

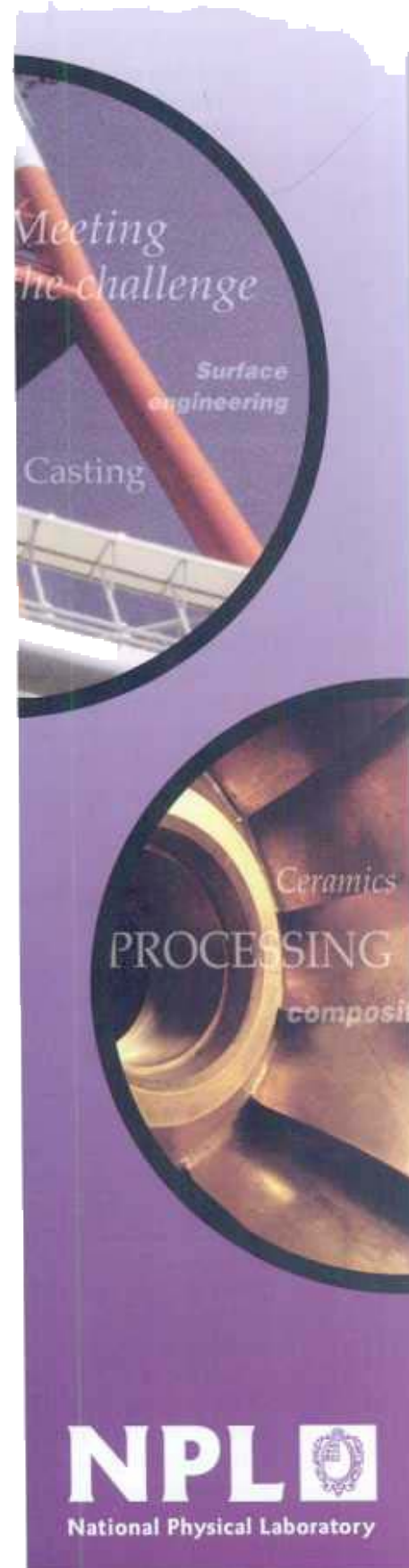
Measurement Note

Determination of Mode I Energy Release Rate in Composite Laminates by the Wedge Test

This document introduces a new test method for the mode I fracture energy of composite laminates. The geometry of the test is adapted from the double cantilever beam (DCB) and the wedge tests used for testing of adhesive joints. The analysis follows the energy release rate approach, and several models are compared. An experimental study, using a simple optical method, and electronic speckle pattern interferometry (ESPI), was conducted to validate the models. It is shown that the models based on classical or augmented beam theory fail too describe the bend profiles of the loading arms without the use of adjusting parameters. Conversely, finite element analyses are in good agreement with experimental results. The results of the wedge test and a classical DCB test are compared and it is shown that they yield different values. A number of solutions is proposed to give a better interpretation of the test.

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1. Introduction

1.1. Background

The properties of ply interfaces in any fibre-reinforced composite material have significant influence on the resultant properties of the composite itself. For most polymer composites the requirement is for a very strong interface so that the properties of the composite become susceptible to microstructural damage formation only at relatively high loading. All unidirectional fibre-reinforced composite materials exhibit significantly different axial and transverse properties. During loading the transverse properties are degraded more than axial properties because of interface failure and its consequences. The development of a strong ply interface helps to maintain transverse properties, and to minimise the differences between axial and transverse composite properties, as well as the rate of accumulation of damage during loading. The anisotropic behaviour of the laminate is the cause of mixed mode failure, that is a combination of mostly mode I (tensile crack opening) and mode II (shear crack opening). The extent to which mode III influences mixed mode failure is yet to be determined. Various methods of measuring these properties have been developed, most of which require the use of a micro-mechanical model to interpret the results of the experiments in terms of so-called interface properties. In the present work, the focus is on the determination of the interfacial fracture energy.

A number of tests have been devised to determine this parameter experimentally, the most famous being the double cantilever beam (DCB) test and the wedge test. While the DCB test has been widely studied [1-7] and is an established standard [8], little attention has been paid to the wedge test [9, 10]. Effectively, the wedge geometry is mainly used for long-term environmental testing of adhesive-bonded lap joints, and is essentially qualitative, as described by the ASTM D3762 [11].

1.2. Goals

The objective of this paper is the measurement and modelling of the energy release rate of a crack propagating essentially in mode I between two adjacent plies of a composite laminate. The geometry is that of the wedge test. First, the specimen and test geometry are introduced. Second, a number of existing models are adapted for the particular geometry of the test, and compared. The theory is then applied to practical results, making use of a simple optical method, and of a fine displacement measurement technique, electronic speckle pattern interferometry (ESPI).

2. Problem Addressed and Associated System Geometry

In this study we consider a typical double cantilever beam geometry in which a crack is propagated by advancing a wedge along the interface line, as shown in Figure 1. The beam of length L , section of surface Ω (in case of a rectangular section, width b and thickness h) is subjected to a load P applied at a distance a from its support of dimensions c and h . The crack tip is taken as the origin of coordinates and downward deflections are considered as positive. A point on the beam may be identified by four quantities of which only one is independent: the two rectangular coordinates x and y , the arc length s and the deflection angle ψ which is the angle of the tangent at the point under discussion and the horizontal. Additionally, one defines the horizontal component Δ of the displacement of the loaded side of the beam, and the corresponding displacement δ , as shown in Figure 1. The beam has a uniform cross-section of moment of inertia I and is composed of a material whose modulus of elasticity is E (isotropic material or anisotropic material whose x -axis corresponds to one principal direction) or a function of two or more elastic constants (anisotropic materials in general orientation).

Because composite laminates are generally of high dimensional aspect ratio, the corresponding displacement of the loaded end is often large compared to the dimensions of the beam section. Therefore, the elementary beam theory cannot be used in most cases, as the basic assumption that the slope of the deflection dy/dx is small, is not verified in principle. The next section summarises the most important approaches that have been made at the determination of the mode I energy release rate. A number of other models [2, 4-6] have been shown to yield similar results as the ones introduced and will not be discussed.

