

**Project MMS11
Report 4**

**Assessment of Predictive Analysis
for Bonded and Bolted T-Joints**

**W R Broughton, L E Crocker,
M R L Gower and R M Shaw**

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W R Broughton, L E Crocker, M R L Gower and R M Shaw
NPL Materials Centre, National Physical Laboratory
Teddington, Middlesex
TW11 0LW, UK

ABSTRACT

This report presents the results of research conducted at the National Physical Laboratory (NPL) into developing models and test methods for accurately determining deformation and failure in bonded and bolted structures under static and cyclic loading conditions. Tests were conducted on aluminium and glass fibre-reinforced plastic (GRP) T-joints and compared with predictive analysis for this purpose. Finite element analysis (FEA) was used to predict joint stiffness, joint strength and failure location. The study involved T-joints subjected to direct tension and lateral tension loading conditions. The FEA results provide further evidence that the combination of a new cavitation model, developed at NPL, and the hydrostatic stress yield criterion can be used to predict deformation, failure load and failure location within joints bonded with rubber-toughened epoxy adhesives. Closed form solutions were also derived to predict joint stiffness for the basic T-joint configurations and loading modes. Good agreement was obtained between the closed form solutions and experimental results.

The report includes an evaluation of strain gauges and linear voltage displacement transducers (LVDTs) used for monitoring local and global changes in joint stiffness under quasi-static and cyclic loads. The results show that strain gauges strategically placed can detect in many cases the onset of damage. A number of methods of restraining joint movement were also considered (i.e. filling of internal cavities of bonded joints, and side clamping and multi-bolt arrays for bolted joints) with the intention of increasing joint stiffness and strength. The report also examines the relationship between stiffness loss and fatigue life of bonded and bolted T-joints.

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National Physical Laboratory
Teddington, Middlesex, UK, TW11 0LW

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Approved on behalf of Managing Director, NPL,
by Dr C Lea, Head of NPL Materials Centre.

CONTENTS

1	INTRODUCTION	1
2	NUMERICAL MODELLING AND TESTING.....	1
2.1	MODELLING OF BONDED T-JOINTS	1
2.2	FAILURE CRITERIA FOR BONDED T-JOINTS	4
2.3	MODELLING OF BOLTED T-JOINTS	5
2.4	QUASI-STATIC TESTING.....	7
3	EXPERIMENTAL RESULTS.....	9
3.1	BONDED T-JOINTS	9
3.2	BOLTED T-JOINTS	11
4	CYCLIC FATIGUE	14
5	DISCUSSION AND CONCLUDING REMARKS	17
	ACKNOWLEDGEMENTS.....	18
	REFERENCES	18
	APPENDIX 1: MATERIALS DATABASE	20
	APPENDIX 2: YIELD CRITERIA FOR RUBBER-TOUGHENED ADHESIVES.....	23
	APPENDIX 3: CLOSED FORM SOLUTIONS FOR BONDED T-JOINTS	26
	APPENDIX 4: CLOSED FORM SOLUTIONS FOR BOLTED T-JOINTS	31

1 INTRODUCTION

Safe and reliable design of bonded structures is dependent on the availability of accurate materials models and failure criteria that can be used to predict deformation and failure behaviour in bonded and bolted structures. Bonded and bolted joints are frequently expected to sustain static or cyclic fatigue loads for considerable periods of time without any adverse effect on the load-bearing capacity of the structure (e.g. 120 years or greater for railroad bridges). A major concern is that under dynamic fluctuating or constant loads, joints will often fail at stress levels much lower than the strength of the joint determined under quasi-static, monotonic loading.

This report presents the results of research conducted at the National Physical Laboratory (NPL) into developing models and test methods for accurately characterising deformation and failure behaviour of bonded and bolted structures under static and cyclic loading conditions. The study was conducted on aluminium and glass fibre-reinforced plastic (GRP) T-joints subjected to direct tension and lateral tension loading conditions. Finite element analysis (FEA) was used to numerically model joint behaviour. The exponent Drucker-Prager and a new cavitation model, developed at NPL, were used to characterise deformation behaviour of the rubber-toughened epoxy adhesives used for bonding the T-joints. The cavitation model accounts for the influence of rubber particle cavitation on the yield behaviour of adhesives. The research presented in this report forms part of the DTI funded Measurement for Materials Systems Project “Design for Fatigue and Creep in Joined Systems (MMS11)”.

The report is divided into five (5) sections (including Section 1, Introduction). Section 2 describes the numerical modelling and testing of bonded and bolted T-joints. Section 3 compares predicted analysis and experimental results for different joint and loading configurations under quasi-static (monotonic) loading. The effect of cyclic loading on the life expectancy and residual stiffness of bonded and bolted T-joints is considered in Section 4. Discussion and concluding remarks are given in Section 5. Material property data used in the FEA is presented in Appendix 1. Yield criteria, such as von Mises, linear Drucker-Prager, exponent Drucker-Prager and cavitation materials models are described in Appendix 2. Closed form solutions for predicting deformation of bonded and bolted T-joints are given in Appendices 3 and 4.

2 NUMERICAL MODELLING AND TESTING

2.1 MODELLING OF BONDED T-JOINTS

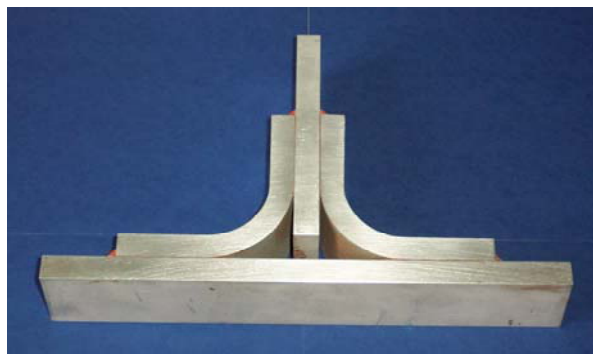
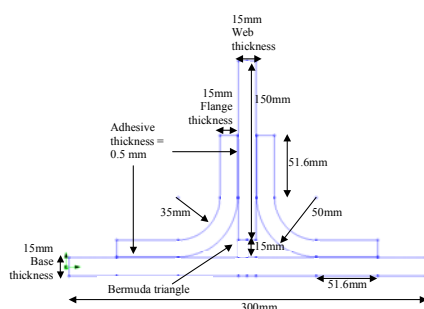


Figure 1: Standard bonded aluminium T-joint used in FEA (width = 50 mm).

FEA was conducted on bonded aluminium and GRP T-joint configurations to determine joint strength, joint stiffness and failure location [1-2]. The basic aluminium bonded T-joint configuration used in the study is shown in Figure 1. The GRP T-joint geometry and dimensions were similar to the aluminium T-joint shown in Figure 1. The main difference between the two joint types being that the GRP T-joints (Figure 2) were constructed from 12.16 mm instead of 15 mm thick sections. Direct tension (vertical) and transverse (lateral) loading configurations were assessed (Figure 3).

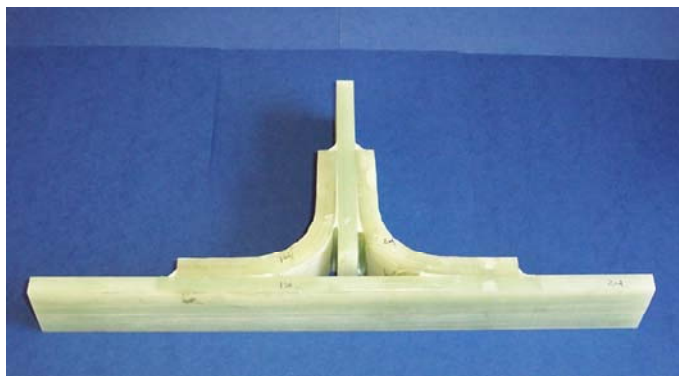


Figure 3: Bonded GRP T-joint.

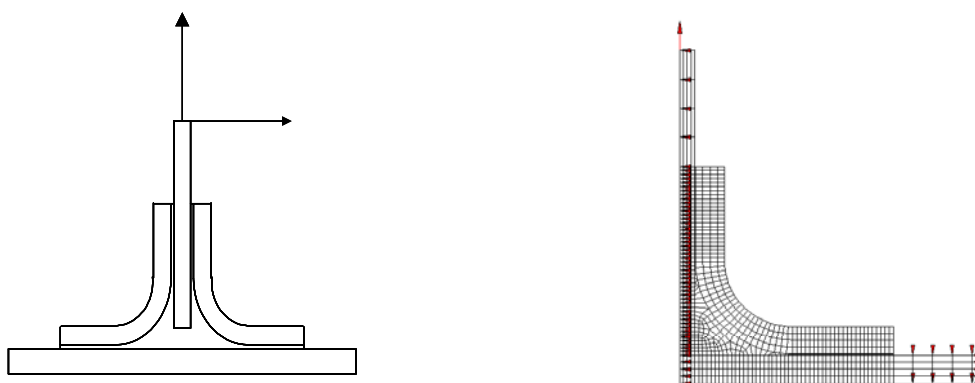


Figure 3: Loading modes (left); and half-model FEA mesh for vertical loading (right).

FE models were constructed and solved using ABAQUS V6.2 [3] standard (implicit) solver while mesh generation was carried out using the FEMGV pre-processor [4]. For vertical loading, a half model was used due to symmetry conditions. For the other loading configurations a full mesh was required. A displacement was applied to a point on the top face of the web in either the 2-direction (vertical loading) or 1-direction (transverse loading). All nodes along the top face of the web were coupled to the displaced node so they all moved the same distance. For the half model symmetry conditions were imposed by constraining the nodes at the symmetry plane against movement in the 1-direction. The boundary and loading conditions applied to the bonded meshes are shown in Figure 3. An example of a deformed mesh for the full model is shown in Figure 4.

Initial analyses were carried out using a three-dimensional (3-D) mesh, but the computer processing time was excessive so a two-dimensional (2-D) mesh was constructed using CPE4 continuum elements. Analyses showed there was little difference between stress and strain predictions for the two cases.

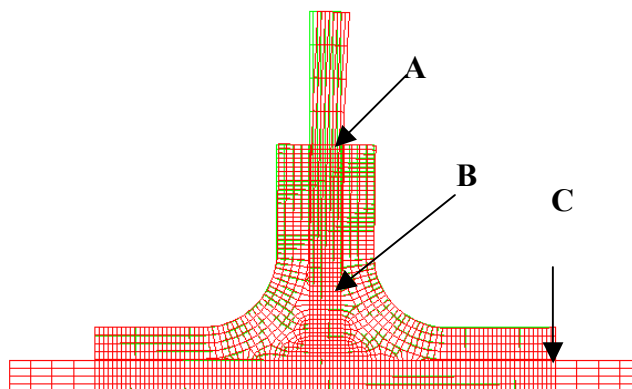


Figure 4: Transverse tension – deformed mesh (red) over original mesh (green).

The aluminium T-joints were constructed from 2014-T6 aluminium alloy. For the bonded T-joints, the adhesive was XD4601, a one-part epoxy with a cure temperature of 180°C (supplied by Dow chemicals). The laminated sections used to construct the GRP T-joints consisted of a combination of 0.64 mm thick unidirectional (UD) plies (layers) and 0.4 mm thick biaxial woven plies (double bias biaxial woven fabric where the glass fibres are oriented at +45° and –45° to the section edges). The standard lay-up is given in Table 1. SP Systems supplied the GRP prepreg material for manufacturing laminated panels. The adhesive used for bonding GRP sections was DP460, a two-part room-temperature curing epoxy (supplied by 3M, UK).

Table 1: Basic Laminate Configuration

Layer Number	Ply Orientation	Total Thickness (mm)
1-6	0	3.84
7-10	Biaxial Fabric	1.60
11-12	90	1.28
13-16	Biaxial Fabric	1.60
17-22	0	3.84

Note: Unidirectional 0° and 90° layers consist of PG51-531 SE90/L/EGL/1200/1000/32%
Fabric layers consist of PG21-532 SE90/M/XE603D/1250/35%.

