

REPORT

Determination of Mode II Energy Release Rate in Composite Laminates by the Modified Double-Lap Shear Test

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Abstract

This document introduces a new test method for the mode II fracture energy of composite laminates. The geometry of the test is adapted from the modified double-lap shear test used for adhesion testing of lap-joints. It minimises peel stresses and facilitates specimen fabrication, and thereby increases test reproducibility. The analysis follows the energy release rate approach, based on the shear lag theory in multiple layer systems, and includes internal stresses and also friction stresses when debonding of an interface is introduced. Axial and shear stresses, as well as displacements, are calculated for specimens stressed in the elastic regime. The energy release rate, depending only on average axial stresses and axial displacements, is also obtained. It is shown that the present model captures all the essential features of the energy release rate, including the effects of internal stresses and of friction at the debonded interface, within a certain range of debond length and friction stress. An experimental study, using electronic speckle pattern interferometry (ESPI), is conducted to validate the models.

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Contents

1. Introduction	2
1.1. Background	
1.2. Interface strength	
1.3. Interfacial fracture energy	
1.4. Goals	
2. Problem Addressed and Associated System Geometry	3
3. Optimal Shear-Lag Analysis	4
4. Determination of Stresses in the 3 Zones of a Cracked Specimen	6
4.1. Zone I	
4.2. Zone II	
4.3. Zone III	
4.4. Boundary Conditions	
5. Energy Release Rate	8
5.1. Exact Solution	
5.2. Approximate Solution	
5.2.1. Calculation of Displacements in the 3 Zones	
5.2.2. Phase Averaged Stresses	
6. Model Analysis	11
7. Experimental	11
7.1. Materials	
7.2. Equipment	
7.3. Loading Sequence & Analysis	
8. Results - ESPI	16
9. Results - Energy Release Rate	19
10. Conclusions	21
11. Acknowledgements	21

1. Introduction

1.1. Background

The properties of ply interfaces in any fibre reinforced composite material have a significant influence on the resultant properties of the composite itself. For most polymer composites the requirement is for a very strong interface so that the properties of the composite become susceptible to microstructural damage formation only at relatively high loading. All unidirectional fibre reinforced composite materials exhibit significantly different axial and transverse properties. During loading both transverse and axial properties are degraded because of interface failure and its consequences. The development of a strong ply interface helps to maintain mechanical properties during loading, and to minimise the differences between axial and transverse composite properties, as well as the rate of accumulation of damage during loading. The property difference of the laminate is the cause of mixed mode failure, that is a combination of mode I (tensile crack opening) and mode II (shear crack opening) [1]. Various methods of measuring these properties have been developed, most of which require the use of a micro-mechanical model to interpret the results of the experiments in terms of so-called interface properties.

A key problem regarding the failure of interfaces between fibre and matrix in unidirectional fibre reinforced composites is defining exactly the meaning of the term 'interface failure'. The following concepts need to be addressed, namely, interface strength and interface fracture energy.

1.2. Interface strength

The strength of an interface is often regarded as being a key parameter characterising its failure. The term 'strength' implies a critical value of a component of stress - but which one? In many interfacial tests the loading of the interface is dominated by shear deformation that leads on to the concept of an interfacial shear strength. Indeed it is often assumed, that at the point of shear dominated interfacial failure, the shear stress on the interface has reached a critical value known as the shear strength. Frequently it is assumed that the shear stress on the interface is uniform enabling the easy calculation of an apparent interfacial shear strength. It has been demonstrated that this assumption is false for sub-millimetre composites [2, 3], and does not hold for laminates either [4, 5]. Refined models have been used with success, although it is still not clear if the results are relevant to other test configurations [6].

The interesting question is: what happens if the interface is subject to combined tensile and shear loading? This is often the case in interfacial test method configurations. There is no easy answer to this question using strength concepts.

1.3. Interfacial fracture energy

Interfacial failure is also characterised by an interfacial fracture energy that ideally quantifies the degree of bonding between two plies. This concept is ideal for the consideration of the propagation of a debonded zone in the interface due to progressive axial loading. Such a phenomenon occurs in mode I and mode II tests. A physically based approach will attempt to balance the energy available from the environment of debonding surfaces, taking account of the potential energy of the loading and of energy dissipation due to friction at the interface in the debonded region, with the energy required to overcome cohesive interface forces during the creation of new areas of debonded interface. This technique therefore solves the problem of taking account of the effects of combined tensile and shear loading of the interface. The difficulty is that it is not yet clear how the initiation of debonding can be considered. One viable approach taken in the study of $[0_m; 90_n]_s$ laminates subjected to transverse ply cracking in tension is to assume that the interface in a composite has a large number of small defects in the form of local defects, whose crack propagation energy is distributed following a certain law [7]. The initiation of interfacial debonding is thought to arise from the initiation of the growth of the defects governed by fracture mechanics principles. The initiation of debonding may be such that the initial stages of debond growth may be highly unstable so that a significant portion of the interface suddenly debonds followed by arrest. Subsequent debond growth then requires additional loading. Another difficulty is that debond growth involves mixed mode fracture.

The criterion for fracture in mode I (tensile opening of a crack) involves the mode I fracture toughness K_{Ic} such that fracture occurs when the mode I intensity factor K_I attains the value K_{Ic} . The fracture toughness is related to cohesive forces (characterising adhesion) acting at the debond tip, and to small scale yielding plasticity mechanisms that must occur in the debond tip region. Similarly the

criterion for fracture in mode II (in-plane shear opening of a crack) involves the mode II fracture toughness K_{IIc} such that fracture occurs when the mode II intensity factor K_{II} attains the value K_{IIc} . The measurement of K_{IIc} for a material involves difficulties of attaining pure mode II conditions which are not usually realised in practice. For mixed mode opening where the crack faces simultaneously exhibit in-plane open and shear opening, the correct fracture criterion to use does not seem to be agreed. An approach often made is to assume that mixed mode fracture is governed by a relation of the form [8]

$$\left(\frac{K_I}{K_{Ic}}\right)^2 + \left(\frac{K_{II}}{K_{IIc}}\right)^2 = 1 \quad (1)$$

which has no physical justification except that it is one method of interpolating between the limiting cases of pure mode I and mode II fracture. The following equivalent relation involving strain energy release rates is also used

$$\frac{G_I}{G_{Ic}} + \frac{G_{II}}{G_{IIc}} = 1 \quad (2)$$

Many other variations of this theme have been proposed, and a generalisation of the above expression is often employed for describing the failure of composite laminates:

$$\left(\frac{G_I}{G_{Ic}}\right)^\alpha + \left(\frac{G_{II}}{G_{IIc}}\right)^\beta = 1 \quad (3)$$

where α and β are materials constants. It must be said that, although this last equation has been used with relative success in various works, it has never been established on the grounds of physical principles. At best, it provides a way to interpolate between experimental results, but there is no absolute proof that the materials constants are independent of the test geometry and loading conditions.

1.4. Goals

The objective of this paper was the measurement and modelling of the energy release rate of a crack propagating essentially in mode II between two adjacent plies of a composite laminate. First, the specimen geometry and a general shear lag model were introduced. This model was then applied to a multi-layered symmetric composite laminate with two cracks. Stresses were derived for this particular geometry, and the calculation of the displacements enabled the calculation of the strain energy release rate of the crack propagating between two plies. The model was used in a theoretical study of the effect of internal stresses and friction at the debonding interface. The theory was then applied to practical results, making use of a refined displacement measurement technique, electronic speckle pattern interferometry (ESPI). This technique was used to validate the shear lag model, and determine the friction coefficient at the debonded interface. Finally, experimental and theoretical energy release rates were compared for various laminates.

2. Problem Addressed and Associated System Geometry

In this work, an extension of the modified double-lap shear specimen presented in refs. [4, 5] was adapted for a composite laminate geometry. Figure 1 presents the geometry for a mode II energy release rate specimen with cracks of length a opened at two ply interfaces. The system shown has N orthotropic layers of thickness $t_i = x_i - x_{i-1}$, mechanical properties $E_x^{(i)}, E_y^{(i)}, G_{xy}^{(i)}, \nu_{xy}^{(i)}$ (respectively Young's modulus in the axial and transverse direction, shear modulus and Poisson's ratio of the layer i), and coefficient of thermal expansion in the axial and transverse direction of $\alpha_x^{(i)}, \alpha_y^{(i)}$, respectively. The boundary conditions are defined by a simple compression on the N_a inner layers of magnitude σ_0 while the remaining N_b layers are simply supported. Additionally, two lateral pressures can be applied on the outer adherents, of magnitude σ_t . The coordinates (x, y, z) have their origin on the mid-plane of the system such that it is coplanar with the debond crack tips.

Clearly, the problem is symmetrical about the y -axis, so that it can be solved for one half of the system (which indeed has the advantage to have automatically a zero shear stress for the plane $x = 0$). The width of the specimen W is supposed large enough to be in plane strain conditions with respect to the z -direction. This was verified in previous works by examining the fracture surfaces of the specimen [9, 10]. The shear-lag analysis consists in determining the averaged axial stresses $\langle \sigma_{yy}^{(i)} \rangle$ in the layers, and the interfacial shear stresses $\tau_{xy}^{(i)}$.

3. Optimal Shear-Lag Analysis

A modified shear-lag theory was introduced recently [4] for determining stresses in a multi-layered structure of n orthotropic layers. A similar -simplified- analysis, involving isotropic materials only, has been validated and used in a number of studies to analyse the effects of internal stresses and physical aging on the mechanical behaviour of bonded structures. Based on the analysis of the most common assumptions used in shear-lag models, it was proposed that an optimal shear-lag analysis can be derived by assuming that the shear stresses are the product of two functions, one which depends only on x , and one which depends only on y . As a result the shear stress $\tau_{xy}^{(i)}$ in layer i was written as a function of two unknown functions of x only, L_i and R_i (respectively left and right), and the interfacial shear stresses $\tau_{xy}(x_{i-1})$ and $\tau_{xy}(x_i)$, so that:

$$\tau_{xy}^{(i)} = \tau_{xy}(x_{i-1})L_i(x) + \tau_{xy}(x_i)R_i(x) \quad (4)$$

with

$$L_i(x) = \begin{cases} 1 & \text{at } x = x_{i-1} \\ 0 & \text{at } x = x_i \end{cases} \quad (5a)$$

$$R_i(x) = \begin{cases} 0 & \text{at } x = x_{i-1} \\ 1 & \text{at } x = x_i \end{cases} \quad (5b)$$

