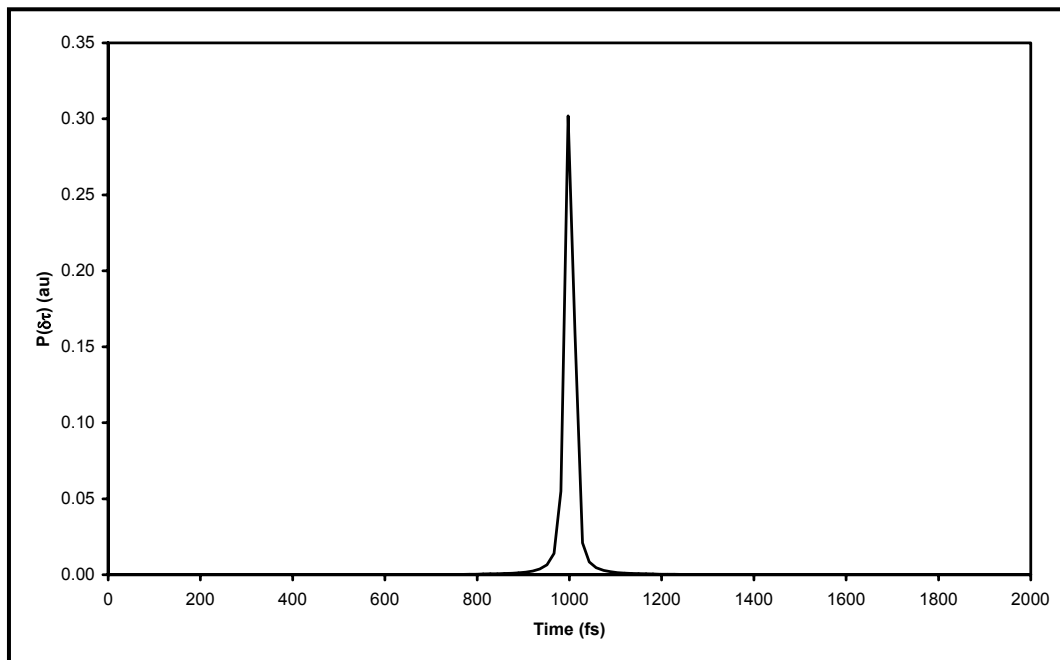


Figure 38 Fourier transform of fixed analyser scan of a low polarisation mode coupled fibre with 1000 fs DGD.

For low polarisation mode coupled fibres the DGD is given by

$$\Delta\tau = \frac{\int_0^\infty P(\delta\tau) \delta\tau \partial\delta\tau}{\int_0^\infty P(\delta\tau) \partial\delta\tau}, \quad (96)$$

where in practical measurement situations the range over which the integral is taken is restricted to avoid integrating the noise in $P(\delta\tau)$.

The Fourier transform of the high polarisation mode coupled fibre shown in Figure 37 is shown in Figure 39. The RMS DGD is given by the second moment of the time domain distribution given by the Fourier transform. For highly polarisation mode coupled fibres the distribution is Gaussian and could be fitted to a Gaussian equation.

$$\langle \Delta\tau^2 \rangle^{1/2} = \sigma_R = \frac{\int_0^\infty P(\delta\tau) \delta\tau^2 \partial\delta\tau}{\int_0^\infty P(\delta\tau) \partial\delta\tau}. \quad (97)$$

Again in practical measurement situations in order to avoid integrating any residual DC component and noise the range of integration is restricted. The second moment, σ_R , is then calculated by iteration of the following equation,

$$\frac{\int_a^b P(\delta\tau) \delta\tau^2 \partial\delta\tau}{\int_a^b P(\delta\tau) \partial\delta\tau} = \int_a^b \sqrt{\frac{2}{\pi}} \frac{1}{\sigma_R} e^{-\left(\frac{\delta\tau^2}{2\sigma_R^2}\right)} \delta\tau^2 \partial\delta\tau. \quad (98)$$

where a and b are chosen such that a is just greater than the any DC peak and b is just less than the point where the Fourier transform amplitude is more than 2 times the RMS noise.

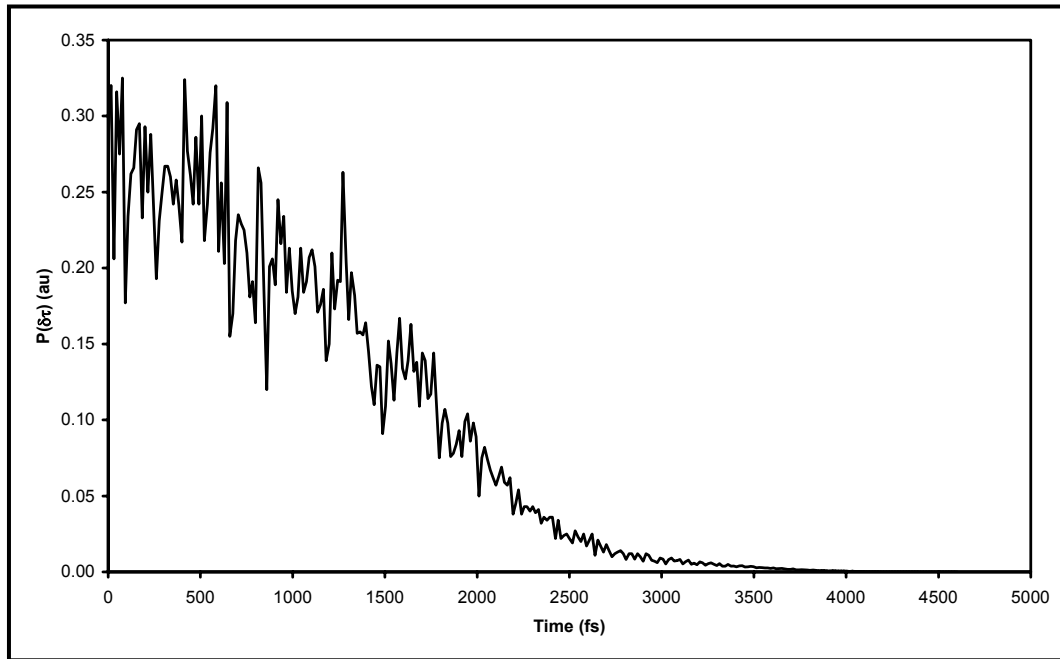


Figure 39 Fourier transform of fixed analyser scan of a high polarisation mode coupled fibre with ~ 1000 fs PMD.

5.3.3 Interferometric Method to measure PMD

The interferometric measurement method is a time domain method to measure PMD^[33]. A polarised broadband source is transmitted through the fibre or component under test and the output signal analysed through a linear polariser and a Michelson interferometer. A basic configuration is shown in Figure 40. The polarisation controllers allow the input and analysed states to be adjusted. The input signal is split into the principle states of polarisation and undergoes PMD. The interferometer provides additional delay and allows interference between the polarisation modes that fall within the coherence time of the source. The analyser mixes the output states to generate the interference signal. Interference occurs when the sum of the delay in the fibre or component under test and the delay in the interferometer is equal to zero. For a low mode coupled device this occurs when the delay in the interferometer is equal to the DGD or when the delay in the interferometer is zero when the autocorrelation of the source coherence function is obtained. For low polarisation-mode coupled fibre and components the polarisation controllers are adjusted to maximize the intensity of the satellite peaks. With high polarisation-mode coupling the polarisation controllers are used to reduce the measurement uncertainty by averaging over the 9 state Mueller set, that is for all combinations of three input states orthogonal on the Poincaré sphere and three analyser states orthogonal on the Poincaré sphere.

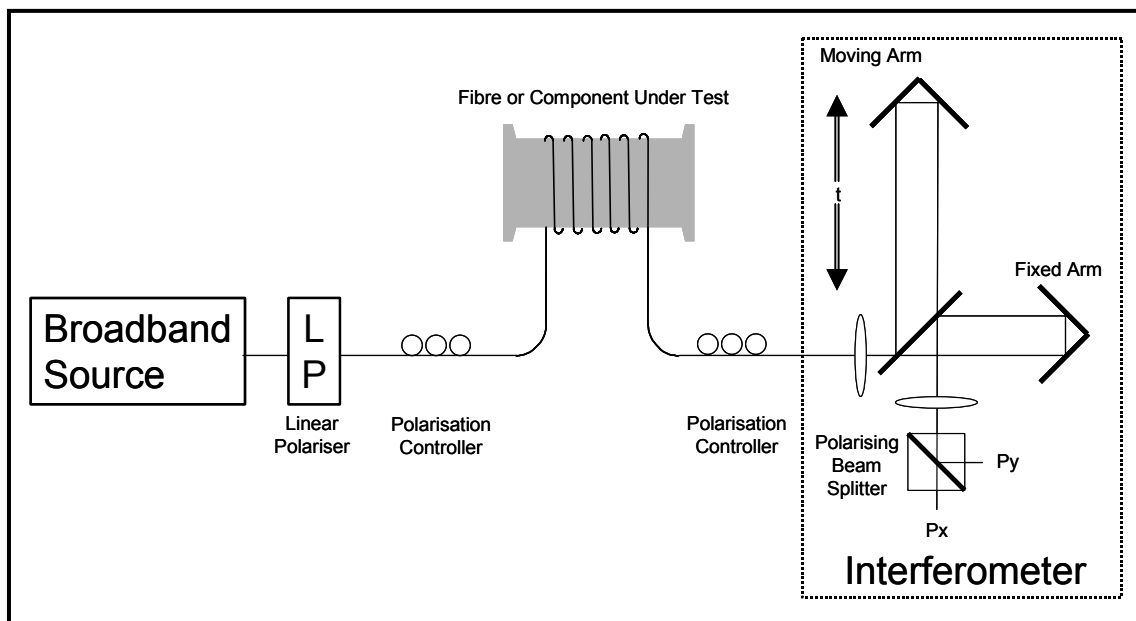


Figure 40 Layout of Interferometric measurement of PMD

There are two forms of data analysis; traditional analysis that calculates the standard deviation of the cross correlation envelope and general analysis that calculates the standard deviation of the square of the cross correlation envelope less the standard deviation of the square of the autocorrelation envelope.

