

A Comparison of the Data Requirements and Predictive Accuracy of the Cavitation Model and a Simple Model for Non-Linear Behaviour in Finite Element Systems

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SUMMARY

Two elastic-plastic models are considered here for describing the non-linear deformation of tough plastics at large strains. These models can be used to predict the impact performance of a component up to failure. One is a relatively simple model that is commonly implemented in commercial finite element systems. The limitations of this model for use with certain types of plastics are illustrated with results for an ABS and a propylene-ethylene copolymer. The limitations arise with materials that undergo stress whitening through cavitation under tensile stress states. The second model has been developed to describe the influence of cavitation on deformation behaviour. Two formulations of this model have been developed and are needed to model the behaviour of two types of cavitating material, rubber-toughened plastics and semi-crystalline polymers with a rubbery amorphous phase.

The determination of parameters for each model is described and their predictive accuracies are compared with experimental data from uniaxial tension, uniaxial compression and constrained tension tests.

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CONTENTS

1	INTRODUCTION	1
2.	ELASTIC-PLASTIC MODELS	2
2.1	THE LINEAR DRUCKER-PRAGER MODEL.....	2
2.2	THE CAVITATION MODEL (1, 2)	4
2.2.1	Rubber-toughened plastics.....	4
2.2.2	The propylene copolymer	6
3.	DETERMINATION OF MODEL PARAMETERS.....	6
3.1	THE LINEAR DRUCKER-PRAGER MODEL.....	7
3.1.1	Young's modulus E and Poisson's ratio ν^e	7
3.1.2	Tensile yield stresses σ_T	7
3.1.3	The hydrostatic stress sensitivity parameter μ	7
3.1.4	The flow parameter μ'	8
3.2	THE CAVITATION MODEL.....	9
3.2.1	Young's modulus and Poisson's ratio	9
3.2.2	Effective yield stresses σ_o	9
3.2.3	The hydrostatic stress sensitivity parameter μ	9
3.2.4	The volume fraction of rubber, v_{Ro} , and parameters σ_{o1} , q_1 and k for rubber-toughened materials	10
3.2.5	The volume fraction of the amorphous phase susceptible to cavitation, v_A , and parameters σ_{o1} , q and k_1 for semicrystalline polymers.	11
3.2.6	The flow parameter μ'	12
3.2.7	The cavity nucleation parameters ϵ_{1v} , ϵ_{2v} and β	12
4.	COMPARISONS OF PREDICTIONS OF STRESS/STRAIN BEHAVIOUR FROM BOTH MODELS WITH EXPERIMENTAL RESULTS	13
4.1	UNIAXIAL TENSION	14
4.2	POISSON'S RATIO	14
4.3	UNIAXIAL COMPRESSION	15
4.4	CONSTRAINED TENSION	15
5.	CONCLUSIONS	16
6	ACKNOWLEDGEMENTS.....	17
7	REFERENCES	17

1 INTRODUCTION

Most engineering plastics are tough materials that can sustain large strains through plastic deformation before failure. For certain applications, especially those involving accidental impact loading, there is a need for information on the forces and displacements associated with the deformation of a component. In general, this will be required for loading under multiaxial stress states. Methods are available, based on finite element analysis, for determining this information by calculation. This is potentially more rapid and less costly than through testing trials.

Elastic-plastic models are used by finite element methods to describe large-strain, non-linear behaviour when this is caused by yielding and flow. The dependence of plastic deformation on strain rate can be included in the analysis. These models have been developed for metals and other materials but can be used with plastics. However, for some types of plastics, the accuracy of predicted results is uncertain. This uncertainty arises for those materials where plastic deformation under tensile stress states is associated with the nucleation of cavities. This is indicated by the appearance of stress whitening. Cavitation occurs in rubber-toughened plastics where the rubber particles are sites for cavity nucleation. Cavities are also created in polyethylenes and polypropylenes and copolymers of these materials. Here the nucleation sites are believed to be the amorphous (rubbery) regions between the crystalline phase. The formation of cavities produces an increase in plastic deformation in the material between cavities. This lowers the tensile yield stress and is responsible for enhanced toughness in these materials.

Although the deformation behaviour of these types of polymer can be modelled approximately using available models in finite element systems, the determination of model parameters is ambiguous and the accuracy of predictions is uncertain and may be low under some circumstances. A new model (the cavitation model) was therefore developed in project CPM1.1 that takes account of the influence of cavitation on plastic deformation. The predictive capabilities of both models are illustrated and compared in this report using results for a rubber-toughened plastic (ABS) and a copolymer of propylene and ethylene. The determination of properties and parameters for each model is also explained.

The version of the cavitation model developed to describe the rubber-toughened plastic was unable to provide accurate predictions of the copolymer behaviour. Modifications have recently been made to the original version that enable its application to the copolymer. These changes are also described here.

2. ELASTIC-PLASTIC MODELS

2. THE LINEAR DRUCKER-PRAGER MODEL

With elastic-plastic models, behaviour at low strains is taken to be linear elastic characterised by two materials parameters which are commonly the Young's modulus E and the elastic Poisson's ratio ν^e . The onset of non-linearity in a stress-strain curve is attributed to plastic deformation and occurs at a stress level regarded as the first yield stress. The subsequent increase of stress with strain is associated with plastic strain hardening. Stress analysis calculations involve the use of a yield criterion which for plastics is known to be sensitive to the hydrostatic component of stress. The simplest of these criteria takes the form

$$\sigma_e = \sigma_0 + \mu \sigma_k \quad (1)$$

where σ_e is the effective shear stress and σ_k is the hydrostatic stress. These are related to components of principal stress σ_1 , σ_2 and σ_3 by the equations

$$\sigma_e = \left\{ \frac{1}{2} \left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 \right] \right\}^{1/2} \quad (2)$$

and

$$\sigma_k = \frac{1}{3} (\sigma_1 + \sigma_2 + \sigma_3) \quad (3)$$

The quantities σ_0 and μ are material parameters. σ_0 is a hardening function and is related to the yield stress in shear σ_s by

$$\sigma_o = \sqrt{3} \sigma_s \quad (4)$$

The quantity μ is a measure of the sensitivity of yielding to the hydrostatic component of stress.

The measurement of yield stresses in shear is less routine than it is under uniaxial tension where test methods are well established. For this reason, equation (1) may be more conveniently expressed in the form

$$\frac{(\mu+3)}{3} \sigma_T = \sigma_e + \mu \sigma_k$$

where σ_T is a yield stress under uniaxial tension.