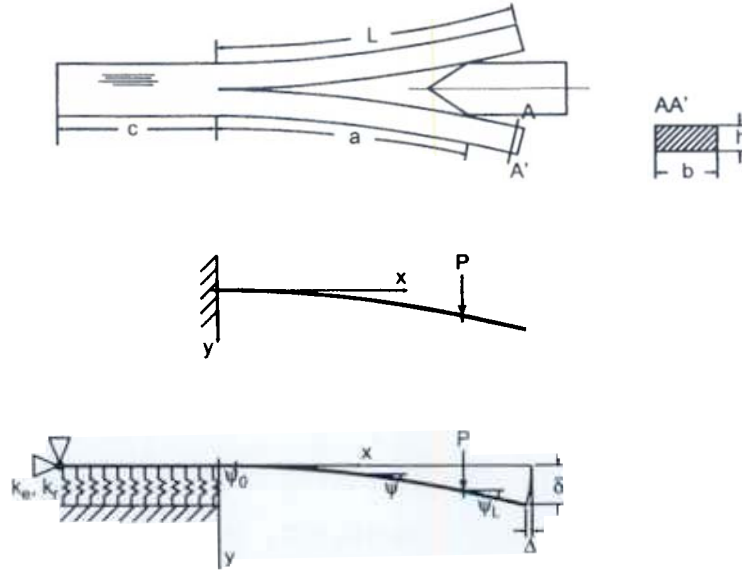


Fig. 1. Double cantilever beam specimen used in the wedge test, with a support of dimensions c and h , and associated problems: cantilever with a fixed end at support, and cantilever on an elastic foundation with stiffness per unit length characteristics of k_e in extension and k_r in rotation

3. Crack Propagation Models for DCB

3.1 Linear Analysis

The most basic form of the bent profile equation is obtained by expressing the curvature of the beam $1/R$, where R is the radius of curvature, as a function of the moment of the force applied at the end of the cantilever:

$$\frac{1}{R} = \frac{y''}{(1 + y'^2)^{3/2}} = -\frac{M}{EI} \quad (1)$$

where I is the moment of inertia of the beam, and equals to $bh^3/12$ in the simple case of a rectangular cross-section, as shown in Figure 1. In the case of small deflections, it is usual to treat the squared derivative as small compared to 1, and the equation of the beam profile is therefore given by, after two integrations:

$$y = -\frac{M}{2EI}x^2 + C_1x + C_2 \quad (2)$$

The constants C_1 and C_2 are determined by the boundary conditions, which are zero displacement and rotation at $x = 0$, i.e. $y(x = 0) = 0$ and $y'(x = 0) = 0$. As a result, both constants are equal to zero and the profile is described as:

$$y = -\frac{M}{2EI}x^2 \quad (3)$$

The strain energy due to bending of a beam of length a is, according to elementary beam theory:

$$W = \int_0^a \frac{M^2}{2EI} ds \quad (4)$$

from which the classical result for the linear theory can be obtained

$$\frac{Wa}{2EI} = \frac{3}{2} \left(\frac{\delta}{a} \right)^2 \quad (5)$$

The energy release rate is defined as:

$$G \equiv \frac{1}{b} \frac{\partial W}{\partial a} \quad (6)$$

which yields the non-dimensional form:

$$\frac{Gba^2}{2EI} = \frac{9}{2} \left(\frac{\delta}{a} \right)^2 \quad (7)$$

3.2. Large Deformation Analysis

It was argued by Devitt et al. that as a result of the low flexural rigidity of thin composite specimens, large deflections and rotations may be present during the test and need to be accounted for in the analysis [1]. These corrections were introduced separately by Barten [12, 13] and by Bisshopp and Drucker [14], their theory allowing for arbitrarily large rotations and bending deflections. The model resulted in expressing the ratio δ/a as a function of elliptic integrals:

$$\frac{\delta}{a} = 1 - 2 \frac{E(k) - E(k, \theta_1)}{F(k) - F(k, \theta_1)} \quad (8)$$

where $F(k)$ and $E(k)$ are the complete elliptic integrals of the first and second kind, respectively, and $F(k, \theta_1)$ and $E(k, \theta_1)$ are the corresponding incomplete elliptic integrals. The elliptic parameters k and θ_1 are related to ψ_0 by:

$$k \equiv \frac{1}{\sqrt{2}} (1 + \sin \psi_0)^{\frac{1}{2}}, \quad \theta_1 \equiv \sin^{-1} (\sqrt{2}k)^{-1} \quad (9)$$

The total strain energy is given by:

$$\frac{Wa}{2EI} = 2(F(k) - F(k, \theta_1))^2 \left[\frac{E(k) - E(k, \theta_1)}{F(k) - F(k, \theta_1)} + k^2 - 1 \right]$$

and the strain energy release rate is therefore, with applying Eq. 6:

$$\frac{Gba^2}{2EI} = \frac{Wa}{2EI} \left(1 + \frac{d(\log(f))}{d(\log(\delta/a))} \right)$$

with $f = f(\delta/a) = \frac{Wa}{2EI}$.

3.3. Augmented Models

3.3.1. Beam on a Winkler Foundation

A more rigorous crack propagation analysis was proposed by Kanninen [15], where the beam model takes direct account of the region beyond the crack. This is obtained by treating a finite length beam which is partly free and partly supported by an elastic foundation. The theory was derived from the Euler-Bernoulli beam theory and the Winkler foundation. This type of foundation exhibits an extensional stiffness k_e , and is modelled as shown in Figure 1. Solving the governing differential equation for the beam deflection with appropriate boundary conditions, and matching the values of the deflection and its first three derivatives at $x = 0$, this author obtained:

$$y(x) = \frac{6P}{Ebh^3\lambda^3} \left(\frac{\lambda^3 x^3}{3} + a\lambda^3 x^2 - A\lambda x + B \right), \quad x \geq 0 \quad (12)$$

and

$$y(x) = \frac{6P}{Ebh^3\lambda^3} \left(a\lambda \sin \lambda x \sinh \lambda x - \frac{A-1}{2} \sin \lambda x \cosh \lambda x + B \cos \lambda x \cosh \lambda x - \frac{A+1}{2} \cos \lambda x \sinh \lambda x \right) \quad x \leq 0$$