In traditional analysis the interferogram P_x or P_y is analysed, examples of interferograms for low polarisation-mode coupled fibre and high polarisation-mode coupled fibre are shown in figures 41 and 42.

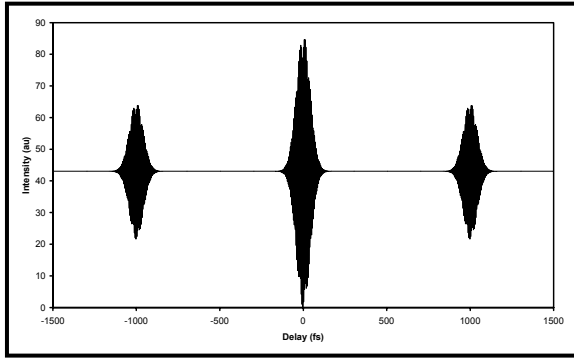


Figure 41 Low Polarisation-mode coupled fibre interferogram

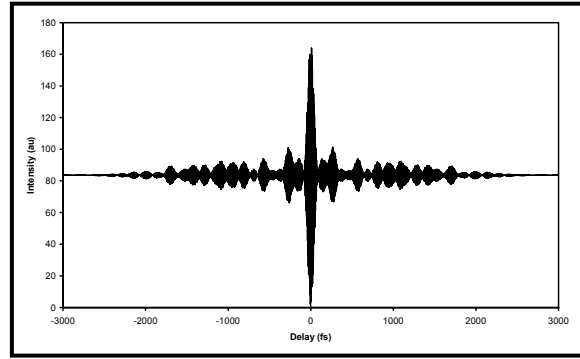


Figure 42 High Polarisation-mode coupled fibre interferogram

The envelope of the interferogram is analysed to obtain the PMD, thus the first stage is to calculate or measure directly the interferogram envelope, as shown in figures 43 and 44.

For low polarisation-mode coupled fibre the polarisation cross correlation peaks are separated by the DGD from the central autocorrelation peak. The DGD is given by half the delay between the two satellite peaks.

For high polarisation-mode coupled fibre the PMD (RMS DGD) is calculated from the second moment of the polarisation cross correlation envelope, σ_E , using^[34],

$$\langle \Delta\tau^2 \rangle^{1/2} = \sqrt{\frac{3}{4}} \sigma_E \quad (99)$$

The interferogram is first centred such that the centre of the autocorrelation peak is at zero delay. The second moment of the cross correlation envelope, σ_E , is then calculated by iteration of the following equation,

$$\frac{\int_{-b}^{-a} P_E(\delta\tau) \delta\tau^2 \partial\delta\tau + \int_a^b P_E(\delta\tau) \delta\tau^2 \partial\delta\tau}{\int_{-b}^{-a} P_E(\delta\tau) \partial\delta\tau + \int_a^b P_E(\delta\tau) \partial\delta\tau} = \int_{-b}^{-a} \sqrt{\frac{2}{\pi}} \frac{1}{\sigma_E} e^{-\left(\frac{\delta\tau^2}{2\sigma_E^2}\right)} \delta\tau^2 \partial\delta\tau + \int_a^b \sqrt{\frac{2}{\pi}} \frac{1}{\sigma_E} e^{-\left(\frac{\delta\tau^2}{2\sigma_E^2}\right)} \delta\tau^2 \partial\delta\tau \quad (100)$$

where P_E is the envelope of the interferogram and a and b are chosen such that a is just greater than the maximum width of the autocorrelation envelope and b is just less than the point where the envelope is more than 4 times the RMS noise.

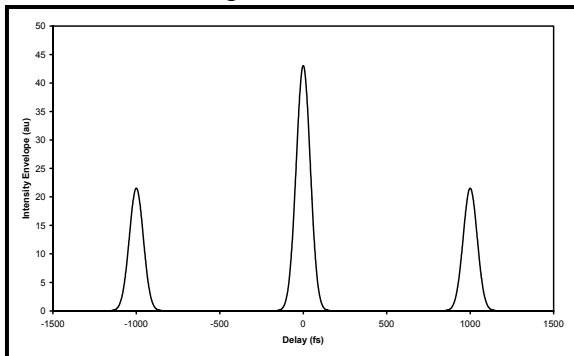


Figure 43 Envelope of low polarisation-mode coupled fibre.

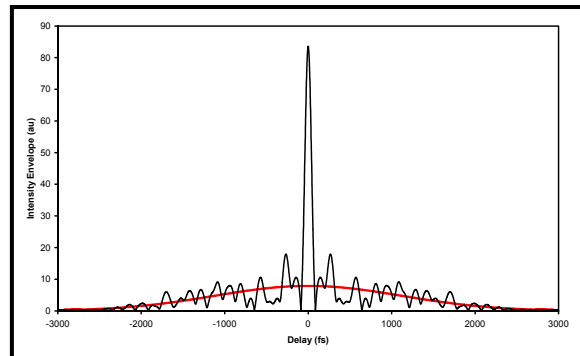


Figure 44 Envelope of high polarisation-mode coupled fibre (black) with calculated Gaussian cross correlation envelope in red.

In general analysis the DC part of each of the x and y interferogram is first removed. Thence the autocorrelation interferogram, P_0 , and cross correlation interferograms, P_1 , are separated by using

$$P_0(\delta\tau) = [P_x(\delta\tau) - P_x(\infty)] + [P_y(\delta\tau) - P_y(\infty)] , \quad (101)$$

and

$$P_1(\delta\tau) = [P_x(\delta\tau) - P_x(\infty)] - [P_y(\delta\tau) - P_y(\infty)] . \quad (102)$$

The envelope of the autocorrelation and cross correlation interferograms P_{0E} and P_{1E} is calculated, for the interferograms shown in figures 41 and 42 these are shown in figures 45 and 46. For low polarisation-mode coupled fibre and components the DGD is given by half the separation of the cross correlation peaks. For high polarisation-mode coupled fibre the RMS DGD is calculated from the second moments of the square envelopes using

$$\langle \Delta\tau^2 \rangle^{1/2} = \sqrt{\frac{3}{2}(\sigma_1^2 - \sigma_0^2)} , \quad (103)$$

where σ_0 and σ_1 are given by

$$\sigma_0^2 = \frac{\int_{-\infty}^{\infty} \delta\tau^2 P_{0E}(\delta\tau)^2 d\delta\tau}{\int_{-\infty}^{\infty} P_{0E}(\delta\tau)^2 d\delta\tau} , \quad (104)$$

and

$$\sigma_1^2 = \frac{\int_{-\infty}^{\infty} \delta\tau^2 P_{1E}(\delta\tau)^2 d\delta\tau}{\int_{-\infty}^{\infty} P_{1E}(\delta\tau)^2 d\delta\tau} . \quad (105)$$

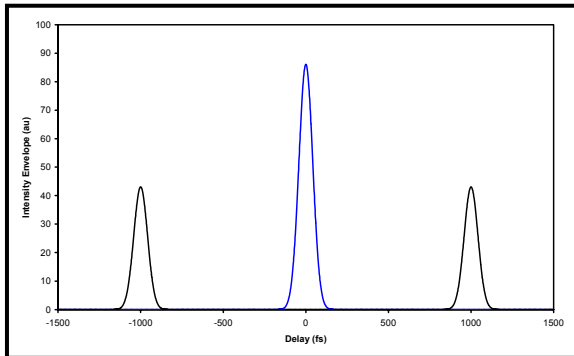


Figure 45 Cross correlation envelope and autocorrelation envelope (blue) for low polarisation-mode coupled fibre.

