

Determination of the Properties of Plastics at High Strain Rates

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SUMMARY

Computerised design methods are available for predicting the performance of tough plastics under impact loading. These methods require stress/strain data over a wide range of strain rates. These data are most conveniently measured in tension, but ISO standards for tensile property measurement are currently only applicable for strain rates up to around 0.1 s^{-1} . To simulate behaviour under impact, data are required at rates that are 3 or 4 orders of magnitude higher than this. For accurate data acquisition at these higher rates, attention needs to be paid to apparatus design in order to minimise contributions from transient forces arising from resonances and the propagation of a shock wave pulse in the specimen. In addition, procedures and extensometers are not routinely available for determining strains at the higher rates of deformation.

Furthermore, even at low strain rates, there are no standard procedures for measuring the strain in specimens that undergo non-uniform deformation (necking is an extreme example) in the specimen gauge length at strains above that corresponding to the stress maximum. ISO standards specify the determination of a nominal or average strain.

The attached paper illustrates the acquisition of data over a wide range of strain rate through a combination of measurements at low and moderate strain rates with extrapolation of these data to higher rates. Extrapolation is achieved by the use of mathematical functions to model the stress/strain curves and the rate dependence of parameters. A new tensile test specimen is proposed for the measurement of behaviour at moderate and large strains.

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ISSN 1473 2734

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Approved on behalf of Managing Director, NPL,
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1 DATA REQUIREMENTS FOR PERFORMANCE PREDICTION

Since the need for data at high strain rates is predominantly for performance calculations under impact, it is constructive to direct the test procedures and data analyses in the proposed Standard to the requirements of the materials models used in these calculations. The minimum data requirements are:

- Young's modulus and Poisson's ratio at an appropriate strain rate
- A plot of true stress against true strain to a strain level that is equal to or above the maximum predicted in the calculation.

The derivation of true stresses requires simultaneous measurements of lateral strain (Poisson's ratio) with the longitudinal strain in the specimen. In order to supply data for a variety of impact loading situations and to allow for a distribution of strain levels and hence strain rates in a component under load, it is necessary to determine the above properties over a range of strain rates.

2. ACQUISITION OF DATA

2.1 APPARATUS

The measurement of properties up to test speeds of 100 mm/s is relatively straightforward. There are several precautions that could be adopted to minimise errors and maximise the strain rate limit. These include use of

- a high stiffness, piezoelectric force transducer
- small specimens
- light-weight extensometers or non-contacting devices for strain measurement
- low mass components in the test assembly.

At test speeds above this, problems and errors arise in the measurement of force and strain. Errors in force measurement are associated with the high accelerations generated by load

application that give rise to transient forces. These transient forces arise from excitation of a resonance of the force transducer and from multiple reflections of the displacement pulse generated at one end of the specimen by the sudden load application. These errors can be minimised by using components of low mass connected to the force transducer, a high stiffness transducer and by filtering the force signal.

The development of high-speed extensometers that operate at higher strain rates has been reported, although these devices are not used routinely for data acquisition. A simpler approach to determining strain in such tests is to derive values from cross-head movement or the change in grip separation. An apparent or average strain is obtained from the ratio of the change in grip separation to the length of the specimen between the grips prior to load application. [Note, if the measurement of cross-head movement is used then a correction will, in general, be necessary arising from the compliance of the test assembly.] Since the test specimen is usually waisted in the gauge region, this average strain will differ from the actual strain in the gauge region of the specimen, where the corresponding stress is calculated. It is possible to estimate the actual strain values from the average strain using a correlation of these quantities determined from lower speed tests, where actual strain can be measured with extensometers. The correlation obtained will be specific to the type of polymer, the specimen size and geometry and the strain rate and temperature. The correlation may also vary from one test to another under identical conditions.

Because of these problems with force and strain measurement at high speeds, it is proposed that properties at high strain rates are obtained by modelling behaviour at low and moderate strain rates and deriving high rate data by extrapolation. This is considered next.

2.2 DETERMINATION OF PROPERTIES AT LOW AND MODERATE STRAIN RATES

The measurement of stresses and strains can be considered in two stages

measurements up to the maximum in stress or, if no clear maximum exists, to strains where the gradient in the stress/strain curve is approximately zero.

2. measurements at strains above this.

Measurements in the linear region of strain and the initial non-linear region up to the stress maximum are relatively straightforward. The preferred specimen is the small ISO tensile specimen type 1BA or 1BB. This should allow higher strain rates to be achieved compared with use of the larger specimen type 1A or 1B because the acceleration on any contacting extensometer will be less. A suitable gauge length for strain measurements is between 10 and 20 mm although use of a gauge length nearer the lower limit should enable higher strain rates to be achieved because, once again, of the lower acceleration on any contacting extensometer.

Measurements at strains above the region corresponding to the maximum in stress will in general be unreliable because the strain distribution along the length of the specimen will be non-uniform. The extent of non-uniformity in strain may be relatively small, as illustrated for a talc-filled, rubber toughened polypropylene in figure 1, or extreme in the case of the rubber-toughened polycarbonate supplied by Dow in figure 2. In these tests, simultaneous strain measurements were made using a video extensometer in separate regions of length 10 mm near the centre of an ISO standard tensile specimen. The extent of the strain localisation depends on the flow behaviour of the polymer and hence the test temperature and speed. In extreme situations, the localisation is clearly visible through necking as seen in figure 2 but for many materials the extent of the non-uniformity in strain is less extreme and less apparent. Under these circumstances, conventional extensometry can give only an average strain that will depend on the chosen gauge length and location on the specimen.

In order to obtain more meaningful measurements of strain, and, in particular, of strain at failure, an alternative specimen geometry has been chosen that is illustrated in figure 3. The waisted region is defined by a constant radius that locates the maximum stress, and hence onset of flow, at the centre of the specimen. The dimensions shown enable the specimen to be machined from the centre of a multipurpose test specimen. There may be scope for optimising the values chosen for the radius and depth of the waist. Finite element analysis calculations have been carried out on this specimen geometry to give an indication of stress and strain distributions in the centre of the specimen. The material used for these calculations is a propylene-ethylene copolymer supplied by BASF. Deformation was

described by an elastic-plastic model with a hydrostatic-stress-sensitive yield criterion. The dependence of yield behaviour on strain rate (rate-dependent plasticity) was included in the analysis.

Calculations of the stress distribution show that there is a region of essentially uniform, uniaxial stress in the specimen centre. The stress state is not pure uniaxial since the analysis indicates a small, transverse, in-plane stress rising to 1 MPa at the specimen centre. The stress in the through-thickness direction is zero. The small transverse stress is attributable to the high strain gradient during flow. Distributions of the components of strain along the specimen axes are shown in figure 4. The results indicate that the strain is essentially uniform over a length around 3 to 5 mm in the specimen centre. This gives an indication of the gauge length that should typically be used for strain measurements. It is worth noting that the transverse strain in the plane of the specimen is predicted to decrease through the width and is on average less than the strain in the through-thickness direction. This is presumably a result of the non-zero, transverse in-plane stress discussed above. The predicted strain distributions illustrated in figure 4 are supported by strain measurements in the plane of the specimen shown in figure 5 using advanced video extensometry.