A system of $(n-1)$ second-order differential equations with $(n-1)$ unknowns $\tau_{xy}(x_i)$ was obtained:

$$\begin{aligned} \frac{\tau_{xy}''(x_{i+1})}{G_{xy}^{(i+1)}} t_{i+1} \langle (1 - \xi_{i+1}) R_{i+1} \rangle + \tau_{xy}''(x_i) \left(\frac{t_{i+1} \langle (1 - \xi_{i+1}) L_{i+1} \rangle}{G_{xy}^{(i+1)}} + \frac{t_i \langle \xi_i R_i \rangle}{G_{xy}^{(i)}} \right) + \frac{\tau_{xy}''(x_{i-1})}{G_{xy}^{(i)}} t_i \langle \xi_i L_i \rangle \\ + \frac{\tau_{xy}(x_{i+1})}{E_y^{(i+1)} t_{i+1}} - \tau_{xy}(x_i) \left(\frac{1}{E_y^{(i)} t_i} + \frac{1}{E_y^{(i+1)} t_{i+1}} \right) + \frac{\tau_{xy}(x_{i-1})}{E_y^{(i)} t_i} = -\frac{\tau_0}{E_y^{(1)} t_1} - \frac{\tau_n}{E_y^{(n)} t_n} \end{aligned} \quad (6)$$

where τ_0 and τ_n are non-zero boundary conditions on the left edge of layer 1 and on the right edge of layer n respectively, and the local, normalised abscissa ξ_i relative to layer i is defined as:

$$\xi_i = \frac{x - x_{i-1}}{t_i} \quad (7)$$

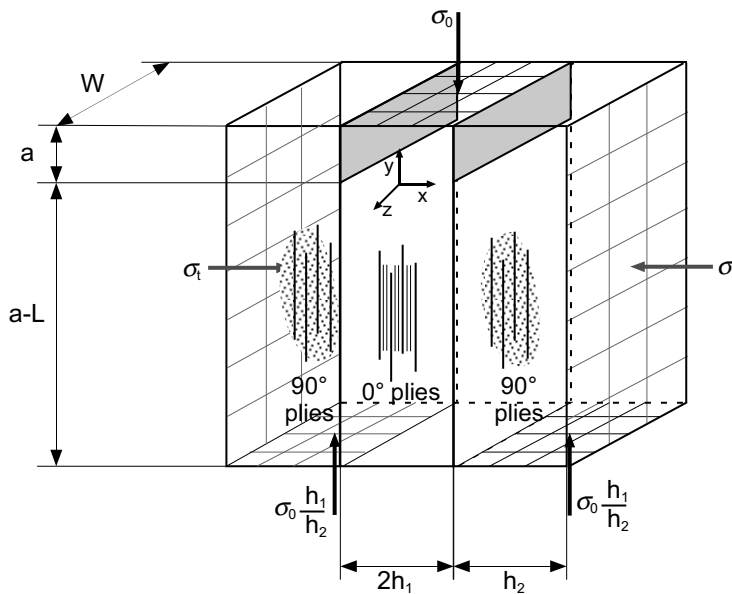


Fig. 1. Geometry of the modified double lap-shear specimen with a crack of length a

This system is solved by writing the shear stresses between consecutive layers as a vector and determining the roots of the system matrix (i.e. eigenvalues + eigenvectors). Defining $\tau = (\tau_{xy}(x_1), \tau_{xy}(x_2), \dots, \tau_{xy}(x_n))$ and $\tau_0 = \left(-\frac{\tau_0}{E_y^{(1)}t_1}, 0, \dots, -\frac{\tau_n}{E_y^{(n)}t_n}\right)$, Equation (6) can be written as:

$$[\mathbf{A}] \frac{\partial^2 \tau}{\partial y^2} - [\mathbf{B}] \tau = \tau_0 \quad (8)$$

where $[\mathbf{A}]$ and $[\mathbf{B}]$ are both tridiagonal matrices with elements

$$\begin{aligned} \mathbf{A}_{i,i-1} &= \frac{t_i \langle \xi_i L_i \rangle}{G_{xy}^{(i)}}, \quad \mathbf{A}_{i,i} = \frac{t_{i+1} \langle (1 - \xi_{i+1}) L_{i+1} \rangle}{G_{xy}^{(i+1)}} + \frac{t_i \langle \xi_i R_i \rangle}{G_{xy}^{(i)}}, \quad \mathbf{A}_{i,i+1} = \frac{t_{i+1} \langle (1 - \xi_{i+1}) R_{i+1} \rangle}{G_{xy}^{(i+1)}} \\ \mathbf{B}_{i,i-1} &= -\frac{1}{E_y^{(i)} t_i}, \quad \mathbf{B}_{i,i} = \frac{1}{E_y^{(i+1)} t_{i+1}} + \frac{1}{E_y^{(i)} t_i}, \quad \mathbf{B}_{i,i+1} = -\frac{1}{E_y^{(i+1)} t_{i+1}} \end{aligned} \quad (9)$$

For solution purposes, Equation (8) is rewritten as:

$$\frac{\partial^2 \tau}{\partial y^2} - [\mathbf{M}_\tau] \tau = -[\mathbf{M}_\tau][\mathbf{B}]^{-1} \tau_0 \quad (10)$$

where $[\mathbf{M}_\tau] = [\mathbf{A}]^{-1}[\mathbf{B}]$. The solution of Equation (10) is found by an eigen-analysis, leading to:

$$\tau_{xy}(x_i) = \tau_0 + \sum_{j=1}^i \frac{t_j E_y^{(j)}}{t E_y^{(0)}} (\tau_n - \tau_0) + \sum_{j=1}^{n-1} (a_j e^{\lambda_j y} - b_j e^{-\lambda_j y}) \omega_{i,j} \quad (11)$$

where $t = \sum_{i=1}^n t_i$ is the total thickness of the composite, $E_y^{(0)} = \frac{1}{t} \sum_{i=1}^n t_i E_y^{(i)}$ is the rule-of-mixtures effective composite modulus of an undamaged structure in the y direction, λ_j^2 for $j = 1$ to $n - 1$ are the eigenvalues of the $[\mathbf{M}_\tau]$ matrix, $\omega_{i,j}$ is the i^{th} element of the eigenvector of $[\mathbf{M}_\tau]$ associated with λ_j^2 , and a_j and b_j for $j = 1$ to $n - 1$ are constants to be determined by boundary conditions. Note that internal stresses do not appear explicitly in this equation, but are included through a_j and b_j , calculated as shown below. Additionally, by making use of the equation of equilibrium, it was shown that the average axial stresses are obtained from:

$$\langle \sigma_{yy}^{(i)} \rangle = E_y^{(i)} \left[\frac{(\tau_0 - \tau_n) + t \sigma_0(0)}{t E_y^{(0)}} + (\alpha_y^{(0)} - \alpha_y^{(i)}) \Delta T \right] + \sum_{j=1}^{n-1} (a_j e^{\lambda_j y} + b_j e^{-\lambda_j y}) \frac{\omega_{j,i-1} - \omega_{j,i}}{t_i \lambda_j} \quad (12)$$

where $\alpha_y^{(0)}$ is the rule-of-mixtures effective y -thermal expansion coefficient of an undamaged structure given by

$$\alpha_y^{(0)} = \frac{1}{t E_y^{(0)}} \sum_{i=1}^n \alpha_y^{(i)} t_i E_y^{(i)} \quad (13)$$

Note that $\sigma_0(y) = (\tau_0 - \tau_n)y + \sigma_0(0)$ is the total applied axial stress in the y -direction for problems with constant, non-zero shear stresses applied to the sides of the specimen. When there are no shear-stress boundary conditions, $\sigma_0(y)$ is constant and equal to the total applied axial stress.

The average displacements in the y -direction in each layer i can be obtained either as a function of the axial displacement at the interface x_i :

$$\langle v_i \rangle = v(x_i) - \frac{t_i \tau_{xy}(x_{i-1})}{G_{xy}^{(i)}} \langle \xi_i L_i \rangle - \frac{t_i \tau_{xy}(x_i)}{G_{xy}^{(i)}} \langle \xi_i R_i \rangle \quad (14)$$

or, alternatively, as a function of the average displacement in layer $i + 1$:

$$\begin{aligned} \langle v_{i+1} \rangle - \langle v_i \rangle &= \frac{t_{i+1} \langle (1 - \xi_{i+1}) R_{i+1} \rangle}{G_{xy}^{(i+1)}} \tau_{xy}(x_{i+1}) + \frac{t_i \langle \xi_i L_i \rangle}{G_{xy}^{(i)}} \tau_{xy}(x_{i-1}) \\ &+ \left(\frac{t_{i+1} \langle (1 - \xi_{i+1}) L_{i+1} \rangle}{G_{xy}^{(i+1)}} + \frac{t_i \langle \xi_i R_i \rangle}{G_{xy}^{(i)}} \right) \tau_{xy}(x_i) \end{aligned} \quad (15)$$

In the following section these equations are applied to the case of a symmetric laminate with two cracks. Due to the large number of layers and their small thickness, it is convenient to use linear shape functions, a common assumption in all shear-lag methods: the x -variation in all layers is linear, which implies $L_i = 1 - \xi_i$ and $R_i = \xi_i$, and results in

$$\langle \xi_i L_i \rangle = \frac{1}{6}; \langle (1 - \xi_i) L_i \rangle = \frac{1}{3}; \langle \xi_i R_i \rangle = \frac{1}{3}; \langle (1 - \xi_i) R_i \rangle = \frac{1}{6} \quad (16)$$

4. Determination of Stresses in the 3 Zones of a Cracked Specimen

In the case of a specimen with a cracked interface, one can distinguish three elastic problems, identified from three zones shown in Figure 2. The origin of the y axis is progressing with the debond, i.e. $y = 0$ is located at the crack tip.