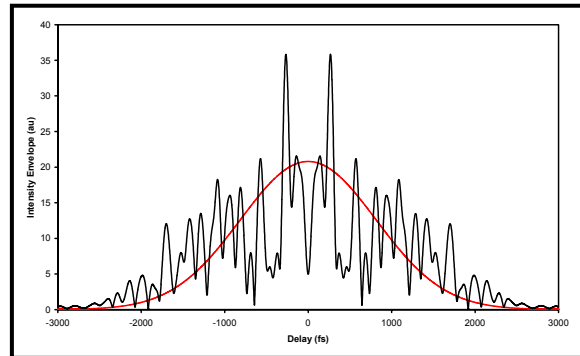


Figure 46 Cross correlation envelope and gaussian approximation (red) for high polarisation-mode coupled fibre.

This method is quick and is ideal for installed systems where there is no control of the mechanical stability of the fibres under test.

5.3.4 Phase Shift measurement of DGD and PMD

The phase shift measurement technique provides a time domain measurement of DGD and PMD. The method measures the phase shift of the modulation on an optical signal as the optical carrier polarisation state is changed. The phase shift is directly related to the transmission time so that the DGD can be easily measured. The basic layout is shown in Figure 47.

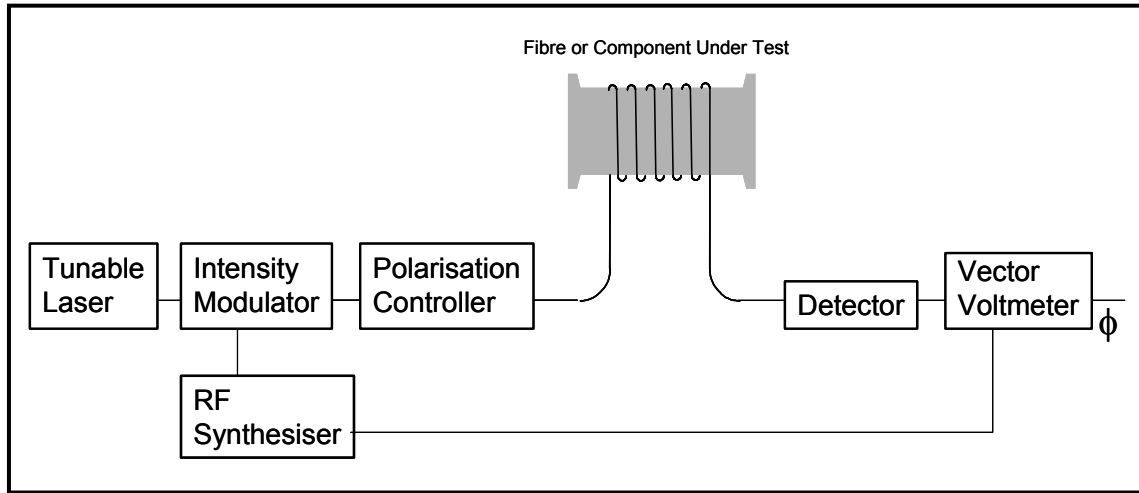


Figure 47 Basic layout of a phase shift system to measure DGD.

The system can be operated in two ways, by scanning the input polarisation state through all possible states and measuring the maximum and minimum phase and by measuring the output phase for a number of set input polarisation states. The first technique requires greater stability of the measurement equipment since the limited number of states in the second technique can be measured more quickly.

For the random scanning technique the DGD, $\Delta\tau$, is given by

$$\Delta\tau = \frac{\phi_{\text{Max}} - \phi_{\text{Min}}}{2\pi\nu}, \quad (106)$$

where ϕ_{Max} and ϕ_{Min} are the maximum and minimum measured phases in radians and ν is the modulation frequency. The DGD can be measured as a function of optical frequency by changing the optical frequency of the tunable laser source; thence the PMD can be calculated as the RMS average over optical frequency.

For the set state method the output phase is measured for a Mueller set of input polarisation states. These input polarisation states have three states orthogonal on the Poincaré sphere and a fourth which is opposite, for example Linear horizontal (H), Linear at $+45^\circ$ (+), Right hand circular (R) and linear vertical (V). The respective measured phases are ϕ_H , ϕ_+ , ϕ_R , and ϕ_V . From these the average phase, ϕ_0 , for a depolarised signal is calculated as

$$\phi_0 = \frac{1}{2}(\phi_H + \phi_V). \quad (107)$$

The DGD is then given by^[35]

$$\Delta\tau = \frac{1}{\pi\nu} \arctan \left\{ \sqrt{\tan^2(\phi_H - \phi_0) + \tan^2(\phi_+ - \phi_0) + \tan^2(\phi_R - \phi_0)} \right\}. \quad (108)$$

A slightly better technique is to measure the phase output for six polarisation states that are orthogonal and opposite on the Poincaré sphere, for example linear horizontal (H), linear vertical (V), linear at $+45^\circ$ (+), linear at -45° (-), right hand circular (R) and left hand circular (L). It is preferred to record the phase of orthogonal polarisation states consecutively to reduce the effect of any system drift. The DGD is then given by

$$\Delta\tau = \frac{1}{\pi\nu} \arctan \left\{ \sqrt{\tan^2\left(\frac{\phi_H - \phi_V}{2}\right) + \tan^2\left(\frac{\phi_+ - \phi_-}{2}\right) + \tan^2\left(\frac{\phi_R - \phi_L}{2}\right)} \right\}. \quad (109)$$

The technique is ideally suited to narrow band optical components since the optical bandwidth required to make a DGD measurement is only that required by the modulation frequency, ν .

5.3.5 Statistical Uncertainty in measurement of PMD

The uncertainty associated with a measurement of PMD is a combination of the equipment accuracy and the statistical nature of PMD. For low mode coupled devices this statistical uncertainty is low but for highly mode coupled fibres and components this statistical uncertainty will dominate the measurement uncertainty.

The DGD of a highly mode coupled fibre will change with environmental influence such that any single measurement of DGD will be a sample from the Maxwellian distribution. The PMD of such a fibre fully describes the distribution and is considered to be independent of minor changes in environmental conditions. Consider the polarimetric methods to measure PMD, although the uncertainty applies equally to all measurement methods. The DGD will be measured as a function of optical frequency and the PMD calculated as the RMS DGD. Each DGD value is a sample from the Maxwellian distribution and the PMD is the RMS mean of the distribution. The Maxwellian distribution has an RMS mean, $\langle \Delta\tau^2 \rangle^{1/2}$, given by,

$$\langle \Delta\tau^2 \rangle = \int_0^\infty P(\Delta\tau) \Delta\tau^2 \partial(\Delta\tau) = \int_0^\infty \sqrt{\frac{2}{\pi}} \frac{\Delta\tau^2}{\alpha^3} e^{-\frac{\Delta\tau^2}{2\alpha^2}} \Delta\tau^2 \partial(\Delta\tau) = 3\alpha^2 \quad (110)$$

and a standard deviation of the square DGD, $\sigma_{\Delta\tau^2}$, given by

$$(\sigma_{\Delta\tau^2})^2 = \int_0^\infty P(\Delta\tau) \Delta\tau^4 \partial(\Delta\tau) - \left(\int_0^\infty P(\Delta\tau) \Delta\tau^2 \partial(\Delta\tau) \right)^2 = 15\alpha^4 - 9\alpha^4 = 6\alpha^4 \quad (111)$$

In the measurements there will only be a finite number of DGD measurements and the uncertainty on the RMS mean, $\sigma_{\langle \Delta\tau^2 \rangle^{1/2}}$, will be given by

$$(\sigma_{\langle \Delta\tau^2 \rangle^{1/2}})^2 = \frac{(\sigma_{\Delta\tau^2})^2}{4 \langle \Delta\tau^2 \rangle N} = \frac{\alpha^2}{2N} = \frac{\langle \Delta\tau^2 \rangle}{6N} \quad (112)$$

where N is the number of independent measurements of DGD, $\Delta\tau$.

N Gisin^[36] et al propose the argument that there are two independent measurements points per cycle in the DGD curve as per the Nyquist sampling theorem, giving the number of independent measurements as

$$N = \sqrt{\frac{8}{3\pi}} \langle \Delta\tau^2 \rangle^{1/2} \frac{\Delta\omega}{\pi} = \sqrt{\frac{8}{3\pi}} \langle \Delta\tau^2 \rangle^{1/2} \left[\frac{2c(\lambda_2 - \lambda_1)}{\lambda_1 \lambda_2} \right] \quad (113)$$

where the RMS DGD average $\langle \Delta\tau^2 \rangle^{1/2}$ is taken over the optical frequency window, $\Delta\omega$, or over the wavelength range from λ_1 to λ_2 . More recent studies of the autocorrelation of the polarisation dispersion vector^[37] suggests that the actual number of independent measurements, N, is approximated by

$$N = \langle \Delta\tau^2 \rangle^{1/2} \frac{\Delta\omega}{2.5\pi} = \langle \Delta\tau^2 \rangle^{1/2} \left[\frac{c(\lambda_2 - \lambda_1)}{1.25\lambda_1 \lambda_2} \right] \quad (114)$$

for $10 < N < 100$ and tends to underestimate N outside this range.

