

Testing of IEC 61220 criteria using the NPL-developed hydrophone-amplifier model

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ABSTRACT

The curve fitting method outlined in Annex A of IEC 61220 describes a simplified procedure intended to correct for the effects of finite hydrophone bandwidth and active element size when determining the peak-positive acoustic pressure in an ultrasonic field. In this report, this method has been tested using four types of ultrasonic measurement systems, comprising resonant, band-limited and wide-band systems. The reference acoustic pressure waveforms used have been theoretically generated and include an asymmetric distorted pressure pulse with 3.5 MHz fundamental frequency, with and without added band-limited white noise, and distorted sinusoidal waveforms, also of 3.5 MHz fundamental frequency. The waveforms are convolved with the impulse responses of the measurement systems resulting in the output voltage. The correction procedure outlined in Annex A of IEC 61220 is then applied to provide an estimate of the peak-positive pressure, which is then compared with the peak-positive pressure of the original acoustic waveform. This report shows that this correction procedure is no substitute for knowledge of the transfer characteristics of the measurement system when it comes to providing an estimate of the peak-positive acoustic pressure, although an exponential curve fit can provide an estimate of the peak-positive acoustic pressure within $\pm 5\%$ of its true value, provided the system is of sufficient bandwidth to encompass the dominant frequency content of the original pressure waveform and does not cause excessive ringing in the hydrophone voltage waveform.

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0. PROJECT DETAILS

This report carried out by the National Physical Laboratory (NPL) for the DTI National Measurement System Policy Unit describes the work carried out as part of Milestone 4.4.2.3 of deliverable AL044020: **Complete implementation and dissemination of hydrophone-amplifier model**. This milestone aims to **Complete testing of IEC 61220 criteria using model, and report**.

1. INTRODUCTION

Due to effects of finite bandwidth, a wide-band voltage waveform measured by a given hydrophone-amplifier combination will require corrections so that an estimate of the peak-positive pressure is obtained. If the transfer characteristics of the measurement system are known, in magnitude and phase, it is possible to remove their effects through deconvolution of the measured voltage waveform, and the true pressure waveform may be estimated. If these transfer characteristics are unknown, and only an estimate of the peak pressure is required, Annex A of IEC 61220: Ultrasonics – Fields – Guidance for the measurements and characterisation of ultrasonic equipment using hydrophones in the frequency range 0.5 MHz to 15 MHz [1], describes a simplified procedure to correct measurements made using hydrophones of finite bandwidth.

Some points made by Annex A of the Technical Report [1] are listed below:

- The simplified procedures are based on situations where the hydrophone-amplifier measurement system resulted in only a small amount of distortion – using a 9 μm film thickness membrane hydrophone (probably a coplanar device, as it claims that the overall measurement bandwidth of the system was 75 MHz);
- The report claims that the procedures are also applicable to situations in which the distortion is more appreciable;
- The effect of the variation of sensitivity with frequency of the measurement system is not considered within the analysis;
- It is not intended as a rigorous method, but the report states that it can be quickly applied and provides reliable results.

The aim of the work described in this report is to investigate the accuracy of applying the procedures described in the report.

A literature survey was carried out and no publications that reviewed [1] were encountered.

2. METHODOLOGY

2.1 CHOICE OF REFERENCE WAVEFORM

In order to investigate the correction procedure in [1], a reference pressure waveform was required. A simulated distorted waveform was used, based on a mathematical representation of a saw-tooth waveform [2].

An asymmetric distorted pressure pulse may be modelled as [2]:

$$p_D = P_F \left(\sum_{k=1}^{\infty} \frac{\sin[k\omega(t-\tau)+\varphi]}{k} \right) \exp[-Q^2(t-t_1)^2] \quad (1)$$

Where:

- ω is the angular frequency;
- φ is a phase shift added to all the harmonics generated and has been set to $\pi/4$;
- τ is a time shift chosen to normalise the initial conditions to $p|_{t=0} = 0$ and $dp/dt|_{t=0} < 0$, and has been set to $0.65\pi/\omega_0$, [2] where $\omega_0 = 2\pi f_0$, where f_0 corresponds to the fundamental frequency of the pulse;
- t_1 locates the centre of the pulse and has been set to $2.7/f_0$;
- P_F is the pressure amplitude of the fundamental frequency and is set to 0.84 MPa;
- Q fixes the width of the pulse $W = 2/Q$; W is set to 1 μs ;
- k varies between 1 and 1000, so that the first 1000 harmonics are accounted for;
- t is time and varies between 0 and $6T$, where $T = 1/f_0$.

Note that the pulse width W is defined as the time interval between the $1/e$ points of the Gaussian envelope.

The pulse is shown as a function of time in Figure 2.1.

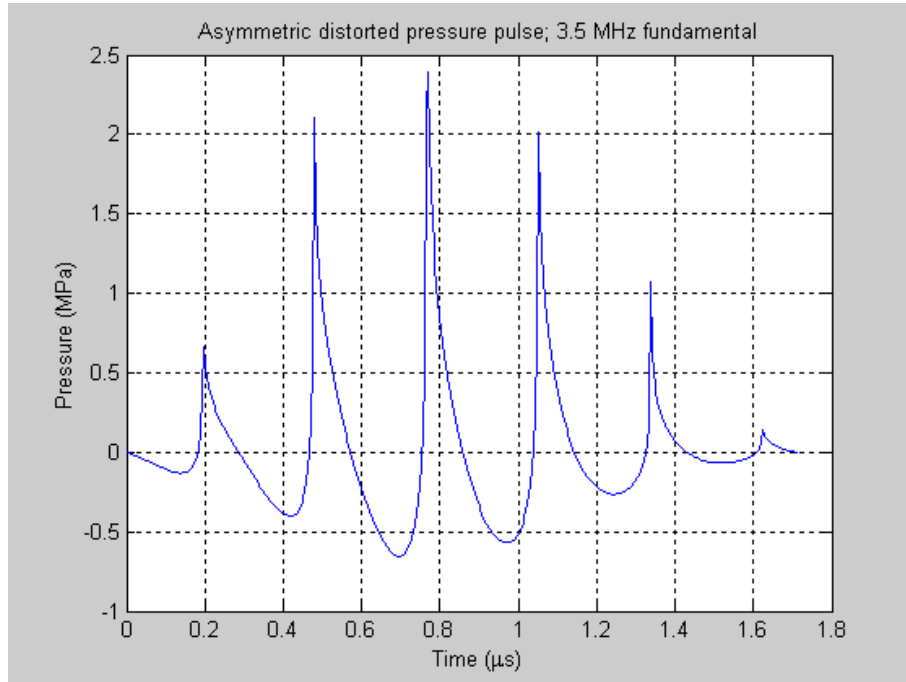


Figure 2.1 Asymmetric distorted pressure pulse with 3.5 MHz fundamental frequency to be used as reference acoustic pressure.

Note that, in order to have a more accurate representation of experimental conditions, the waveform will not be represented as a continuous function of time. The pulse will be sampled every 4 ns, starting at $t = 0$, hence giving a sampling frequency of 250 MHz and a Nyquist frequency of 125 MHz.

2.2 HYDROPHONE-AMPLIFIER SYSTEMS

Four types of hydrophone-amplifier combinations were investigated, all comprising bilaminar membrane hydrophones:

- 25 μm *pvd*f thickness, 0.5 mm element diameter membrane hydrophone with matched amplifier, producing a combined response of ± 1 dB between 2 MHz and 16 MHz.
- 15 μm *pvd*f thickness, 0.4 mm element diameter membrane hydrophone with amplifier. It should be noted that the amplifier loading correction has not been accounted for here.
- 9 μm *pvd*f thickness, 0.5 mm element diameter membrane hydrophone with amplifier.
- 5 μm *pvd*f thickness, 0.5 mm element diameter theoretical membrane hydrophone, used to mimic a wide bandwidth system.

2.2.1 25 μm *pvd*f thickness, 0.5 mm element diameter membrane hydrophone with matched amplifier 55641X (gain $\times 1$)

The hydrophone and amplifier considered here are NPL devices IP001 and 55641X (with voltage gain set to 1), respectively. In order to simulate the transfer characteristics of the hydrophone, the following steps are taken:

1. The hydrophone is calibrated using the NPL multiple frequency method [3], in this case between 1 MHz and 20 MHz, in steps of 1 MHz.
2. In order to obtain magnitude and phase information over frequencies of interest, the NPL membrane hydrophone model is used [4]. Since the transfer characteristics of membrane hydrophones have a tendency to vary from one hydrophone to another, it is difficult to get the model in [4] to accurately fit the measured magnitude characteristics. Instead, the calibration data is used to form the objective function ψ .

$$\psi(d_{33}, R_{leg}, w_{leg}) = \sum_{i=1}^N \left| |V_{pre}(f_i)| - |V_{cal}(f_i)| \right|^2 \quad (2)$$

Where:

- d_{33} is the piezoelectric coefficient;
- R_{leg} is the hydrophone leg resistance;
- w_{leg} is the hydrophone leg width;
- V_{pre} is the hydrophone sensitivity predicted by the model in [4] for specific values of d_{33} , R_{leg} and w_{leg} ;
- V_{cal} is the calibration data;
- f_i correspond to the calibration frequencies;

3. The objective function is minimised and optimal values for d_{33} , R_{leg} and w_{leg} are obtained, which are then fed back into the hydrophone model, so that magnitude and phase data may be obtained over the desired frequency range.

The input parameters fed into the software before minimisation of the objective function are displayed in Table 2.1.

c : velocity of sound (m s^{-1})	water: 1482.36; gold: 3240; chromium: 6608; <i>pvd</i> : 2300; glue: 1600
ρ : material density (kg m^{-3})	water: 998.2; gold: 19281; chromium: 7194; <i>pvd</i> : 1780; glue 1000
t : thickness of layer (μm)	gold: 0.1; chromium: 0.05; <i>pvd</i> (poled+depoled): 23.5; depoled <i>pvd</i> region: 0 per membrane; glue: 0.5
α : attenuation coefficient (neper $\text{m}^{-1} \text{MHz}^{-1}$)	<i>pvd</i> : 111; glue: 104
d_{33} : piezoelectric coefficient (pC N^{-1})	14
ϵ_0 : dielectric constant (pF m^{-1})	8.85
$ \epsilon_r $: modulus of relative permittivity of <i>pvd</i> (as a function of frequency f in Hz)	$-2.82 \log_{10}(f) + 26.7$
$\tan \delta$: loss tangent of relative permittivity of <i>pvd</i> (as a function of frequency f in MHz)	$0.22 + 0.07 \log_{10}(f) - 0.055 \log_{10}(f)^2$
r : element radius (mm)	0.25
w_{leg} : leg width (mm)	0.2
L_{leg} : leg length (mm)	56.5
R_{leg} : leg resistance (Ω)	142
c_{cable} : velocity of wave propagation in cable (m s^{-1})	2×10^8
L_{cable} : length of cable (cm)	71
$Z_{0,\text{cable}}$: cable characteristic impedance (Ω)	75

Table 2.1: Model input parameters for prediction of IP001 transfer characteristics.

Note that the *pvd* thickness has been adjusted to 23.5 μm prior to the minimisation process, since this initially gave a better fit to the calibration data, in terms of the resonance position.

After convergence of the minimisation algorithm, the following results are obtained:

- $d_{33} = 7.17 \cdot 10^{-12} \text{ pC N}^{-1}$
- $R_{\text{leg}} = 751 \Omega$
- $w_{\text{leg}} = 6.12 \mu\text{m}$

Note that these results, in particular the value for the leg width, are not representative of what is expected for this type of hydrophone. The measured open-circuit sensitivity did not include the hydrophone resonance, and this has an effect on the optimisation procedure. Nevertheless, the transfer characteristics depicted in Figure 2.2 are representative of this type of measurement system, and for the purpose of this analysis, this is what matters.

A comparison of the optimised sensitivity and the calibration data are given in Figure 2.2, together with an estimate of the phase.

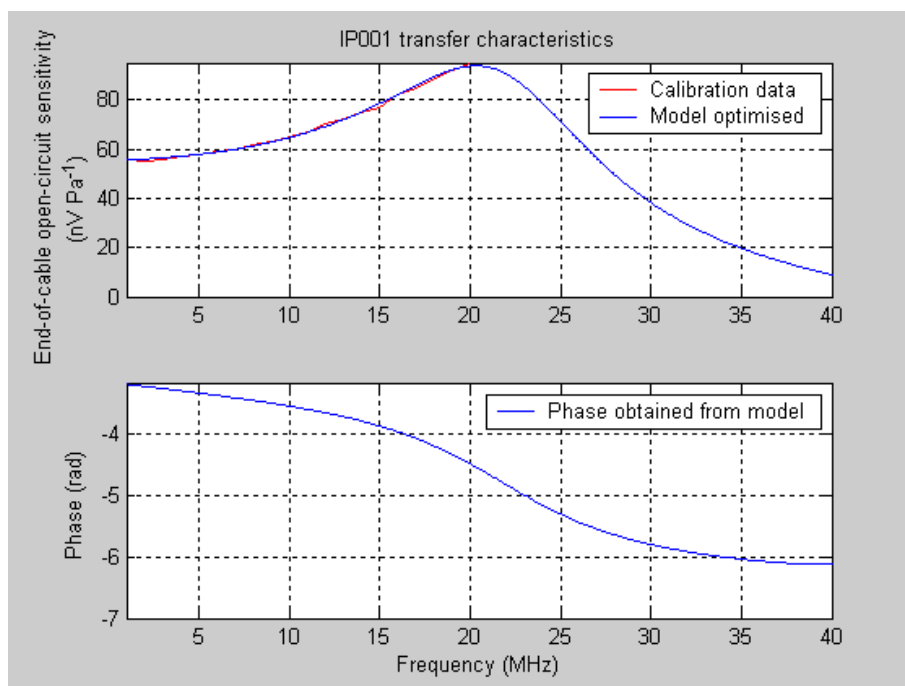


Figure 2.2: IP001 transfer characteristics – Calibration data and optimised sensitivity.

The amplifier transfer characteristics are then measured, as well as the loading correction associated with hydrophone IP001. This is done between D.C. and 80 MHz. The transfer characteristics of the complete hydrophone-amplifier system may then be obtained, as shown in Figure 2.3.

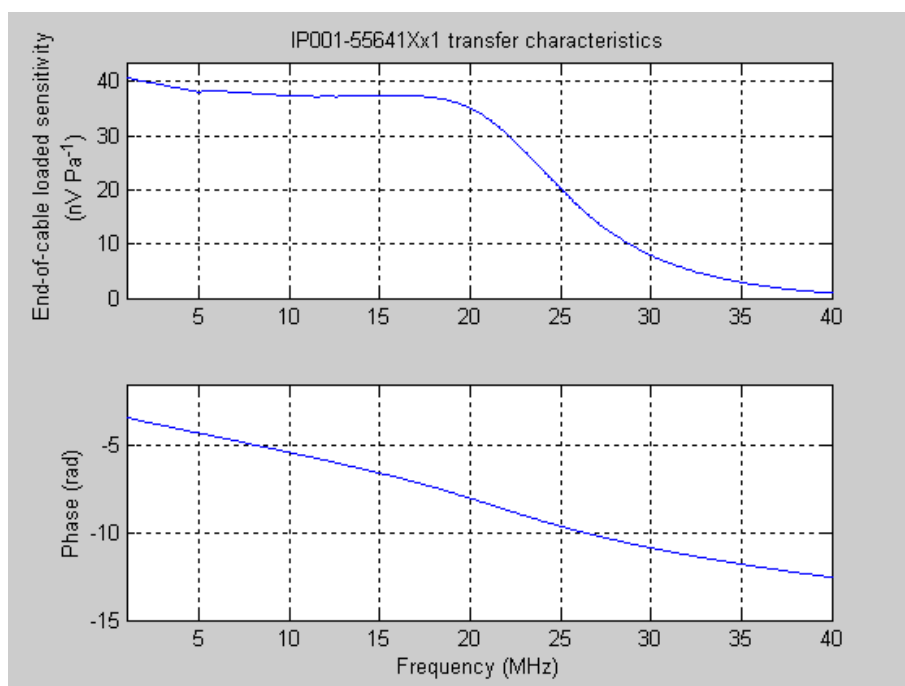


Figure 2.3: IP001- 55641Xx1 transfer characteristics.