where

$$A = \frac{\sinh^2 \lambda c + \sin^2 \lambda c}{\sinh^2 \lambda c - \sin^2 \lambda c} + 2a\lambda \frac{\sinh \lambda c \cosh \lambda c + \sin \lambda c \cos \lambda c}{\sinh^2 \lambda c - \sin^2 \lambda c}$$

and

$$B = \frac{\sinh \lambda c \cosh \lambda c - \sin \lambda c \cos \lambda c}{\sinh^2 \lambda c - \sin^2 \lambda c} + a\lambda \frac{\sinh^2 \lambda c + \sin^2 \lambda c}{\sinh^2 \lambda c - \sin^2 \lambda c}$$

$$\lambda^4 = \frac{k}{4EI} \quad (16)$$

Adapting the solution to the geometry of the wedge test problem, with $\delta = y(a)$, the relation for the specimen compliance, i.e. the unit deflection at the point where the load is applied, can be written:

$$\frac{\delta}{P} = \frac{\Phi}{Eb} \quad (17)$$

where

$$\Phi = \frac{2}{\lambda^3 h^3} \left[2\lambda^3 a^3 + 6\lambda^2 a^2 \frac{\sinh \lambda c \cosh \lambda c + \sin \lambda c \cos \lambda c}{\sinh^2 \lambda c - \sin^2 \lambda c} + 6\lambda a \frac{\sinh^2 \lambda c + \sin^2 \lambda c}{\sinh^2 \lambda c - \sin^2 \lambda c} + 3 \frac{\sinh \lambda c \cosh \lambda c - \sin \lambda c \cos \lambda c}{\sinh^2 \lambda c - \sin^2 \lambda c} \right] \quad (18)$$

The stiffness parameter of the foundation is not independent of the beam parameters, because it compensates for the missing half of the specimen. The approach followed by the author and others is to relate the ratio of the beam deflection to the half height of the beam with the average transverse strain in the specimen, and the foundation reaction with the transverse stress [15, 3]. This semi-empirical approach was shown to compare well with finite element (FE) and experimental results with isotropic materials [15]. To a first approximation, it can be written:

$$\sigma = \frac{ky(x)}{b} = E\epsilon = \frac{Ey(y)}{h/2} \quad (19)$$

As a result,

$$k = \frac{2Eb}{h} \quad (20)$$

and

$$\lambda = \frac{6^{1/4}}{h} \quad (21)$$

Finally, it is possible to write the elastic strain energy of the beam $W(a)$ as:

$$W(a) = \frac{Eb\delta^2}{\Phi} \quad (22)$$

This form is introduced for crack extension under constant displacement conditions. The energy release rate is then obtained from Equation 6, and as a result:

$$G = \frac{E\delta^2}{\Phi^2} \dot{\Phi} \quad (23)$$

$$\dot{\Phi} = \frac{\partial \Phi}{\partial a} - \frac{\partial \Phi}{\partial c} = \frac{12}{\lambda^2 h^3} \left[\lambda a \frac{\sinh^2 \lambda c + \sin^2 \lambda c}{\sinh^2 \lambda c - \sin^2 \lambda c} + \frac{\sinh \lambda c \cosh \lambda c - \sin \lambda c \cos \lambda c}{\sinh^2 \lambda c - \sin^2 \lambda c} \right]^2 \quad (24)$$

3.3.2. Beam on a Generalised Foundation

In order to deal with dynamic, unstable crack propagation, Kanninen developed a more advanced formulation of the problem, where the rotation of the beam is accounted for [16]. The model consists in a Timoshenko beam [17, 18] on a generalised foundation of extension and rotation stiffness of k_e and k_r respectively, whose geometry is shown in Figure 1. In the Timoshenko beam equations, the effective transverse shear strain is taken to be equal to the average shear stress on a cross section divided by the product of the shear modulus and a dimensionless shear coefficient. To evaluate this latter quantity, Kanninen used the results of Cowper [19], which permitted the calculation of the rotation stiffness of the foundation k_r . Eventually, an approximate expression for the strain energy release rate G was obtained, together with an expression for the compliance of the specimen δ/P :

$$G = \frac{P^2 a^2}{bEI} \left[1 + \frac{\sqrt{5} h}{2 a} + \frac{1}{2} \left(\frac{h}{a} \right)^2 \right] \quad (25)$$

$$\frac{\delta}{P} = \frac{a^3}{3EI} \left[1 + \frac{3\sqrt{5} h}{4 a} + \frac{2}{3} \left(\frac{h}{a} \right)^2 + \frac{3\sqrt{5}}{16} \left(\frac{h}{a} \right)^3 \right] \quad (26)$$

3.4. Model Analysis

To summarise the different approaches, it is interesting to compare the non-dimensional energy release rate $\frac{Gba^2}{2EI}$ obtained from the various models, as shown in Figure 2. As expected, the models converge for small δ/a ratios, obtained when the crack length a is large compared to the arms opening δ . In particular, the non-linear beam theory converges towards the classical beam theory for small δ/a ratios, as $\frac{d(\log(f))}{d(\log(\delta/a))}$ tends towards a value of 2 in Equation 11. It is important to note that the non-linear beam theory has been shown to provide a better description of the bending profile of thin laminates, but has not been tested versus other models for the energy release rate. Because it accounts for beam rotations and therefore corresponds to the maximum displacement of the arm that can be obtained, the non-linear beam theory is an upper-bound to the value of the energy release rate. By comparison, the displacement of the arm is decreased when a foundation is introduced. The same trend as with the built-in beam theories is observed, i.e. the energy release rate of the Timoshenko beam on generalised foundation model, which accounts for rotation of the arm, is always larger than that of the Euler-Bernoulli beam on a Winkler foundation. Although the Euler-Bernoulli beam is a lower bound of the problem here, a number of other foundation models have been devised [20], and models that allow for viscous dissipation should yield lower values of the non-dimensional energy release rate. It is worth noting that most of the crack propagation studied in this document were obtained for ratios of $0.22 \leq \delta/a \leq 0.3$. At the higher end of this interval, the difference between the two foundation-based models is about 20%, while it is more than 200% between foundation and built-in models. At the lower end of this interval, these values reduce to ca. 10% and 140%. Therefore, a large difference between the calculated values can be expected.

