

Comparison of the Measured and Predicted Deformation of a Circular Plate of an ABS Polymer Under Centre Loading

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SUMMARY

Apparatus is described for deforming plate specimens that are supported on their lower surface by a circular ring of diameter 100 mm. The plates are loaded by a rigid, hemispherical shape contacting the centre of the upper surface of the plate. The apparatus has been instrumented to enable simultaneous measurements of the applied force, the central deflection and the displacement of reference marks on the lower surface of the plate.

Predictions of strains and deformations have been made using an elastic-plastic model in the Abaqus finite element system. The measurement of materials properties and parameters required by this model is described. The analysis of these data for an ABS polymer demonstrates limitations in the model for describing the non-linear deformation behaviour of rubber-toughened plastics. The cause of this is traceable to the nucleation of cavities in the rubber particles under certain stress states.

A new, elastic-plastic model is described that takes account of the influence of cavitation on yield and flow behaviour. Predictions of force against central deflection and surface strains using both models are compared with experimental values at a low test speed. Comparisons of force against central deflection measurements at higher test speeds are also made with predictions based on analyses that account for rate-dependent plasticity.

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1 INTRODUCTION

A previous report (1) described the use of finite element analyses to predict deformation of a circular plate of a ductile thermoplastic that was supported near the circumference of one face and loaded at the centre by a rigid hemisphere (see figure 1). Strain distributions in the lower surface of the plate were also calculated. At moderate to large deflections of the plate, the strains produced were above the limit for linear behaviour of the polymer. An elastic-plastic model was therefore used in the analyses to describe the deformation behaviour of the plates. The model used was the linear Drucker-Prager model in which the yield criterion takes account of the sensitivity of plastic deformation in polymers to the hydrostatic component of stress. In Abaqus, this model is implemented with rate-dependent plasticity which made possible the prediction of deformation and strain distributions at different loading speeds. For this purpose, experimental hardening curves were measured over a range of plastic strain rates. Calculated force against central deflection curves were compared with measurements at loading speeds of 0.1 mm/s, 10 mm/s and 1000 mm/s. The agreement was good.

At the lowest speed, the displacements of reference marks on the lower surface of the plate were measured by a photographic method. These were compared with values calculated by the FE analysis and were found to be consistently lower than predictions.

In this report, previous results are compared with predictions made using a new elastic-plastic model (2-4) that includes the influence of cavitation in the polymer under certain stress states on plastic deformation. This model is able to accurately predict the stress/strain behaviour of an ABS polymer under tension and compression using hardening data obtained in shear. In this polymer, cavities nucleate in the rubber particles, and so, in the absence of cavity growth, the effective volume fraction of cavities is determined by the volume fraction of rubber. The model has been able to successfully describe the stress peak and the region of negative slope in the tensile curve and the large fall in Poisson's ratio with strain that are attributable to cavity nucleation.

In addition, the influences of the magnitudes of the flow parameter in elastic-plastic models and the coefficient of friction between the polymer and the loading and support surfaces on predicted results are explored.

2 TESTS ON PLATE SPECIMENS

2.1 APPARATUS

The apparatus developed for loading the plate specimens is shown in figure 1. The specimen has a diameter of 120 mm and rests on a circular support of diameter 100 mm which is the upper edge of a cylindrical tube. The edge has a radius of 5 mm. For tests at low speed, the cylinder is fixed to the moving cross-head of a universal testing machine. The specimen is loaded through a hemispherical steel surface that is bolted to the upper part of the loading assembly. Clamped to this part is a bracket which passes through a slot in the wall of the cylinder and supports a mirror at an angle of 45° to the vertical. This allows the lower surface of the plate to be viewed by a camera positioned remote from the test assembly. The specimen is loaded by raising the supporting cylinder. In this way, the centre of the specimen remains fixed with respect to the photographic plane of the camera. The central deflection of the plate is determined from measurement of the cross-head movement.

Measurements of surface displacement were made at a single speed of 0.1 mm/s. Load against central deflection measurements were made over a range of loading speeds. For speeds up to 1 mm/s, the apparatus was mounted in a screw driven tensile testing machine. For higher speeds up to 1 m/s, a servohydraulic machine was used and the support cylinder was fixed whilst the hemispherical surface was attached to the actuator.

2.2 MEASUREMENT OF DEFORMATION IN THE PLATE SURFACE

The strain distribution in the lower surface of the specimen has been estimated by a photographic method which involves measuring the movement of reference marks on this surface with increasing deflection of the plate. Reference marks were positioned along 2 perpendicular lines through the centre of the face of the specimen. These lines were drawn with a black felt-tip pen. A very fine line was scribed through the centre of these lines using a

sharp stylus. The stylus was also used to define a cross at radial distances of 5, 10, 15, 20, 25 and 30 mm from the centre of the plate. The specimen surface was photographed using a Nikon D1 digital camera with a Sigma 180 mm macro lens and a dedicated flash. The light sensing element in this camera consists of an array of 1320 x 2010 pixels. The camera was positioned at a distance of 930 mm from the plate surface at which point the reference marks on the specimen filled the field of view. Simple image processing software was used to display a photographic image on a screen. Each fine cross appeared lighter than the surrounding area of black ink, and its centre could be located by eye to a single pixel enabling the coordinates to be identified. The photographic system was calibrated to enable horizontal displacements of the reference points during plate deformation to be determined from the changes in pixel coordinates.

3 ANALYSIS OF PLATE DEFORMATION USING THE LINEAR DRUCKER-PRAGER MODEL

3 THE YIELD CRITERION

The non-linear behaviour of solid materials in finite element systems is generally described using elastic-plastic models. In these models, behaviour at low strains is taken to be linear elastic and characterised by two materials parameters which are commonly the Young's modulus E and the elastic Poisson's ratio ν_e . The onset of non-linearity in a stress-strain curve is attributed to plastic deformation and occurs at a stress level regarded as the first yield stress. The subsequent increase of stress with strain is associated with plastic strain hardening. Stress analysis calculations involve the use of a yield criterion which for plastics is known to be sensitive to the hydrostatic component of stress. The simplest of these criteria takes the

$$\sigma_o = \sigma_e + \mu \sigma_k \quad (1)$$

where σ_e is the effective shear stress and σ_k is the hydrostatic stress. These are related to components of principal stress σ_1 , σ_2 and σ_3 by the equations

$$\sigma_e = \left\{ \frac{1}{2} \left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 \right] \right\}^{1/2} \quad (2)$$

and
$$\sigma_k = \frac{1}{3} (\sigma_1 + \sigma_2 + \sigma_3)$$

The quantities σ_o and μ are material parameters. σ_o is a hardening function (see section 3.3) and is related to the yield stress in shear σ_s by

$$\sigma_o = \sqrt{3} \sigma_s$$

The quantity μ is a measure of the sensitivity of yielding to the hydrostatic component of stress.

Equation (1) is identical to the yield criterion in the linear Drucker-Prager model in Abaqus where it takes the form

$$d = q - p \tan \beta \quad (5)$$

where
$$\begin{aligned} q &= \sigma_e & p &= -\sigma_k \\ d &= \sigma_o & \text{and } \mu &= \tan \beta \end{aligned} \quad (6)$$

The measurement of yield stresses in shear is less routine than it is under uniaxial tension where test methods are well established. For this reason, equation (1) may be more conveniently expressed in the form

$$\frac{(\mu+3)}{3} \sigma_T = \sigma_e + \mu \sigma_k \quad (7)$$

where σ_T is a yield stress under uniaxial tension.

3.2 THE FLOW POTENTIAL

The flow rule is used in elastic-plastic models for the determination of plastic strains through definition of the flow potential F . The most general form for the flow potential in the linear Drucker-Prager model is given by

$$F = \sigma_e + \mu' \sigma_k - \sigma_o \quad (8)$$

Here, the flow parameter μ' differs from the value of μ in equation (1) in the case of non-associated flow. In Abaqus, μ' is replaced by $\tan\Psi$.

3.3 DETERMINATION OF MODEL PARAMETERS

The yield stresses σ_T and σ_s (and hence σ_o) will increase with plastic strain because of strain hardening and plastic flow. Yielding is then characterised by the hardening functions $\sigma_T(\epsilon_T^p)$ or $\sigma_s(\gamma^p/\sqrt{3})$ where ϵ_T^p and γ^p are components of plastic strain under tension and shear respectively. Either of these functions is required by a finite element analysis and can be determined experimentally using procedures described elsewhere (4). Examples of hardening curves for the ABS polymer studied here are given in figure 2b and have been derived from the experimental data in figure 2a.

The magnitude of the hydrostatic stress sensitivity parameter $\mu(= \tan \beta)$ can be determined experimentally from measurements of yield stress under two different stress states. Uniaxial tension is an obvious choice for one stress state and the additional test is commonly either shear or uniaxial compression. If σ_C is a yield stress under uniaxial compression at the same effective plastic strain as σ_T and σ_s , then the parameter μ can be calculated using either of the following equations

$$\mu = 3 \left[\left(\sqrt{3} \sigma_s / \sigma_T \right) - 1 \right]$$

$$\mu = 3 \left[\frac{(\sigma_C / \sigma_T) - 1}{(\sigma_C / \sigma_T) + 1} \right]$$

If the data in figure 2b are used to determine μ , it is found that the derived value depends on whether tension and shear or tension and compression results are used. The value also depends on the strain level chosen for the determination of σ_s , σ_T and σ_C . Both of these observations arise because the mechanisms of plastic deformation under tension are different from those under shear or compression, and this gives rise to the different shapes of the tensile

and shear curves observed in figure This is discussed further in section 4.1 The value for μ quoted in table 1 has been derived using stresses from the flow region of the curves in figure 2b where yield stresses are relatively independent of plastic strain.