Equation (1) is identical to the yield criterion in the linear Drucker-Prager model in Abaqus where it takes the form

$$d = q - p \tan \beta$$

where

$$\begin{aligned} q &= \sigma_e & p &= -\sigma_k \\ d &= \sigma_o & \text{and } \mu &= \tan \beta \end{aligned}$$

The flow rule (1) is used in elastic-plastic models for the determination of plastic strains through definition of the flow potential F . The most general form for the flow potential in the linear Drucker-Prager model is given by

$$F = \sigma_e + \mu' \sigma_k - \sigma_o$$

Here, the flow parameter μ' is a material property that differs from the value of μ in equation (1) in the case of non-associated flow. In Abaqus, μ' is replaced by $\tan \Psi$.

2.2 THE CAVITATION MODEL (1, 2)

In rubber-toughened plastics such as the ABS, cavities are nucleated in the rubber particles under stress states in which there is a significant hydrostatic stress component. Cavities are also formed in the propylene copolymer but the nucleation sites are now the amorphous regions between the crystal polypropylene phase. There are small differences in the formulation of the model for these two types of polymer. The application to rubber-toughened plastics is considered in the next section, and the changes needed to model the copolymer are explained subsequently

2.2. Rubber-toughened plastics

In the absence of cavitation, equation (1) is a satisfactory criterion for yielding. This equation is now modified to include the effects of cavitation as follows

$$\frac{\sigma_e^2}{\sigma_M^2} - (q_1 f)^2 + 2q_1 f \cosh \frac{3\sigma_k}{2\sigma_M} = \left(\frac{\mu\sigma_k}{\sigma_M} \right)^2$$

Here f is the effective volume fraction of cavities which at small strains is zero but increases over some characteristic strain region associated with cavity nucleation. The parameter q_1 has been introduced to account for the effect of void interactions on the stress distribution in the matrix between cavities. σ_M is the effective yield stress of the matrix polymer between cavities and is equal to σ_0 in the absence of cavities but rises with increasing f as the volume fraction of the rubber in the matrix decreases. The increase in σ_M as cavities nucleate is given by the expression

$$\sigma_M = \frac{\sigma_0}{1 - kv_{R0}} \left[1 - k \frac{(v_{R0} - f_n)}{1 - f_n} \right]$$

where v_{R0} is the volume fraction of rubber and f_n is the effective volume fraction of cavities created by cavity nucleation in rubber particles. At large strains, the quantity f may be larger than f_n as a result of cavity growth (1,5) subsequent to nucleation. The parameter k relates the

shear yield stress of an uncavitated rubber-toughened polymer to the volume fraction of rubber v_{R0} by the equation

$$\sigma_o = \sigma_{o1} (1 - kv_{R0}) \quad (11)$$

where σ_{o1} is the shear yield stress of the untoughened polymer. In the cavitation model, $\sigma_o = \sqrt{3} \sigma_s$ is derived from experimental data obtained in shear and replaces σ_T as the basic hardening function for the polymer. The nucleation of a cavity in a rubber particle is assumed to occur at some critical volumetric strain that decreases with increasing particle diameter. For a distribution of particle sizes, the cavity nucleation should then occur over a range of total volumetric strain ϵ_v related to the critical strain range for the rubber particles.

Through comparisons with experimental data for a range of plastics, it has been shown that the increase in the volume fraction of nucleated cavities f_n with volumetric strain is given with satisfactory accuracy by the expressions

$$f_n = 0 \quad \text{for } \epsilon_v \leq \epsilon_{1V}$$

$$f_n = v_{R0} \left\{ 1 - \exp \left(- \frac{(\epsilon_v - \epsilon_{1V})}{\epsilon_{2V}} \right)^\beta \right\} \quad \text{for } \epsilon_v > \epsilon_{1V}$$

The parameters ϵ_{1V} , ϵ_{2V} and β determine the location and breadth of the volumetric strain range over which cavity nucleation occurs.

The results of axial and transverse strain measurements in ABS specimens under uniaxial tension has revealed that flow in this polymer is non-associated. The flow potential is therefore assumed to be

$$F = \frac{\sigma_e^2}{\sigma_M^2} - (q_1 f)^2 + 2q_1 f \cosh \frac{3\sigma_k}{2\sigma_M} \left(\frac{\mu' \sigma_k}{\sigma_M} \right)^2$$

2.2.2 The propylene copolymer

The yield criterion in the cavitation model for the copolymer is given by equation (9) except that the volume fraction of rubber v_{R0} is replaced by the effective volume fraction of the amorphous phase v_A where cavities are assumed to nucleate. This effective volume fraction may be less than the amorphous volume fraction since the smaller, more localised, amorphous regions or the less mobile layer around crystallite boundaries may evade cavitation. Consequently, the value for v_A may vary with temperature and stress state. As cavity nucleation proceeds and the amorphous regions become replaced by cavities, the yield stress of the polymer material between cavities σ_M will increase as with the rubber-toughened materials. However, the results of experiments and model calculations on the copolymer reveal that equation (11) and hence equation (10) are not suitable for describing the changes for the copolymer. It is necessary to replace equation (11) by a non-linear dependence of yield stress σ_o on the volume fraction of the amorphous phase as follows

$$\sigma_o = \sigma_{o1} \exp \left[- (k_1 v_A) \right]$$

where σ_{o1} is now the effective shear yield stress of the crystalline phase that is linked by the residual amorphous regions. This leads to the alternative expression for the yield stress σ_M of the material between cavities

$$\sigma_M = \frac{\sigma_o}{\exp \left[- (k_1 v_A) \right]} \exp \left[\frac{v_A - f_n}{1 - f_n} \right]$$

3. DETERMINATION OF MODEL PARAMETERS

In this section, methods are described for determining the parameters in each model from measurements of stress/strain curves under tension, shear and uniaxial compression. For illustration, results are shown in figure 1a for the ABS, which contains 38% by volume of rubber, and figure 1b for the propylene-ethylene copolymer which has 8% ethylene. The measurements under each stress state were made at the same effective plastic strain rate of 0.004 s^{-1}

3. THE LINEAR DRUCKER-PRAGER MODEL

Young's modulus E and Poisson's ratio ν^e

These are determined from tensile data at small strains where behaviour is assumed to be linear elastic.

3.2 Tensile yield stresses σ_T

For strain hardening materials, tensile yield stresses vary with plastic strain ϵ_T^p and are expressed as a hardening curve $\sigma_T(\epsilon_T^p)$ corresponding to a particular plastic strain rate. This curve is derived from the tensile stress/strain curve by subtracting the elastic strain component from the total strain ϵ_T through the expression

$$\epsilon_T^p = \epsilon_T - \frac{\sigma_T}{E} \quad (16)$$

Tensile hardening curves for both materials derived from the curves in figures 1a and 1b are included in figures 2a and 2b. Hardening curves obtained similarly from compressive data (σ_C v ϵ_C^p) and from shear data (σ_o v ϵ_o^p - see eq (19) below) are also included.

