

Predictive Models for Assessing Long-Term Performance of Polymer Matrix Composites

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ABSTRACT

This report reviews life prediction methods used for evaluating long-term properties, residual strength and lifetime of fibre bundles, unidirectional composites and composite laminates exposed to heat, moisture or aggressive chemicals, and static or dynamic fatigue loads. A number of different approaches, including physics/mechanistic based theories (including Monte Carlo concepts) and empirical modelling (eg wear-out and micromechanical models) are considered. Materials models for characterising progressive failure of unbonded fibre bundles and impregnated strands will be developed and evaluated within a subsequent experimental programme. These models will be extended to laminated composites and will include the effects of non-uniform moisture and temperature gradients. The models and analyses developed will be required to assess the results of the parametric studies to be carried out to determine the effects of temperature and level of degrading agent on the degree of degradation, identifying any synergistic or superimposed effects between environmental and loading parameters.

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1. INTRODUCTION

This report reviews physics/mechanistic based theories and empirical models that can be used to relate the degree of degradation of composite laminates with the level of degrading agent, exposure time and loading conditions. The work reviewed is relevant to the direction of the perceived future developments within the DTI funded project "Composite Performance and Design - Life Assessment and Prediction (CPD2)". A distillation of relevant features for each methodology is included along with the essential mathematical relationships on which the methodology is based. The review considers both environmental degradation and fatigue damage of fibre bundles (unbonded and impregnated) and laminated composites.

The report is divided into four sections (including the Introduction). Section 2 describes mechanistic modelling approaches to be developed within the project. The review covers environmental effects on unidirectional composites (single fibres and fibre bundles/strands), the application of models to laminates and the effect of fatigue loading on cross-ply laminates. Section 3 reviews various non-mechanistic (ie empirical and phenomenological) theories that have been used for predicting material degradation and life expectancy of composite laminates due to fatigue or exposure to heat, moisture and/or aggressive chemicals (including weathering). Conclusions are given in Section 4. Appendix A summarises the weakest link/extreme failure hypothesis, a probabilistic approach to predicting fracture in brittle fibrous materials. Appendix B presents the mathematical analysis for determining fatigue crack growth in cross-ply laminates

2. MATERIALS MODELS

2.1 UNIDIRECTIONAL COMPOSITES

The behaviour of unidirectional composites when loaded parallel to the fibres is dominated by the behaviour of the fibres, especially for relatively high fibre volume fractions of relatively high modulus fibres (eg glass and carbon) in a relatively low modulus matrix resin. When such composites are loaded transversely, the role of the fibre/matrix interface becomes very important. For unidirectional composites loaded in axial shear, the behaviour of the composite can be significantly affected by the matrix and the interface, with the result that composite properties depend in a complex way on fibre, matrix and interface properties. The planned technical activities in the CPD2 Project are such that the axial behaviour of unidirectional composites needs to be thoroughly understood. For this reason, the following review of materials models for unidirectional composites will be devoted only to the axial mode of loading.

For polymer composite materials of structural interest, the fibre moduli are significantly greater than those of the matrix so that the majority of any uniaxial load applied to the composite is carried by the fibres. The property degradation of a uniaxially loaded unidirectional composite is thus very dependent upon the behaviour of the fibres. There are two different types of degradation behaviour that are to be considered in the CPD2 Project. The first concerns the behaviour of a composite in an aggressive environment where the influence of the environment (in aqueous form) on fibre strength is such that the stiffness and both the residual and ultimate strengths of a unidirectional composite all degrade with time, even if the applied stress was held constant (ie static fatigue). The second mode of degradation arises from the application of cyclically varying fatigue loads (ie dynamic fatigue). This topic will not be included in the review of continuous aligned composites as the axial properties of these materials are not expected to degrade significantly during fatigue loading. However, this mechanism of degradation will be considered when reviewing the fatigue performance of cross-ply laminates. The subsequent discussion of this section will, therefore, concentrate mainly on the effects of aggressive environments on the degradation of composite properties.

Environmental Degradation in a Single Fibre

The static strength of fibres is thought to be governed by the size and distribution of microdefects which might be associated with the fibre surface and/or bulk structure. Statistical methods using the weakest link concepts (see Appendix A) are used to characterise the strength of individual fibres. The key characteristics of the approach are that the strength of an individual fibre length is statistically distributed, and that the cumulative failure probability is strongly dependent upon the length of the fibre tested. The Weibull distribution (see Appendix A) is most frequently used to characterise the variability of fibre strength. One reason for its success is that the Weibull distribution is the only asymptotic form from extreme value theory that is defined over the positive real axis. The other two asymptotic forms, shown in Appendix A, are defined over regions that involve negative values; physically unreasonable when considering fibre tensile strengths.

The effect of the environment on a stressed glass fibre (not carbon fibres) is to cause slow growth in the size of surface defects in the fibres leading to a time-dependent loss in strength. When the defect size reaches a critical value, determined by the fracture toughness of the fibre material, the fibre fails catastrophically. Stress corrosion cracking of the fibre defects leads to a progressive loss of residual strength with catastrophic failure occurring when the residual strength has declined to the value of the stress applied to the fibre.

Behaviour of Unimpregnated Fibre Bundles

Consider first of all the static strength of a loose (unimpregnated) bundle of fibres whose strengths are statistically distributed. As the number of fibres in a bundle increases, the strength distribution of the fibre bundle becomes asymptotic to a normal distribution [1, 2]. The modelling assumes that whenever a fibre breaks, the load that was carried by that fibre is shared equally between all surviving fibres - a situation known as equal load (or global load) sharing. The analysis provides an accurate value for the mean strength of the bundle and indicates that the standard deviation tends to zero as the number of the fibres tends to infinity. The important result is that, if the number of fibres in a bundle is large enough, then the strength of the bundle has a determinate value that is easily derived from the strength distribution for individual fibres. Extensive computer-based calculations have shown that the strength distribution can be accurately approximated by a normal distribution whenever the number of fibres in a bundle exceeded about two hundred [3, 4]. The strength properties of the fibres do not have a significant effect on this number.

The behaviour of loose bundles of fibres has been considered [5, 6] when exposed to an aggressive environment where fibres progressively fail in time due to degradation of the residual strength of the individual fibres by a stress corrosion mechanism. It was assumed that the number of fibres in the bundle was very large so that the static strength of the bundle had a determinate value. Whenever a fibre failed, the load carried by the fibre was assumed to be shared equally between all unbroken fibres. This assumption is not valid when considering the failure of a impregnated strand, as the load will be shared only amongst the fibres in the vicinity of the fibre fracture. The strength of each individual fibre in the bundle was associated with a particular largest defect size in the fibre. The growth in size of the defect, arising from stress corrosion cracking under load, was assumed to be governed by a defect growth law where crack growth rate (da/dt) is proportional to the stress intensity factor K raised to a power (ie $da/dt \propto K^n$). The catastrophic failure of an individual fibre was assumed to occur when the stress intensity factor for a particular defect reached the fracture toughness of the fibre material (assumed to be given by bulk properties when considering glass fibres). By constructing a mathematical model to describe the time-dependent behaviour of the fibre bundle, it was possible to predict the lifetime of a bundle subject to a fixed applied stress, and the time dependence of its residual strength, in terms of the fibre strength distribution, the defect growth law parameters and the fracture toughness of the material.

2.1.3 Behaviour of Composite Strands

2.1.3.1 Static Strength

While a very tractable mathematical model could be developed for the degradation of a loose bundle of fibres, as described in Section 2.1.2, the modelling of the degradation of a strand of fibres impregnated by matrix is much more complex. The modelling of the strength of a composite using statistical principles has been forced to make simplifications that enable the analysis to be tractable. The first simplification is to assume that the fibres support all the load followed by the assumption that the load carried by a breaking fibre is shared equally amongst the surviving fibres of the composite. These assumptions lead to modelling that is identical to the loading of a loose bundle of fibres.

For most composites, the concept of equal load sharing following fibre fracture is unreasonable as the matrix transfers load locally to the extent that the load carried by the broken fibre sufficiently well away from the location of fracture is not affected by the fracture event. Statistical models have been developed based on the local load sharing concept where the load carried by a broken fibre is shared equally amongst only the neighbouring fibres. Such models can be applied only to very small lengths of composite material (determined by the stress transfer distance) which are of the order of several fibre diameters. As macroscopic lengths of composite need to be considered, the models have to be modified so that the composite is represented by a series of fibre bundles, where each bundle is subject to local load sharing. These models are often referred to as series/parallel models and can be analyzed for both equal and local load sharing when fibres break.

