

REPORT

The traceability of density measurements via solid density reference standards using the NPL 1 kg hydrostatic weighing apparatus

D J Bayliss and I Severn

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ABSTRACT

This report describes a method for making traceable measurements of the density of water by hydrostatic weighing using calibrated spherical reference standards of nominal mass 1 kg manufactured from Zerodur and silica. The results obtained were in good agreement with values calculated from recently published formula. A traceable density measurement of a stainless steel 1 kg artefact by hydrostatic weighing using the method above to determine the density of the water is also described.

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Approved on behalf of Managing Director, NPL,
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1 INTRODUCTION

The National Physical Laboratory owns two non-metallic precision ground and polished spheres of nominal mass 1 kg, one made of Zerodur (the proprietary name for a ceramic glass produced by Schott) identified as NPLW S01, and the other from silica identified as NPLW S02. Both spheres have recently been calibrated for mass and volume by two European national metrology institutes - NPLW S01 by the IMGC¹ in March 2001, and NPLW S02 by the PTB² in January 2001.

This report describes experiments carried out to assess the suitability of these spheres as volume standards³ to provide traceable measurements of the density of water and, via this value, the density of artefacts - principally standard weights of nominal mass 1 kg. A density determination was made of a weight (identified as A1), which had previously been well characterised in terms of density, to establish consistency with the earlier NPL hydrostatic measurements that relied on values for the density of water obtained from tables or calculated from formula.

These experiments also provided an opportunity for further assessment of the NPL's existing 1 kg Solid Density Apparatus; the weight exchanger on this equipment was modified to accept spherical artefacts in 1998, but had not previously been tested in this configuration.

2 EXPERIMENTAL PROCEDURE

The following measurements were carried out:

2.1 Mass determination:

The mass value of each sphere was determined by making comparison weighings on a Mettler AT 10005 comparator against two NPL stainless steel Reference Standards and applying corrections for air buoyancy.

¹ Istituto di Metrologia "G. Colonnetti", Turino, Italy.

² Physikalisch-Technische Bundesanstalt, Braunschweig, Germany

³ NPL has recently manufactured two spheres from pure single crystal silicon for experimental work related to the Avogadro project. It is expected that one of these spheres will become available for use as a volume standard.

2.2 Density of water:

Each sphere was used as a volume standard to calculate the density of water by hydrostatic weighing. The values obtained were compared with those calculated using formula for the thermal dilatation of water published by Patterson and Morris [1].

2.3 Density measurement of a stainless steel artefact:

The volume of a stainless steel cylindrical artefact of nominal mass 1 kg was determined by hydrostatic weighing in water whose density had been determined using the Zerodur sphere (NPLW S01). This value was compared with values previously obtained by NPL and other laboratories.

3 MASS MEASUREMENT

Initial mass values of both spheres were determined by NPL on a Mettler AT10005 comparator (Figure 1) against two NPL stainless steel Reference Standards. The Zerodur sphere was then cleaned by washing with an ethanol/ether mixture and rinsing with distilled water whilst the silica sphere was left uncleaned as a control. Subsequent comparison weighings showed that the cleaning process had reduced the mass value of the Zerodur sphere by about 0.24 mg.

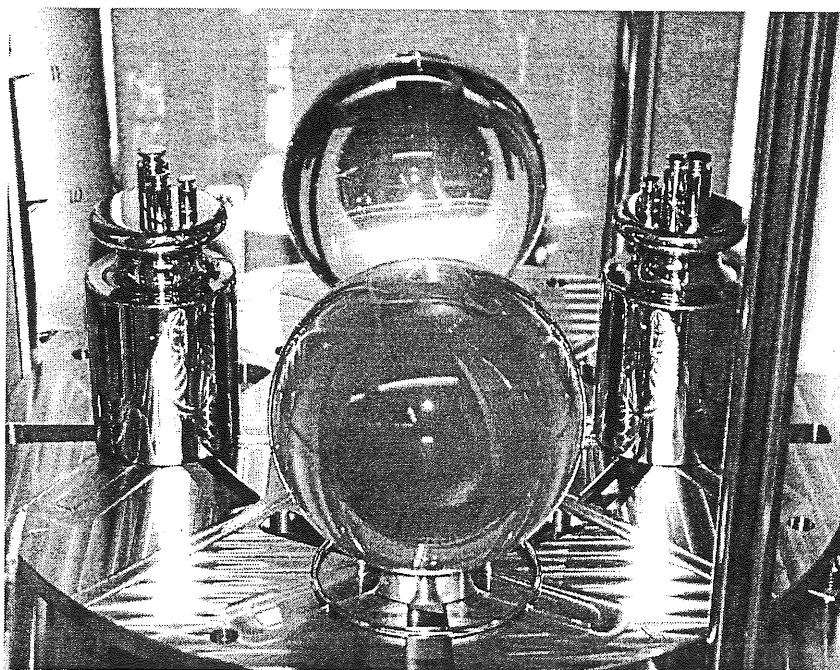


Figure 1. Weighing spheres on Mettler AT10005

The silica sphere was also cleaned at NPL prior to sending it to PTB for calibration. PTB cleaned it with ethanol before both their initial and final mass determinations (i.e. before and after hydrostatic weighing). They provided two mass values with their associated uncertainties using two different methods to calculate the air buoyancy correction. The first value was obtained by using the conventional CIPM formula (Giacomo [2], revised by Davis [3]) to determine air density and hence the buoyancy correction, while the second value was based on the use of air buoyancy artefacts calibrated in a vacuum. Although considerable smaller uncertainties were obtained by the second method, the mass value obtained using the CIPM formula was used for calculating water density in the investigation described in this report, since it was more consistent with the normal practice used for mass evaluation by NPL.

After washing the Zerodur sphere in a methanol/ether mixture and rinsing in distilled water, IMGC calculated two mass values with their associated uncertainties using a Mettler H315 comparator and a Mettler AT10005 comparator. As shown in Figure 2, the values agreed to within 23 μg . The mass value obtained with the Mettler AT10005 was used for calculating water density in this investigation described in this report.

Following their return from IMGC and PTB and prior to hydrostatic weighing by NPL, mass values for both spheres were again determined on a Mettler AT10005 comparator against two NPL stainless steel Reference Standards. The results of these weighings are given in Appendix 1. The work of Gläser [4] shows that physical differences in the shape of artefacts under comparison may give rise to erroneous results due to convection effects caused by changes in air temperature. The comparison of spheres with cylindrical mass standards is particularly prone to this problem. Because measurements have shown that ambient air temperature in the laboratory where NPL's Mettler AT10005 comparator is located broadly follows a diurnal cycle, it was decided to use only multiples of 24 hour measurement runs for the final calculations. One 48 hour and two 24 hour measurement runs (a total 160 comparison weighings) were used for each sphere.

A summary of mass values and uncertainties is shown in Figure 2. After the initial cleaning by NPL, both spheres showed a significant mass loss (approximately 0.24 mg for the Zerodur sphere and 0.40 mg for the silica sphere), which can reasonably be attributed to the removal of surface contamination. There is some evidence of a subsequent mass gain by both spheres

following their calibration by IMGC and PTB possibly due to re-contamination occurring, but further investigation would be required to substantiate this.

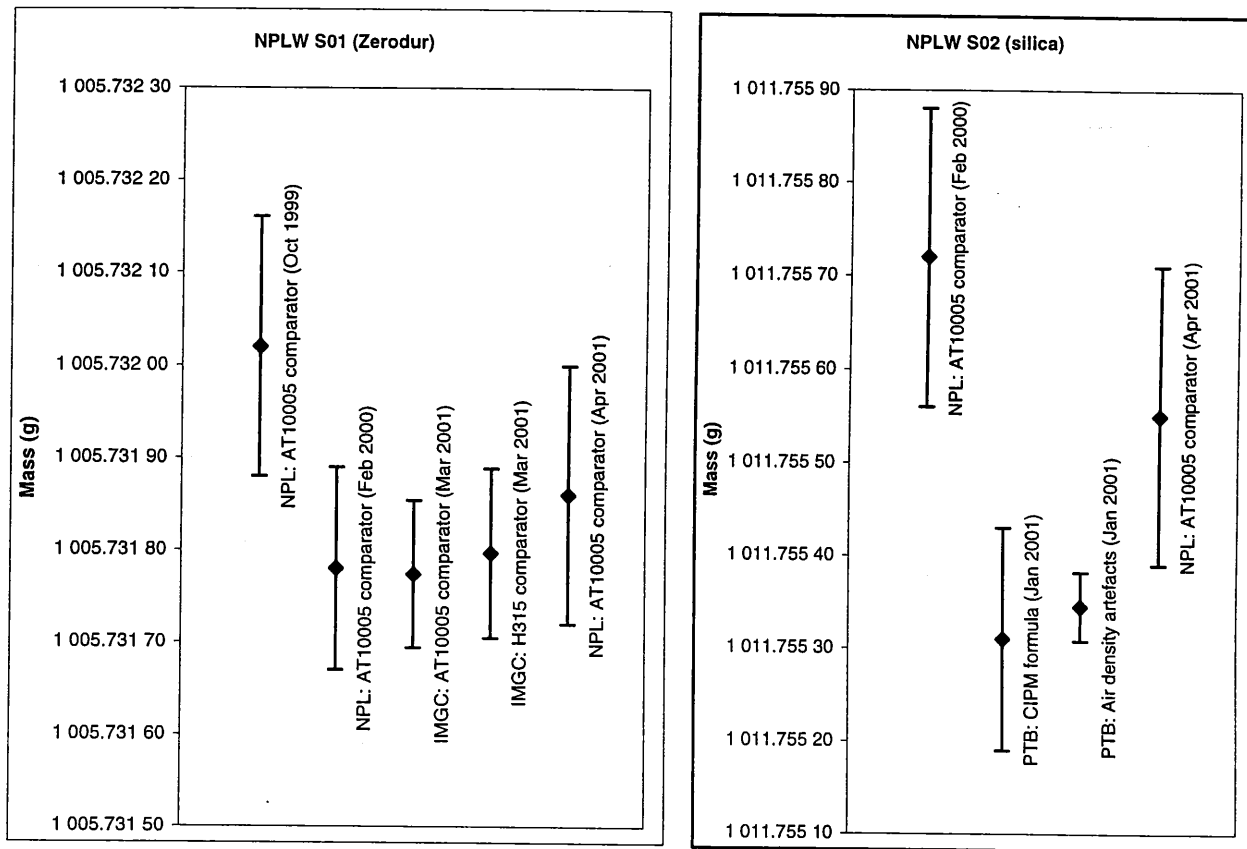


Figure 2. Mass values obtained for spheres NPLW S01 and NPLW S02.

The error bars represent the combined uncertainties of the mass values (at $k=2$).

4 DENSITY OF WATER

4.1 Water

Pure water was prepared by the de-ionisation and single distillation of tap water in accordance with standard NPL procedure described in NPL Report MOT 3[5]. After filling the vessel, the water was allowed to stand for two days before starting the measurement runs to allow it to approach air saturation equilibrium. Conductivity of the water was checked at intervals during the runs. The measurements obtained ranged from $1.4 \mu\text{S}$ to $2.1 \mu\text{S}$ - a good indication that no significant contamination of the water had occurred.