3 MODELLING STRESS/STRAIN BEHAVIOUR

3.1 LOW-STRAIN BEHAVIOUR

At low strains, deformation behaviour in constant strain-rate tests is usually assumed to be linear elastic and characterised by a modulus value. Since polymers are viscoelastic materials, a slight curvature will be apparent in the stress/strain relationship and the derived modulus value will depend on strain rate. This dependence will be small at temperatures well below the glass-to-rubber transition temperature and may be modelled for the purpose of modulus determination at high rates by extrapolation (section 5). Viscoelastic effects will give rise to a decrease in Poisson's ratio with strain rate but this is generally very small. A single value is usually sufficient and should be determined at a suitable test speed that permits the accurate measurement of transverse strain.

3.2 DETERMINATION OF TRUE STRESSES AND PLASTIC STRAINS

At larger values of strain, where the deformation behaviour is non-linear, true values for stress and strain need to be determined. Figure 6 shows stress/strain curves for the rubber-toughened polycarbonate in figure 2 obtained over a range of strain rates using the small ISO tensile specimen. The Poisson's ratio result has been obtained at a single rate of 0.004 s^{-1} . Figure 7 shows similar results but to higher strains obtained on the new tensile specimen in figure 3. To meet the requirements for data acquisition for finite element analysis, it is necessary to express these data as plots of true stress (σ_T) against true strain (ϵ). These are calculated from the measured engineering stresses (σ_E) and engineering strains (ϵ') in figures 6 and 7 as follows

$$\sigma_T = \frac{\sigma_E}{(1 - v\epsilon')^2} \quad (1)$$

$$\epsilon = \log_e (1 + \epsilon') \quad (2)$$

where v is Poisson's ratio and varies with strain as shown in figure 7.

The determination of true stresses for the data in figure 6 is relatively straightforward since v has an approximately constant value of 0.4. At higher strains, v decreases with strain ϵ' and the accurate determination of true stresses requires the appropriate v value to be used with each value of ϵ' . If a small error in σ_T can be tolerated, the calculation can be simplified by use of an effectively constant value of v equal to 0.24 according to the results in figure 7. This will result in a small under estimate in σ_T at strains between 0.05 and 0.1.

Mathematical functions are now considered for modelling the curves at different strain rates to explore the prediction of curves at higher strain rates by extrapolation. For this purpose, it is proposed to separate the effects of elasticity and plasticity. It is then possible to identify a function that describes the shape of the curves over a wide range of strain and whereby only one parameter shows any significant variation with strain rate. Furthermore, the separation of elastic and plastic behaviour to generate curves of true stress against true plastic strain

produces data in a form (hardening functions) required by finite element analyses of deformation of ductile materials.

The total true strain ϵ is considered to be the sum of true elastic and true plastic components, ϵ^e and ϵ^p respectively, so that

$$\epsilon = \epsilon^e + \epsilon^p \quad (3)$$

where
$$\epsilon^e = \frac{\sigma_T}{E} \quad (4)$$

and E is the Young's modulus appropriate to the curve being analysed (see figure 8). Note, more accurate values for ϵ^p are likely to be derived if E is taken as the gradient of the linear region of the data from the test being analysed rather than an average value from many tests. It is now possible to present data as curves of true stress σ_T against true plastic strain ϵ^p . These are shown in figures 9 and 10 derived from the data in figures 6 and 7.

3.3 MODELLING BEHAVIOUR AT INTERMEDIATE STRAINS

The experimental data in figure 9 have been modelled using the function

$$\sigma_T = \sigma_o + (\sigma_f - \sigma_o) \left(1 - \exp - (\epsilon^p / \epsilon_o^p)^\beta \right) \quad (5)$$

where σ_o is the stress at zero plastic strain

σ_f is the limiting stress at high plastic strains

and ϵ_o^p and β are parameters that determine the mean strain and strain range over which the increase in σ_T with ϵ^p occurs. Table 1 shows the values for the parameters in equation (5) used to obtain the fits to the data shown in figure 9.

Table 1. Values for the parameters in equation (5) used to obtain the fits to tensile data from small ISO specimens in figure 9

Test speed (mm/s)	Strain rate (s ⁻¹)	σ_f (MPa)	σ_o (MPa)	ϵ_o^p	β
0.1	0.004	54.5	25	0.004	0.8
1	0.045	59.5	25	0.004	0.8
10	0.35	64	25	0.004	0.8
100	3	67.5	25	0.004	0.8

It can be seen that excellent fits are achieved with only one parameter σ_f changing with strain rate. The variation of this parameter with strain rate is considered in section 4. The observation that the other 3 parameters show little or no variation with strain rate will not be generally found for all polymers. Materials behaviour in which the strain at peak stress shifts with strain rate can be modelled by allowing a small variation in ϵ_o^p with strain rate.

3.4 MODELLING BEHAVIOUR AT LARGE STRAINS

Equation 5 gives a good description of stress/plastic strain curves up to total strains of 0.05 (plastic strains of 0.03). This may be satisfactory over a wider strain range for materials that show little change in σ_T with higher plastic strains or for calculations in which the effect of any subsequent variation in σ_T on accuracy can be neglected. However, the shape of the $\sigma_T(\epsilon^p)$ curve at higher strains may have a significant influence on predicted behaviour in some applications and a more precise description of the curve shape is then needed. This can be achieved by a small modification to equation (5). For slightly higher values of plastic strain, the increase or decrease in σ_T can be represented by a linear function that is multiplied by, or added to, the expression for σ_T thus

$$\sigma_T = \sigma_o + (\sigma_f^a - \sigma_o) \left(1 - \exp - (\epsilon^p / \epsilon_o^p)^\beta \right) (1 + g \epsilon^p)$$

$$\sigma_T = \sigma_o + (\sigma_f^a - \sigma_o) \left(1 - \exp - (\epsilon^p / \epsilon_o^p)^\beta \right) + f \epsilon^p$$

where g and f are additional parameters that are negative if σ_T decreases with ϵ^p . Note that when these terms are included, then values for the parameter σ_f^a will differ slightly from values for σ_f obtained using equation (5).