Finally, it is important to mention that the models presented do not account for residual stresses. That is essentially because symmetric, balanced laminated are used in the study, and residual stresses do not have an influence with such stacking sequences. By opposition, if non-symmetric or unbalanced laminates were to be used, the energy release rate would need to be corrected, e.g. following the approach introduced by Nairn [7]. In the case of non-symmetric laminates, the correction terms are expected to have a large influence on the results, because residual stresses contribute to the energy release rate by releasing strain energy and doing external work as the crack grows. This author further

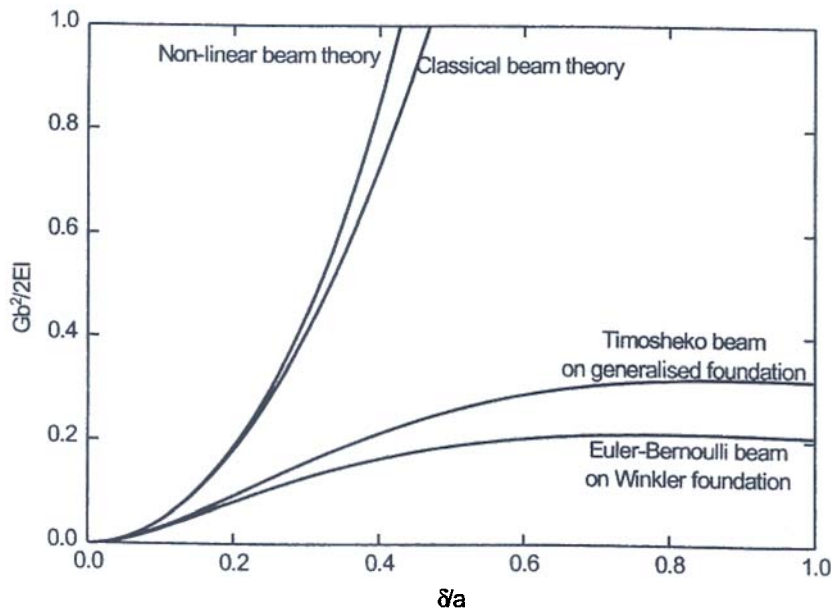


Fig. 2. Non-dimensional energy release rate for cantilever beam

showed that the energy release rate can be readily evaluated from the expression:

$$G = \left(\sqrt{G_{mech}} + \frac{P \frac{dv_r}{da}}{\sqrt{G_{mech}}} \right)^2 \quad (27)$$

where v_r is the y -displacement due to residual stresses, and G_{mech} is the energy release rate calculated from mechanical loading (i.e. obtained following the approach described in this section). This last expression is exact, however the remaining equations of Ref. [7] should not be used in all cases without further validation, since they are all based on linear beam theory. The evaluation of v_r by means of non-linear or foundation theories is clearly beyond the scope of the present work.

4. Experimental

4.1 Materials

Symmetric glass fibre-reinforced epoxy laminates were manufactured from a Hexcel-913 prepreg and tested using the wedge test. This material was used as its thermo-mechanical properties have been extensively documented in previous studies at NPL [21, 22]. These values are reported in Table I.

The specimen dimensions were $125 \times 20 \times 3 \text{ mm}^3$, and the stacking sequence was $[0_{12}]_s$. An approximate initial crack length of 60 mm was fixed by the length of teflon film inserts. Composites plates of dimensions of approximately $250 \times 200 \times 4 \text{ mm}^3$, having teflon film inserts of 120 mm width, were manufactured in an autoclave following the processing conditions prescribed by the manufacturer, which are 1 hour at 120°C and 7 bar pressure (heating up rate of $3^\circ\text{C}/\text{min}$). The plates were then cooled down to the room temperature at a rate of approximately $2.5^\circ\text{C}/\text{min}$, and specimens were cut to the specified dimensions. The specimens were then stored in a dry environment until the tests were carried out.

4.2 Equipment

Electronic speckle pattern interferometry (ESPI) offers very precise measurement of the thermal expansion coefficient and elastic properties of composite specimens with relatively large dimensions and aspect ratios, such as plates or entire components, or locally within a larger specimen or part. It is a full field measurement method with which deformations under mechanical and thermal loads can be measured along three perpendicular axes without contact with the specimen [23–27]. Sample preparation is easy and requires a minimum of handling [28].

The speckle effect is produced by the interference between the reflected rays when a non-specular, dispersing surface is illuminated with a laser beam [23]. The pattern is defined by the microscopically rough topography of the specimen surface. Any deformation of the surface results in a change of the speckle pattern. A correlation between the speckle patterns for an object in a reference state and in a deformed state gives information on the deformation of the object's surface. This is typically done by gray level difference of two digital video interferograms and some additional digital image analysis. For each interferogram, three or four images of the interference pattern are taken with a phase shift of the laser beam of 120° or 90° respectively, and they are combined to give a phase image, which holds information not only on the amount, but also on the direction of the deformation.

The ESPI installation used in this study (Ettemeyer AG, Germany) consists of an ESPI head, a hardware control and data acquisition system, and an image analysis facility. The software Istra[®] v.3.3 was used for the image processing. A compact unit, described in a previous work [22], allows the quasi simultaneous measurement of in- and out-of-plane displacements. The in-plane displacements are given by the relative phase shift of two imaging beams. The out-of-plane displacements, i.e. parallel to the optical axis, are obtained from the amount of phase shift of an imaging beam with respect to a reference

Table I. Thermo-mechanical properties of GF/epoxy laminate Hexcel-913

$E_{11} = 42.288 \text{ GPa}$	$E_{22} = 14.034 \text{ GPa}$
$G_{12} = 6.465 \text{ GPa}$	$G_{13} = 4.600 \text{ GPa}$
$\nu_{12} = 0.271$	
$\alpha_{11} = 7.052 \times 10^{-6} \text{ m/m/K}$	$\alpha_{22} = 17.695 \times 10^{-6} \text{ m/m/K}$