The adhesive was modelled as an elastic-plastic material using exponent Drucker-Prager and cavitation models, while the aluminium was modelled as an elastic material. The parameters used for the materials models are given in Appendix 1. All aluminium and adhesive properties used in the modelling work were generated from material tests performed at NPL. Preliminary FE analyses using elastic-plastic data for the aluminium produced similar results to those obtained using the elastic model.

Modelling a composite material is more complicated than for metals, such as aluminium, due to the layered nature of a composite. It was decided that for the composite T-joint analyses the individual laminate plies would not be modelled; as initial FE analyses indicated that computer run times would be excessive. Instead global layers were modelled. For example, the inner two 90° plies were modelled as a single layer. When defining the geometry, the adherends were divided into separate element sets of the correct thickness (see Table 1).

The use of a 2-D geometry required transforming the composite ply data to account for a new x, y orientation. The data was transformed such that the through-thickness (T-T) direction became the x-direction; the longitudinal direction became the y-direction and the transverse direction was the z-direction (Figure 5). The composite sections in the T-joint are not all oriented the same way. For example, in the base the 0° fibres are horizontal, whereas in the web the 0° fibres are vertical. To account for differences in orientation, local 1, 2 coordinate systems were applied to each different composite section. Care was needed when generating the local coordinate system for the angle section of the flange. The local coordinate systems were set-up so that in all sections the 0° fibres were parallel to the 2-direction. The 1-direction was in the T-T direction of each section (Figure 5). Bulk composite data was used for each layer of the composite laminate. Unidirectional and biaxial (woven) layer properties, and composite laminate property data are given in Appendix 1. Appendix 1 also includes predicted laminate properties used in the FEA. This data was derived using NPL Composite Design and Analysis (CODA) software.

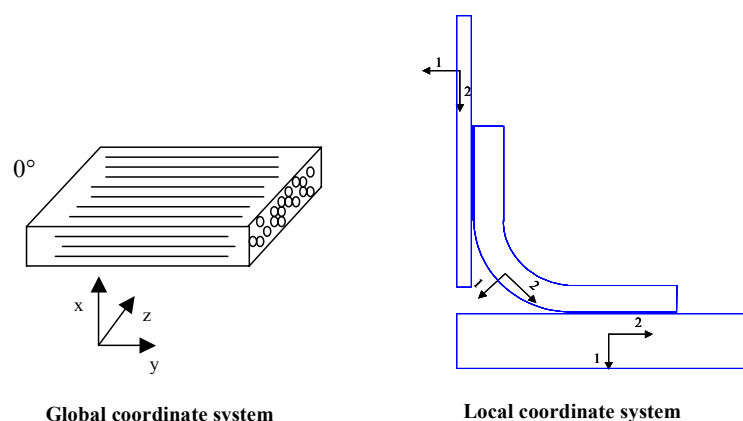


Figure 5: Global and local composite material property coordinate systems used for FEA.

2.2 FAILURE CRITERIA FOR BONDED T-JOINTS

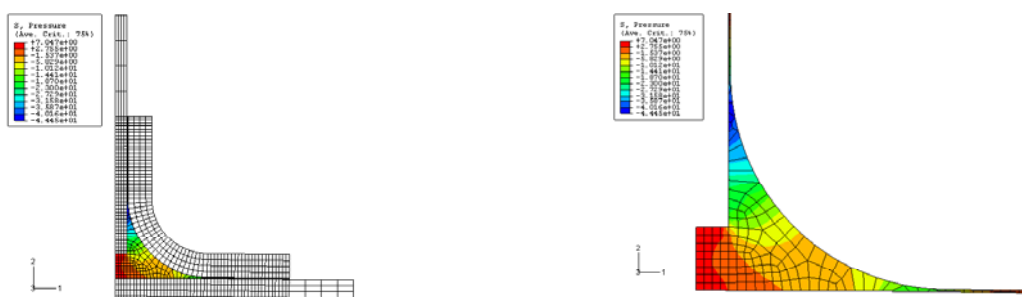


Figure 6: Contour plot of hydrostatic stress at the predicted failure load.

In all analyses, the required output was load-displacement (force-extension) curve and predicted loads corresponding to the appropriate failure criteria for the adhesive and each layer of the composite. There are possibly numerous failure criteria for adhesives. Comparisons of maximum values of selected components of stress and strain in the adhesive, at extensions where failure is considered to initiate in different types of joints, reveal that the maximum principle stress and hydrostatic stress are approximately constant at failure [5]. This applies to bulk adhesives and adhesive joints. Furthermore, the location of failure initiation has been shown to occur in adhesive joints at the location predicted by the cavitation model in combination with the hydrostatic stress criterion (see Appendix 2). Figure 6 shows a contour plot of hydrostatic stress for the half mesh. The stress contours are shown for the adhesive only.

For the composite material, the failure strengths of the different plies were used. The stress components required were S_{11} , S_{22} and S_{12} (local orientations). The failure strengths in these directions were known for the individual plies. Although composite failure criteria are available in ABAQUS, it was not possible to use them in these analyses. For the failure criteria to work correctly, the 1- and 2-directions of the material constants defined for the composite sections must align with the fibre and the transverse to the fibre directions respectively.

It was necessary to obtain the increments when the criteria reached a critical value. A Python script file was written to output the force-extension data. Another Python script file was used to output the maximum and minimum values calculated at integration points for every increment in the analysis. The script file is run through ABAQUS/viewer post processor and the results are output to a report file. The result files were opened in a text file editor such as Microsoft Word and searched for each variable in turn, noting the increments either side of the critical value. The load is known at each increment from the output force-extension curve so the data can be interpolated to obtain the failure load at the critical value of each variable. The location at which failure was predicted to occur was noted. It was found that there were three common locations: top flange-web interface; bottom flange-web interface and flange-base plate interface. These locations are referred to as locations A, B and C respectively and are shown on a deformed mesh in Figure 4. In some cases, the analysis failed to converge before the critical value was reached. Further information on the use of FEA for design with adhesives can be found in NPL Measurement Good Practice Guide No. 48 [6].

2.3 MODELLING OF BOLTED T-JOINTS

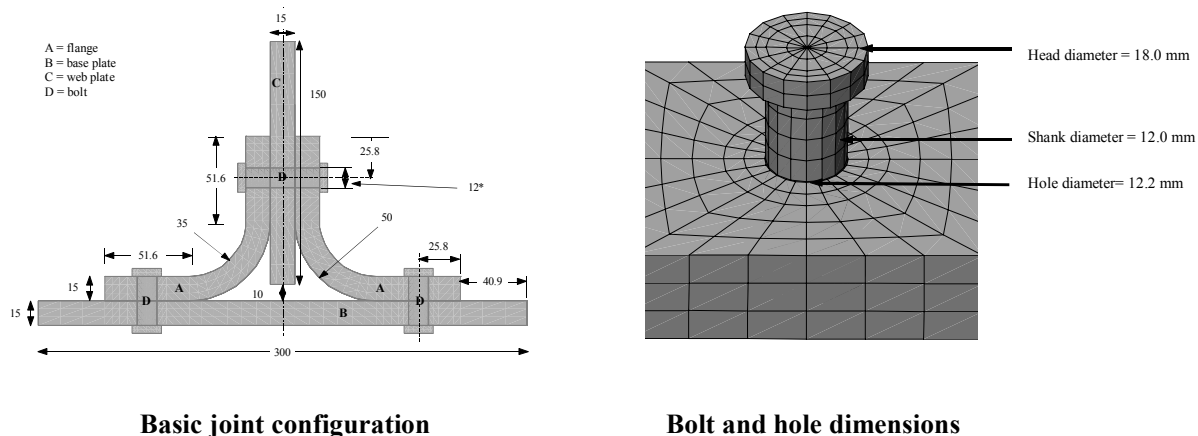


Figure 7: Standard bolted aluminium T-joint configuration used in FEA (width = 50 mm).

FEA modelling of the bolted T-joints was carried out using the LUSAS Composite (Version 13.4) FEA package [7]. The basic configurations for the aluminium and GRP bolted T-joints are shown in Figures 7 and 8. The GRP T-joints were constructed from sections with a nominal thickness of 12.16 mm, and not 15 mm; as in the case for the aluminium T-joints. Figure 9 shows photographs of manufactured specimens. The aluminium and GRP properties, and laminate lay-up are identical to those described in Section 2.1 (see also Appendix 1).

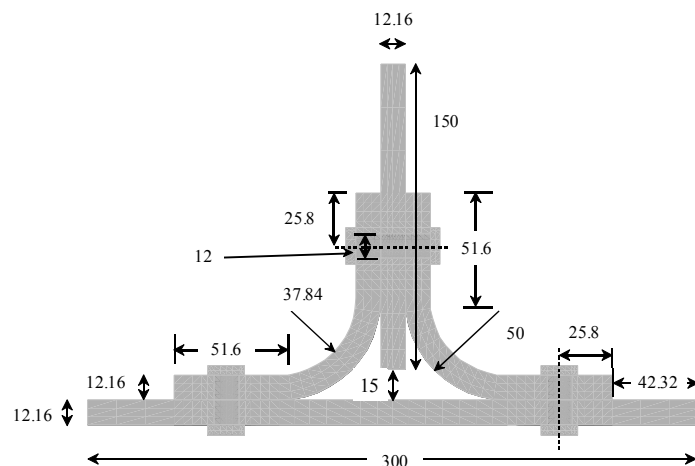


Figure 8: Standard bolted GRP T-joint configuration used in FEA (width = 50 mm).

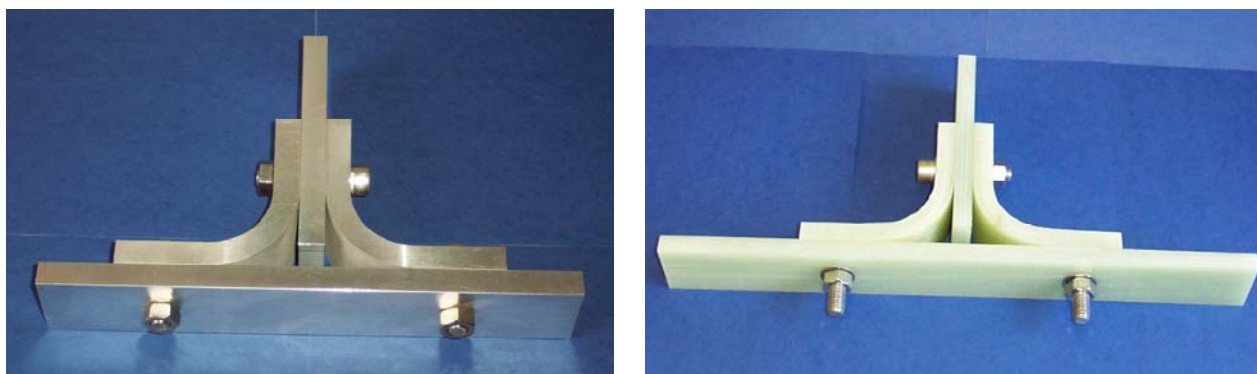


Figure 9: Bolted aluminium (left) and GRP (right) T-joints.

For the bolted joint, all analyses were performed using 3-D models (Figure 10). The geometry was meshed using a combination of PN6 and HX8M, linear order three dimensional (3-D) solid continuum elements. In order to model the clamping force of the torqued bolts, a pre-stress was applied to elements in the shank of each bolt (29.5 MPa in the bolt through the web and 28 MPa in the bolts through the base plate). In the analyses, the bolt pre-stresses (load case 1) were applied first, followed by application of the global load (load case 2). Due to time constraints it was only possible to analyse a configuration consisting of single bolts connecting left and right flange plates to the base plate, and a single bolt connecting both flange plates to the web plate. In order to reduce the complexity of the analyses, washers were not modelled.