4.2 Weighing procedure

The object of the weighing procedure was to compare the weight in air of the standards (with the spheres supported free of the cradle by the lifting arms) with the weight in water of one sphere at a time loaded on the cradle. The two weighing configurations are shown schematically in Figure 3. A series of measurement runs, each comprising two comparisons was made. At the commencement of a run the balance reading with the standards (S1) on the comparator pan and the cradle empty was adjusted to within 20 mg of the reading with the cradle loaded with the sphere undergoing comparison, by placing suitable makeweights on the comparator pan.

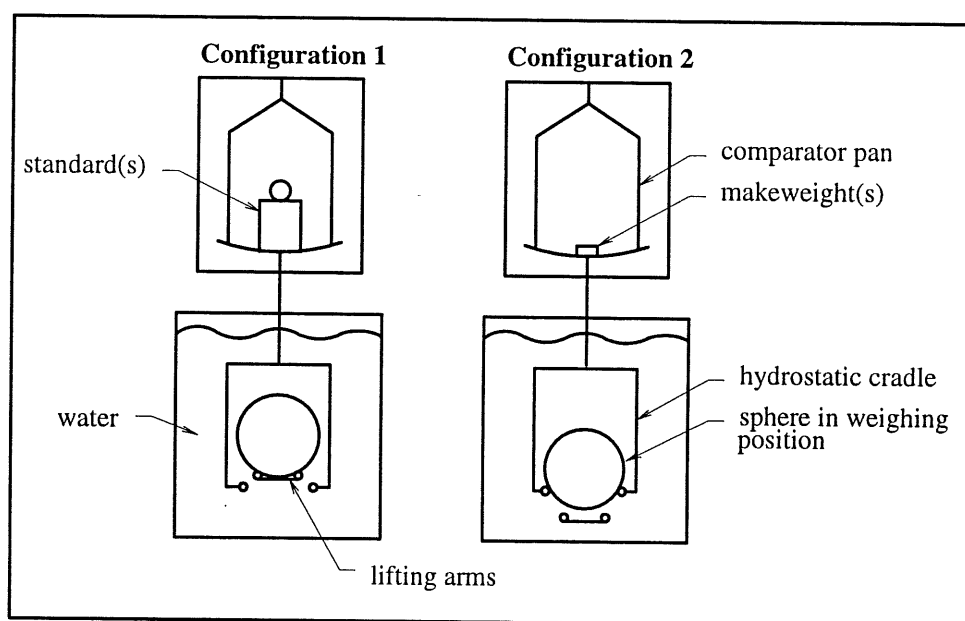


Figure 3. Schematic diagram of weighing configurations.

Configuration 1 – standards (S1) on comparator pan, sphere (S01) clear of cradle.

Configuration 2 – makeweights (S2) on comparator pan, sphere on cradle.

Initially, the silica sphere was loaded in the lower station of the exchanger and the Zerodur sphere in the upper station (Figure 4). Two Tinsley model 5187SA platinum resistance thermometers were positioned at either side of the silica sphere. A series of measurement runs were made. The air temperature, pressure and dew point, and the water temperature were recorded at each stage.

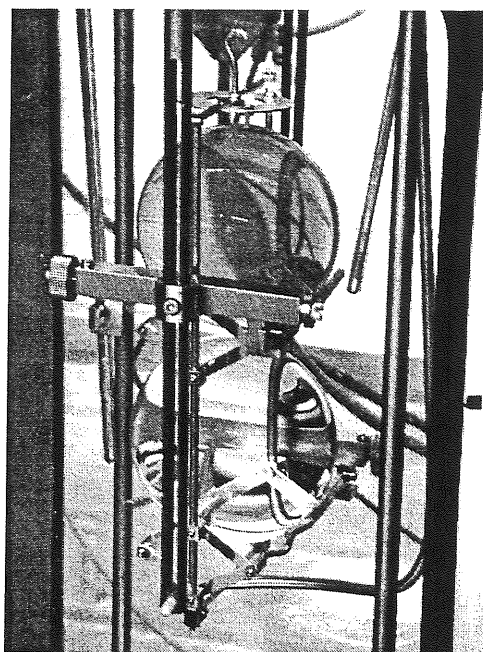


Figure 4. Zerodur sphere (top) and silica sphere (bottom) loaded in apparatus

After weighing the silica sphere, the thermometers were repositioned on either side of the Zerodur sphere and another series of measurement runs were made. To reduce the possibility of systematic errors the measurement procedure was then repeated with the silica sphere loaded in the upper station and the Zerodur sphere in the lower. Details of these measurements are given in Appendix 2.1. Finally, the whole experiment was repeated in a fresh sample of pure water (Appendix 2.2).

4.3 Calculation of water density

Let W_1 be the apparent mass in air of the standards (S_1) (kg)
 W_2 be the apparent mass in water of the sphere plus the apparent mass in air of the makeweights (S_2) (kg)
 M_{S1} be the mass of the standards (kg)
 V_{S1} be the volume of the standards (m^3)
 M_{S2} be the mass of the makeweights (kg)
 V_{S2} be the volume of the makeweights (m^3)
 M_{S01} be the mass of the sphere (kg)
 V_{S01} be the volume of the sphere (m^3)
 ρ_A be the air density ($kg\ m^{-3}$)
 ρ_W be the water density ($kg\ m^{-3}$)

Since by Archimede's principle the upthrust must equal the weight of fluid displaced

then $W_1 = (M_{S1} - \rho_A \cdot V_{S1})$ (kg) (1)

and $W_2 = (M_{S2} - \rho_A \cdot V_{S2}) + (M_{S01} - \rho_W \cdot V_{S01})$ (kg) (2)

However, $(W_1 - W_2)$ is equal to the difference in balance readings (ΔW) for the two configurations because the apparent mass of the cradle and suspension cancels out as it is the same in both cases.

Therefore $\Delta W = W_1 - W_2$ (kg) (3)

by substituting equations (1) and (2) in (3)

$$\Delta W = (M_{S1} - \rho_A \cdot V_{S1}) - (M_{S2} - \rho_A \cdot V_{S2}) - (M_{S01} - \rho_W \cdot V_{S01})$$

or $\rho_W \cdot V_{S01} = (M_{S2} - M_{S1}) + \rho_A (V_{S1} - V_{S2}) + M_{S01} + \Delta W$

$$\rho_W = \frac{\rho_A (V_{S1} - V_{S2}) + (M_{S2} - M_{S1}) + M_{S01} + \Delta W}{V_{S01}} \quad (kg\ m^{-3}) \quad (4)$$

4.4 Corrections to the water density equation

Several corrections must be considered in respect of Equation (4). Because of its compressibility, the density of water increases with depth due to the weight of the column of water above it. As a value for the water density at standard atmosphere pressure (101 325.0 Pa) is required, a negative correction must be applied to compensate for the depth of immersion of the spheres. The correction required was determined using the compressibility data published by Kell [6]. This is equivalent to $4.7 \times 10^{-5} \text{ kg m}^{-3}$ per cm of immersion depth. As the centre of the upper sphere was 12 cm below the water surface and the centre of the lower sphere 22 cm below the water surface, immersion depth corrections of $-0.000 56 \text{ kg m}^{-3}$ and $-0.001 03 \text{ kg m}^{-3}$ respectively were applied.

Small corrections were applied to compensate for changes in the volume of the silica sphere due to its thermal expansion; the Zerodur sphere has a negligible coefficient of thermal expansion at 20 °C. Variations in the ambient air pressure also have a small effect on the water density, but within the uncertainties applicable to these experiments, the effects are insignificant.

Corrections were also applied to compensate for the difference in local gravity caused by the vertical separation between the spheres and standards. The correction required on the mass of the standards (ΔM_s) is given by the formula

$$\Delta M_s = \frac{1}{g} \times \frac{\partial g}{\partial z} \times h \text{ (kg)}$$

where $\frac{\partial g}{\partial z}$ is the gravitational gradient⁴ (s^{-2})

g is the acceleration due to gravity (m s^{-2})

h is the vertical separation between the centres of gravity of the spheres and standards (m)

The thermal expansivity of the water can be assumed to be linear since it was maintained within the narrow range of $20 \text{ °C} \pm 0.1 \text{ °C}$ during measurement runs. The best-fit line from a regression analysis within this temperature range using the water density equation derived by Patterson and

⁴ A gravitational gradient value of $2 948 \times 10^{-9} \text{ s}^{-2}$ measured at NPL in 1993 by the University of Edinburgh Department of Geology and Geophysics was used in the calculations. [7]

Morris [1] gave a value for the thermal expansivity of $2.066\,36 \times 10^{-4} \text{ }^{\circ}\text{C}^{-1}$. Using this value, the experimental results of the water density were corrected to a temperature of $20 \text{ }^{\circ}\text{C}$.

4.5 Results of water density measurements

The results obtained using each sphere were tabulated in a spreadsheet and values for the water density obtained using equation (4). The corrected values determined by using each sphere for both water samples are shown in Figure 5.

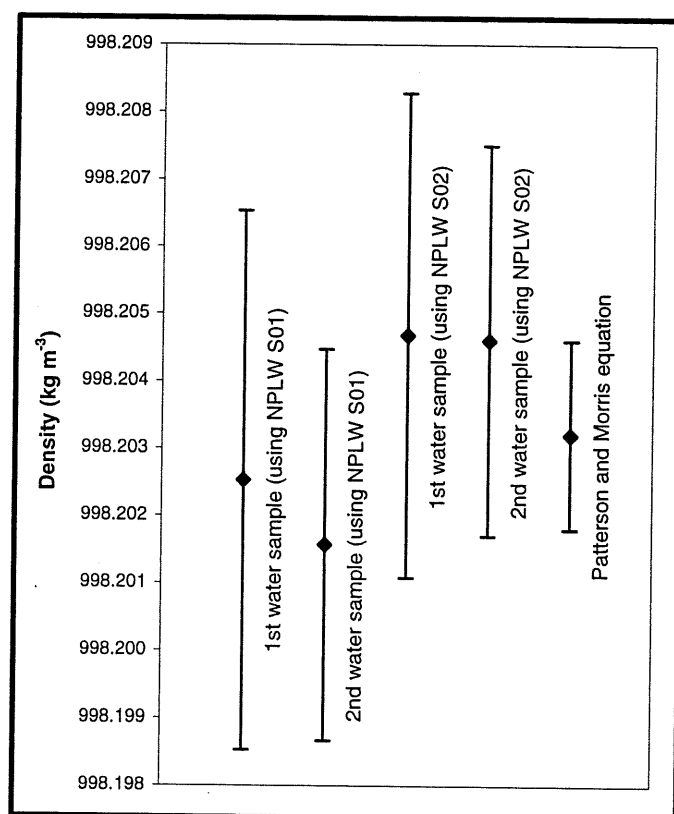


Figure 5. Density values of air saturated water corrected to $20 \text{ }^{\circ}\text{C}$ and standard atmospheric pressure.

The error bars represent the combined uncertainties at $k=2$.