Table II. Optical configurations of the ESPI set-up

Lens	Distance to object (mm)	Imaging field (mm × mm)	Resulting calibration (mm/pixel × mm/pixel)	s_x, s_y $\mu\text{m}/\text{fringe}$
12mm F*8	392	122 × 91.7	0.163 × 0.158	2.22
16mm, 2* extender	352	37.4 × 28.4	$4.98 \cdot 10^{-2} \times 4.88 \cdot 10^{-2}$	2.00

beam. One can thus measure the 3-D deformation of an object under a load. This integrated ESPI head includes the laser source, mirrors, piezos, shutter, and video camera. The measurement sensitivity of the system, defined as the amount of relative displacement giving rise to an additional interference fringe, is

$$\text{out-of-plane: } s_z = \frac{\lambda}{2}; \quad \text{in-plane: } s_{x,y} = \frac{\lambda}{2 \sin \theta} \quad (28)$$

where λ is the wavelength of the laser and θ the angle between the imaging beams and the optical axis. The speckle interferometer uses near infrared laser diodes ($\lambda = 0.780 \mu\text{m}$), giving an out-of-plane measurement sensitivity of $0.390 \mu\text{m}/\text{fringe}$. The distance between the front mirrors forming the imaging beam sources was 140 mm and the distance to the object was varied following the values given in Table II. Table II presents a range of different configurations successfully used with NPL's set-up, and indicates the in-plane measuring sensitivity for both in-plane axes. Since information on the displacement gradient is carried by the entire phase image, and not just by the fringes, the effective in-plane sensitivity is about an order of magnitude better. The limiting factor of the sensitivity is the inherently high noise of the method resulting from the speckle pattern.

Images were recorded with a 752×582 pixel built-in CCD camera operating at 25 Hz. A set of Pentax Cosmimar[®] macro lenses was used on the front end of the viewing system, giving an imaging field presented in Table II. The resulting calibration of the monitor screen in the x and y -directions, together with the lateral resolution achieved are also given in Table II. The time required for a measurement, determined by the frequency of the video camera and the speed of the shutters and phase shifting piezo-devices, was approximately a third of a second for each axis.

4.3. Loading Sequence & Analysis

Given the finite time required for each measurement, the mechanical tests were conducted under quasi-static conditions. The loading sequence processed from 0 kN to the complete failure of the specimen by imposed steps of $60 \mu\text{m}$ at a loading rate of 1 mm/s. The resulting time between two consecutive load-steps was approximately 5 s, during which the measurement of the displacements in the three directions were triggered to start after the load had been reached for 1 s, so as to reduce possible interference from the vibrations of the machine. Data reduction was carried-out by applying a phase filter to the raw data, followed by the unwrapping sequence. Since the noise level - inherent to the method - was low, higher order filters were not needed. The median filter used and its characteristics are documented elsewhere [29]. The deformation fields can be evaluated from the filtered phase images along a line or over the entire area. A full description of the method of analysis has been given in a recent Measurement Note [22].

5. Results - ESPI

5.1. Wedge Test

A typical force-displacement graph obtained from the wedge test of a $[0_{12}]_s$ laminate is presented in Figure 3. First of all, it must be remarked that the measurement involves a large noise factor, associated with both the friction of the wedge on the inside edges of the laminate, and the sensitivity of the load cell. The latter could be greatly improved, as a 100 kN load cell was used in this study, which shows some limitations below 100 N. However, the data reduction is not based on the load since the friction coefficient is unknown, which minimises the importance of this experimental error. As the wedge is inserted along the interface line, the load increases progressively until the crack starts to propagate. This occurs as a plateau limit value of the load is reached, and the load could therefore be considered as

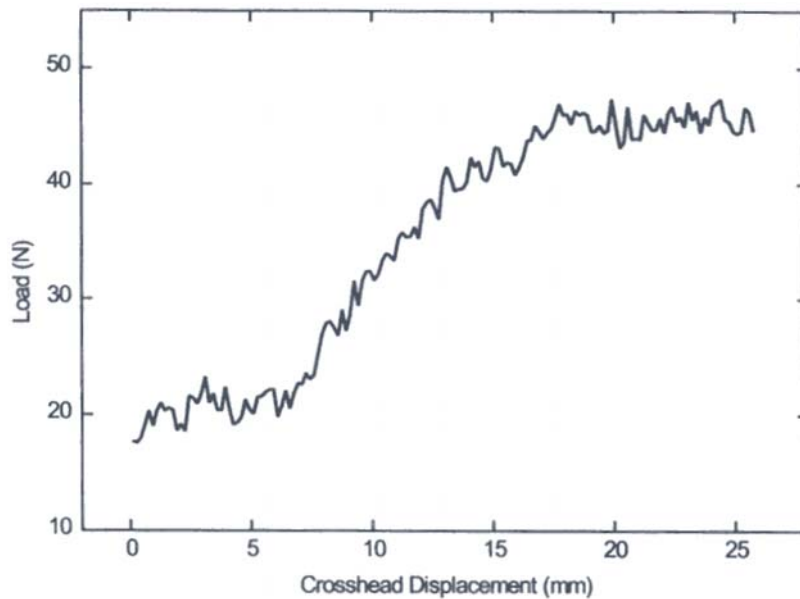


Fig. 3. Experimental load vs. displacement curve of a wedge test of a $[012]_s$ Hexcel-913 laminate

a constant while the crack propagates. It is also important to notice that the crack propagation is stable in this experiment.