This gives the number of independent measurements within a single measurement scan. In order to improve the statistical uncertainty it is necessary to make several independent measurement scans. It is not sufficient to simply repeat a measurement scan as this is unlikely

to produce a statistically independent scan. It is necessary to change the mechanical or environmental conditions of the fibre under test such that the polarisation mode coupling is altered. It has been suggested that by changing the temperature of the fibre under test by at least 5 °C will give an independent measurement however further study is required to rigorously confirm this. An installed cable measured on separate occasions is likely to give independent measurement scans. However this should be confirmed by examining the DGD curves.

5.4 Polarisation Dependent Loss and Polarisation Dependent Response

Polarisation dependent loss and polarisation dependent response require basically identical measurements, the first being related to the transmission of a passive device and the latter to the output of an active device. The basic layout is shown in Figure 48. The measurements can be made in two ways, by random scanning of the input polarisation state or through fixed known input polarisation states.

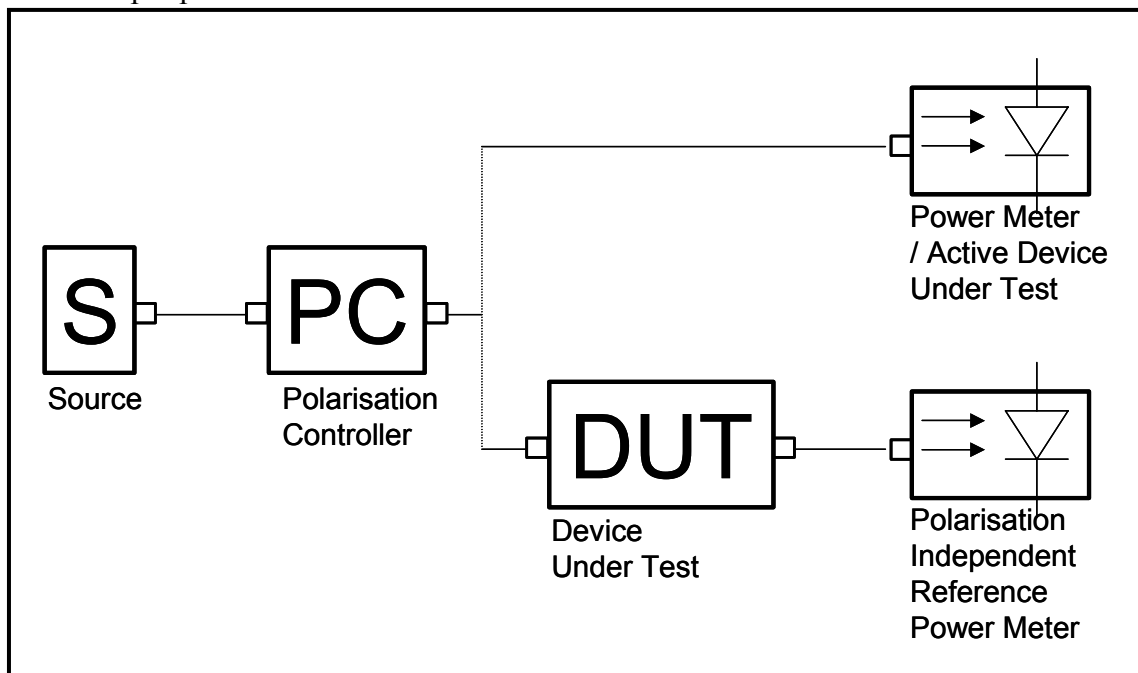


Figure 48 Basic technique to measure PDL or PDR.

With the random scanning technique the detected output power through the device under test or the detected output signal from the device under test is recorded as the input state of polarisation is scanned through all possible states. The PDL or PDR is then given by

$$\text{PDL}_{\text{dB}} = 10 \text{ LOG} \left[\frac{P_{\text{Max}}}{P_{\text{Min}}} \right]. \quad (115)$$

This technique relies on the stability of the input power to the device under test while the polarisation controller is scanned through all states. Any variation in transmitted power of the polarisation controller will lead to an error in the measured PDL or PDR. Likewise any PDR of the polarisation independent power meter will also give an error in the measurement of PDL. It is possible to assess the combined effect of these uncertainties by connecting the output of the polarisation controller directly to the polarisation independent power meter. It is not possible to compensate for this error since depending upon the alignment of the polarisation axis of the device under test the error may add or subtract from the measured PDL or PDR. Any instability in the source power and noise in the detected signal will tend to

increase the PDL or PDR since these effects will always increase the maximum and reduce the minimum.

By using a number of known fixed input polarisation states the measurement of PDL and PDR can be improved by allowing a reference measurement to remove the polarisation effects of the measurement system. The most common technique is to use a Mueller set of input polarisation states for example linear horizontal (H), linear at $+45^\circ$ (+), right hand circular (R) and linear vertical (V)^[38]. For PDL the transmitted signal through the device under test is measured using a polarisation independent detector, for PDR the signal measured on the active device under test is measured. The reference signal is then measured for each input polarisation state on the polarisation independent detector. The signal transmitted or recorded by the device under test is normalised to the reference signal to remove the variation in transmission of the polarisation controller. The respective normalised measured signal are P_H , P_+ , P_R , and P_V , where

$$P_H = \frac{P_{\text{trans\_H}}}{P_{\text{ref\_H}}} \quad (116)$$

From these the normalised measured signals the average transmission, P_0 , for a depolarised signal is calculated as

$$P_0 = \frac{1}{2}(P_H + P_V) \quad (117)$$

The linear PDL magnitude is given by

$$|\Gamma| = \sqrt{\left[\frac{P_H - P_0}{P_0}\right]^2 + \left[\frac{P_+ - P_0}{P_0}\right]^2 + \left[\frac{P_R - P_0}{P_0}\right]^2} \quad (118)$$

and the PDL or PDR is then given by

$$\text{PDL}_{\text{dB}} = 10 \text{ LOG} \left[\frac{1 + |\Gamma|}{1 - |\Gamma|} \right] \quad (119)$$

A slightly better technique is to measure the transmitted signal for six polarisation states that are orthogonal and opposite on the Poincaré sphere, for example linear horizontal (H), linear vertical (V), linear at $+45^\circ$ (+), linear at -45° (-), right hand circular (R) and left hand circular (L). It is preferable to record the signal of orthogonal polarisation states consecutively to reduce the effect of any system drift. The measured transmitted signal is normalised to the reference signal as before. The linear PDL magnitude is then given by

$$|\Gamma| = \sqrt{\left[\frac{P_H - P_V}{P_H + P_V}\right]^2 + \left[\frac{P_+ - P_-}{P_+ + P_-}\right]^2 + \left[\frac{P_R - P_L}{P_R + P_L}\right]^2} \quad (120)$$

The PDL_{dB} or PDR_{dB} is then given by equation (119).

This improved technique is better able to withstand instabilities in the source and variations in the transmission of the polarisation controller. It is possible to improve the measurement system further by introducing a pick off to monitor the source power and while this pick off may drift it allows any fast changes in source power to be normalised. The measurement of orthogonal input polarisation states quickly allows the normalisation of slower drift in the source power and the use of fixed known input states allows for a reference measurement that can normalise the changes in transmission of the polarisation controller. The measurement accuracy relies on the polarisation independent detector having a low or zero PDL since the device under test is being compared directly with this detector.

5.5 Polarisation Extinction Ratio

The polarisation extinction ratio of a source or component coupled with HiBi fibre is a measure of the quality of the alignment of the birefringence axis of the HiBi fibre and the strength of any external perturbations on the HiBi fibre. The definition of extinction ratio is in terms of the ratio of the power in the fast to the power in the slow axis as per equation (63). In many instances it is not possible to determine the fast and slow axes and so the measurement is made by looking for the maximum and minimum signal that is transmitted through a linear polariser.

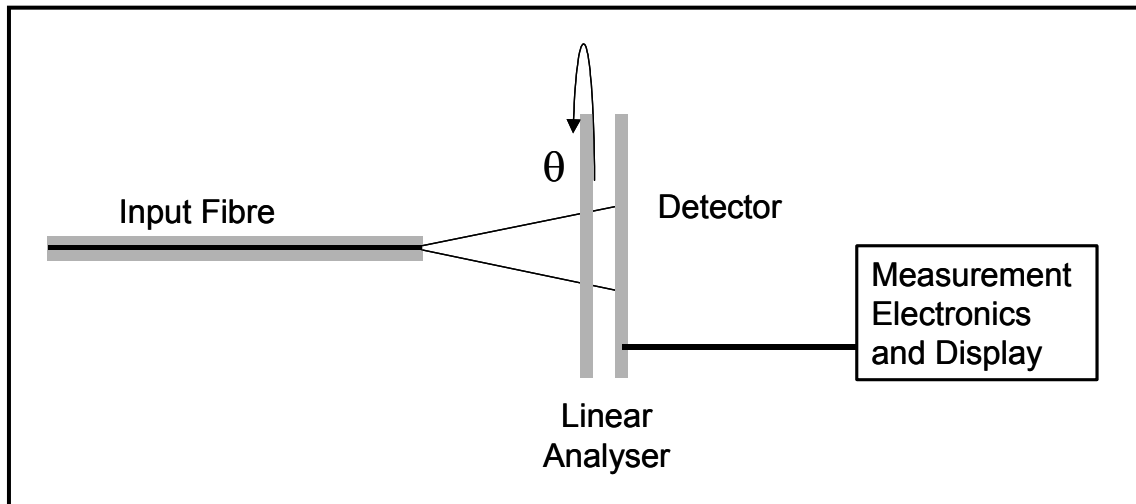


Figure 49 An arrangement to measure polarisation extinction ratio.