For comparison, a value calculated from the Patterson and Morris equation is included which has been adjusted for the effect of air saturation using Bignell's equation [8]. It should be noted that for the hydrostatic weighing of an artefact, further type B uncertainties mainly arising from water temperature measurement would need to be added to the Patterson and Morris value, increasing the uncertainty to approximately 0.003 kg m^{-3} (at $k=2$).

The density equation given by Patterson and Morris [1] gives values based on water having the oxygen isotope ($^{18}\text{O}:^{16}\text{O}$) ratio and deuterium - hydrogen (D:H) ratio of Standard Mean Ocean Water (SMOW)⁵. To enable local NPL water to be directly compared, a correction of $+ 0.002\,0\,\text{kg m}^{-3}$ has been applied to the NPL results⁶ shown in the Figure 5.

Spreadsheets and graphs of the results are given in Appendix 2.1. An explanation of the evaluation of the uncertainties is included in Appendix 4 and the uncertainty budgets are given as Appendices 5.1 to 5.4.

⁵ This standard was superseded by Vienna Standard Mean Ocean Water (VSMOW) in 1995. None of the changes made to the standard are significant in the context of this report.

⁶ This value was determined from a sample of NPL tap water analysed at the BIPM in 1986. See NPL Report MOT 3 [5].

5 DENSITY OF AN ARTEFACT

5.1 Weighing procedure

A fresh sample of de-ionised and distilled water was prepared for this experiment. The Zerodur sphere (NPLW S01) was loaded in the lower station of the exchanger and the stainless steel artefact A1 in the upper station (Figure 6). One Tinsley model 5187SA platinum resistance thermometer was positioned level with the centre of the sphere and another level with the centre of the artefact.

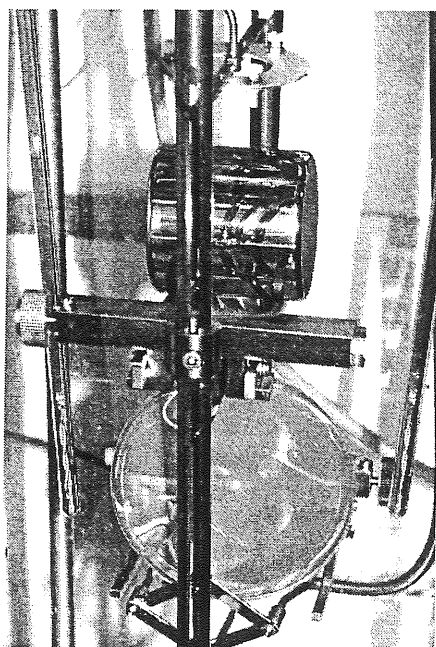


Figure 6. Artefact A1 and Zerodur sphere loaded in apparatus

The density of the water at 20 °C was first calculated by hydrostatic weighing of the Zerodur sphere using the method described in Section 4.

A series of measurement runs using the artefact A1 were then performed. The air temperature, pressure and dew point, and the water temperature were recorded at each stage. At the commencement of a run the balance reading with the standards (S1) on the comparator pan and the cradle empty was adjusted to within 20 mg of the reading with the artefact A1 on the cradle, by placing suitable makeweights on the comparator pan.

Finally, as there appeared to be a systematic difference between the values obtained for the water density when using the Zerodur and silica spheres (Figure 5), a further determination of the water density was made by hydrostatic weighing of the silica sphere.

5.2 Calculation of the density of an artefact

Let W_1 be the apparent mass in air of the standards (S_1) (kg)
 W_2 be the apparent mass in water of the sphere plus the apparent mass in air of the makeweights (S_2) (kg)
 M_{S1} be the mass of the standards (kg)
 V_{S1} be the volume of the standards (m^3)
 M_{S2} be the mass of the makeweights (kg)
 V_{S2} be the volume of the makeweights (m^3)
 M_{A1} be the mass of the artefact (kg)
 V_{A1} is the volume of the artefact (m^3)
 ρ_A is the air density ($kg\ m^{-3}$)
 ρ_W is the water density ($kg\ m^{-3}$)

Since by Archimede's principle the upthrust must equal the weight of fluid displaced

$$W_1 = (M_{S1} - \rho_A \cdot V_{S1}) \text{ (kg)} \quad (5)$$

and $W_2 = (M_{S2} - \rho_A \cdot V_{S2}) + (M_{A1} - \rho_W \cdot V_{A1}) \text{ (kg)} \quad (6)$

However, ($W_1 - W_2$) is equal to the difference in balance readings (ΔW) for the two configurations because the apparent mass of the cradle and suspension cancels out as it is the same in both cases.

Therefore $\Delta W = W_1 - W_2 \text{ (kg)} \quad (7)$

by substituting equations (5) and (6) in (8)

$$\Delta W = (M_{S1} - \rho_A \cdot V_{S1}) - (M_{S2} - \rho_A \cdot V_{S2}) - (M_{A1} - \rho_W \cdot V_{A1})$$

or $\rho_W \cdot V_{A1} = \rho_A (V_{S1} - V_{S2}) + (M_{S2} - M_{S1}) + M_{A1} + \Delta W$

$$V_{A1} = \frac{\rho_A (V_{S1} - V_{S2}) + (M_{S2} - M_{S1}) + M_{A1} + \Delta W}{\rho_W} \text{ cm}^3 \quad (8)$$

5.3 Results of the density determination of A1

The results of the water density measurements are given in Appendix 2.2 and the uncertainty budgets in Appendices 5.5 and 5.6.

For each measurement of artefact A1, temperature corrections were made to the value of the density of water obtained experimentally, by applying the thermal expansivity value calculated previously (Section 4.4). After corrections were made for immersion depth and the difference in local gravity caused by the vertical separation between the artefact and standards, the volume was calculated using Equation (8). The mass value of the artefact (measured prior to this experiment) was then used to calculate its density at the measured temperature, and corrected to the density at 20 °C by using the coefficient of cubic expansion of $4.7 \times 10^{-5} \text{ °C}^{-1}$ which had been determined during earlier work at NPL.

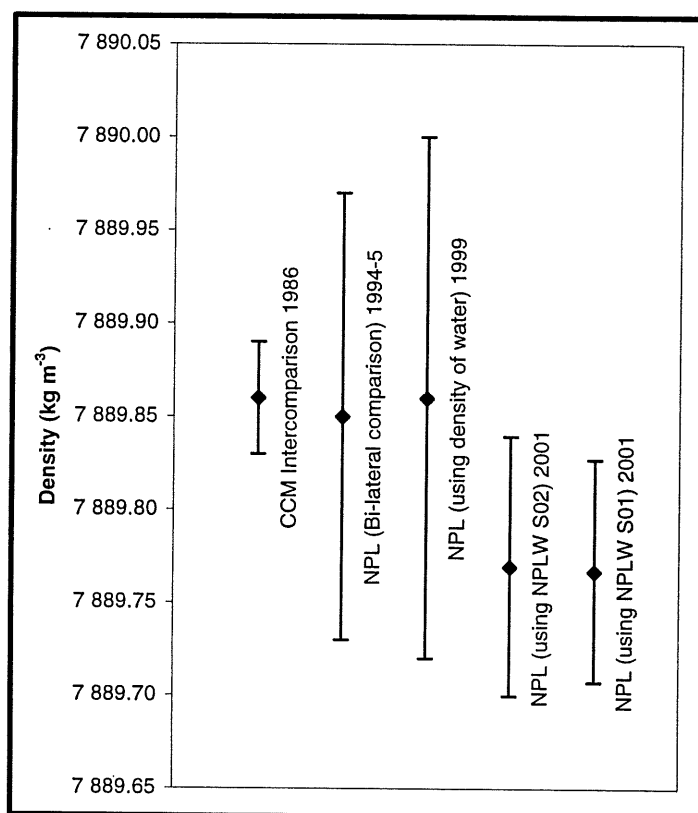


Figure 7. Density values for artefact A1⁷.

The error bars represent the combined uncertainties (at $k=2$).

⁷ The CCM Intercomparison shows Type A uncertainty contributions only

A summary comparing the results with previously obtained values is shown in Figure 7. Since some of the earlier results used different formula to determine the density of water, corrections have been made to compensate where applicable.

The spreadsheets and graphs are shown as Appendix 3. An explanation of the evaluation of the uncertainties is described in Appendix 4 and the uncertainty budget is given in Appendix 5.5.

6 CONCLUSIONS

The aims of the project were satisfied in so far as values for the density of water were obtained which were traceable to the calibrated mass and volume of the spheres. These values agreed, within their calculated uncertainties, with each other and (with appropriate corrections applied) to the value calculated using the Patterson and Morris formula. However, for both water samples, the values obtained using the Zerodur sphere (NPLW S01) were about 2-3 ppm lower than those obtained using the silica sphere (NPLW S02), suggesting the possibility of a systematic error.

Although, because of limitations of the equipment and the laboratory environment, the uncertainties achieved were inevitably inferior to those obtainable by the use of the Patterson and Morris formula, the method described has the advantage that the result is a measure of the actual density of the water (or other liquid) that will be used for hydrostatic weighing. It is therefore independent of any factors (e.g. contamination, degree of air saturation) that could affect the integrity of a density value determined from formula or tables.

Values for the density of the stainless steel artefact A1 were also obtained which were traceable via the density of the water to the calibrated mass and volume of the spheres. These agreed, within their calculated uncertainties, with each other and with values obtained from previous calibrations. Although the calculated uncertainties were lower than those previously obtained (using the same artefact and apparatus but traceable to the calculated density of water), it was established that this was the result of tighter control of the uncertainty budget for the new determination. Reprocessing the earlier data using the current method for determining uncertainties showed a similar improvement to the overall uncertainties.

The main contributions to the uncertainties were the standard deviation of the measurements and the gradient and instability of the water temperature within the vessel. Both are the result of limitations inherent in the apparatus and ancillary equipment. Nevertheless, the experiments provided valuable information about the methodology and calculations required and established that the use of the spheres was viable as a traceable method for determining the density of a liquid. It is anticipated that much of this knowledge will be directly applicable to the operation of NPL's new 1 kg hydrostatic apparatus, which is expected to be operational early next year.

7 FUTURE WORK

Although the new 1 kg hydrostatic apparatus is expected to become NPL's primary equipment for the high accuracy density determination of standards up to 1 kg, the existing equipment used for the work described in this report will still remain useful since it is very versatile as its configuration is easily changed to accommodate artefacts of non-standard shape and/or mass.

Experiments have shown that, because of its proximity to the surface, the water adjacent to the upper station is appreciably more unstable in temperature than that adjacent to the lower station. It is planned to modify the apparatus by extending the lifting arms so that the stations are lower in the water. It is believed that this will reduce the temperature gradient of the water in the vicinity of the upper station and lead to a significant improvement in the overall uncertainties obtainable.

8 ACKNOWLEDGEMENTS

The authors wish to thank Mr S Downes for his help and advice throughout this project. Financial support was provided by the National Measurement System Policy unit of the UK Department of Trade and Industry.