With most polymers that show a small decrease in stress with plastic strain beyond the stress peak, this is followed by an increase in stress at larger strains which is attributed to the stiffening effect of molecular orientation. This is illustrated in the results shown in figure 10. Satisfactory curve fits can then be obtained by replacing linear modifications to equation 5, shown in equation 6, by a parabolic expression thus

$$\sigma_T = \sigma_o + (\sigma_f^a - \sigma_o)(1 - \exp(-(\epsilon^p / \epsilon_o^p)^\beta)) \left(\epsilon_s^p \right)$$

or

$$\sigma_T = \sigma_o + (\sigma_f^a - \sigma_o)(1 - \exp(-(\epsilon^p / \epsilon_o^p)^\beta)) - 2 \frac{\delta}{\epsilon_s^p} \epsilon^p + \frac{\delta}{\epsilon_s^{p^2}} \epsilon^{p^2}$$

where γ , δ and ϵ_s^p are identifiable parameters. δ is the fall in stress after the stress peak and ϵ_s^p is the plastic strain at the minimum in stress.

Figure 10 shows fits to the tensile data using equation (7b). Values for the parameters used to obtain these fits are given in table 2.

Table 2. Values for the parameters in equation (7b) used to obtain the fits to data shown in figure 10 from the new tensile specimen geometry

Test speed (mm/s)	Strain rate (s ⁻¹)	σ_f^a (MPa)	σ_o (MPa)	ϵ_o^p	β
0.1	0.02	58.5	25	0.004	0.8
1	0.15	61	25	0.004	0.8
10	1.5	65	25	0.004	0.8

100	10	70.5	25	0.004	0.8
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Note again that σ_f^a is the only parameter to change with strain rate. These values are slightly higher than the values of σ_f in table 1. This is mainly because the strain rates with the new specimen are higher than values obtained with the small ISO specimen at the same test speed (see table 1). It can be seen that values for the other parameters are the same as those obtained using the small ISO specimen and do not change significantly with strain rate.

TEMPERATURE INCREASE DURING PLASTIC DEFORMATION

The plastic work involved with deformation of a specimen will be dissipated as heat in the specimen. Simple calculations indicate that, even under adiabatic conditions, the temperature rise will have an insignificant effect on properties for tensile strains up to about 0.2. At higher strains, the temperature rise in the specimen will depend on the local strain rate and hence test speed. With results in figures 7 and 10, the dissipation of plastic work will be more adiabatic at the higher speeds, and the associated greater temperature rise at these speeds should cause the curves to converge at the higher strains. This is not clearly evident, and the effect of temperature rise on results at high strain rates needs further clarification.

4 DEPENDENCE OF THE PEAK STRESS σ_f ON STRAIN RATE

The variation of strain with time in a test on the rubber-toughened polycarbonate using the new tensile specimen is shown in figure 11. The strain rate values given with the tensile data in figures 6, 7, 9 and 10 have been obtained from the gradient of similar curves for each test at the strain corresponding to the peak in stress. The strain rate is observed to increase most rapidly in this region of the test. Values for the peak true stress obtained from all tests on both types of specimen are plotted against the logarithm of the plastic strain rate $\dot{\epsilon}^p$ in figure 12. Note that around the peak in stress, the total strain rate and the plastic strain rate are the same. It is apparent that, to a good approximation, the variation of σ_f with $\dot{\epsilon}^p$ can be modelled using the Eyring equation.

$$\begin{aligned}\sigma_f &= \sigma_{f0} + a \log \dot{\epsilon}^p \\ &= \sigma_{f0} (1 + \alpha \log \dot{\epsilon}^p)\end{aligned}\tag{8}$$

where σ_{f0} is the intersection with the axis at zero log strain rate and a or α are related to the gradient. The parameter σ_f^a in equations (6) and (7) will have a similar dependence on strain rate so that

$$\sigma_f^a = \sigma_{f0}^a (1 + \alpha \log \dot{\epsilon}^p)$$

where σ_{f0}^a differs very slightly from σ_{f0} because a small change to σ_f is required by the introduction of the linear or parabolic terms and by the different value for Poisson's ratio used to determine σ_T for the two specimen types (see section 3.2).

If it can be assumed that equations (8) and (9) are valid at higher strain rates, then substitution of equation (8) into equation (5) or equation (9) into equations (6) or (7) will enable stress/ plastic strain curves to be predicted over a wide range of strain rates including high rates where problems with measurement arise. The curves in figure 10 at the two higher strain rates have been predicted this way using values for $\sigma_{f0}^a = 66$ MPa and $\alpha = 0.071$ determined from figure 12. The validity of equations (8) and (9) for extrapolation to higher strain rates can be explored through tests at a lower temperature which are equivalent to results at higher rates at ambient temperature. This will be the subject of another study.

5 DEPENDENCE OF MODULUS ON STRAIN RATE

A linear viscoelastic analysis of the deformation of a glassy polymer at constant strain rate indicates that the tensile modulus E should vary with strain rate according to the relationship

$$E = E_o \left(\frac{k}{\dot{\epsilon}} \right)$$

where E_0 , k and n are parameters. This equation may be used for extrapolating modulus measurements to higher strain rates.

6 CONCLUSIONS AND RECOMMENDATIONS

- For the measurement of tensile stress/strain curves up to the strain where the maximum in stress occurs, the large or small ISO standard tensile specimens should be used. With suitable attention to the choice of force transducer and extensometer and the design of the test assembly, measurements can be made up to test speeds around 100 mm/s. Use of the small tensile specimen as opposed to the large specimen may enable results at slightly higher strain rates to be obtained.
- By separating elastic and plastic components of the total strain and presenting data as curves of true stress against true plastic strain, data can be modelled over a wide range of strain rates using equations (5) and (8). Stress/strain curves can then be conveniently predicted at higher strain rates. Equation (5) is only an approximate description of stress/strain behaviour beyond the strain at the stress maximum since it predicts a stress that does not change subsequently with strain.
- The ISO standard specimens are generally unsuitable for the measurement of stress/strain behaviour at strains above the region where the maximum in stress occurs because the strain in the specimen is then not uniform. This leads to errors in the measurement of strain and true stress which can be minimised through the use of a new tensile specimen geometry shown in figure 3
- Equations (6), or (7) and (9) can be used to model stress/strain behaviour over wide ranges of strain and strain rate and thereby to predict behaviour at high strain rates where there are measurement problems.
- The validity of equations (8) and (9) for extrapolation can be explored through the analysis of tensile results at lower temperatures.

7 ACKNOWLEDGEMENT

This work was funded by the Department of Trade and Industry as part of an interim programme between the CPM and MPP programmes.