Figure 49 shows an arrangement to measure polarisation extinction ratio. The maximum and minimum signal is recorded and used to calculate the extinction ratio. This gives the instantaneous degree of linear polarisation of the beam. The assumption is that a linear polarised output must have been aligned with the birefringence axis, there is however, the ambiguity of which, fast or slow, axis. There is also the ambiguity of the length of HiBi fibre being such that the output phase difference between the light propagating in the fast and slow axis is zero or $2n\pi$. If this was the case the output state of polarisation would be linearly polarised. To overcome this it is necessary to ensure that the phase difference between the fast and slow axis is changed to explore all possibilities and the measured extinction ratio taken as the worst case over these phase possibilities. The phase difference between the fast and slow axis can be change by mechanically stretching the HiBi fibre or by warming or cooling the HiBi fibre. It may also be possible to explore the phase difference by changing the operating wavelength of the source or by using a broadband source.

6. Calibration of Polarisation Measurement Equipment

6.1 Calibration of Polarisation State Analysers

A polarisation state analyser is essential for the full characterisation of polarisation properties of optical fibres and devices. In order to make accurate measurements it is necessary to fully characterise the polarisation state analyser. Polarisation state analysers are relatively complex measurement instruments with a number of settings and ranges. Thus to fully characterise a polarisation state analysers it is necessary to check the polarisation states across the whole Poincaré sphere at wavelengths across the range of operation, for powers across the range of operation and for input from fibre and or free space beams. In order to reduce this to a manageable problem the polarisation state is checked across the whole of the Poincaré sphere at one wavelength, one power and one input condition. The difference between these reference conditions and other wavelengths, powers and input conditions are then taken into account.

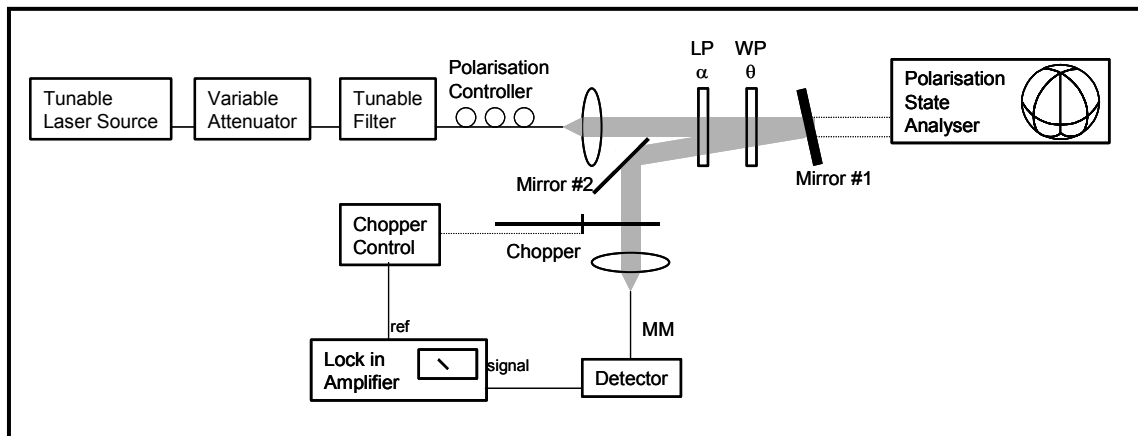


Figure 50 Experimental Layout used to calibrate a Polarisation State Analyser

Figure 50 shows a possible characterisation arrangement for a free space collimated input. The polarisation state presented to the polarisation state analyser is determined from the quality of the linear polariser and quarter waveplate and from the azimuth of these two elements. The extinction ratio of the linear polariser is assessed by measuring the transmitted power of a similar crossed linear polariser. Referring to the arrangement of figure 50, with the mirror #1 in position the optical beam is back reflected through the quarter waveplate and linear polariser before being diverted by mirror #2 to the detector. The azimuth of the quarter waveplate and the wavelength of the source are adjusted to minimise the back reflected signal. This occurs when the state impinging on mirror #1 is a circular polarised state. A small error occurs since the mirror #1 is not at normal incidence to allow the back reflected beam to be separated at mirror #2. This allows the wavelength of operation to be set such that the retardance of the quarter waveplate is exactly $\frac{1}{2}\pi$ radians and the relative azimuth between the linear polariser and the quarter waveplate to be assessed.

The Stokes parameter of the polarisation state generated at the mirror #1 by the linear polariser at azimuth, α , and the quarter waveplate at azimuth, θ , is given by

$$s = \begin{pmatrix} s_1 \\ s_2 \\ s_3 \end{pmatrix} = \begin{pmatrix} \cos(2\alpha)\cos^2(2\theta) + \sin(2\alpha)\cos(2\theta)\sin(2\theta) \\ \cos(2\alpha)\cos(2\theta)\sin(2\theta) + \sin(2\alpha)\sin^2(2\theta) \\ \sin(2\alpha)\cos(2\theta) - \cos(2\alpha)\sin(2\theta) \end{pmatrix}. \quad (121)$$

The uncertainty on this polarisation state is given by the extinction ratio of the linear polarisers, E dB, the isolation of the back reflected circular state, I dB, the accuracy of the angular setting σ_α & σ_θ , and the uncertainty due to non-normal reflection from the mirror #1, σ_M ,

$$\sigma_s^2 = \sigma_{s1}^2 + \sigma_{s2}^2 + \sigma_{s3}^2 = \frac{1}{3} \left[10^{-\frac{E}{20}} + 10^{-\frac{I}{20}} \right]^2 + \sin^2(2\sigma_\alpha) + \sin^2(2\sigma_\theta) + \sigma_M^2. \quad (122)$$

For extinction and isolation ratio > 45 dB, angular uncertainty $< 6'$ and an angle of incidence $< 1^\circ$ at mirror #1 the uncertainty on the set polarisation state, σ_s , is less than 0.01 rad.

The azimuth of the linear polariser and quarter waveplate are adjusted to give polarisation states over the whole sphere that are used to test the polarisation state analyser. The average deviation of the measured state from the set polarisation state weighted by the area of the Poincaré sphere marked out by the nearest points is calculated as the uncertainty. At NPL the azimuth was set from -90° to 90° in 15° steps giving 169 measurements. This gives 62 different states on the Poincaré sphere which are weighted by the area of the sphere marked out by arcs $\pm 15^\circ$ from the centre point.

The effect of the input power level on the polarisation state analyser can be easily assessed by changing the input optical power using the variable attenuator. The actual polarisation state entering the polarimeter will be fixed provided the azimuth of the linear polariser and quarter waveplate remain unchanged. It may be necessary to perform this test for a number of input polarisation states to discover the worst case. For the polarimeter at NPL the worst case input occurred for an input with $s_3 \sim 1/2$.

In order to assess the polarisation state analyser accuracy with wavelength it is necessary to present the polarisation state analyser with a known state as the wavelength is changed. This is relatively easy for a linear polarised state but may not highlight problems with measuring circular states. By noting the wavelength at which the quarter waveplate had exactly $1/2\pi$ rad of retardance it is possible within an offset of $n\pi$ to calculate the exact thickness based on published data for the birefringence and the known material of the waveplate. Given this thickness and the published birefringence data, it is possible to calculate the retardance as a function of wavelength and thus the polarisation state presented to the polarisation state analyser.

At NPL a final uncertainty of ± 0.03 rad was achieved on the measured polarisation state for a single mode fibre input in the range -5 to -45 dBm, 1270 to 1640 nm.

6.2 Calibration of PMD Measurement Equipment

For a full and thorough calibration of any PMD measurement equipment it will be necessary to perform a calibration on each part of the measurement system. Thus for a Poincaré sphere based method it will be necessary to calibrate the polarisation state analyser, the wavelength of the source and aspects of the polarisation controller; For a fixed analyser type method it will be necessary to calibrate the optical spectrum analyser and aspects of the polarisation controllers; for the interferometric method it will be necessary to calibrate the motion of the

moving arm and the linearity of the detector and for the phase shift technique it will be necessary to calibrate the frequency, phase accuracy and input polarisation states. It is likely that only the measurement system manufacturer can perform this kind of full calibration, however the end user may still wish to perform some check to prove the overall measurement capability. This can be done with standard artefacts.

It is relatively easy to provide standard artefacts that have no polarisation mode coupling and give a single DGD value, see Figure 51. These could be a single birefringent waveplate or a short length of HiBi fibre. Such a low mode coupled artefact will give a single well-defined DGD that is repeatable under moderate environmental changes and between all PMD measurement methods. A selection of such devices can provide a sequence of calibration points along the DGD scale.