**Table 1 Values for the parameters for ABS
in the linear Drucker-Prager model**

E(GPa)	2.3	$\mu' = \tan \Psi$	0.9
ν_e	0.39	$\sigma_T(\epsilon_T^p)$	see fig 2b
$\mu = \tan \beta$	1.55	$\sigma_T(\epsilon_T^p, \dot{\epsilon}_T^p)$	see fig 3

A value for μ' can be determined experimentally from measurements of the plastic component of Poisson's ratio ν_p using the equation

$$\mu' = \frac{3(1 - 2\nu_p)}{2(1 + \nu_p)} \quad (11)$$

The dependence of the measured Poisson's ratio ν on strain is shown in figure 2a from which the plastic component ν_p has been calculated. A value for μ' derived using equation (11) is shown in table

3.4 RATE DEPENDENT BEHAVIOUR

Some of the parameters in elastic-plastic models are dependent on strain rate. Since the strain rate in the plate specimen depends upon location in the specimen and the speed of testing, rate-dependent data should be supplied for accurate predictions of performance. The property showing the highest dependence on strain rate is the hardening function $\sigma_T(\epsilon_T^p)$ or $\sigma_s(\gamma^p)$. Experimental hardening curves for the ABS polymer are shown in figure 3b. These were obtained from the results of tensile tests at different test speeds shown in figure 3a, which produced the plastic strain rates shown with each curve. These tests also reveal some sensitivity of the Young's modulus to strain rate as shown in table 2. An analysis with rate-

dependent elasticity has not been possible, but a different value for the modulus was selected for the analysis at each test speed appropriate to the mean strain rate in the specimen at that speed.

Table 2 Values for Young's modulus E at different strain rates determined from the experimental data in figure 3a

E(GPa)	Strain rate (s^{-1})
2.27	0.0006
2.30	0.004
2.34	0.06
2.36	0.2
2.40	2.3
2.44	33
2.47	104

3.5 COMPARISON OF PREDICTIONS OF PLATE DEFORMATION WITH EXPERIMENTAL RESULTS

Finite element analyses of the deformation of plate specimens were carried out at loading speeds of 0.1 mm/s, 10 mm/s and m/s. Figure 4 compares predicted curves of force against central deflection with experimental values. A single measured curve is shown at each speed that was selected as representative of repeated test results at each speed.

In figure 5, predicted and measured values for the displacements (horizontal component) of reference points at 5 mm and 10 mm from the plate centre are compared at discrete values for the deflection of the plate at a speed of 0 mm/s. Predictions compare well with experiment at small deflections where the deformation is predominantly elastic but calculated displacements lie progressively above measured values at larger deflections. These results indicate that predicted strains have a significant error. This may be attributable to limitations in the validity of the linear Drucker-Prager model, but there may be other contributing factors. It was therefore considered informative to explore the sensitivity of strain calculations to the coefficient of friction and the flow parameter $\mu' (= \tan \psi)$. A simple experiment was carried

out to determine the coefficient of friction between the plate surface and the support cylinder under a relatively small load. A value of 0.25 was obtained. Figure 6a shows the influence of the friction coefficient on predicted distributions of the radial and circumferential components of strain with distance from the plate centre. Figure 6b shows predicted strain distributions for different values of the flow parameter μ' .

The influence of including friction in the analysis is seen to lower strain predictions but not enough to make a significant improvement in the comparison with experimental data in figure 5. Decreasing the flow parameter below the measured value of 0.9 is seen to increase strain predictions whilst values above 0.9 are seen to have little effect.

4. ANALYSIS OF PLATE DEFORMATION USING THE CAVITATION MODEL

4.1 THE CAVITATION MODEL

The determination of some of the parameters in the linear Drucker-Prager model using the tensile and shear data in figure 2 has revealed potential limitations in the validity of this model for stress analysis in ABS. Furthermore, the value derived for the hydrostatic stress sensitivity parameter μ is quite different if tension and compression or compression and shear data are employed for the derivation. The inability of the linear Drucker-Prager model to accurately describe the deformation behaviour of ABS arises because of the nucleation of cavities in the rubber particles in ABS under stress states where there is a significant hydrostatic component. The presence of cavities promotes plastic deformation in the regions between cavities and lowers the applied stress to cause yielding. Cavity nucleation therefore gives rise to the fall in the tensile yield stress at a tensile strain of 0.02 and also to the decrease in Poisson's ratio with strain (see figure 2a).

4.2 THE YIELD CRITERION

In the absence of cavitation, equation (1) is a satisfactory criterion for yielding. This equation is now modified (2-4) to include the effects of cavitation as follows

$$\frac{\sigma_e^2}{\sigma_M^2} - (q_1 f)^2 + 2q_1 f \cosh \frac{3\sigma_k}{2\sigma_M} = \left(1 - \frac{\mu\sigma_k}{\sigma_M}\right)^2$$

Here f is the effective volume fraction of cavities which at small strains is zero but increases over some characteristic strain region associated with cavity nucleation. The parameter q_1 has been introduced to account for the effect of void interactions on the stress distribution in the matrix between cavities. σ_M is the effective yield stress of the matrix polymer between cavities and is equal to σ_o in the absence of cavities but rises with increasing f as the volume fraction of the rubber in the matrix decreases. The increase in σ_M as cavities nucleate is given by the expression

$$\sigma_M = \frac{\sigma_o}{1 - kv_{R0}} \left[-k \frac{(v_{R0} - f_n)}{1 - f_n} \right]$$

where v_{R0} is the volume fraction of rubber and f_n is the effective volume fraction of cavities created by cavity nucleation in rubber particles. At large strains, the quantity f may be larger than f_n as a result of cavity growth subsequent to nucleation. The parameter k relates the shear yield stress of a rubber-toughened polymer to the volume fraction of rubber v_{R0} (2,4) by the equation

$$\sigma_o = \sigma_{o1} (1 - kv_{R0})$$

where σ_{o1} is the shear yield stress of the untoughened polymer. In the cavitation model, $\sigma_o = \sqrt{3}\sigma_s$ is derived from experimental data obtained in shear and replaces σ_T as the basic hardening function for the polymer. As in equation (1), the parameter μ in equation (12) specifies the sensitivity of yielding to the hydrostatic stress in the absence of cavitation. It is therefore determined from experimental data in shear and compression (see figure 2b) using

$$\mu = 3 \left(1 - \frac{\sqrt{3}\sigma_s}{\sigma_c} \right)$$

Values for the parameters v_{Ro} , q_1 , k and μ for the ABS polymer studied here are given in table 3

Table 3 Values for the parameters for ABS in the cavitation model

E(GPa)	2.3	ϵ_{1V}	0.0036
v_e	0.39	ϵ_{2V}	0.002
v_{Ro}	0.38	β	0.62
k	1.35	μ_1'	0.16
μ	0.22	$\Delta\mu'_R$	0.39
q_1	1.57	$\sigma_o (\gamma^p / \sqrt{3})$	see fig 2b

4.3 CAVITY NUCLEATION

Following the work of Bucknall and co-workers, it is assumed that a cavity is nucleated in a rubber particle at some critical volumetric strain that decreases with increasing particle diameter. For a distribution of particle sizes, the cavity nucleation should then occur over a range of total volumetric strain ϵ_V related to the critical strain range for the rubber particles. Different expressions were considered for the cavity nucleation function and were evaluated by comparing the predicted tensile curve with the experimental data shown in figure 2b. The best fit was obtained using the function

$$\begin{aligned}
 f_n &= 0 \quad \text{for } \epsilon_V \leq \epsilon_{1V} \\
 f_n &= v_{Ro} \left\{ 1 - \exp \left(- \left(\frac{(\epsilon_V - \epsilon_{1V})}{\epsilon_{2V}} \right)^\beta \right) \right\} \quad \text{for } \epsilon_V > \epsilon_{1V}
 \end{aligned} \tag{16}$$

The parameters ϵ_{1V} , ϵ_{2V} and β determine the location and breadth of the volumetric strain range over which cavity nucleation occurs and are determined through an iterative process to give the best prediction of tensile behaviour. This is shown in figure 7(a). Values for the parameters are given in table 3.