4.1. Zone I

Stresses in zone I can be obtained by considering that i) by symmetry conditions, the shear stress is zero in the $x = 0$ plane, and ii) assuming a constant shear on the debonded interface, which is a parameter of the model, denoted τ_f . In doing so, two singularities of the problem are neglected at $y = a$ and $y = 0$, where the shear stress of the cracked interface should be zero and singular, respectively. At $y = a$, it is not necessary to refine the problem by introducing a zone of infinitely small length, as was done for other types of load and geometry [11, 12, 13], because the energy release rate will not depend on the shear stress $\tau_{xy}(a)$. The singularity at $y = 0$ is more of a problem, and the accuracy of the model should always be checked at that location. However, previous work on isotropic materials have shown that the effect of the singularity was small provided the crack length is large enough, that is the singularity is not close to the top of the specimen. From Eq. (12) the average axial stress is determined, with $\tau_0 = 0$ and $\tau_n = \tau_f$:

$$\langle \sigma_{yy}^{(i)} \rangle = E_y^{(i)} \left[\frac{-\tau_f + h_1 \sigma_0(0)}{h_1 E_y^{(0)I}} + (\alpha_y^{(0)I} - \alpha_y^{(i)}) \Delta T \right] + \sum_{j=1}^{N_a-1} \left(a_j^I e^{\lambda_j^I y} + b_j^I e^{-\lambda_j^I y} \right) \frac{\omega_{j,i-1}^I - \omega_{j,i}^I}{t_i \lambda_j^I} \quad (17)$$

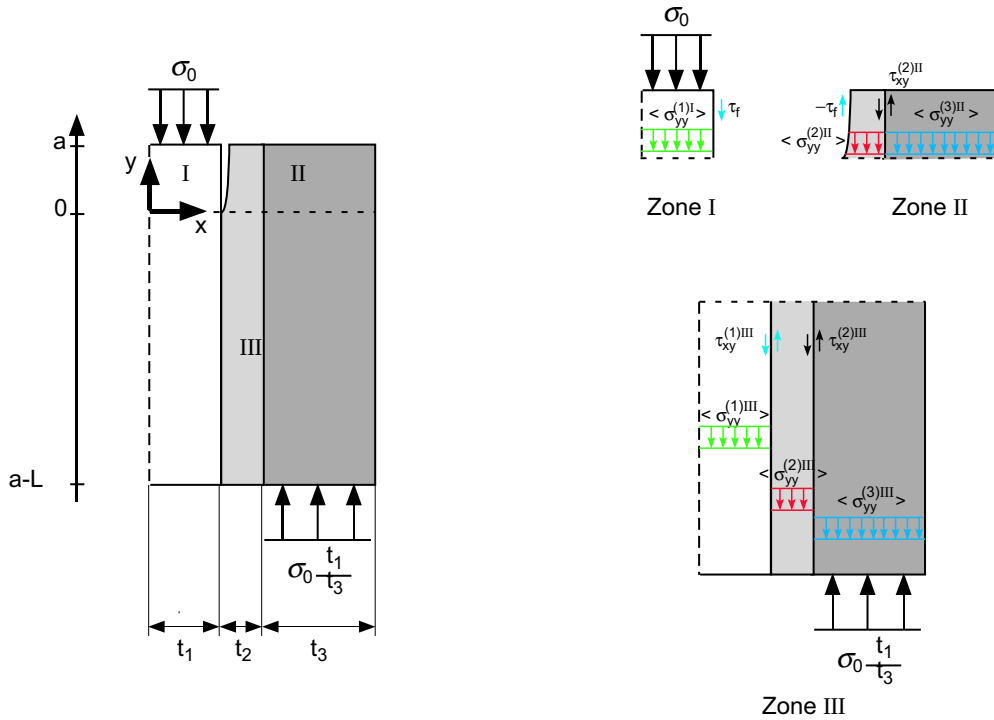


Fig. 2. MDLS specimen with a crack, definition of the three zones. For readability, a specimen with 3 layers only is represented here

where

$$\begin{aligned} h_1 &= \sum_{i=1}^{N_a} t_i \\ E_y^{(0)I} &= \frac{1}{h_1} \sum_{i=1}^{N_a} t_i E_y^{(i)} \\ \alpha_y^{(0)I} &= \frac{1}{h_1 E_y^{(0)I}} \sum_{i=1}^{N_a} t_i E_y^{(i)} \alpha_y^{(i)} \end{aligned} \quad (18)$$

Similarly, using Eq. (11) the shear stress is given by:

$$\tau_{xy}(x_i) = \sum_{j=1}^i \frac{t_j E_y^{(j)}}{h_1 E_y^{(0)I}} \tau_f + \sum_{j=1}^{N_a-1} \left(a_j^I e^{\lambda_j^I y} - b_j^I e^{-\lambda_j^I y} \right) \omega_{i,j}^I \quad (19)$$

4.2. Zone II

In zone II, the friction stress in the debonded interface is of opposite sign to that of zone I, and therefore we have $\tau_0 = -\tau_f$, and $\tau_n = 0$. The same approach leads to:

$$\begin{aligned} \langle \sigma_{yy}^{(i)} \rangle &= E_y^{(i)} \left[\frac{\tau_f + h_2 \sigma_0(0)}{h_2 E_y^{(0)II}} + (\alpha_y^{(0)II} - \alpha_y^{(i)}) \Delta T \right] \\ &+ \sum_{j=1+N_a}^{N_a+N_b-1} \left(a_{j-N_a}^{II} e^{\lambda_{j-N_a}^{II} y} + b_{j-N_a}^{II} e^{-\lambda_{j-N_a}^{II} y} \right) \frac{\omega_{j-N_a,i-1}^{II} - \omega_{j-N_a,i}^{II}}{t_i \lambda_{j-N_a}^{II}} \end{aligned} \quad (20)$$

and the shear stress is given by:

$$\tau_{xy}(x_i) = -\tau_f - \sum_{j=1+N_a}^i \frac{t_j E_y^{(j)}}{h_2 E_y^{(0)II}} \tau_f + \sum_{j=1+N_a}^{N_a+N_b-1} \left(a_{j-N_a}^{II} e^{\lambda_{j-N_a}^{II} y} - b_{j-N_a}^{II} e^{-\lambda_{j-N_a}^{II} y} \right) \omega_{j-N_a,i}^{II} \quad (21)$$

where

$$\begin{aligned} h_2 &= \sum_{i=1+N_a}^{N_a+N_b} t_i \\ E_y^{(0)II} &= \frac{1}{h_2} \sum_{i=1+N_a}^{N_a+N_b} t_i E_y^{(i)} \\ \alpha_y^{(0)II} &= \frac{1}{h_2 E_y^{(0)II}} \sum_{i=1+N_a}^{N_a+N_b} t_i E_y^{(i)} \alpha_y^{(i)} \end{aligned} \quad (22)$$

4.3. Zone III

In zone III, all stresses can be determined using the formalism developed in the previous section, with $\tau_0 = \tau_n = 0$. That is,

$$\langle \sigma_{yy}^{(i)} \rangle = E_y^{(i)} \left[\frac{\sigma_0(0)}{E_y^{(0)}} + (\alpha_y^{(0)} - \alpha_y^{(i)}) \Delta T \right] + \sum_{j=1}^{N-1} \left(a_j e^{\lambda_j y} + b_j e^{-\lambda_j y} \right) \frac{\omega_{j,i-1} - \omega_{j,i}}{t_i \lambda_j} \quad (23)$$

and

$$\tau_{xy}(x_i) = \sum_{j=1}^{N-1} \left(a_j e^{\lambda_j y} - b_j e^{-\lambda_j y} \right) \omega_{i,j} \quad (24)$$

4.4. Boundary Conditions

Based upon the preceding equations, $2(N_a - 1) + 2(N_b - 1) + 2(N_a + N_b - 1)$ unknowns are to be determined, $a_i, b_i, a_j^I, b_j^I, a_k^{II}, b_k^{II}$, where $i = 1..(N_a + N_b - 1), j = 1..(N_a - 1)$ and $k = 1..(N_b - 1)$. The following set of boundary conditions was found compatible with the problem:

$$\begin{aligned}
 \left\langle \sigma_{yy}^{(i)I} \right\rangle \Big|_{y=a} &= -\sigma_0, i = 1..N_a \\
 \left\langle \sigma_{yy}^{(i)I} \right\rangle \Big|_{y=0} &= \left\langle \sigma_{yy}^{(i)III} \right\rangle \Big|_{y=0}, i = 1..N_a \\
 \left\langle \sigma_{yy}^{(i)III} \right\rangle \Big|_{y=a-L} &= 0, i = 1..N_a \\
 \tau_{xy}^I(x_i) \Big|_{y=0} &= \tau_{xy}^{III}(x_i) \Big|_{y=0}, i = 1..N_a \\
 \left\langle \sigma_{yy}^{(i)II} \right\rangle \Big|_{y=a} &= 0, i = 1 + N_a..N_a + N_b \\
 \left\langle \sigma_{yy}^{(i)II} \right\rangle \Big|_{y=0} &= \left\langle \sigma_{yy}^{(i)III} \right\rangle \Big|_{y=0}, i = 1 + N_a..N_a + N_b \\
 \left\langle \sigma_{yy}^{(i)III} \right\rangle \Big|_{y=a-L} &= \sigma_0 \frac{h_1}{h_2}, i = 1 + N_a..N_a + N_b \\
 \tau_{xy}^{II}(x_i) \Big|_{y=0} &= \tau_{xy}^{III}(x_i) \Big|_{y=0}, i = 1 + N_a..N_a + N_b
 \end{aligned} \tag{25}$$

The resulting system of equations can be solved analytically for a small number of layers (typically three, c.f. refs [4, 5]), but in most cases a numerical implementation is more straightforward. Once these constants have been obtained, it is possible to calculate the energy release rate of a crack propagating along the interface between two plies, which is the subject of the next section.

5. Energy Release Rate

5.1. Exact Solution

The calculation of the approximate energy release rate proceeds in two steps. First, an exact expression of the energy release rate is obtained, which involves axial displacements and average axial stresses only. Second, the approximate shear lag equations of stresses and displacements are used to evaluate the energy release rate. The energy release rate for the double lap-shear specimen is obtained from the expression derived for a general composite geometry [4]:

$$G = \frac{1}{2} \frac{\partial}{\partial A} \left(\int_{S+S_c} \vec{T}^0 \cdot \vec{u} dS + \int_V \sigma \alpha T dV \right) \tag{26}$$

where A is the cracked area, S denotes the outer surface, S_c stands for the cracked or imperfect interfaces, and V denotes the volume. $\vec{T}^0$ is the vector of stresses applied to S and S_c , and $\vec{u} = (u \ v \ w)$ where u, v and w are the displacements in the x, y , and z directions respectively. The vectors $\vec{T}^0$ and $\vec{u}$ are evaluated for each surface of the specimen as given below, and their contribution to the surface integral on $S + S_c$ are summed afterwards. On the top surface, in zone I:

$$\begin{aligned}
 \vec{T}^0 &= (0, -\sigma_0, 0) \\
 \vec{u} &= (u_i, v_i(x, a), w_i) \\
 \int_{S+} \vec{T}^0 \cdot \vec{u} dS &= -\sigma_0 W \sum_{i=1}^{N_a} t_i \langle v_i(a) \rangle
 \end{aligned} \tag{27}$$

On the bottom surface, in zone III:

$$\begin{aligned}\vec{T}^0 &= \left(0, \sigma_0 \frac{h_1}{h_2}, 0\right) \\ \vec{u} &= (u_i, v_i(x, a-L), w_i) \\ \int_{S^-} \vec{T}^0 \cdot \vec{u} dS &= \sigma_0 \frac{h_1}{h_2} W \sum_{i=1+N_a}^{N_a+N_b} t_i \langle v_i(a-L) \rangle\end{aligned}\quad (28)$$

On the fracture surface, in zone I:

$$\begin{aligned}\vec{T}^0 &= (-\sigma_t, \tau_f, 0) \\ \vec{u} &= (u_{N_a}(h_1, y), v_{N_a}(h_1, y), w_{N_a})\end{aligned}\quad (29)$$

On the fracture surface, in zone II:

$$\begin{aligned}\vec{T}^0 &= (\sigma_t, -\tau_f, 0) \\ \vec{u} &= (u_{1+N_a}(h_1, y), v_{1+N_a}(h_1, y), w_{1+N_a})\end{aligned}\quad (30)$$