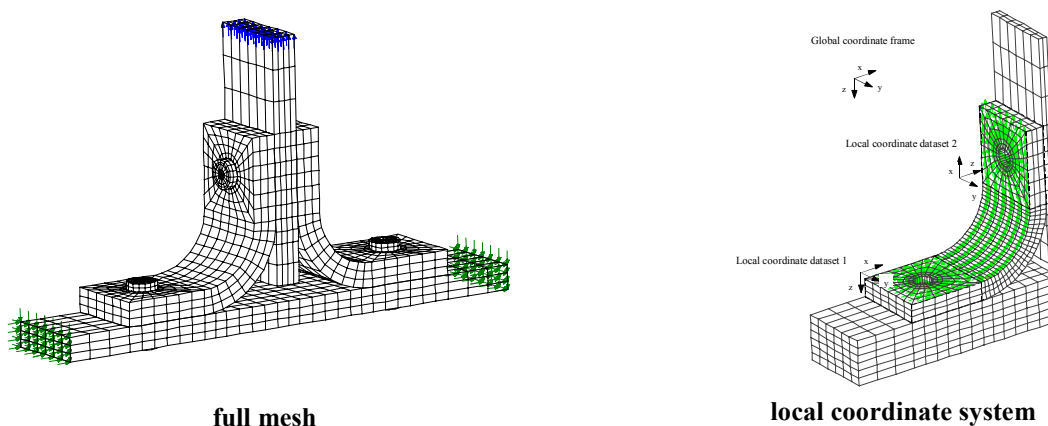


Figure 10: FEA mesh with local coordinate datasets for bolted T-joints.

An important feature of the bolted configuration being analysed was that of contact between component parts. Parts of the model that were in contact before a load was applied or would come into contact during loading were identified. These regions were:

- Initial contact between web, base and flange plates,
- Initial contact between the underside of the bolt heads to web, base and flange plates,
- Loss of, or increased contact between flanges and web/base plates,
- Potential contact between the bolt shanks and holes.

LUSAS slidelines were used to model the existing or potential areas of contact, as these are ideal for modelling situations where there is no exact prior knowledge of the contact process.

Local coordinate datasets (Figure 10) were defined and assigned to the relevant sections in the model in order to ensure that the ply orientations of the laminate coincided with the correct orientations of the model. For the curved sections of the angle flanges (where it was not possible to define an appropriate local coordinate dataset), the volume orientation axes were positioned such that they coincided with the respective orientations of the laminate. The arrows shown in Figure 10 indicate the direction of the 0° fibres in the top ply. The steel bolts were considered to behave elastically.

The onset of failure was regarded as yielding of aluminium for the aluminium joints or the failure strengths of the different plies for the composite material joints. A predicted failure load was obtained corresponding to the onset of failure (or yielding) at either position A, B or C (see Figure 11). In some cases, yielding was predicted to occur at more than one position and in most cases the joint was capable of withstanding substantial additional load following the onset of localised yielding or damage.

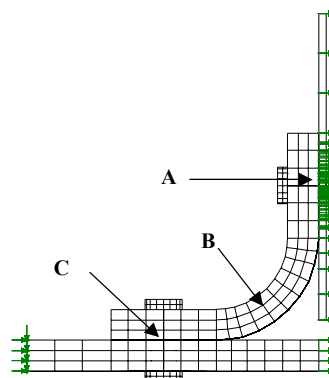


Figure 11: Direct tension – half model showing FEA mesh and symmetry support.

2.4 QUASI-STATIC TESTING

Mechanical tests were conducted on all T-joint configurations to assess the accuracy of the FEA modelling. The T-joints were tested under standard laboratory conditions ($23 \pm 2^\circ\text{C}/50 \pm 5\%$ relative humidity) using a 4507 Instron test frame at a cross-head displacement rate of 0.5 mm/min. The loading arrangements used for applying pull-off and lateral loads to the T-joints are shown in Figures 12 and 13. An RDP spring return LVDT with an operating range of 0 to 25 mm was used to measure displacement in the direction of applied load. Strain gauges were also used to monitor local deformations on three T-joints (bonded aluminium, bolted aluminium and bolted GRP) subjected to direct tension.

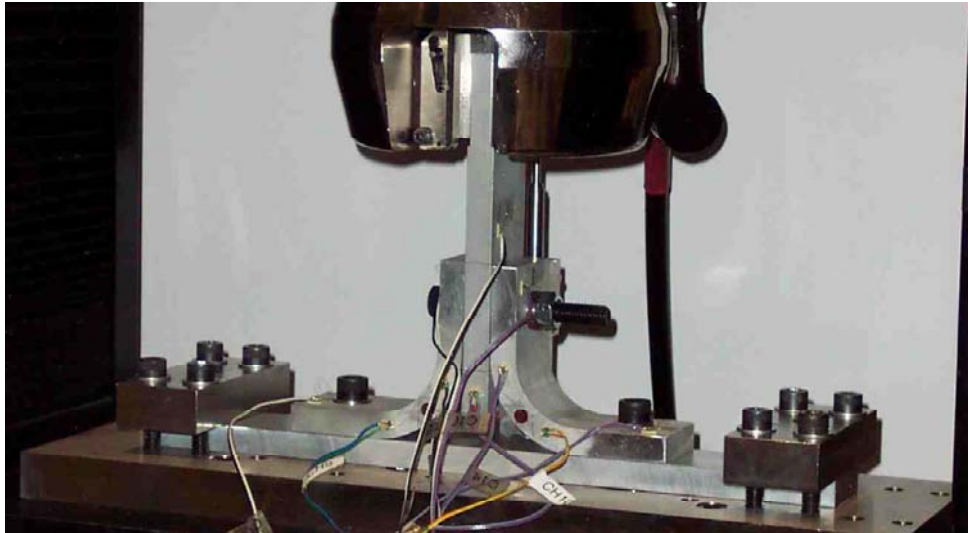


Figure 12: Direct tension loading of bolted aluminium T-joint (with strain gauges).

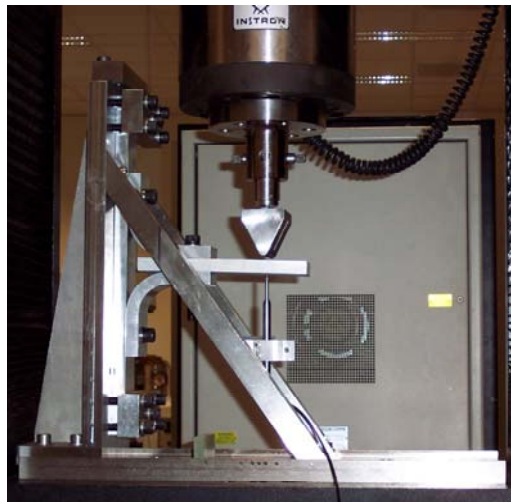


Figure 13: Lateral loading of bonded aluminium T-joint.

Figure 14 shows the location of strain gauges used for measuring local strain. Two types of strain gauge were used:

- 5 mm length uniaxial (Radio Spares)
- 2 mm length 0°/90° biaxial (Kyowa)

For the bonded aluminium T-joint, uniaxial gauges were centrally located on the two flanges and central web section at positions **E**, **F**, **H**, **I** and **J**. The distance between the gauge centre and the flange end was 7 mm for gauges located at **E** and **F**, and 45 mm for gauges located at **H**, **I** and **J**.

For the bolted aluminium T-joint, uniaxial gauges were located at **A**, **B**, **E**, **F**, **G** and **K** with biaxial gauges located at the apex of the flanges at **C** and **D**. The distance between the centre of the gauges located at **A** and **B**, and the flange end was 7 mm. The distance between the centre of the gauge at **G** and the flange end was 26 mm. The centre of the gauge at **K** was 25 mm from the bottom end of the web. The biaxial gauges at **C** and **D** were oriented perpendicular to the flange edge as shown in Figure 14. For the bolted GRP T-joint, uniaxial gauges were located at **E**, **F**, **G** and **K**, and biaxial gauges at **A**, **B**, **C** and **D**.

Uniaxial gauges were also located at 0° (F), 90° (M), 180° (N), and 270° (O) on the flat surface about the upper bolt securing the right hand side flange to the central web (see Figure 14). The gauge centres were approximately equidistant (17 to 20 mm) from the centre of the bolt hole with the top gauge located at position F. The bolt hole diameter was 12 mm with the hole centre located 26 mm from the flange edge.

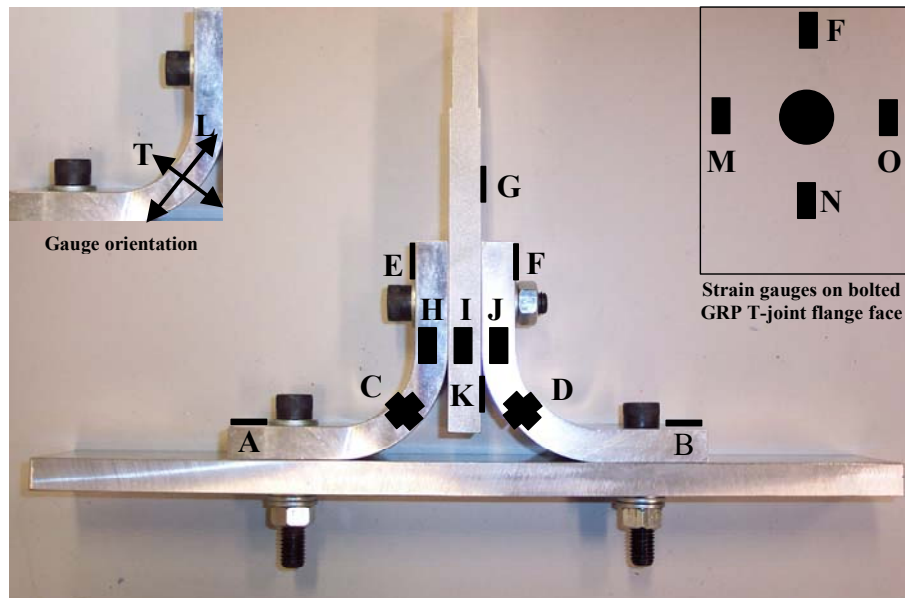


Figure 14: Schematic of strain gauge location on T-joints.
(strain gauges centrally located on either an edge or face of a flange or web section)

3. EXPERIMENTAL RESULTS

3.1 BONDED T-JOINTS

Direct Tension: FEA results showed that load-displacement response; initial stiffness and failure load for both aluminium and GRP T-joints could be predicted with reasonable accuracy using the combination of cavitation model and hydrostatic stress failure criterion (see Figure 14 and Table 2). Cohesive failure occurred as predicted in the internal adhesive fillets between the flanges and centre web. Failure was sudden and catastrophic. Differences in predicted and measured stiffness for the GRP T-joints can be attributed to the fact that the actual flange thickness was ~13 mm and not 12.16 mm (as modelled).

Table 2: Strength (P_{MAX}) and Stiffness (S) for Direct and Transverse Tension Loading
(* FEA failed to converge)

Configuration	Experiment		FEA/Analytical Solution	
	P_{MAX} (kN)	S (kN/mm)	P_{MAX} (kN)	S (kN/mm)
Direct				
Aluminium	30.76 ± 3.54	58.58 ± 4.38	31.16	56.80/59.88
GRP	14.27 ± 4.53	17.15 ± 2.38	15.44	16.05/15.41
Transverse				
Aluminium	13.60 ± 0.51	3.88 ± 0.38	7.41*	3.74/3.90
GRP	6.27 ± 1.54	1.25 ± 0.11	5.13*	1.06/1.00

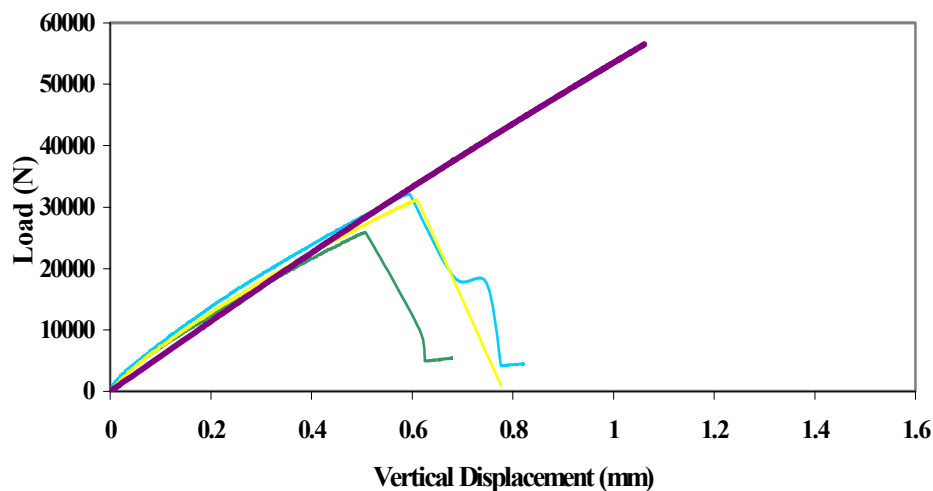


Figure 15: Direct tension load-displacement curves for bonded aluminium T-joints.