4.4 THE FLOW POTENTIAL

The results of axial and transverse strain measurements in ABS specimens under uniaxial tension has revealed that, using the linear Drucker-Prager model, flow in this polymer is non-associated (see section 3.2). The flow potential is therefore assumed to be

$$F = \frac{\sigma_e^2}{\sigma_M^2} - (q_1 f)^2 + 2q_1 f \cosh \frac{3\sigma_k}{2\sigma_M} - \left(1 - \frac{\mu' \sigma_k}{\sigma_M}\right)^2 \quad (17)$$

Although equation (11) is now not valid for the determination of μ' , a related expression for a cavitating material can be derived using equation (17) in the flow rule and enables a value for μ' to be determined from plastic Poisson's ratio measurements. A value for $\mu' = 0.16$ gives the Poisson's ratio predictions shown in figure 7b which compare well with experimental values. This value for μ' gives rise to a significant volumetric strain ϵ_v during plastic deformation even under stress states such as shear and compression. According to the proposed nucleation function equation (16), this plastic expansion is sufficient to induce cavitation under shear and compression. Further tests have been carried out involving measurements of axial and transverse strain under uniaxial compression. These indicate a small plastic expansion under compression (and therefore in the absence of cavitation) but not as large as predicted with the value of $\mu' = 0.16$. An expression was therefore developed that allows μ' to be dependent on the stress state and the volume fraction of cavities (3). This has the form

$$v_R^* \sinh \left[\frac{3\sigma_k (1 - kv_R^*)}{2\sigma_M (1 - f)} \right]$$

where v_R^* is the instantaneous volume fraction of unvoided rubber in the polymer surrounding any cavities and is given by

$$v_R^* = \frac{v_{Ro} - f_n}{1 - f_n} \quad (19)$$

4.5 RATE-DEPENDENT BEHAVIOUR

The cavitation model has been coded to interface with the Abaqus finite element system for the solution of analyses by the standard or explicit solver. At present, the implementation does not include rate-dependent plasticity. For the purpose of comparisons of predictions with the linear Drucker-Prager model, hardening curves at a single effective plastic strain rate of 0.004 s^{-1} have been used. These are shown in figure 2b, the tensile curve being used with the linear Drucker-Prager model and the shear curve with the cavitation model.

4.6 CAVITY GROWTH AND ORIENTATION HARDENING

At strains above the range responsible for cavity nucleation, cavity growth occurs through plastic deformation within the matrix material. This leads to an increase in f above f_n , the magnitude of which depends on the stress state. Figure 8 shows comparisons of predictions of tensile stress/strain curves, with and without cavity growth, with experimental data out to 30% strain. The prediction with growth falls below experimental data, but both predicted curves neglect the effect of orientation hardening that is visible in experimental data at the higher strains. Orientation hardening is caused by the alignment of molecules at high plastic strains which produces an additional stress during plastic flow. Its contribution to behaviour under uniaxial tension has been considered in an earlier report (5) but no method has yet been developed for including the effects of orientation hardening for other stress states. The observation that the predicted curve without cavity growth gives a better fit to the tensile data implies that the effects of cavity growth and orientation hardening tend to cancel each other, at least up to strains of about 0.25 under uniaxial tension. Accordingly, stress analysis calculations of the deformation of plate specimens using the cavitation model have been carried out neglecting cavity growth.

5 COMPARISON OF PREDICTIONS OF PLATE DEFORMATION USING THE CAVITATION AND THE LINEAR DRUCKER-PRAGER MODELS

5 FORCE AGAINST CENTRAL DEFLECTION PREDICTIONS

Figure 9 compares force/deflection predictions using both models with the respective parameters given in tables 2 and 3. A single hardening curve, shear for the cavitation model

and tensile for the linear Drucker-Prager, at an effective plastic strain rate of 0.004 s^{-1} and a coefficient of friction of 0.25 were employed with each model

5.2 PREDICTIONS OF STRAIN DISTRIBUTIONS

Figure 10 compares predicted distributions of principal strain components from each model with radial position in the lower surface of the plate. The strain levels are fairly insensitive to the plastic strain rate of the hardening curve and hence on test speed. It can be seen that the maximum and intermediate principal strains predicted by the cavitation model are about 10% lower than those calculated using the linear Drucker-Prager model. The former are therefore closer to measured values but, by inference from figure 5, are still significantly above these values. Inclusion of cavity growth in the strain predictions using the cavitation model has the effect of increasing all principal strain components.

Predicted strains at the plate centre in the thickness direction of the plate are sensitive to the value used for the flow parameter. According to model calculations, this strain component decreases with position through the plate thickness by about 15%. Measurements of changes in the plate thickness during a test might therefore give an additional method for estimating or verifying the value of μ' .

6 CONCLUSIONS

The linear Drucker-Prager model has a limited ability to describe the deformation behaviour of rubber-toughened plastics. Deformations under shear and compression can be modelled with satisfactory accuracy as long as model parameters are determined from shear and compression data. Under tensile stress states, however, the rubber particles cavitate producing enhanced plastic deformation around the cavities which is not described by the model.

The linear Drucker-Prager model has been modified to account for the influence of cavity nucleation on yield stresses and flow behaviour. A cavity nucleation criterion is proposed related to a critical range of volumetric strain for cavity formation. This cavitation model is able to describe the different shape of the stress/strain curve for an ABS polymer under

uniaxial tension (caused by cavitation) compared with the shape under shear and uniaxial compression. It also predicts the decrease in Poisson's ratio in this material with strain which is attributable partly to cavity formation and partly to the high value for the flow parameter.

Both models give similar and accurate predictions of force against central deflection of plate specimens of the ABS. Parameters for the linear Drucker-Prager model were determined from tensile and shear test data. Different predictions are expected from the use of data from tension and compression or compression and shear tests for obtaining the material parameters. The use of hardening curves measured over a range of strain rates with rate-dependent plasticity in the linear Drucker-Prager model gives good predictions of force/deflection curves at different test speeds. The determination of parameters for the cavitation model requires test data under all 3 stress states of tension, shear and compression.

Predictions of strain levels in the lower surface of plate specimens are higher than measured values for both models. These predictions are sensitive to the values used for the friction coefficient and the flow parameter. Predictions using the cavitation model are slightly closer to measured values than those from the linear Drucker-Prager model

Further measurements of surface strain or displacement near the centre of the plate should be made to confirm the above conclusion

ACKNOWLEDGEMENTS

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8 REFERENCES

- Deformation of a centrally loaded circular plate of a thermoplastic material: comparison of measurements with predictions using a finite element analysis. G Dean and L Wright. NPL Report CMMT(A)299, December 2000.
2. A proposed model for the non-linear, stress/strain behaviour of rubber-toughened plastics. B Read, G Dean and L Wright. NPL Report CMMT(A)301, December 2000.
 3. A further evaluation of the cavitation model for rubber-toughened plastics. G Dean, B Read and L Wright. NPL Report MATC(A)75, January 2002.
 4. Modelling non-linear stress/strain behaviour of rubber-toughened plastics. B Read, G Dean and L Wright; *Plastics, Rubber and Composites*, Vol 30, No 7, p 328 (2001).
 5. Effects of cavity growth and orientation hardening on the stress/strain behaviour of rubber-toughened plastics. B Read, G Dean and L Wright. NPL Report MATC(A)35, July 2001