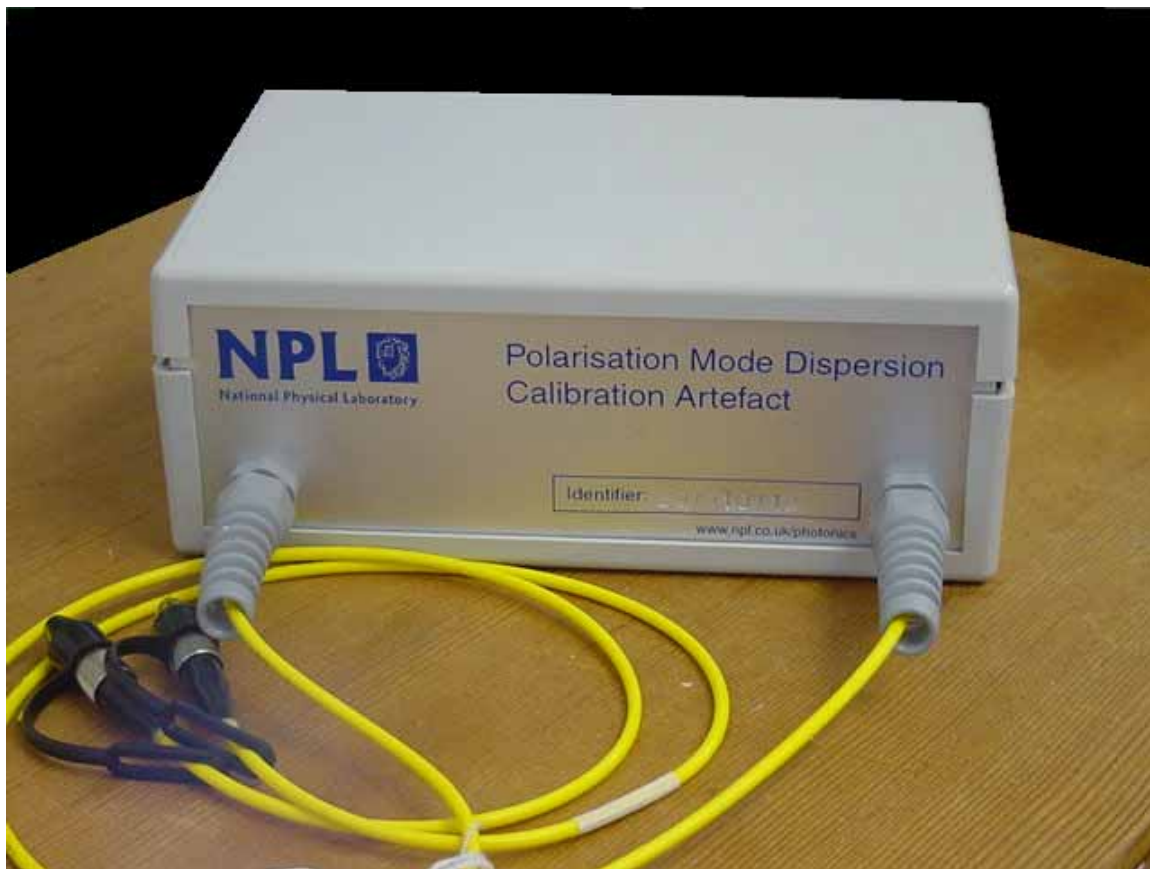


Figure 51 A low polarisation mode coupled artefact

The more interesting situation is that of highly mode coupled regime. A fibre based standard will possess a high degree of polarisation mode coupling but the exact DGD will depend heavily on environmental conditions. This means that the calibration accuracy will be limited by the statistical nature of PMD as outlined in section 5.3.5. An alternative highly mode coupled artefact can be made by creating a stack of birefringent waveplates that have random azimuth and thickness, see Figure 52. This produces a DGD vs wavelength curve similar to randomly mode coupled real fibre that is stable under moderate environmental changes, see Figure 53. This is ideal for the calibration of polarimetric or phase shift based measurement systems where the DGD vs wavelength curve can be compared to assess both the wavelength and DGD scales. For interferometric and fixed analyser based methods that measure only an average PMD the exact average value depends on the wavelength range of the source and the exact input and analysed states and this leads to a similar statistical uncertainty as for the real

fibre. However as the standard is stable under environmental change it is not possible to alter the statistical distribution to obtain more independent measurements that would allow for a reduced uncertainty on the average, where as with a real fibre measurements could be repeated at a number of temperatures to reduce the overall statistical uncertainty.

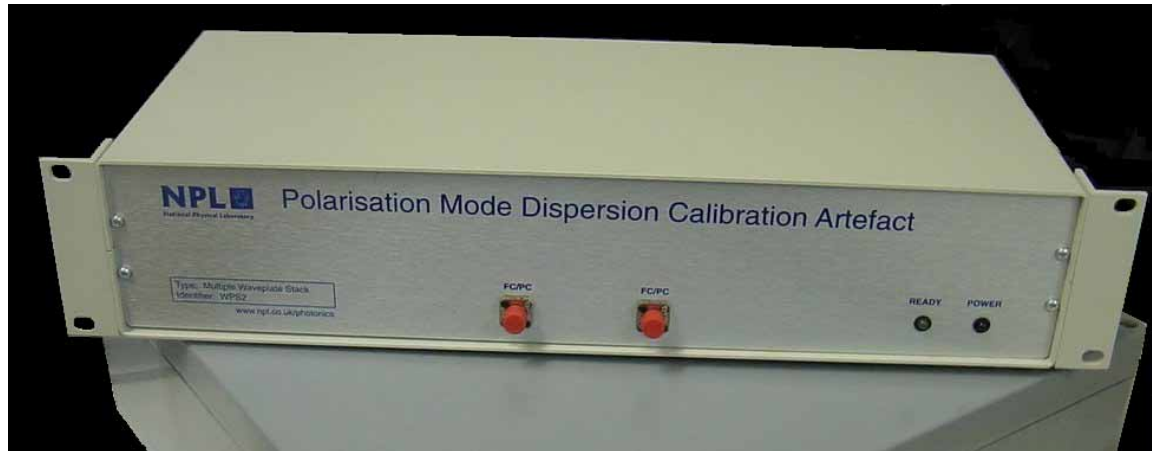


Figure 52 High Polarisation mode coupled artefact

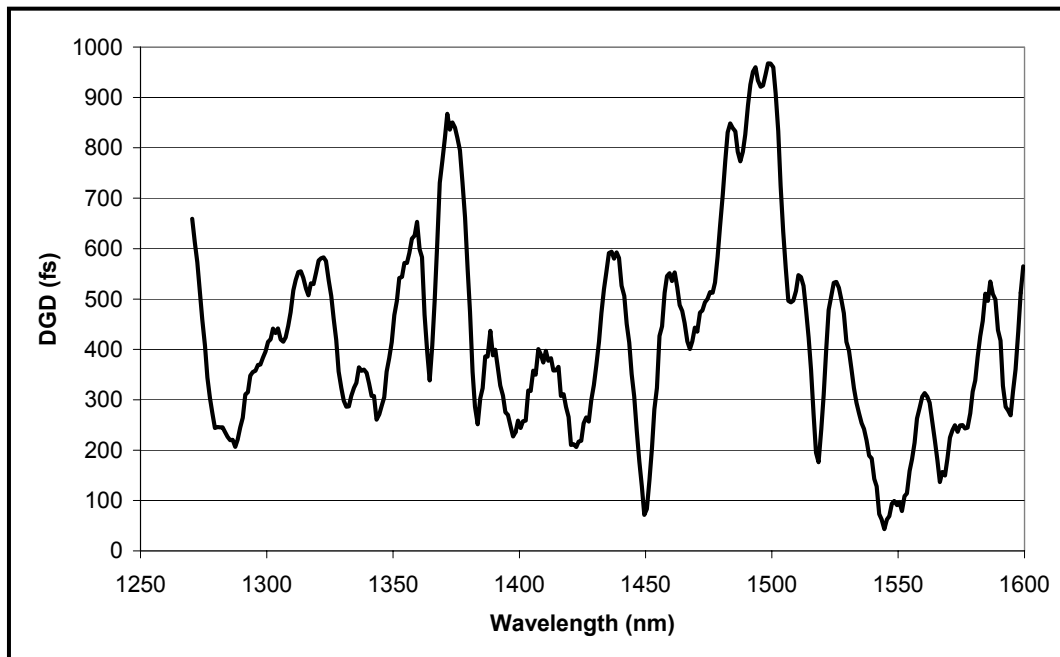


Figure 53 DGD vs wavelength curve for multi-waveplate stack artefact.

6.3 Calibration of PDL Meters

The accuracy of PDL meters relies on the stability of the optical source and the polarisation independence of the detector. A basic check to assess the uncertainty can be made by measuring the PDL of a very low PDL artefact for example a good quality patchlead. This will show any zero offset in the measured PDL due to variation in transmission of the polarisation controller or PDR of the detector. This zero offset can only be used as an uncertainty and will limit the minimum measureable PDL. A further check with a PDL standard, see Figure 54, with PDL in the normal measured range should then be made.