The contributions of the two crack surfaces are then assembled, and noting that

$$\vec{T}^0 \cdot \vec{u} = -\sigma_t u_{N_a}(h_1) + \tau_f v_{N_a}(h_1, y) - \tau_f v_{1+N_a}(h_1, y) + \sigma_t u_{1+N_a}(h_1) \quad (31)$$

and that $u_{N_a}(h_1) = u_{1+N_a}(h_1)$ (surfaces in contact), or $\sigma_t = 0$ (surfaces not in contact), leads to the same result:

$$\int_{S_c} \vec{T}^0 \cdot \vec{u} dS = W \int_0^a \tau_f (v_{N_a}(h_1, y) - v_{1+N_a}(h_1, y)) dy \quad (32)$$

Finally, on the side surface ($S^>$, which has $N = N_a + N_b$):

$$\begin{aligned}\vec{T}^0 &= (-\sigma_t, 0, 0) \\ \vec{u} &= (u_N(t, y), v_N, w_N) \\ \int_{S^>} \vec{T}^0 \cdot \vec{u} dS &= -\sigma_t L W \langle u_N(t) \rangle\end{aligned}\quad (33)$$

with σ_t the transverse stress (or other normal stress) as shown in Figure 1, and the crack area $dA = W da$ (Notice τ_f is positive to resist compression), G is evaluated from:

$$\begin{aligned}G &= \frac{1}{2} \frac{d}{da} \left[-\sigma_0 \sum_{i=1}^{N_a} t_i \langle v_i(a) \rangle + \sigma_0 \frac{h_1}{h_2} \sum_{i=1+N_a}^{N_a+N_b} t_i \langle v_i(a-L) \rangle - \sigma_t L \langle u_N(t) \rangle \right. \\ &\quad \left. + \int_0^a \tau_f (v_{N_a}(h_1, y) - v_{1+N_a}(h_1, y)) dy + L \Delta T \sum_{i=1}^N \alpha_y^{(i)} t_i \overline{\sigma_{yy}^{(i)}} \right]\end{aligned}\quad (34)$$

where the bars denote phase averaged stresses, calculated as:

$$\overline{\sigma_{yy}^{(i)}} = \int_V \sigma_{yy}^{(i)} dv = \frac{1}{L} \int_{a-L}^a \langle \sigma_{yy}^{(i)} \rangle dy \quad (35)$$

The expression for the energy release rate G determined by Equation (34) is exact and depends only on axial stresses and displacements in the three layers. The optimal shear-lag analysis is used in the following section to solve for G numerically.

5.2. Approximate Solution

5.2.1. Calculation of Displacements in the 3 Zones

Zone I In its most general form,

$$\langle v_1(y) \rangle = \langle v_1(0) \rangle + \int_0^y \langle \varepsilon_{yy}^{(1)} \rangle dy = \langle v_1(0) \rangle + \int_0^y \left(\frac{\sigma_{yy}^{(1)}}{E_y^{(1)}} + \alpha_y^{(1)} \Delta T \right) dy \quad (36)$$

Taking as a reference for displacements $\langle v_1(0) \rangle = 0$, and making use of Equation (17) one obtains

$$\langle v_1(y) \rangle = \left[\frac{-\tau_f + h_1 \sigma_0(0)}{h_1 E_y^{(0)I}} + \alpha_y^{(0)I} \Delta T \right] y + \frac{1}{E_y^{(1)}} \sum_{j=1}^{N_a-1} \left(a_j^I e^{\lambda_j^I y} - b_j^I e^{-\lambda_j^I y} \right) \frac{\omega_{j,i-1}^I - \omega_{j,i}^I}{t_i \lambda_j^{I2}} \quad (37)$$

from which $\langle v_1(y=a) \rangle$ can be calculated. The other displacements at $y = a$, $\langle v_i(y=a) \rangle$, $i = 2..N_a$ in zone I are obtained directly from Eq. (15), and $\langle v_{N_a}(y=a) \rangle$ is determined. The edge displacement $v_{N_a}(h_1, y)$ also needs to be calculated, and from Eq. (14):

$$v_{N_a}(h_1, y) = \langle v_{N_a}(y) \rangle + \frac{t_{N_a} \tau_{xy}^I(x_{N_a-1})}{G_{xy}^{(N_a)}} \langle \xi_{N_a} L_{N_a} \rangle + \frac{t_{N_a} \tau_f}{G_{xy}^{(N_a)}} \langle \xi_{N_a} R_{N_a} \rangle \quad (38)$$

Zone II By definition:

$$\langle v_{1+N_a}(y) \rangle = \langle v_{1+N_a}(0) \rangle + \int_0^y \left(\frac{\sigma_{yy}^{(1+N_a)II}}{E_y^{(1+N_a)}} + \alpha_y^{(1+N_a)} \Delta T \right) dy \quad (39)$$

with $\langle \sigma_{yy}^{(1+N_a)II} \rangle$ given by Equation (20). To obtain the $\langle v_{1+N_a}(0) \rangle$ term, the edge displacement $v_{1+N_a}(h_1, y)$ needs to be calculated first. Again, from Equation (14), and accounting for shear stresses on the edges of zone II:

$$v_{1+N_a}(h_1, y) = \langle v_{1+N_a}(y) \rangle + \frac{t_{1+N_a} \tau_f}{G_{xy}^{(1+N_a)}} \langle \xi_{1+N_a} L_{1+N_a} \rangle - \frac{t_{1+N_a} \tau_{xy}^{II}(x_{1+N_a})}{G_{xy}^{(1+N_a)}} \langle \xi_{1+N_a} R_{1+N_a} \rangle \quad (40)$$

The continuity of axial displacement between zones I and II at $y = 0$:

$$v_{1+N_a}(h_1, 0) = v_{N_a}(h_1, 0) \quad (41)$$

is used to obtain an expression for $v_{1+N_a}(h_1, y)$ that depends only on known variables and functions, and to further calculate the difference of displacements of the fracture surface and its integral between 0 and a .

Zone III The last displacements that need to be evaluated in Equation (34) are $\langle v_i(a-L) \rangle$, which is done by using Equation (15) and a cascade elimination.

5.2.2. Phase Averaged Stresses

All the axial displacements and their averages have been evaluated, and only the phase averaged axial stresses $\overline{\sigma_{yy}^{(i)}}$ still need to be determined. By definition, they are obtained from:

$$\overline{\sigma_{yy}^{(i)}} = \frac{1}{L} \int_{a-L}^a \langle \sigma_{yy}^{(i)} \rangle dy = \frac{1}{L} \left(\int_{a-L}^0 \langle \sigma_{yy}^{(i)III} \rangle dy + \int_0^a \langle \sigma_{yy}^{(i)I} \rangle dy \right), i = 1..N_a \quad (42)$$

and

$$\overline{\sigma_{yy}^{(i)}} = \frac{1}{L} \int_{a-L}^a \langle \sigma_{yy}^{(i)} \rangle dy = \frac{1}{L} \left(\int_{a-L}^0 \langle \sigma_{yy}^{(i)III} \rangle dy + \int_0^a \langle \sigma_{yy}^{(i)II} \rangle dy \right), i = 1 + N_a..N_a + N_b \quad (43)$$

Because the constants a_i^k, b_i^k are functions of crack length a , there is no other choice but to differentiate numerically the expression obtained by substituting phase averaged stresses (Eq. (42)-(43)) into the energy release rate equation (Eq. (34)). Therefore the expression of G is convenient for evaluation by numerical differentiation.

6. Model Analysis

Figures 3-6 present the energy release rate G calculated from the model for a number of lay-ups, with and without internal stresses, for various friction stresses, and with laminate thermo-mechanical properties given in the next section (Table I). The ensemble of the simulations shown was obtained for a fixed value of $\sigma_0 = 20MPa$. In Figure 3, for a friction stress of $\tau_f = 0$, the energy release rate increases steadily from 18 to $21kJ/m^2$. When τ_f is not zero, the energy release rate results in a maximum for a certain crack length, which was not described satisfactorily by the simplified model with isotropic materials, although it was obtained [5]. In practice, this phenomenon is generally observed, as the load increases to propagate a longer crack, with the energy release rate being constant versus crack length. Therefore, a minimum crack length corresponding to the maximum G should be chosen in an experiment involving steady state crack propagation, so that the load can be increased monotonously. This behaviour is well marked on the other figures as well, and from the simulation results of Figs. 3-6 it was decided that the minimum crack length should be at least of $3mm$ in order to guarantee a steady state crack propagation. This provides another advantage: from Equation (34), the result depends on the value of the derivatives $d\tau_{xy}^{(i)}(a-l)/da$, which are not approximated well by shear lag at small crack lengths [5].

In ref. [5] it was also shown that the effect of internal stresses is accurately modelled by the shear lag approximation. This is because the approximation of average axial stresses by shear lag theory is very accurate, and $\overline{\sigma_{yy}^{(i)}}$ dominates the internal stress response of the system at long crack lengths. The effect of internal stresses was well quantified and the accuracy of its description by the model did not depend on the level of the friction stress. In the case of isotropic materials, the authors analysed assemblies of aluminium adherents and thin adhesive layers, and it was shown that the influence of internal stresses on the energy release rate was small (contrary to their effect on adhesive strength, as shown in Refs. [9, 10]). In the present case, however, a considerably larger effect was found, as witnessed by comparing the values from Figures 3 and 4, and from Figures 5-6 as well. For instance, for a crack length of $5mm$ in Fig. 6 one obtains an error of 300% by neglecting internal stresses for the smallest friction coefficient. The magnitude of the effect is typical of orthotropic materials, and has been observed on several other occasions [6, 14].

The effect of the friction at the debonded surface is also well marked, as it was observed previously [5]. In the case of fibre-composites, by analogy to mode I crack propagation, it is wise to keep in mind that the initial crack propagation involves a low friction (that of the laminate on a teflon insert), whereas further crack propagation might be accompanied by fibre bridging, and fibre-matrix interactions, which all tend to increase friction. Thus, two different coefficients of friction might be used to describe a single experiment where steady state crack propagation can be observed, and the energy release rate values obtained at initiation should be differentiated from the ones obtained at propagation.

The model accuracy is influenced to a large extent by the choice of shape functions, and it is certainly possible to increase the accuracy of the result by adjusting them. Another solution could therefore be to let the shape functions be linear, but increase the number of layers, so as to create a vertical mesh. The model's accuracy does not truly benefit from this approach, as layers are thin enough, and it was therefore chosen to use one layer to represent one ply.

To conclude, both the friction stress and the internal stress levels have a large influence on the energy release rate, and they should therefore be determined accurately in an experimental study.

7. Experimental

7.1. Materials

Symmetric glass fibre-epoxy laminates were manufactured from a Hexcel-913 prepreg and tested in compression double-lap shear. This material was used as its thermo-mechanical properties have been extensively documented in previous studies at NPL [15, 16]. These values are reported in Table I.