FEA modelling predicted elastic-plastic yielding would occur within the adhesive layer prior to failure and would have a minimal effect on the linearity of the load-displacement response (see extended line in Figure 15). The test results confirmed the FEA predictions. Cohesive failure was observed to occur in the adhesive layer and adhesive deformation had a small to moderate effect on the linearity of the load-displacement response of aluminium and GRP T-joints. There was no clear evidence from the LVDT data to indicate imminent failure of the T-joints, although there seems to be a point of inflection at a load of ~ 22 kN. The maximum surface strains measured on the flange and central web sections (Figure 14) were small (less than $200 \mu\epsilon$), which made data interpretation difficult. As expected, the strain at **I** (web) was approximately twice that for **H** and **J** (flange). Failure occurs within the adhesive fillets between the flanges and web, leading to separation of the adjacent sections. Onset of failure is manifested by a change in slope of the load-strain response of gauges at **H** and **J**, which occurs at ~ 22 kN. Failure was sudden and catastrophic.

Transverse Tension: FEA stiffness predictions were in good agreement with measured values (see Table 2). Strength predictions for the two materials were unsatisfactory, as the FEA failed to reach the prescribed hydrostatic stress using the cavitation model (i.e. FEA failed to converge). The force-displacement responses for the two materials were essentially linear-elastic to failure. The bonded aluminium T-joints fail as a result of cohesive failure occurring within the adhesive fillets between the base plate and flange. The GRP T-joints also fail in the same location, however failure was observed to occur at the adherend/adhesive interface or within the surface layers of the composite laminate. Failure was sudden and catastrophic.

Table 2 also includes predicted stiffness results using closed form solutions derived for the aluminium and GRP T-joints (see Appendix 3). The closed form solutions are in reasonable agreement with measured and FEA stiffness values.

Additional tests were conducted on an aluminium T-joint with the entire internal volume (i.e. Bermuda triangle) filled with XD4601 epoxy adhesive (see Figure 16). The results presented in Table 3 show good agreement between predicted and measured stiffness for the fully filled T-joint. However, the measured strength was considerably lower than the predicted value. The test specimen contained voids, which resulted in premature failure of the joint. As expected, stiffness and strength properties are strongly enhanced by filling the Bermuda triangle with adhesive.

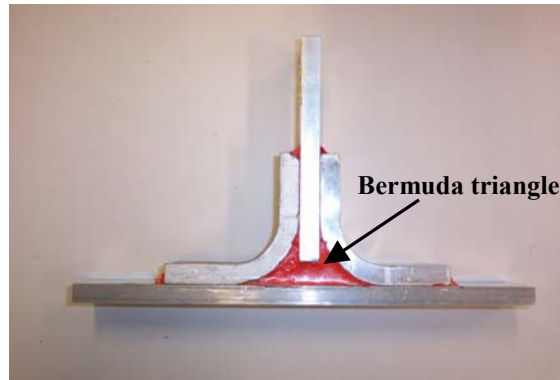


Figure 16: Fully filled bonded aluminium T-joint.

Table 3: Structural Properties for Direct Tension Loading of Aluminium T-Joints

Configuration	Experiment		FEA/Analytical Solution	
	P_{MAX} (kN)	S (kN/mm)	P_{MAX} (kN)	S (kN/mm)
Unfilled	30.76 ± 3.54	58.58 ± 4.38	31.16	56.80/59.88
Filled	54.18	85.51	87.38	85.80

3.2 BOLTED T-JOINTS

Direct Tension: FEA has shown for bolted aluminium T-joints under direct tension that damage in the form of hole elongation would first initiate in the base plate and flanges at **A** (Figure 17) followed by similar damage in the flanges and web at **B**. Ultimate failure was predicted to occur in the flange at **C** as a result of high stress and strain concentrations in this region. The sequence and form of damage predicted was also observed experimentally. Generally, predictive analysis and experimental results were in reasonable agreement.

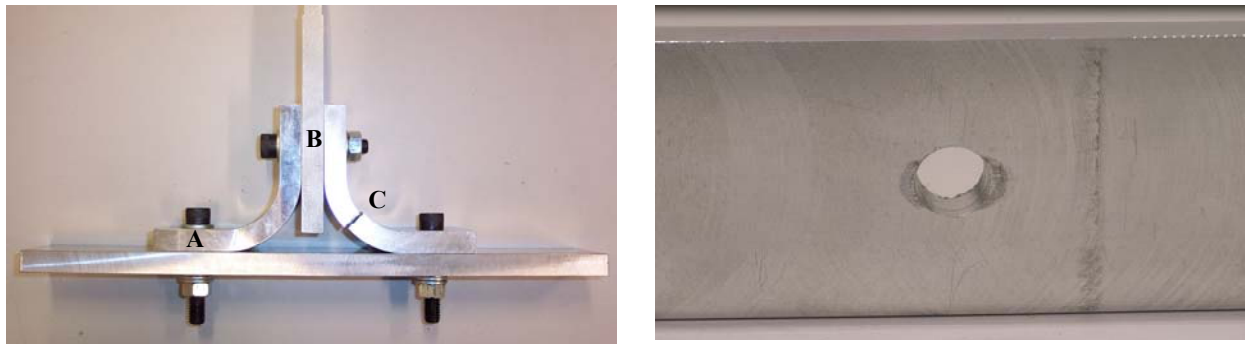


Figure 17: Direct tension failure of bolted aluminium T-joints (left: flange failure at C; right: bolt hole elongation at A).

The load-displacement response measured using the LVDT showed that non-linearity occurs at ~ 48 kN, which agrees with the FEA prediction (see Figure 17 and Table 4). FEA also predicts localised damage initiation around the bolt hole at **A** when the load is ~ 38 kN. In fact, hole elongation had minimal effect on the load-displacement response. The surface strain measurements (Figure 18) for the bolted aluminium T-joint also deviated from linearity at ~ 50 kN. The longitudinal strain at **C** (CL - see Figure 14) shows a marked deviation from linearity at ~ 50 kN as a result of localised yielding of the aluminium flange. Although local deformation occurred around the bolt holes at loads less than 50 kN, it was impossible to isolate hole elongation from global effects using the strain gauges. At failure, hole elongation in the base plate was ~ 0.1 - 0.2 mm (1-2% of hole diameter).

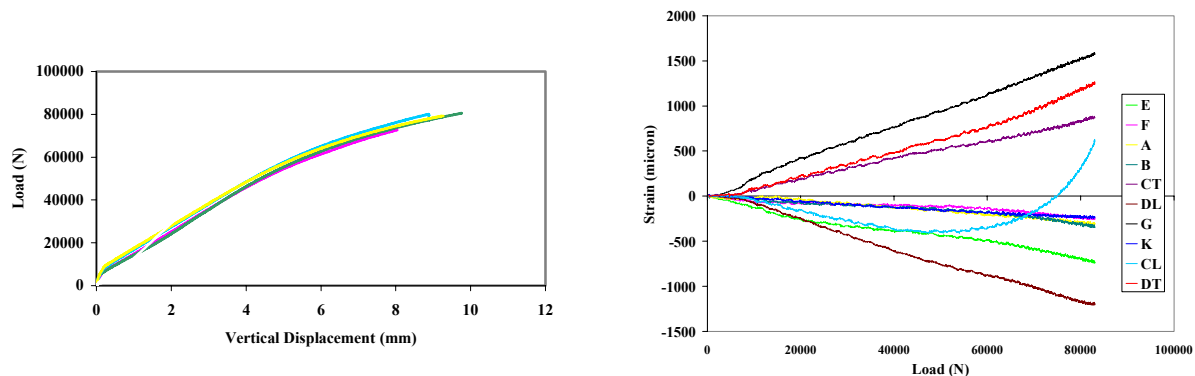


Figure 18: Direct tension of bolted aluminium T-joints.
(left: load-displacement curves; right: load-strain curves for a single T-joint)

Table 4: Structural Properties for Direct Tension Loading of Bolted T-Joints

Configuration	Experiment		FEA/Analytical Solution	
	P (kN)	S (kN/mm)	P (kN)	S (kN/mm)
Aluminium				
Ultimate	79.75 ± 0.67	10.93 ± 0.48	96.13	14.72/9.82
Gross Yielding	48.43 ± 3.01		49.20	
Localised Yielding	-		31.80	
GRP				
Ultimate	20.47 ± 3.79	2.94	18.53	2.50/3.31
Gross Yielding	-		-	
Localised Yielding	~ 10.00		10.52	

Multiple modes of damage in the composite (i.e. longitudinal splitting and delaminations) were observed for bolted GRP T-joints in direct tension (Figure 19). As predicted, delaminations causing catastrophic failure of the joint, initiate in the vicinity of the bolts securing the flanges to the base plate at ~ 20 kN. These delaminations are a result of combined through-thickness shear and tensile stresses (see fractographic image in Figure 19). Delaminations originate in the upper half of the laminate at the interface between a biaxial fabric 45° ply and a 90° UD ply. The crack then propagates through the thickness to the laminate centre and around the flange circumference. Although the LVDT data was insensitive to localised damage around the bolt holes, strain gauges in these regions detected changes at ~ 10 kN; as predicted by FEA.



Figure 19: Damage induced in a bolted GRP T-joint resulting from direct tension loading.
(left: longitudinal splitting around bolt hole; right: delamination in flange)

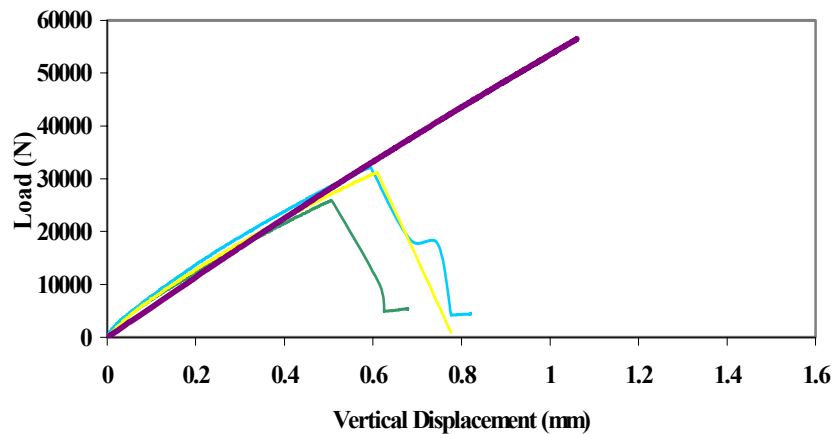


Figure 20: Lateral tension of bolted aluminium T-joints.

Transverse Tension: The web section of bolted aluminium T-joints tended to undergo considerable deformation (i.e. bending), thus most tests were stopped before failure occurred within the structure. For lateral loads, failure results from high shear stresses at the flange apex causing sudden, rapid delamination growth in the flanges. Table 5 compares predictive analysis with FEA. Reasonable agreement was achieved between the measured and predicted stiffness and strength for the aluminium T-joint (Figure 20), although the results for the GRP proved less satisfactory. Strength predictions were also in reasonable agreement.