This hydrophone-amplifier combination behaves as a band-limited system, rather than a resonant system.

2.2.2 15 μm *pvd*f thickness, 0.4 mm element diameter membrane hydrophone with amplifier

This type of hydrophone corresponds to the new generation of Precision Acoustics membrane hydrophones. [5]

The transfer characteristics have been obtained exclusively through use of the hydrophone model described in [4], by using the input parameters listed in Table 2.2.

c : velocity of sound (m s^{-1})	water: 1482.36; gold: 3240; chromium: 6608; <i>pvd</i> f: 2300; glue: 1600
ρ : material density (kg m^{-3})	water: 998.2; gold: 19281; chromium: 7194; <i>pvd</i> f: 1780; glue 1000
t : thickness of layer (μm)	gold: 0.1; chromium: 0.05; <i>pvd</i> f (poled+depoled): 15; depoled <i>pvd</i> f region: 0.7 per membrane; glue: 0.5
α : attenuation coefficient (neper $\text{m}^{-1} \text{MHz}^{-1}$)	<i>pvd</i> f: 111; glue: 104
d_{33} : piezoelectric coefficient (pC N^{-1})	14
ϵ_0 : dielectric constant (pF m^{-1})	8.85
$ \epsilon_r $: modulus of relative permittivity of <i>pvd</i> f (as a function of frequency f in Hz)	$-2.82 \log_{10}(f) + 26.7$
$\tan \delta$: loss tangent of relative permittivity of <i>pvd</i> f (as a function of frequency f in MHz)	$0.22 + 0.07 \log_{10}(f) - 0.055 \log_{10}(f)^2$
r : element radius (mm)	0.2
w_{leg} : leg width (mm)	0.3
L_{leg} : leg length (mm)	50
R_{leg} : leg resistance (Ω)	22
c_{cable} : velocity of wave propagation in cable (m s^{-1})	2×10^8
L_{cable} : length of cable (cm)	2
$Z_{0,\text{cable}}$: cable characteristic impedance (Ω)	75

Table 2.2 Model input parameters for prediction of Precision Acoustics membrane hydrophone transfer characteristics.

The predicted transfer characteristics are shown in Figure 2.4.

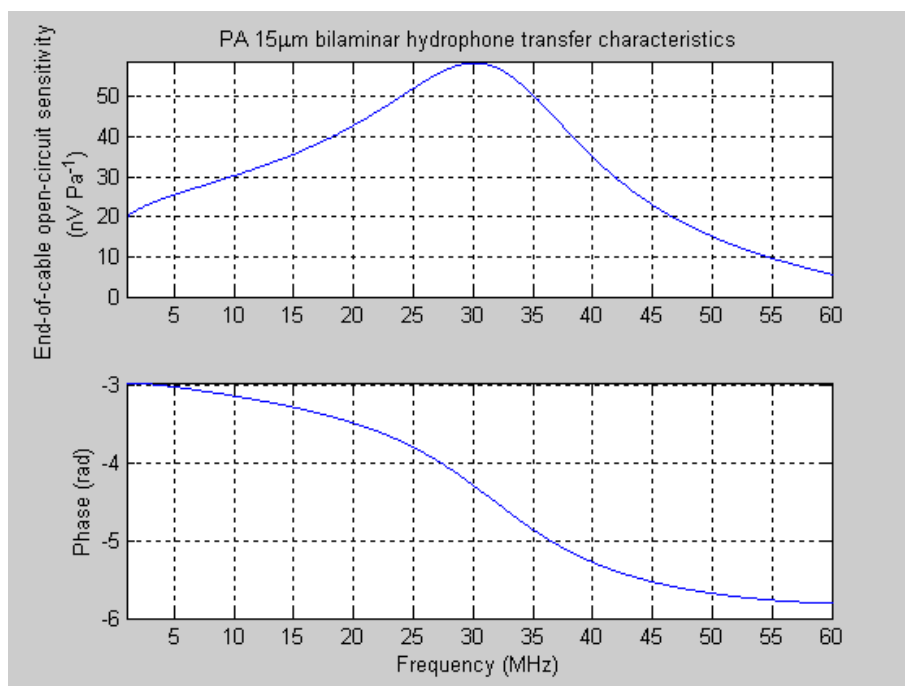


Figure 2.4: Precision Acoustics 15 μm *pvdF* film thickness, 0.4 mm element diameter transfer characteristics – theoretical.

This type of hydrophone contains a built-in amplifier, the response of which was measured between D.C. and 60 MHz. The hydrophone-amplifier transfer characteristics are shown in Figure 2.5. Note that loading correction was not accounted for here (the amplifier is designed to take high impedance load). The amplifier reference is w235056.

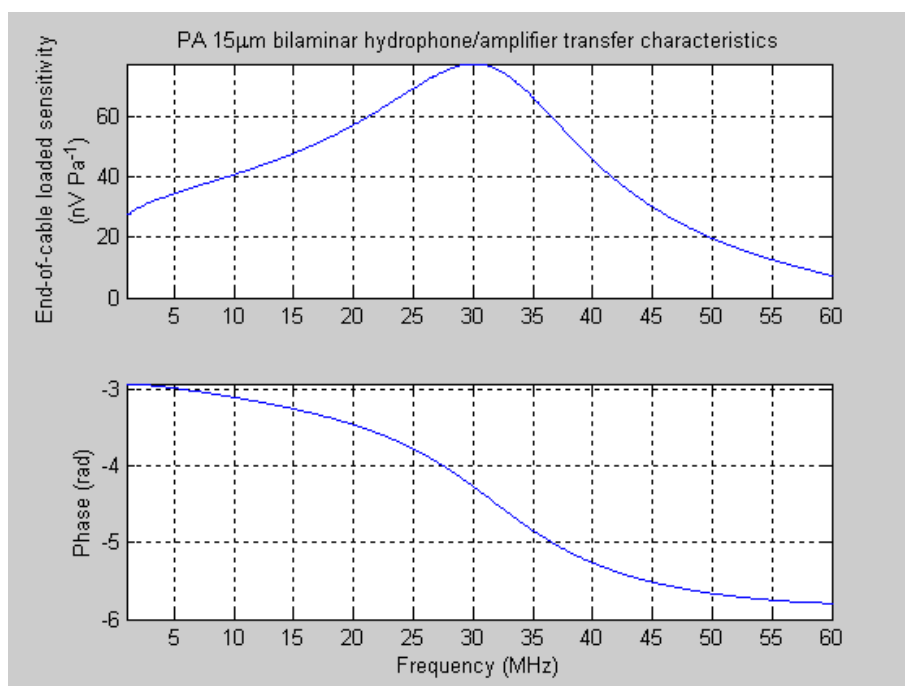


Figure 2.5: Precision Acoustics hydrophone-amplifier transfer characteristics.

2.2.3 9 μm *pvd*f thickness, 0.5 mm element diameter membrane hydrophone with amplifier

The hydrophone and amplifier considered here are IP016 and 55391001, respectively. In order to simulate the transfer characteristics of the hydrophone, the same steps as in Section 2.2.1 are taken.

c : velocity of sound (m s^{-1})	water: 1482.36; gold: 3240; chromium: 6608; <i>pvd</i> f: 2300; glue: 1600
ρ : material density (kg m^{-3})	water: 998.2; gold: 19281; chromium: 7194; <i>pvd</i> f: 1780; glue 1000
t : thickness of layer (μm)	gold: 0.1; chromium: 0.05; <i>pvd</i> f (poled+depoled): 9; depoled <i>pvd</i> f region: 0 per membrane; glue: 0.5
α : attenuation coefficient ($\text{neper m}^{-1} \text{MHz}^{-1}$)	<i>pvd</i> f: 111; glue: 104
d_{33} : piezoelectric coefficient (pC N^{-1})	14
ϵ_0 : dielectric constant (pF m^{-1})	8.85
$ \epsilon_r $: modulus of relative permittivity of <i>pvd</i> f (as a function of frequency f in Hz)	$-2.82 \log_{10}(f) + 26.7$
$\tan \delta$: loss tangent of relative permittivity of <i>pvd</i> f (as a function of frequency f in MHz)	$0.22 + 0.07 \log_{10}(f) - 0.055 \log_{10}(f)^2$
r : element radius (mm)	0.25
w_{leg} : leg width (mm)	0.1
L_{leg} : leg length (mm)	56.5
R_{leg} : leg resistance (Ω)	142
c_{cable} : velocity of wave propagation in cable (m s^{-1})	2×10^8
L_{cable} : length of cable (cm)	15
$Z_{0,\text{cable}}$: cable characteristic impedance (Ω)	75

Table 3: Model input parameters for prediction of IP016 transfer characteristics.

After convergence of the minimisation algorithm, the following results are obtained:

- $d_{33} = 17.3 \cdot 10^{-12} \text{ pC N}^{-1}$
- $R_{\text{leg}} = 115 \Omega$
- $w_{\text{leg}} = 0.132 \text{ mm}$

A comparison of the optimised sensitivity and the calibration data are given in Figure 2.6, together with an estimate of the phase.

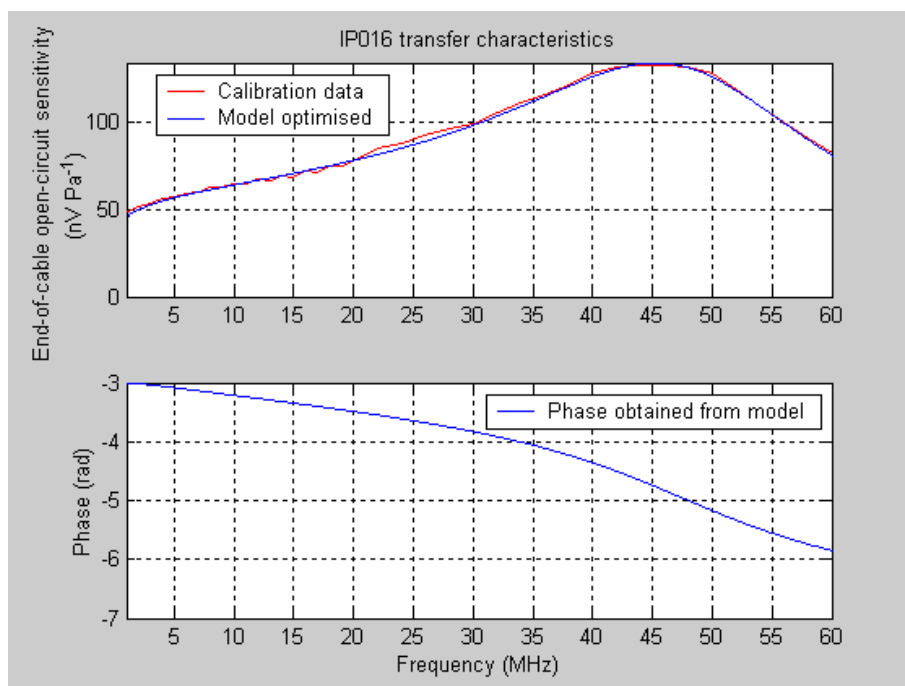


Figure 2.6: IP016 transfer characteristics – Calibration data and optimised sensitivity.

The amplifier transfer characteristics are then measured, as well as the loading correction associated with hydrophone IP016. This is done between D.C. and 80 MHz. The transfer characteristics of the complete hydrophone-amplifier system may then be obtained, as shown in Figure 2.7.

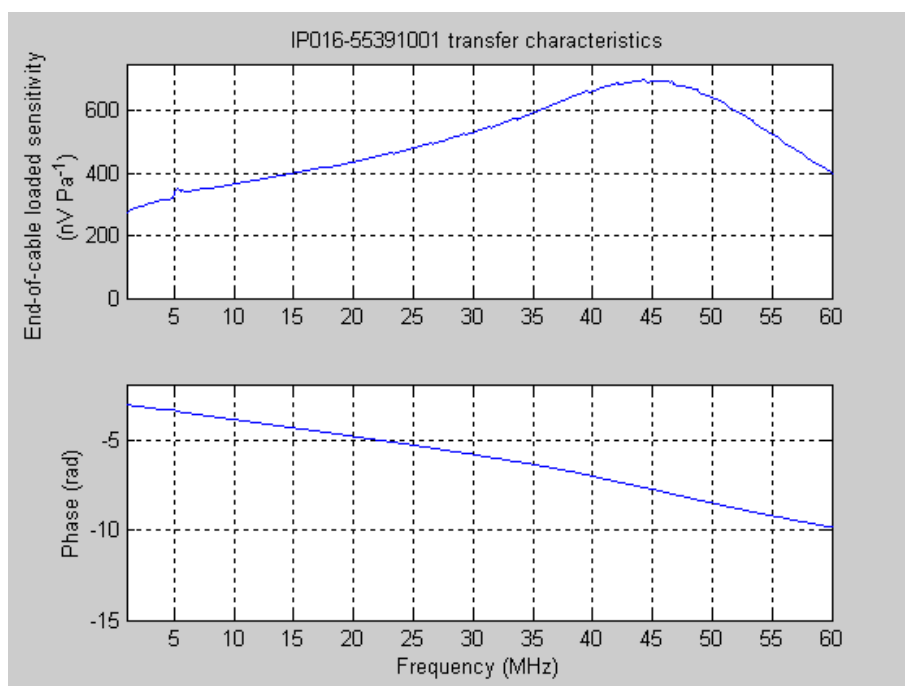


Figure 2.7: IP016- 55391001 transfer characteristics.

2.2.4 5 μm *pvd*f thickness, 0.5 mm element diameter theoretical membrane hydrophone

The objective of the following analysis is to obtain a wide bandwidth measurement system. In order to achieve this, a theoretical 5 μm *pvd*f thickness, 0.5 mm element diameter membrane hydrophone is considered. This leads to a resonance at a frequency around 80 MHz. The model input parameters are listed in Table 4.

c : velocity of sound (m s^{-1})	water: 1482.36; gold: 3240; chromium: 6608; <i>pvd</i> f: 2300; glue: 1600
ρ : material density (kg m^{-3})	water: 998.2; gold: 19281; chromium: 7194; <i>pvd</i> f: 1780; glue 1000
T : thickness of layer (μm)	gold: 0.1; chromium: 0.05; <i>pvd</i> f (poled+depoled): 4.3; depoled <i>pvd</i> f region: 0.7 per membrane; glue: 0.5
α : attenuation coefficient (neper m^{-1} MHz^{-1})	<i>pvd</i> f: 111; glue: 104
d_{33} : piezoelectric coefficient (pC N^{-1})	14
ϵ_0 : dielectric constant (pF m^{-1})	8.85
$ \epsilon_r $: modulus of relative permittivity of <i>pvd</i> f (as a function of frequency f in Hz)	$-2.82 \log_{10}(f) + 26.7$
$\tan \delta$: loss tangent of relative permittivity of <i>pvd</i> f (as a function of frequency f in MHz)	$0.22 + 0.07 \log_{10}(f) - 0.055 \log_{10}(f)^2$
r : element radius (mm)	0.25
w_{leg} : leg width (mm)	0.2
L_{leg} : leg length (mm)	56.5
R_{leg} : leg resistance (Ω)	142
c_{cable} : velocity of wave propagation in cable (m s^{-1})	2×10^8
L_{cable} : length of cable (cm)	2
$Z_{0,\text{cable}}$: cable characteristic impedance (Ω)	75

Table 4: Model input parameters for prediction of transfer characteristics of theoretical 5 μm *pvd*f film thickness, 0.5 mm diameter bilaminar membrane hydrophone.