For an analysis with rate-dependent plasticity, hardening curves are needed over a range of strain rate. Note also that the measurement of stress/strain curves at strains beyond the strain corresponding to the peak in stress may require the use of a non-standard specimen (3).

The hydrostatic stress sensitivity parameter μ

This parameter determines the sensitivity of yielding to the hydrostatic component of stress and can be obtained from measurements of yield stress under two different stress states. Tension and shear have been used here so that if σ_T and σ_S are yield stresses at the same effective plastic strain (1), then μ is given by

$$\mu = 3 \left[(\sqrt{3} \sigma_S / \sigma_T) - 1 \right] \quad (17)$$

The value for μ determined from the tensile and shear data in figure 2 depends on the strain level at which the pair of stresses were selected for the calculation. This is particularly noticeable with the ABS material and arises because of the different shapes of the tensile and shear curves. Furthermore, different values for μ are obtained if tensile and compressive or compressive and shear data are used. These uncertainties in the determination of μ are attributable to the nucleation of cavities under tension which is not explained by this model.

3.1.4 The flow parameter μ'

From the flow rule, it follows that a value for μ' can be determined from the plastic component of Poisson's ratio ν^p using the equation

$$\mu' = \frac{3(1-2\nu^p)}{2(1+\nu^p)} \quad (18)$$

ν^p is the ratio of the plastic component of the transverse strain to the plastic component of the axial strain and is calculated from the Poisson's ratio measurements in figure

Values for the parameters in the linear Drucker-Prager model for both plastics are recorded in table 1

Table 1 Values for the parameters in the linear Drucker-Prager model at a strain rate of 0.004 s^{-1}

	ABS	Propylene copolymer
E (GPa)	2.3	1.6
ν^e	0.39	0.39
σ_T	fig 2a	fig 2b
$\mu = \tan \beta$	1.55	0.88
$\mu' = \tan \psi$	0.9	0.75

3.2 THE CAVITATION MODEL

Young's modulus and Poisson's ratio

These parameters are required for the characterisation of elastic behaviour in all elastic-plastic models and are determined as explained in section 3.1

Effective yield stresses σ_o

For the cavitation model, hardening behaviour is expressed by a curve of the effective stress σ_o against the effective plastic strain ϵ_o^p . These quantities are obtained from shear stress (σ_s) and shear strain (γ) data in figures 1a and 1b using

$$\begin{aligned} \sigma_o &= \sqrt{3} \sigma_s \\ \text{and} \quad \epsilon_o^p &= \gamma^p / \sqrt{3} \end{aligned} \quad (19)$$

where γ^p is the plastic component of the shear strain and is given, by analogy with equation (16), by

$$\gamma^p = \gamma - \frac{\sigma_s}{G} \quad (20)$$

where G is the shear modulus.

Hardening curves for the effective stress are shown in figures 2a and 2b.

The hydrostatic stress sensitivity parameter μ

For the cavitation model, the parameter μ is determined from measurements of yield stresses under two different stress states for which no cavities are present. Compression and shear data are used which enable μ to be calculated from

$$\mu = 3(1 - \sigma_o / \sigma_c) \quad (21)$$

The stresses σ_o and σ_C must refer to the same effective plastic strain. Using the plastic work equivalence,

$$\sigma_C \epsilon_C^p = \sigma_o \epsilon_o^p$$

and equation (21), equivalent plastic strains ϵ_C^p and ϵ_o^p are related by

$$\epsilon_C^p = \frac{(3-\mu)}{3} \epsilon_o^p$$

The determination of a value for μ is now a process of iteration to find points σ_o and ϵ_o^p on the effective shear curve and σ_C and ϵ_C^p on the compression curve that satisfy equations (21) and (23). To minimise the sensitivity of the process to the derivation of equivalent strains, a point (σ_o, ϵ_o^p) on the effective shear curve should be selected in the region just beyond the maximum in stress where σ_o varies little with ϵ_o^p . Using a starting estimate for μ of 0.25, a value for ϵ_C^p can be derived from equation (23) and hence a value for σ_C from the compression curve. Equation (21) will give a revised estimate for μ from which better estimates can be determined from successive iterations.

3.2.4 The volume fraction of rubber, v_{Ro} , and parameters σ_{ol} , q_1 and k for rubber-toughened materials

The determination of these parameters can present problems which make the definition of a procedure impractical. A consistent set of values must satisfy equations (9), (10) and (11). For this purpose, equation (9) is related to the situation of uniaxial tension at strain levels where cavitation is complete and cavity growth is not significant. Equation (9) then becomes

$$\frac{\sigma_T^2}{\sigma_{ol}^2} - (q_1 v_{Ro})^2 + 2q_1 v_{Ro} \cosh\left(\frac{\sigma_T}{2\sigma_{ol}}\right) \left(1 - \frac{\mu\sigma_T}{3\sigma_{ol}}\right)^2 = 0$$

where σ_{01} is given by equation (11). Equations (24) and (11) cannot be used to determine all the parameters and so some must be obtained from separate experiments or by estimation. For example, the volume fraction of rubber may be available from the material supplier. A typical value for $q_1 = .5$ may then be assumed, enabling σ_{01} to be obtained from equation (24) and then k from equation (11)

Alternatively, if specimens of the untoughened matrix polymer are available, a value for σ_{01} may be equated to $\sqrt{3} \times$ the measured flow stress in a shear test. This will not be possible if brittle fracture precedes yielding and flow. In this situation, the flow stress measured in a test under uniaxial compression may enable a value for σ_{01} to be deduced using equation (21), with $\sigma_o = \sigma_{01}$ and assuming a value for μ is the same as that for the toughened polymer. If values for σ_{01} and v_{R0} are known, then values for k and q_1 can be obtained from equations (11) and (24), respectively.

In the absence of values for v_{R0} and σ_{01} , then a representative value for $\sigma_{01} = 100$ MPa may be assumed for a glassy polymer. A value for v_{R0} can then be determined from equation (24) taking $q_1 = .5$

3.2.5 The volume fraction of the amorphous phase susceptible to cavitation, v_A and parameters σ_{01} , q and k_1 for semicrystalline polymers.