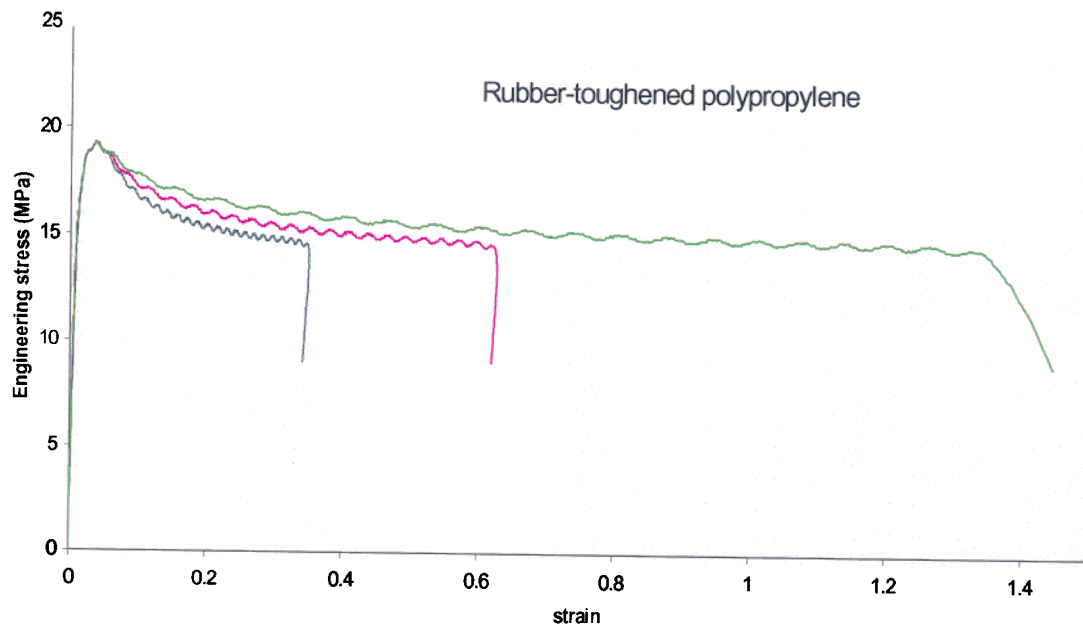


Figure 1 Tensile stress/strain curves for a rubber-toughened polypropylene measured using a single ISO standard specimen of type 1B. A gauge length of 10 mm was used at 3 separate locations along the length of the specimen.

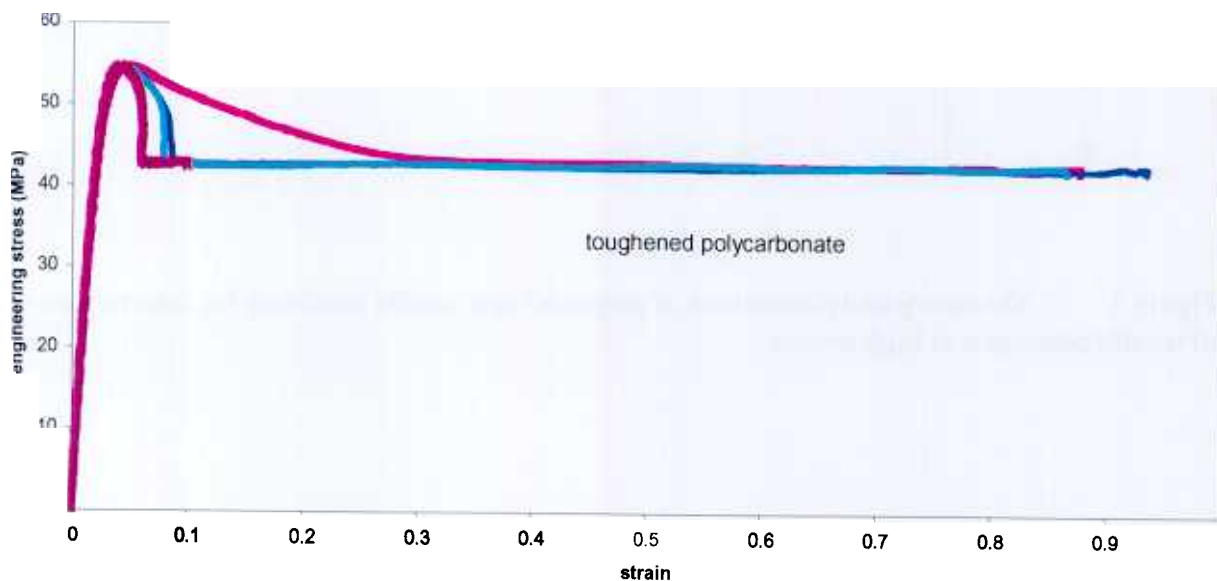


Figure 2 Tensile curves for a toughened polycarbonate specimen of type 1B obtained using 4 separate gauge lengths of 10 mm along the specimen length.

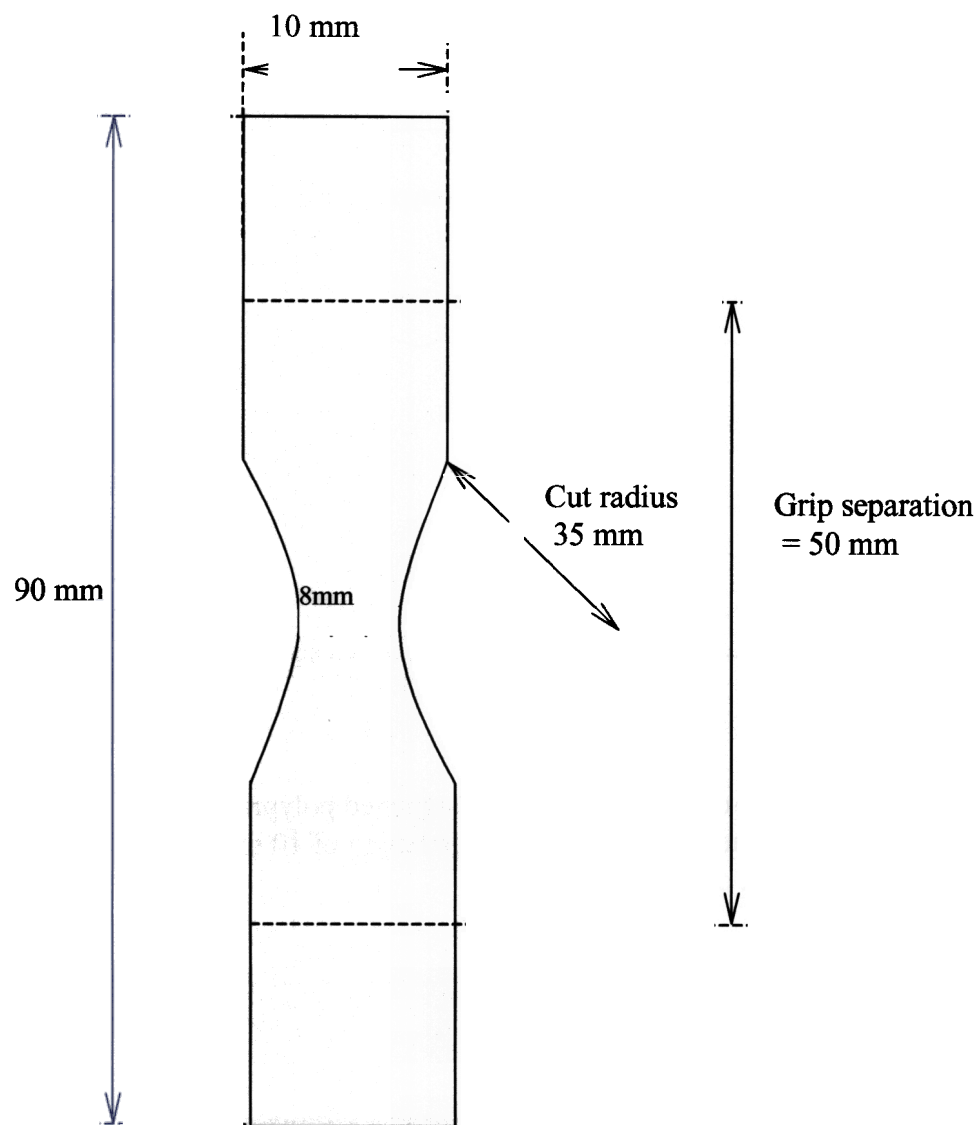
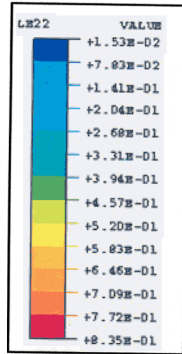
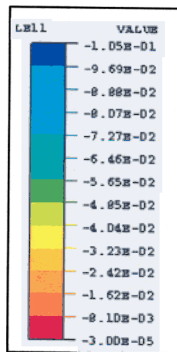
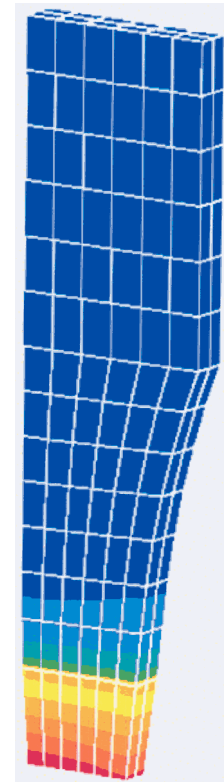