The predicted transfer characteristics are shown in Figure 2.8.

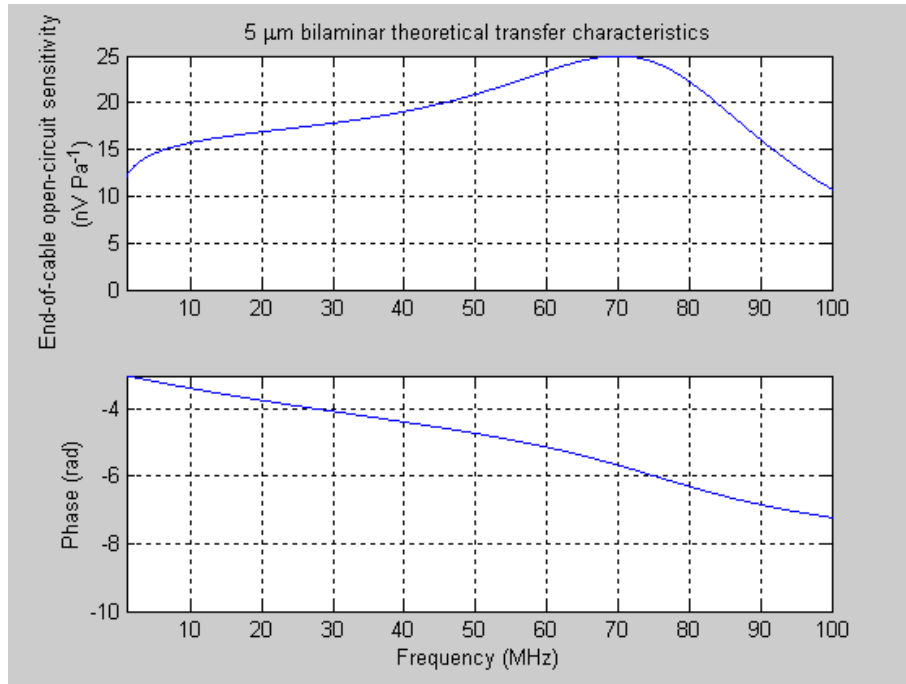


Figure 2.8: 5 μm *pvd*f thickness, 0.5 mm element diameter theoretical membrane hydrophone transfer characteristics.

2.3 DISCUSSION

It should be noted that the hydrophone-amplifier characteristics are prone to uncertainties. In sections 2.2.1 and 2.2.3, the hydrophone responses have been obtained by attempting to match a theoretical magnitude response to a calibration at specific frequencies through variation of specific input parameters (d_{33} , R_{leg} and w_{leg}). This approach is not without limitations.

Firstly, this approach relies on the accuracy of the calibration data. It is known that there is an uncertainty associated with the calibration results at each frequency, and this constitutes a source of error in the estimate of the theoretical hydrophone transfer characteristics when minimising the objective function described in equation (2).

Also, there is no guarantee that the predicted transfer characteristics will match those of the actual device outside the calibration frequency range. The phase response obtained using the hydrophone model in [4] has not yet been validated against measurement, and this constitutes a further source of uncertainty in the estimate of the overall transfer characteristics of the device.

The fact that the parameters d_{33} , R_{leg} and w_{leg} were chosen as the parameters to be varied is possibly not the most thorough approach. Ideally, all hydrophone parameters with which there is a source of uncertainty should be given a plausible range so as to create a variable space in which a global minimum is obtained.

Nevertheless, such an approach also presents difficulties, as it is not possible to assert that the optimal values of the variables correspond to the values of the physical parameters: there may be several solutions in which the obtained theoretical transfer characteristic lies within the range of calibration uncertainty and the model may not be truly representative of the device.

However, the aim of this work is not primarily to find an accurate representation of hydrophone-amplifier transfer characteristics, but to produce plausible complex frequency responses of systems routinely used so that IEC 61220 may be investigated. Hence, the systems depicted in Sections 2.2

and 2.4 are based on configurations likely to be encountered experimentally, although they have not been fitted to known experimental systems in the same way as those in 2.1 and 2.3.

Other uncertainties that arise are those associated with the amplifier complex gain measurements and the loading correction measurements. In particular, there is some quantisation noise present in the gain measurements of amplifiers 55641X (Section 2.2.1) and 55391001 (Section 2.2.3).

3. DESCRIPTION OF CURVE FITTING METHODS

3.1 ELLIPTICAL FIT

3.1.1 Choice of points

According to [1], the points required in order to fit the ellipse to the pulse waveform are as follows:

- Point P_1 : point where ringing has "stopped";
- Point P_2 : zero crossing of pulse;
- Point P_3 : minimum (zero slope);

Figure 3.1 shows the peak positive half cycle of acoustic pressure pulse described in equation (1) convolved with impulse response of hydrophone-amplifier combination IP016 – 55391001.

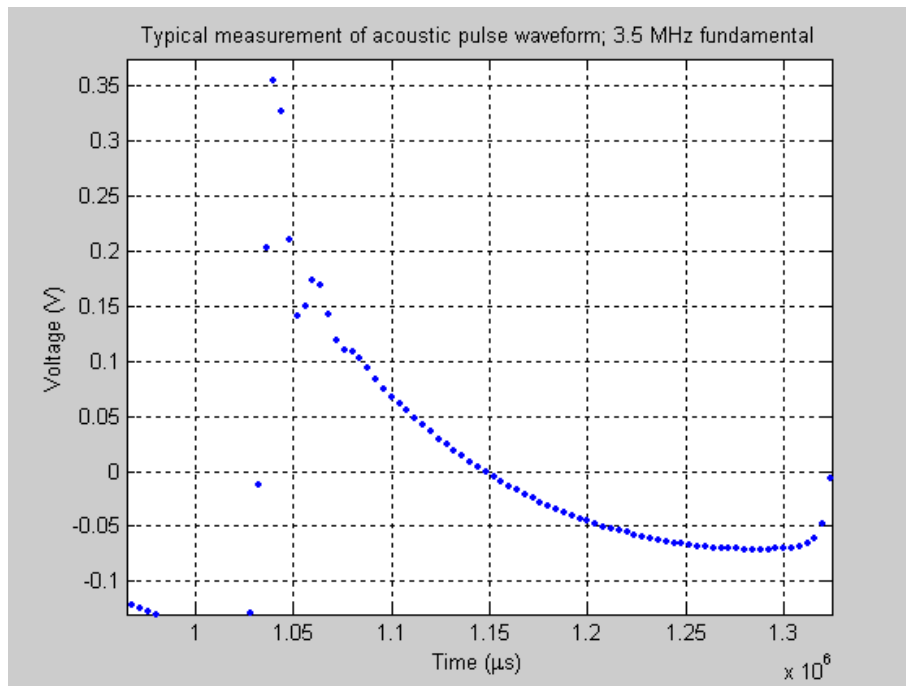


Figure 3.1: Peak positive half cycle of acoustic pressure pulse described in equation (1) convolved with impulse response of hydrophone-amplifier combination IP016 – 55391001.

The point P_3 determines the x coordinate M of the centre of the ellipse. This point also satisfies the ellipse equation:

$$\frac{(t - M)^2}{a^2} - \frac{(v - N)^2}{b^2} = 1 \quad (3)$$

Where (M, N) are the coordinates of the centre of the ellipse.

The points that are chosen to lie on the ellipse are as follows:

- The point at the pulse minimum, P_3 ;
- The point on the pulse midway between the P_2 and P_3 ;
- The point on the pulse midway between the above point and point P_1 .

3.1.2 Method used for solving the ellipse equations

Let these three points on the ellipse have the coordinates (t_1, v_1) , (t_2, v_2) and (t_3, v_3) , respectively. These coordinates are known, as is the value of M . Finding an ellipse that contains these points and has a centre whose x -coordinate is M is therefore equivalent to solving the following system of equations for N , a and b :

$$\begin{cases} \frac{(t_1 - M)^2}{a^2} - \frac{(v_1 - N)^2}{b^2} = 1 \\ \frac{(t_2 - M)^2}{a^2} - \frac{(v_2 - N)^2}{b^2} = 1 \\ \frac{(t_3 - M)^2}{a^2} - \frac{(v_3 - N)^2}{b^2} = 1 \end{cases} \quad (4)$$

In order to do this, the following approach has been used. The variables a^2 and b^2 are eliminated, which leads to solving:

$$\frac{(t_1 - M)^2(v_1 - N)^2 - (t_1 - M)^2(v_2 - N)^2}{(t_2 - M)^2(v_1 - N)^2 - (t_1 - M)^2(v_2 - N)^2} - \frac{(t_1 - M)^2(v_1 - N)^2 - (t_3 - M)^2(v_1 - N)^2}{(t_1 - M)^2(v_3 - N)^2 - (t_3 - M)^2(v_1 - N)^2} - 1 = 0 \quad (5)$$

Subject to the conditions in equation (4).

Equation (5) is solved numerically by searching for zero crossings of the function of N equal to the left hand side of equation (5). This is done for a range of contiguous plausible intervals and values for a^2 and b^2 are obtained by substituting the value for N in any two of the equations in (4). Note that a and b are both positive. A solution that satisfies the conditions in equation (4) is retained.

Once the ellipse has been characterised, [1] specifies that the peak pressure can be estimated by considering the intercept of the ellipse with the ordinate axis passing through the zero-crossing point immediately before the temporal-peak acoustic pressure.

3.2 EXPONENTIAL FIT

An elliptical fit may not always yield useful results, as the ellipse does to always intersect the ordinate axis. In such cases, [1] recommends fitting an exponential curve to the same three points that were used for the ellipse. The following system of equations must then be solved:

$$\begin{cases} v_1 = a \exp(-\alpha t_1) + b \\ v_2 = a \exp(-\alpha t_2) + b \\ v_3 = a \exp(-\alpha t_3) + b \end{cases} \quad (6)$$

Where the unknowns are a , b and α . Note that $\alpha > 0$.

In order to do this, the following approach has been used. The variables a and b are eliminated, which leads to solving:

$$\frac{\exp(-\alpha t_1) - \exp(-\alpha t_2)}{\exp(-\alpha t_1) - \exp(-\alpha t_3)} - \frac{v_1 - v_2}{v_1 - v_3} = 0 \quad (7)$$

Subject to the conditions in equation (6).

Equation (5) is solved numerically by searching for the zero crossing of the function of α equal to the left hand side of equation (7). This is done for an interval with plausible upper and lower limits and values for a and b are obtained by substituting the value for α in any two of the equations in (6).

Although the second and third points can be obtained in a straightforward way, locating the first point is not so obvious, and the IEC Report [1] recommends taking an average of the estimated peak pressures obtained for various positions of P_1 .

4. RESULTS

In this section, the elliptical and exponential fits will be applied to the waveform in equation (1) after it has been convolved with the impulse responses of the systems described in Sections 2.2.1 – 2.2.4. The voltage waveforms are then scaled through division by the combined hydrophone-amplifier response value at the fundamental frequency, i.e. 3.5 MHz.

It was mentioned in Section 3.1 that the point P_1 , where the ringing has “stopped”, is not always obvious to locate. The curve-fitting procedures will therefore be repeated for five locations of P_1 .

The zero crossing point of the scaled voltage curve is located using linear interpolation instead of, for example, cubic-spline, as rise-times are usually short. A more precise value of the intercept could be obtained by using equation (1), but this would not be possible in a measurement situation, hence the use of the digitised waveform.

The intercept of the ellipse and exponential curve with the ordinate axis passing through the zero-crossing point immediately before the temporal-peak is located by cubic-spline interpolation.

The estimated peak pressures are then tabulated for all points for each curve fit and an average is obtained. Percentage errors when compared with the true peak pressure obtained from the waveform described by equation (1) are also listed.

In many cases, the elliptical fit does not intersect the ordinate axis passing through the zero-crossing point immediately before the temporal-peak. Also, an elliptical fit does not always appear to be attainable. Solving equation (5) for N does not always yield a solution within an interval in which lies a plausible centre for the ellipse to be fitted. Further investigation would be needed to confirm this, but this is outside the scope of this work. In either of those cases, the expression “No fit” will be used when tabulating the results. These cases are ignored when computing the average peak pressure for a given fit. If no fit for any point is obtainable, N/A (not applicable) will appear in the table cell corresponding to the mean.

In Section 4.5, high-frequency noise is added to the waveform described by equation (1) and the accuracy of the fitting procedure under these circumstances investigated.

In Section 4.6, the input pressure waveform will consist of a six cycle sinusoidal waveform propagated in a non-compressible viscous dissipative fluid.

4.1 25 μm pvdf thickness, 0.5 mm element diameter membrane hydrophone with matched amplifier

The five locations for P_1 are chosen as shown in Figure 4.1.

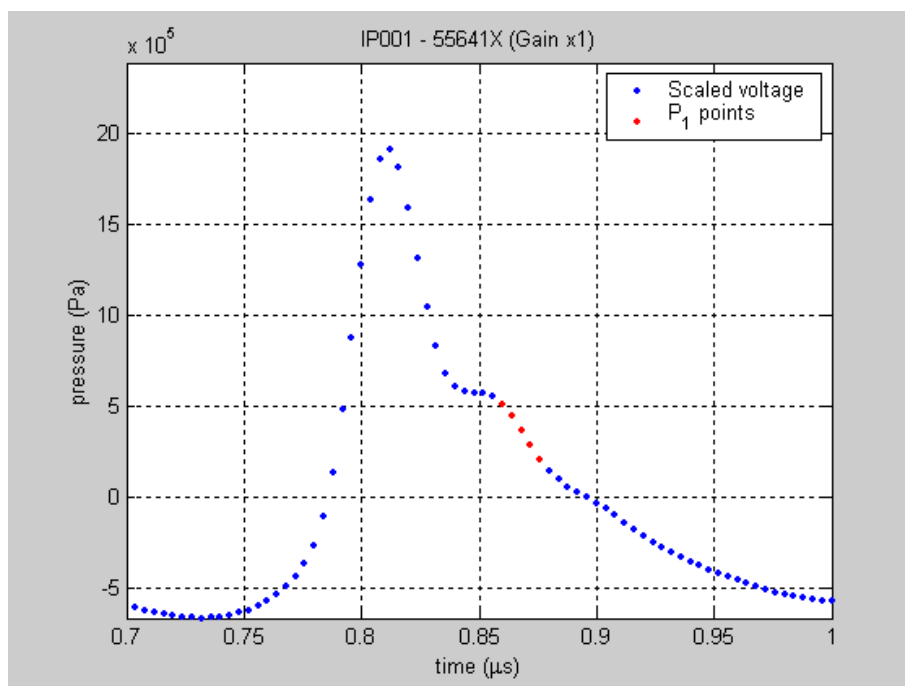


Figure 4.1: Locations of P_1 points for IP001 – 55641X (Gain $\times 1$) combination.