In the ceramic matrix composites (CMCs) field, and for fibre reinforced metals (FRMs) where the matrix plasticity can be neglected, fibre fracture is accompanied by a significant degree of interface debonding that initiates at the location of the fibre fracture. For many such composites, there is frictional contact between fibre and matrix, which has been extensively modelled by assuming that the interfacial shear stress in the region of debonding has a constant value. This is often regarded as a materials constant even though it is obvious that temperature changes leading to changes in interfacial normal stresses will affect the interfacial shear stress. The statistical modelling of fibre fracture, including analyses of the fibre fragmentation of monofilaments, has been based on the assumption of a constant interfacial shear stress and the use of a shear lag stress analysis to account for the stress transfer between fibre and matrix in the neighbourhood of fibre fractures.

Hui et al [7] have shown that a simple statistical approach used by Curtin [8], which takes into account stress transfer effects between fibre and matrix and involves fibre fragmentation, is valid only if the Weibull shape parameter is large enough (> 0.75 as is the case for most practical applications). When applying such statistical approaches to a unidirectional composite the problem arises of dealing with stress transfer associated with statistically distributed fibre fractures. Curtin and Zhou [9] considered a continuous aligned composite with a brittle matrix as a set of parallel fibres with randomly distributed fibre breaks. The fibres are held together by matrix and the stress transfer, associated with the fibre fractures, is modelled using the constant interfacial shear stress concept. As a result, the stress distribution along any fibre is a piecewise linear function, which is zero at the fibre break locations and a constant value (equal to that fibres of an undamaged composite subject to the same stress) in regions away from the stress transfer zones. When estimating the overall strength using statistical principles it was assumed that the composite was infinite in extent both along the fibres and in the plane normal to the fibres.

It is worth noting that the assumption of neglecting the matrix stress is inconsistent with the existence of a non-uniform stress along the fibre. A non-uniform stress can arise only if there is some stress in the matrix, such that the effective stress (averaged over the cross-sectional area) carried by the fibres and matrix on the selected plane must equal the applied stress.

In spite of this inconsistency, the model does have one very useful feature that is worth emphasising. Since the composite is infinite in extent, it means that it can be assumed that all possible fibre fracture configurations for a single fibre must appear in the composite. Furthermore, for a given configuration of fibre breaks in a single fibre, every fibre stress that occurs along the length of this fibre will also be sampled somewhere in the selected plane of cross-section. It can be then deduced that the average stress in the fibres in the selected plane is equal to the average stress in the fibres, where the average is taken over all possible configurations of fibre breaks in a single fibre. This technique offers some potential for being able to predict reliably the average stress applied to a composite that is compatible with a given state of fibre damage including at the point of fracture. More work is needed in order to form a definitive view on the current status of this approach.

Statistical approaches to static strength prediction using equal or local load sharing concepts with series/parallel structures are very rudimentary when dealing with stress transfer from fibre to matrix caused by fibre fracture. Imposing a more realistic load sharing methods leads to intractable statistical analysis. As a result, Monte Carlo concepts have been developed for static strength prediction. The methodology is based upon randomly distributing fibre strengths to a large number of fibre elements into which the composite is divided, creating a series/parallel structure. For each simulation, a different distribution of fibre strengths will occur, leading to a different load at which the composite fails. The statistical variability of strength can only be determined by carrying out many repeated simulations of composite failure.

It is clear that developing a statistical model for the prediction of the static strength of a unidirectional composite, taking proper account of stress transfer arising from the fibre fracture, is exceedingly difficult. The more tractable approach is to modify the Monte Carlo approach to the prediction of static strength. A preliminary model, based on a Defence Evaluation and Research Agency (DERA) approach [10, 11], was developed as part of the PREDICT Project that was supported by the Link Structural Composites Programme. In the model, unidirectional composite was regarded as being divided along its length into a set of layers (bundles of very short fibres in series), and the effects of the matrix were ignored, except for its effect on the load sharing rules to be applied when a fibre fractured. Whenever a fibre was considered to have failed, the load it carried was distributed to neighbouring hexagonal rings of fibres within a layer according to the load sharing rule determined by an appropriate stress transfer model [12]. Load sharing to any number of neighbouring fibre layers (along the length of the composite) was allowed using a different load sharing rule determined by the same stress transfer model. In contrast to most Monte Carlo approaches, a new method of assessing composite failure was developed, where fibre failures leading to composite collapse could occur in a prescribed number of neighbouring layers along the length of the strand. This approach led to predictions of fibre pull-out; at least for some values of the materials parameters of the system. The methodology developed resulted in reasonable predictions of composite strength, although the experimentally measured strength distribution of fibres had to be extrapolated in order that the values were relevant to very small lengths of the fibre layers in series; as assumed in the Monte Carlo simulation.

Monte Carlo modelling to-date has ignored the effect of the matrix, as it was considered to support very little load due to its low modulus compared to that of the fibre, except for its role in controlling how the load, carried by a breaking fibre, was shared between its neighbours. An important next step is to include within the simulation the load carrying capacity of the matrix. This should be relatively straightforward in principle, as the network method used to label and locate fibres has an equal number of unused sites that can be used to record the local state of the matrix. The completion of this extension to the model, which is part of the work in the CPD2 project, should lead to the ability to predict the static strength of a unidirectional composite in terms of statistically variable fibre strengths, and the properties of the matrix.

2.1.3.2 Composite Degradation

The principal technical challenge awaiting a satisfactory solution is the development of a suitable model to predict the time dependent degradation of a unidirectional composite. There appears to be two different approaches that can be made, and these will now be described. The first approach assumes that one effect of the aggressive environment is to destroy all adhesion at the interfaces between the fibres and matrix. Assuming no friction acts at debonded interfaces then the fibres and matrix act independently within the composite. The fibres and matrix interact only within the grip region and the system can be represented by the well known two-bar model, where the parallel bars are regarded as being fixed to two rigid supports that are pulled apart in a direction parallel to the fibre direction. One bar of the model represents all the fibres in the composite so that it behaves as a loose fibre bundle subject to stress corrosion cracking. The other bar represents the matrix material. Time dependent matrix properties arising from viscoelasticity, and from the effects of the environment, can be modelled. This type of representation is recommended as the first approach to modelling of stress corrosion failure of unbonded fibre bundles and impregnated strands to be carried out in the CPD2 Project.

The second approach assumes that a good bond is maintained between fibres and matrix. The first development will be to neglect the load bearing capacity of the matrix, but include the time dependence of fibre strengths arising from fibre defect growth arising from a stress corrosion mechanism. Assuming the same load sharing rules that are applied when predicting static strength, it will be possible to predict the lifetime of the composite as a function of applied stress, and the time dependence of the residual strength of the bundle, as a function of applied stress. The second development will be to include the load carrying capacity of the matrix, together with its time dependence arising from exposure to an aggressive environment and possibly from matrix viscoelasticity.

2.2 APPLICATION OF MODELS TO LAMINATES

Through previous DTI funded projects, especially "Design of Composite Components - Failure of Brittle Matrix Composites" Project DCC3, a thorough understanding of ply crack formation in laminates has been developed, together with associated software to predict damage formation for a variety of loading conditions. As the literature on the topic of damage formation in laminates subject to monotonic loading is vast, a thorough review of the technical literature is considered to be of little value to this review due to the excessive number of differing *approximate* approaches that have been used. For example, there have been many approaches based on shear-lag principles which are now well known to be far too approximate for use in an engineering design environment. The subject has now progressed, principally through work carried out at both NPL and the Wright Laboratories (USA), to the point where models and associated software are available that can predict very accurately the stress and displacement distributions in multiple ply cross-ply laminates having ply cracks. A similar situation exists also for symmetric laminates where NPL has developed a model that can handle ply cracks that form in plies having a single orientation.

The accurate stress analyses are used as the basis of predictions for the effective thermoelastic constants as a function of crack density, and when combined with energy balance principles, the constants can be used to predict the progressive formation of ply cracking in laminates during loading, which can be complex involving combined in-plane biaxial and shear loading modes, and combined axial and bending loading for the case of cross-ply laminates which might be non-symmetrical. The accuracy of the analyses has led to the development of relatively simple analytical results that have very good potential for being used in engineering design. The review of durability modelling for laminates will build on the significant progress that has already been made by describing how the current capabilities will be extended to deal with fibre fracture in laminates, and the effects of aggressive environments and fatigue loading.

Having outlined in Section 2.1 the expected developments for the prediction of the static strength, residual strength and lifetimes of a unidirectional composite subjected to an aggressive environment, it is now appropriate to indicate how these unidirectional models will be incorporated into models that can predict the performance of laminates. As transverse and shear loading modes have been avoided, it is sensible to restrict the application of the unidirectional models to cross-ply laminates. In a cross-ply laminate subject to uniaxial loading, the 0° plies will behave in a way that can be modelled by the unidirectional composite models described in Section 2.1. Thus, the effects of fibre fracture resulting from both static fracture and stress corrosion cracking can be incorporated into the laminate model. The 90° plies will be subject to transverse cracking that is governed by the effective fracture energy for ply cracking, a materials parameter that will be affected by the environment. The stress distributions in the 0° plies arising from fibre fracture will be so localised that their effect can be taken into account by modifying the effective longitudinal stiffness of the 0° plies during loading.