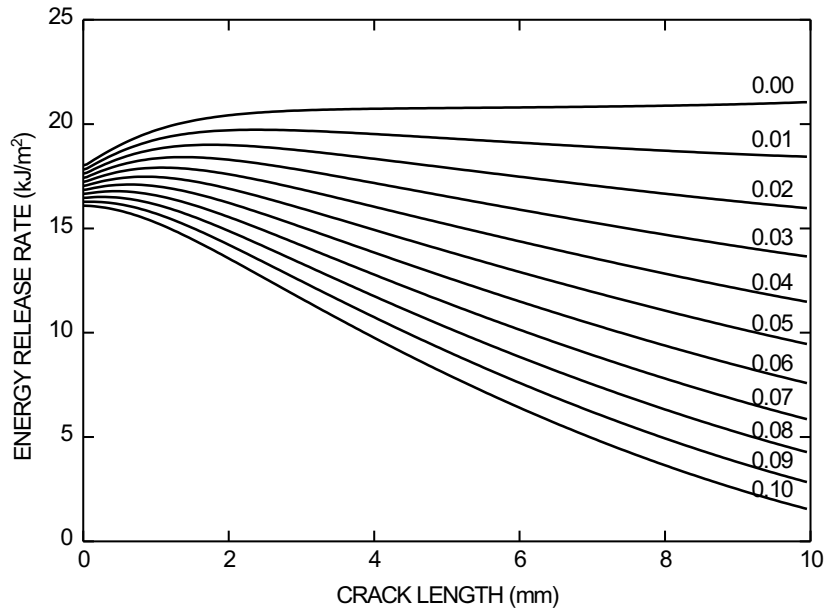


Fig. 3. Energy release rate of a debonding interface of a $[0_{32}; 90_{16}]_s$ epoxy-glass fibre laminate: model results, for various coefficients of friction $\chi = \tau_f/\sigma_0$ as indicated on the figure, and no internal stresses

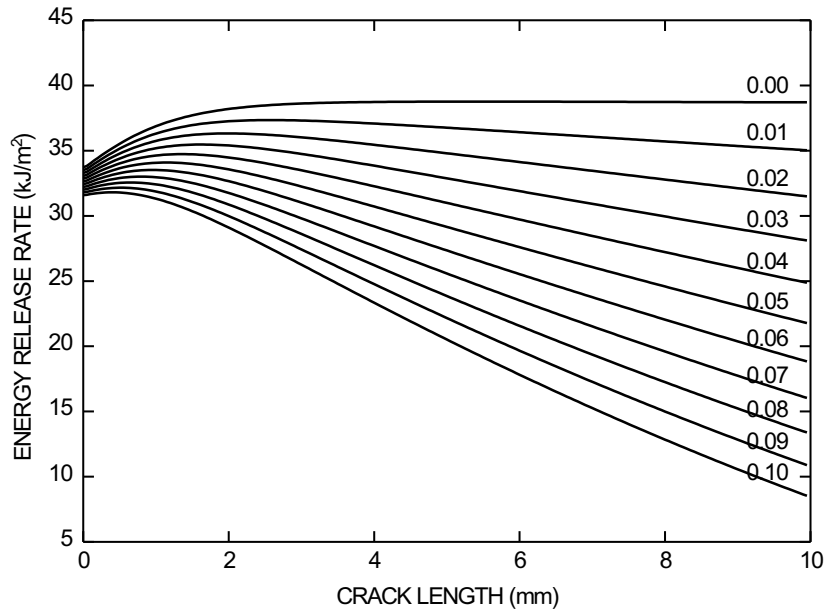


Fig. 4. Energy release rate of a debonding interface of a $[0_{32}; 90_{16}]_s$ epoxy-glass fibre laminate: model results, for various coefficients of friction $\chi = \tau_f/\sigma_0$ as indicated on the figure, and an internal stress level characterised by a temperature difference $\Delta T = -100^\circ C$

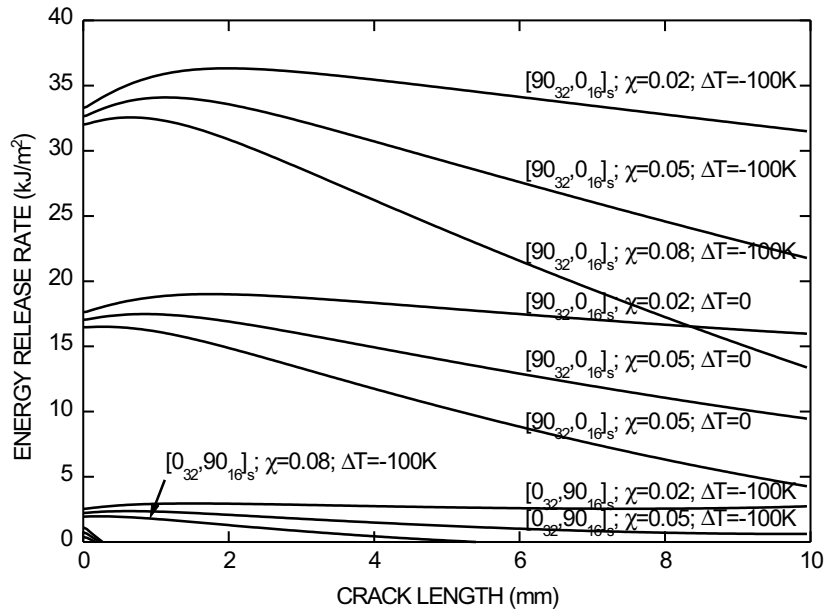


Fig. 5. Energy release rate of a debonding interface of $[0_{32}; 90_{16}]_s$ and $[90_{32}; 0_{16}]_s$ epoxy-glass fibre laminates with various coefficients of friction $\chi = \tau_f / \sigma_0$ and internal stress levels as indicated on the figure

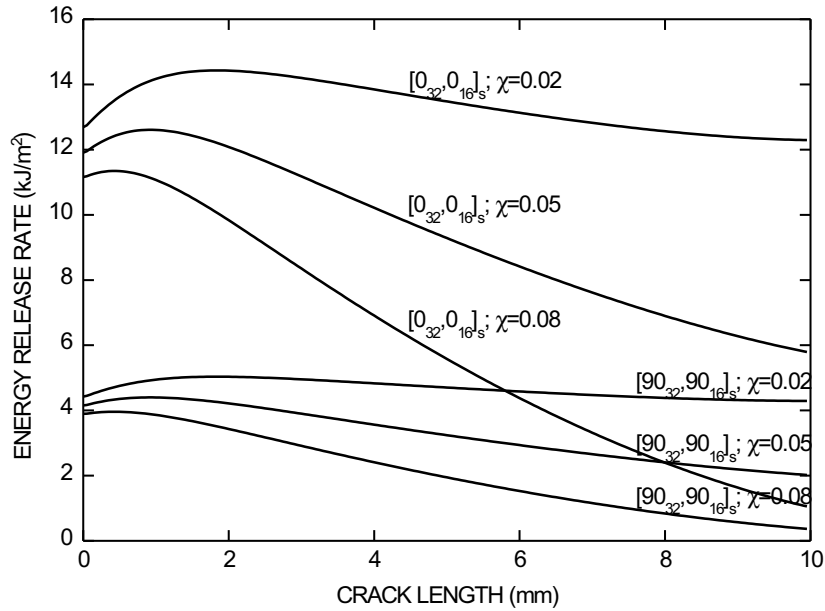


Fig. 6. Energy release rate of a debonding interface of $[0_{32}; 0_{16}]_s$ and $[90_{32}; 90_{16}]_s$ epoxy-glass fibre laminates with various coefficients of friction $\chi = \tau_f / \sigma_0$. Notice there is no effect of internal stresses with these lay-ups

The specimen dimensions were $25 \times 20 \times 12\text{mm}^3$, and the stacking sequences were $[0_{32}; 90_{16}]_s$, $[0_{32}; 0_{16}]_s$, and $[90_{32}; 0_{16}]_s$. Various crack lengths were fixed by the lengths of teflon film inserts. Composites plates of dimensions of approximately $300 \times 200 \times 12\text{mm}^3$, having teflon film inserts of 20mm width, were manufactured in an autoclave following the processing conditions prescribed by the manufacturer, which are 1 hour at 120°C and 7 bar pressure (heating up rate of $3^\circ\text{C}/\text{min}$). The plates were then cooled down to the room temperature at a rate of approximately $2.5^\circ\text{C}/\text{min}$, and specimens were cut to the specified dimensions. The specimens were then stored in a dry environment until the tests were carried out.

7.2. Equipment

Electronic speckle pattern interferometry (ESPI) offers very precise measurement of average strains that enable the calculation of the thermal expansion coefficient and elastic properties of composite specimens with relatively large dimensions and aspect ratios, such as plates or entire components, or locally within a larger specimen or part. It is a full field measurement method in which deformations under mechanical and thermal loads can be measured along three orthogonal directions without contact with the specimen [17–21].

Sample preparation is easy and requires a minimum of handling [22]. In the present work, a simple chalk spray was used to obtain a reproducible surface roughness, independent of the specimen.

The speckle effect is produced by the interference between the reflected rays when a non-specular, dispersing surface is illuminated with a laser beam [17], as shown in Figure 7. The pattern is defined by the microscopically rough topography of the specimen surface. Any deformation of the surface results in a change of the speckle pattern. A correlation between the speckle patterns for an object in a reference state and in a deformed state gives information on the deformation of the object's surface. This is typically done by gray level difference of two digital video interferograms and some additional digital image analysis. For each interferogram, three or four images of the interference pattern are taken with a phase shift of the laser beam of 120° or 90° respectively, and they are combined to give a phase image, which holds information not only on the amount, but also on the direction of the deformation. The ESPI installation used in this study (Ettemeyer AG, Germany) consists of an ESPI head, a hardware control and data acquisition system, and an image analysis facility, as depicted in Figure 7. The software Istra[®] v.3.3 was used for the image processing. A compact unit, described in a previous work [16], allows the quasi simultaneous measurement of in- and out-of-plane displacements. The in-plane displacements are given by the relative phase shift of two imaging beams. The out-of-plane displacements, i.e. parallel to the optical axis, are obtained from the amount of phase shift of an imaging beam with respect to a reference beam. One can thus measure the 3-D deformation of an object under a load. This integrated ESPI head includes the laser source, mirrors, piezos, shutter, and video camera. The measurement sensitivity of the system, defined as the amount of relative displacement giving rise to an additional interference fringe, is

$$\text{out-of-plane: } s_z = \frac{\lambda}{2}; \quad \text{in-plane: } s_{x,y} = \frac{\lambda}{2 \sin \theta} \quad (44)$$

where λ is the wavelength of the laser and θ the angle between the imaging beams and the optical axis. The speckle interferometer uses near infrared laser diodes ($\lambda = 0.780\mu\text{m}$), giving an out-of-plane measurement sensitivity of $0.390\mu\text{m}/\text{fringe}$. The distance between the front mirrors forming the imaging beam sources was 140mm and the distance to the object was varied following the values given in Table II. Table II presents a range of different configurations successfully used with NPL's set-up, and indicates the in-plane measuring sensitivity for both in-plane axes. Since information on the displacement gradient is carried by the entire phase image, and not just by the fringes, the effective in-plane sensi-