These points outside this range either do not correspond to values of time where one can consider that the ringing has “stopped”, or do not produce curve fits with plausible intercepts

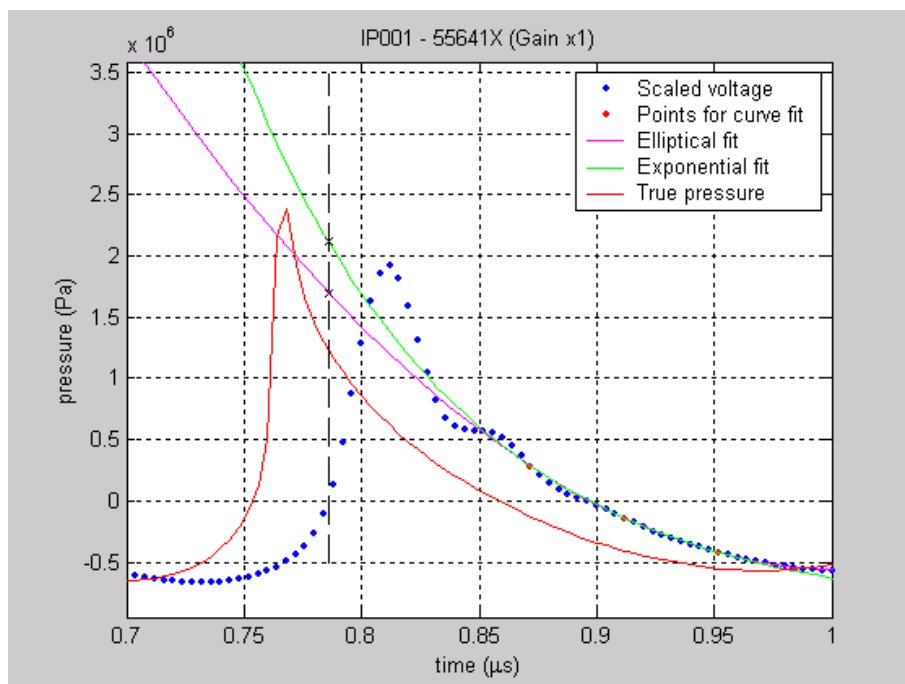


Figure 4.2: Example of curve fits and intercepts for IP001 – 55641X (Gain $\times 1$) combination.

	Peak pressure estimated from elliptical fit (MPa)	Error in elliptical fit	Peak pressure estimated from exponential fit (MPa)	Error in exponential fit
$P_{1,1}$	3.34	39.7%	2.61	9.38%
$P_{1,2}$	2.65	11.1%	2.58	7.96%
$P_{1,3}$	2.11	-11.7%	2.39	1.73%
$P_{1,4}$	1.70	-29.0%	2.11	-11.5%
$P_{1,5}$	No fit	N/A	1.76	-26.2%
Mean	2.45	2.54%	2.29	-4.11%

Table 4.1: Estimates of the peak pressure for IP001 – 5564X (Gain $\times 1$) combination.

The peak-positive acoustic pressure is 2.39 MPa.

4.2 15 μm pvdf thickness, 0.4 mm element diameter membrane hydrophone with amplifier

The five locations for P_1 are chosen as shown in Figure 4.3.

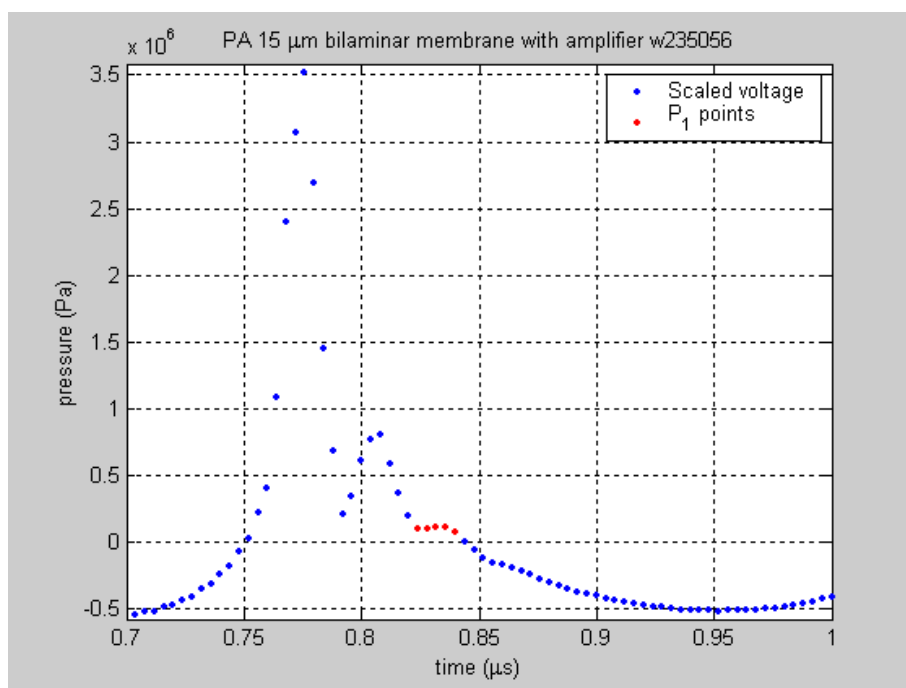


Figure 4.3: Locations of P_1 points for PA 15 μm membrane hydrophone with amplifier w235056.

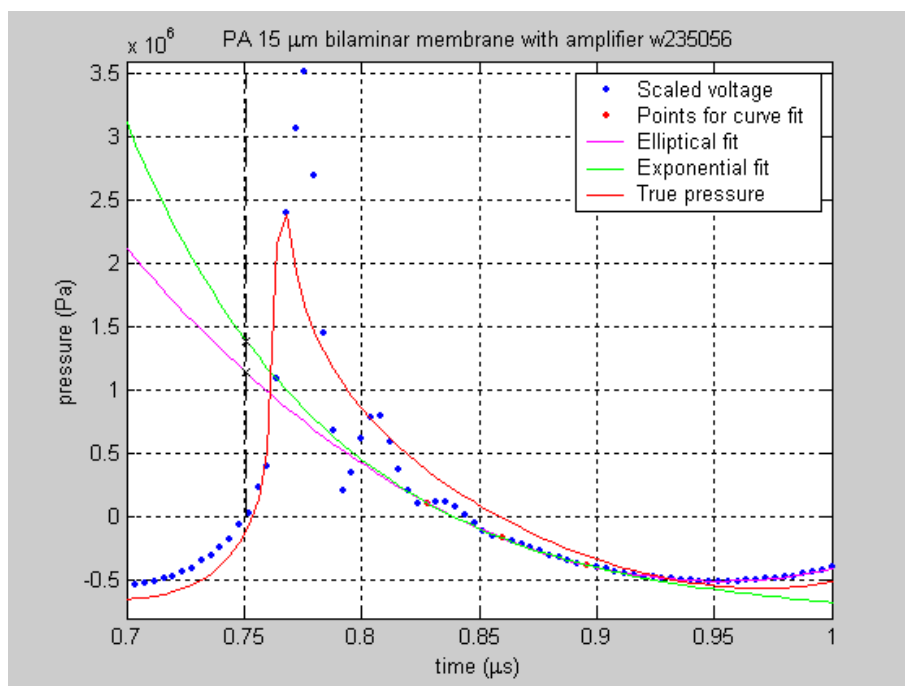


Figure 4.4: Example of curve fit and intercept for PA 15 μm membrane hydrophone with amplifier w235056.

	Peak pressure estimated from elliptical fit (MPa)	Error in elliptical fit	Peak pressure estimated from exponential fit (MPa)	Error in exponential fit
$P_{1,1}$	No fit	N/A	0.957	-59.9%
$P_{1,2}$	1.13	-52.5%	1.38	-42.1%
$P_{1,3}$	No fit	N/A	2.01	-16.0%
$P_{1,4}$	No fit	N/A	3.29	37.9%
$P_{1,5}$	No fit	N/A	3.18	33.2%
Mean	1.13	-52.5%	2.17	-9.37%

Table 4.2: Estimates of the peak pressure for PA 15 μm membrane hydrophone with amplifier w235056

The peak-positive acoustic pressure is 2.39 MPa.

4.3 9 μm pvdF thickness, 0.5 mm element diameter membrane hydrophone with amplifier

The five locations for P_1 are chosen as shown in Figure 4.5.

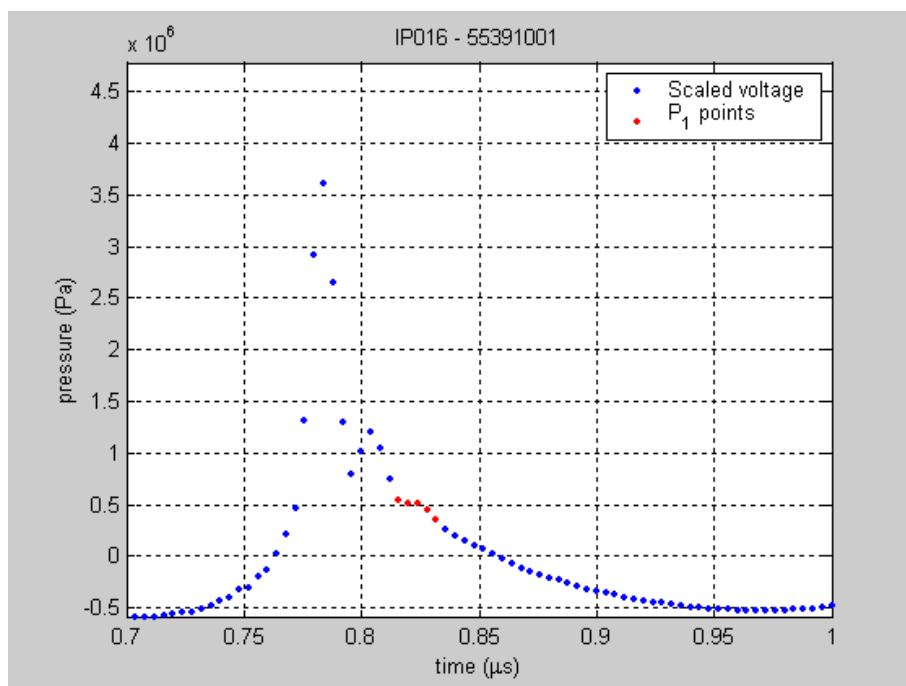


Figure 4.5: Locations of P_1 points for IP016 – 55391001 combination.

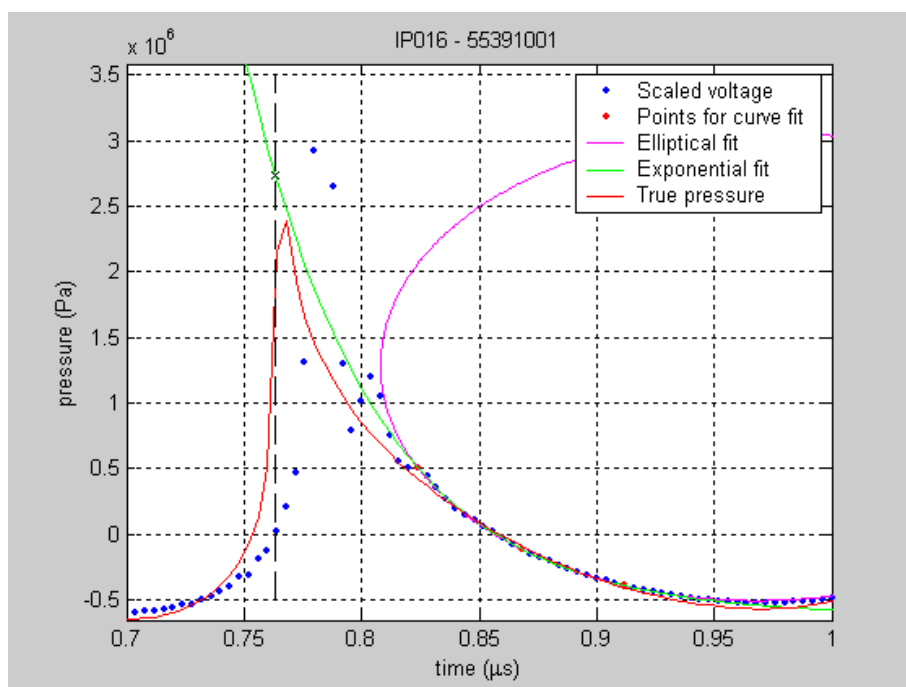


Figure 4.6: Example of curve fits and intercepts for IP016 – 55391001 combination.

	Peak pressure estimated from elliptical fit (MPa)	Error in elliptical fit	Peak pressure estimated from exponential fit (MPa)	Error in exponential fit
$P_{1,1}$	No fit	N/A	1.91	-20.2%
$P_{1,2}$	No fit	N/A	2.09	-12.4%
$P_{1,3}$	No fit	N/A	2.73	14.4%
$P_{1,4}$	No fit	N/A	2.98	24.8%
$P_{1,5}$	No fit	N/A	2.72	13.7%
Mean	N/A	N/A	2.49	4.03%

Table 4.3: Estimates of the peak pressure for IP016 – 55391001 combination.

The peak-positive acoustic pressure is 2.39 MPa.

4.4 5 μm pvdf thickness, 0.5 mm element diameter theoretical membrane hydrophone

The five locations for P_1 are chosen as shown in Figure 4.7.

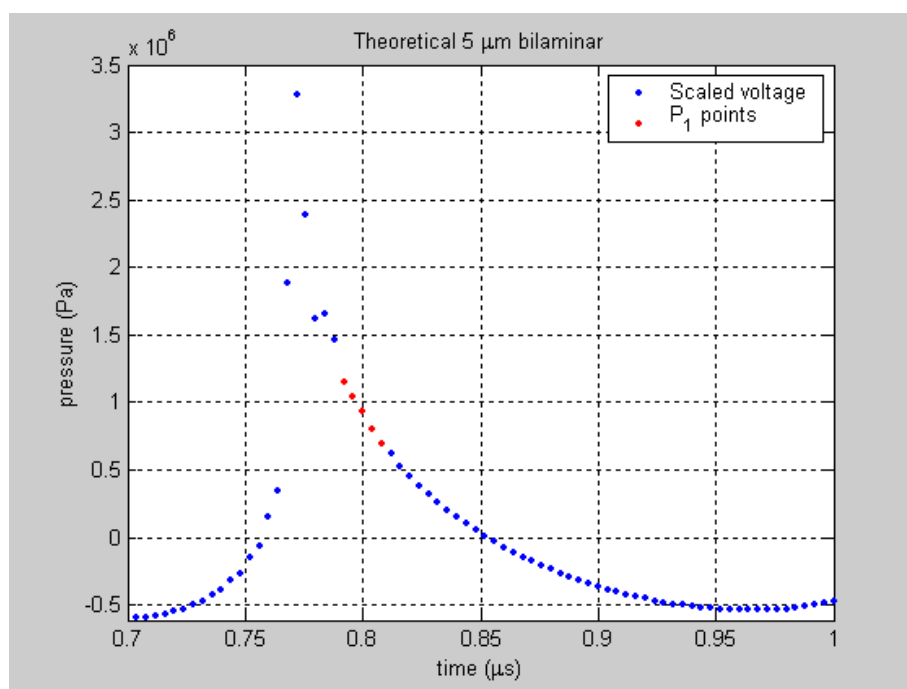


Figure 4.7: Locations of P_1 points for 5 μm pvdf thickness, 0.5 mm element diameter theoretical membrane hydrophone.