A more difficult aspect is the effect of ply cracks in 90° plies on the stress distributions in the fibres lying in the 0° plies. This effect can only be modelled by increasing the axial stresses in the fibres and matrix of the 0° plies in a way that is consistent with the stress expected in these plies when modelled as homogeneous anisotropic materials. The effects on radial and shear stresses in the fibre and matrix of the 0° plies will therefore be neglected. Another problem that arises, is that the predicted stress in the 0° plies in the plane of a crack in the 90° ply will be singular at the interface, implying immediate fibre failure. In reality the singularity does not exist in practice (even for elastic materials), but is merely a consequence of solving the cracked laminate problem assuming that the individual plies can be modelled as homogeneous anisotropic materials. When the ply crack is considered within the fibre/matrix model (not feasible in practice due to the large number of fibres that have to be considered) the interface between the plies does not exist because of the presence of the matrix, and the singularity normally encountered would not exist.

To overcome the problem of the singularity, while making use of homogeneous anisotropic properties of the plies, the stress field in the 0° ply of a cracked laminate in the region of the singularities needs to be averaged over some characteristic distance, such as the diameter of the representative volume element for a single fibre in a 0° ply. It is clear that the simulation of fibre failures in the 0° plies of a cross-ply laminate will involve the use of two stress transfer models. The first [13] is applied at the laminate level in order to estimate the distribution of stress increase in the 0° plies arising from the formation of ply cracks in the 90° plies. The second [14] is applied at the fibre level in the 0° plies in order to distribute realistically the stress that was carried by a breaking fibre. The non-uniform stress distributions in the fibres arising from ply cracking can therefore be modelled and used in conjunction with Monte carlo simulations of fibre fracture in the 90° plies, taking into account stress corrosion cracking in the fibres for cases where environmental effects are important.

2.3 EFFECT OF FATIGUE LOADING ON CROSS-PLY LAMINATES

The estimation of the life of a composite, when subjected to fatigue or creep deformations, is perhaps the most difficult prediction an engineer has ever to make. In order to develop suitable models, it has to be recognised that composite failure is generally preceded by a multitude of localised damage effects that are strongly interactive, subject to statistical uncertainty, and that lead to an increasing deterioration in performance to the point of catastrophic failure. If a practically useful life prediction design methodology is to be developed then it is vital that the initiation and growth of damage preceding failure is capable of being predicted with confidence. The overall objective of this section is, therefore, to outline a useful method that can be recommended with confidence for use when predicting the growth and effects of fatigue induced damage on the mechanical performance of cross-ply laminates. The modelling to be discussed in this report does not consider the prediction of the initiation of fatigue damage; a phenomenon of great complexity that is

highly statistical in nature, and not amenable to reliable detailed modelling due to the complex geometrical configurations of the initial forms of fatigue damage (eg localised fibre/matrix debonds). The modelling will assume that the fatigue damage has grown to the point where microcracks have formed that traverse the thickness of the 90° plies.

During fatigue loading (at modest stress levels) of cross-ply laminates, ply cracking normal to the loading direction is encountered. This leads to cracks in the 90° plies of the composite, normal to the loading direction, that are bridged by 0° plies. The presence of these cracks leads to a degradation of composite properties, such as elastic moduli, Poisson's ratio and coefficients of thermal expansion. It is therefore important to understand the factors that determine the formation and growth of these cracks during fatigue loading. The stress intensity factors at the tips of bridged ply cracks are influenced strongly by the bridging 0° plies, and it is important to be able to predict the dependence of stress intensity factors on crack bridging forces. This leads to the need to be able to relate the effective stress, p , carried by the bridging 0° plies to the effective crack opening displacement, Δv of the 90° plies.

The concept of crack bridging stress and crack opening displacement has been discussed in detail elsewhere [15, 16]. It has been shown how crack bridging relations derived from micro-mechanical models of ply cracking can be used to develop an equivalent continuum model that can be treated using fracture mechanics methods suitably modified to account for the effect of crack bridging. The methods described here extend the models so that the growth of fatigue cracks can be treated. A quantitative knowledge of the $p - \Delta v$ relation is the key to the understanding and prediction of the effect of bridging plies on the stress intensity factors for ply cracks, and consequently on ply crack growth rates during fatigue loading, and for calculating critical stress intensity factors for crack instability. The methodology applies a stress transfer model, for ply cracks in a 0°-90°-0° cross-ply laminate, using the derived $p - \Delta v$ relation defined in terms of the thermo-mechanical properties of the laminate plies, in order to:

- (a) predict the stress intensity factors of ply cracks for the case when 0°-plies remain intact during ply crack growth,
- (b) determine the point of instability of long ply cracks.

For cross-ply laminates, where ply cracking in the 90°-plies is the only form of damage and where the interfaces between the 0° and 90° plies remain perfectly bonded, the $p - \Delta v$ relation is linear due to the assumption that the stress analysis is based on linear elasticity theory.

The application of non-linear crack bridging relations to fatigue crack growth has been considered in detail for the case of fibre-reinforced titanium composites [17]. The approach for PMCs is to adopt the same general philosophy that was described in [17], but to develop a different approach to mathematical modelling that can be made only for linear $p - \Delta v$ relations. The model will be developed for the case of 0°-90°-0° laminates so that an analytical approach can be used. The details of the approach are given in Appendix B. It is worth emphasising here that, for ply cracks that have grown beyond their initiation point, the effective stress range intensity factor is independent of crack length. To consider multiple ply laminates would involve using numerical methods; a topic beyond the scope of any preliminary investigation. Constant crack growth can be expected.

The approach described above attempts to link the more accurate analytical methods of laminate stress analysis with the conventional approach to the prediction of fatigue damage where ΔK is assumed to be the parameter controlling fatigue crack growth. Benefic et al [18] have considered fatigue damage growth where less reliable shear lag stress analysis has been used to work out strain energy release rates for ply cracks subject to monotonic loading. They have suggested, based on an examination of experimental results, that fatigue crack growth can be correlated using the parameter ΔG in place of ΔK . Both approaches have merit, but to date it is debatable which approach is more valid. It is concluded that the

subject requires a thorough mechanics based approach for the prediction of the behaviour of fatigue cracks in laminates that is consistent with the well established mechanics for predicting the behaviour of laminates during monotonic loading.

Mention must also be made of the damage accumulation / damage mechanics approach to the prediction of fatigue damage in laminates. Beaumont [19] has described various approaches that have been made to develop methodology based on 'model-informed empiricism', leading on to the concept of continuum damage mechanics based on internal state variables (see for example papers by Talreja and Allen [20]). The field of damage mechanics, which is currently being reviewed as part of another project (CPD4c), attempts to establish a mechanics framework for predicting the effects of damage on macroscopic engineering properties. The approach is based on the use of internal state variables which are not necessarily interpreted in terms of physical phenomena. The approach is, therefore, classified as being phenomenological rather than mechanistic. An important merit of the continuum damage mechanics approach is that it obeys the various conservation laws of mechanics, and is formulated to be consistent with the principles of thermodynamics, especially the second law that is concerned with irreversible processes. Recent unpublished work at NPL has attempted to seek the link between continuum damage mechanics methodology and the mechanistic approach for the special case where the damage is in the form of ply cracks in cross-ply laminates. Very useful results have been obtained that indicate that the continuum damage mechanics approach has validity, but that much of the literature requires re-examination and re-formulation to benefit from the rigorous results based on mechanistic modelling that lead to dramatic simplifications to the damage mechanics model. A detailed description of these new results will be given in a subsequent review.

3. NON-MECHANISTIC MODELLING

This section considers a number of non-mechanistic models that have been proposed to predict the residual strength and life expectancy of composites following exposure to combinations of heat, applied loads (static and fatigue) and moisture, or natural weathering.

3.1 TEMPERATURE-MOISTURE-STRESS SUPERPOSITION

The modelling of any degradation process requires information on the change in material properties with time, and the rate of change of those properties with the level of degrading agent(s). A number of semi-empirical relations (both linear and logarithmic) for property degradation have been suggested [21-24]. These are usually of the form:

$$P(t, T) = P(\infty, T) + [P(0, T) - P(\infty, T)] \exp[-k(T)t^n] \quad (1)$$

where k is the reaction rate (or degradation rate), P is the material property (eg strength or stiffnesses), T is the ageing temperature (usually absolute), t is the ageing time and n is an experimentally determined constant. The strength decays exponentially with time to an asymptotic value (usually zero). This approach assumes only one time-dependent process is occurring when in reality there can be several processes occurring simultaneously.