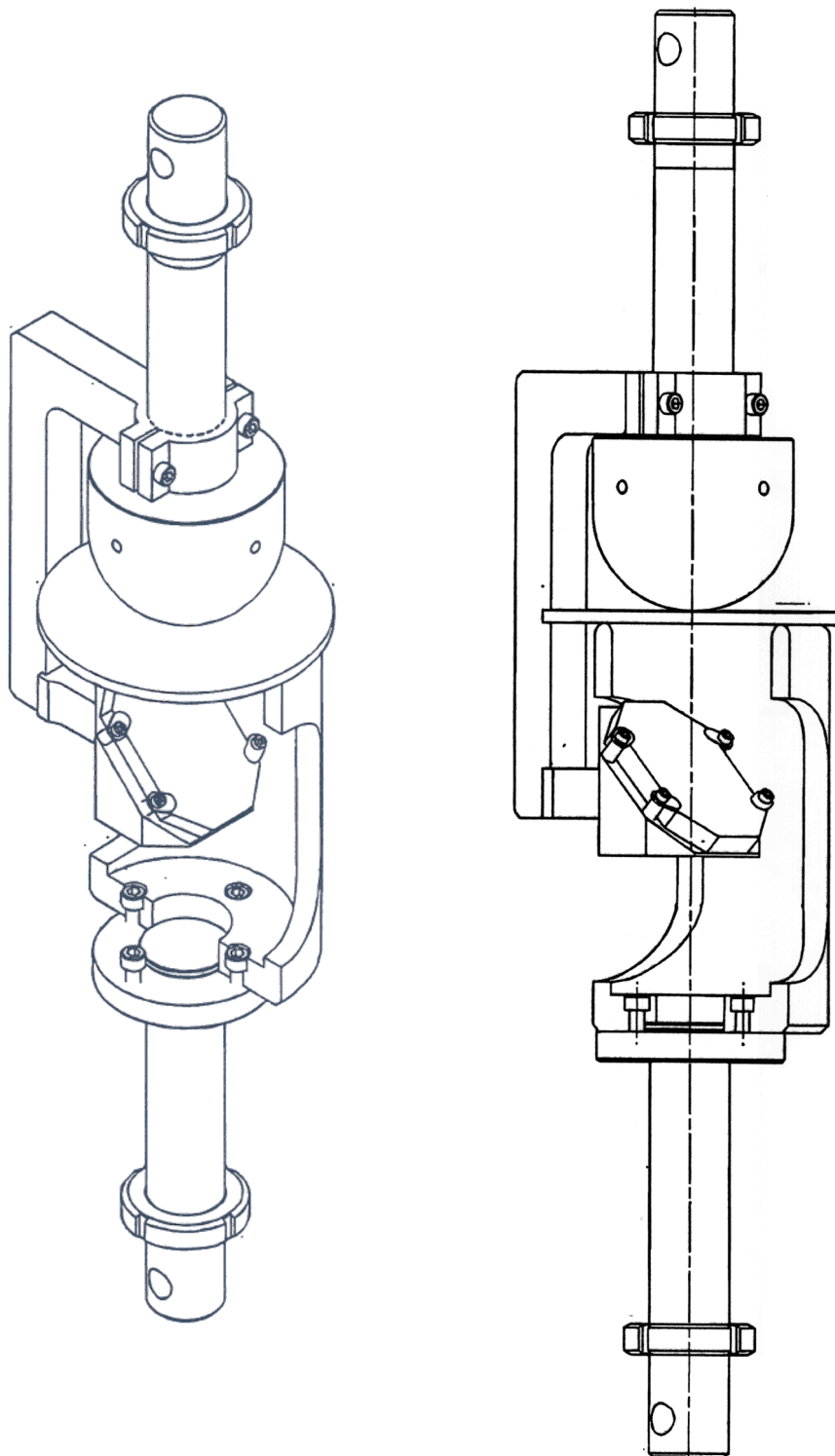
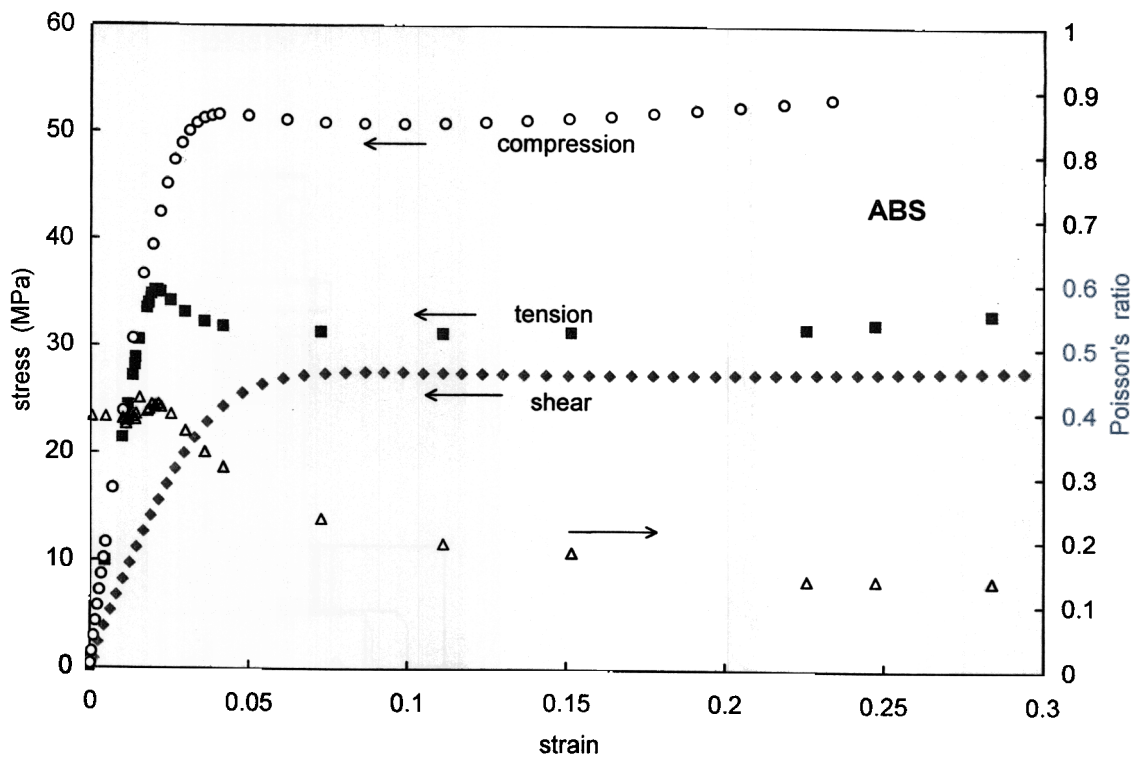
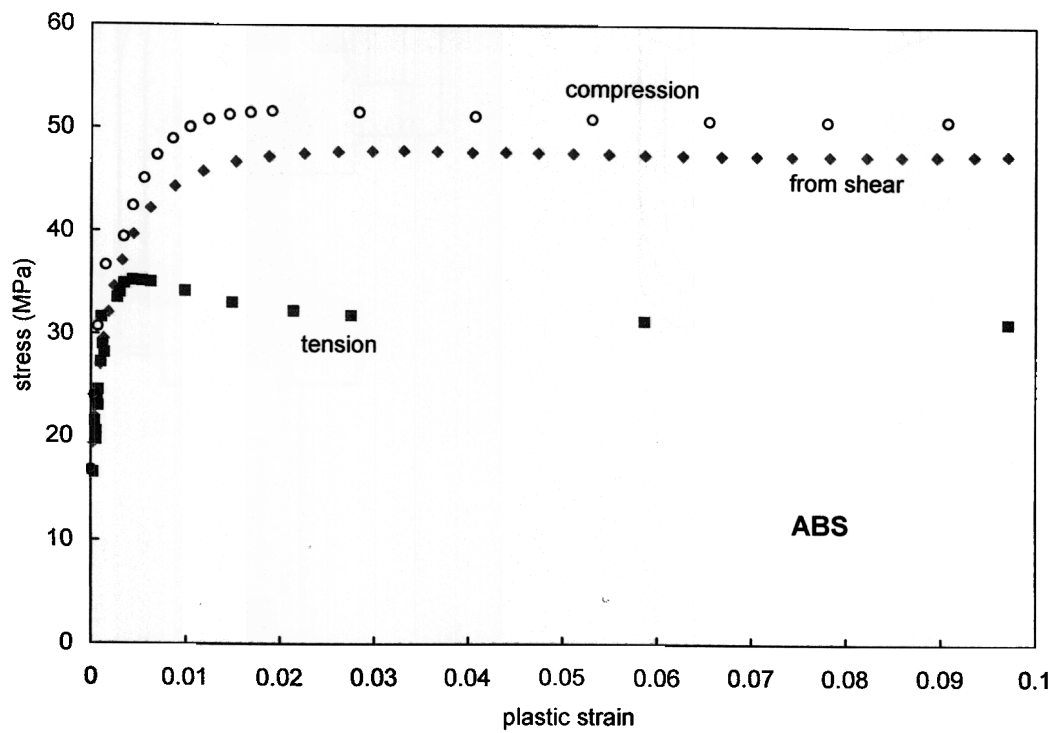


Figure 1. Apparatus for testing plate specimens. The mirror beneath the specimen enables measurements to be made of surface displacements in the deformed specimen.



(a)



(b)

Figure 2. (a) True stress/true strain curves and Poisson's ratio for ABS measured in tension, compression and shear at an effective plastic strain rate of 0.004 s^{-1} and (b) plastic strain hardening curves derived from these data.