A consistent set of these parameters must satisfy equation (24), with v_{R0} replaced by v_A , equation (14) and equation (15). Assuming a value for q_1 of 1.5, solutions to equation (24) give pairs of possible values for v_A and σ_{01} . Plausible values are constrained by the requirement that v_A must be somewhat less than the volume fraction of the amorphous phase. Furthermore, a suitable value for σ_{01} probably lies in the range 90 MPa to 100 MPa. Values of k_1 may then be estimated from σ_{01} , σ_o and v_A using equation (14)

3.2.6 The flow parameter μ'

A value for the flow parameter is best determined from Poisson's ratio measurements under tension out to strain levels for which cavity nucleation is complete. The plastic component of Poisson's ratio is then calculated, and a value for μ' can be derived from the relationship.

$$v^p = \frac{\left[\frac{\sigma_T}{\sigma_{ol}} - q_l v_{Ro} \sinh\left(\frac{\sigma_T}{2\sigma_{ol}}\right) - \frac{2\mu'}{3} \left(1 - \frac{\mu'\sigma_T}{3\sigma_{ol}}\right) \right]}{\left[\frac{2\sigma_T}{\sigma_{ol}} + q_l v_{Ro} \sinh\left(\frac{\sigma_T}{2\sigma_{ol}}\right) + \frac{2\mu'}{3} \left(1 - \frac{\mu'\sigma_T}{3\sigma_{ol}}\right) \right]}$$

where σ_T is the tensile stress at the strain level associated with v^p .

The results of measurements of Poisson's ratio prior to cavity nucleation in tension and under uniaxial compression (where $f = 0$) (4) indicate that μ' has a relatively low value when $f = 0$ which then increases with the cavity volume fraction. This observation has lead to the following proposed expression for μ'

$$\mu' = \mu'_1 \frac{(1 - v_{Ro})}{(1 - f_n)}$$

where μ'_1 is the value when cavitation is complete. Note that μ'_1 characterises the volumetric plastic strain in the rigid component of the matrix. Equation (26) is then consistent with an increased matrix expansion (or increased μ') as the rubber content in the matrix decreases with increasing f_n .

3.2.7 The cavity nucleation parameters ε_{1v} , ε_{2v} and β

These parameters are obtained by an iterative process to achieve satisfactory predictions of the shape of the tensile stress/strain curve and the Poisson's ratio vs strain curve in the strain range associated with cavity nucleation. This requires a method for obtaining solutions for the case of uniaxial tension for comparison with experiment. The solutions can be achieved by a finite element analysis applied to a single element with appropriate boundary conditions but a

routine coded on a PC is more convenient. An optimisation procedure for finding the parameter values is not necessary since this is best done by an interactive method for rapidly exploring the effect of changes to the parameters on curve shapes.

Values for the parameters in the cavitation model for the ABS and the propylene copolymer are recorded in table 2

**Table 2 Values for parameters in the cavitation
model at a strain rate of 0.004 s^{-1}**

	ABS	Propylene copolymer
E (GPa)	2.3	1.6
ν^c	0.39	0.39
σ_0 (MPa)	fig 2a	fig 2b
μ	0.22	0.24
ν_{R0}, ν_A	0.38	0.4
σ_{01} (MPa)	99	92
k, k_1	1.35	2.4
q_1	1.57	1.5
μ_1'	0.17	0.16
ε_{1v}	0.0035	0.005
ε_{2v}	0.002	0.007
β	0.6	0.7

4. COMPARISONS OF PREDICTIONS OF STRESS/STRAIN BEHAVIOUR FROM BOTH MODELS WITH EXPERIMENTAL RESULTS

For purposes of comparisons of predictions with experiments, finite element analyses have been carried out under uniaxial tension, uniaxial compression and uniaxial tension with lateral constraint to maintain zero transverse strain. Measurements and stress analyses have been

carried out at a constant plastic strain rate of 0.004 s^{-1} . Model parameters for each polymer are given in table 1 for the linear Drucker-Prager model and table 2 for the cavitation model.

4.1 UNIAXIAL TENSION

Hardening behaviour for the cavitation model is described by an effective shear stress/effective plastic strain curve as shown for each polymer in figure 2. Tensile stress/strain curves predicted using this model are compared with experimental data in figure 3.

Cavity growth (1.5) has been included in the calculation and gives rise to a progressive increase in cavity volume fraction f which becomes significant following the cavity nucleation phase. This increase in f leads to a steady reduction in tensile stress during flow and a departure from experimental data. The change in f through cavity growth is less if a higher value for μ'_1 were adopted but this would influence the agreement with Poisson's ratio data (see figure 4 and section 3.2.6). A more plausible explanation for the departure in predictions of tensile behaviour at large strains is that this is compensated by a small increase in tensile stress at these strains caused by molecular orientation (5), which has not at this stage been included in the model.

4.2 POISSON'S RATIO

Predictions of the dependence of Poisson's on tensile strain using both models are compared with measured values in figure 4. The small rise in experimental values at low strains is caused by plastic deformation. It is only predicted for small values of μ' which are present in the cavitation model prior to cavity nucleation (see section 3.2.6 and equation (26)). The predicted decrease in Poisson's ratio with strain using the cavitation model has contributions related to the value for μ' and the volume fraction of cavities (see equation (25)). With the Drucker-Prager model, the decrease is caused by a constant value for the parameter μ' and the shape of the variation of Poisson's ratio with strain is not accurately described.

4.3 UNIAXIAL COMPRESSION

Comparisons of predicted compressive behaviour are compared with experimental data in figure 5. The limitations in the predictive accuracy of the linear Drucker-Prager model are evident here and arise because of the large value for μ determined for this model using tensile and shear data (see table 1). Much better agreement would have been obtained if tensile and compressive data had been employed to derive the μ value but predictions of behaviour under shear would then be inaccurate. The inappropriate shape of the predicted compressive curve for ABS using the Drucker-Prager model arises because the hardening behaviour is characterised by tensile data which is influenced by cavity nucleation.

4.4 CONSTRAINED TENSION

Experimental data under this stress state have been obtained using a butt-joint tensile test. In this test, a disc of the polymer, 25 mm in diameter and to 2 mm thick, is adhesively bonded between two steel cylinders of 25 mm diameter to produce a butt-joint. Lateral contraction of the polymer is constrained by the steel and is essentially zero except in a narrow region around the circumference of the disc. Calculated stresses are compared with measurements of axial stress in figure 6. For both materials, the influence of cavity growth is larger under this stress state than under uniaxial tension. The cavitation model predicts that the lateral stress during plastic deformation and flow is about half the axial value (see figure 6b). The linear Drucker-Prager model over predicts the axial stress and predicts that the lateral stress becomes equal to the axial stress (equi-triaxial tension) at large strains. If a value for the parameter μ had been derived for the Drucker-Prager model using tensile and compressive data, then the predicted axial stress using this model would have been somewhat higher.