An alternative approach is to plot material property data against time for one temperature-moisture level; the data being represented by one of the following empirical relations:

$$\log P(t, T) = A(T) - B(T)t \quad (2)$$

$$P(t, T) = P(0, T) \exp[-k(T)t] \rightarrow P(\infty, T) = 0 \quad (3)$$

where B is the degradation rate and A is a constant. Similar data are generated at different temperatures. The time required for the strength to degrade to a pre-determined or limit value (usually half its original value (half-life) at each temperature is then calculated from the fitted equations. The next step is to plot the limit value as a function of the reciprocal of the ageing temperature (ie $1/T$). The half-life $t_{1/2}$ is related to the ageing temperature T as follows:

$$\ln t_{1/2} = C + \frac{D}{T} \quad \rightarrow \quad k(T) = \ln 2 \exp \left[-C - \frac{D}{T} \right] \quad (4)$$

where C and D are material constants. The half-life at service temperature can be estimated by extrapolation from the plot of $\log t_{1/2}$ versus $1/T$ (a straight line) or by fitting the data to Equation (4). It is important that the test temperatures are kept moderate ($< 100^\circ\text{C}$) to ensure that chemical reactions (eg thermal oxidation) that occur at higher temperatures are avoided, and that the dominant mode of failure is identical at all temperatures and stress levels.

The above approach has been extended to determining the combined effect of temperature, moisture and stress with the failure time t_f given by [25]:

$$\log t_f = C - \log T + \frac{\Delta H}{2.303RT} - b \frac{S}{T} \quad (5)$$

where C and b are constants, T is absolute temperature, ΔH is the activation energy, R is the universal gas constant, T is the absolute temperature ($^\circ\text{K}$) and S is the uniaxial stress. At constant temperatures, the $\log t_f$ versus S/T relation is a straight line.

Strength and stiffness property degradation due to the combined effects of hygrothermal ageing and repeated or cyclical fatigue loading has been approximated using a simple algebraic relationship of the form shown below [22]:

$$\frac{P}{P_0} = \left(\frac{T_{GW} - T}{T_{GD} - T_0} \right)^{1/2} - A \log N \quad (6)$$

where P denotes the material property (usually strength) at temperature T , and T_{GD} and T_{GW} are the glass transition temperatures (T_G) of dry and conditioned (ie wet) material, A is obtained by curve-fitting experimental data and N is the number of cycles. The value of constant A will vary between fibre/matrix systems and mechanical properties. T_{GW} can be expressed in terms of T_{GD} as follows:

$$T_{GW} = (AM^2 + BM + C)T_{GD} \quad (7)$$

where M is the moisture content in the composite in percent weight, and A , B and C are constants obtained by curve fitting experimental data. The use of the term $\log N$ in Equation (5) is unfortunate because when $N = 0$, P is infinite! This problem, which often arises when empirically fitting data, is easily solved by replacing N by $N+1$.

3.2 ARTIFICIAL WEATHERING

Weathering is of considerable complexity due to natural fluctuations in temperature, humidity, ultraviolet radiation and other environmental factors, and to the interaction of these factors, and is therefore impossible to simulate. Attempts have been made to determine the rate of degradation of a material property P due to the combined effect of temperature and sunlight in a similar manner to temperature-moisture superposition relations shown in Section 3.1. A number of empirical relations have been proposed [26]:

- (i) Linear (without temperature effects)

$$P = P_0 + bD \quad (8)$$

where P_0 is the initial property value, D is the ultraviolet radiation dose and b is a constant.

- (ii) Linear with temperature effects (ie Arrhenius relationship)

$$P = P_0 + D \exp(-\Delta H/RT) \quad (9)$$

- (iii) Power law (without temperature effects)

$$P = P_0 + bD^n \quad (10)$$

where n is a constant.

- (iv) Exponential (without temperature effects)

$$P = P_0 + A \exp(D) \quad (11)$$

where A is a constant.

The above relations only apply to one set of conditions, and therefore extrapolating short-term data to long-term property prediction would be ill advised. A design of experiment approach using statistical analysis in the form of the Taguchi method could possibly be used to identify critical interactions and to determine their relative magnitude [27].

3.3 FATIGUE DAMAGE MODELS

Fatigue behaviour is usually expressed in terms of the maximum applied fatigue stress S and fatigue life (number of cycles to failure N). For intercomparative purposes, fatigue strength data is normalised with respect to the ultimate strength of identically conditioned specimens measured at the fatigue test loading rate. Quantification and prediction of fatigue performance is largely empirical and based on experiment with the high degree of uncertainty being a major concern. This section reviews empirical and semi-empirical methods that have been used to predict residual strength, stiffness degradation and fatigue life of composites. Due to the substantial number of methods in existence, the review will be limited to prime examples of the different extant modelling approaches [28-31].

3.3.1 Empirical Fatigue Theories

There are many empirical fatigue theories that are used to characterise S-N curves for PMCs. A number of these are shown below [29] :

$$\sigma_{ULT} = \sigma_s N^{-m} \quad (12)$$

$$\sigma_s = \sigma_{ULT} - b \log N \quad (13)$$

$$\sigma_{range} = a + \frac{b}{N} \quad (14)$$

$$\sigma_{RANGE} = a + \frac{b}{N^x} - \frac{c}{N} \quad (15)$$

where:

$$A = \frac{1 - R}{1 + R} = \frac{\sigma_{RANGE}}{\sigma_{MEAN}} \quad (16)$$

$$R = \frac{\sigma_{MIN}}{\sigma_{MAX}}; \quad -1 < R \leq 1 \quad (17)$$

σ_{MIN} and σ_{MAX} and σ_{MEAN} are the minimum, maximum stresses and mean stresses, respectively for stress cycling. It is worth mentioning that these empirical fatigue equations only apply to constant amplitude uniaxial loading conditions and are generally not applicable to in-service situations. The relations (12-15) lead to the physically unreasonable situation of unbounded predictions when the number of cycles tends to zero. It would be much more sensible to replace N by $N+1$ in these relations; a change that leads to imperceptible differences when applied in a fatigue context.

Strength Degradation - Deterministic Approach

The residual strength, $S(N)$, initially equals the static strength σ_{ULT} , and is assumed to monotonically decrease with fatigue cycles. Ignoring environmental and frequency effects, the rate of degradation should be dependent on σ_{ULT} , the magnitude of the maximum stress σ_{MAX} and the stress ratio R . The residual strength $S(N)$ has been written in the form of [30]:

$$S(N) = \sigma_{ULT} - (\sigma_{ULT} - \sigma_{MAX}) \left(\frac{N}{N_F} \right)^v \quad (18)$$

where N is the number of constant amplitude loading cycles before carrying out the residual strength test, N_F is the constant amplitude fatigue life and v is the "strength degradation parameter". Failure is assumed to occur when the residual strength $S(N)$ is equal to the maximum stress σ_{MAX} . The peak stress σ_{MAX} is usually 50% of σ_{ULT} . Equation (18) defines a family of $S(N)$ curves for residual strength as a function of fatigue loading cycles prior to strength testing. The path of each curve is dependent on the strength degradation parameter v with: (i) linear degradation occurring for $v = 1$; (ii) sudden death behaviour for $v \gg 1$; and (iii) rapid initial loss of strength for $v < 1$.

Strength Degradation - Statistical Approach

Due to the inherent variability in residual strength and life expectancy, the deterministic approach described in Section 3.3.2 has generally proved inaccurate, necessitating statistical analysis of the data. A widely used approach has been to represent the residual strength and life expectancy by two parameter Weibull functions. Using this approach, the probability of failure for constant amplitude loading (ie the probability that the residual strength after N cycles is less than the maximum applied stress σ_{MAX}) can be expressed as [30]:

$$P[S(N) \leq \sigma_{MAX}] = 1 - \exp \left[- \frac{\sigma_{MAX}}{\sigma_{ULT} - (\sigma_{ULT} - \sigma_{MAX}) \left(\frac{N}{N_F} \right)^v} \right]^{B_f(N)} \quad (19)$$

where $B_f(N)$ is the Weibull shape parameter for residual strength.

At zero cycles, Equation (19) should reduce to the static strength Weibull cumulative distribution function, which describes the probability that the initial strength S_0 is less than or equal to the maximum stress σ_{MAX} . This function has the following form [30]:

$$P[\sigma_{ULT} \leq \sigma_{MAX}] = 1 - \exp \left[- \left(\frac{\sigma_{MAX}}{\sigma_{ULT}^0} \right)^{B_s} \right] \quad (20)$$

where B_s is the shape parameter for static strength and σ_{ULT}^0 is a scaling parameter. Thus, a required scaling factor seems to be missing from Equation (19) and one might expect that $B_f(0) = B_s$. Equation (19) predicts that $P[S(N_F) \leq \sigma_{MAX}] = 1 - 1/e = 0.632$ as $N \rightarrow N_F$. This is a very peculiar result as one would expect the probability to have some dependence on material properties. The identified problems relating to the use of Equation (19) seem to arise from substituting the deterministic relation given by Equation (18) for the residual strength in place of a required scaling factor.

A relation for the Weibull fatigue life distribution is given by [30]:

$$P[N_F \leq N] = 1 - \exp \left[- \left(\frac{N}{N_F^0} \right)^{B_L} \right]$$

where B_L is the shape parameter for fatigue life and where N_F^0 is a scaling parameter.