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APPENDIX 1

MASS DETERMINATION OF SPHERES NPLW S01 AND NPLW S02

Silica sphere NPLW S02

Summary of weighings on Mettler AT10005

Run	Series	Comparison 55DD v S02 (mg)	Mass (g)	Comparison S02 v 55TD (mg)	Mass (g)
Run 2	Series 1:	1054.241 5	1 011.755 3	-1054.420 5	1 011.755 3
	Series 2:	1054.361 4	1 011.755 4	-1054.520 9	1 011.755 4
	Series 3:	1054.345 3	1 011.755 4	-1054.540 9	1 011.755 4
	Series 4:	1054.372 7	1 011.755 4	-1054.601 7	1 011.755 5
	Series 5:	1054.435 2	1 011.755 5	-1054.658 0	1 011.755 6
	Series 6:	1054.443 7	1 011.755 5	-1054.696 9	1 011.755 6
	Series 7:	1054.517 8	1 011.755 6	-1054.716 2	1 011.755 6
	Series 8:	1054.521 0	1 011.755 6	-1054.747 1	1 011.755 7
	Series 9:	1054.542 0	1 011.755 6	-1054.694 5	1 011.755 6
	Series 10:	1054.530 8	1 011.755 6	-1054.720 8	1 011.755 6
	Series 11:	1054.591 1	1 011.755 6	-1054.731 0	1 011.755 6
	Series 12:	1054.596 6	1 011.755 7	-1054.749 4	1 011.755 7
	Series 13:	1054.486 9	1 011.755 5	-1054.668 0	1 011.755 6
	Series 14:	1054.555 7	1 011.755 6	-1054.721 7	1 011.755 6
	Series 15:	1054.566 2	1 011.755 6	-1054.725 9	1 011.755 6
	Series 16:	1054.581 6	1 011.755 6	-1054.737 1	1 011.755 6
	Series 17:	1054.592 6	1 011.755 7	-1054.761 5	1 011.755 7
	Series 18:	1054.563 2	1 011.755 6	-1054.783 8	1 011.755 7
	Series 19:	1054.581 4	1 011.755 6	-1054.746 8	1 011.755 7
	Series 20:	1054.610 7	1 011.755 7	-1054.745 9	1 011.755 7
Run 3	Series 1:	1054.424 3	1 011.755 5	-1054.649 1	1 011.755 6
	Series 2:	1054.500 4	1 011.755 6	-1054.671 7	1 011.755 6
	Series 3:	1054.574 2	1 011.755 6	-1054.793 7	1 011.755 7
	Series 4:	1054.551 8	1 011.755 6	-1054.667 9	1 011.755 6
	Series 5:	1054.459 8	1 011.755 5	-1054.598 7	1 011.755 5
	Series 6:	1054.402 5	1 011.755 5	-1054.613 4	1 011.755 5
	Series 7:	1054.473 1	1 011.755 5	-1054.634 1	1 011.755 5
	Series 8:	1054.447 1	1 011.755 5	-1054.685 2	1 011.755 6
	Series 9:	1054.535 3	1 011.755 6	-1054.671 0	1 011.755 6
	Series 10:	1054.535 0	1 011.755 6	-1054.722 3	1 011.755 6
Run 4	Series 1:	1054.431 0	1 011.755 5	-1054.589 5	1 011.755 5
	Series 2:	1054.430 0	1 011.755 5	-1054.629 4	1 011.755 5
	Series 3:	1054.431 8	1 011.755 5	-1054.584 4	1 011.755 5
	Series 4:	1054.379 9	1 011.755 4	-1054.589 2	1 011.755 5
	Series 5:	1054.416 2	1 011.755 5	-1054.565 7	1 011.755 5
	Series 6:	1054.393 2	1 011.755 5	-1054.581 1	1 011.755 5
	Series 7:	1054.385 3	1 011.755 4	-1054.531 8	1 011.755 4
	Series 8:	1054.375 6	1 011.755 4	-1054.609 0	1 011.755 5
	Series 9:	1054.394 9	1 011.755 5	-1054.554 8	1 011.755 5
	Series 10:	1054.493 1	1 011.755 6	-1054.631 7	1 011.755 5
Average (g):		1054.476 80	1 011.755 53	-1054.656 56	1 011.755 56
σ (±mg):		0.087 2	0.089 6	0.083 3	0.085 2
Using results for both standards					Average (g): 1 011.755 55
					σ (±mg): 0.081 3
					$\sigma/180$ (±mg): 0.009 1

0.00946549

Zerodur sphere NPLW S01

Summary of weighings on Mettler AT10005

Run	Series	Comparison 55DD v S01 (mg)	Mass (g)	Comparison S01 v 55TD (mg)	Mass (g)
Run 2	Series 1:	-997.287 6	1 005.731 7	-997.482 3	1 005.731 7
	Series 2:	-997.324 8	1 005.731 7	-997.466 7	1 005.731 7
	Series 3:	-997.378 2	1 005.731 8	-997.523 1	1 005.731 7
	Series 4:	-997.431 2	1 005.731 8	-997.628 7	1 005.731 9
	Series 5:	-997.391 4	1 005.731 8	-997.623 5	1 005.731 8
	Series 6:	-997.463 7	1 005.731 8	-997.659 6	1 005.731 9
	Series 7:	-997.488 1	1 005.731 9	-997.691 2	1 005.731 9
	Series 8:	-997.572 8	1 005.732 0	-997.733 8	1 005.732 0
	Series 9:	-997.525 1	1 005.731 9	-997.695 7	1 005.731 9
	Series 10:	-997.584 3	1 005.732 0	-997.716 7	1 005.731 9
	Series 11:	-997.539 8	1 005.731 9	-997.767 3	1 005.732 0
	Series 12:	-997.553 4	1 005.731 9	-997.688 6	1 005.731 9
	Series 13:	-997.530 6	1 005.731 9	-997.701 2	1 005.731 9
	Series 14:	-997.533 8	1 005.731 9	-997.751 6	1 005.732 0
	Series 15:	-997.569 3	1 005.732 0	-997.712 1	1 005.731 9
	Series 16:	-997.548 2	1 005.731 9	-997.731 2	1 005.732 0
	Series 17:	-997.583 4	1 005.732 0	-997.749 8	1 005.732 0
	Series 18:	-997.558 1	1 005.731 9	-997.763 1	1 005.732 0
	Series 19:	-997.602 9	1 005.732 0	-997.810 5	1 005.732 0
	Series 20:	-997.585 7	1 005.732 0	-997.742 9	1 005.732 0
Run 3	Series 1:	-997.420 7	1 005.731 8	-997.628 6	1 005.731 9
	Series 2:	-997.493 5	1 005.731 9	-997.646 6	1 005.731 9
	Series 3:	-997.515 8	1 005.731 9	-997.708 3	1 005.731 9
	Series 4:	-997.590 6	1 005.732 0	-997.689 8	1 005.731 9
	Series 5:	-997.477 5	1 005.731 9	-997.668 5	1 005.731 9
	Series 6:	-997.449 7	1 005.731 8	-997.603 3	1 005.731 8
	Series 7:	-997.423 0	1 005.731 8	-997.586 9	1 005.731 8
	Series 8:	-997.485 9	1 005.731 9	-997.677 1	1 005.731 9
	Series 9:	-997.482 1	1 005.731 9	-997.653 6	1 005.731 9
	Series 10:	-997.564 2	1 005.731 9	-997.693 4	1 005.731 9
Run 4	Series 1:	-997.435 9	1 005.731 8	-997.596 2	1 005.731 8
	Series 2:	-997.403 7	1 005.731 8	-997.580 1	1 005.731 8
	Series 3:	-997.389 9	1 005.731 8	-997.588 2	1 005.731 8
	Series 4:	-997.423 3	1 005.731 8	-997.623 9	1 005.731 8
	Series 5:	-997.409 5	1 005.731 8	-997.605 1	1 005.731 8
	Series 6:	-997.340 2	1 005.731 7	-997.555 6	1 005.731 8
	Series 7:	-997.392 9	1 005.731 8	-997.527 1	1 005.731 8
	Series 8:	-997.335 3	1 005.731 7	-997.540 0	1 005.731 8
	Series 9:	-997.407 3	1 005.731 8	-997.520 2	1 005.731 7
	Series 10:	-997.392 3	1 005.731 8	-997.609 6	1 005.731 8
Average (g):		-997.471 89	1 005.731 85	-997.648 04	1 005.731 87
σ (±mg):		0.085 7	0.088 5	0.084 2	0.086 3
Using results for both standards					Average (g): 1 005.731 86
					σ (±mg): 0.084 7
					$\sigma/180$ (±mg): 0.009 5

APPENDIX 2.1

DETERMINATION OF DENSITY OF WATER – 1st SAMPLE

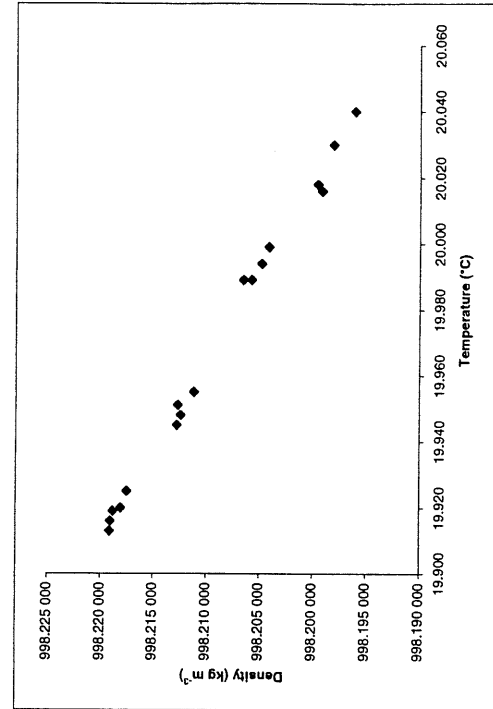
Using silica sphere NPLW S02

Parameters	Mass (g)	Volume (cm ³)	Expansion coeff. (°C ⁻¹)
Water Sphere	-	-	2.07E-04
Standards (S1)*	1 011.755 310	459.718 75	1.40E-06
Makeweights (S2)*	603.003 198	77.225 9	-
	50.050 158	6.376 5	-

* 0.000 109 g added to S1 and 0.000 009 g added to S2 to compensate for gravity gradient

Run	ΔW (g)	Air density (kg m ⁻³)	Water temp (°C)	Water density* (kg m ⁻³)	Water density at 20 °C (kg m ⁻³)	Corrected for depth (kg m ⁻³)
1	0.010 66	1.194 5	19.945	998.213 788	998.202 424	998.201 394
2	0.010 47	1.194 8	19.948	998.213 417	998.202 672	998.201 642
3	0.010 73	1.192 9	19.951	998.213 685	998.203 561	998.202 531
4	0.010 10	1.192 0	19.955	998.212 171	998.202 873	998.201 843
5	0.012 62	1.207 4	19.913	998.220 084	998.202 108	998.201 078
6	0.012 65	1.206 6	19.916	998.220 022	998.202 666	998.201 636
7	0.012 56	1.206 4	19.919	998.219 791	998.202 025	998.201 485
8	0.012 26	1.205 8	19.920	998.219 045	998.202 515	998.201 485
9	0.012 03	1.205 2	19.925	998.218 445	998.202 948	998.201 918
10	0.004 99	1.186 8	20.016	998.200 169	998.203 475	998.202 915
11	0.004 96	1.186 9	20.018	998.200 116	998.203 835	998.203 275
12	0.004 27	1.187 2	20.030	998.198 645	998.204 843	998.204 283
13	0.003 35	1.187 1	20.040	998.196 614	998.204 879	998.204 319
14	0.006 94	1.203 7	19.989	998.207 053	998.204 780	998.204 220
15	0.006 67	1.202 7	19.989	998.206 311	998.204 038	998.203 478
16	0.006 33	1.201 5	19.994	998.205 380	998.204 140	998.203 560
17	0.006 08	1.200 7	19.999	998.204 706	998.203 939	998.203 939
Average:						998.202 680
σ (±) :						0.001 135