The cavitation model provides a closer fit to the axial stress-strain data than the linear Drucker-Prager model. However, there is a need to consider further the void nucleation criterion in an attempt to delay somewhat the predicted onset of cavitation in the constrained tension test. For the propylene-ethylene copolymer, the observed decrease in axial stress at

strains above 0.02 is too large to be accounted for by void growth. This effect may be associated with an experimental artefact such as the debonding of the specimen.

5. CONCLUSIONS

With the linear Drucker-Prager model, the derived value for the parameter μ , that characterises the sensitivity of yielding to hydrostatic stress, depends on which test data are used for the derivation. Using this model, errors arise in predictions which depend upon the stress state in the analysis and the value derived for μ .

Significantly more parameters are involved in the cavitation model, and their derivation requires tests under compression as well as tension and shear. With rubber-toughened materials, there can be some ambiguity in derived parameter values unless additional information is available from separate tests for the parameters v_{R0} and σ_{0I} .

Small changes are needed to the formulation of the cavitation model for rubber-toughened plastics before the model can be applied to the propylene copolymer. Owing to the greater uncertainty in values for some of the model parameters for semicrystalline rubbery polymers, such as the propylene copolymer, the derivation of a valid set of parameters is more problematic with this type of material than for rubber-toughened plastics. Additional data from butt-tension tests would help with deciding a viable set of parameters.

The accuracy of predicted stress and strain distributions from a finite element analysis is expected to be significantly higher using the cavitation model than with the linear Drucker-Prager model for materials that stress whiten (promote cavitation) during plastic deformation.

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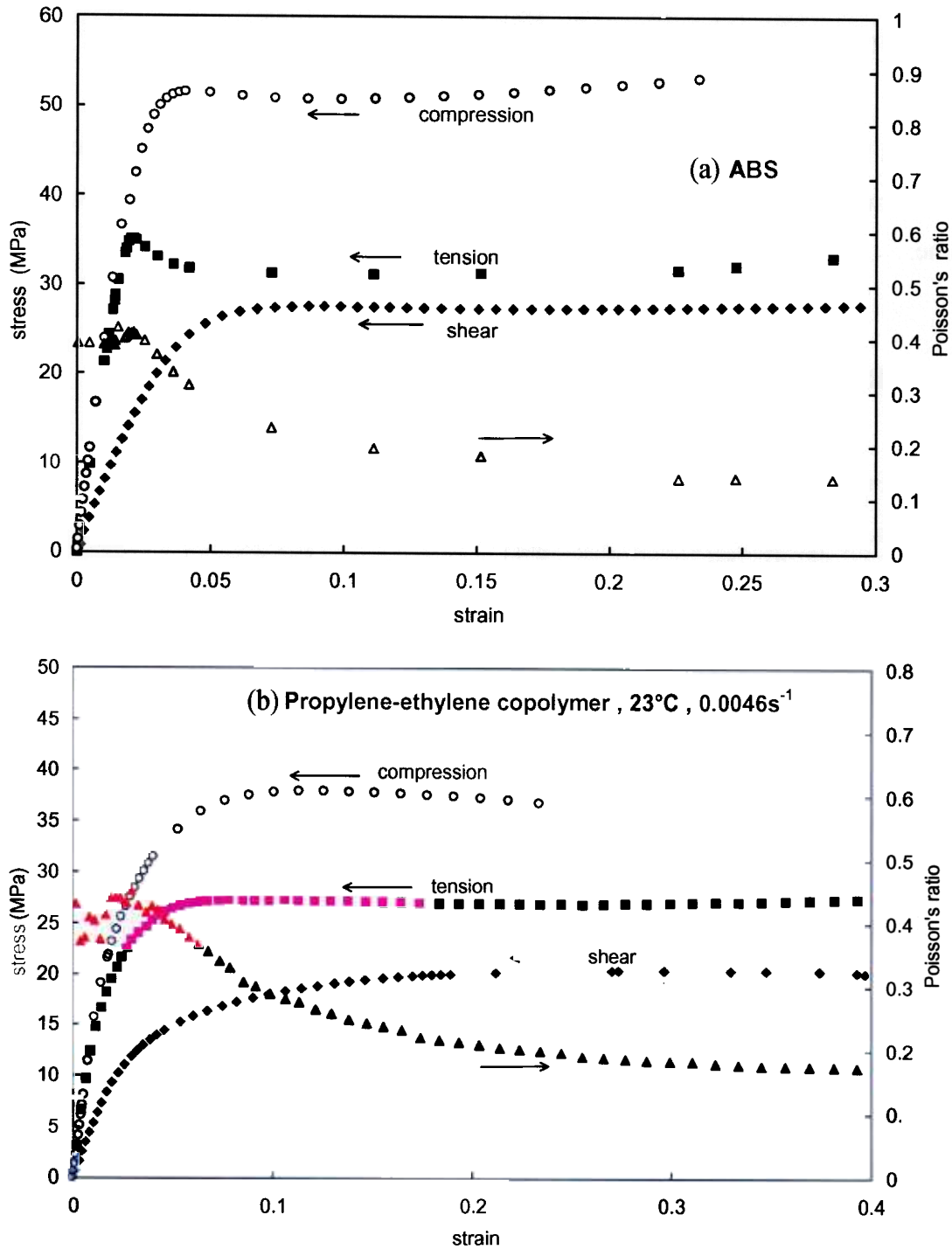


Figure 1. Curves of true stress and Poisson's ratio against true strain measured under tension, compression, and shear for (a) the ABS polymer and (b) the propylene-ethylene copolymer.