Static and fatigue life tests are conducted to obtain N_F^0 and σ_{ULT}^0 , the shape parameters $B_f(N)$, B_L and B_s , and the scaling factors. The "optimum" value of the strength degradation parameter ν is obtained by determining the best predictive fit of Equation (19) to the experimental data. The scatter in residual strength distributions increases with increasing cycles, and hence the shape parameter decreases.

Talreja [29] has considered the statistics of predicting fatigue failure in composites where the following expression, which can be derived from the Paris type of fatigue crack growth law, is used for the residual strength $S(N)$ after the N^{th} fatigue cycle:

$$S^n(N) = S^n(0) - f(\sigma_{MAX})N$$

where n is a constant and f is some function of the maximum cyclic stress σ_{MAX} . The approach of Talreja is to use Equation (22) to find those values of X for which the range $S(0) \leq X$ corresponds to the range of residual strengths $S(N) \leq x$.

The use of the Weibull distribution for the cumulative failure probability $P[S(0) \leq X]$, given by Equation (20), then leads to the result:

$$P[S(N) \leq x] = 1 - \exp \left[- \left(\frac{x^n + f(\sigma_{MAX})N}{(\sigma_{ULT}^0)^n} \right)^{B_L/n} \right]$$

The result given by Equation (23) does not involve N_F , contrasting with Equation (19). Equation (23) predicts that the probability function achieves the value unity when $N \rightarrow \infty$, as to be expected. It is of interest to use the approach of Talreja when using Equation (19) for the residual strength, which is first written in the form:

$$S(N) = S(0) \left[1 - \left(\frac{N}{N_F} \right)^{\nu} \right] + \sigma_{MAX} \left(\frac{N}{N_F} \right)^{\nu}$$

It then follows that $S(N) \leq x$ whenever:

$$S(0) \leq \frac{\sigma_{ULT} - \sigma_{MAX} \left(\frac{N}{N_F} \right)^{\nu}}{1 - \left(\frac{N}{N_F} \right)^{\nu}}$$

On using Equation (25) in conjunction with Equation (20) it follows that:

$$P[S(N) \leq x] = 1 - \exp \left[- \left\{ \frac{x - \sigma_{MAX} (N/N_F)^{\nu}}{\sigma_{ULT} [1 - (N/N_F)^{\nu}]} \right\}^{B_L} \right]$$

When $N \rightarrow 0$ the Weibull distribution for static strength is recovered, but when $N \rightarrow N_f$ the probability tends to unity contrasting sharply with the properties of the result (23). The model based on the use of (18) is such that there exists a deterministic life N_f such that it is certain that all samples in the population will have failed after N_f cycles have been executed. It is concluded that the better approach is to use (22) and (23) in preference to (18), (19) and (26).

3.3.4 Stiffness Degradation

At present, the determination of the remaining life/residual strength due to cumulative damage is impossible using non-destructive inspection techniques. However, cumulative damage can adversely affect elastic properties, which are measurable. Stiffness reduction can be used as a measure of structural health, and is often used in laboratory environments to monitor the onset of damage and loss of strength. The relationship between residual strength and stiffness reduction is relatively complex, material specific, and would need to include interactive components, thus requiring a mechanistic approach as outlined in Section 2.

3.4 CLASSICAL LAMINATE APPROACH

An approach commonly used for determining degradation of multi-directional laminates is to use classical laminate failure analysis (eg Tsai-Wu failure criterion) with derated lamina elastic and strength properties. The lamina data can either be measured or determined using one of the many formulations identified in the previous sections. This approach is compatible with the material synthesis module contained in the NPL Composite Design and Analysis (CoDA) software.

Tsai-Wu proposed the following stress-formulated failure criterion for orthotropic lamina subjected to plane stress:

$$F_1\sigma_{11} + F_2\sigma_{22} + F_{11}\sigma_{11}^2 + F_{22}\sigma_{22}^2 + F_{66}\tau_{12}^2 + 2F_{12}\sigma_{11}\sigma_{22} \geq 1 \quad (27)$$

where σ_{11} , σ_{22} and τ_{12} are the in-plane longitudinal, transverse and shear stresses, respectively. The strength coefficients F_1 , F_2 , F_{11} , F_{22} and F_{66} are expressed in terms of the longitudinal tensile strength $S_{11}(T)$, transverse tensile strength $S_{22}(T)$, longitudinal compressive strength $S_{11}(C)$, transverse compressive strength $S_{22}(C)$ and shear strength S_{12} as follows:

$$\begin{aligned} F_1 &= \frac{S_{11}(C) - S_{11}(T)}{S_{11}(T)S_{11}(C)} & F_2 &= \frac{S_{22}(C) - S_{22}(T)}{S_{22}(T)S_{22}(C)} \\ F_{11} &= \frac{1}{S_{11}(T)S_{11}(C)} & F_{22} &= \frac{1}{S_{22}(T)S_{22}(C)} & F_{66} &= \frac{1}{S_{12}^2} \end{aligned} \quad (28)$$

The interaction term F_{12} in Equation (26) is equal to $-\frac{1}{2}(F_{11}F_{22})^{1/2}$ [32,33]. It is important to realise that the Tsai-Wu failure theory does not provide information on fracture mechanisms, account for interfacial effects and is not directly translatable to laminated structures. Despite this, most current laminate analysis software packages contain both the Tsai-Hill and the Tsai-Wu failure criteria. Both failure criteria ignore the microstructure where failure mechanisms operate and are unable to account for sub-critical events leading to final failure. Failure analysis can be considered essentially a curve fitting exercise with many authors concurring that the Tsai-Wu tensor failure theory provides better agreement with experimental data than the other failure criteria in use today.

4. CONCLUSIONS

In conclusion, the area of modelling the degradation of PMCs due to fatigue and environmental effects is an on-going activity requiring further development. A large proportion of the work to-date has been directed towards statistical modelling of continuous aligned composites (ie fibre bundles) for static fatigue, and to a much less extent dynamic fatigue, with only a limited amount of effort having been spent on analysing and modelling subcritical damage and damage growth in laminated composites. Stress distributions in the fibres and matrix of unidirectional composites due to sub-critical damage need to be analysed and modelled. It is the intention within the CPD2 Project to develop predictive models based on the approaches identified in Section 2. These models will be used to predict stress corrosion failure of unbonded fibre bundles and impregnated strands. The models for unidirectional composites will be extended to simulate progressive degradation of ply properties in laminated composites and will include the effects of fatigue loading. Allowance for non-uniform moisture content and associated expansion effects (ie resin) swelling in addition to interfacial effects are to be included. A parallel test programme will be used to compare theory with experimental data in order to assess model reliability in predicting the residual strength and life expectancy of cross-ply laminates for a number of simple test conditions (eg tension and hot/wet).

The recommended approach when predicting damage formation during the fatigue loading of laminates is to make use of the more accurate stress transfer models that account for the presence of fatigue cracks growing in the transverse plies of cross-ply laminates. An area requiring further investigation is the development of a mechanics-based model that specifies quantitatively the nature of the parameter(s) that control the fatigue crack growth of ply cracks. The damage mechanics approach, while phenomenological in nature, has been shown to have some merit resulting from an examination of its properties in comparison to those which are predicted rigorously using a mechanism-based models for a special case.

Other life prediction methodologies considered in Section 3 of this report are generic and not specifically developed for composite materials. Evidence was available to suggest that efforts, although limited, were being made to incorporate life prediction methodologies into design procedures. Most of the predictive models considered were empirical (ie curve fitting to experimental data). Relating damage mechanisms, such as reduction in cross-linkage density of a polymer to strength reduction through hydrolysis is still in its infancy.

In contrast, statistical life prediction models are well developed and are regularly used by engineers for lifing materials. Again, the analysis is empirical. In practice, there is often considerable scatter in failure data, which increases with exposure time. Hence, consideration needs to be given to data variability. The data need to be subjected to rigorous statistical analysis to ensure meaningful results. Parametric studies are to be carried out within the CPD2 Project to determine the effect of temperature and the level of degrading agent with the degree of degradation, identifying any synergistic or superimposed effects between environmental and loading parameters. It is intended, where possible, to amalgamate these with the micro-mechanics equations that have been included in the NPL Composite Design and Analysis (CoDA) software.