Table I. Thermo-mechanical properties of GF/epoxy laminate Hexcel-913

$E_{11} = 42.288\text{GPa}$	$E_{22} = 14.034\text{GPa}$
$G_{12} = 6.465\text{GPa}$	$G_{13} = 4.600\text{GPa}$
$\nu_{12} = 0.271$	
$\alpha_{11} = 7.052 \times 10^{-6}\text{m/m/K}$	$\alpha_{22} = 17.695 \times 10^{-6}\text{m/m/K}$

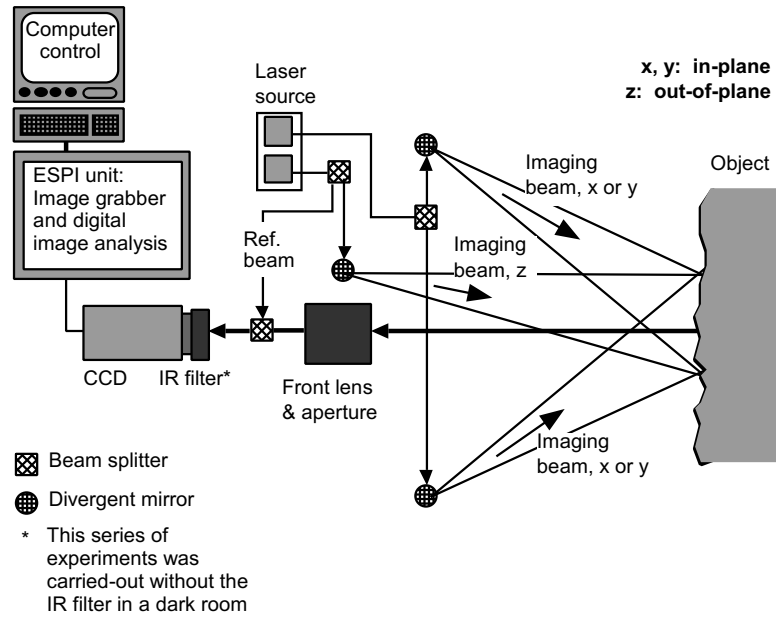
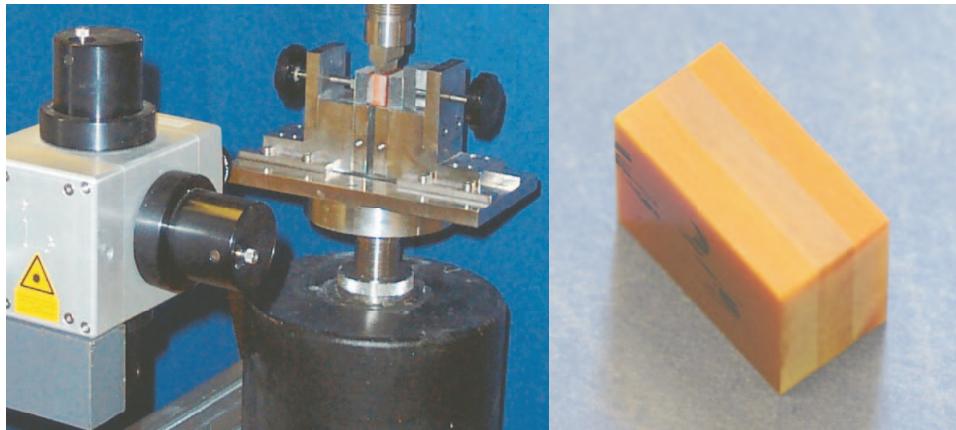


Fig. 7. Layout of the ESPI deformation measurement system

Fig. 8. NPL's experimental modified double-lap shear set-up (left), and a $[90_{32}; 0_{16}]_s$ specimen (right)

tivity is about an order of magnitude better. The limiting factor of the sensitivity is the inherently high noise of the method resulting from the speckle pattern.

Images were recorded with a 752×582 pixel built-in CCD camera operating at $25Hz$. A set of Pentax Cosmimar[®] macro lenses was used on the front end of the viewing system, giving an imaging field as described in Table II. The resulting calibration of the monitor screen in the x and y -directions, together with the lateral resolution achieved are also given in Table II. The time required for a measurement, determined by the frequency of the video camera and the speed of the shutters and phase shifting piezo-devices, was approximately a third of a second for each axis. The experimental modified

Table II. Optical configurations of the ESPI set-up

Lens	Distance to object (mm)	Imaging field (mm $\times$ mm)	Resulting calibration (mm/pixel $\times$ mm/pixel)	s_x, s_y $\mu m/fringe$
12mm F*8	392	122 $\times$ 91.7	0.163 $\times$ 0.158	2.22
16mm, 2* extender	352	37.4 $\times$ 28.4	$4.98 \cdot 10^{-2} \times 4.88 \cdot 10^{-2}$	2.00

double-lap shear (MDLS) set-up available at NPL, together with a close-up of mode II specimen, are shown in Figure 8. In this configuration, the MDLS set-up is screwed on an Instron tensile tester, and the ESPI system is mounted on a rail system screwed to the tensile tester with intermediate cork plates. This ensures a perfect alignment of the ESPI head to the surface of the specimen, and reduced vibrations. A measure of displacements typically produces parallel fringes in an isotropic material without any defects from which quantitative values of displacements and deformations are obtained from the sensitivity by transforming the fringe pattern (commonly called unwrapping of the fringe pattern), as described in the next section.

7.3. Loading Sequence & Analysis

Given the finite time required for each measurement, the mechanical tests were conducted under quasi-static conditions. The loading sequence processed from 0kN to the complete failure of the specimen by imposed crosshead displacement steps of $30\mu m$ at a loading rate of $10\mu m \cdot s^{-1}$. The resulting time between two consecutive load-steps was approximately 6s, during which the measurement of the displacements in the three directions were triggered to start after the load had been reached for 1.5s, so as to reduce possible interference from the vibrations of the machine. The procedure was as follows: the specimen dimensions were measured within $\pm 10\mu m$, and all the specimens per batch were aligned and sprayed with a powder chalk spray at the same time. This ensured similar speckle intensity when they were subsequently mounted. The individual specimen was then placed in the test rig and aligned. A pre-load of approximately 3kN was applied to correct for alignment errors, and removed, to obtain a starting load of approximately 0.2kN. For each individual specimen, a piezo-calibration was then taken (as described in ref. [23]), and the test then proceeded, the tensile tester being under controlled displacement conditions. Data reduction was carried-out by applying a phase filter to the raw results, followed by the unwrapping sequence, the details of which are given in Ref. [23]. In the alternative case, the phase images were filtered with a routine that takes into account a 3×3 pixel area around the pixel of concern and preserves the phase information. Since the noise level - inherent to the method - was low, higher order filters were not needed. The deformation fields can be evaluated from the filtered phase images along a line or over the entire area. A full description of the method of analysis has been given in a recent Measurement Note [16].

8. Results - ESPI

Although tests were carried on three different types of laminate, this section concentrates on $[90_{32}; 0_{16}]_s$ laminates. All the issues discussed in this section are however applicable to the other laminates considered in this study. First, an example of interferograms obtained during the loading sequence of a $[90_{32}; 0_{16}]_s$ specimen is shown in Figure 9. In the Figure, interferograms show increased surface displacement from left to right, as shown by an increasing number of fringes, as the load is increased. It must also be kept in mind that only the first interferogram has a sensitivity per fringe of $2\mu m$, whereas the subsequent have increased sensitivity since fringes represent cumulative displacements. The first loading step (a) correspond to a slight translation combined with a rotation of the specimen. This is mainly due to the geometry of the specimen, which is never perfectly rectangular, due to the machining operation. The interferogram shown in Figure 9-b presents a combination of the fringes obtained in -a and the fringes due to loading of the specimen. This phenomenon clearly reduces the symmetry of the interferogram, and as the loading increases in -c the fringes are not perfectly symmetric, although the effect of misalignment are becoming less important. After these first three steps, a typical fringe pattern such as shown in -d is generally obtained. The good quality of fringes enables quantitative measurement of the axial displacements, which are exploited below to validate the shear lag model. Similar interferograms are obtained until the failure of the specimen is reached, in -e. This last fringe pattern was taken at the onset of failure of the specimen. It can be seen that an excess of displacement takes place on the right side of the specimen at the top, as shown by the three fringes in this region. This is due to the non-symmetric failure of the specimen, which begins to crack on the right interface, which has a slightly smaller crack length than the left one, and therefore a higher stress concentration. It is important to notice that this observation can be made through the use of ESPI only, as macroscopically one observes a simultaneous failure of the interfaces. In fact, as long as the difference between the two crack lengths is smaller than a millimetre, the two interfaces are failing simultaneously. When the specimen is clearly non-symmetric, and the difference between crack lengths is greater than a millimetre,

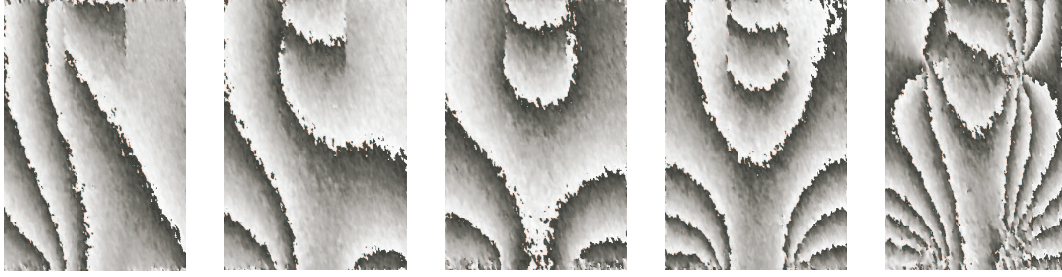


Fig. 9. Typical ESPI fringe patterns obtained during the loading of a specimen (a to e from left to right, see text for details)

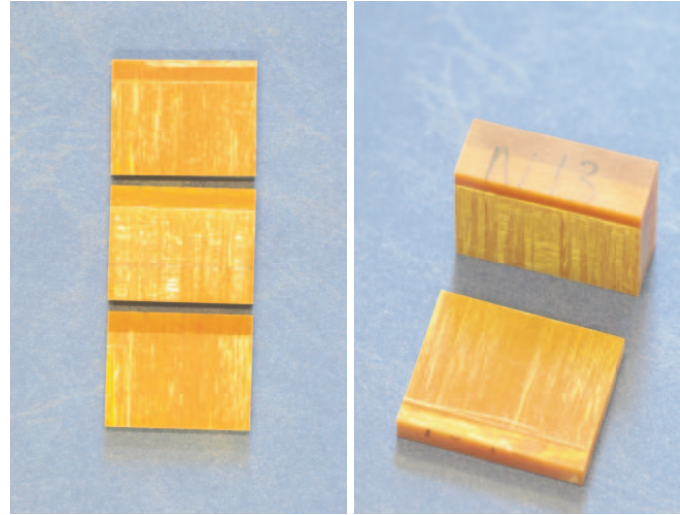


Fig. 10. Cracked specimen: left, valid test, as the two interfaces have cracked at the same time; right, test rejected, only one interface has been cracked

only one interface fails, as shown in Figure 10. Such a specimen is automatically discarded from further analysis.