Figure 54 A PDL standard artefact.

6.4 Calibration of PER Meters

The calibration of a PER meter requires the presentation of an optical beam with known polarisation extinction ratio. Many commercial available PER meters are only able to accept fibre inputs. At NPL an in house measurement system has been constructed based on a high quality polariser and polarisation insensitive detectors with the capability of measuring the PER of light exiting a HiBi fibre. The alignment of the input state of polarisation is adjusted to give the desired PER which can be accurately measured. This known PER beam is then connected to the PER meter under test and compared to the PER meter readings.

7. Glossary and References

7.1 Glossary of Terms

Autocorrelation	The correlation of a function with itself.
Azimuth, θ	The angle, measured in the plane perpendicular to the direction of optical propagation, of the optical axis of a waveplate or linear polariser.
Beat Length, L_b, m	The length over which the birefringent phase shift, $\delta\beta \cdot L_b$ is equal to 2π rad.
Birefringence, $\delta\beta$, rad.m^{-1}	The degree to which a material exhibits different phase velocities for the two orthogonal states of polarisation, see paragraph 1.3 & 3.2.
Bit Error Rate, BER, bits.s^{-1}	The number of errors per second in a transmitted bit sequence.
Brewster's Angle, θ_p	The angle of incidence at which the reflected light from a dielectric material is fully polarised, see paragraph 1.2
Circular Birefringence, $\delta\beta_C$, rad.m^{-1}	The degree to which a material exhibits different phase velocities for the two orthogonal circular states of polarisation. Often referred to as Optical Activity, see paragraph 1.5 & 3.3
Coupling Length, m	The inverse of the coupling parameter, $1/h$, see paragraph 3.5.
Coupling Parameter, h, m^{-1}	The fractional power coupling between polarisation modes per unit length, see paragraph 3.5.
Cross-Correlation	The correlation of one function with another.
Differential Group Delay, DGD, $\delta\tau$, s	The difference in propagation time between the fast and slow principle states of polarisation, see paragraph 3.5.
Electro-Optic Effect	The physical phenomena where an applied electric field changes the refractive index of a material, see paragraph 1.6 & 3.2.
Extinction Ratio, ER	The ratio, often in dB, between the optical power in the slow axis and that in the fast axis of a polarisation maintaining component or fibre, see paragraph 3.6.
Fourier Transform	A mathematical process allowing the transformation of a signal between the time and frequency domains.
Gaussian	Mathematical function of the form $f(x) = e^{-x^2}$.

Half Waveplate	A linear birefringent element that provides a phase difference of π radians, $\lambda/2$, between fast and slow axis at a specified operating wavelength, see paragraph 1.4.
HiBi Fibre	Optical fibre designed to have a large intrinsic linear birefringence, sometimes referred to as polarisation maintaining fibre, see paragraph 3.6.
Interferogram	The output intensity profile from an interferometer.
Jones Matrix	A two by two complex matrix that describes the transfer of an input state of polarisation to its output state of polarisation, see paragraph 2.1 & 2.4.
Jones Vector	A complex vector that describes the polarisation state in terms of two orthogonal components of the electric field, see paragraph 2.1.
Kerr Effect	The physical phenomena where an applied electric field changes the birefringence of a material, see paragraph 1.6 & 3.2.
Linear Birefringence, $\delta\beta_l$, rad.m^{-1}	The degree to which a material exhibits different phase velocities for the two orthogonal linear states of polarisation, see paragraph 1.4 & 3.2.
Magneto-Optic Effect	The physical phenomena where an applied magnetic field changes the circular birefringence of a material, see paragraph 1.6 & 3.3.
Maxwellian	A mathematical function of the form $f(x) = x^2 e^{-x^2}$.
Mode Coupling	The cross coupling of optical power from one polarisation mode to another, see coupling parameter and coupling length.
Modal Dispersion, $\delta\beta'$, s.m^{-1}	The difference in group delay between polarisation modes per unit length, $\delta\beta' = \frac{\partial\delta\beta}{\partial\omega}$, see paragraph 3.5.
Mueller Matrix	A transfer matrix describing the transfer of the input state of polarisation described by a Stokes vector to the output state of polarisation, see 2.2 & 2.4.
Optical Activity	A material that exhibits different phase velocities for the two orthogonal circular states of polarisation. Also referred to as Circular Birefringence, see paragraph 1.5 & 3.3.
PM Fibre	Optical fibre designed to have a large intrinsic linear birefringence, sometimes referred to as HiBi fibre, see paragraph 3.6.

PMD Compensation	The compensation of PMD by active or other techniques, see paragraph 4.4.
Poincaré Sphere	A visualisation of polarisation states based on normalised Stokes vectors that fall on the surface of a sphere, see paragraph 2.3.
Polarisation Analyser	A Polariser used to analyse the amount of light within a given state of polarisation.
Polarisation Dependent Loss, PDL, Γ	The variation of loss of an optical fibre or optical fibre component with the polarisation state of the transmitted light. The vector Γ is parallel to the least attenuated polarisation state. See paragraph 3.7.
Polarisation Dependent Response, PDR	The variation in the response of a detector system to the polarisation state of the incident light. See paragraph 3.8.
Polarisation Dispersion Vector, PDV, Ω, s	The vector in Stokes space that describes the polarisation dispersion. The vector is aligned along the PSP with magnitude equal to the DGD. See paragraph 3.5.
Polarisation Mode Dispersion, PMD	A general term for all dispersive effects caused by polarisation properties. Also specifically used to describe an average DGD, either linear or root mean square average over optical frequency, time or sample. See paragraph 3.5.
Polarised Light	Light where the electric field vector direction is defined, see paragraph 1.1.
Polariser	An optical component that only transmits light with one defined state of polarisation. See paragraph 1.2.
Principle State of Polarisation, PSP	The output polarisation state of a system that is stationary with optical frequency to first order, see paragraph 3.5.
Q Factor	The quality factor of a digital optical communication signal. The signal to noise ratio of the difference between the one and zero levels at the decision point.
Quarter Waveplate	A linear birefringent element that provides a phase difference of $\pi/2$ radians, $\lambda/4$, between fast and slow axis at a specified operating wavelength, see paragraph 1.4.
Retardance Plate	A birefringent element that provides a phase difference between fast and slow axis, also known as a Waveplate, see paragraph 1.4

Second Moment, σ

The root mean square deviation of a signal from the mean signal.

$$\sigma^2 = \frac{1}{P_0} \int_{-\infty}^{\infty} P(x)(x - \langle x \rangle)^2 dx ,$$

$$\langle x \rangle = \frac{1}{P_0} \int_{-\infty}^{\infty} P(x)x dx , P_0 = \int_{-\infty}^{\infty} P(x)dx .$$

Second Order PDV, $\Omega\omega$, s^2

The vector that describes the change of the polarisation dispersion vector with change in optical frequency. It is the differential of the PDV with optical frequency, see paragraph 3.5.

Stokes Vector

A four-element vector (often normalised to three elements) that describes the polarisation state of an optical signal in terms of the power transmitted through polarising analysers. See paragraph 2.2 & 2.4.

Stress-Optic Effect

The physical phenomena where applied mechanical stress changes the refractive index and birefringence of a material.

Verdet Constant

The constant that describes the magnitude of the magneto-optic effect, see paragraph 1.6.

Waveplate

A birefringent element that provides a phase difference between fast and slow axis, also known as a Retardance Plate, see paragraph 1.4

7.2 NPL Publications

D J IVES, "Calibration of a Polarisation State Analyser for Polarisation Mode Dispersion Measurements", *4th Optical Fibre Measurement Conference Digest, NPL UK*, Poster, (October 1997).

D J IVES, "Polarisation Mode Dispersion in Chirped Fibre Bragg Gratings", *1998 Symposium on Optical Fibre Measurement Digest, NIST Boulder USA*, (September 1998).

D J IVES, "Measurement of the Distribution of Polarisation Mode Dispersion (PMD) along an Optical Fibre System", *NPL Report COEM55*, June 2000.

D J IVES, FOTON UK, "An Intercomparison of Polarisation Mode Dispersion Measurements and Calibration Artefacts", *2000 Symposium on Optical Fibre Measurements, NIST Special Publication 953*, (September 2000), pp 91 – 94.

D J IVES, "Measurement of Polarisation Dependent Response of Optical Fibre Power Meters and Optical Spectrum Analysers", *OFMC'01, Girton College, Cambridge*, pp 13 – 16.

S HARRIS, D J IVES, "Polarisation Mode Dispersion in Restricted Optical Bandwidth: An Evaluation of Measurement Techniques", *NPL Report CETM40*, February 2002.

R F STEVENS, "Polarisation Extinction Ratio Measurement Requirements for Optical Communications Systems", *NPL Report CETM41*, March 2002.