Figure 3 Geometry and dimensions of proposed new tensile specimen for determination of tensile behaviour at high strains



Longitudinal strain

ϵ^2



Transverse strain

ϵ^2

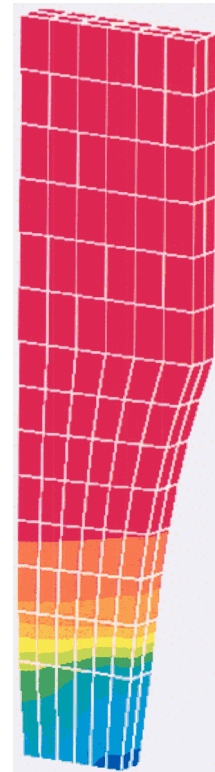


Figure 4 Predicted strain distributions in the new tensile specimen of the propylene-ethylene copolymer

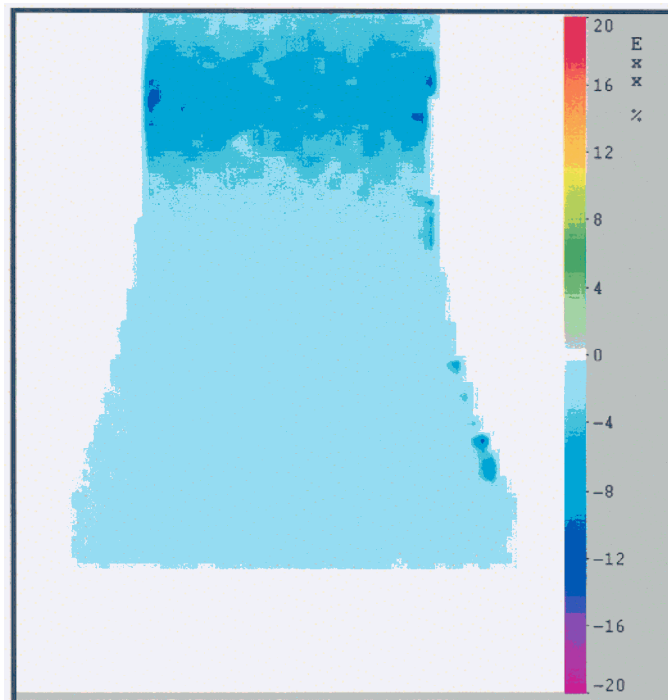
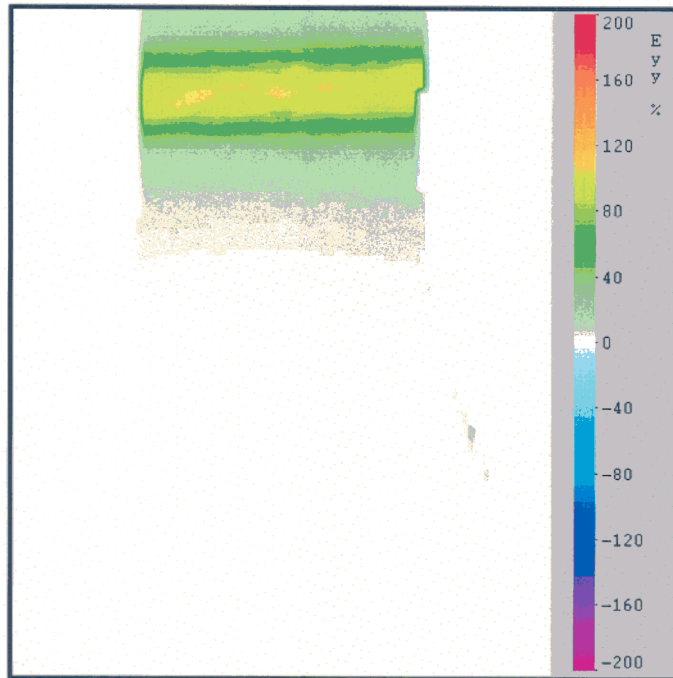


Figure 5 Longitudinal and transverse strain distributions measured using a video extensometer at the same conditions as the predictions in figure 4. Note the strains are engineering values.