Having established the validity of the loading sequence, the "regular" interferograms can now be used to validate the stress transfer model. To do so, the interferogram is unwrapped, and the resulting displacements are plotted along two lines shown in Figure 11, and compared to the model predictions in Figure 12. Because the displacement values obtained from the model are related to the displacement at the crack tip - taken as zero in the analysis of the energy release rate - and not the ESPI software, this value was determined as an adjusting parameter from the displacement profile of $v(x_1)$.

The fringes on the central part of the specimen are not connected to the edges, and are not affected nor affect the outer fringes, so that the friction in the cracked region appears as being negligible. This information is made available only through the use of a full field displacement method, such as ESPI, and could not be obtained through the use of strain gauges for example. The resulting modelled displacements are in close agreement to the experimental values obtained from the ESPI displacement map, as shown in Figure 12. It is very interesting to notice that the linear portion of the curve (in the cracked region) is overestimated by the model. This indicates that residual stresses have been slightly overestimated, as ΔT was taken to be $\sim -100^\circ C$. This value can be refined using approaches similar to those described in Ref. [11]. The close agreement of the remaining parts of the curves indicates that the model is well adapted to describe not only displacement, but also stresses, on which the calculation of displacements is based. From Figure 12 it seems, however, that the axial modulus of the 90° plies is slightly underestimated. The difference is small but noticeable. Further simulations can be used to demonstrate that the value used should be increased by about 5% to obtain a better fit.

Finally, it is important to remark that the displacement profiles presented are raw results, i.e. they have not been smoothed, which is a current practice when analysing ESPI results. Therefore, the imperfection on the curve $v(x = 0)$ at $y = 7mm$ is not removed, and edge effects at $y = 19 - 20mm$ are

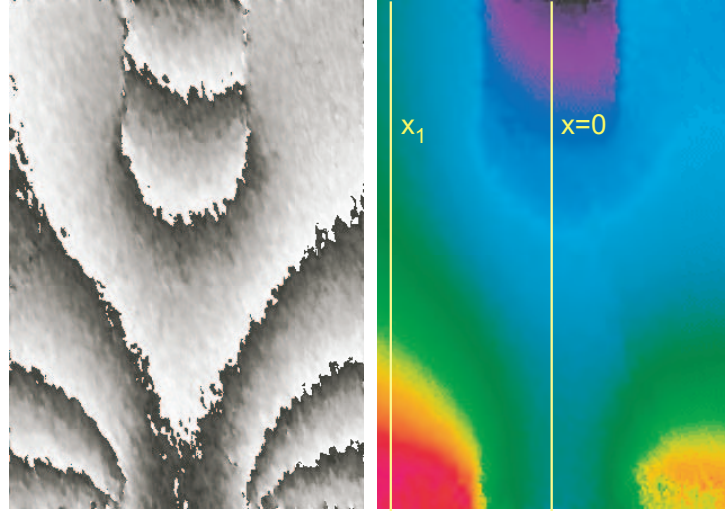


Fig. 11. ESPI fringe pattern obtained for a $[90_{32}; 0_{16}]_s$ laminate loaded between 1 and 26kN (left); resulting displacement map and definition of two lines along which the displacement profile is plotted (right)

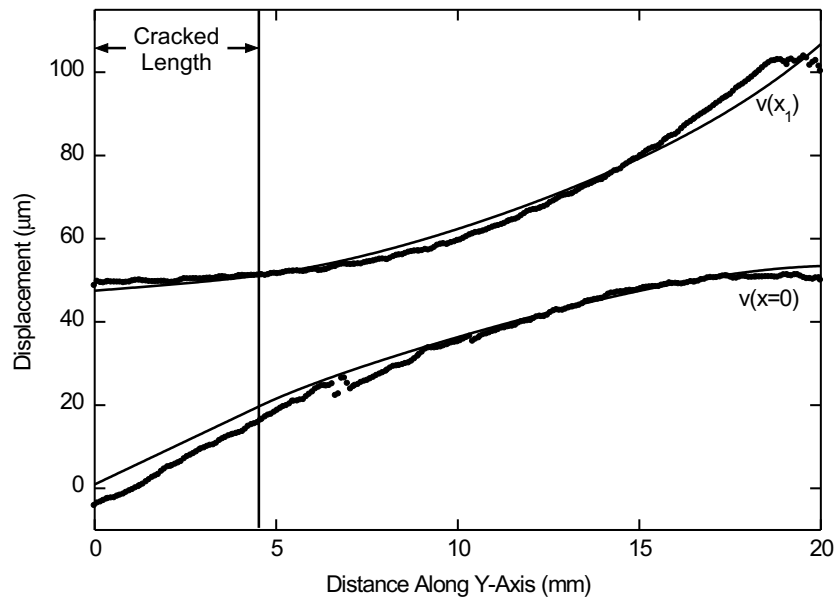


Fig. 12. Displacement profile along lines x_1 and $x = 0$, as defined in Figure 11 (circles), compared to the results of the shear lag model (lines)

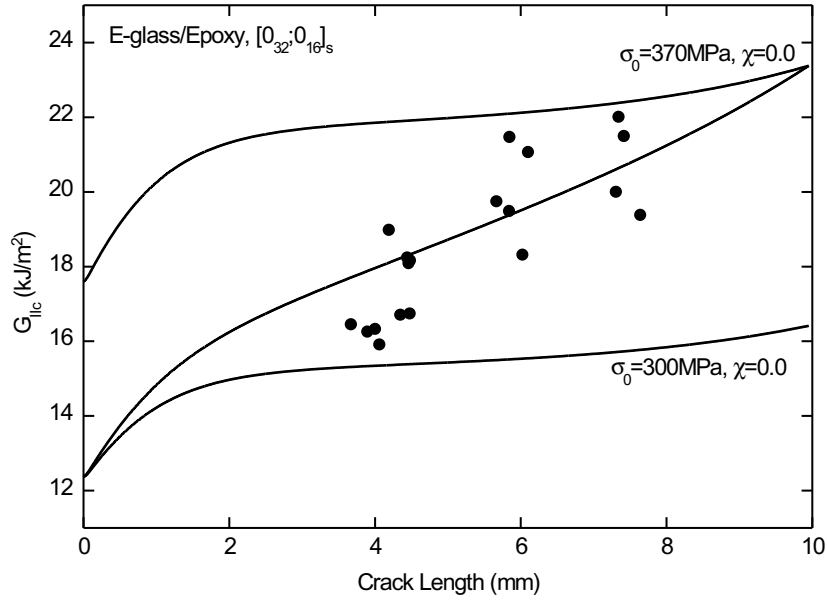


Fig. 13. Experimental vs. modelled mode II energy release rate for an E-glass/epoxy $[0_{32}; 0_{16}]_s$ laminate

also visible. The former is due to the transition between a zone with close fringes to a zone with a long fringe, i.e. smaller displacements, while the latter are generally related to a loss of contrast of the speckle pattern at the edges of the specimen. This cannot be corrected easily, as increasing the contrast generally decreases the resolution of the rest of the fringes. Since this problem also occurs when multiple directions are considered (i.e. deformations on more than one axis are recorded and analysed), the preferred solution should therefore be to modify the software in order to grade the contrast over the imaging field. Another solution is to use smaller mirrors in order to get a larger illuminating area, and therefore a more even distribution of the intensity over the area considered. These matters, as well as the traceability of the measurement, are going to be dealt with in the near future in an European exercise.

9. Results - Energy Release Rate

Now that the accuracy of the stress transfer model has been successfully compared to experimental results of surface displacement, and that it has been established that the friction coefficient of the debonded interface was zero in all cases, these results can be used to model the mode II energy release rate. The experimental energy release rate was obtained from the usual expression:

$$G_{IIc} = \frac{U}{W\Delta a} \quad (45)$$

where U is the elastic energy stored in the specimen before the crack propagates, calculated as the integral of the load vs. displacement curve, and Δa is the variation of crack length, equal to $L - a$ when complete failure occurs. The results for the three stacking sequences are depicted in Figures 13-14. In the three cases, it is observed that the mode II energy release rate G_{IIc} increases with the crack length. This confirms that the friction coefficient χ is zero, as any positive value of χ implies a decrease in G_{IIc} with the crack length. Nevertheless, it can be seen that a fixed stress level does not describe the results accurately. In fact, they would be best described by a negative friction coefficient, which is physically impossible.

The stress levels given on the plots were chosen as follows: the bottom line corresponds to an absolute minimum stress of the experimental series, while the top line corresponds to an absolute maximum.

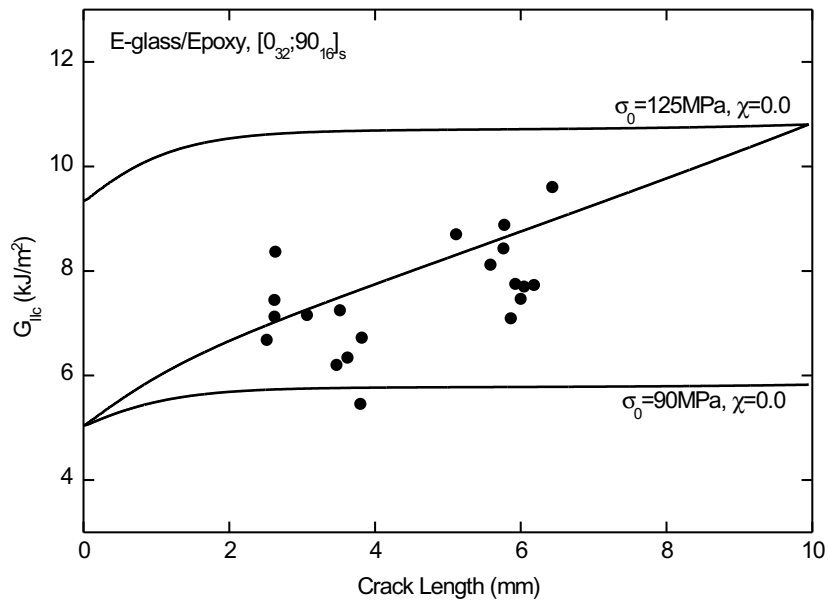


Fig. 14. Experimental vs. modelled mode II energy release rate for an E-glass/epoxy $[0_{32}; 90_{16}]_s$ laminate