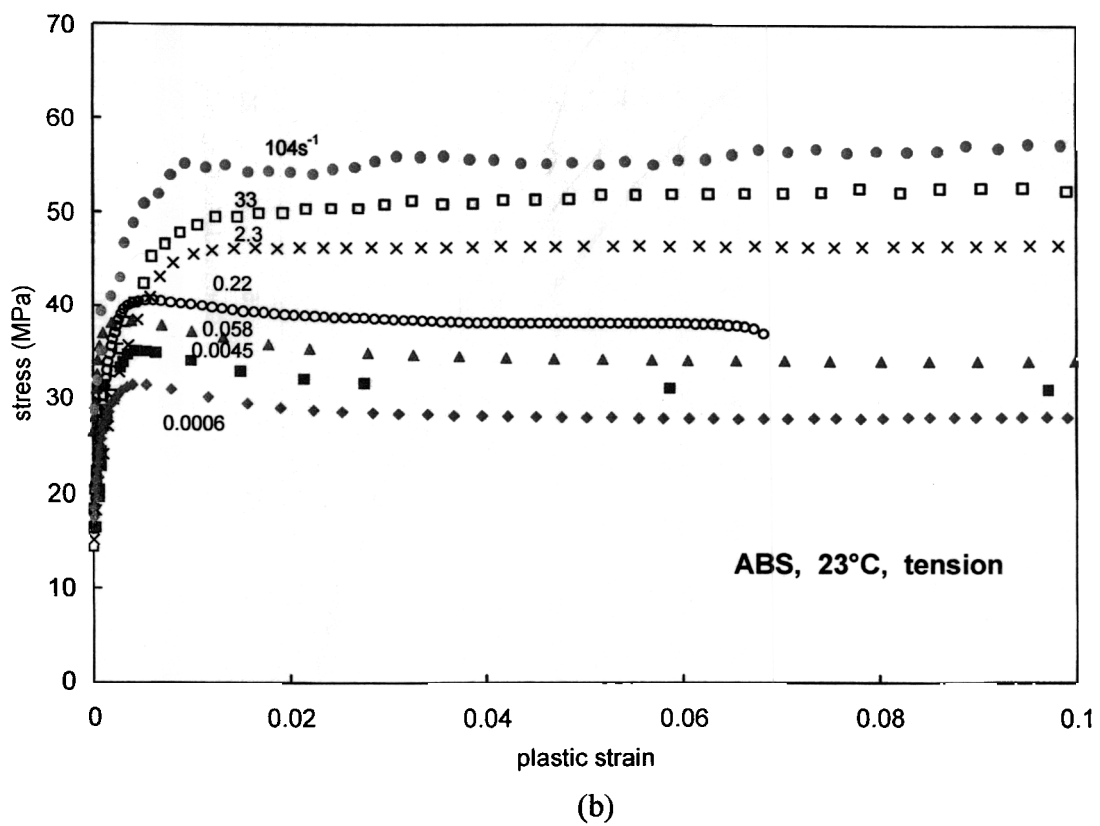
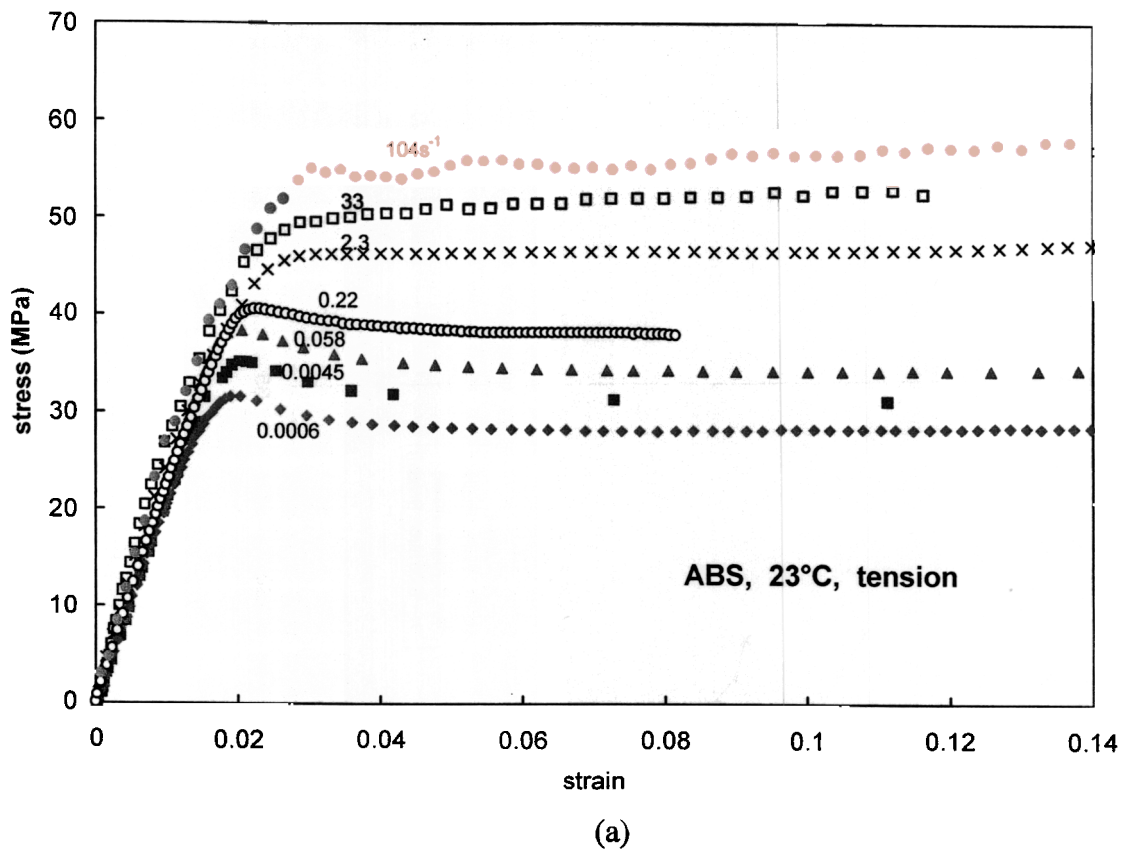


Figure 3. (a) Tensile true stress/true strain curves for the ABS polymer measured at different strain rates and (b) tensile hardening curves derived from these. The plastic strain rates are shown with each curve.

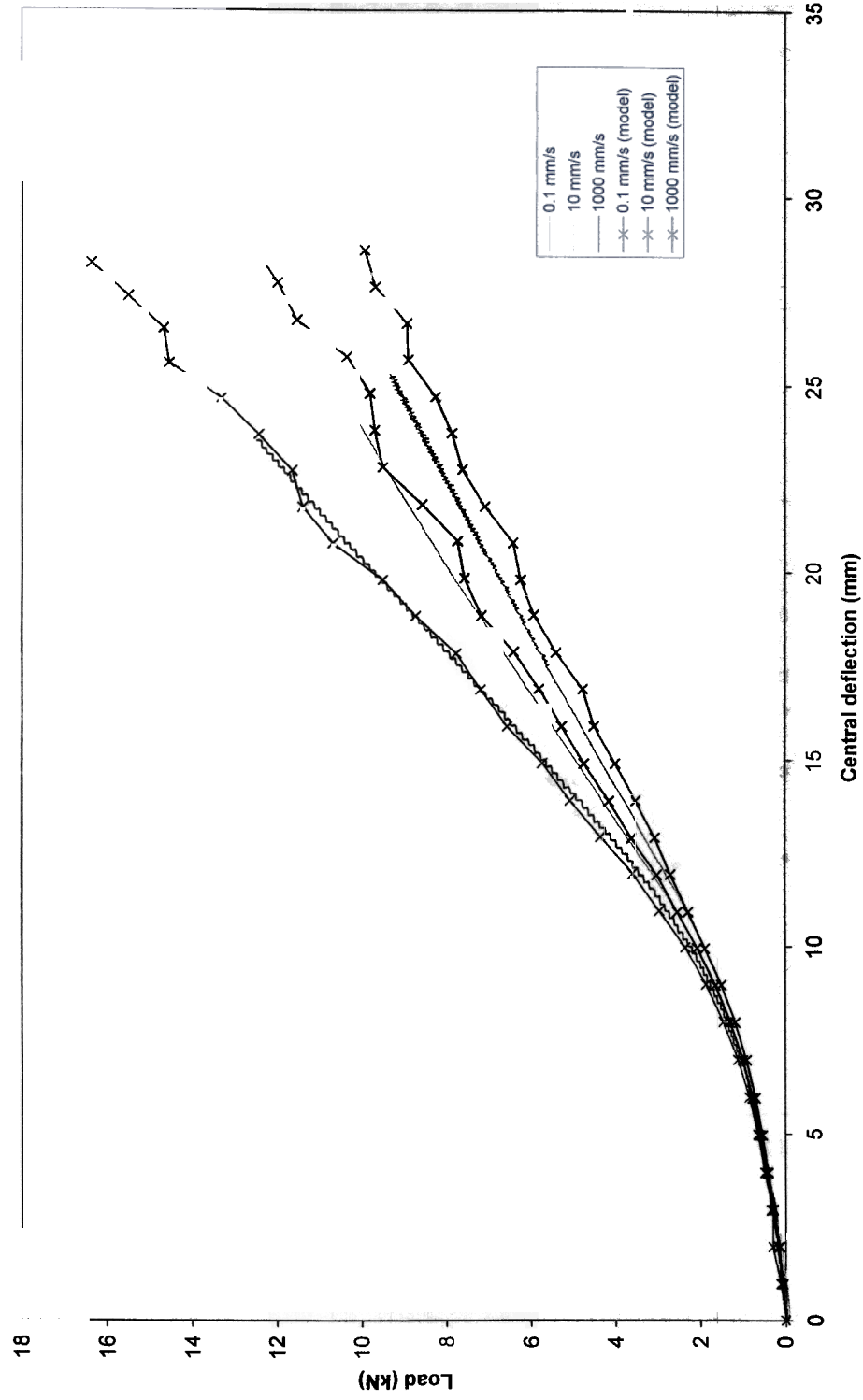


Figure 4: Comparison of measured force against central deflection curves at different speeds with predicted results using the linear Drucker-Prager model.