* Formula includes correction for thermal expansion of sphere

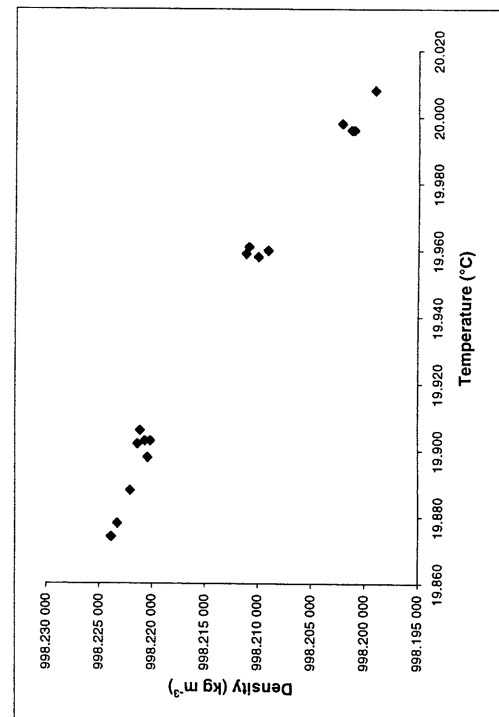


Using Zerodur sphere NPLW S01

Parameters	Mass (g)	Volume (cm ³)	Expansion coeff. (°C ⁻¹)
Water Sphere	-	-	2.07E-04
Standards (S1)*	1 005.731 774	395.998 64	0.000 000
Standards (S1)*	610.553 221	78.195 1	-
Makeweights (S2), runs 1-9	0.000 000	0.000 0	-
Makeweights (S2), runs 9-16	0.019 989	0.002 5	-

* 0.000 110 g added to compensate for gravity gradient

Run	ΔW (g)	Air density (kg m ⁻³)	Water temp (°C)	Water density (kg m ⁻³)	Water density at 20 °C (kg m ⁻³)	Corrected for depth (kg m ⁻³)
1	0.013 46	1.197 9	20.008	998.199 597	998.201 250	998.200 690
2	0.014 39	1.197 3	19.996	998.201 827	998.201 001	998.200 441
3	0.014 74	1.197 4	19.998	998.202 731	998.202 317	998.201 757
4	0.014 33	1.196 7	19.996	998.201 557	998.200 731	998.200 171
5	0.023 53	1.194 9	19.874	998.224 434	998.198 399	998.197 839
6	0.023 32	1.194 6	19.878	998.223 845	998.198 636	998.198 076
7	0.022 93	1.193 3	19.888	998.222 603	998.199 461	998.198 901
8	0.022 29	1.193 0	19.898	998.220 928	998.199 852	998.199 292
9	0.020 71	1.214 2	19.903	998.221 124	998.201 081	998.200 521
10	0.021 26	1.213 4	19.902	998.222 355	998.202 105	998.201 075
11	0.001 06	1.212 6	19.903	998.221 656	998.201 614	998.200 584
12	0.001 30	1.211 9	19.906	998.222 124	998.202 701	998.201 671
13	-0.003 37	1.210 5	19.960	998.210 055	998.201 790	998.200 760
14	-0.002 85	1.208 6	19.958	998.210 993	998.202 314	998.201 284
15	-0.002 31	1.207 5	19.959	998.212 139	998.203 668	998.202 638
16	-0.002 39	1.207 1	19.961	998.211 858	998.203 800	998.202 770
Average:						998.200 529
σ (±) :						0.001 433



APPENDIX 2.2

DETERMINATION OF DENSITY OF WATER – 2nd SAMPLE
Used for density measurement of artefact A1

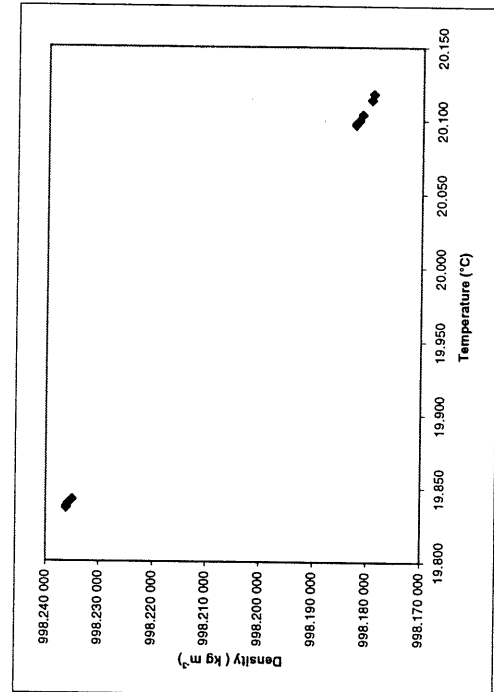
Using silica sphere NPLW S02

Parameters	Mass (g)	Volume (cm ³)	Expansion coeff. (°C ⁻¹)
Water Sphere	1 011.755 310	459.718 75	2.07E-04
Standards (S1)*	603.003 198	77 225 9	1.40E-06
Makeweights (S2)*	50.050 158	6.376 5	-

* 0.000 109 g added to S1 and 0.000 009 g added to S2 to compensate for gravity gradient

Run	ΔW (g)	Air density (kg m ⁻³)	Water temp (°C)	Water density* (kg m ⁻³)	Water density at 20 °C (kg m ⁻³)	Corrected for depth (kg m ⁻³)
1	-0.003 25	1.195 4	20.097	998.183 457	998.203 500	998.202 470
2	-0.003 25	1.194 7	20.098	998.183 348	998.203 597	998.202 567
3	-0.003 46	1.193 9	20.100	998.182 765	998.203 427	998.202 397
4	-0.003 67	1.193 2	20.104	998.182 194	998.203 684	998.202 654
5	-0.004 68	1.196 4	20.114	998.180 477	998.204 032	998.203 002
6	-0.004 75	1.195 3	20.118	998.180 149	998.204 531	998.203 501
7	0.021 29	1.194 9	19.836	998.237 125	998.203 238	998.202 208
8	0.021 39	1.193 3	19.837	998.237 095	998.203 414	998.202 384
9	0.021 36	1.191 8	19.839	998.236 795	998.203 528	998.202 498
10	0.021 07	1.190 8	19.842	998.236 006	998.203 359	998.202 329
Average:						998.202 601
σ (±):						0.000 383

* Formula includes correction for thermal expansion of sphere

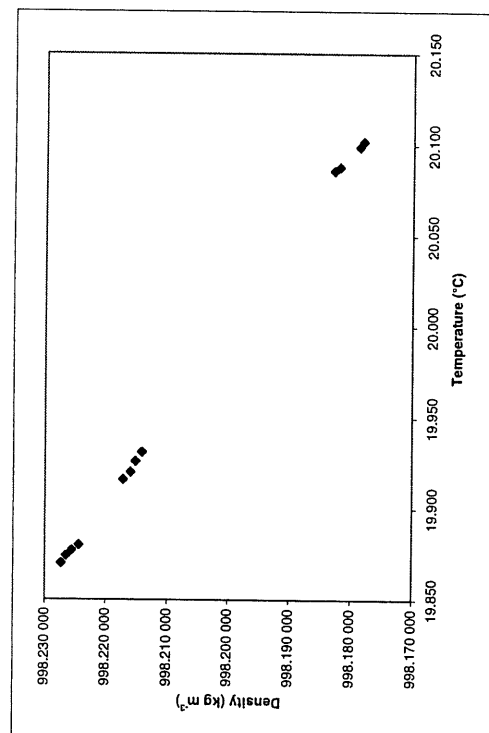


Using Zerodur sphere NPLW S01

Parameters	Mass (g)	Volume (cm ³)	Expansion coeff. (°C ⁻¹)
Water Sphere	1 005.731 774	395.998 64	2.07E-04
Standards (S1)*	610.553 221	78.195 2	0.000 000
Makeweights (S2)	0.019 989	0.002 5	-

* 0.000 110 g added to compensate for gravity gradient

Run	ΔW (g)	Air density (kg m ⁻³)	Water temp (°C)	Water density (kg m ⁻³)	Water density at 20 °C (kg m ⁻³)	Corrected for depth (kg m ⁻³)
1	0.005 63	1.185 6	19.870	998.227 866	998.201 004	998.199 974
2	0.005 44	1.183 9	19.874	998.227 050	998.201 015	998.199 985
3	0.005 19	1.182 5	19.877	998.226 143	998.200 727	998.199 697
4	0.004 83	1.181 3	19.880	998.224 997	998.200 201	998.199 171
5	0.001 86	1.182 4	19.916	998.217 714	998.200 357	998.199 327
6	0.001 51	1.180 8	19.920	998.216 514	998.199 984	998.198 954
7	0.001 31	1.179 4	19.926	998.215 732	998.200 442	998.199 412
8	0.000 97	1.178 4	19.931	998.214 676	998.200 419	998.199 389
9	-0.013 36	1.184 7	20.099	998.179 733	998.200 189	998.199 159
10	-0.013 52	1.183 5	20.102	998.179 092	998.200 168	998.199 138
11	-0.012 40	1.188 5	20.088	998.182 908	998.201 091	998.200 061
12	-0.011 94	1.187 3	20.086	998.183 833	998.201 603	998.200 573
Average:						998.199 570
σ (±):						0.000 487



APPENDIX 3

DETERMINATION OF THE DENSITY OF A STAINLESS STEEL ARTEFACT A1

Parameters	Mass (g)	Volume at 20 °C (cm ³)	Expansion coeff.(°C ⁻¹)	Density at 20 °C (kg m ⁻³)
Water (S01 sphere)	-	-	2.07E-04	998.199 570
Water (S02 sphere)	-	-	2.07E-04	998.202 601
Artefact (A1)	999.557 704	-	4.70E-05	-
Standards (S1)*	1005.222 971	127.683 3	-	-
Standards (S2)*	132.000 108	16.766 9	-	-

* 0.000 180 g added to S1 and 0.000 024 added to S2 to compensate for gravity gradient