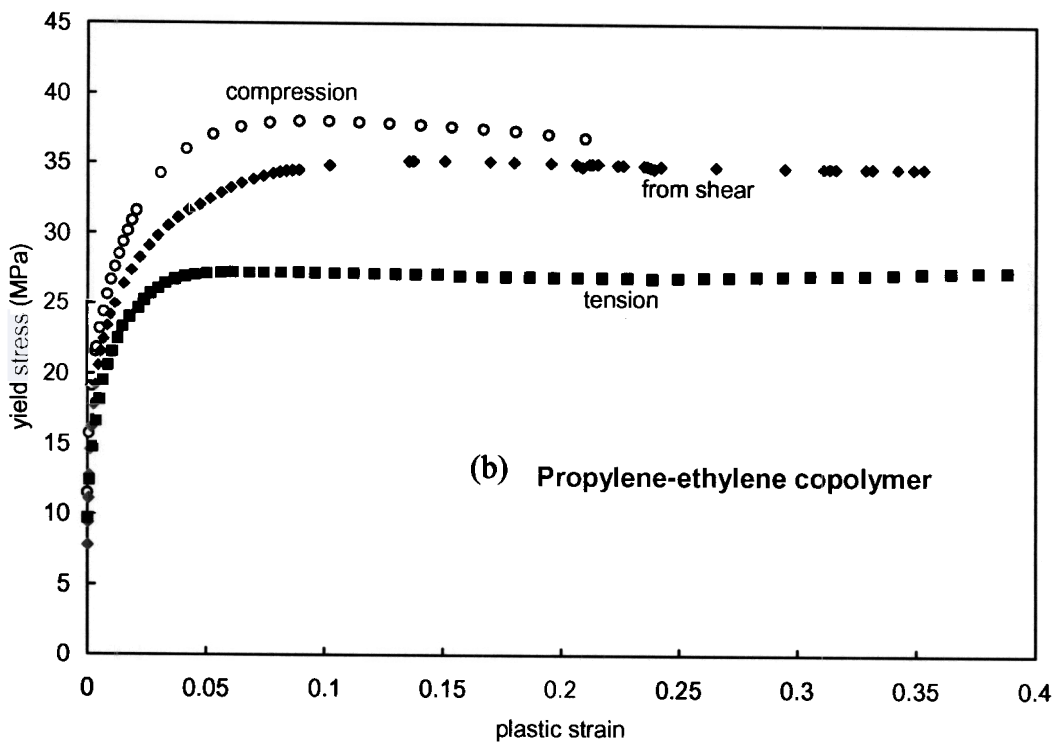
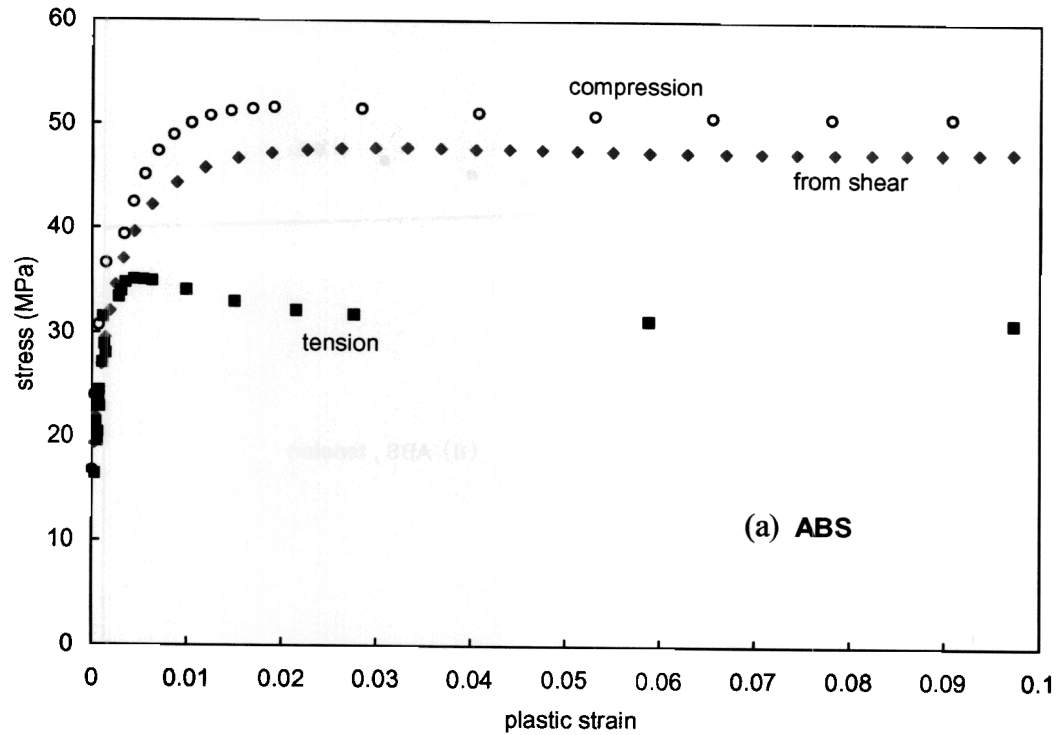


Figure 2. Curves of true yield stress against true plastic strain derived from the results in figure 1. The shear data are expressed as effective values for shear stress σ_0 and shear strain ϵ_0^p (see equations (19)).