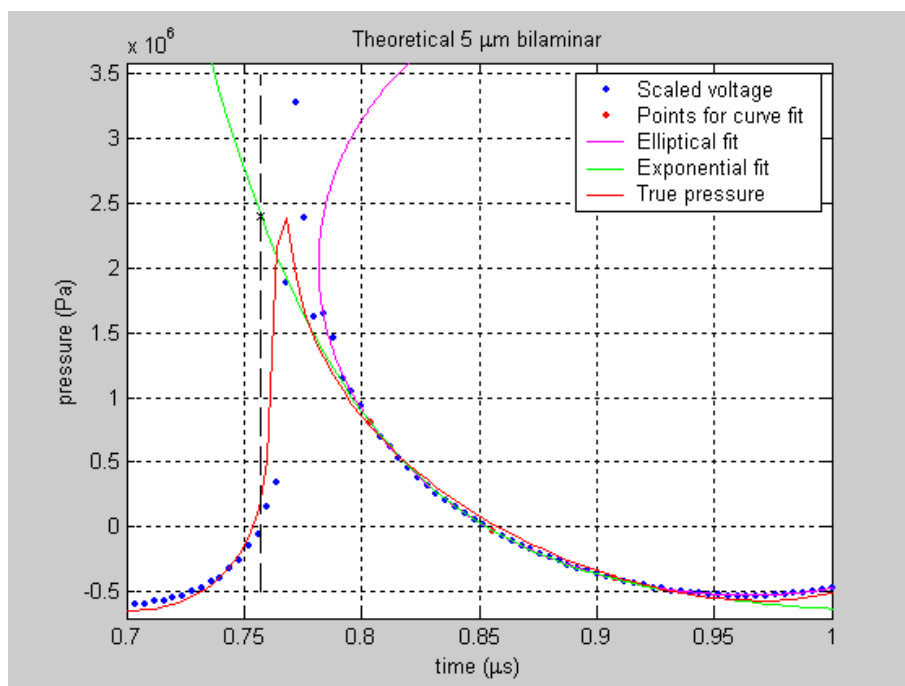


Figure 4.8: Example of curve fits and intercepts for 5 μm *pvd*f thickness, 0.5 mm element diameter theoretical membrane hydrophone.

	Peak pressure estimated from elliptical fit (MPa)	Error in elliptical fit	Peak pressure estimated from exponential fit (MPa)	Error in exponential fit
$P_{1,1}$	No fit	N/A	2.53	6.06%
$P_{1,2}$	No fit	N/A	2.57	7.44%
$P_{1,3}$	No fit	N/A	2.58	7.98%
$P_{1,4}$	No fit	N/A	2.40	0.528%
$P_{1,5}$	No fit	N/A	2.31	-3.30%
Mean	N/A	N/A	2.48	3.74%

Table 4.4: Estimates of the peak pressure for 5 μm *pvd*f thickness, 0.5 mm element diameter theoretical membrane hydrophone.

The peak-positive acoustic pressure is 2.39 MPa.

4.5 ADDED WHITE NOISE

Measured voltages are often affected by high-frequency noise. In order to simulate this, high-pass filtered white noise has been added to the pressure waveform described in equation (1). A pseudo-random number generator has been used, to simulate white noise in the time domain. The noise has been normalised to have zero mean. Four levels of noise have been examined. Each noise waveform has been scaled so that it does not exceed a specific value. This value is set to:

- $0.020 p_{\text{max}}$
- $0.025 p_{\text{max}}$
- $0.030 p_{\text{max}}$
- $0.035 p_{\text{max}}$

Where p_{max} represents the peak-positive acoustic pressure described by equation (1).

This noise has then been filtered using a 100th order Finite Impulse Response high-pass filter with a cut-off at 50 MHz and added to the pressure in equation (1). The three waveforms are shown Figures 4.9 - 4.12.

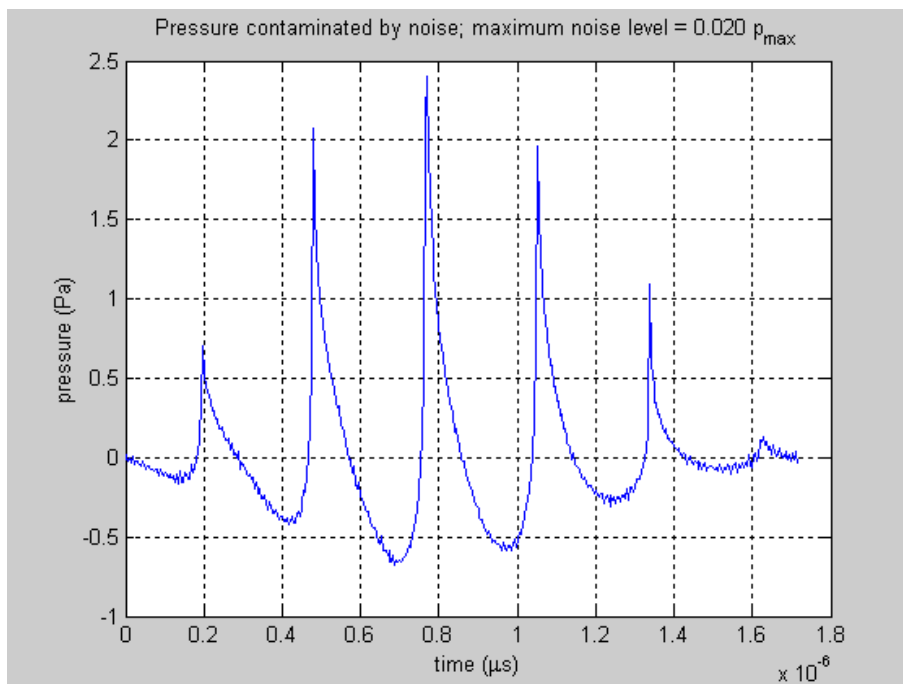


Figure 4.9: Pressure contaminated by high-pass filtered white noise with maximum noise level equal to $0.020 p_{\max}$.

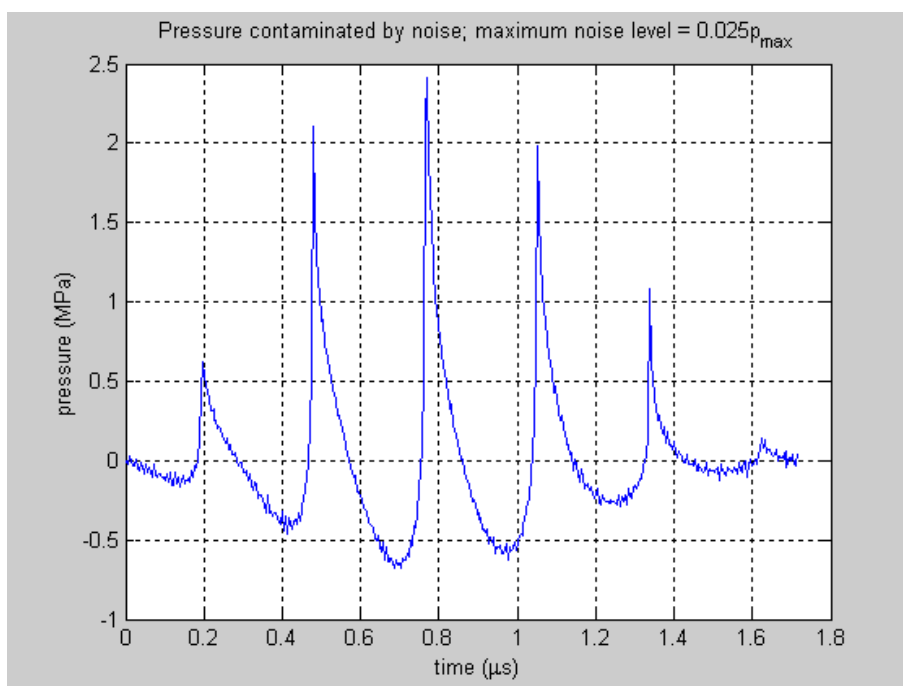


Figure 4.10: Pressure contaminated by high-pass filtered white noise with maximum noise level equal to $0.025 p_{\max}$.

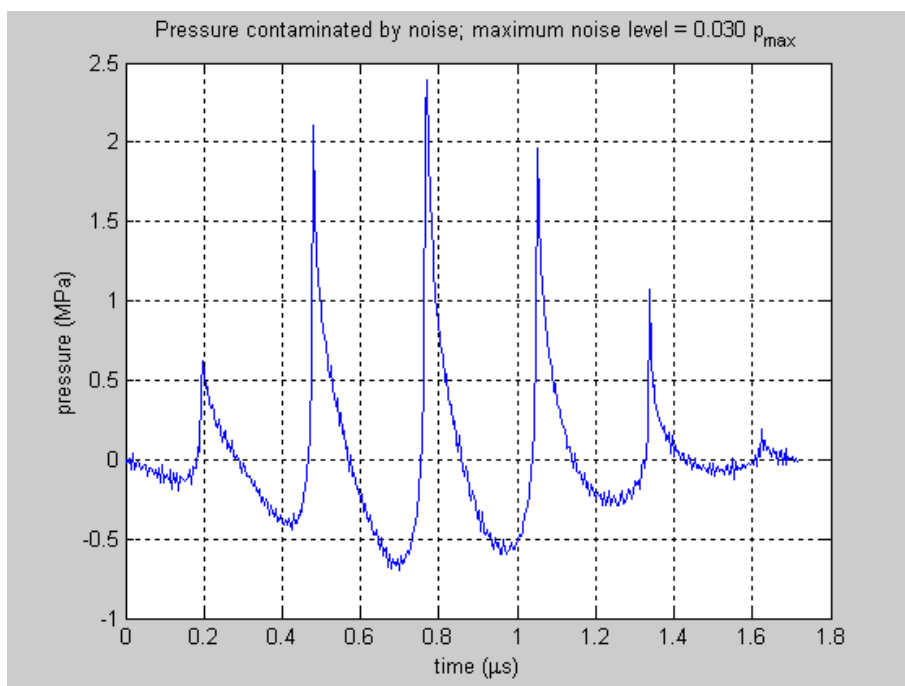


Figure 4.11: Pressure contaminated by high-pass filtered white noise with maximum noise level equal to $0.030 p_{\max}$.

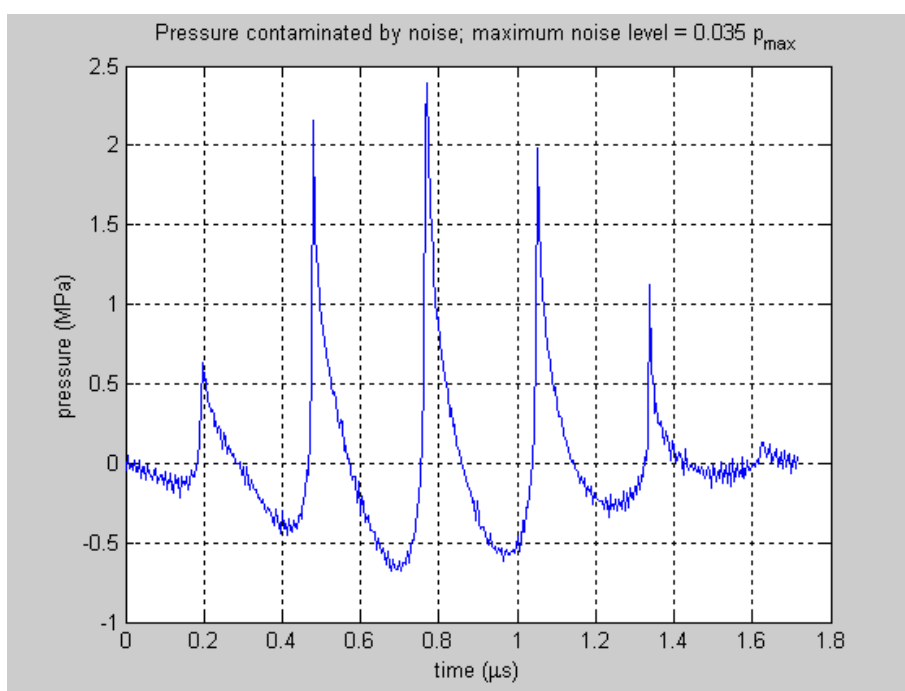


Figure 4.12: Pressure contaminated by high-pass filtered white noise with maximum noise level equal to $0.035 p_{\max}$.

These waveforms will be referred to as $p_{\text{noise},1}$, $p_{\text{noise},2}$, $p_{\text{noise},3}$ and $p_{\text{noise},4}$ respectively.

4.5.1 15 μm *pvd*f thickness, 0.4 mm element diameter membrane hydrophone with amplifier using $p_{\text{noise},1}$

The five locations for P_1 are chosen as shown in Figure 4.13.