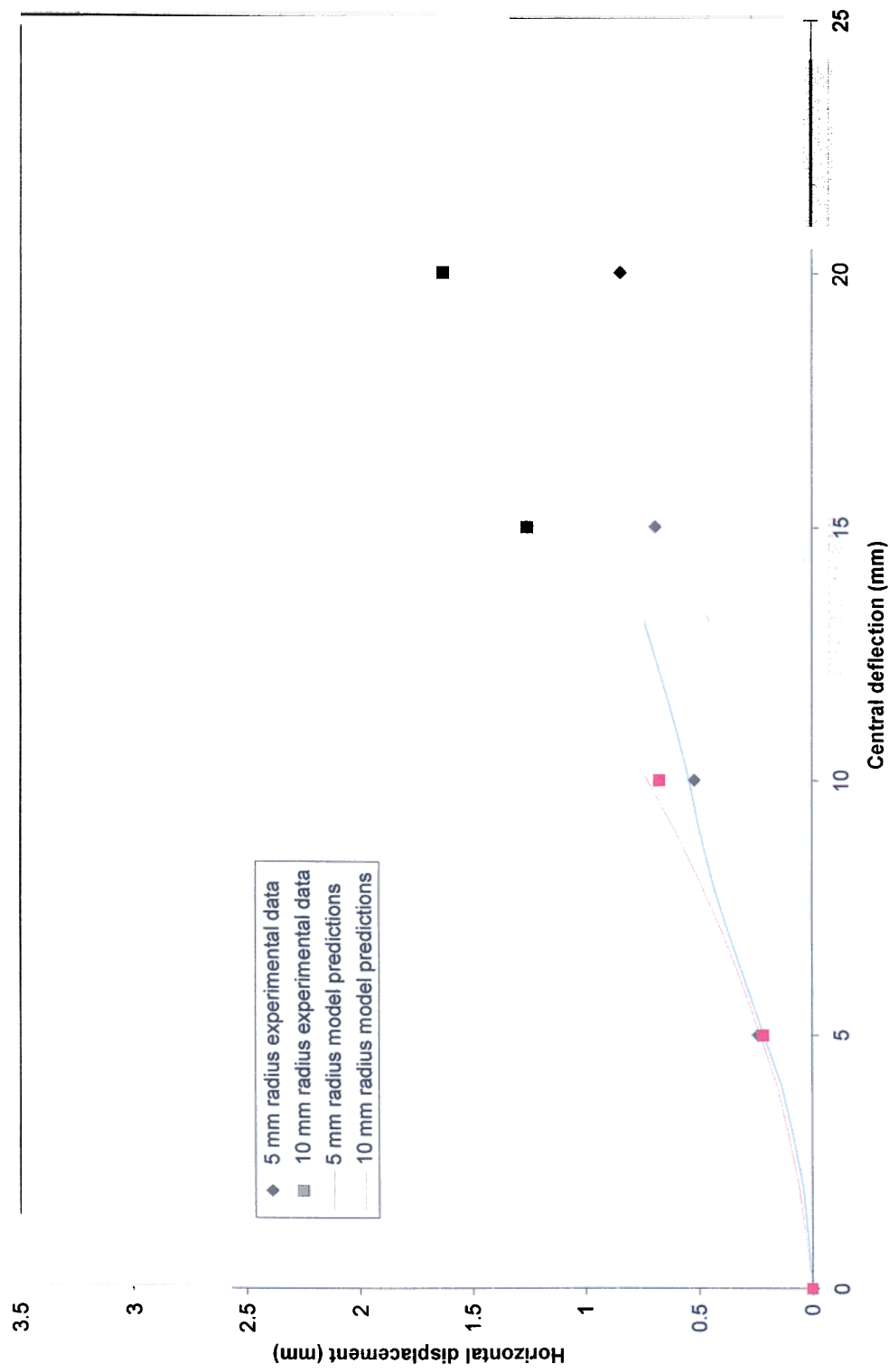


Figure 5. Comparison of measurements of the horizontal displacements of reference points on the lower surface of a plate specimen with predictions made using the linear Drucker-Prager model. Rate dependent plasticity, no friction.

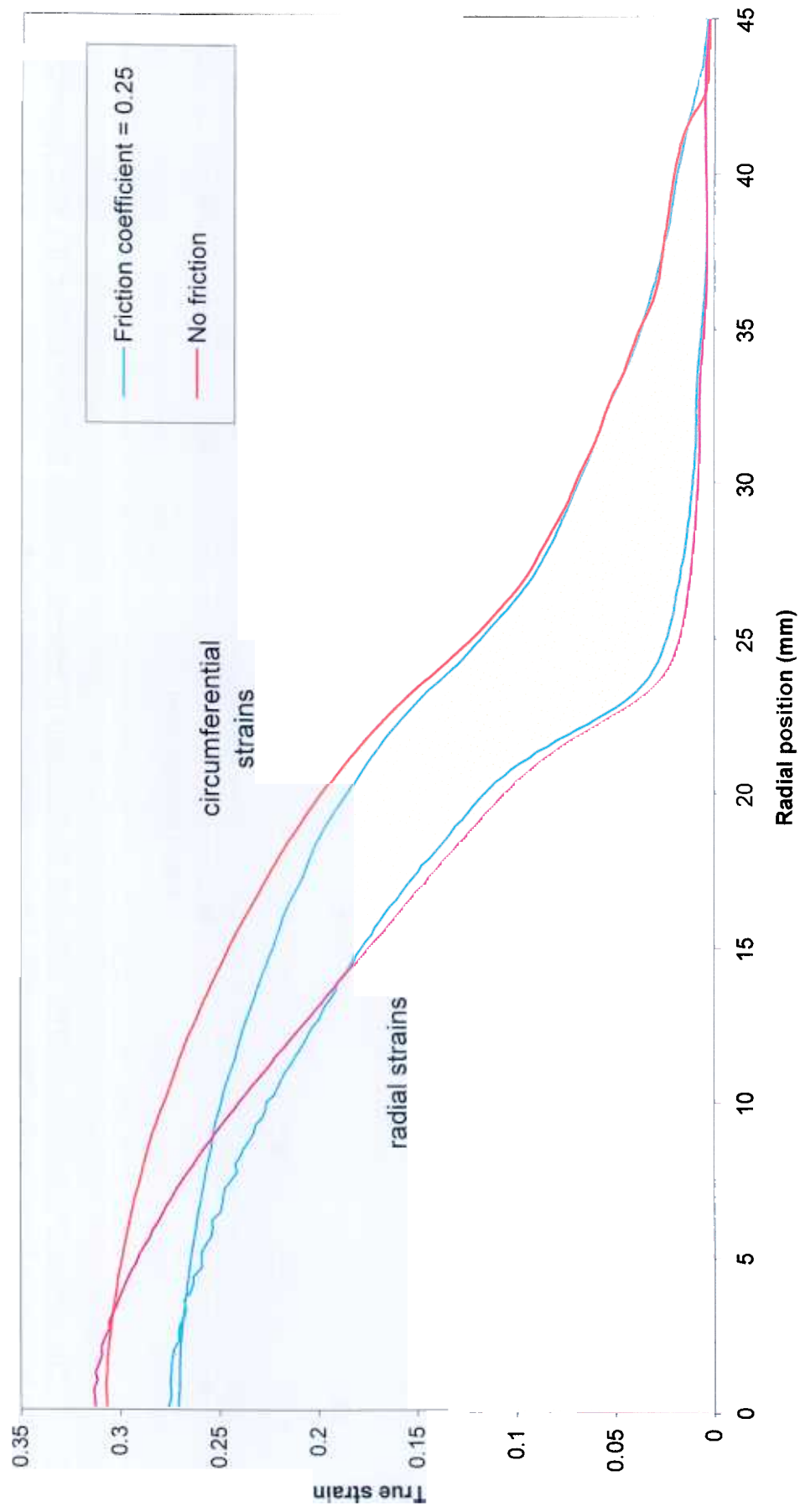


Figure 6a. Influence of friction on predicted strain distributions in the radial and circumferential directions in the lower surface of a plate specimen at a central deflection of 24 mm. Linear Drucker-Prager model