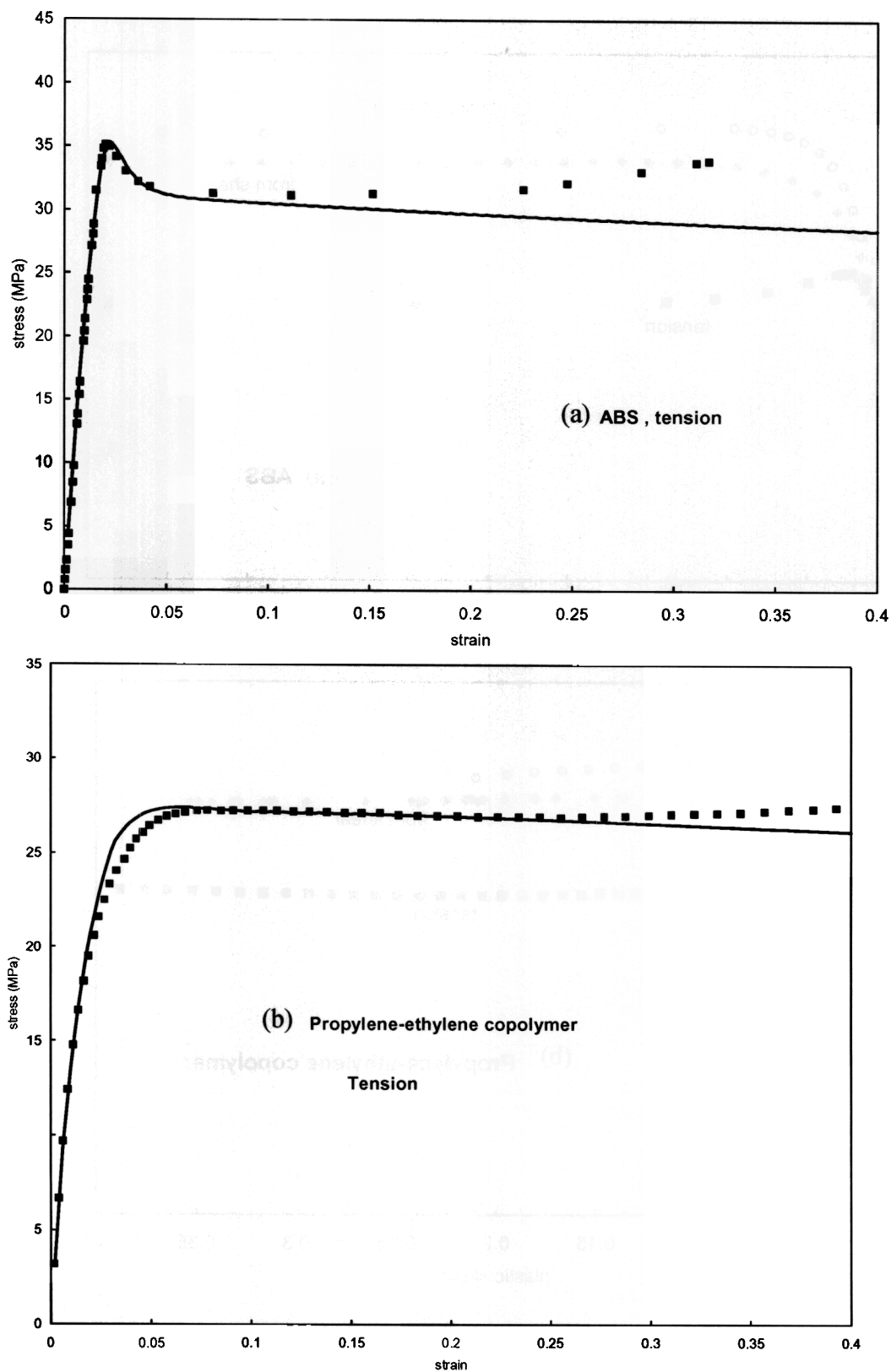


Figure 3. Comparisons of stress/strain curves measured under tension with curves predicted using the cavitation model. Cavity growth is included in the calculations.

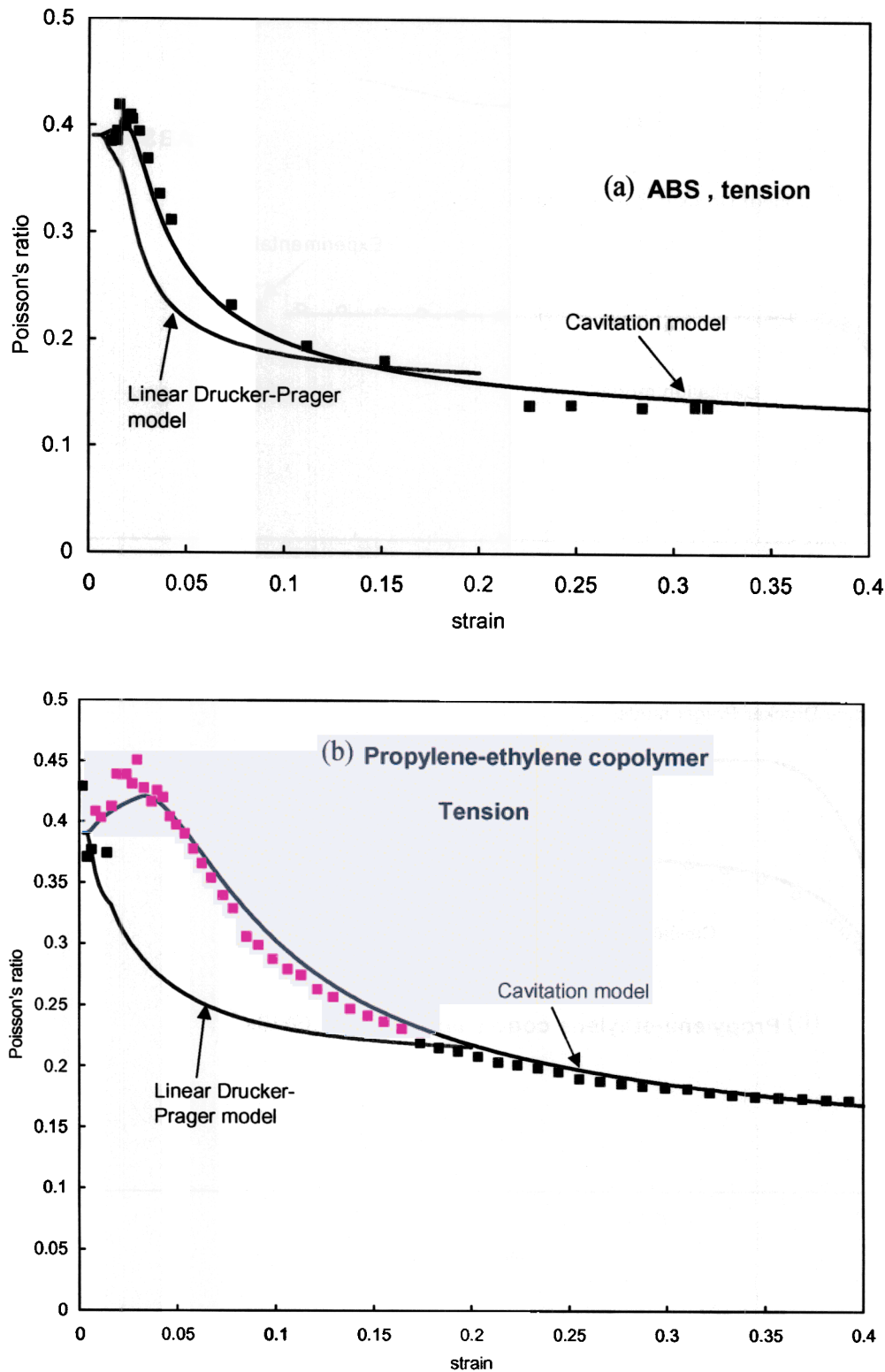


Figure 4. Comparisons of measured Poisson's ratio values with predictions using both models.