Table 5: Structural Properties for Lateral Tension Loading of Bolted T-Joints

Configuration	Experiment		FEA/Analytical Solution	
	P (kN)	S (kN/mm)	P (kN)	S (kN/mm)
<u>Aluminium</u>				
Ultimate	22.88	1.93 ± 0.08	26.33	1.94/1.72
Gross Yielding	15.68 ± 0.47		17.33	
Localised Yielding	-		12.60	
<u>GRP</u>				
Ultimate	9.41 ± 0.83	0.51 ± 0.02	9.08	0.28/0.45
Gross Yielding	-		-	
Localised Yielding	5.00		4.34	

Closed form solutions were derived for both loading modes (see Appendix 4) and these proved satisfactory for predicting the joint stiffness of bolted T-joints (see Tables 4 and 5).

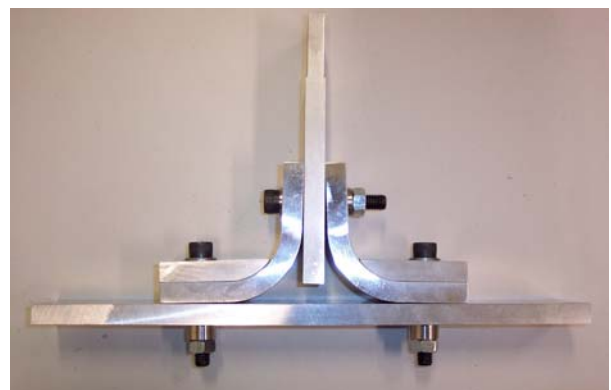
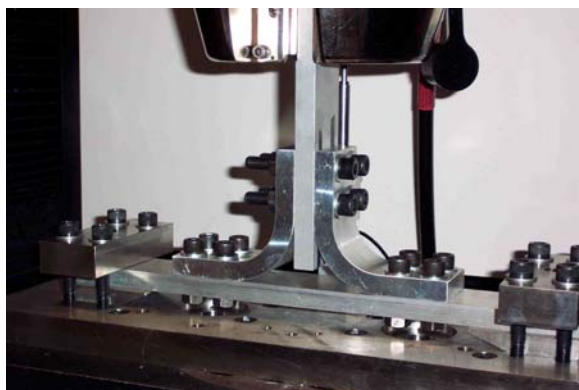


Figure 21: 4-bolt array (left) and side clamping (right).

Direct tension tests were also conducted on aluminium T-joints joined by 4-bolt arrays, consisting of 6 mm and 10 mm diameter bolt, between the central web and flanges, and between the base plate and flanges (Figure 21). Side clamping (Figure 21) was also introduced to determine the restraining affect of the clamps on joint stiffness and strength. The results shown in Table 6 indicate that additional clamping using either 4-bolt arrays or restraining clamps had no affect on joint stiffness. Additional clamping, however, increases joint strength by ~20-25%. The bolted structures underwent considerable deformation, particularly the base plate (i.e. bolt hole elongation and bending) as shown in Figure 22. The side clamped T-joint eventually failed as a result of tensile failure of the bolt joining the central web to the flanges. No attempt was made to model deformation and failure behaviour of the 4-bolt and side clamped configurations. It was evident that insufficient tightening of the bolts could result in low stiffness values.

Table 6: Restraining Effects on Structural Properties of Bolted Aluminium T-Joints

Configuration	P_{MAX} (kN)	S (kN/mm)
Standard (single bolt)	79.75 ± 0.67	10.93 ± 0.48
4-Bolt Array ($\phi = 6$ mm)	-	10.62
4-Bolt Array ($\phi = 10$ mm)	97.85	10.63
Clamped	99.26	9.87

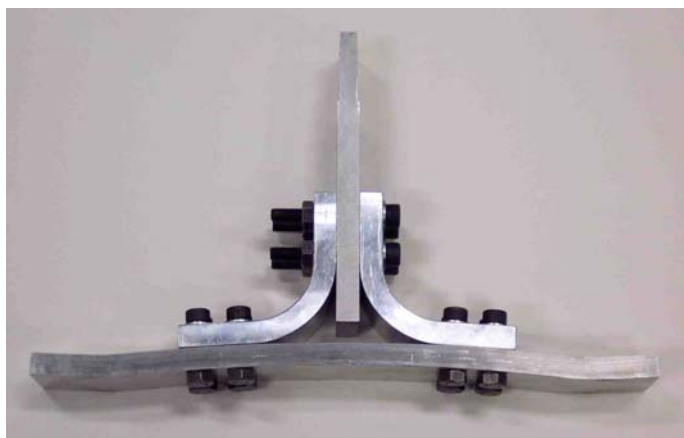


Figure 22: Deformation of 4-bolt array T-joint ($\phi = 10$ mm).

4. CYCLIC FATIGUE

Constant amplitude (sinusoidal waveform) cyclic fatigue tests were conducted on bonded and bolted aluminium and GRP T-joints. The test regimes are described below.

Direct Tension-Tension Loading

- Bonded aluminium/XD4601
- Bonded GRP/DP460
- Bolted GRP

Lateral Tension-Tension Loading

- Bonded aluminium/XD4601
- Bolted GRP

The T-joints were tested in load control using an 8501 Instron servo-hydraulic test machine at a frequency of 3 Hz under standard laboratory conditions. The stress ratio R ($\sigma_{\text{MIN}}/\sigma_{\text{MAX}}$) was equal to 0.1 for direct and lateral tension-tension cyclic loading conditions. Tests were carried out at four different stress levels ($P_{\text{MAX}} = 70\%, 55\%, 40\%$ and $25\% P_{\text{ULT}}$). The ultimate strength P_{ULT} was measured at a displacement rate equivalent to the test frequency.

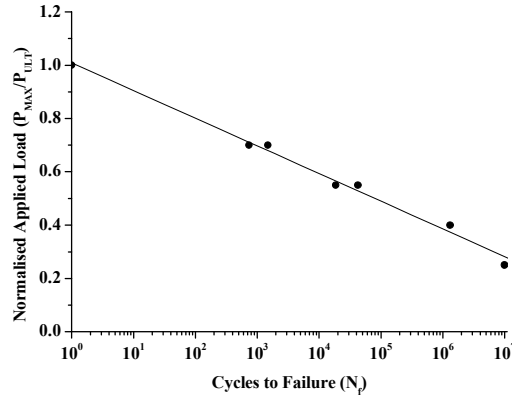


Figure23: Normalised S-N curve for direct tension fatigue of bonded aluminium T-joints.

The normalised stress-fatigue life (S-N) data for all the T-joints tested in cyclic fatigue (see Figure 23) is independent of mode of joining, material or loading mode and can be represented by the following relationship:

$$P_{\text{MAX}} / P_{\text{ULT}} = 1 - k \log N_f \quad (1)$$

where P_{MAX} is the maximum applied load, P_{ULT} is the joint failure load, k (the slope) is the fractional loss in strength per decade of cycles and N_f is the number of cycles to failure. The value of k for the bonded and bolted aluminium and GRP T-joints under the two loading configurations varied from 0.08 to 0.11 (see Table 7).

Table 7: Joint Efficiency Factor and Normalised Residual Stiffness (S_R/S_0) at Failure

T-Joint Configuration	k	Average S_R/S_0 at Failure
Direct Tension - Bonded Aluminium	0.104	0.70
Direct Tension - Bonded GRP	0.106	0.78
Direct Tension - Bolted GRP	0.078	0.74
Lateral Tension - Bonded Aluminium	0.111	0.75
Lateral Tension - Bolted GRP	0.112	0.80

Previous tension-tension fatigue studies carried out on single lap, double lap, double strap and T-peel joints, bulk adhesives and GRP laminates (including coupon containing holes) have shown that $k \approx 0.1$. Unsurprisingly, bonded and bolted T-joints also behave in a similar manner.

Residual stiffness S_R was monitored throughout all the fatigue tests. Figures 24 and 25 show typical normalised residual stiffness (S_R/S_0) plots for bonded aluminium and bolted GRP T-joints under tension-tension fatigue (S_0 is the initial stiffness at $N = 0$). The onset of localised damage (e.g. mode I cracking in the internal fillet of bonded T-joints or flange delamination in bolted GRP T-joints) coincides with a reduction in residual stiffness. Failure modes and locations tended to mirror those observed for quasi-static loading conditions. For bonded aluminium T-joints under lateral tension fatigue loading, S_R/S_0 decreases linearly with N with the rate of decrease dS/dN dependent on $P_{\text{MAX}}/P_{\text{ULT}}$ (see Figure 26).

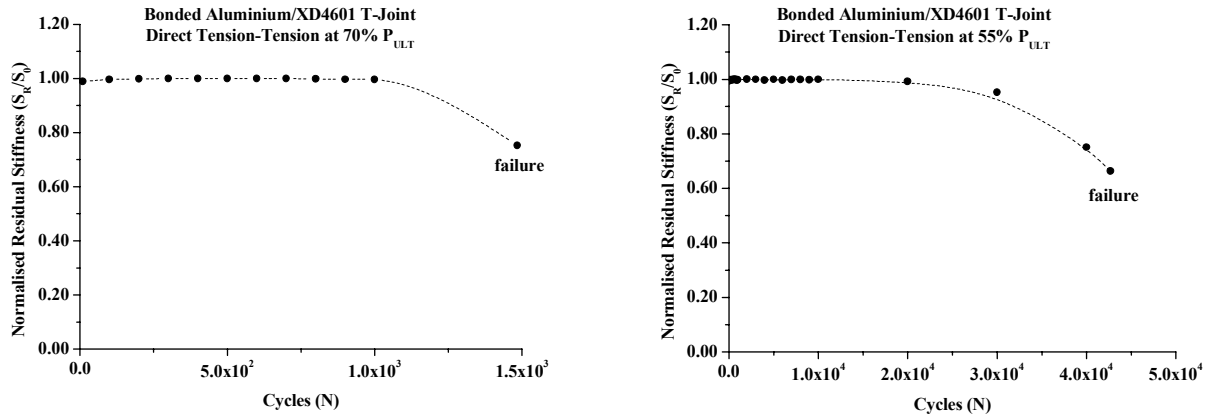


Figure 24: Normalised residual stiffness for fatigue loading of bonded aluminium T-joints.

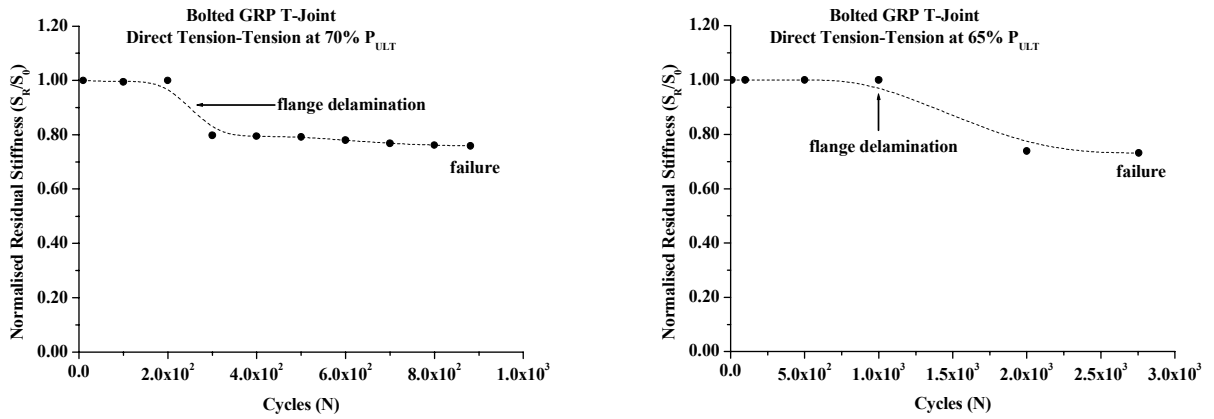


Figure 25: Normalised residual stiffness for fatigue loading of bolted GRP T-joints.