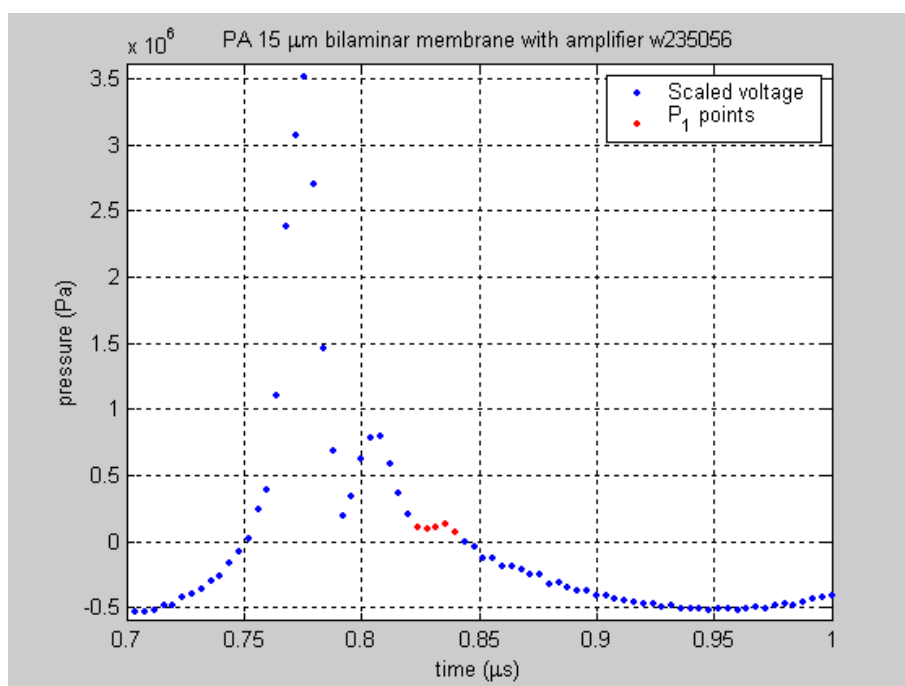


Figure 4.13: Locations of P_1 points for PA 15 μm membrane hydrophone with amplifier w235056 using $p_{\text{noise},1}$ as an input waveform

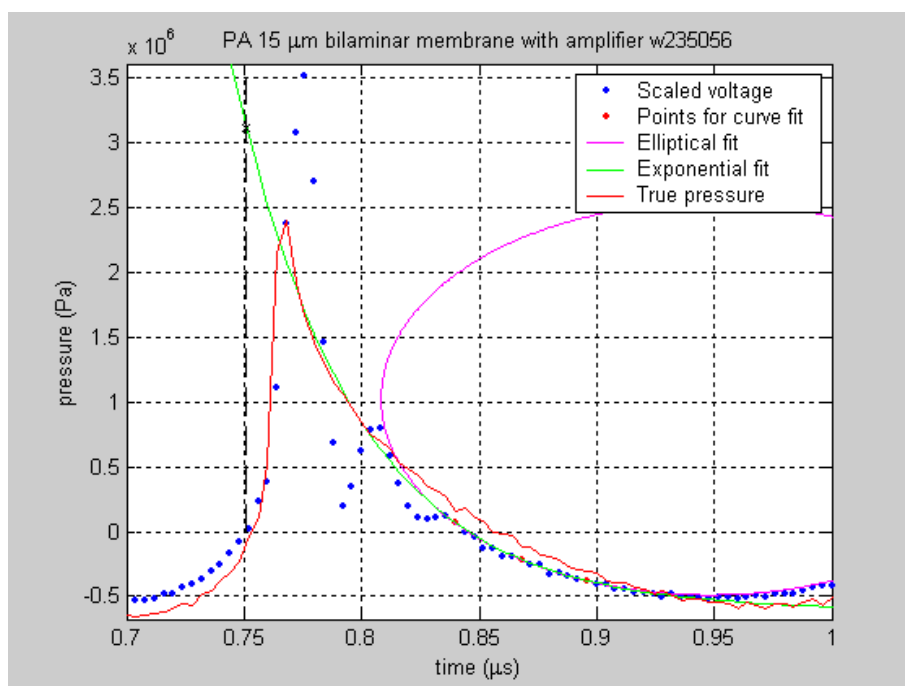


Figure 4.14: Example of curve fit and intercept for for PA 15 μm membrane hydrophone with amplifier w235056 using $p_{\text{noise},1}$ as an input waveform

	Peak pressure estimated from elliptical fit (MPa)	Error in elliptical fit	Peak pressure estimated from exponential fit (MPa)	Error in exponential fit
$P_{1,1}$	1.19	-50.1%	1.38	-42.4%
$P_{1,2}$	No fit	N/A	1.87	-21.8%
$P_{1,3}$	1.53	-36.0%	1.78	-25.7%
$P_{1,4}$	No fit	N/A	3.75	57.0%
$P_{1,5}$	No fit	N/A	3.11	30.0%
Mean	1.36	-43.1%	2.38	-0.678%

Table 4.5: Estimates of the peak pressure for PA 15 μm membrane hydrophone with amplifier w235056 using $p_{\text{noise},1}$ as an input waveform

The peak-positive acoustic pressure is 2.39 MPa.

4.5.2 15 μm *pvd*f thickness, 0.4 mm element diameter membrane hydrophone with amplifier using $p_{\text{noise},2}$

The five locations for P_1 are chosen as shown in Figure 4.15.

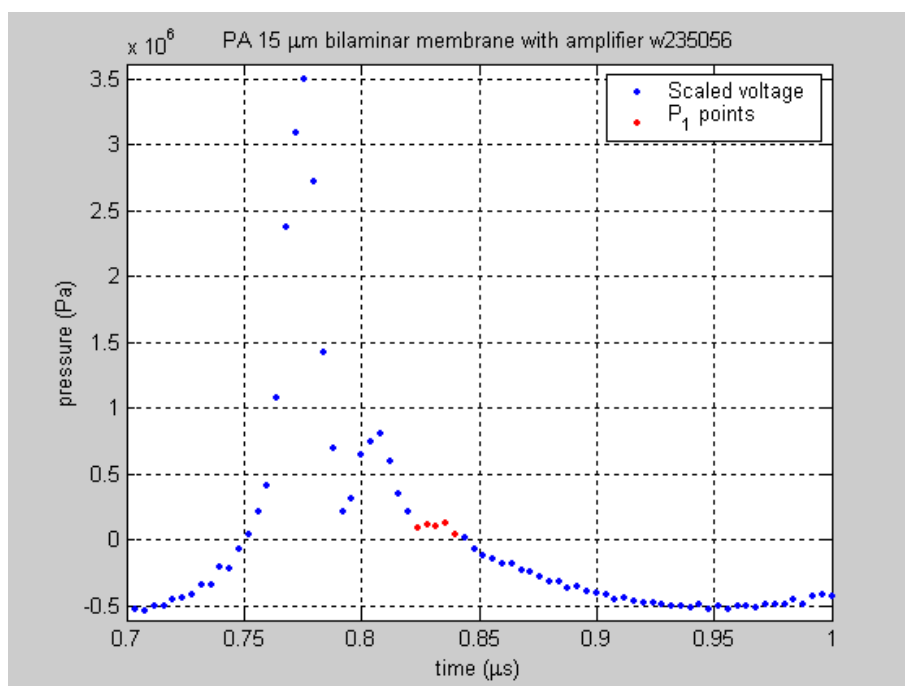


Figure 4.15: Locations of P_1 points for PA 15 μm membrane hydrophone with amplifier w235056 using $p_{\text{noise},2}$ as an input waveform

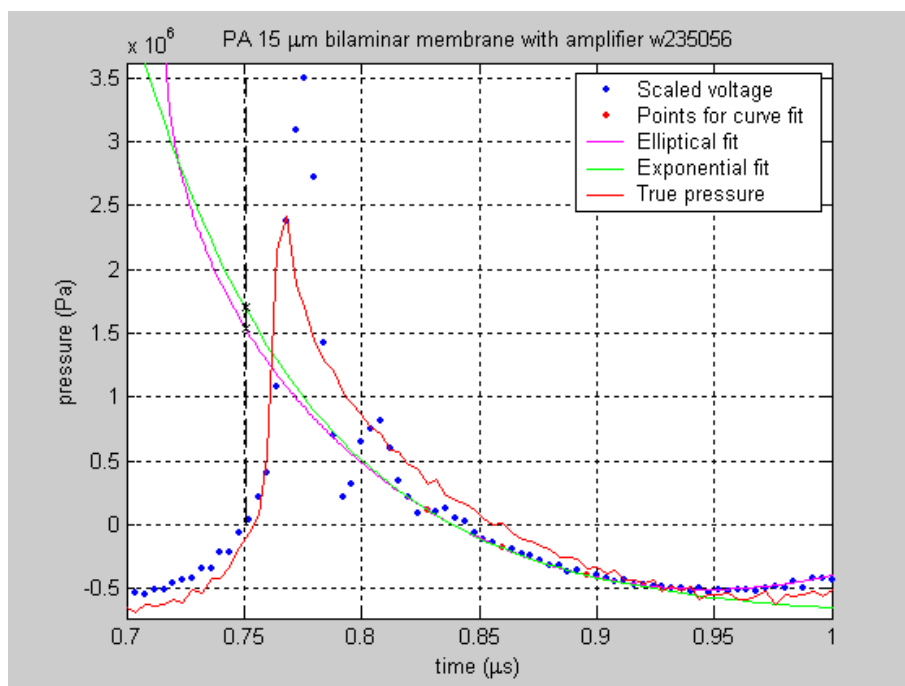


Figure 4.16: Example of curve fit and intercept for for PA 15 μm membrane hydrophone with amplifier w235056 using $p_{\text{noise},2}$ as an input waveform

	Peak pressure estimated from elliptical fit (MPa)	Error in elliptical fit	Peak pressure estimated from exponential fit (MPa)	Error in exponential fit
$P_{1,1}$	No fit	N/A	0.894	-62.6%
$P_{1,2}$	1.54	-35.4%	1.70	-29.0%
$P_{1,3}$	No fit	N/A	1.39	-41.8%
$P_{1,4}$	No fit	N/A	2.97	24.3%
$P_{1,5}$	No fit	N/A	2.60	8.79%
Mean	1.54	-35.4%	1.91	-20.0%

Table 4.6: Estimates of the peak pressure for PA 15 μm membrane hydrophone with amplifier w235056 using $p_{\text{noise},2}$ as an input waveform

The peak-positive acoustic pressure is 2.39 MPa.

4.5.3 15 μm *pvd*f thickness, 0.4 mm element diameter membrane hydrophone with amplifier using $p_{\text{noise},3}$

The five locations for P_1 are chosen as shown in Figure 4.17.

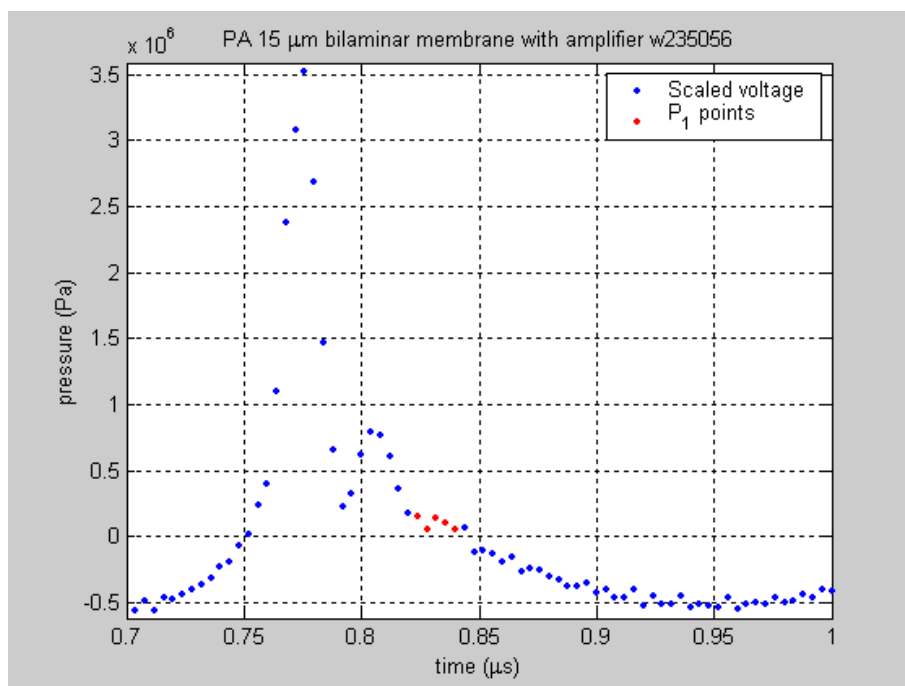


Figure 4.17: Locations of P_1 points for PA 15 μm membrane hydrophone with amplifier w235056 using $p_{\text{noise},3}$ as an input waveform

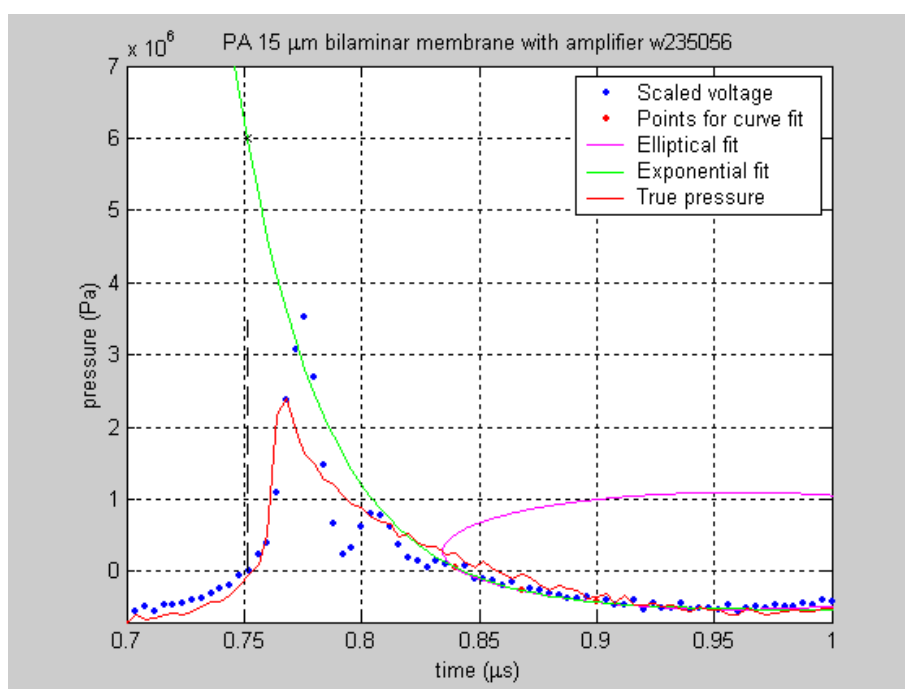


Figure 4.18: Example of curve fit and intercept for PA 15 μm membrane hydrophone with amplifier w235056 using $p_{\text{noise},3}$ as an input waveform

	Peak pressure estimated from elliptical fit (MPa)	Error in elliptical fit	Peak pressure estimated from exponential fit (MPa)	Error in exponential fit
$P_{1,1}$	No fit	N/A	1.71	-28.3%
$P_{1,2}$	No fit	N/A	No fit	N/A
$P_{1,3}$	No fit	N/A	1.16	51.6%
$P_{1,4}$	No fit	N/A	5.31	122%
$P_{1,5}$	No fit	N/A	6.01	151%
Mean	1.54	36.0%	3.55	48.4%

Table 4.7: Estimates of the peak pressure for PA 15 μm membrane hydrophone with amplifier w235056 using $p_{\text{noise},3}$ as an input waveform

The peak-positive acoustic pressure is 2.39 MPa.

Note that, for $P_{1,2}$, no decaying exponential fit can be found.

4.5.4 15 μm *pvd*f thickness, 0.4 mm element diameter membrane hydrophone with amplifier using $p_{\text{noise},4}$

The five locations for P_1 are chosen as shown in Figure 4.19.