S HARRIS, D J IVES, "Polarisation Mode Dispersion in Restricted Optical Bandwidth: An Evaluation of Measurement Techniques", *2002 SOFM, NIST Special Publication 988*, (September 2002), pp 169 – 172.

R F STEVENS, "Measuring Polarisation Extinction Ratio for Optical Fibres and Components", *NPL Report CETM43*, March 2003.

D J IVES, "Polarisation Mode Dispersion Compensation Techniques and Measurement Requirements", *NPL Report CETM48*, May 2003.

D J IVES, "Measurement Requirements and Methods for Optical Fibre Polarisation Controllers", *NPL Report DEM-EM001*, April 2004.

7.3 References

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- [1] M. BORN, E. WOLF, "Principles of Optics", *Pergamon Press*.
 - [2] E. HECHT, "Optics", *Addison-Wesley*.
 - [3] H. G. JERRARD "Modern description of polarized light: matrix methods", *Optics and Laser Tech.*, Dec 1982, pp 309-319.
 - [4] S. C. RASHLEIGH, "Origins and Control of Polarization Effects in Single-Mode Fibers", *J of Light Tech.*, **Vol LT-1 No 2**, June 1983, pp 312-331.
 - [5] M. C. FARRIES, A. J. ROGERS, "Temperature dependence of the Kerr effect in silica optical fibre", *Electron. Lett.*, **Vol 19 No 21**, 1983, pp890-891.
 - [6] A. C. LUI, M. J. F. DIGONNET, G. S. KINO, "Measurement of the dc Kerr and electrostrictive phase modulation in silica", *J. Opt. Soc. Am. B*, **Vol 18 No 2**, 2001, pp187-194.
 - [7] C. D. POOLE , R. E. WAGNER, "Phenomenological approach to polarisation dispersion in long single-mode fibres", *Elect. Lett.*, **22**, 1986, pp1029-1030.
 - [8] H. KOGELNIK, L. E. NELSON, J. P. GORDON, "Emulation and Inversion of Polarization-Mode Dispersion", *J. of Light. Tech.*, **Vol 21 No 2**, 2003, pp482-495.
 - [9] C. D. POOLE, "Statistical treatment of polarization dispersion in single-mode fiber", *Optics. Lett.*, **Vol 13 No 8**, 1988, pp687-689.
 - [10] N. GISIN, J. P. VON DER WEID, J. P. PELLAUX, "Polarization Mode Dispersion of Short and Long Single-Mode Fibres", *J. of Light. Tech.*, **Vol 9 No 7**, 1991, pp821-827.
 - [11] C. D. POOLE , J. H. Winters, J. A. Nagel, "Dynamical equation for polarization dispersion", *Optics. Lett.*, **Vol 16 No 6**, 1991, pp372-374.
 - [12] P. CIPRUT, B. GISIN, N. GISIN, R. PASSY, J. P. VON DER WEID, F. PRIETO, C. W. ZIMMER, "Second-Order Polarization Mode Dispersion: Impact on Analog and Digital Transmissions", *J. of Light. Tech.*, **Vol 16 No 5**, May 1998, pp 757 – 771.
 - [13] G. J. FOSCHINI, R. M. JOPSON, L. E. NELSON, H. KOGELNIK, "The Statistics of PMD-Induced Chromatic Fiber Dispersion", *J. of Light. Tech.*, **Vol 17 No 9**, September 1999, pp 1560 – 1565.
 - [14] G. J. FOSCHINI, C. D. POOLE, "Statistical Theory of Polarization Dispersion in Single Mode Fibres", *J. of Light. Tech.*, **Vol 9 No 11**, 1991, pp1439-1456.
 - [15] N. GISIN, "Statistics of polarization dependent losses", *Optics Comm.*, **114**, 1995, pp399-405.
 - [16] N. GISIN, B. HUTTNER, "Combined effects of polarization mode dispersion and polarisation dependent losses in optical fibers", *Optics Comm.*, **142**, 1997, pp 119 - 125.
 - [17] Y. LI, A. YARIV, "Solutions to the dynamical equation of polarization-mode dispersion and polarization-dependent losses", *J. Opt. Soc. Am. B.*, **Vol 17 No 11**, November 2000, pp 1821 – 1827.

-
- [18] R. FECED, S. J. SAVORY, A. HADJIFOTIOU, “Interaction between polarization mode dispersion and polarization-dependent losses in optical communication links”, *J. Opt. Soc. Am. B.*, **Vol 20 No 3**, March 2003, pp 424 – 433.
 - [19] L. CHEN, X. BAO, “Polarization-Dependent Loss-Induced Pulse Narrowing in Birefringent Optical Fiber with Finite Differential Group Delay”, *J. of Light. Tech.*, **Vol 18 No 5**, May 2000, pp 665 – 667.
 - [20] F. BRUYÈRE, “Impact of First- and Second-Order PMD in Optical Digital Transmission Systems”, *Optical Fibre Tech*, **2**, 1996, pp 269 - 280.
 - [21] L. NELSON, “Polarisation Mode Dispersion Effects” *OFC 2003 Digest*, **1**, 2003, TuO1, pp 258.
 - [22] H. SUNNERUD, C. XIE, M. KARLSSON, R. SAMUELSSON, P ANDREKSON, “A Comparison Between Different PMD Compensation Techniques”, *J. of Light. Tech.*, **Vol 20**, March 2002, pp 368 – 378.
 - [23] P. WAI, C. MENYUK, “Polarization Mode Dispersion, Decorrelation, and Diffusion in Optical Fibers with Randomly Varying Birefringence”, *J. of Light. Tech*, **Vol 14**, February 1996, pp 148 – 157.
 - [24] A. PAPP, H. HARMS, “Polarization optics of index-gradient optical waveguide fibers”, *Appl. Optics*, **Vol 14 No 10**, October 1975, pp 2406- 2411.
 - [25] F. CORSI, A. GALTAROSSA, L. PALMIERI, “Beat Length Characterization Based on Backscatter Analysis in Randomly Perturbed Single-Mode Fibers”, *J of Light. Tech.*, **Vol 17 No 7**, July 1999, pp 1172 – 1178.
 - [26] B. HUTTNER, J. REECHT, N. GISIN, R. PASSY, J. P. VON DER WEID, “Local Birefringence Measurements in Single-Mode Fibres with Coherent Optical Frequency-Domain Reflectometry”, *IEEE Photonics Tech. Lett.*, **Vol 10 No 10**, October 1998, pp 1458 – 1460.
 - [27] TIA/EIA-455-122, “Polarisation mode dispersion measurement for single mode optical fibers by Jones matrix Eigenanalysis”.
 - [28] B. L. HEFFNER, “Automated Measurement of Polarization Mode Dispersion Using Jones Matrix Eigenanalysis”, *IEEE Photonics Tech. Lett.*, **Vol 4 No 2**, September 1992, pp 1066 – 1069.
 - [29] C. D. POOLE, N. S. BERGANO, R.E. WAGNER, H. J. SCHULTE, “Polarization Dispersion and Principle States in a 147 km Undersea Lightwave Cable”, *J. of Light. Tech.*, **Vol 6 No 7**, 1988, pp 1185 – 1190.
 - [30] L.E. NELSON, R. M. JOPSON, H. KOGELNIK “Mueller Matrix Method for Determining Polarization Mode Dispersion Vectors”, ECOC 1999.
 - [31] TIA/EIA-455-113, “Polarisation mode dispersion measurement for single mode optical fibers by the fixed analyser method”.
 - [32] C. D. POOLE, D. L. FAVIN, “Polarization-Mode Dispersion Measurements Based on Transmission Spectra Through a Polarizer”, *J. of Light. Tech.*, **Vol 12 No 6**, June 1994, pp 917 – 929.
 - [33] TIA/EIA-455-124, “Polarisation mode dispersion measurement for single mode optical fibers by interferometry”.

-
- [34] B. L. HEFFNER, “Influence of optical source characteristics on the measurement of polarization-mode dispersion of highly mode-coupled fibers”, *Optics Lett.*, **Vol 21 No 2**, January 1996, pp 113 – 115.
 - [35] P. A. WILLIAMS, “Modulation phase-shift measurement of PMD using only four launched polarisation states: a new algorithm”, *Elect. Lett.*, **Vol 35 No 8**, September 1999, pp 1578 – 1579.
 - [36] N. GISIN, B. GISIN, J. P. VON DER WEID, R. PASSY, “How Accurately Can One Measure a Statistical Quantity Like Polarization-Mode Dispersion?”, *IEEE Photonics Tech. Lett.*, **Vol 8 No 12**, December 1996, pp 1671 – 1673.
 - [37] M. SHTAIF, A. MECOZZI, “Study of the frequency autocorrelation of the differential group delay in fibers with polarization mode dispersion”, *Optics Lett.*, **Vol 25 No 10**, May 2000, pp 707 – 709.
 - [38] R. M. CRAIG, S. L. GILBERT, P. D. HALE, “High-Resolution, Nonmechanical Approach to Polarization-Dependent Transmission Measurements”, *J of Light. Tech.*, **Vol 16 No 7**, July 1998, pp 1285 – 1294.