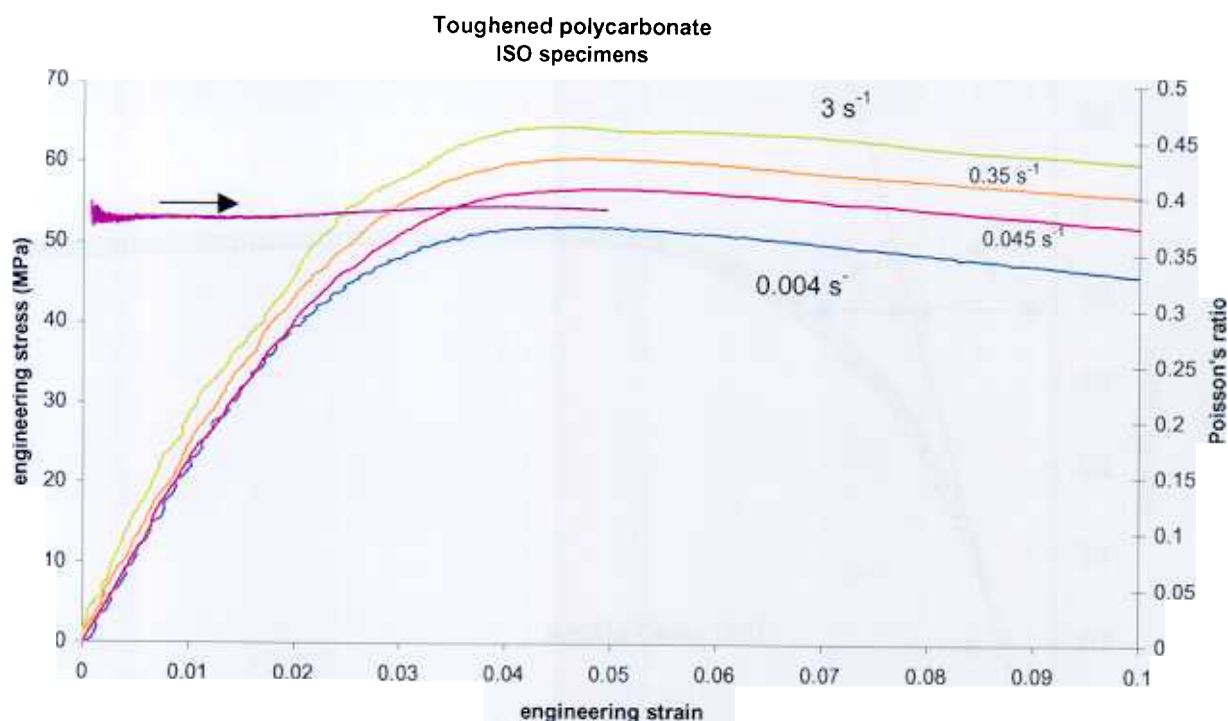


Figure 6 Tensile stress/strain curves and Poisson's ratio for a toughened polycarbonate obtained using small ISO specimens at test speeds ranging from 0.1 to 100 mm/s. The strain rates shown with each curve refer to the value at the peak stress. Poisson's ratio results were obtained at a single speed of 0.1 mm/s.

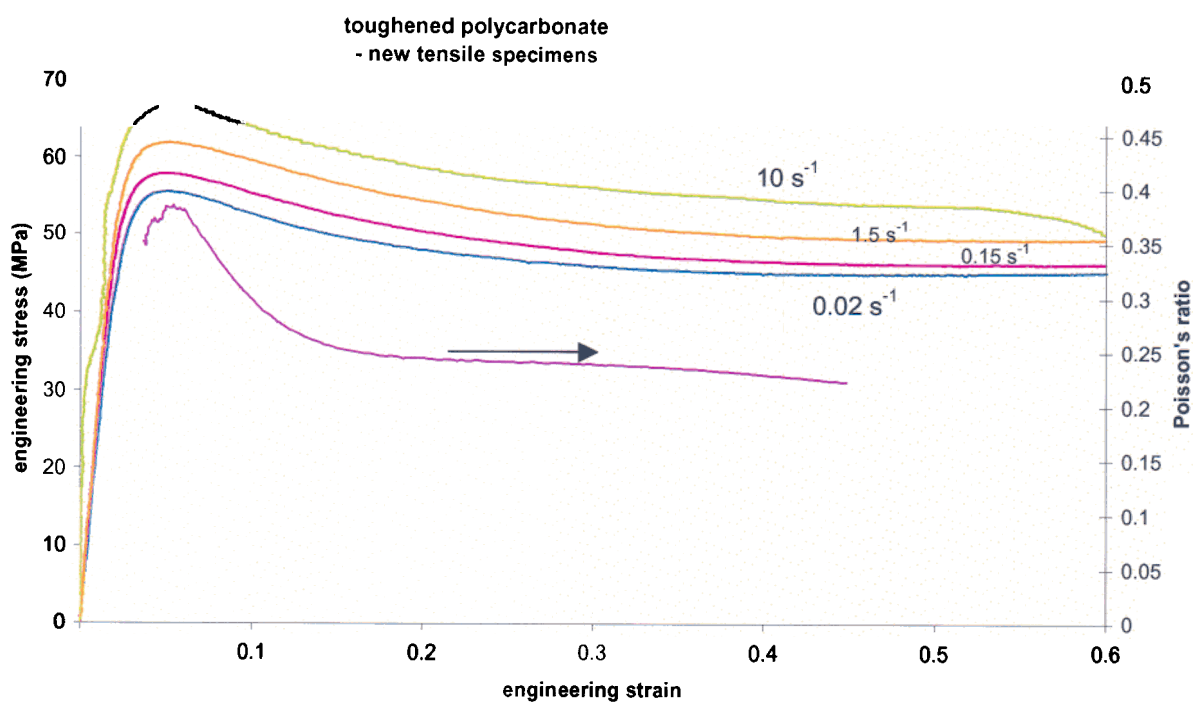


Figure 7 Tensile stress/strain curves and Poisson's ratio obtained using the new tensile specimen at test speeds ranging from 0.1 to 100 mm/s. Strain rates refer to the value at the peak in stress. Poisson's ratio was determined at the single speed of 0.1 mm/s.