5.2. Image Analysis

In order to assess the suitability of the models introduced in the previous section, the profile equations were compared to experimental bending profiles determined from the wedge test. The bend profiles were determined directly from a photograph taken using the CCD camera of the speckle interferometer. The scale factors were known from the optical set-up, as indicated in Table 1. The image treatment was conducted according to the following procedure:

- the position of the edges was measured in pixels and converted to mm
- the data were rotated by $\pi/2$
- the data were centred: an horizontal translation was applied to place the origin at the crack tip
- the slight asymmetry between arms was corrected for by averaging inner borders (left/right) and outer borders (left/right)
- the neutral plane was obtained as the mean of the inner and outer borders

The results of this approach are shown in Figure 4. It is obvious that the models' agreement is rather poor, as none of the models fit the bent profile of the neutral plane satisfactorily. At this point, it must be noted that the axial Young's modulus of the laminate was used, which is not the case of most the approaches of the literature, where a 'bending' or 'flexural' modulus is generally introduced. However, one should be extremely cautious when using such a modulus, because it is at best a way to fit experimental data, and does not have a scientific justification. In fact, the bending modulus is generally justified by the need to accommodate for rotations and distribution of shear within the beam, which would make it similar to an apparent modulus. However, this notion is properly inexact and should be abandoned, as it has already shown crude limitations for the determination of the mode I energy release rate in composite laminates. One problem is then: which theory is the most appropriate to replace it? The four models used here give a poor description of the experimental values, and worryingly spread

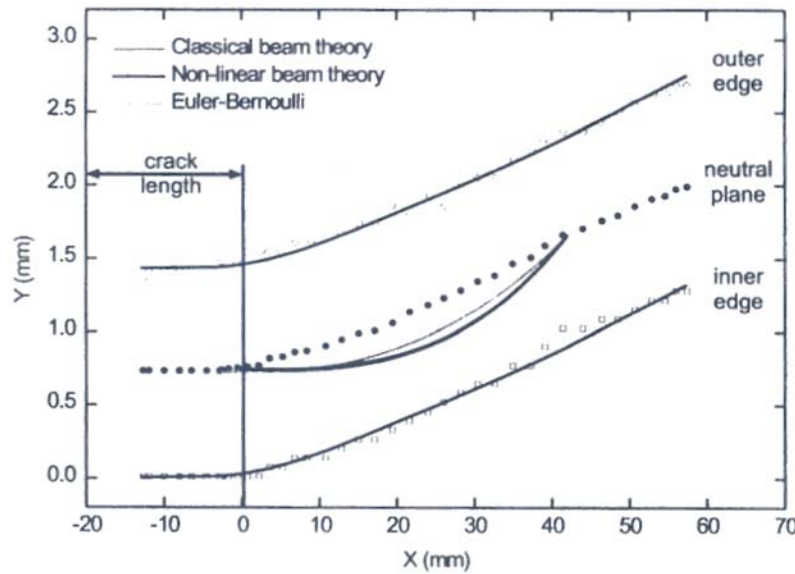


Fig. 4. Experimental bend profiles obtained from image analysis of the wedge test as shown on the right (symbols, lines are guides for the eyes), versus models (lines)

over large differences when used to calculate the energy release rate. Following the trend seen on the energy release rate plots in the previous section, it seems that the Timoshenko beam on a generalised foundation would be closer to experimental values, although the calculation of the complete profile was not carried-out here. At the moment, it seems however that the model needs to be modified to a large extent, and other types of foundation should be used instead.

In particular, the models tested do not accommodate well the rotation at the crack tip, as most of the error on the profile is present close to the origin. This again could well be corrected by using a different value of the Young's modulus of the beam in the x -direction. Also, a better solution would be to tackle the problem at its source, and introduce a more compliant foundation allowing for large rotations at the origin. In order to ascertain these hypotheses, a finite element model of the wedge test was constructed. It consisted in the exact geometry, the mesh comprised 2520 plane strain, quadratic quadrilateral elements. The solver used was LUSAS 134, with QPN8 elements. The mechanical properties of Table 1 were used with a plane strain orthotropic constitutive law. The geometry was formed from two beams held together by 60 interface elements of same stiffness as the composite. Because interface elements were used, LUSAS' non-linear solver was needed: convergence was obtained after one iteration, as the interface toughness was given a very large value so that damage was not allowed to propagate. The results of Figure 5 clearly show an excellent agreement between the experimental and FE bend profiles. This definitely proves that, although the models used poorly describe the bend profiles, the solution is not out of reach. For clarity, the FE model comprised the whole geometry, and did not include any symmetry axis. It is very important to notice that the good agreement between FE and the experiment was obtained with mechanical properties of the laminate that were measured by independent experiments, and that there was no adjustment of the compliance of the beams. In case no better analytical solution is found in the future, finite elements together with an inverse method are definitely able to provide a pertinent analysis of this test. In addition, the displacement profiles obtained by FEA are close to the ones determined using ESPI, in the next section.

5.3. ESPI

Although the models have failed to describe the most basic measurement method available, based on simple optics and described in the previous section, it is still interesting to examine the results of a fine

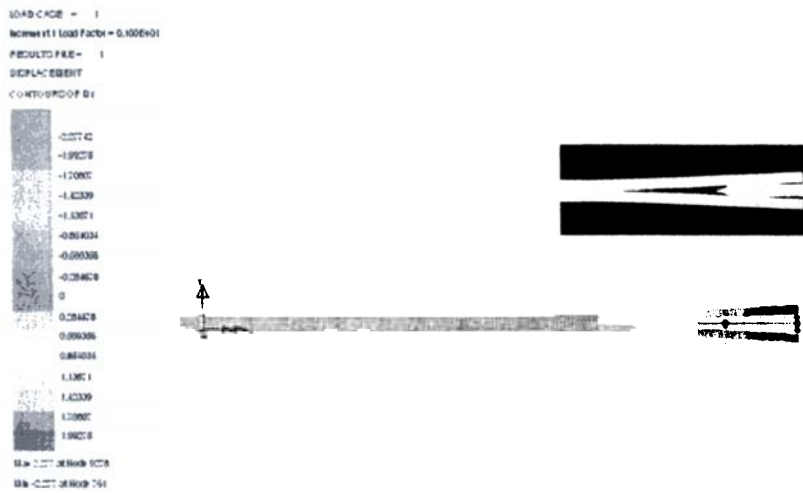


Fig. 5. Comparison of the experimental and FE bend profiles

measurement method to help the development of new models and ultimately check their accuracy. In this section, ESPI is used with the wedge test. Under the same experimental conditions as above, the y -displacements of the wedge system were determined for a number of consecutive displacements of the tensile tester's crossbar, and the results are shown in Figure 6.