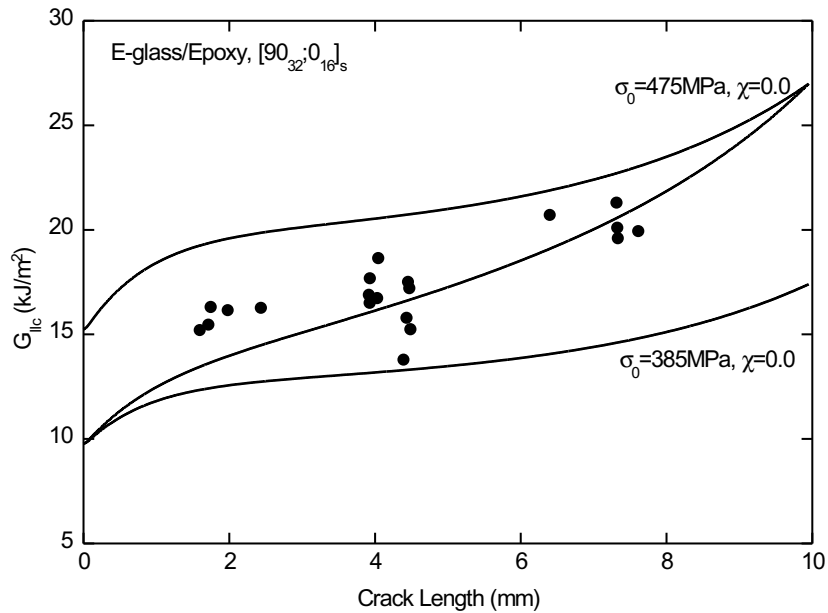


Fig. 15. Experimental vs. modelled mode II energy release rate for an E-glass/epoxy $[90_{32}; 0_{16}]_s$ laminate

The intermediate line corresponds to a linear growth between the minimum and maximum stress levels. The minimum and maximum stress levels were calculated from the minimum and maximum values obtained experimentally, and a residual stress component $E\alpha\Delta T$ was added to them. The stress levels shown on the plots are, therefore, representative of the true stress level, not only the applied stress. This can be understood in the following way: the experimental stress at crack propagation is the result of two components, the intrinsic strength of the interface, and the stress that was present before the loading begins, i.e. the residual stress. Because the model accounts for residual stresses in the calculation of the energy release rate, these cannot be included twice, i.e. the residual stress component must be accounted for in the applied stress, so that it can be deduced in the calculation later. In short, by accounting for residual stresses, the model establishes the correspondence between the intrinsic strength of the interface, and its fracture energy. This is extremely useful since the former depends upon the geometry of the test, while the latter does not. These effects have already been documented in previous studies of lap-joints [24, 25].

The effects of the stacking sequence appear as being large, particularly when comparing the plots of Figures 14 and 15. These are essentially the same type of interface between 0° and 90° plies, the location of these being swapped. When the 0° plies are inside, G_{IIc} is small and increases from $5\text{kJ}/\text{m}^2$ through to $10\text{kJ}/\text{m}^2$, whereas when the 90° plies are inside, G_{IIc} increases from $12\text{kJ}/\text{m}^2$ through to $17\text{kJ}/\text{m}^2$. This is a typical effect of the laminate orthotropy, and the effect of the different mechanical properties in the two principal directions of the ply is further increased by the difference in thermal expansion coefficients, which give rise to large residual stresses. At the present, it is unfortunately not possible to resolve G_{IIc} in two components, one due to mechanical stresses and one due to residual stresses, like it was done in mode I studies [26]. However, using the software developed in this work, it was possible to calculate G_{IIc} free of the residual stress component. Similar values were obtained for the $[0_{32}; 90_{16}]_s$ and $[90_{32}; 0_{16}]_s$ laminates, of about $35\text{kJ}/\text{m}^2$. This value compares well to that of the $[0_{32}; 0_{16}]_s$ laminate, which was found close to $30\text{kJ}/\text{m}^2$. These values are to be taken with caution, as they are approximations. In practice, the software needs to be modified to provide an inverse method, i.e. to have G_{IIc} as input and σ_0 as output. This is current practice for this type of test and analysis, and has been carried out for other geometries in similar works [3], but was not the main purpose of this study.

In this section, it has been shown that the mode II energy release rate can be accurately determined using the present test set-up and geometry, and modelled using the theory introduced in previous sections. The results look promising for establishing an inverse method that allows for the calculation of the intrinsic mode II fracture energy of these laminates, thereby permitting the use of these values for other geometries and loading conditions.

10. Conclusions

This study introduced a new geometry for a lap-shear test, the modified double-lap shear (MDLS) specimen. The stress state in the MDLS specimen, including residual stresses, was obtained from a modified shear lag-theory, extended to account for cracked interfaces. The energy release rate approach was chosen to interpret the test results. It was shown that residual stresses and the frictional stress influence to a great extent the value of G_{II} , and the latter benefited from a thorough, separate experimental study. This was made possible by the use of a powerful non-contact, surface displacement mapping technique: ESPI. This technique was used to show that the friction coefficient in the specimens considered was zero, and it was also used to validate the stress transfer model. The model was further used to interpret the mode II energy release rate of complex interfaces. It was demonstrated that the model captures all the features of the mode II crack propagation in laminates, with a particular emphasis on the effects of residual stresses and ply orientation. Attempts were made to obtain a single value of G_{IIc} representative of the intrinsic properties of the interface only, i.e. free of residual stresses. While the preliminary work was successful, further work is needed to develop the inverse method.

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REFERENCES

1. S. Hashemi, A.J. Kinloch, and J.G. Williams. The analysis of interlaminar fracture in uniaxial fibre-polymer composites. *Proc. R. Soc. Lond.*, A427:173–199, 1990.
2. D.-A. Mendels, Y. Leterrier, and J.-A. E. Månson. The influence of internal stresses on the microbond test 1: Theoretical analysis. *J. Compos. Mater.*, 36(03):347–363, 2002.
3. D.-A. Mendels, Y. Leterrier, and J.-A. E. Månson. The influence of internal stresses on the microbond test 2: Experimental interpretation including the effect of relaxation. *J. Compos. Mater.*, 36:1655–1674, 2002.
4. J.A. Nairn and D.-A. Mendels. On the use of planar shear-lag methods for stress-transfer analysis of multilayered composites. *Mech. Mater.*, 33:335–362, 2001.
5. D.-A. Mendels, S.A. Page, J.-A. E. Månson, and J.A. Nairn. A modified double-lap shear test: Part 1, theory. *Int. J. Adhes. and Adhes.*, Submitted, 2002.
6. D.-A. Mendels. *Coupling of Physical Aging and Internal Stresses in Thermoset-Based Composite Systems*. PhD thesis, Ecole Polytechnique Fédérale de Lausanne, DMX- Laboratoire de Technologie des Composites et Polymères, No 2358 2001.
7. L.N. McCartney. Model to predict effects of triaxial loading on ply cracking in general symmetric laminates. *Comp. Sci. Tech.*, 60:2255–2279, 2000.
8. R.W. Hertzberg. *Deformation and Fracture Mechanics of Engineering Materials*. John Wiley & Sons, Inc., New York, 4th edition, 1996.
9. D.-A. Mendels, S.A. Page, Y. Leterrier, and J.-A. E. Månson. A modified double lap-shear test as a mean to measure intrinsic properties of adhesive joints. Brighton, UK, June 4-7 2000. IOM Publ. Ltd.
10. S.A. Page, D.-A. Mendels, L. Boogh, and J.-A.E. Månson. Influence of the internal stress state on the adhesive performance of epoxy films determined by a new bonded-joint test. Brighton, UK, June 4-7 2000. IOM Publ. Ltd.
11. D.-A. Mendels. Analysis of the single-fibre fragmentation test. Technical Report MATC (A)17, National Physical Laboratory, Teddington, Middlesex, United Kingdom, TW11 0LW, 2001.
12. M.J. Lodeiro, S. Maugdal, L.N. McCartney, R. Morrell, and B. Roebuck. Critical review of interface methods for composites. NPL Report CMMT(A)101, National Physical Laboratory, Teddington, Middlesex, TW11 0LW, UK, 1998.
13. L.N. McCartney. Stress transfer mechanics for systems of transverse isotropic concentric cylinders, I- basic theory. In preparation, 2002.
14. L. Lundquist, Y. Leterrier, P.W. Sunderland, and J.-A. E. Månson. *Life Cycle Engineering*. Elsevier Science, London, 2000.
15. W.R. Broughton and M.J. Lodeiro. Life assessment and prediction: Cyclic fatigue of continuous unidirectional polymer matrix composites. NPL Report CMMT (A)249, National Physical Laboratory, Teddington, Middlesex, TW11 0LW, UK, 2000.
16. D.-A. Mendels and R.M. Shaw. Detection of sub-surface defects in multi-layered polymer composites by speckle pattern interferometry. NPL Measurement Note MATC(MN)17, National Physical Laboratory, Teddington, United Kingdom TW110LW, 2002.
17. J.A. Leendertz. Interferometric displacement measurement on scattering surfaces utilizing speckle effect. *J. Phys. E.: Scientific Instruments*, 3:214–218, 1970.
18. O. Løkberg. Electronic speckle pattern interferometry. *Physics in Technology*, 11:16 – 22, 1980.

19. R. Hofling and P. Aswendt. Interferometric studies on thermal expansion behaviour of carbon fibre-reinforced composites. *Measurement*, 12:85–93, 1990.
20. P. Kim, S. Toll, and J.-A. E. Månson. Measuring the thermal expansion of polymers and composites by speckle pattern interferometry. In *ICCM'10*, Vancouver, Canada, 1995. Whistler.
21. P. Kim, S. Toll, and J.-A. E. Månson. Characterization of the dimensional stability of thermoplastic matrix composites by speckle pattern interferometry. In A. Miravete, editor, *ICCM'9*, pages 333–340, Madrid, Spain, July 12-16 1993. Woodhead Publishers.
22. P. Kim. *The dimensional stability of fiber-reinforced thermoplastic composites*. PhD thesis, No 1427, Ecole Polytechnique Fédérale de Lausanne, 1995.
23. Ettemeyer Gmbh. *Istra V3.3 User's Manual*, 2001.
24. S.A. Page, D.-A. Mendels, L. Boogh, and J.-A. E. Månson. A modified double lap-shear test: Part 3, the influence of internal stresses on practical adhesion. *Int. J. Adhes. Adhes.*, to be Submitted, 2001.
25. D.-A. Mendels, Y. Leterrier, and J.-A. E. Månson. A modified double lap-shear test: Part 4, the influence of physical aging on practical adhesion. *Int. J. Adhes. and Adhes.*, to be Submitted, 2001.
26. J.A. Nairn. Energy release rate analysis for adhesive and laminate double cantilever beam specimens emphasizing the effect of residual stresses. *Int. J. Adh. Adhes.*, 20:59–70, 2000.