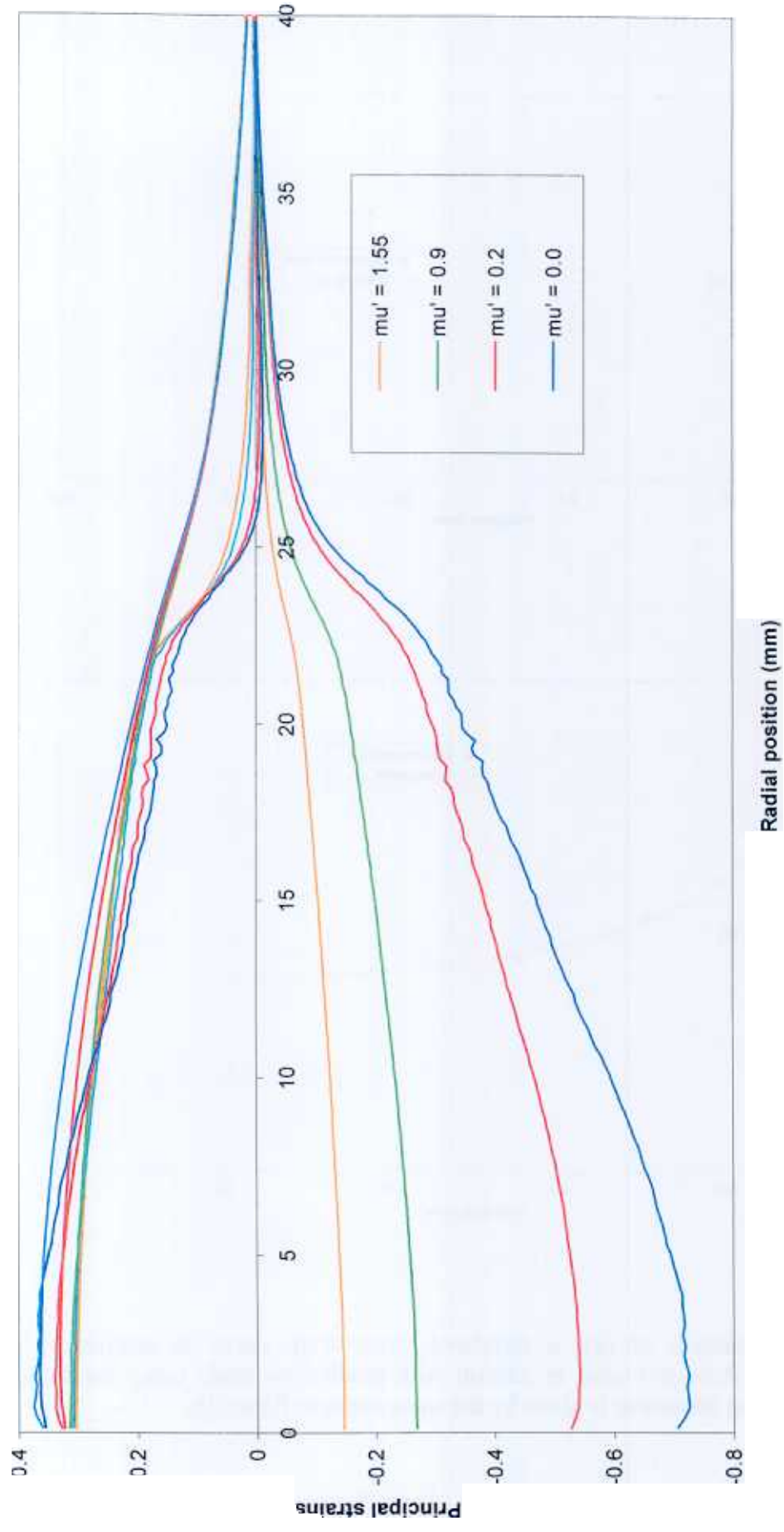


Figure 6b. Influence of the flow parameter μ' on predicted components of principal strain at a central deflection of 24 mm. Linear Drucker-Prager model

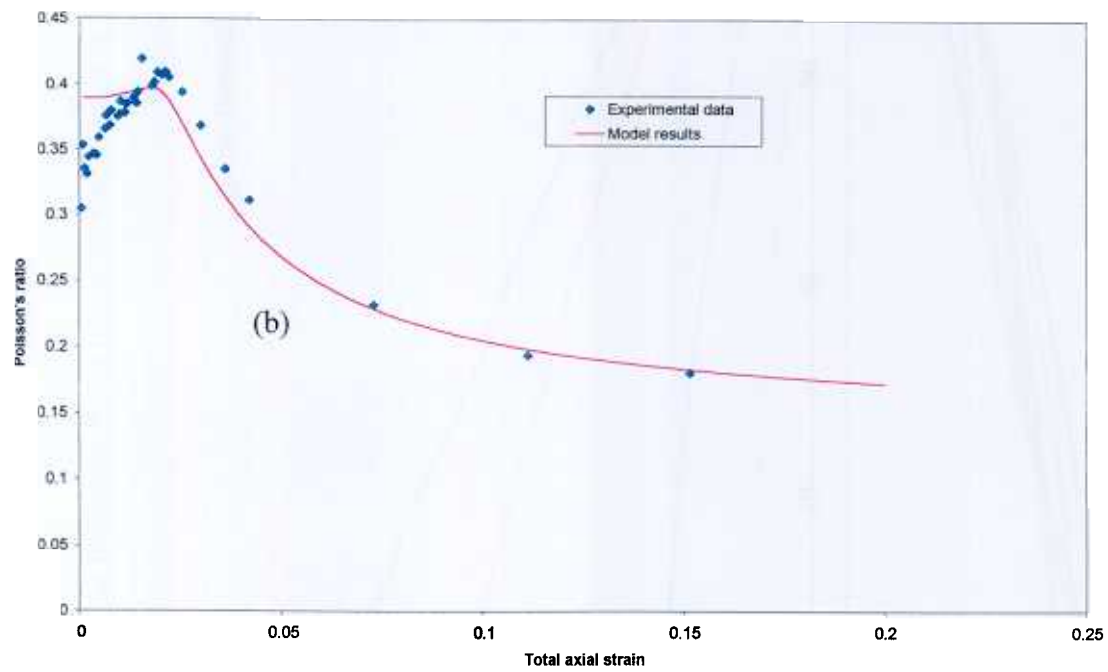
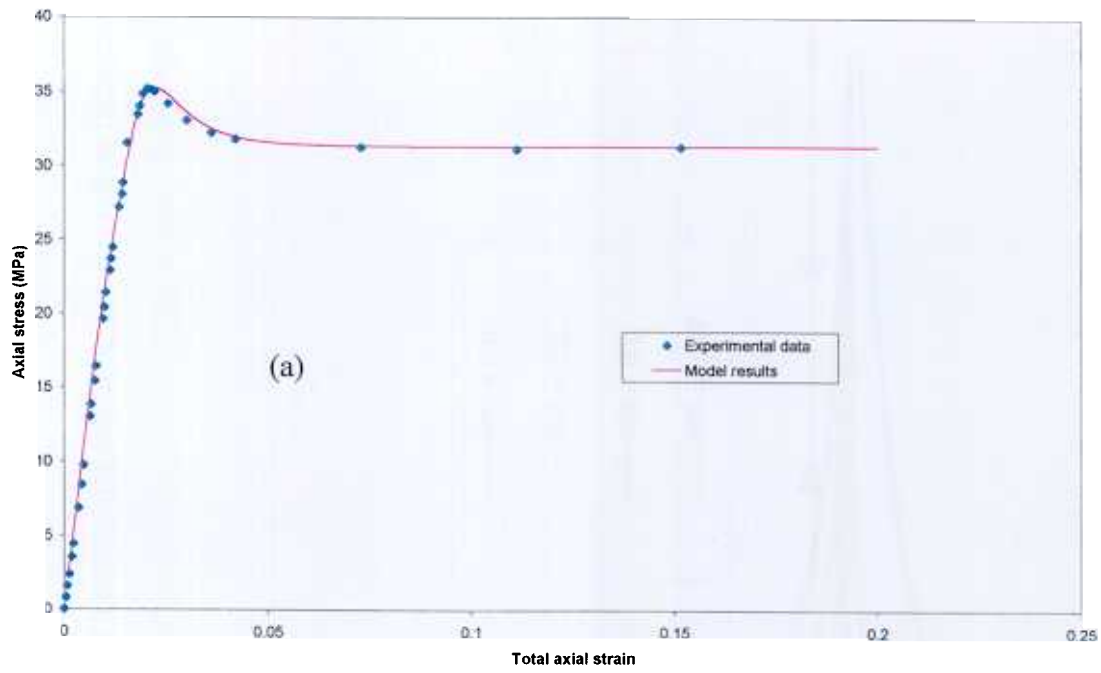


Figure 7. Comparison of (a): a measured stress-strain curve in tension and (b): measured values of Poisson's ratio in tension with predictions made using the cavitation model. The hardening behaviour is given by the shear curve in figure 2b.

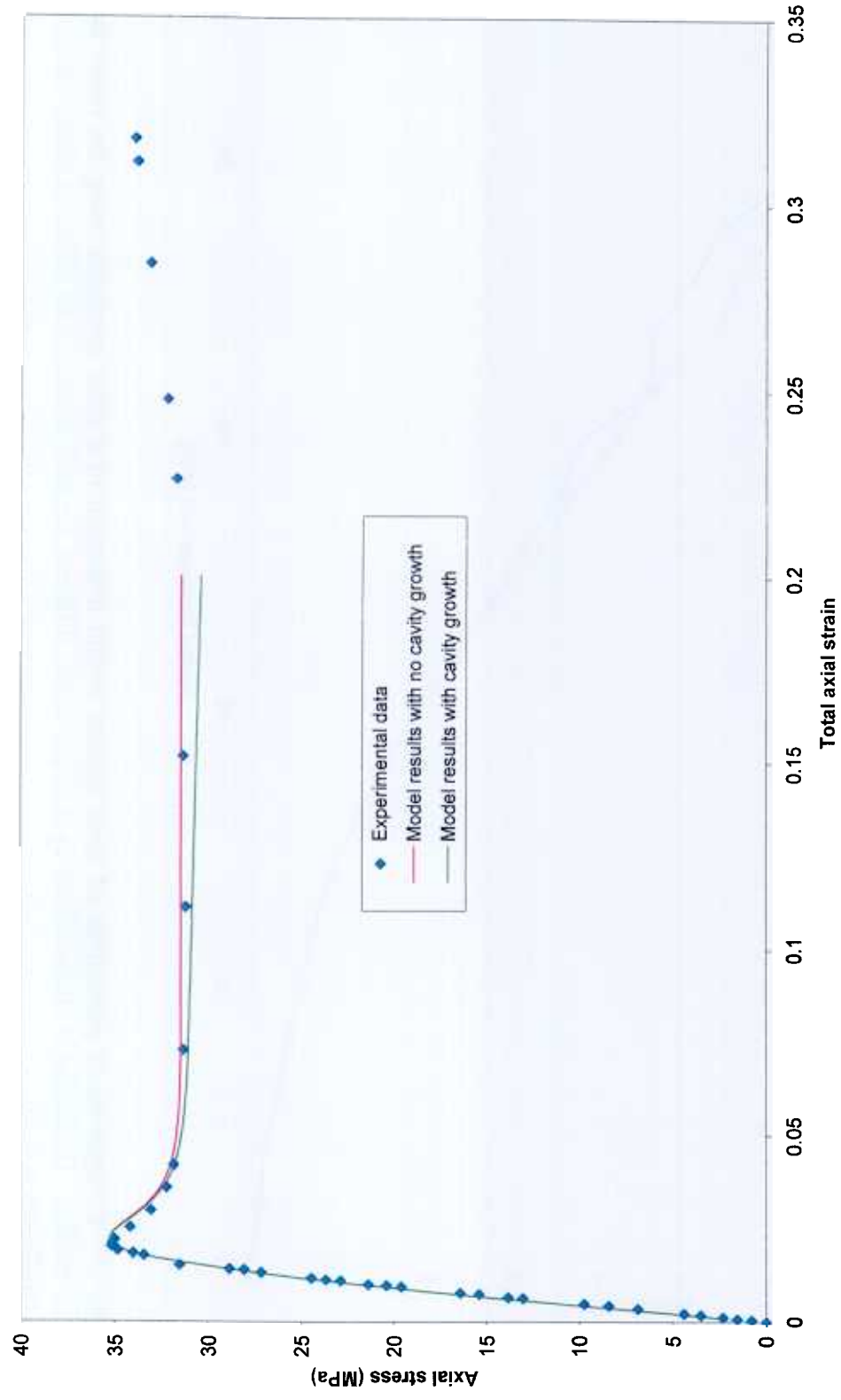


Figure 8. Comparison of an experimental tensile stress/strain curve for the ABS with predictions made using the cavitation model with and without cavity growth.