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REFERENCES

1. Daniels, H.E., "The Statistical Theory of the Strength of Bundles of Threads", *Proceedings of the Royal Society of London*, 183, 1945, pp 405-435.
2. Coleman, B.D., "On the Strength of Classical Fibres and Fibre Bundles", *Journal of Mechanics and Physics of Solids*, 7, 1958, pp 60-70.
3. McCartney, L.N., "Statistics for the Strength of Fibre Bundles", *Metal Science*, 15, 1981, pp 471-474.
4. McCartney, L.N. and Smith, R.L., "Statistical Theory of the Strength of Fibre Bundles", *Journal of Applied Mechanics*, 50, 1983, pp 601-608.
5. Kelly, A. and McCartney, L.N., "Failure by Stress Corrosion of Bundles of Fibres", *Proceedings of the Royal Society of London*, A374, 1981, pp 475-489.
6. McCartney, L.N., "Time Dependent Strength of Large Bundles of Fibres Loaded in Corrosive Environments", *Fibre Science and Technology*, 16, 1995, pp 95-104.
Hui, S.Y., Phoenix, S.L., Ibnabdeljalil, M. and Smith, R.L., "An Exact Closed Form Solution for Fragmentation of Weibull Fibres in a Single Filament Composite with Applications to Fibre Reinforced Ceramics", *Journal of Mechanics and Physics of Solids*, 43, 1995, pp 1551-1585.
8. Curtin, W.A., "Theory of Fibre Fragmentation in a Single Filament", *Journal of Materials Science*, 26, 1991, pp 5239-5253.
9. Curtin, W.A. and Zhou, S.J., "Influence of Processing Damage on Performance of Fibre Reinforced Composites", *Journal of Mechanics and Physics of Solids*, 43, 1995, pp 343-363.
10. Curtis, P.T., "A Computer Model of the Tensile Failure Process in Unidirectional Fibre Composites", *Composite Science and Technology*, 27, 1986, pp 63-86.
11. McCartney, L.N., "Simulation of Fibre failure During the Tensile Loading of a Unidirectional Composite", Technical Report PREDICT/TR/NPL/930629/1.
12. McCartney, L.N., "Stress Transfer Mechanics for Systems of Transverse Isotropic Concentric Cylinders I: Basic Theory", 1998, in preparation.
13. McCartney, L.N., "A Recursive Technique of Calculating Stress Transfer in Multiple-Ply Cross-Ply Laminates Subject to Biaxial Loading I: Development of Model", NPL Report DMM(A)150, 1995.
14. McCartney, L.N., "Stress Transfer Mechanics for Systems of Transverse Isotropic Concentric Cylinders I: Basic Theory", 1998, in preparation.
15. McCartney, L.N., "Mechanics for the Growth of Bridged Cracks in Composite Materials: I - Basic Principles", *Journal of Composite Technology and Research*, 14, 1992, pp 133-146.
16. McCartney, L.N., "Mechanics for the Growth of Bridged Cracks in Composite Materials: II - Applications", *Journal of Composite Technology and Research*, 14, 1992, pp 147-154.
17. McCartney, L.N., "Stress Transfer Mechanics: Models that should be the Basis of Life Prediction Methodology", ASTM STP 1253 on "Life Prediction Methodology for Titanium Matrix Composites", 1996, pp 85-113.
18. Boniface, L., Ogin, S.L. and Smith, P.A., 'Strain Energy Release Rates and the Fatigue Growth of Matrix Cracks in Model Arrays in Composite Laminates', *Proceedings of the Royal Society of London*, A432, 1991, pp 427-444.
19. Beaumont, P.W.R., "Damage Accumulation", Chapter 5, "Damage Mechanics of Composite Materials", 1994, Editor Talreja, R., Elsevier Science B.V.
20. Talreja, R., "Damage Characterisation by Internal Variables", Chapter 2, and Allen, D.H., "Damage Evolution in Laminates", Chapter 3, "Damage Mechanics of Composite Materials", 1994, Editor Talreja, R., Elsevier Science B.V.
21. Ha, S.K. and Springer, G.S., "Nonlinear Mechanical Properties of a Thermoset Matrix Composite at Elevated Temperatures", *Journal of Composite Materials*, 23, 1989, pp 1130-1158.

22. Chamis, C.C. and Murthy, P.L.N., "Simplified Procedures for Designing Bonded Composite Joints", *Journal of Reinforced Plastics and Composites*, Volume 10, 1991, pp 29-41.
23. Collings, T.A., Harvey, R.J. and Dalziel, A.W., "The Use of Elevated Temperature in the Structural Testing of FRP Components for Simulating the Effects of Hot and Wet Environmental Exposure", *Composites*, 24(8), 1993, pp 625-634.
24. Ghorbel, I. and Spiteri, P., "Durability of Closed-End Pressurized GRP Pipes under Hygrothermal Conditions. Part I: Monotonic Tests", *Journal of Composite Materials*, 30(14), 1996, pp 1562-1580.
25. Levi, D.W., Wegman, R.F., Ross, M.C. and Garnis, E.A., "Use of Hot Water Aging for Estimating Lifetimes of Adhesive Bonds to Aluminium", *SAMPE Quarterly*, 7(3), 1976, pp 1-4.
26. Brown, R.P., Kockott, D., Trubiroha, P., Ketola, W. and Shorthouse, J., "A Review of Accelerated Durability Tests", VAMAS Report No.18, Edited by Brown, R.P., Versailles Project on Advanced Materials and Standards, 1995.
27. Roy, R., "A Primer on the Taguchi Method", *Society of Manufacturing Engineers*, 1990.
28. Talreja, R., "Fatigue of Composite Materials", Technomic Publishing Company, 1987.
29. Talreja, R., 'Statistical considerations', Chapter 11, "Fatigue of Composite Materials", Editor, Reifsnider, K.L., Elsevier Science Publishers, Amsterdam-Oxford-New York-Tokyo, 1990.
30. Schaff, J.R. and Davidson, B.D., "Life Prediction Methodology for Composite Structures. Part I - Constant Amplitude and Two-Stress Level Fatigue", *Journal of W.*, "Accelerated Testing: Statistical Models, Test Plans, and Data Analyses", John Wiley & Sons, 1990.
31. "Fatigue of Composite Materials", Editor Reifsnider, K.L., Volume 4, Composite Materials Series, Series Editor Pipes, R.B., Elsevier, 1991.
32. Tsai, S.W. and Wu, E.M., "A General Theory of Strength for Anisotropic Materials", *Journal of Composite Materials*, 5, 1971, pp 58-80.
33. Wu, Ru-Yu. and Stachurski, Z., "Evaluation of the Normal Stress Interaction Parameter in the Tensor Polynomial Strength Theory for Anisotropic Materials" *Journal of Composite Materials*, 18, 1984, pp 456-463.

APPENDIX A

PROBABILISTIC APPROACH TO FRACTURE IN BRITTLE FIBROUS MATERIALS
(Modified extract from an IACFA Report)

The objective of this Appendix is therefore to distil relevant features from the literature, and to indicate how the results may be applied to the prediction of the behaviour of fibres in composite materials.

The weakest-link failure hypothesis

The vast majority of the literature regards materials strength as a random variable without inquiring too deeply into the causes of material failure. The weakest link failure hypothesis resulted because of this approach. A chain breaks in tension when one or more of the links fail. The failure event is thus controlled by the weakest link (or links) in the chain. Any structure can be considered as an assembly of small elements in which the stress state is more or less uniform. The strength of such elements is a random variable. For materials in fibrous form the elements are considered to be aligned in series along the length of the fibres (ie forming a chain).

Most subsequent statistical treatments of failure adhere strictly to this hypothesis. Suppose that $F(s)$ is the cumulative strength distribution for one element of the system (ie $F(s)$ is the probability that the strength of the element is less than s). The quantity $1 - F(s)$ is, therefore, the probability that the strength of the element is greater than s . For an assembly of N elements arranged as a chain (as would be the case for a fibre), each with statistically independent strength characteristics and each subjected to a uniform stress s , the probability that the assembly survives is $[1 - F(s)]^N$ since each element must have strength greater than s . The probability that the application of the stress s causes the failure of at least one element is then given by:

$$P_f(\sigma) = 1 - [1 - F(\sigma)]^N \quad (A1)$$

Equation (A1) is a mathematical statement of the weakest link failure hypothesis.

Extreme value theory

When the number of elements N of a structure is large, it is sensible to seek asymptotic forms for the weakest link, hence Equation (A1) is valid as $N \rightarrow \infty$. The asymptotic theory of extreme values leads to just three limiting distributions as follows :

$$i) P_f(\sigma) = 1 - \exp\left[-N \exp\left(-\frac{\sigma - \sigma_0}{\sigma_c}\right)\right] \quad \sigma_c > 0, \quad |\sigma| < \infty \quad (A2)$$

$$ii) P_f(\sigma) = 1 - \exp\left[-N \left(\frac{\sigma - \sigma_0}{\sigma_c}\right)^m\right], \quad \sigma_c > 0, \quad m > 0, \quad \sigma \geq \sigma_0, \quad P_f(\sigma) = 0 \text{ if } \sigma < \sigma_0 \quad (A3)$$

$$iii) P_f(\sigma) = 1 - \exp\left[-N \left(\frac{\sigma_0 - \sigma}{\sigma_c}\right)^m\right], \quad \sigma_c > 0, \quad m > 0, \quad \sigma \leq \sigma_0, \quad P_f(\sigma) = 0 \text{ if } \sigma > \sigma_0 \quad (A4)$$

For applications to strength distributions under discussion, the limiting relation as shown in Equation (A4) can be discarded immediately, as it implies a physical absurdity that the system survives for all stresses $\sigma > \sigma_0$. The limiting form as shown in Equation (A2) can

also be discarded for strength applications as the variable σ can have negative values which is again absurd when considering strength distributions. The remaining distribution, Equation (A3), is the well known Weibull formula introduced for reasons of mathematical simplicity.