Using the value for the density of water determined using the Zerodur sphere

Run	ΔW (g)	Air density (kg m ⁻³)	Water temp. (°C)	Water density* (kg m ⁻³)	Volume (cm ³)	Artefact A1 Volume at 20 °C (cm ³)	Density at 20 °C (kg m ⁻³)
1	-0.001 24	1.183 9	19.913 0	998.218 21	126.690 65	126.690 13	7 889.783 3
2	-0.000 91	1.182 6	19.914 0	998.218 00	126.690 86	126.690 35	7 889.769 7
3	-0.001 08	1.181 5	19.918 0	998.217 17	126.690 68	126.690 19	7 889.779 9
4	-0.001 18	1.180 5	19.923 0	998.216 14	126.690 60	126.690 14	7 889.783 1
5	-0.000 66	1.181 2	19.926 0	998.215 52	126.691 27	126.690 83	7 889.739 8
6	-0.000 61	1.180 0	19.928 0	998.215 11	126.691 24	126.690 81	7 889.741 0
7	-0.000 60	1.178 7	19.933 0	998.214 07	126.691 24	126.690 84	7 889.739 3
8	-0.000 84	1.177 7	19.938 0	998.213 04	126.691 02	126.690 65	7 889.751 2
9	-0.007 18	1.185 4	20.092 0	998.181 22	126.689 56	126.690 11	7 889.784 8
10	-0.006 96	1.184 4	20.092 0	998.181 22	126.689 67	126.690 22	7 889.778 0
Average:							7 889.765 0
$\sigma (\pm)$:							0.019 8

* Formula includes correction for depth of immersion of + 0.000 66 kg m⁻³

Using the value for the density of water determined using the silica sphere

Run	ΔW (g)	Air density (kg m ⁻³)	Water temp. (°C)	Water density* (kg m ⁻³)	Volume (cm ³)	Artefact A1 Volume at 20 °C (cm ³)	Density at 20 °C (kg m ⁻³)
1	-0.001 24	1.183 9	19.913 0	998.221 24	126.690 27	126.689 75	7 889.807 3
2	-0.000 91	1.182 6	19.914 0	998.221 03	126.690 48	126.689 97	7 889.793 7
3	-0.001 08	1.181 5	19.918 0	998.220 20	126.690 29	126.689 80	7 889.803 9
4	-0.001 18	1.180 5	19.923 0	998.219 17	126.690 21	126.689 75	7 889.807 0
5	-0.000 66	1.181 2	19.926 0	998.218 55	126.690 89	126.690 45	7 889.763 7
6	-0.000 61	1.180 0	19.928 0	998.218 14	126.690 86	126.690 43	7 889.764 9
7	-0.000 60	1.178 7	19.933 0	998.217 11	126.690 85	126.690 46	7 889.763 3
8	-0.000 84	1.177 7	19.938 0	998.216 07	126.690 63	126.690 27	7 889.775 1
9	-0.007 18	1.185 4	20.092 0	998.184 25	126.689 18	126.689 72	7 889.808 8
10	-0.006 96	1.184 4	20.092 0	998.184 25	126.689 29	126.689 83	7 889.802 0
Average:							7 889.789 0
$\sigma (\pm)$:							0.019 8

* Formula includes correction for depth of immersion of + 0.000 66 kg m⁻³

APPENDIX 4

EVALUATION OF UNCERTAINTIES

1 MASS VALUES

The uncertainties for NPL's determination of the mass values of the spheres were obtained using the methods described in NPL Report CMAM 22.

2 WATER DENSITY

CONTRIBUTIONS FOR THE EXPANSIVITY OF WATER

Over the limited temperature range in which this work was carried out (19.9 °C to 20.1 °C) the expansivity of water was assumed to be linear (Section 4.4). Its mean value was evaluated using the fifth order polynomial of Patterson and Morris [1]. Calculations showed that worst case errors due to the assumption of a linear relationship are 2 parts in 10^8 for the data under consideration here.

Therefore if

ρ_{20} is the calculated water density at 20 °C (kg m^{-3})

ρ_T is the measured water density at T °C (kg m^{-3})

ΔT is (20-T) (°C)

α is the expansivity

all of the measured values can be corrected to 20 °C using the following equation:

$$\rho_{20} = \rho_T (1 + \alpha \cdot \Delta T) \quad (\text{kg m}^{-3})$$

and if

u_c is the combined standard uncertainty of the input standard uncertainties

$u(x)$ is the input uncertainty of quantity x

$c_1, c_2, c_3, \dots, c_n$ are the sensitivity coefficients of the input uncertainties

the uncertainty in ρ_{20} may be evaluated as follows:

$$\begin{aligned}
u_c(\rho_{20})^2 &= c_1^2 u(\rho_T)^2 + c_2^2 u(\alpha)^2 + c_3^2 u(T)^2 \\
&= \left(\frac{\partial \rho_{20}}{\partial \rho_T} \right)^2 u(\rho_T)^2 + \left(\frac{\partial \rho_{20}}{\partial \alpha} \right)^2 u(\alpha)^2 + \left(\frac{\partial \rho_{20}}{\partial T} \right)^2 u(T)^2 \\
&= (1 + \alpha \Delta T)^2 u(\rho_T)^2 + (\rho_T \Delta T)^2 u(\alpha)^2 + (\rho_T \alpha)^2 u(T)^2
\end{aligned}$$

For the purpose of this work α was calculated to be $2.066\,36 \times 10^{-4} \text{ }^\circ\text{C}^{-1}$ from the equation of Patterson and Morris in the range $19.9 \text{ }^\circ\text{C}$ to $20.1 \text{ }^\circ\text{C}$.

MEASUREMENT CONTRIBUTIONS

- If
- M_{S1} is the mass of the standards (kg)
 - V_{S1} is the volume of the standards (m^3)
 - M_{S2} is the mass of the makeweights (kg)
 - V_{S2} is the volume of the makeweights (m^3)
 - M_{SP} is the mass of the sphere (kg)
 - V_{SP} is the volume of the sphere (m^3)
 - ρ_A is the air density (kg m^{-3})
 - ρ_W is the water density (kg m^{-3})
 - ΔW is the difference between balance readings (kg)

then (from Section 4.3, Equation 4) the measured water density at temperature T ($^\circ\text{C}$) is given by

$$\rho_{W(T)} = \frac{\rho_A (V_{S1} - V_{S2}) + (M_{S2} - M_{S1}) + M_{SP} + \Delta W}{V_{SP}} \quad (\text{kg m}^{-3})$$

and if Δ_{ST} is the uncertainty contribution of the surface tension force acting on the suspension wire, then

$$\rho_{W(T)} = \frac{\rho_A (V_{S1} - V_{S2}) + (M_{S2} - M_{S1}) + M_{SP} + \Delta W + \Delta_{ST}}{V_{SP}} \quad (\text{kg m}^{-3})$$

therefore the uncertainty in the measured density may be expressed as follows:

$$u_c(\rho_{w(T)})^2 = c_4^2 u(\rho_A)^2 + c_5^2 u(V_{S1})^2 + c_6^2 u(V_{S2})^2 + c_7^2 u(M_{S2})^2 + c_8^2 u(M_{S1})^2 \\ + c_9^2 u(M_{SP})^2 + c_{10}^2 u(\Delta D)^2 + c_{11}^2 u(\Delta W)^2 + c_{12}^2 u(\Delta_{ST})^2 + c_{13}^2 u(V_{SP})^2$$

where

$$c_4 = \left(\frac{\partial \rho_T}{\partial \rho_A} \right) = \frac{V_{S1} - V_{S2}}{V_{SP}}$$

$$c_5 = \left(\frac{\partial \rho_T}{\partial V_{S1}} \right) = \frac{\rho_A}{V_{SP}}$$

$$c_6 = \left(\frac{\partial \rho_T}{\partial V_{S2}} \right) = -\frac{\rho_A}{V_{SP}}$$

$$c_7 = \left(\frac{\partial \rho_T}{\partial M_{S2}} \right) = \frac{1}{V_{SP}}$$

$$c_8 = \left(\frac{\partial \rho_T}{\partial M_{S1}} \right) = -\frac{1}{V_{SP}}$$

$$c_9 = \left(\frac{\partial \rho_T}{\partial M_{SP}} \right) = \frac{1}{V_{SP}}$$

$$c_{10} = \left(\frac{\partial \rho_T}{\partial \Delta W} \right) = \frac{1}{V_{SP}}$$

$$c_{11} = \left(\frac{\partial \rho_T}{\partial \Delta_{ST}} \right) = \frac{1}{V_{SP}}$$

$$c_{12} = \left(\frac{\partial \rho_T}{\partial V_{SP}} \right) = \frac{\rho_A (V_{S2} - V_{S1}) + (M_{S1} - M_{S2}) - M_{SP} - \Delta W - \Delta_{ST}}{V_{SP}^2}$$

AIR DENSITY CONTRIBUTIONS

The dependence of the uncertainty in air density on the parameters that are measured to calculate it, i.e. temperature, pressure, dew point and carbon dioxide concentration, are taken from Giacomo [2].

This can be expressed as

$$u_c(\rho_A)^2 = c_{13}^2 u(\rho_{A(p)})^2 + c_{14}^2 u(\rho_{A(T)})^2 + c_{15}^2 u(\rho_{A(d)})^2 + c_{16}^2 u(\rho_{A(CO2)})^2$$

where $c_{13} = \left(\frac{\partial \rho_A}{\partial p_{A(p)}} \right) = +1 \times 10^{-5} \text{ Pa}$

$$c_{14} = \left(\frac{\partial \rho_A}{\partial p_{A(T)}} \right) = -4 \times 10^{-3} \text{ } ^\circ\text{C}^{-1}$$

$$c_{15} = \left(\frac{\partial \rho_A}{\partial p_{A(d)}} \right) = -3 \times 10^{-4} \text{ } ^\circ\text{C}^{-1}$$

$$c_{16} = \left(\frac{\partial \rho_A}{\partial p_{A(x_{CO2})}} \right) = +0.4 \text{ } \partial x_{CO2}$$

3 DENSITY OF AN ARTEFACT

CONTRIBUTIONS FOR THE DENSITY FORMULA

If M_{A1} is the mass of the artefact (kg)

$V_{A1(20)}$ is the volume of the artefact at 20 °C (m³)

then $\rho_{A1(20)}$, the calculated density of the artefact at 20°C, is expressed by

$$\rho_{A1(20)} = \frac{M_{A1}}{V_{A1(20)}} \text{ kg m}^{-3}$$

and the uncertainty is

$$\begin{aligned} u_c(\rho_{A1(20)})^2 &= c_1^2 u(V_{A1(20)})^2 + c_2^2 u(M_{A1})^2 \\ &= \left(\frac{\partial \rho_{A1(20)}}{\partial V_{A1(20)}} \right)^2 u(V_{A1(20)})^2 + \left(\frac{\partial \rho_{A1(20)}}{\partial M_{A1(20)}} \right)^2 u(M_{A1})^2 \\ &= \left(\frac{-M_{A1(20)}}{V_{A1(20)}^2} \right)^2 u(V_{A1(20)})^2 + \left(\frac{1}{V_{A1(20)}} \right)^2 u(M_{A1})^2 \end{aligned}$$

CONTRIBUTIONS FOR THERMAL EXPANSION OF THE ARTEFACT

If $V_{AI(20)}$ is the calculated volume at 20 °C (g cm^{-3})

$V_{AI(T)}$ is the measured volume at T °C (g cm^{-3})

ΔT is (20-T) (°C)