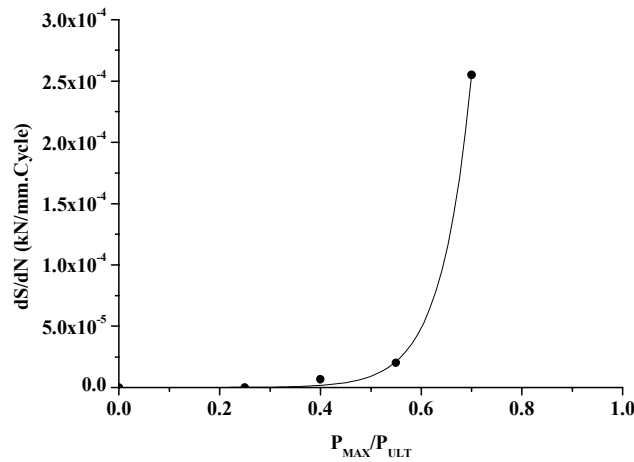


Figure 26: dS/dN versus P_{MAX}/P_{ULT} for lateral tension fatigue of bonded aluminium T-joints.

Normalised residual stiffness S_R/S_0 for bonded T-joints under direct tension fatigue can be estimated for N cycles using the following function:

$$S_R/S_0 = 1 - Ae^{N/B} \quad (2)$$

where **A** and **B** are empirical constants dependent on P_{MAX}/P_{ULT} .

The normalised residual stiffness at failure tends to be:

- Approximately the same for different values of P_{MAX}/P_{ULT} (see Table 7); and
- Equivalent to the final stiffness measured for quasi-static tests.

Preliminary trials indicate that the trend in normalised residual strength (monotonic loading) after N cycles tends to mirror the trend in normalised residual stiffness (i.e. $\approx S_R/S_0$).

5. DISCUSSION AND CONCLUDING REMARKS

Although joint stiffness and strength under quasi-static loading conditions can be determined for complex structures, such as T-joints, with reasonable confidence, further work is required to model fatigue behaviour. The combination of cavitation model and hydrostatic stress failure criteria proved satisfactory in determining deformation and failure behaviour of bonded aluminium and GRP T-joints under direct tension. The FE implementation of the cavitation model proved less satisfactory for predicting the behaviour of the joints under transverse tension. The FEA failed to converge and steps will need to be taken to ensure robustness of the model for all loading conditions. Closed form solutions derived for the basic T-joint configurations proved effective in predicting joint stiffness. This approach enables rapid assessment of the effects of material and geometric properties, such as adhesive layer thickness, web thickness, base plate thickness, separation distance between web and base plate, and flange radius and thickness on joint stiffness.

Joint stiffness and strength can be enhanced in the following ways.

- Completely filling the internal volume of bonded T-joints with adhesive enhances stiffness and strength, although care needs to be taken to prevent void formation and exothermic reactions.
- Side clamping and multi-bolt arrays reduce stress concentrations in the flanges, thus resulting in higher failure loads. No obvious effect was observed on joint stiffness.

The results of the study show that it is possible to monitor the effects of damage on the stiffness of bonded and bolted T-joints. It is difficult, however, to detect the onset of localised damage that can result in the eventual failure of the structure. Strain gauges in some cases proved effective in detecting the onset of localised damage. Non-contact full field strain techniques (e.g. Digital Image Correlation (DIC) and electronic speckle pattern interferometry (ESPI)) may prove more successful in detecting localised strain concentrations and damage onset in joined systems.

Empirical relationships can be used to determine the life expectancy and residual stiffness of these bonded and bolted structures subjected to fatigue loading conditions. As previously mentioned, future efforts need to be directed towards developing physical-based models relating changes in material properties to accumulated damage. It is evident from the plethora of data for both adhesive joints and GRP laminates, that these materials exhibit similar fatigue behaviour and that it may be possible to relate properties, such as unrecoverable strain, to the loss of stiffness and strength of the adhesive or composite. One of the main aims of the *Measurements for Materials Performance* project “F9: Prediction of the lifetime of adhesive joints under sustained loading” funded by the United Kingdom Department of Trade and Industry is to develop and validate a model for predicting residual properties and life expectancy of structural adhesive joints under cyclic loading conditions.

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APPENDICES

APPENDIX 1: MATERIALS DATABASE**XD4601 at 23°C****Tensile Data**

Displacement Rate (mm/min)	Modulus (MPa)	Poisson's Ratio	Maximum Stress (MPa)	Maximum Strain (%)
10	2955	0.35	57.64	8.40

Shear Data

Displacement Rate (mm/min)	Modulus (MPa)	Maximum Stress (MPa)	Maximum Strain (%)
2.0	1031	42.3	0.146

Drucker-Prager Parameters

E (MPa)	ν_e	$\beta(^{\circ})$	$\psi(^{\circ})$	a
2955	0.35	40	26	0.009

Additional Parameters Required for Cavitation Model (* see Appendix 2)

μ	$\sigma_o(\epsilon_o^p)$ (MPa)	ν_{Ro}	q_1	k	ϵ_{1v}	ϵ_{2v}	β	μ'
0.30	*	0.16	1.60	1.35	0.003	0.004	0.90	0.17

DP460 at 23°C**Tensile Data**

Rate (mm/min)	Modulus (MPa)	Poisson's Ratio	Maximum Stress (MPa)	Maximum Strain
10	2288	0.45	43.96	0.210

Shear Data

Rate (mm/min)	Modulus (MPa)	Strength (MPa)	Failure Strain
2.0	623	30.85	0.37

Drucker-Prager Parameters

E (MPa)	ν_e	$\beta(^{\circ})$	$\psi(^{\circ})$	a
2288	0.45	31	8.35	0.0168

Additional Parameters Required for Cavitation Model (* see Appendix 2)

μ	$\sigma_o(\epsilon_o^p)$ (MPa)	ν_{Ro}	q_1	k	ϵ_{1v}	ϵ_{2v}	β	μ'
0.27	*	0.15	1.60	2.62	0.004	0.002	0.90	0.10

2014-T6 Aluminium Alloy

Property	Measured Values
E_{11} (GPa)	77 ± 2
G_{12} (GPa)	28 ± 1
ν_{12}	0.40 ± 0.01
σ_T^U (MPa)	492 ± 2
σ_T^Y (MPa)	440 ± 3
ϵ_T^U (%)	10.6 ± 0.6
ϵ_T^Y (%)	0.63 ± 0.01

SP SYSTEMS GLASS FIBRE-REINFORCED EPOXY ($V_f = 0.56$)**Measured Elastic Properties**

Elastic Property	Material			
	Unidirectional	Woven Fabric	Laminate	
			Measured	Predicted
E_{xx}^T (GPa)	48.0 ± 1.7	14.6 ± 0.4	38.8 ± 1.1	33.5
E_{yy}^T (GPa)	13.4 ± 3.6	14.6 ± 0.4	18.0 ± 0.7	17.9
E_{zz}^T (GPa)	11.6 ± 0.1	14.6 ± 0.7	[14.6 ± 0.7]	13.5
E_{xx}^C (GPa)	44.4 ± 2.9	13.8 ± 0.4	38.8 ± 1.1	33.5
E_{yy}^C (GPa)	12.9 ± 1.0	13.8 ± 0.4	18.0 ± 0.7	17.9
E_{zz}^C (GPa)	13.3 ± 0.3	15.8 ± 0.4	[14.6 ± 0.7]	13.5
G_{xy} (GPa)	4.81 ± 0.38	13.1 ± 0.4	6.50*	6.50
G_{xz} (GPa)	4.75 ± 0.09	4.81 ± 0.38	[4.81 ± 0.38]	3.71
G_{yz} (GPa)	2.84*	4.81 ± 0.38	[4.81 ± 0.38]	3.37
ν_{xy}^T	0.31 ± 0.04	0.58 ± 0.02	0.28 ± 0.01	0.30
ν_{yx}^T	0.08 ± 0.01	0.58 ± 0.02	0.12 ± 0.01	0.16
ν_{xz}^T	0.25 ± 0.01	0.17 ± 0.01	[0.17 ± 0.01]	0.35
ν_{zx}^T	0.07 ± 0.01	0.17 ± 0.01	[0.17 ± 0.01]	0.14
ν_{yz}^T	0.42 ± 0.01	0.16 ± 0.02	[0.16 ± 0.02]	0.24
ν_{zy}^T	0.48 ± 0.01	0.16 ± 0.02	[0.16 ± 0.02]	0.18
ν_{xy}^C	0.33 ± 0.02	0.57 ± 0.02	[0.57 ± 0.02]	0.30
ν_{yx}^C	0.09 ± 0.01	0.57 ± 0.02	[0.57 ± 0.02]	0.16
ν_{xz}^C	0.23 ± 0.01	0.17 ± 0.01	[0.17 ± 0.01]	0.35
ν_{zx}^C	0.07 ± 0.01	0.17 ± 0.01	[0.17 ± 0.01]	0.14
ν_{yz}^C	0.46 ± 0.01	0.20 ± 0.02	[0.20 ± 0.02]	0.24
ν_{zy}^C	0.45 ± 0.01	0.20 ± 0.02	[0.20 ± 0.02]	0.18
$E_{xx}(\text{OHT})$	-	14.2 ± 0.7	37.7 ± 0.1	33.5
$E_{yy}(\text{OHT})$	-	14.1 ± 0.4	14.9 ± 0.4	17.9
$E_{xx}(\text{filled OHT})$	-	19.6 ± 1.9	36.4 ± 0.9	33.5
$E_{yy}(\text{filled OHT})$	-	19.5 ± 0.8	16.2 ± 0.2	17.9
$E_{xx}(\text{OHC})$	-	14.6 ± 0.4	40.0 ± 2.0	33.5
$E_{yy}(\text{OHC})$	-	14.6 ± 0.4	21.0 ± 0.5	17.9
$E_{xx}(\text{filled OHC})$	-	14.6 ± 0.7	40.0 ± 0.8	33.5
$E_{yy}(\text{filled OHC})$	-	13.8 ± 0.4	21.0 ± 1.0	17.9

OHT = open hole tension; OHC = open hole compression; * = predicted value

[] Assumed symmetric in the x and y directions or along the x-z and y-z planes

SP SYSTEMS GLASS FIBRE-REINFORCED EPOXY ($V_f = 0.56$)**Strength Properties**

Strength Property	Material			
	Unidirectional	Woven Fabric	Laminate	
			Measured	Predicted
S_{xx}^T (MPa)	1133 ± 82	190 ± 3	889 ± 33	697
S_{yy}^T (MPa)	43.1 ± 5.0	190 ± 3	211 ± 18	151
S_{zz}^T (MPa)	27.2 ± 1.0	26.9 ± 1.8	[26.9 ± 1.8]	61.5
S_{xx}^C (MPa)	627 ± 25	156 ± 2	964 ± 28	551
S_{yy}^C (MPa)	141 ± 3	156 ± 2	238 ± 16	227
S_{zz}^C (MPa)	153 ± 2	611 ± 13	[611 ± 13]	513
S_{xy} (MPa)	56.5 ± 1.7	56.8 ± 1.5	67.4*	67.4
S_{xz} (MPa)	104 ± 3	57.5 ± 0.5	[57.5 ± 0.5]	52.0
S_{yz} (MPa)	51.9*	57.5 ± 0.5	[57.5 ± 0.5]	52.0
S_{xx} (OHT)	-	154 ± 3	510 ± 11	244
S_{yy} (OHT)	-	143 ± 6	145 ± 5	178
S_{xx} (filled OHT)	-	[154 ± 3]	500 ± 19	396
S_{yy} (filled OHT)	-	[143 ± 6]	182 ± 6	125
S_{xx} (OHC)	-	119 ± 6	606 ± 32	355
S_{yy} (OHC)	-	149 ± 5	217 ± 2	154
S_{xx} (filled OHC)	-	[119 ± 6]	579 ± 13	355
S_{yy} (filled OHC)	-	[149 ± 5]	222 ± 3	154
Pin Bearing _{xx}	-	416 ± 11	446 ± 4	513
Pin Bearing _{yy}	-	668 ± 14	422 ± 7	513
Torque Pin-Bearing _{xx}	-	[416 ± 11]	711 ± 11	820
Torque Pin-Bearing _{yy}	-	[668 ± 14]	665 ± 21	820

OHT = open hole tension; OHC = open hole compression; * = predicted value
 [] Assumed symmetric in the x and y directions or along the x-z and y-z planes

APPENDIX 2: YIELD CRITERIA FOR RUBBER-TOUGHENED ADHESIVES

A2.1 VON MISES YIELD CRITERION

The most simple yield criterion interprets yielding as a purely shear deformation process, which occurs when the effective shear stress σ_e reaches a critical value. This effective stress is defined in terms of principal stress components σ_i ($i = 1, 2$ or 3) by [5-6]:

$$\sigma_e = \left\{ \frac{1}{2} \left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 \right] \right\}^{1/2} \quad (\text{A2.1})$$

The von Mises criterion then relates σ_e to the yield stress in tension σ_T by:

$$\sigma_e = \sigma_T \quad (\text{A2.2})$$

A2.2 LINEAR DRUCKER-PRAGER YIELD CRITERION

A simple modification of the von Mises criterion that includes hydrostatic stress sensitivity, known as the linear Drucker-Prager model, follows from Equation (A2.2) [5-6]:

$$\sigma_e = \sigma_o - \mu \sigma_m \quad (\text{A2.3})$$

where σ_e is the effective shear stress and σ_o is a material property, which is related to the shear yield stress σ_S by:

$$\sigma_o = \sqrt{3} \sigma_S \quad (\text{A2.4})$$

and σ_m is the hydrostatic stress given in terms of principal stresses by:

$$\sigma_m = \frac{1}{3} (\sigma_1 + \sigma_2 + \sigma_3) \quad (\text{A2.5})$$

The parameter μ characterises the sensitivity of yielding to hydrostatic stress. A value for μ can be determined from yield stresses under two or more different stress states.