Although a slight misalignment was found in the image analysis, and could be corrected as part of the experimental error by averaging the left and right arms bend profiles, this cannot be done anymore. The displacement profiles shown in Figure 6 show some large differences and therefore, ESPI can be used as a qualitative measure of the validity of the test. In the present case, the test should be discarded and the geometry changed. It must be noticed as well that the displacements measured are small, as the mismatch between the two arms is about $5\mu\text{m}$ at the end.

Furthermore, displacements are measured accurately and a smooth profile is obtained in the two arms. This is particularly encouraging, as the displacements are negligible in the bonded part of the joint far from the origin, but begin to increase before the crack tip: a change of slope is seen for $-2 \leq x \leq 0$. Moreover, the fact that this result was noticeable even for a rough quality of fringes and a set-up that was not optimised, tends to prove that it is possible to extract quantitative data to help determine the values for an improved foundation model. The confidence in this method is further increased by the similarity of the displacement patterns shown in Figures 5 and 6.

6. Results - Energy Release Rate

In this final section, the mode I energy release rate is evaluated based on the models introduced. Firstly, a classical DCB experiment was carried-out for comparison purposes. This test followed the ASTM D5528-94a, and the energy release rate was determined following the "modified compliance calibration method (MCC)" [5, 6, 8]. The test was carried-out for five specimens, and the results are shown in Figure 7. On this graph, the usual trend is observed, that the energy release rate increases with increasing crack length. Large variations were also observed between specimens, so that the resulting confidence interval is important. Nevertheless, all specimens show the same trend, and this type of result is generally considered as being acceptable in the literature. However, it must be noticed that the regression coefficients calculated to correct the compliance of the specimens were all close to unity, and do not reflect the large variations between specimens. The reproducibility of the test is therefore considered as poor, and the method of analysis is clearly not adapted to the description of the experiment.

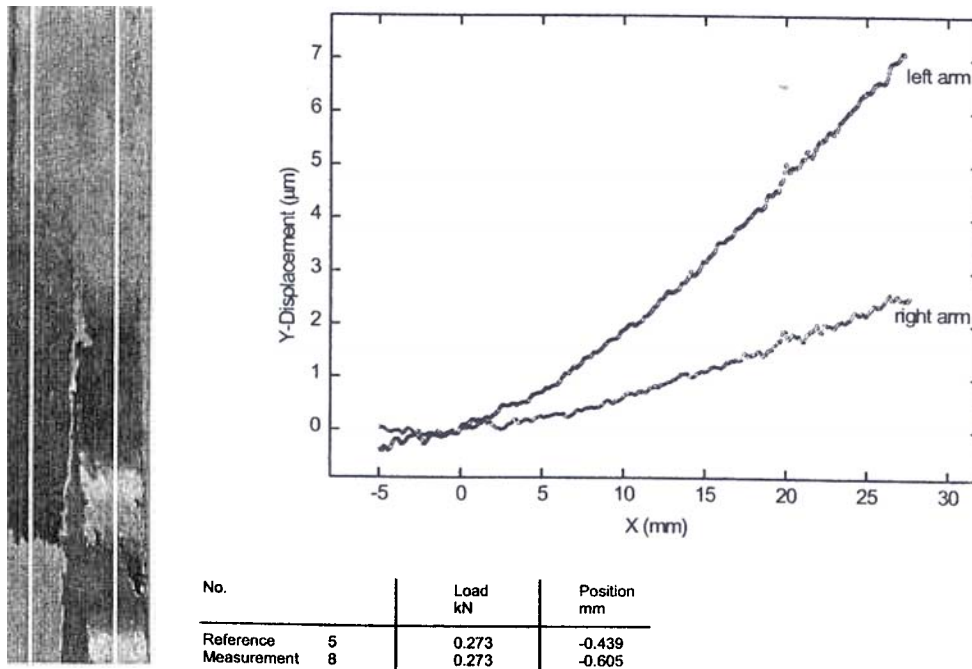


Fig. 6. Displacements in the y-direction obtained by ESPI for the wedge test

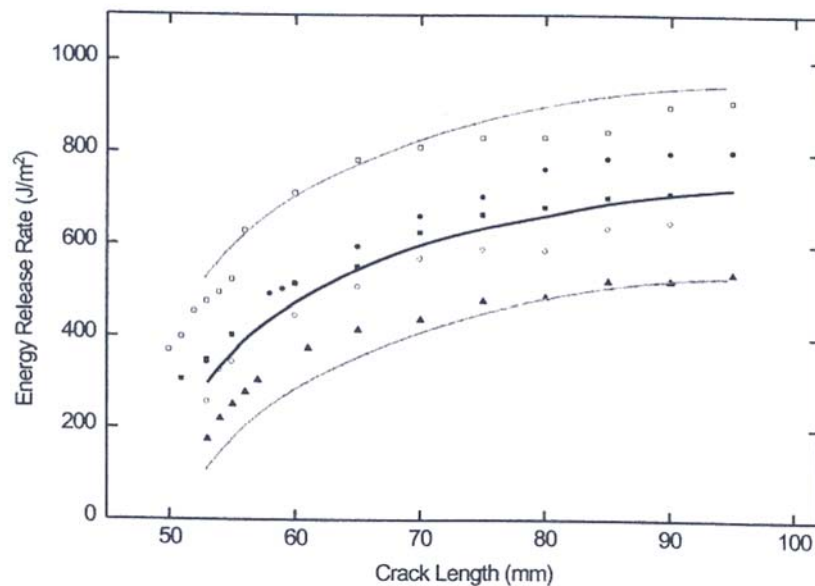


Fig. 7. Mode I energy release rate of a $[012]_s$ Hexcel-913 laminate by the DCB with end-blocks method. The black line is the average of all experimental values, while the grey lines fix the confidence interval