Accounting for non-uniform stress distributions

The results above are valid only for components comprising N equal elements, each of volume V_0 and each subjected to the same stress σ . Since the total volume $V = NV_0$, the result Equation (A3) can be expressed in the form :

$$P_F(\sigma, V) = 1 - \exp \left[-\frac{V}{V_0} \left(\frac{\sigma - \sigma_0}{\sigma_c} \right)^m \right] \quad \sigma \geq \sigma_0$$

indicating a dependence of the failure probability on component size. It should be noted that $P_F(\sigma, V) \rightarrow 1$ for all $\sigma > \sigma_0$ when $V \rightarrow \infty$. When the stress distribution in the component is non-uniform each volume element V_0 may be subjected to a different stress (ie maximum principal stress) and this must be taken into account when estimating the failure probability. It is easy to show that for non-uniform stress distributions that result from Equation (A1) must be replaced by :

$$P_F = 1 - \prod_{i=1}^N [1 - F(\sigma_i, V_0)]$$

where the component comprises N elements of volume V_0 such that σ_i is the stress in the i^{th} element.

As $V_0 \rightarrow 0$, $F(\sigma_i, V_0) \approx V_0 f(\sigma_i)$ and it is easily shown that when applied to a fibre having a non-uniform stress distribution along its length :

$$P_F = 1 - \exp \left\{ - \int_V f(\sigma(z)) dV \right\}$$

where $\sigma(z)$ denotes the stress distribution which varies along the length of a fibre in the z -direction. This extremely important result shows how the failure probability of a region of fibre having a non-uniform stress distribution may be calculated. For most engineering applications it is assumed that :

$$f(\sigma) = \left(\frac{\sigma - \sigma_0}{\sigma_c} \right)^m, \quad \text{if } \sigma \geq \sigma_0, \quad f(\sigma) = 0, \quad \text{otherwise.}$$

If the stress distribution is uniform then the substitution of Equation (A8) into Equation (A7) leads to Equation (A3) derived from the asymptotic theory of extreme values. Contrary to popular belief the representation as given by Equation (A8) is not merely an empirical relation introduced to simplify the mathematics. The relationship of equation (A8) to extreme value theory probably explains why this simple formula has been so successful in representing the distribution of strength data in a wide variety of brittle materials, including materials in fibrous form.

It should be noted that if the defects causing failure are distributed only on the external surface of the component then the volume integral appearing in Equation (A7) is replaced by a surface integral. If defects are also distributed in the bulk then the sum of a surface integral and a volume integral appears in the Equation (A7).

Description in terms of defect distribution

For many practical applications using glass and ceramics fibres, the defects, particularly those on the surface, can grow during service because of environmental effects. The formulation of failure probability outlined above is not suitable when trying to take proper account of defect growth and nucleation. It is much more sensible to regard defect size as the random variable instead of the strength of a suitably small sample of material. When formulated in this way two different approaches have emerged. The first approach assumes that each volume element into which the component is divided contains just one defect. The size of the defect is a random variable. This model is valid only for uniform distributions of defects. The second approach is only valid for randomly distributed defects having random size.

It can be shown that the Equation (A7) results when a statistical theory of brittle fracture considers defect size rather than strength as a random variable. The result corresponding to Equation (A7) is :

$$P_F = 1 - \exp \int_V \left\{ \int_{X(\sigma(z))} q(x) dx \right\} dV \quad (A9)$$

where $q(x) dx$ is the expected number of defects in a unit volume of material having size in the range $x \rightarrow x + dx$. The function $X(\sigma)$ is obtained from a fracture criterion of the form :

$$x_c = X(\sigma) \quad (A10)$$

where the critical defect size x_c is associated with a local maximum principal stress σ . For brittle materials the Griffith fracture criterion is appropriate so that $X(s)$ is inversely proportional to the square of the stress σ ; the constant of proportionality depending upon the fracture toughness of the material.

For a tensile test specimen, having volume V containing a distribution of defects whose sizes are time dependent, it can be shown that the probability that the specimen will have failed at time t is :

$$P_F(t) = 1 - \exp \left[-V \int_{X_0(t)} q(x) dx \right] \quad (A11)$$

where $X_0(t)$ is the initial defect size requiring time t to grow to the critical size $X(\sigma)$ when the fixed stress σ is applied. It then follows that the hazard function $H(t)$ associated with the time dependent cumulative failure probability can be interpreted in terms of the defect distribution as follows :

$$\int_0^t H(\tau) d\tau \equiv V \int_{X_0(t)} q(x) dx \quad (A12)$$

APPENDIX B

FATIGUE CRACK GROWTH IN CROSS-PLY LAMINATES

Fatigue crack growth law for ply cracks

When carrying out fatigue life calculations for monolithic unreinforced materials three essential ingredients are needed. The first is a crack growth law that is usually assumed to have the form:

$$\frac{dc}{dN} = A(\Delta K)^n$$

where A and n are empirical constants, N is the number of cycles and K is the stress range intensity factor of the form:

$$\Delta K = \Delta \sigma \sqrt{\pi c} Y$$

where $\Delta \sigma$ is the applied stress range of uniform amplitude fatigue loading, $2c$ denotes the crack length and Y is a factor that takes geometrical effects into account. The second ingredient, for loading cases where small scale yielding conditions prevail, is a knowledge of the fracture toughness of the material K_{Ic} at which unstable fracture occurs. The third ingredient is a knowledge of initial defect sizes that can be used, together with the instability condition and crack growth law, to predict fatigue lives.

For cracks in the 90° plies of cross-ply laminates, a fatigue crack growth law of the form:

$$\frac{dc}{dN} = A(\Delta K^{(90)})^n$$

is assumed, where $\Delta K^{(90)}$ is the effective stress range intensity factor for the fatigue crack growth of a ply crack in the 90° ply of the laminate. This parameter, valid only for small scale yielding conditions, will be influenced by the 0° plies that bridge the ply crack and by the fact that only the 90° plies are cracking during fatigue loading. It is important that a rational approach is taken to defining the effective stress range intensity factor $\Delta K^{(90)}$. As crack instability needs to be characterised it is useful to consider the fracture of 0° - 90° - 0° cross-ply laminates under monotonic loading conditions in order to define the key fracture mechanics parameters on which the definition of $\Delta K^{(90)}$ can be based.

Fracture criterion for monotonic loading

Consider a large rectangular anisotropic plate which is made of a cross-ply laminate material. The axes of material symmetry are parallel to the edges of the plate. On introducing a rectangular set of co-ordinate axes (x , y) such that the x and y axes denote the axial and transverse directions respectively, the in-plane stress/strain/temperature relations for the laminate are of the form [1]:

$$\epsilon_{xx} = a_{11}\sigma_{xx} + a_{12}\sigma_{yy} + \alpha_T \Delta T \quad (B4)$$

$$\epsilon_{yy} = a_{12}\sigma_{xx} + a_{22}\sigma_{yy} + \alpha_A \Delta T \quad (B5)$$

$$2\epsilon_{xy} = a_{66}\sigma_{xy} \quad (B6)$$

The parameters a_{11} , a_{12} , a_{22} and a_{66} are elastic constants for a transverse isotropic solid having the following values,

i) for generalised plane stress conditions (thin sheets) :

$$a_{11} = \frac{1}{\epsilon} \quad a_{22} = \frac{1}{\epsilon}$$

$$a_{12} = -\frac{\nu_A}{E_A}, \quad a_{66} = \frac{1}{\epsilon'}$$

ii) for plane strain conditions (thick plates) :

$$a_{11} = \frac{1 - \nu_T^2}{E_T} \quad a_{22} = \frac{1}{\epsilon} \left(1 - \nu_A^2 \frac{E_T}{E_A} \right)$$

$$a_{12} = -\frac{\nu_A(1 + \nu_T)}{E_A} \quad a_{66} = \frac{1}{\epsilon'}$$

It has been shown [1] that the equation, valid for non-linear $p - v$ relations, determining the stress for the instability of long cracks in brittle matrix composites subject to monotonic loading is given by the simple relation:

$$\sigma \Delta v_0 - \int_0^{\Delta v_0} p(\Delta v) d(\Delta v) = 2\gamma \quad (B9)$$

where 2γ is the effective fracture energy for ply crack growth and Δv_0 is the effective crack opening for a fully developed ply crack in a laminate subject to an applied stress σ .