A2.3 EXPONENT DRUCKER-PRAGER YIELD CRITERION

The exponent Drucker-Prager (ED-P) criterion is significantly more accurate for predicting deformation than the linear version under stress conditions where there is a high hydrostatic stress component (e.g. adhesive joints). This model can be written in the form [5-6]:

$$\sigma_e^2 = \lambda \sigma_T^2 - 3(\lambda - 1) \sigma_T \sigma_m \quad (\text{A2.6})$$

where λ is another hydrostatic-stress-sensitivity parameter. The ED-P criterion is implemented in ABAQUS with an exponent parameter of 2.

Equation (A2.6) then can be expressed in the form:

$$aq^2 = p + p_1 \quad (\text{A2.7})$$

where $q = \sigma_e$ and $p = -\sigma_m$. Comparison of Equations (A2.6) and (A2.7) then gives the following relationships between model parameters:

$$a = \frac{1}{3\sigma_T(\lambda - 1)} \quad (\text{A2.8})$$

$$p_1 = a\lambda\sigma_T^2 \quad (\text{A2.9})$$

A2.4 CAVITATION MODEL

Equation (A2.3) is now modified to include the effects of cavitation of the rubber particles in toughened adhesive as follows [5]:

$$\frac{\sigma_e^2}{\sigma_M^2} - (q_1 f)^2 + 2q_1 f \cosh \frac{3\sigma_m}{2\sigma_M} = \left(1 - \frac{\mu\sigma_m}{\sigma_M}\right)^2 \quad (\text{A2.10})$$

Here f is the effective volume fraction of cavities, which at small strains is zero but increases over some characteristic strain region associated with cavity nucleation. The parameter q_1 has been introduced to account for the effect of void interactions on the stress distribution in the matrix between cavities. σ_M is the effective yield stress of the matrix polymer between cavities and is equal to σ_o (see Equation (A2.4)) in the absence of cavities. The effective yield stresses rises with increasing f as the volume fraction of the rubber in the matrix decreases. The increase in σ_M as cavities nucleate is given by the expression [5]:

$$\sigma_M = \frac{\sigma_o}{1 - kv_{R0}} \left[1 - k \frac{(v_{R0} - f_n)}{1 - f_n} \right] \quad (\text{A2.11})$$

where v_{R0} is the volume fraction of rubber and f_n is the effective volume fraction of cavities created by cavity nucleation in rubber particles. At large strains, the quantity f may be larger than f_n as a result of cavity growth [5] subsequent to nucleation. The parameter k relates the shear yield stress of an uncavitated rubber-toughened adhesive to the volume fraction of rubber v_{R0} by the equation [5]:

$$\sigma_o = \sigma_{o1}(1 - kv_{R0}) \quad (\text{A2.12})$$

where σ_{o1} is the effective shear yield stress of the untoughened adhesive. It can now be seen that, for uncavitated material, $f = 0$ and Equation (A2.10) reduces to Equation (A2.3). In the cavitation model, $\sigma_o = \sqrt{3}\sigma_s$ is derived from experimental data obtained in shear and replaces σ_T as the basic hardening function for the polymer.

The nucleation of a cavity in a rubber particle is assumed to occur at some critical volumetric strain that decreases with increasing particle diameter. For a distribution of particle sizes, the cavity nucleation should then occur over a range of total volumetric strain ϵ_V related to the critical strain range for the rubber particles. Through comparisons with experimental data for a number of plastics and adhesives materials, it has been shown that the increase in the volume fraction of nucleated cavities f_n with volumetric strain is given with satisfactory accuracy by the expressions [5]:

$$\begin{aligned} f_n &= 0 \quad \text{for } \epsilon_V \leq \epsilon_{1V} \\ f_n &= v_{Ro} \left\{ 1 - \exp \left[- \left(\frac{(\epsilon_V - \epsilon_{1V})}{\epsilon_{2V}} \right)^\beta \right] \right\} \quad \text{for } \epsilon_V > \epsilon_{1V} \end{aligned} \quad (\text{A2.13})$$

The parameters ϵ_{1V} , ϵ_{2V} and β determine the location and breadth of the volumetric strain range over which cavity nucleation occurs.

The flow potential is assumed to be [5]:

$$F = \frac{\sigma_e^2}{\sigma_M^2} - (q_1 f)^2 + 2q_1 f \cosh \frac{3\sigma_m}{2\sigma_M} - \left(1 - \frac{\mu' \sigma_m}{\sigma_M} \right)^2 \quad (\text{A2.14})$$

APPENDIX 3: CLOSED FORM SOLUTIONS FOR BONDED T-JOINTS

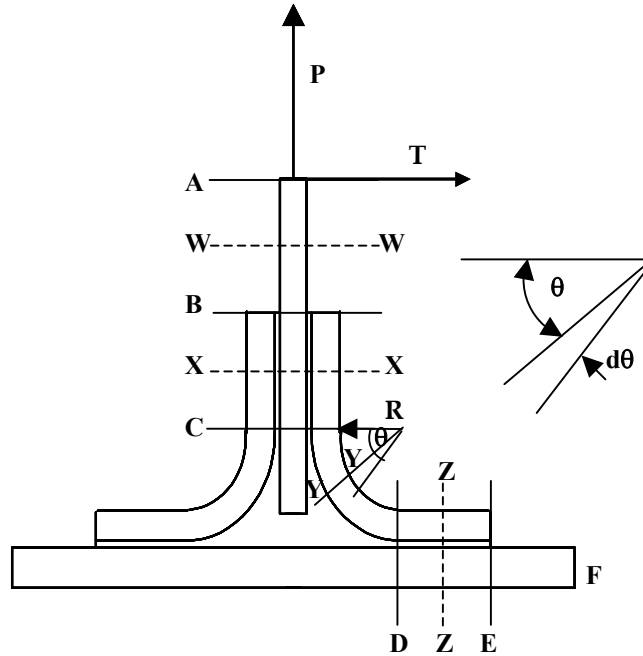


Figure A3.1: Schematic of bonded T-joint.

Parameters

- a = width of flange, web and base plate
- b_1 = web thickness
- b_2 = flange thickness
- b_3 = base plate thickness
- E = longitudinal stiffness
- I = second moment of area = $ab^3/12$
- L_{AB} = length AB
- L_{BC} = length BC
- L_{CD} = length CD
- L_{DE} = length DE
- L_{EF} = length EF (fully clamped)
- M = bending moment
- P = vertical load
- R = inner radius of flange
- s_1 = length AW
- s_2 = length BX
- s_3 = length CY
- s_4 = length DZ
- T = transverse load
- δ = displacement
- θ = angle

A3.1 VERTICAL DEFLECTION

The following analysis uses Castigliano's energy method to analyse one-half of the T-joint, which is assumed to be symmetric about the vertical axis.

$$\delta = \int \frac{M}{EI} \frac{\partial M}{\partial P} ds \quad (\text{A3.1})$$

Deflection for AB

$$\delta_{AB} = \int \frac{M_{WW}}{EI} \frac{\partial M_{WW}}{\partial P} ds_1 = 0; \quad M_{WW} = 0 \quad (\text{A3.2})$$

Deflection BC

$$\delta_{BC} = \int \frac{M_{XX}}{EI} \frac{\partial M_{XX}}{\partial P} ds_2 = 0; \quad M_{XX} = 0 \quad (\text{A3.3})$$

Deflection for DE

$$\delta_{DE} = \int \frac{M_{ZZ}}{EI} \frac{\partial M_{ZZ}}{\partial P} ds_4 \quad (\text{A3.4})$$

$$M_{ZZ} = P(R + s_4); \quad \frac{\partial M_{ZZ}}{\partial P} = R + s_4 \quad (\text{A3.5})$$

$$\delta_{DE} = \frac{P}{EI} \int_0^{L_{DE}} (R + s_4)(R + s_4) ds_4 \quad (\text{A3.6})$$

$$\delta_{DE} = \frac{P}{EI} \int_0^{L_{DE}} [R^2 + 2Rs_4 + (s_4)^2] ds_4 \quad (\text{A3.7})$$

$$\delta_{DE} = \frac{P}{EI} \left[R^2 s_4 + R(s_4)^2 + \frac{(s_4)^3}{3} \right]_0^{L_{DE}} \quad (\text{A3.8})$$

$$I = I_{DE} = \frac{a(b_2 + b_3)^3}{12} \quad (\text{A3.9})$$

Deflection for CD

$$\delta_{CD} = \int \frac{M_{YY}}{EI} \frac{\partial M_{YY}}{\partial P} ds_3 \quad (\text{A3.10})$$

$$M_{YY} = P(R - R \cos \theta); \quad \frac{\partial M_{YY}}{\partial P} = R - R \cos \theta; \quad ds_3 = R d\theta \quad (\text{A3.11})$$

$$\delta_{CD} = \frac{P}{EI} \int_0^{\pi/2} (R - R \cos \theta)(R - R \cos \theta) R d\theta \quad (\text{A3.12})$$

$$\delta_{CD} = \frac{PR^3}{EI} \int_0^{\pi/2} \left[1 - 2\cos\theta + \frac{1 + \cos 2\theta}{2} \right] d\theta \quad (A3.13)$$

$$\delta_{CD} = \frac{PR^3}{EI} \left[\theta - 2\sin\theta + \frac{\theta}{2} + \frac{\sin 2\theta}{4} \right]_0^{\pi/2} d\theta \quad (A3.14)$$

$$\delta_{CD} = \frac{PR^3}{EI} \left[\frac{\pi}{2} - 2 + \frac{\pi}{4} \right] = \frac{PR^3}{EI} \left[\frac{3\pi}{4} - 2 \right] \quad (A3.15)$$

$$I = I_{CD} = \frac{a(b_2)^3}{12} \quad (A3.16)$$

The total vertical deflection of the full T-joint at A is given by:

$$\delta_{TOTAL} = \frac{(\delta_{AB} + \delta_{BC} + \delta_{CD} + \delta_{DE})}{2} \quad (A3.17)$$

A3.2 HORIZONTAL OR LATERAL DEFLECTION

Deflection for DE

$$\delta_{DE} = \int \frac{M_{ZZ}}{EI} \frac{\partial M_{ZZ}}{\partial T} ds_4 = 0; \quad M_{ZZ} = 0 \quad (A3.18)$$