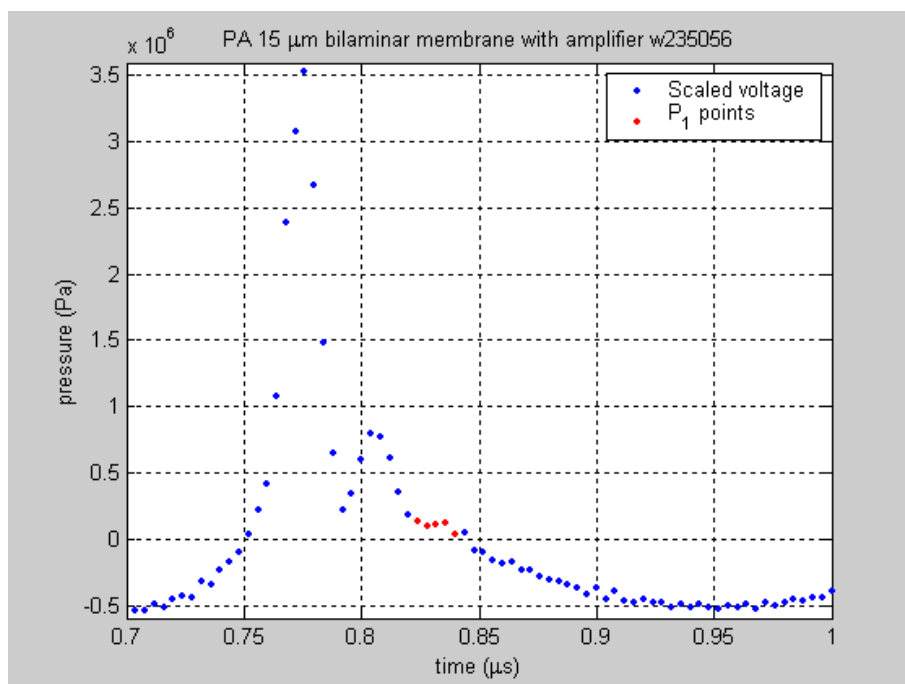


Figure 4.19: Locations of P_1 points for PA 15 μm membrane hydrophone with amplifier w235056 using $p_{\text{noise},4}$ as an input waveform

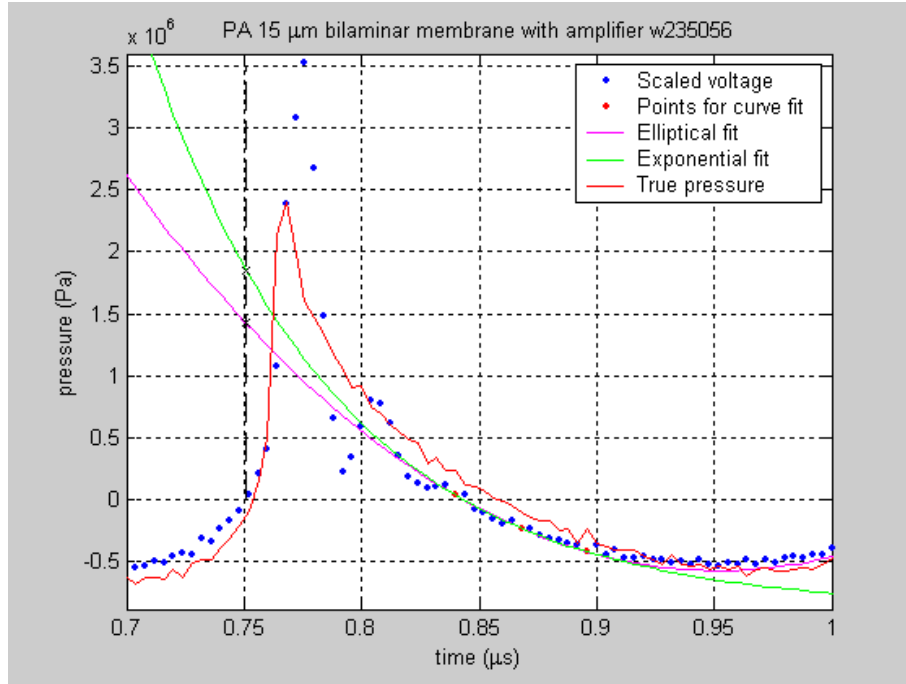


Figure 4.20: Example of curve fit and intercept for for PA 15 μm membrane hydrophone with amplifier w235056 using $p_{\text{noise},4}$ as an input waveform.

	Peak pressure estimated from elliptical fit (MPa)	Error in elliptical fit	Peak pressure estimated from exponential fit (MPa)	Error in exponential fit
$P_{1,1}$	No fit	N/A	1.22	-49.0%
$P_{1,2}$	No fit	N/A	1.34	-43.9%
$P_{1,3}$	No fit	N/A	0.953	-60.1%
$P_{1,4}$	No fit	N/A	1.73	-27.7%
$P_{1,5}$	1.42	-40.4%	1.85	-22.5%
Mean	1.42	-40.4%	1.42	-40.6%

Table 4.8: Estimates of the peak pressure for PA 15 μm membrane hydrophone with amplifier w235056 using $p_{\text{noise},4}$ as an input waveform

The peak-positive acoustic pressure is 2.39 MPa.

4.6 DISTORTED SINUSOIDAL WAVEFORM AS PRESSURE INPUT

In this section, six cycles of a finite amplitude plane wave of frequency 3.5 MHz and amplitude p_0 will be propagated a distance of $x_0 = 0.05$ m in a non-compressible viscous dissipative fluid. The solution for the pressure as a function of time at a distance x is given by [6]:

$$p(x, \tau) = \frac{\rho_0 \delta_0}{\beta} \frac{\partial}{\partial \tau} \ln \left[\left(\frac{c_0^3}{2\pi x \delta_0} \right)^{\frac{1}{2}} \int_{-\infty}^{+\infty} e^{-\frac{c_0^3(\tau-\tau')^2}{2x\delta_0} + \frac{\beta}{\rho_0 \delta_0} \int^{\tau'} p(0, \tau'') d\tau''} d\tau' \right] \quad (8)$$

Where

- τ, τ' and τ'' are time-related variables

- $\delta_0 \approx \frac{2\alpha c_0^3}{\omega^2}$ [7]
- α is the attenuation coefficient
- $\beta=1+B/(2A)$ (≈ 3.5 for water)
- c_0 is the velocity of sound in the unperturbed medium.
- ρ_0 is the density of the medium

The values of the constants are those for water. Refer to [8] for more details.

The pressure magnitude p_0 will assume values of 0.7 MPa, 0.9 MPa and 1.2 MPa in order to represent various degrees of non-linearity. The pressure waveforms after propagation are shown in Figure 4.21.

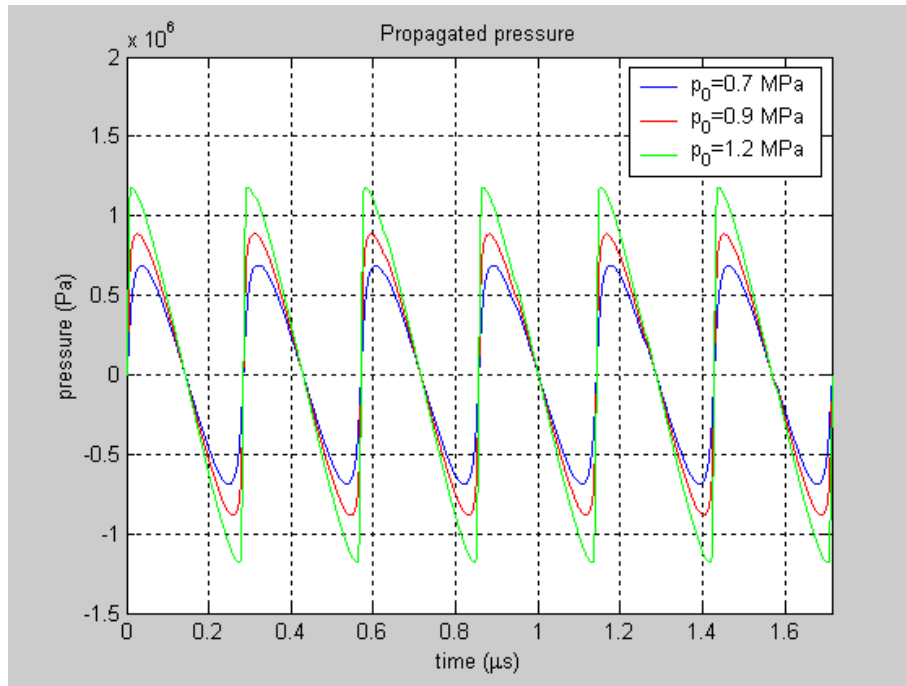


Figure 4.21: Six periods of a finite amplitude plane wave of frequency 3.5 MHz and amplitude p_0 will be propagated a distance of $x_0 = 0.05$ m in a non-compressible viscous dissipative fluid.

The waveforms in Figure 4.21 will constitute the input to the PA 15 μm membrane hydrophone with amplifier w235056, and the output voltage then obtained will be used to estimate the peak pressure.

It should be noted that the waveforms generated by the algorithm in equation (8) are very different from those produced by that in equation (1). The former produces waveforms with more “rounded” maxima and minima whilst the latter attempts to reproduce the effects of diffraction in the acoustic field, resulting in a greater magnitude of the peak positive pressure compared to the peak negative pressure.

4.6.1 Distorted sinusoidal waveform as pressure input; $p_0 = 0.7$ MPa

The five locations for P_1 are chosen as shown in Figure 4.22.

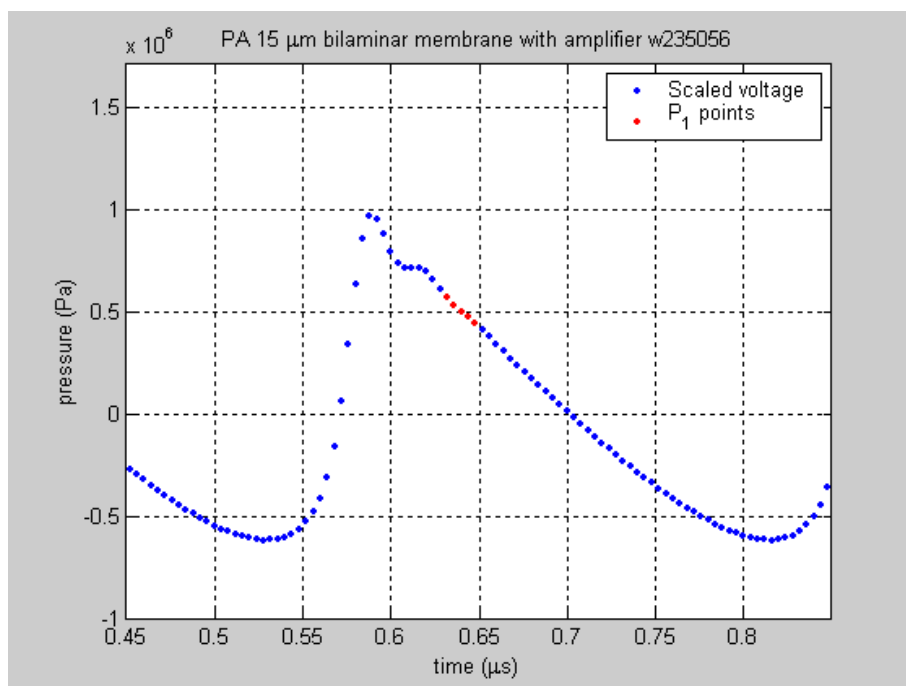


Figure 4.22: Locations of P_1 points for PA 15 μm membrane hydrophone with amplifier w235056 using distorted a 3.5 MHz sinusoidal as an input waveform; $p_0 = 0.7$ MPa.

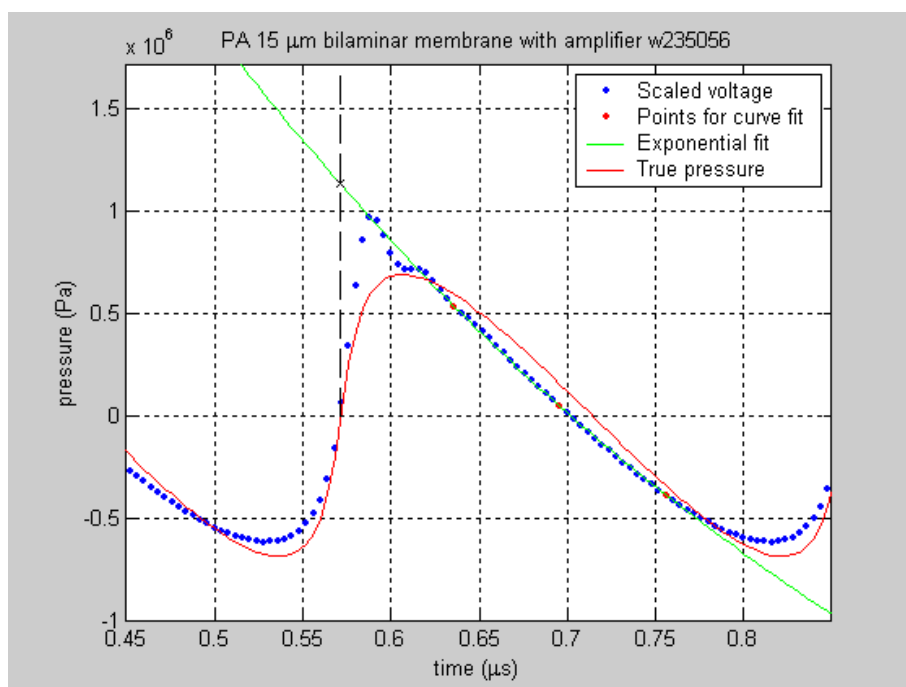


Figure 4.23: Example of curve fit and intercept for PA 15 μm membrane hydrophone with amplifier w235056 using distorted a 3.5 MHz sinusoidal as an input waveform; $p_0 = 0.7$ MPa.

	Peak pressure estimated from elliptical fit (MPa)	Error in elliptical fit	Peak pressure estimated from exponential fit (MPa)	Error in exponential fit
$P_{1,1}$	No fit	N/A	1.14	65.4%
$P_{1,2}$	No fit	N/A	1.14	64.9%
$P_{1,3}$	No fit	N/A	1.07	54.7%
$P_{1,4}$	No fit	N/A	1.18	71.4%
$P_{1,5}$	No fit	N/A	1.21	75.6%
Mean	N/A	N/A	1.15	66.4%

Table 4.9: Estimates of the peak pressure for PA 15 μm membrane hydrophone with amplifier w235056 using distorted a 3.5 MHz sinusoidal as an input waveform; $p_0 = 0.7$ MPa.

The peak-positive acoustic pressure is 0.689 MPa.

4.6.2 Distorted sinusoidal waveform as pressure input; $p_0 = 0.9$ MPa

The five locations for P_1 are chosen as shown in Figure 4.24.

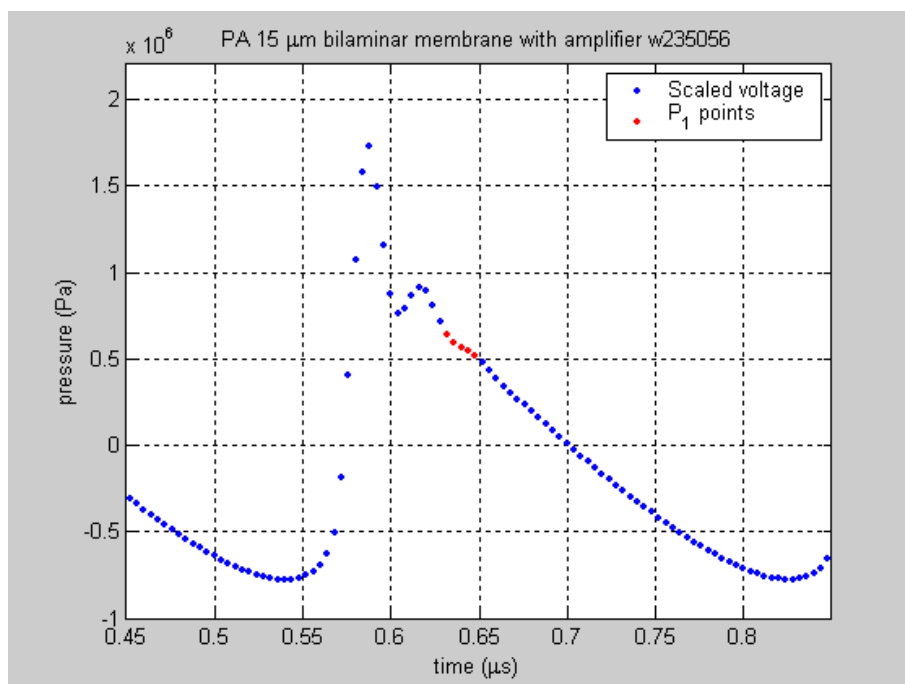


Figure 4.24: Locations of P_1 points for PA 15 μm membrane hydrophone with amplifier w235056 using a distorted 3.5 MHz sinusoidal as an input waveform; $p_0 = 0.9$ MPa.