α is the cubic coefficient of thermal expansion

The volume of the artefact at 20°C is found from the formula

$$V_{AI(20)} = V_{AI(T)}(1 + \alpha \Delta T) \quad (\text{cm}^{-3})$$

and the uncertainty in measured volume may be expressed as follows

$$\begin{aligned} u_c(V_{AI(20)})^2 &= c_3^2 u(V_{AI(T)})^2 + c_4^2 u(\alpha)^2 + c_5^2 u(T)^2 \\ &= \left(\frac{\partial V_{AI(20)}}{\partial V_{AI(T)}} \right)^2 u(V_{AI(T)})^2 + \left(\frac{\partial V_{AI(20)}}{\partial \alpha} \right)^2 u(\alpha)^2 + \left(\frac{\partial V_{AI(20)}}{\partial T} \right)^2 u(T)^2 \\ &= (1 + \alpha \Delta T)^2 u(V_{AI(T)})^2 + (V_{AI(T)} \Delta T)^2 u(\alpha)^2 + (V_{AI(T)} \alpha)^2 u(T)^2 \end{aligned}$$

MEASUREMENT CONTRIBUTIONS

If M_{S1} is the mass of the standards (kg)

V_{S1} is the volume of the standards (m^3)

M_{S2} is the mass of the makeweights (kg)

V_{S2} is the volume of the makeweights (m^3)

M_{SP} is the mass of the sphere (kg)

V_{SP} is the volume of the sphere (m^3)

ρ_A is the air density (kg m^{-3})

ρ_W is the water density (kg m^{-3})

ΔW is the difference between balance readings (kg)

then (from equation 8) the measured volume of the artefact at temperature T (°C) is given by

$$V_{AI(T)} = \frac{\rho_A (V_{S1} - V_{S2}) + (M_{S2} - M_{S1}) + M_{AI} + \Delta W}{\rho_{W(T)}} \quad (\text{m}^3)$$

and if Δ_{ST} is the uncertainty contribution of the surface tension force acting on the suspension, (i.e. the wetting check), then

$$V_{AI(T)} = \frac{\rho_A (V_{S1} - V_{S2}) + (M_{S2} - M_{S1}) + M_{A1} + \Delta W + \Delta_{ST}}{\rho_{W(T)}} \quad (\text{m}^3)$$

therefore the uncertainty in the measured volume may be expressed as follows

$$u_c(V_{AI(T)})^2 = c_6^2 u(\rho_A)^2 + c_7^2 u(V_{S1})^2 + c_8^2 u(V_{S2})^2 + c_9^2 u(M_{S2})^2 + c_{10}^2 u(M_{S1})^2 \\ + c_{11}^2 u(M_{A1})^2 + c_{12}^2 u(\Delta W)^2 + c_{13}^2 u(\Delta_{ST})^2 + c_{14}^2 u(\rho_{W(T)})^2$$

where

$$c_6 = \left(\frac{\partial V_{AI(T)}}{\partial \rho_A} \right) = \frac{V_{S1} - V_{S2}}{\rho_{W(T)}}$$

$$c_7 = \left(\frac{\partial V_{AI(T)}}{\partial V_{S1}} \right) = \frac{\rho_A}{\rho_{W(T)}}$$

$$c_8 = \left(\frac{\partial V_{AI(T)}}{\partial V_{S2}} \right) = -\frac{\rho_A}{\rho_{W(T)}}$$

$$c_9 = \left(\frac{\partial V_{AI(T)}}{\partial M_{S2}} \right) = \frac{1}{\rho_{W(T)}}$$

$$c_{10} = \left(\frac{\partial V_{AI(T)}}{\partial M_{S1}} \right) = -\frac{1}{\rho_{W(T)}}$$

$$c_{11} = \left(\frac{\partial V_{AI(T)}}{\partial M_{A1}} \right) = \frac{1}{\rho_{W(T)}}$$

$$c_{12} = \left(\frac{\partial V_{AI(T)}}{\partial \Delta W} \right) = \frac{1}{\rho_{W(T)}}$$

$$c_{13} = \left(\frac{\partial V_{AI(T)}}{\partial \Delta_{ST}} \right) = \frac{1}{\rho_{W(T)}}$$

$$c_{14} = \left(\frac{\partial V_{AI(T)}}{\partial \rho_{W1}} \right) = \frac{\rho_A (V_{S2} - V_{S1}) + (M_{S1} - M_{S2}) - M_{A1} - \Delta W - \Delta_{ST}}{\rho_{W(T)}^2}$$

AIR DENSITY CONTRIBUTIONS

As shown previously

$$u_C(\rho_A)^2 = c_{15}^2 u(\rho_{A(p)})^2 + c_{16}^2 u(\rho_{A(T)})^2 + c_{17}^2 u(\rho_{A(d)})^2 + c_{18}^2 u(\rho_{A(CO_2)})^2$$

therefore

$$c_{15} = \left(\frac{\partial \rho_A}{\partial \rho_{A(p)}} \right) = +1 \times 10^{-5} \text{ Pa}$$

$$c_{16} = \left(\frac{\partial \rho_A}{\partial \rho_{A(T)}} \right) = -4 \times 10^{-3} \text{ }^\circ\text{C}^{-1}$$

$$c_{17} = \left(\frac{\partial \rho_A}{\partial \rho_{A(d)}} \right) = -3 \times 10^{-4} \text{ }^\circ\text{C}^{-1}$$

$$c_{18} = \left(\frac{\partial \rho_A}{\partial \rho_{A(x_{CO_2})}} \right) = +0.4 \text{ } \partial^x_{CO_2}$$

4 REFERENCES

- [1] PATTERSON, J.B. and MORRIS, E.C. Measurement of Absolute Water Density, 1 °C to 40°C. *Metrologia*, 1994, **31**, 277-288.
- [2] GIACOMO, P. Equation for the Determination of the Density of Moist Air (1981). *Metrologia*, 1982, **18**, 33-40.

APPENDIX 5.1

UNCERTAINTY BUDGET FOR THE DENSITY OF WATER Using Zerodur sphere (1st water sample)

All uncertainties calculated at $k=1$

Parameter		Uncertainty Contribution		Divisor	Standard uncertainty (u) (source unit)	Sensitivity coefficient (c_x)	Standard uncertainty (u) $\pm(\text{kg m}^{-3})$
Symbol	Nominal value		Value				

Table A - contributions to air density uncertainty

Pressure	$P_A(p)$	1 020 hPa	Stability Calibration	20.000 000 Pa	$\sqrt{3}$	11.547 01 Pa	c_{13} 0.000 01	0.000 12
Temperature	$P_A(T)$	20 °C	Stability Gradient Calibration	6.000 00 Pa 0.050 00 °C 0.200 00 °C 0.030 00 °C	2 $\sqrt{3}$ $\sqrt{3}$ 2	3.000 00 Pa 0.028 87 °C 0.115 47 °C 0.015 00 °C	0.000 01 -0.004 00 -0.004 00 -0.004 00	0.000 03 -0.000 12 -0.000 46 -0.000 06
Dew point	$P_A(d)$	7 °C	Stability Gradient Calibration	2.000 00 °C 2.000 00 °C 0.200 00 °C	$\sqrt{3}$ $\sqrt{3}$ 2	1.154 70 °C 1.154 70 °C 0.100 00 °C	-0.000 30 -0.000 30 -0.000 30	-0.000 35 -0.000 35 -0.000 03
Carbon dioxide	$P_A(\text{CO}_2)$	400 ppm		0.000 04 kg m ⁻³	$\sqrt{3}$	0.000 02 kg m ⁻³	c_{16} 0.400 00	0.000 01
CIPM equation				0.000 12 kg m ⁻³	1	0.000 12 kg m ⁻³	1.000 00	0.000 12

Combined air density uncertainty: 0.000 71

Table B - contributions to water density at T °C

Air density	P_A	0.001 2 g cm ⁻³	From table A	0.000 71 kg m ⁻³	1	0.000 71 kg m ⁻³	c_4 0.196 97	0.000 14
Volume of standards	V_{S1}	78 cm ³	Calibration	0.010 00 cm ³	2	0.005 00 cm ³	c_5 0.000 00	0.000 02
Volume of makeweights	V_{S2}	0 cm ³	Calibration	0.000 00 cm ³	2	0.000 00 cm ³	c_6 0.000 00	0.000 00
Mass of makeweights	M_{S2}	0 g	Calibration	0.000 00 g	2	0.000 00 g	c_7 0.002 53	0.000 00
Mass of standards	M_{S1}	610 g	Calibration	0.000 02 g	2	0.000 01 g	c_8 -0.002 53	-0.000 03
Mass of sphere	M_{SP}	1 005 g	Calibration	0.000 05 g	2	0.000 02 g	c_9 0.002 53	0.000 06
Balance difference	ΔW	0.024 g	Calibration	0.000 19 g	2	0.000 10 g	c_{10} 0.002 53	0.000 24
Surface tension	Δ_{ST}	0.000 2 g	Repeatability	0.000 21 g	$\sqrt{3}$	0.000 12 g	c_{11} 0.002 53	0.000 31
Volume of sphere	V_{SP}	396 cm ³	Calibration	0.000 06 cm ³	2	0.000 03 cm ³	c_{12} -0.002 52	-0.000 08

Combined water density uncertainty at T °C: 0.000 43

Table C - contributions to water density at 20 °C

Density at t °C	$\rho_w(T)$	0.998 g cm ⁻³	From table B	0.000 43 kg m ⁻³	1	0.000 43 kg m ⁻³	c_1 1.000 03	0.000 43
Expansion	α	2.07E-04 °C ⁻¹		0.000 00 °C ⁻¹	1	0.000 00 °C ⁻¹	c_2 0.125 75	0.000 03
Temperature difference	ΔT	0.126 °C	Stability Gradient Calibration	0.005 00 °C 0.010 00 °C 0.001 00 °C	$\sqrt{3}$ $\sqrt{3}$ 2	0.002 89 °C 0.005 77 °C 0.000 50 °C	0.000 21 0.000 21 0.000 21	0.000 60 0.001 19 0.000 10
Standard deviation	σ		Repeatability	0.001 44 kg m ⁻³	1	0.001 44 kg m ⁻³	1.000 00	0.001 44

Combined water density uncertainty at 20 °C: 0.002 01

APPENDIX 5.2

UNCERTAINTY BUDGET FOR THE DENSITY OF WATER
Using Zerodur sphere (2nd water sample)

All uncertainties calculated at $k=1$

Parameter		Uncertainty Contribution		Divisor	Standard uncertainty (u) (source unit)	Sensitivity coefficient (c_i)	Standard uncertainty (u) $\pm(\text{kg m}^{-3})$
Symbol	Nominal value		Value				

Table A - contributions to air density uncertainty

Pressure	$P_A(p)$	1 020 hPa	Stability Calibration	20,000 000 Pa	$\sqrt{3}$	11.547 01 Pa	c_{13} 0.000 01	0.000 12
Temperature	$P_A(T)$	20 °C	Stability Gradient Calibration	6,000 00 Pa 0.050 00 °C 0.200 00 °C 0.030 00 °C	2 $\sqrt{3}$ $\sqrt{3}$ 2	3.000 00 Pa 0.028 87 °C 0.115 47 °C 0.015 00 °C	0.000 01 -0.004 00 -0.004 00 -0.004 00	0.000 03 -0.000 12 -0.000 46 -0.000 06
Dew point	$P_A(d)$	7 °C	Stability Gradient Calibration	2,000 00 °C 2,000 00 °C 0.200 00 °C	$\sqrt{3}$ $\sqrt{3}$ 2	1.154 70 °C 1.154 70 °C 0.100 00 °C	-0.000 30 -0.000 30 -0.000 30	-0.000 35 -0.000 35 -0.000 03
Carbon dioxide CIPM equation	$p_{A(\text{CO}_2)}$	400 ppm		0.000 04 kg m ⁻³ 0.000 12 kg m ⁻³	$\sqrt{3}$ 1	0.000 02 kg m ⁻³ 0.000 12 kg m ⁻³	c_{16} 0.400 00 1.000 00	0.000 01 0.000 12