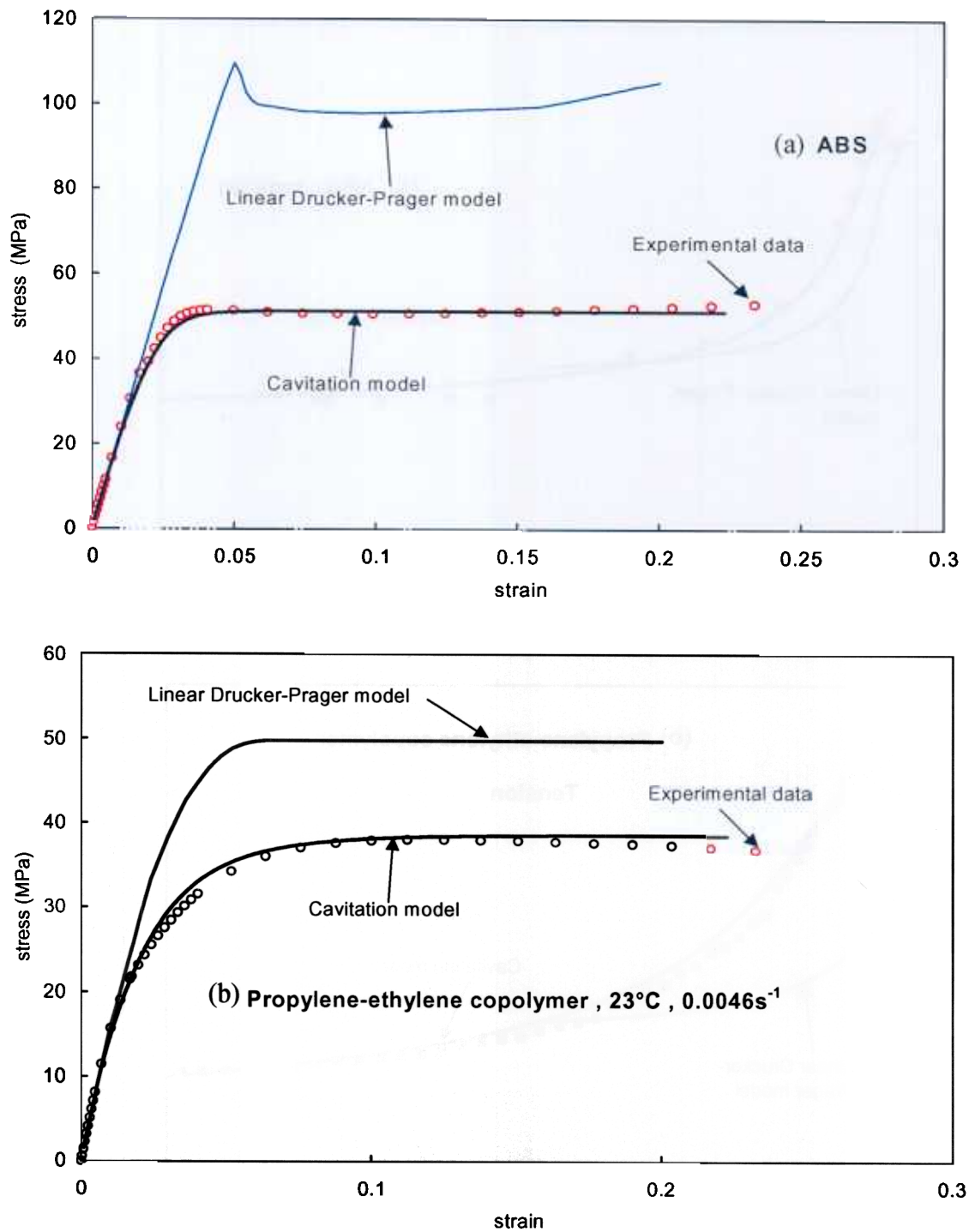


Figure 5. Comparisons of measured data under compression with predictions using both models.

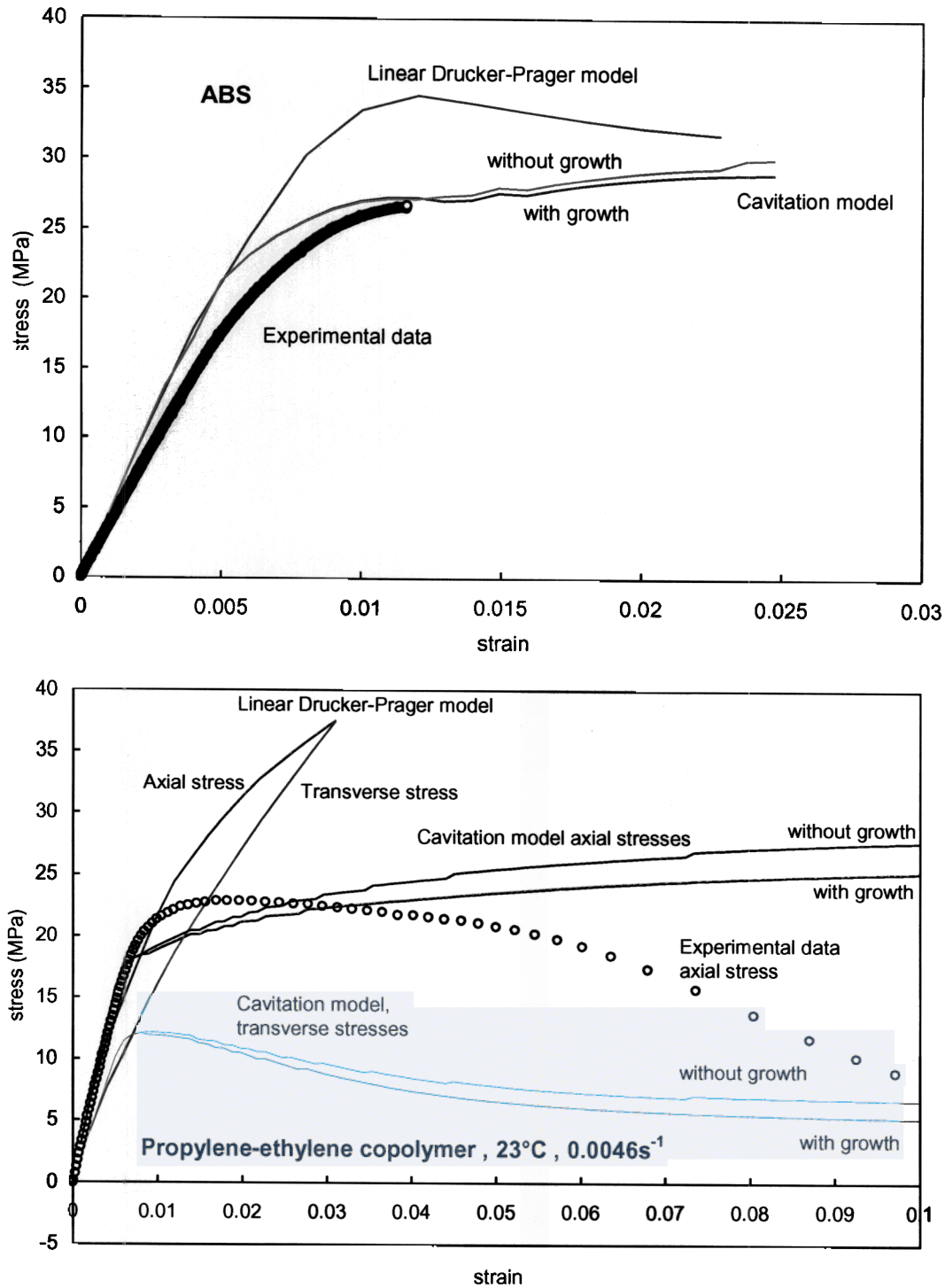


Figure 6. Comparisons of data measured under tension with lateral constraint with predictions using both models.