Deflection for AB

$$\delta_{AB} = \int \frac{M_{WW}}{EI} \frac{\partial M_{WW}}{\partial T} ds_1 \quad (A3.19)$$

$$M_{WW} = Ts_1; \quad \frac{\partial M_{WW}}{\partial T} = s_1 \quad (A3.20)$$

$$\delta_{AB} = \frac{T}{EI} \int_0^{L_{AB}} (s_1)^2 ds_1 \quad (A3.21)$$

$$\delta_{AB} = \frac{T}{EI} \left[\frac{(s_1)^3}{3} \right]_0^{L_{AB}} \quad (A3.22)$$

$$I = I_{AB} = \frac{a(b_1)^3}{12} \quad (A3.23)$$

Deflection for BC

$$\delta_{BC} = \int \frac{M_{XX}}{EI} \frac{\partial M_{XX}}{\partial T} ds_2 \quad (A3.24)$$

$$M_{XX} = Ts_2; \quad \frac{\partial M_{XX}}{\partial T} = s_2 \quad (A3.25)$$

$$\delta_{BC} = \frac{T}{EI} \int_0^{L_{BC}} (s_2)^2 ds_2 \quad (A3.26)$$

$$\delta_{BC} = \frac{T}{EI} \left[\frac{(s_2)^3}{3} \right]_0^{L_{BC}} \quad (A3.27)$$

$$I = I_{BC} = \frac{a(0.5b_1 + b_2)^3}{12} \quad (A3.28)$$

Deflection for CD

$$\delta_{CD} = \int \frac{M_{YY}}{EI} \frac{\partial M_{YY}}{\partial T} ds_3 \quad (A3.29)$$

$$M_{YY} = T(R + R \sin \theta); \quad \frac{\partial M_{YY}}{\partial T} = R + R \sin \theta; \quad ds_3 = R d\theta \quad (A3.30)$$

$$\delta_{CD} = \frac{T}{EI} \int_0^{\pi/2} (R + R \sin \theta)(R + R \sin \theta) R d\theta \quad (A3.31)$$

$$\delta_{CD} = \frac{TR^3}{EI} \int_0^{\pi/2} \left[1 + 2 \sin \theta + \frac{1 - \cos 2\theta}{2} \right] d\theta \quad (A3.32)$$

$$\delta_{CD} = \frac{TR^3}{EI} \left[\theta - 2 \cos \theta + \frac{\theta}{2} - \frac{\sin 2\theta}{4} \right]_0^{\pi/2} d\theta \quad (A3.23)$$

$$\delta_{CD} = \frac{TR^3}{EI} \left[\frac{\pi}{2} + \frac{\pi}{4} \right] = \frac{TR^3}{EI} \left[\frac{3\pi}{4} \right] \quad (A3.34)$$

$$I = I_{CD} = \frac{a(b_2)^3}{12} \quad (A3.35)$$

The total horizontal deflection of the full T-joint at A is given by:

$$\delta_{TOTAL} = \frac{(\delta_{AB} + \delta_{BC} + \delta_{CD} + \delta_{DE})}{2} \quad (A3.36)$$

A3.3 TANGENTIAL LOADING (ϕ)

For the case where load V is applied at an angle ϕ , the load can be resolved into vertical and horizontal components. The vertical and horizontal load components P and T can be determined using the following relationships:

$$P = V \sin \phi; \quad T = V \cos \phi \quad (\text{A3.37})$$

The tangential displacement δ_V can be calculated as follows:

$$\delta_V = \sqrt{(\delta_P)^2 + (\delta_T)^2} \quad (\text{A3.38})$$

where the vertical deflection δ_P and horizontal deflection δ_T are calculated using the analyses in Sections A3.1 and A3.2, respectively.

APPENDIX 4: CLOSED FORM SOLUTIONS FOR BOLTED T-JOINTS

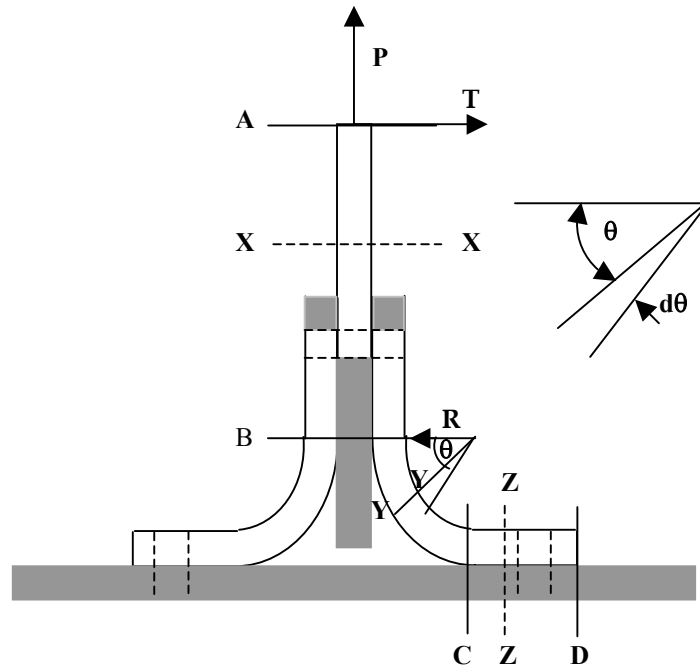


Figure A4.1: Schematic of bolted T-joint.

Parameters

- a = width of flange, web and base plate
- b_1 = web thickness
- b_2 = flange thickness
- b_3 = base plate thickness
- E = longitudinal stiffness
- I = second moment of area = $ab^3/12$
- L_{AB} = length AB
- L_{BC} = length BC
- L_{CD} = length CD
- I = bending moment
- P = vertical load
- R = inner radius of flange
- s_1 = length AX
- s_2 = length BY
- s_3 = length CZ
- T = transverse load
- δ = displacement
- θ = angle

Note: For simplicity, the white regions in Figure A4.1 are assumed to form a continuous section of uniform thickness and width. The regions shaded grey are assumed to have a negligible effect on the stiffness, and hence ignored in the analysis. A more comprehensive analysis would need to account for differences in thickness between the central web, flanges and base plate.

A4.1 VERTICAL DEFLECTION

Using Castigliano's energy method to analyse one-half of the T-joint, which is assumed to be symmetric about the vertical axis.

$$\delta = \int \frac{M}{EI} \frac{\partial M}{\partial P} ds \quad (\text{A4.1})$$

Deflection for AB

$$\delta_{AB} = \int \frac{M_{xx}}{EI} \frac{\partial M_{xx}}{\partial P} ds_1 = 0; \quad M_{xx} = 0 \quad (\text{A4.2})$$

Deflection for BC

$$\delta_{BC} = \int \frac{M_{yy}}{EI} \frac{\partial M_{yy}}{\partial P} ds \quad (\text{A4.3})$$

$$M_{yy} = P(R + s_2); \quad \frac{\partial M_{yy}}{\partial P} = R + s_2 \quad (\text{A4.4})$$

$$\delta_{BC} = \frac{P}{EI} \int_0^{L_{BC}} (R + s_2)(R + s_2) ds_2 \quad (\text{A4.5})$$

$$\delta_{BC} = \frac{P}{EI} \int_0^{L_{BC}} [R^2 + 2Rs_2 + (s_2)^2] ds_2 \quad (\text{A4.6})$$

$$\delta_{BC} = \frac{P}{EI} \left[R^2 s_2 + R(s_2)^2 + \frac{(s_2)^3}{3} \right]_0^{L_{BC}} \quad (\text{A4.7})$$

$$I = I_{BC} = \frac{a(b_2)^3}{12} \quad (\text{A4.8})$$

Deflection for CD

$$\delta_{CD} = \int \frac{M_{zz}}{EI} \frac{\partial M_{zz}}{\partial P} ds_3 \quad (\text{A4.9})$$

$$M_{zz} = P(R - R \cos \theta); \quad \frac{\partial M_{zz}}{\partial P} = R - R \cos \theta; \quad ds_3 = R d\theta \quad (\text{A4.10})$$

$$\delta_{CD} = \frac{P}{EI} \int_0^{\pi/2} (R - R \cos \theta)(R - R \cos \theta) R d\theta \quad (\text{A4.11})$$

$$\delta_{CD} = \frac{PR^3}{EI} \int_0^{\pi/2} \left[1 - 2 \cos \theta + \frac{1 + \cos 2\theta}{2} \right] d\theta \quad (\text{A4.12})$$

$$\delta_{CD} = \frac{PR^3}{EI} \left[\theta - 2\sin\theta + \frac{\theta}{2} + \frac{\sin 2\theta}{4} \right]_0^{\pi/2} d\theta \quad (A4.13)$$

$$\delta_{CD} = \frac{PR^3}{EI} \left[\frac{\pi}{2} - 2 + \frac{\pi}{4} \right] = \frac{PR^3}{EI} \left[\frac{3\pi}{4} - 2 \right] \quad (A4.14)$$

$$I = I_{CD} = \frac{a[0.5(b_2 + b_3)]^3}{12} \quad (A4.15)$$

The total vertical deflection of the full T-joint at A is given by:

$$\delta_{TOTAL} = \frac{(\delta_{AB} + \delta_{BC} + \delta_{CD})}{2} \quad (A4.16)$$

A4.2 HORIZONTAL OR LATERAL DEFLECTION

Deflection for CD

$$\delta_{CD} = \int \frac{M_{ZZ}}{EI} \frac{\partial M_{ZZ}}{\partial T} ds_3 = 0; \quad M_{ZZ} = 0 \quad (A4.17)$$

Deflection for AB

$$\delta_{AB} = \int \frac{M_{XX}}{EI} \frac{\partial M_{XX}}{\partial T} ds_1 \quad (A4.18)$$

$$M_{XX} = Ts_1; \quad \frac{\partial M_{XX}}{\partial T} = s_1 \quad (A4.19)$$

$$\delta_{AB} = \frac{T}{EI} \int_0^{L_{AB}} (s_1)^2 ds_1 \quad (A4.20)$$

$$\delta_{AB} = \frac{T}{EI} \left[\frac{(s_1)^3}{3} \right]_0^{L_{AB}} \quad (A4.21)$$

$$I = I_{AB} = \frac{a[0.5(b_1 + b_2)]^3}{12} \quad (A4.22)$$

Deflection for BC

$$\delta_{BC} = \int \frac{M_{YY}}{EI} \frac{\partial M_{YY}}{\partial T} ds_2 \quad (A4.23)$$

$$M_{YY} = T(R + R \sin \theta); \quad \frac{\partial M_{YY}}{\partial T} = R + R \sin \theta; \quad ds_2 = R d\theta \quad (A4.24)$$

$$\delta_{BC} = \frac{T}{EI} \int_0^{\pi/2} (R + R \sin \theta)(R + R \sin \theta) R d\theta \quad (A4.25)$$

$$\delta_{BC} = \frac{TR^3}{EI} \int_0^{\pi/2} \left[1 + 2 \sin \theta + \frac{1 - \cos 2\theta}{2} \right] d\theta \quad (A4.26)$$

$$\delta_{BC} = \frac{TR^3}{EI} \left[\theta - 2 \cos \theta + \frac{\theta}{2} - \frac{\sin 2\theta}{4} \right]_0^{\pi/2} d\theta \quad (A4.27)$$

$$\delta_{BC} = \frac{TR^3}{EI} \left[\frac{\pi}{2} + \frac{\pi}{4} \right] = \frac{TR^3}{EI} \left[\frac{3\pi}{4} \right] \quad (A4.28)$$

$$I = I_{BC} = \frac{a(b_2)^3}{12} \quad (A4.29)$$

The total horizontal deflection of the full T-joint at A is given by:

$$\delta_{TOTAL} = \frac{(\delta_{AB} + \delta_{BC} + \delta_{CD})}{2} \quad (A4.30)$$

A4.3 ADDITIONAL COMMENTS

As previously mentioned, the analysis described above simplifies the joint into three sections when in fact the loading path encounters at least five distinct sections. A more accurate analysis would require a more detailed subdivision of these geometric features, which is not easily modelled using an analytical approach. Tangential loading analysis described in Section A3.3 also applies to bolted T-joints.