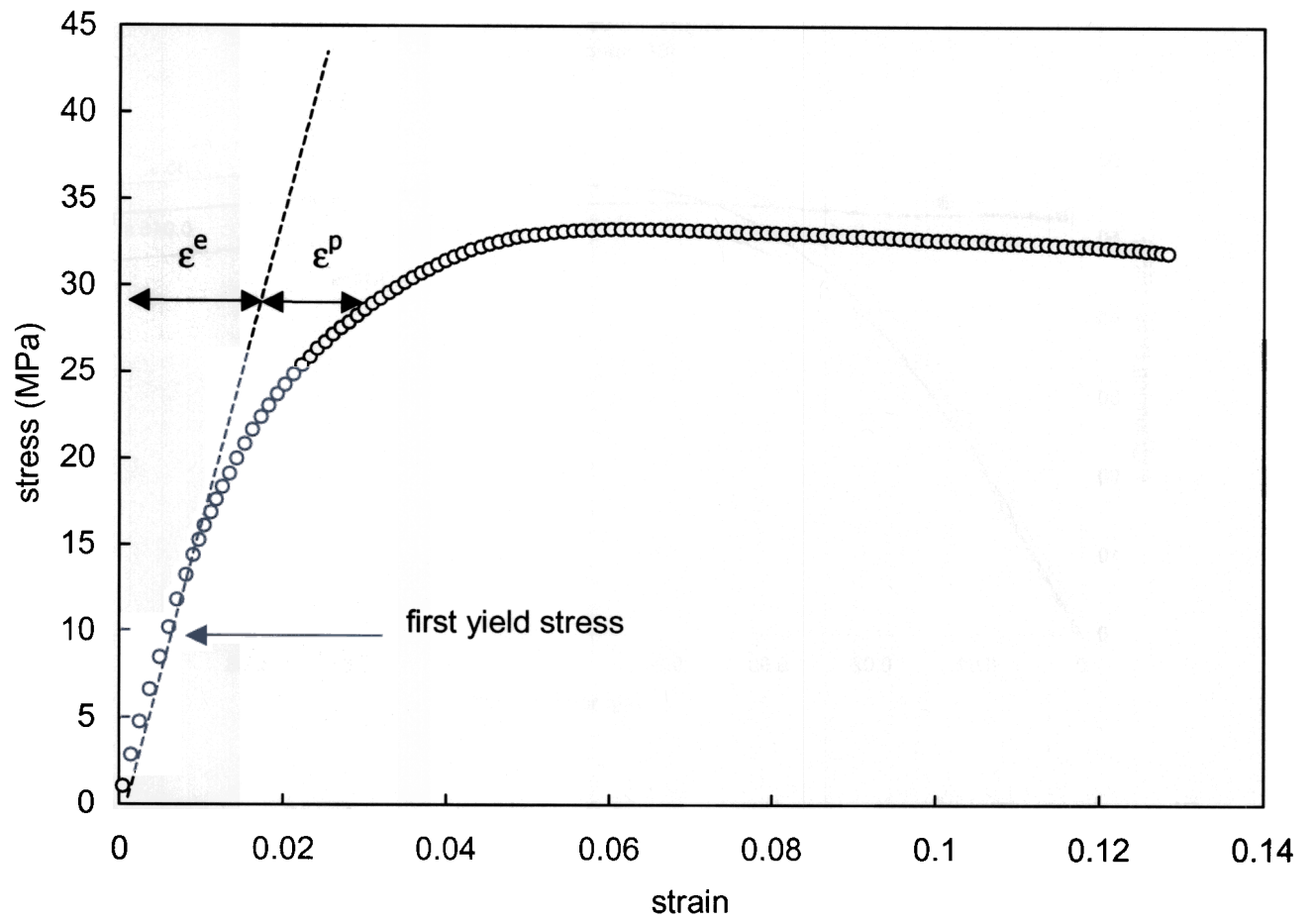


Figure 8 Schematic curve illustrating the separation of elastic and plastic components of strain for elastic-plastic models of non-linear deformation

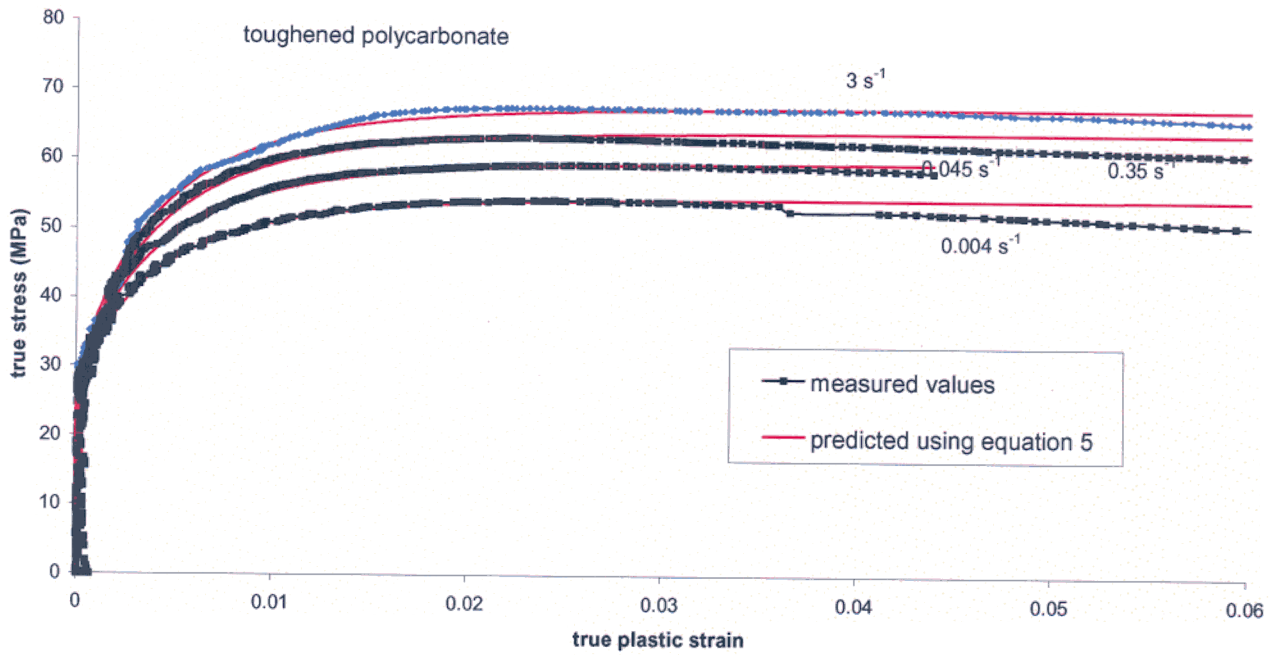


Figure 9 Stress/plastic strain curves derived from the data on small ISO specimens at different strain rates in figure 6. These have been modelled using equation 5.