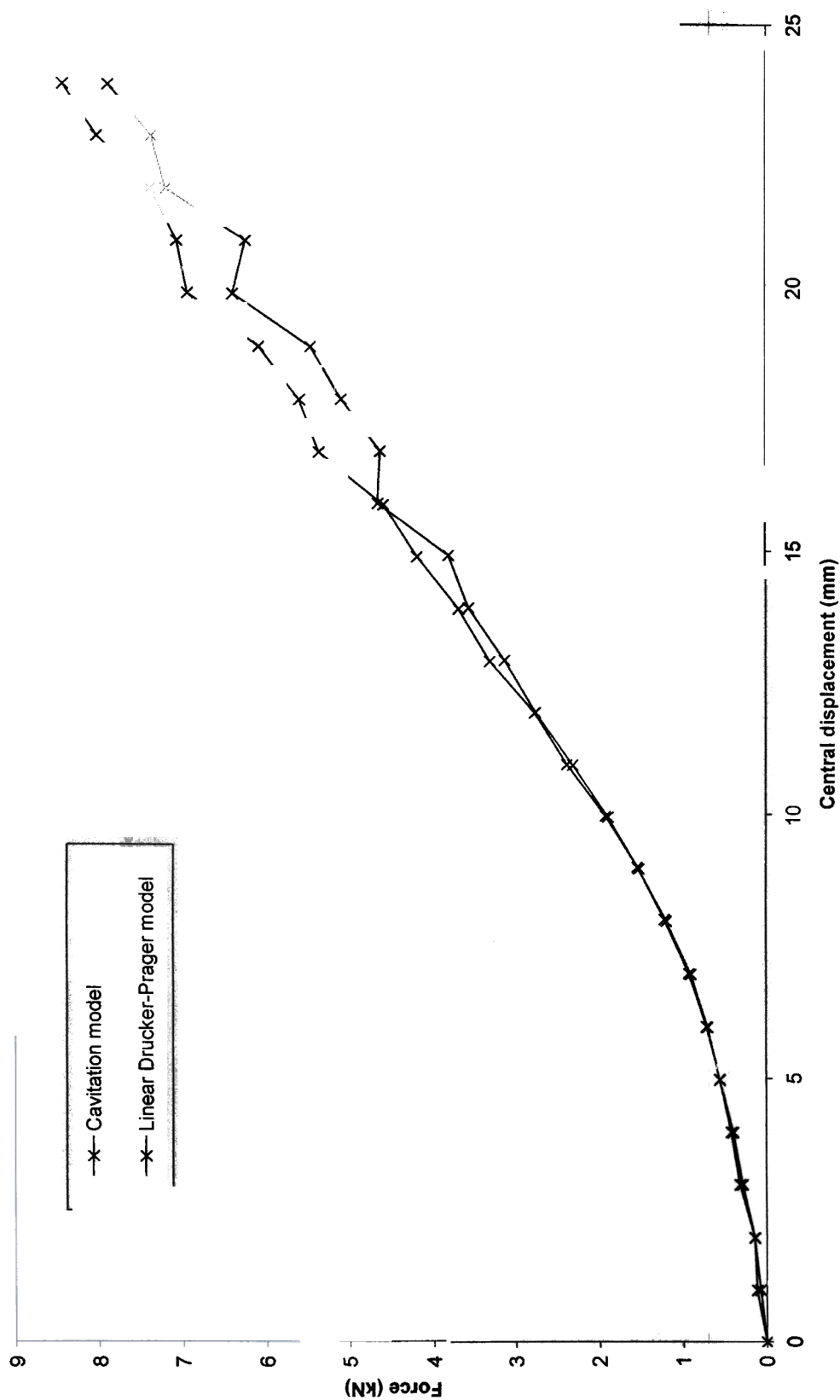


Figure 9. Comparison of predictions of force against central deflection of a plate specimen using the linear Drucker-Prager and the cavitation models. Hardening is characterised by a single curve, different for each model, at an effective plastic strain rate of 0.004 s^{-1} . The friction coefficient is 0.25

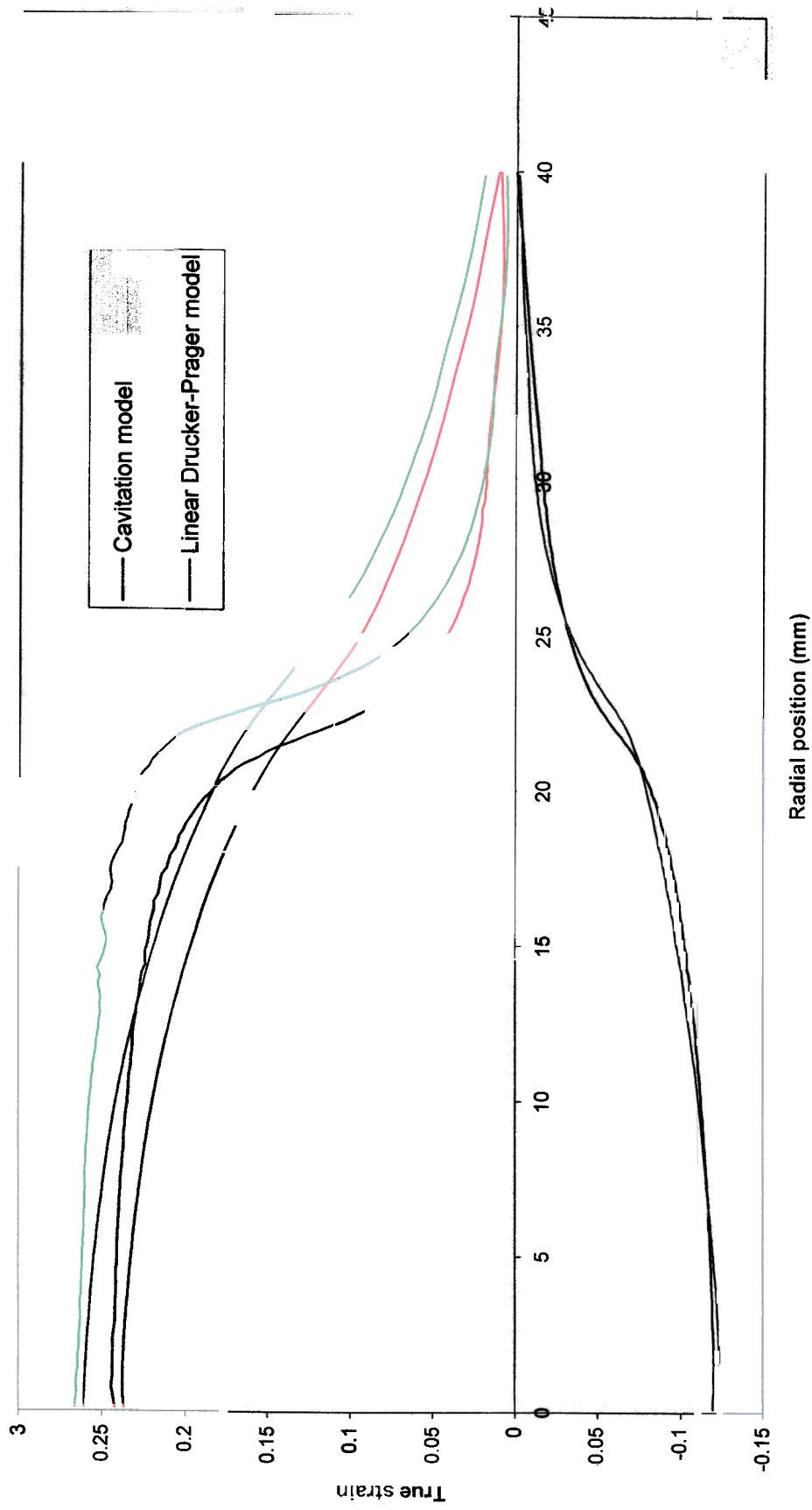


Figure 10. Comparison of predictions of principal strain components in the lower surface of a plate specimen using the linear Drucker-Prager and the cavitation models. Central deflection 24 mm; friction coefficient 0.25; no cavity growth.