It has also been shown [1], that the fracture criterion for the unstable growth of bridged ply cracks has the following simple form:

$$K^2 = \frac{2\gamma E_A}{\Omega} = K_{Ic}^2 \quad (B10)$$

where 2γ is the effective fracture energy for a composite and E_A is the axial Young's modulus of the laminate. The dimensionless parameter Ω is defined [1] by:

$$\Omega^2 = \frac{E_A^2 a_{22}}{4} \left[2\sqrt{a_{11}a_{22}} + 2a_{12} + a_{66} \right] \quad (B11)$$

The parameter K_{Ic} given in Equation (B10) is the effective mode I fracture toughness of the composite. The parameter is effective in the sense that its role is defined to be identical to that for unbridged cracks while its implementation takes full account of the fact that the ply crack is bridged by the 0° plies, and cracking is allowed only in the 90° ply. Equation (B10) corresponds to the well known Griffith fracture criterion for unbridged cracks when the material is isotropic so that $\Omega = 1$ (generalised plane stress) or $\Omega = 1 - \nu^2$ (plane strain), and when $E_A = E$, where ν is Poisson's ratio and E is Young's modulus for an isotropic solid. Equation (10) relates K_{Ic} to the effective fracture energy 2γ . For ply cracks in the 90° plies of a 0° - 90° - 0° cross-ply laminate $\gamma = (h^{(90)}/h) \gamma^{(90)}$ where $2h$ is the laminate thickness, $2h^{(90)}$ is the total thickness of the 90° ply and where $2\gamma^{(90)}$ is the fracture energy for cracking parallel to the fibres in a monolithic sample of 90° ply material.

When applied to monolithic 90° ply material, Equation (B10) is written:

$$[K_{Ic}^{(90)}]^2 = \frac{2\gamma^{(90)} E_T^{(90)}}{\Omega^{(90)}}$$

where the superscript $^{(90)}$ denotes that the quantity is associated with the property of the 90° ply. It follows from Equations (B10) and (B12) and the relationship between γ and $\gamma^{(90)}$ that the effective fracture toughness for a ply crack in a laminate may be calculated using the relation:

$$K_{Ic}^2 = \frac{h^{(90)}}{h} \frac{\Omega^{(90)}}{\Omega} \frac{E_A}{E_T^{(90)}} [K_{Ic}^{(90)}]^2$$

Equation (B13) can be used as the basis for the following definition of the effective stress intensity factor $K^{(90)}$:

$$[K^{(90)}]^2 = \frac{h}{h^{(90)}} \frac{\Omega}{\Omega^{(90)}} \frac{E_T^{(90)}}{E_A} K^2$$

Consider the case of linear $p - \Delta v$ relationships which can be expressed [1,2] in the general form:

$$p = \Lambda \Delta v + \sigma_i \quad (B15)$$

where $2c$ is the length of the ply crack and σ_i is the value of the applied stress for which the axial stress in the 90° plies of an undamaged composite would be zero so that the ply crack is just closed. The parameter Λ depends upon the ply properties and their thicknesses, and may be derived from a suitable stress transfer model [3]. As Δv_0 is the crack opening displacement when the applied stress is σ , it follows from (B15) that:

$$\sigma = \Lambda \Delta v_0 + \sigma_i$$

On substituting Equation (B15) into Equation (B9), and making use of Equation (B16), it can be shown that the instability condition for long ply cracks is given by:

$$\frac{(\sigma - \sigma_i)^2}{2\Lambda} = 2\gamma \quad (B17)$$

which is independent of crack length.

Stress intensity factors for long ply cracks

As Equation (B17) is equivalent to Equation (B10) for the case of long cracks, it follows that the effective stress intensity factor for a long ply crack is given by the simple relation:

$$K = \sqrt{\frac{E_A}{2\Omega\Lambda}} (\sigma - \sigma_i) \quad (B18)$$

It is clear that the stress intensity factor for long ply cracks is independent of crack length.

As fatigue crack growth can be thought of as resulting from energy dissipation (or energy absorption) in the plastic zones (see for example [4]), it is reasonable to use energy concepts to define the effective stress range intensity factor ΔK . Equation (B18), which was derived for brittle matrix composites by energy balance arguments, is the basis for the following definition of the effective stress range intensity factor:

$$\Delta K = \sqrt{\frac{E_A}{2\Omega\Lambda}} \Delta\sigma \quad (B19)$$

where $\Delta\sigma$ is the stress range for uniform amplitude stress cycling. The definition given in Equation (B19) is consistent with the following relationship [5]:

$$\Delta K(\Delta\sigma, \Delta T) = 2 K(\Delta\sigma / 2, 0) \quad (B20)$$

On using Equation (B20) it follows from Equation (B14) that:

$$[\Delta K^{(90)}]^2 = \frac{h}{h^{(90)}} \frac{\Omega}{\Omega^{(90)}} \frac{E_T^{(90)}}{E_A} \Delta K^2 \quad (B21)$$

The effects of accumulating fatigue damage

For structural panels subjected to fatigue loading at modest stress levels, a series of parallel cracks can be encountered leading to a degradation of the mechanical properties of the panel. The final objective of this Appendix is to indicate how micromechanical models may be used to predict the change of properties such as moduli, Poisson's ratios and thermal expansion coefficients, as a function of crack density. Such calculations usually make the assumption that ply cracks are uniformly spaced, but it has been shown [6] that transverse cracking in cross-ply laminates having non-uniform crack spacings can be treated.

Consider a composite panel containing an array of $n + 1$ parallel ply cracks labelled by the parameter $i = 1 \dots n + 1$, such that $2l_i$ denotes the separation of the i^{th} and $i+1^{\text{th}}$ cracks and $2l$ denotes the distance between the first and $(n+1)^{\text{th}}$ cracks. From the micromechanical models of laminated composites, it is possible to calculate the effective thermo-elastic constants of each segment of the panel lying between neighbouring cracks. Thus for that part of the panel lying between the i^{th} and $i+1^{\text{th}}$ cracks the following thermo-elastic constants can be calculated:

$$E_A(l_i), E_T(l_i), \nu_A(l_i), \alpha_A(l_i), \alpha_T(l_i),$$

where E_A and E_T are the axial and transverse Young's moduli, ν_A is the axial Poisson's ratio, and where α_A and α_T are the axial and transverse thermal expansion coefficients.

The effective thermo-elastic constants for a cracked panel containing n parallel ply cracks that may be spaced non-uniformly are denoted by:

$$E_A(n), E_T(n), \nu_A(n), \alpha_A(n), \alpha_T(n)$$

where the argument n is used to indicate average properties when n randomly spaced cracks are present. By considering the whole assembly of n elements defined by the $n + 1$ ply cracks it can be shown [6] that the effective thermo-elastic constants for the panel as a whole are given by the following addition formulae:

$$\begin{aligned} E_T(n) &= \sum_{i=1}^n \frac{l_i}{l} E_T(l_i) \\ E_A(n) &= \sum_{i=1}^n \frac{\lambda_i}{E_A(l_i)} \\ \frac{\nu_A(n)}{E_A(n)} &= \sum_{i=1}^n \lambda_i \frac{\nu_A(l_i)}{E_A(l_i)} \\ \alpha_A(n) &= \sum_{i=1}^n \lambda_i \alpha_A(l_i) \\ \alpha_T(n) &= \sum_{i=1}^n \lambda_i \alpha_T(l_i) \end{aligned} \tag{B23}$$

where the parameters λ_i , $i = 1 \dots n$ are defined by:

$$\lambda_i = \frac{l_i}{l} \frac{E_T(l_i)}{E_T(n)}, \quad \text{such that} \quad \sum_{i=1}^n \lambda_i = 1$$

REFERENCES FOR APPENDIX B

1. McCartney, L.N., "Mechanics for the Growth of Bridged Cracks in Composite Materials: I - Basic Principles", Journal of Composite Technology and Research, 14, (1992), pp 133-146.
2. McCartney, L.N., "Mechanics for the Growth of Bridged Cracks in Composite Materials: II - Applications", Journal of Composite Technology and Research, 14, (1992), 147-154.
3. McCartney, L.N., "Theory of Stress Transfer in a 0° - 90° - 0° Cross-Ply Laminate Containing a Parallel Array of Transverse Cracks", Journal of Mechanics and Physics of Solids, 40, (1992), pp. 27-68.
4. Lindley, T.C. and McCartney, L.C., "Mechanics and Mechanisms of Fatigue Crack Growth", Chapter 5 in "Developments in Fracture Mechanics, 2nd Edition", Chell, G.G., Applied Science Publishers, London - New Jersey, 1981.
5. McCartney, L.N., "Stress Transfer Mechanics: Models that should be the Basis of Life Prediction Methodology", ASTM STP 1253 on "Life Prediction Methodology for Titanium Matrix Composites", 1996, pp.85-113.
6. McCartney, L.N., "The Prediction of Non-Uniform Cracking in Biaxially Loaded Cross-Ply Laminates", NPL Report DMM(A)142, 1994.