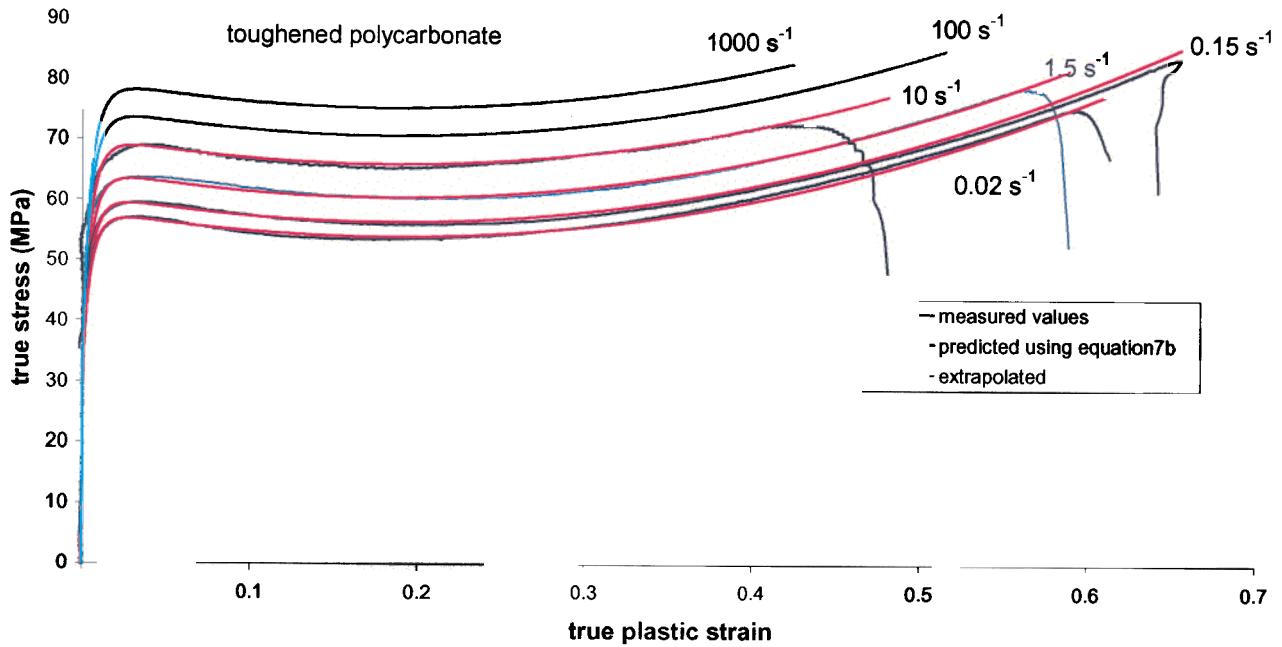


Figure 10 Stress/plastic strain curves derived from the data on new tensile specimens at different strain rates in figure 7. Curves in red have been modelled using equation 7b. Curves in blue have been extrapolated using equations 7b and 9.

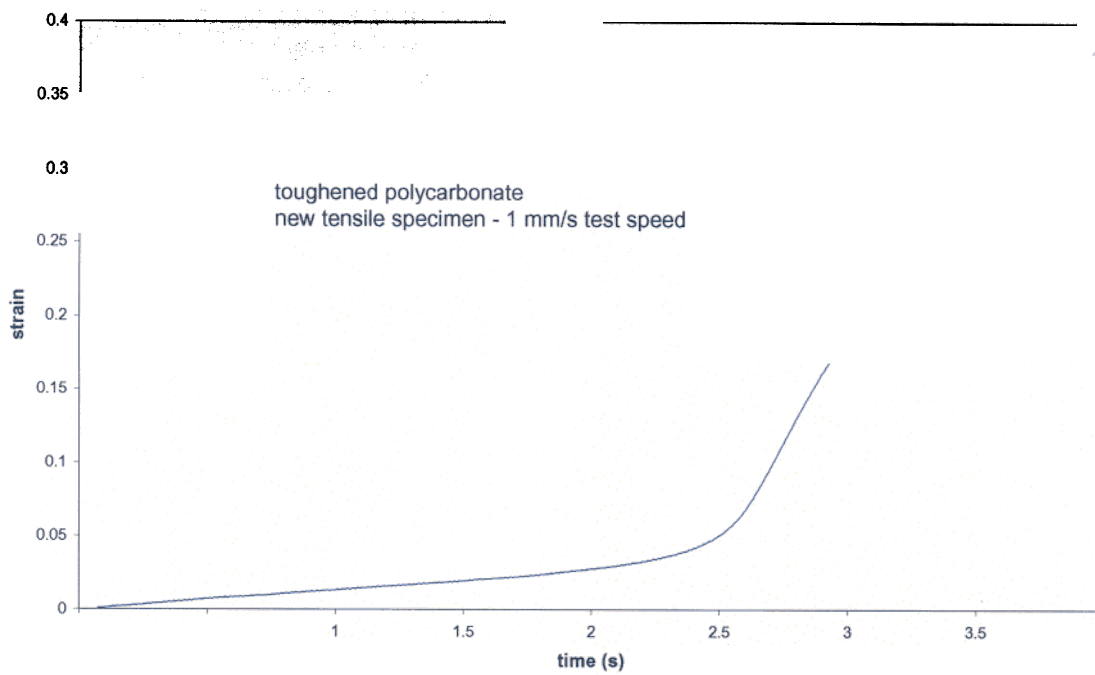


Figure 1 Strain against time for a test on the new tensile specimen at a speed of 1 mm/s

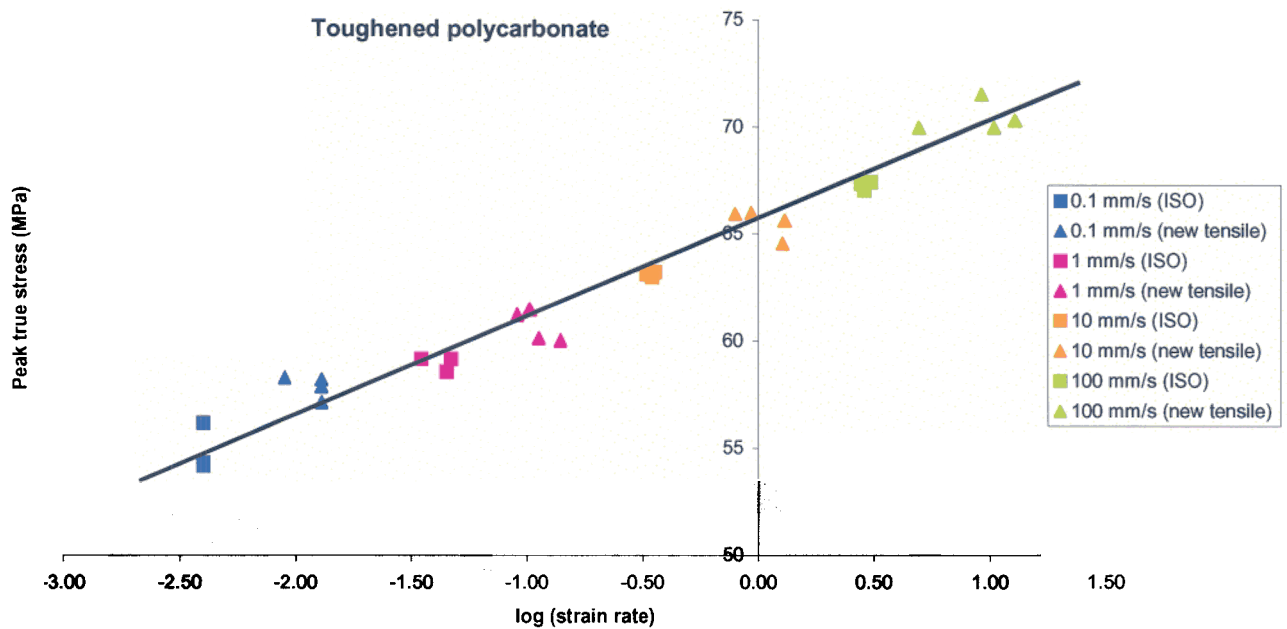


Figure 12 Plot of peak stress against $\log_{10}$ strain rate for all tests using both specimen types.