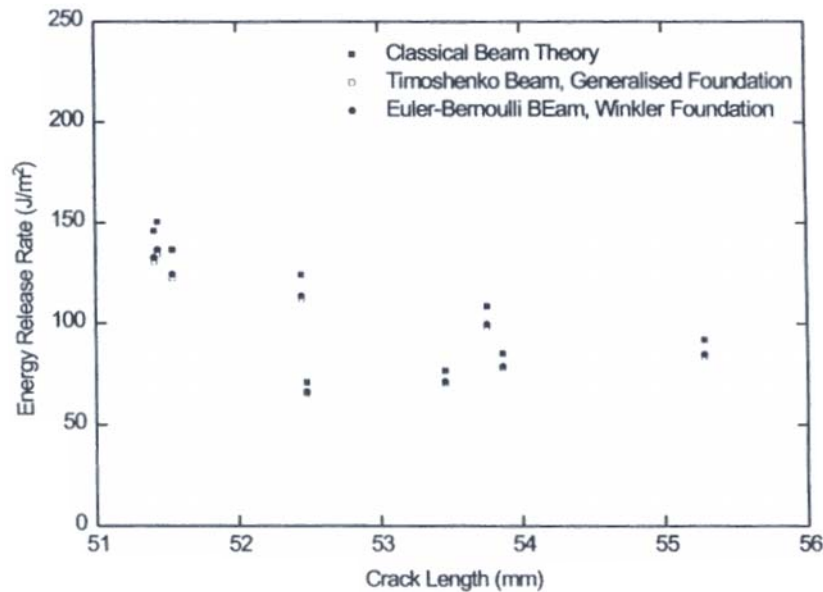


Fig. 8. Mode I energy release rate of a $[0_{16}]_s$ Hexcel-913 laminate by the wedge test. Results are presented as a function of the crack length as defined in Fig. 1

Other methods of interpretation described in the ASTM standard, i.e. the modified beam theory and the compliance calibration methods, essentially yielded the same results.

The wedge test is an alternative method, which corrects for one major disadvantage of the DCB geometry, that is the lack of control of the position of the free end of the DCB. In the wedge configuration this end is clamped in the jaws of the tensile tester. However, the analysis is based on the geometry alone, as the load applied in the y -direction is unknown, because the friction coefficient between the wedge and the two arms is unknown. Keeping in mind that the bend profiles are not well-defined, as shown in the previous section, the energy release rate is plotted versus the crack length, as defined in Figure 1, and versus the distance of the crack tip to the edge of the split-arms. The results are shown in Figures 8-9. By using two different plots, it is possible to make a comparison between the wedge and DCB test methods. In Figure 8 the energy release rate is shown to decrease in average, but has an erratic behaviour. This decrease in energy release rate with increasing crack length is better seen in Figure 9, where the crack length is taken as the distance between the crack tip and the point of application of the wedge. The decrease is essentially due to the non-accurate description of the bend profile by the models, which results in a large underestimation of the elastic energy stored in the loading arms. Nevertheless, it is worth noting that the two foundation-based models fall close to each other, while the classical beam theory and the non-linear beam theory yielded essentially the same result. Although the present theories do not provide an acceptable description of the wedge test, the experimental results are thought to be promising enough to further develop micro-mechanical models which account for the peculiarities of the boundary conditions of this test. The methodology introduced above can be used to validate these models, which should in turn yield a constant value of the energy release rate versus the crack length. ESPI, together with the R-curves, should also be used to ascertain the validity of the concept of a crack length.

Finally, it is important to notice that the results from the two methods, namely the DCB and the wedge test, have little in common. Prospectively, it is possible to say the wedge test values should be higher, because of the poor description of the deformed beam profiles by the models. However, the error obtained from the DCB test does not permit to disclose one method for the other, and an optimisation of the DCB test geometry is definitely needed to make these results comparable.

Mode I Energy Release Rate

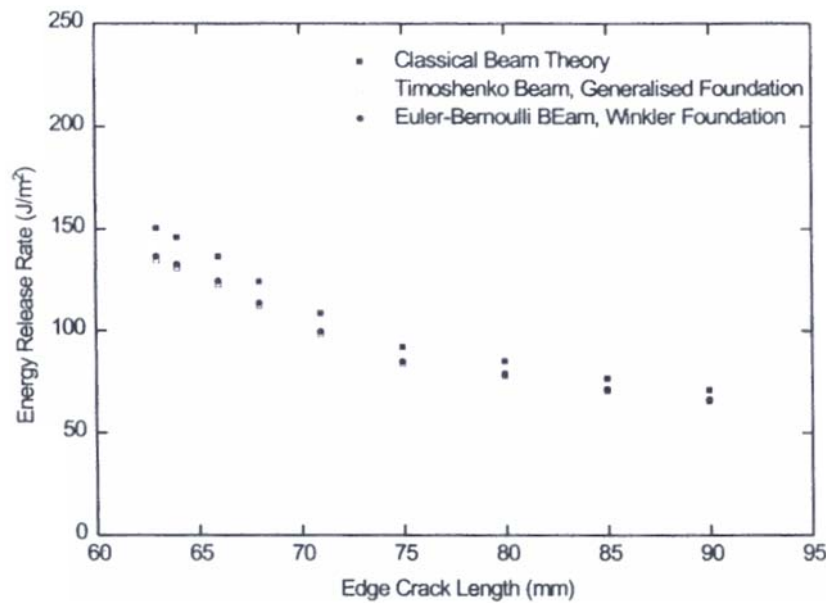


Fig. 9. Mode I energy release rate of a $[0_{16}]_s$ Hexcel-913 laminate by the wedge test. Results are presented as a function of the distance of the crack tip to the edge of the split arms

7. Conclusions

This study introduced a new geometry for the determination of the mode I energy release rate, that of the wedge test. Four alternative theories were introduced to interpret the test results, based on:

- the classical beam theory
- the non-linear beam theory
- the Euler-Bernoulli beam on elastic foundation theory
- the Timoshenko beam on generalised foundation theory.

It was shown that none of these theory could be used to predict the bend profiles of the arms in the wedge test configuration. The importance of working with a well-defined Young's modulus, and not with a corrected compliance, was discussed. Better fits could have been obtained by modifying the Young's modulus accordingly, which is in violation with the basic hypotheses of the models. Furthermore, such modification is not needed to obtain a good description of the bent profiles by finite elements. The feasibility of the wedge test was dully proven, and it was shown that ESPI could be used as a very simple means to validate the test geometry. Conversely, ESPI could not be used with the classical DCB test, which has poorly defined boundary conditions.

Although the determination of the energy release rate was greatly impeded by the bad description of the bend profiles by the models, the experimental results were found good enough to encourage further modelling work on this test.

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