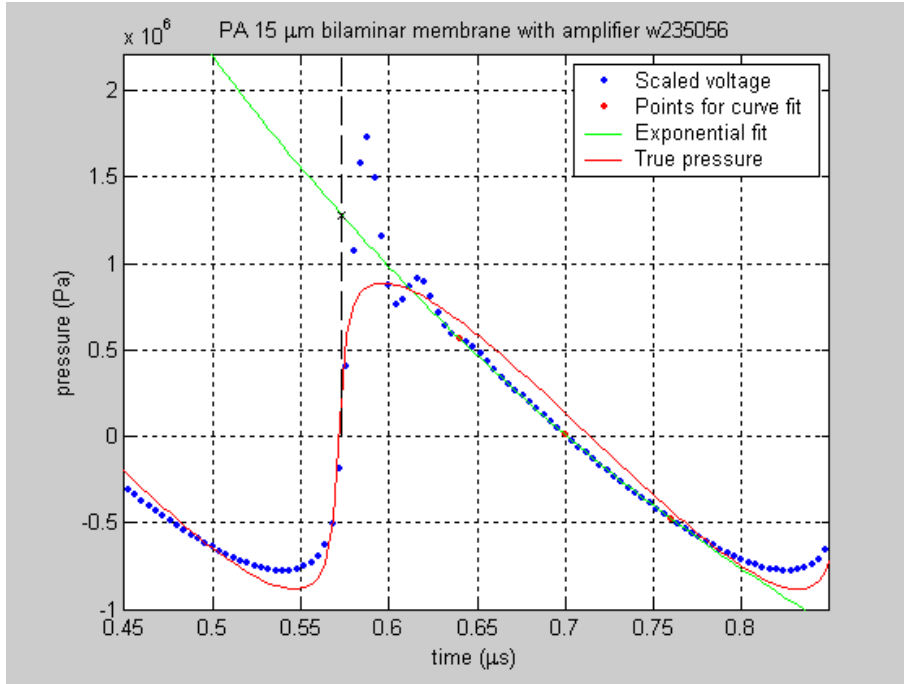


Figure 4.25: Example of curve fit and intercept for PA 15 μm membrane hydrophone with amplifier w235056 using distorted a 3.5 MHz sinusoidal as an input waveform; $p_0 = 0.9 \text{ MPa}$.

	Peak pressure estimated from elliptical fit (MPa)	Error in elliptical fit	Peak pressure estimated from exponential fit (MPa)	Error in exponential fit
$P_{1,1}$	No fit	N/A	1.26	41.7%
$P_{1,2}$	No fit	N/A	1.23	39.0%
$P_{1,3}$	No fit	N/A	1.28	44.7%
$P_{1,4}$	No fit	N/A	1.36	53.7%
$P_{1,5}$	No Fit	N/A	1.42	61.1%
Mean	N/A	N/A	1.31	48.0%

Table 4.10: Estimates of the peak pressure for PA 15 μm membrane hydrophone with amplifier w235056 using a distorted 3.5 MHz sinusoidal as an input waveform $p_0 = 0.9 \text{ MPa}$.

The peak-positive acoustic pressure is 0.886 MPa.

4.6.3 Distorted sinusoidal waveform as pressure input; $p_0 = 1.2 \text{ MPa}$

The five locations for P_1 are chosen as shown in Figure 4.26.

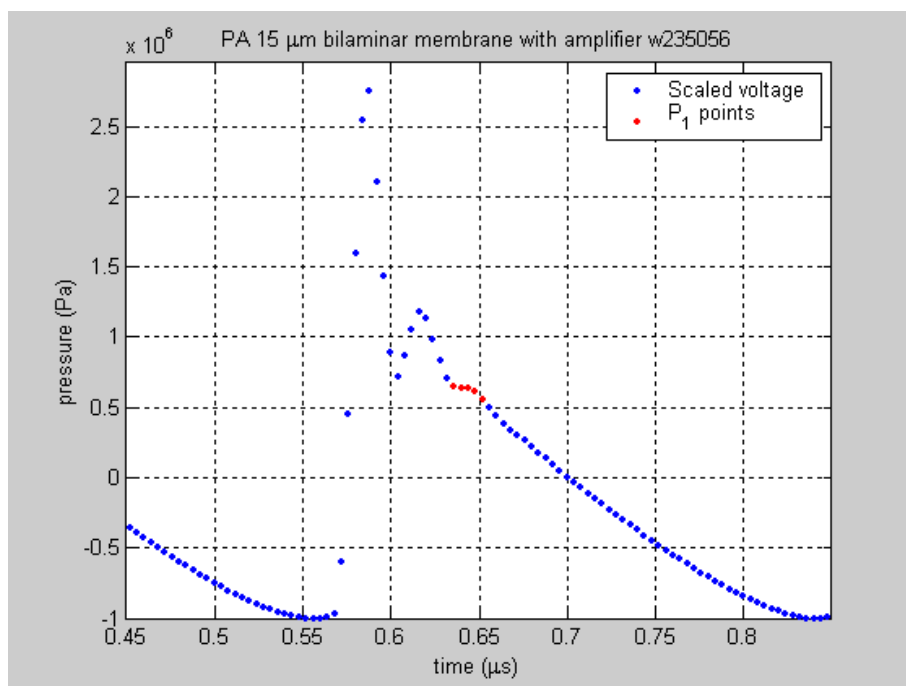


Figure 4.26: Locations of P_1 points for PA 15 μm membrane hydrophone with amplifier w235056 using a distorted 3.5 MHz sinusoidal as an input waveform; $p_0 = 1.2$ MPa.

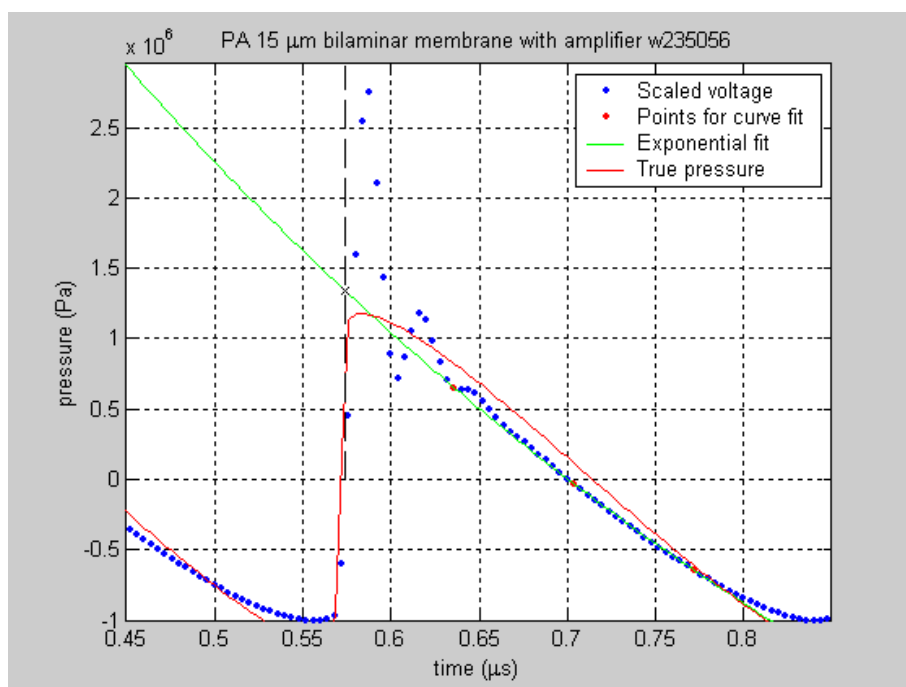


Figure 4.27: Example of curve fit and intercept for PA 15 μm membrane hydrophone with amplifier w235056 using distorted a 3.5 MHz sinusoidal as an input waveform; $p_0 = 1.2$ MPa.

	Peak pressure estimated from elliptical fit (MPa)	Error in elliptical fit	Peak pressure estimated from exponential fit (MPa)	Error in exponential fit
$P_{1,1}$	No fit	N/A	1.34	13.1%
$P_{1,2}$	No fit	N/A	1.44	21.8%
$P_{1,3}$	No fit	N/A	1.58	33.6%
$P_{1,4}$	No fit	N/A	1.67	42.9%
$P_{1,5}$	No fit	N/A	1.67	42.9%
Mean	N/A	N/A	1.55	31.0%

Table 4.11: Estimates of the peak pressure for PA 15 μm membrane hydrophone with amplifier w235056 using a distorted 3.5 MHz sinusoidal as an input waveform $p_0 = 1.2$ MPa.

The peak-positive acoustic pressure is 1.18 MPa.

5. DISCUSSION

It should first be noted that choosing adequate positions of P_1 (see Section 3) for the curve-fitting procedures is not an obvious task, despite the guidance provided in [1]. Experimentation with various positions for P_1 has shown that more plausible results for the peak-positive acoustic pressure are obtained when choosing points where ringing is still slightly visible on the voltage waveform.

Overall, it appears that the exponential fit can provide an estimate of the peak pressure within $\pm 5\%$ of the true value for systems that are of sufficient bandwidth to encompass the dominant frequency content of the original pressure waveform without causing excessive resonance. Band limited systems and resonant systems with a resonance at a relatively high frequency fall under this category. Hence, systems like the 25 μm *pvd*f thickness, 0.5 mm element diameter membrane hydrophone with matched amplifier, producing a combined response of $\pm 1\text{dB}$ between 2 MHz and 16 MHz, as well as the theoretical 5 μm *pvd*f thickness, 0.5 mm element diameter theoretical membrane hydrophone, used to mimic a wide bandwidth system respond particularly well. The estimated peak pressure is respectively within 4.1% and within 3.7% of the true pressure for each of these systems (see tables 4.1 and 4.4).

In the other two systems considered here, i.e. the 15 μm *pvd*f thickness, 0.4 mm element diameter membrane hydrophone with amplifier and the 9 μm *pvd*f thickness, 0.5 mm element diameter membrane hydrophone with amplifier, the resonance is much more pronounced relative to the ‘flat’ region in the frequency response (see Figures 2.5 and 2.7). This causes more pronounced ripples in the output voltage waveform, and the exponential fit generally fails to provide a good estimate of the peak-positive acoustic pressure when abiding by the guidelines in [1]. Nevertheless, it appears that the exponential fit gives best results when the point P_1 is close to the centre of the oscillatory part of the waveform. For example, the point $P_{1,2}$ in Table 4.3 provides the best estimate out of the five points considered, giving an error in the peak-positive acoustic pressure of 12.2%.

Although further work would be necessary in order to confirm this, it appears that the points P_1 should be chosen so that they are located close to the centre of the oscillatory part of the waveform rather than after the ringing in the voltage waveform has sufficiently decayed, as the guidelines in [1] suggest. In Figure 5.1, for example, by selecting the position of P_1 towards the central part of the second oscillation, the error in the estimate of the peak-positive pressure turns out to be -14.6%, compared with a minimum value of -16.0% in Table 4.2.

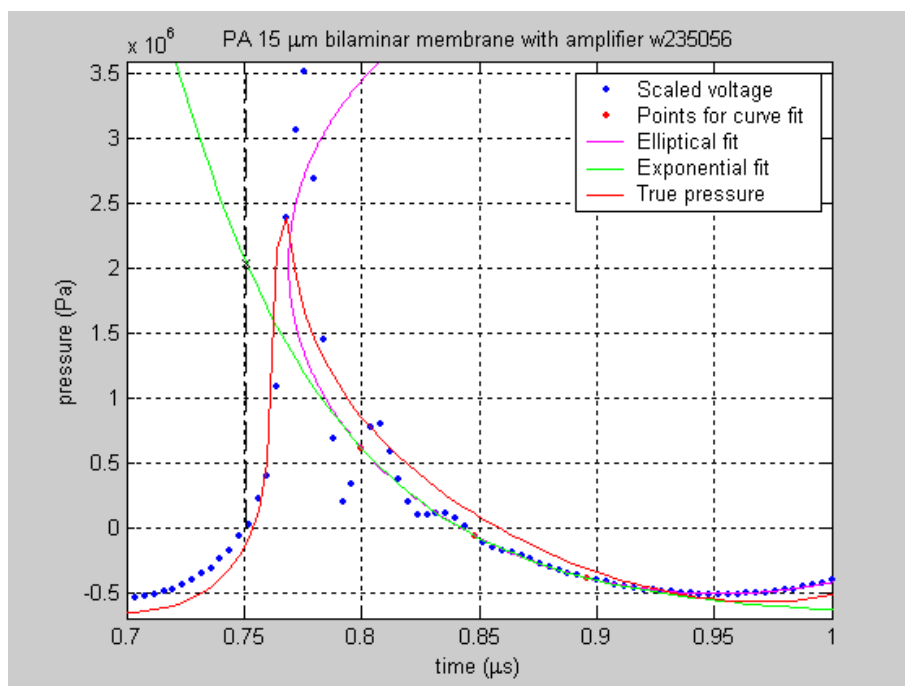


Figure 5.1: Example of curve fit and intercept for PA 15 μm membrane hydrophone with amplifier w235056.

Also, the mean value of the error in the peak-positive pressure is surprisingly low in Tables 4.2 and 4.3, considering the rather large variance in the results. This indicates that it may be sufficient to have samples that occur throughout the whole of a single oscillation of the waveform in order to obtain good results.

The elliptical fit however, provides limited success in estimating the peak-positive acoustic pressure, either because of the lack of intercept between the ordinate axis passing through the zero-crossing point immediately before the temporal-peak acoustic pressure, or simply because the algorithm described in Section 3.1.2 fails to converge (see beginning of Section 4). When a solution is attainable, the exponential fit tends to provide better results.

When band-limited white noise is superimposed on the pressure waveform described by equation (1), obtaining the estimate of the peak-positive pressure becomes increasingly worse as the noise level is increased (see Tables 4.5 – 4.8).

Using a distorted sinusoidal waveform as a pressure pulse appears to worsen the predictions made by the curve fitting routines for the peak-positive pressure. Elliptical fits are not attainable and exponential fits produce estimates of the peak-positive pressures with substantial error (e.g. 66.4 % in Table 4.9). This further highlights the limitations of the curve-fitting processes described in [1] for this purpose.

6. CONCLUSION

- The curve-fitting methods outlined in Annex A of IEC 61220 [1] fail to provide a robust means of estimating the peak-positive acoustic pressure for the acoustic waveforms and measurement systems considered here.
- However, the exponential fit can provide an estimate of the peak-positive acoustic pressure within $\pm 5\%$ of its true value, provided the system is of sufficient bandwidth to encompass the dominant frequency content of the original pressure waveform without causing excessive ringing. Results obtained using this method are nevertheless error-prone and should be used only as a guideline.
- Based on the limited amount of information from the present study, it appears that in cases where significant ringing occurs in the hydrophone voltage waveform, if the choice of fitting points on the waveform covers a full oscillation of the tail of the ringing, the mean value of the estimated peak-positive acoustic pressure will be more accurate.
- In order to obtain a more reliable estimate of the peak-positive acoustic pressure, it is more advisable to obtain the magnitude and phase characteristics of the measurement system across the bandwidth of the measured acoustic signal.

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