Combined air density uncertainty: **0.000 71**

Table B - contributions to water density at T °C

Air density	ρ_A	0.001 2 g cm ⁻³	From table A	0.000 71 kg m ⁻³	1	0.000 71 kg m ⁻³	c_4 0.196 97	0.000 14
Volume of standards	V_{S1}	78 cm ³	Calibration	0.010 00 cm ³	2	0.005 00 cm ³	c_5 0.000 00	0.000 02
Volume of makeweights	V_{S2}	0 cm ³	Calibration	0.000 00 cm ³	2	0.000 00 cm ³	c_6 0.000 00	0.000 00
Mass of makeweights	M_{S2}	0 g	Calibration	0.000 00 g	2	0.000 00 g	c_7 0.002 53	0.000 00
Mass of standards	M_{S1}	610 g	Calibration	0.000 02 g	2	0.000 01 g	c_8 -0.002 53	-0.000 03
Mass of sphere	M_{SP}	1 005 g	Calibration	0.000 05 g	2	0.000 02 g	c_9 0.002 53	0.000 06
Balance difference	ΔW	-0.013 g	Calibration	0.000 19 g	2	0.000 10 g	c_{10} 0.002 53	0.000 24
Surface tension	Δ_{ST}	0.000 2 g	Repeatability	0.000 21 g	$\sqrt{3}$	0.000 12 g	c_{11} 0.002 53	0.000 31
Volume of sphere	V_{SP}	396 cm ³	Calibration	0.000 06 cm ³	2	0.000 03 cm ³	c_{12} -0.002 52	-0.000 08

Combined water density uncertainty at T °C: **0.000 43**

Table C - contributions to water density at 20 °C

Density at t °C	$\rho_{WT(t)}$	0.998 g cm ⁻³	From table B	0.000 43 kg m ⁻³	1	0.000 43 kg m ⁻³	c_1 1.000 03	0.000 43
Expansivity	α	2.07E-04 °C ⁻¹		0.000 00 °C ⁻¹	1	0.000 00 °C ⁻¹	c_2 0.129 74	0.000 03
Temperature difference	ΔT	0.13 °C	Stability Gradient Calibration	0.005 00 °C 0.010 00 °C 0.001 00 °C	$\sqrt{3}$ $\sqrt{3}$ 2	0.002 89 °C 0.005 77 °C 0.000 50 °C	c_3 0.000 21 c_{10} 0.000 21 c_{11} 0.000 21	0.000 60 0.001 19 0.000 10
Standard deviation	σ		Repeatability	0.000 48 kg m ⁻³	1	0.000 48 kg m ⁻³	1.000 00	0.000 48

Combined water density uncertainty at 20 °C: **0.001 48**

APPENDIX 5.3

UNCERTAINTY BUDGET FOR THE DENSITY OF WATER Using silica sphere (1st water sample)

All uncertainties calculated at $k=1$

Parameter	Uncertainty Contribution		Divisor	Standard uncertainty (u) (source unit)	Sensitivity coefficient (c_i)	Standard uncertainty (u) $\pm(\text{kg m}^{-3})$
	Symbol	Nominal value				

Table A - contributions to air density uncertainty

Pressure	$P_{A(p)}$	1 020 hPa	Stability Calibration	20.000 000 Pa	$\sqrt{3}$	11.547 01 Pa	c_{13}	0.000 01	0.000 12
Temperature	$P_{A(T)}$	20 °C	Stability Gradient Calibration	6.000 00 Pa 0.050 00 °C 0.200 00 °C 0.030 00 °C	2 $\sqrt{3}$ $\sqrt{3}$ 2	3.000 00 Pa 0.028 87 °C 0.115 47 °C 0.015 00 °C		0.000 01 -0.004 00 -0.004 00 -0.004 00	0.000 03 -0.000 12 -0.000 46 -0.000 06
Dew point	$P_{A(d)}$	7 °C	Stability Gradient Calibration	2.000 00 °C 2.000 00 °C 0.200 00 °C	$\sqrt{3}$ $\sqrt{3}$ 2	1.154 70 °C 1.154 70 °C 0.100 00 °C	c_{14} c_{15}	-0.000 30 -0.000 30 -0.000 30	-0.000 35 -0.000 35 -0.000 03
Carbon dioxide	$P_{A(\text{CO}_2)}$	400 ppm		0.000 04 kg m ⁻³	$\sqrt{3}$	0.000 02 kg m ⁻³	c_{16}	0.400 00	0.000 01
CIPM equation				0.000 12 kg m ⁻³	1	0.000 12 kg m ⁻³		1.000 00	0.000 12

Combined air density uncertainty: 0.000 71

Table B - contributions to water density at T °C

Air density	ρ_A	0.001 2 g cm ⁻³	From table A	0.000 71 kg m ⁻³	1	0.000 71 kg m ⁻³	c_4	0.154 35	0.000 11
Volume of standards	V_{S1}	77 cm ³	Calibration	0.010 00 cm ³	2	0.005 00 cm ³	c_5	0.000 00	0.000 01
Volume of makeweights	V_{S2}	6 cm ³	Calibration	0.000 00 cm ³	2	0.000 00 cm ³	c_6	0.000 00	0.000 00
Mass of makeweights	M_{S2}	50 g	Calibration	0.000 00 g	2	0.000 00 g	c_7	0.002 17	0.000 00
Mass of standards	M_{S1}	603 g	Calibration	0.000 02 g	2	0.000 01 g	c_8	-0.002 17	-0.000 03
Mass of sphere	M_{SP}	1 012 g	Calibration	0.000 05 g	2	0.000 02 g	c_9	0.002 17	0.000 05
Balance difference	ΔW	0.013 g	Calibration	0.000 19 g	2	0.000 10 g	c_{10}	0.002 17	0.000 21
Surface tension	Δ_{ST}	0.000 2 g	Repeatability	0.000 21 g	$\sqrt{3}$	0.000 12 g	c_{11}	0.002 17	0.000 26
Volume of sphere	V_{SP}	460 cm ³	Calibration	0.000 06 cm ³	2	0.000 03 cm ³	c_{12}	-0.002 17	-0.000 07

Combined water density uncertainty at T °C: 0.000 36

Table C - contributions to water density at 20 °C

Density at t °C	$\rho_{W(t)}$	0.998 g cm ⁻³	From table B	0.000 36 kg m ⁻³	1	0.000 36 kg m ⁻³	c_1	1.000 02	0.000 36
Expansivity	α	2.07E-04 °C ⁻¹		0.000 00 °C ⁻¹	1	0.000 00 °C ⁻¹	c_2	0.086 83	0.000 02
Temperature difference	ΔT	0.087 °C	Stability Gradient Calibration	0.005 00 °C 0.010 00 °C 0.001 00 °C	$\sqrt{3}$ $\sqrt{3}$ 2	0.002 89 °C 0.005 77 °C 0.000 50 °C	c_3	0.000 21 0.000 21 0.000 21	0.000 60 0.001 19 0.000 10
Standard deviation	σ		Repeatability	0.001 14 kg m ⁻³	1	0.001 14 kg m ⁻³		1.000 00	0.001 14

Combined water density uncertainty at 20 °C: 0.001 79

APPENDIX 5.4

UNCERTAINTY BUDGET FOR THE DENSITY OF WATER Using silica sphere (2nd water sample)

All uncertainties calculated at $k=1$

Parameter		Uncertainty Contribution		Divisor	Standard uncertainty (u) (source unit)	Sensitivity coefficient (c_x)	Standard uncertainty (u) $\pm(\text{kg m}^{-3})$
Symbol	Nominal value		Value				

Table A - contributions to air density uncertainty

Pressure	$P_{A(p)}$	1 020 hPa	Stability Calibration	$\sqrt{3}$	11.547 01 Pa	c_{13}	0.000 12
Temperature	$P_{A(T)}$	20 °C	Stability Gradient Calibration	2 $\sqrt{3}$ 2	3.000 00 Pa 0.028 87 °C 0.115 47 °C 0.015 00 °C	0.000 01 -0.004 00 -0.004 00 -0.004 00	0.000 03 -0.000 12 -0.000 46 -0.000 06
Dew point	$P_{A(d)}$	7 °C	Stability Gradient Calibration	$\sqrt{3}$ $\sqrt{3}$ 2	2.000 00 °C 2.000 00 °C 0.200 00 °C	c_{14} c_{15} -0.000 30 -0.000 30 -0.000 30	-0.000 35 -0.000 35 -0.000 03
Carbon dioxide CIPM equation	$P_{A(\text{CO}_2)}$	400 ppm		$\sqrt{3}$	0.000 04 kg m ⁻³	c_{16}	0.000 01
				1	0.000 12 kg m ⁻³	1.000 00	0.000 12

Combined air density uncertainty: 0.000 71

Table B - contributions to water density at T °C

Air density	ρ_A	0.001 2 g cm ⁻³	From table A	1	0.000 71 kg m ⁻³	c_4	0.000 11
Volume of standards	V_{S1}	77 cm ³	Calibration	2	0.005 00 cm ³	c_5	0.000 01
Volume of makeweights	V_{S2}	6 cm ³	Calibration	2	0.000 00 cm ³	c_6	0.000 00
Mass of makeweights	M_{S2}	50 g	Calibration	2	0.000 00 g	c_7	0.000 00
Mass of standards	M_{S1}	603 g	Calibration	2	0.000 01 g	c_8	-0.000 03
Mass of sphere	M_{SP}	1 012 g	Calibration	2	0.000 02 g	c_9	0.000 05
Balance difference	ΔW	0.024 g	Calibration	2	0.000 10 g	c_{10}	0.000 21
Surface tension	Δ_{ST}	0.000 2 g	Repeatability	$\sqrt{3}$	0.000 12 g	c_{11}	0.000 26
Volume of sphere	V_{SP}	460 cm ³	Calibration	2	0.000 03 cm ³	c_{12}	-0.000 07

Combined water density uncertainty at T °C: 0.000 36

Table C - contributions to water density at 20 °C

Density at t °C	$\rho_{W(t)}$	0.998 g cm ⁻³	From table B	1	0.000 36 kg m ⁻³	c_1	0.000 36
Expansivity	α	2.07E-04 °C ⁻¹		1	0.000 00 °C ⁻¹	c_2	0.000 05
Temperature difference	ΔT	0.164 °C	Stability Gradient Calibration	$\sqrt{3}$ $\sqrt{3}$ 2	0.002 89 °C 0.005 77 °C 0.000 50 °C	c_3 0.000 21 0.000 21	0.000 60 0.001 19 0.000 10
Standard deviation	σ		Repeatability	1	0.000 39 kg m ⁻³	1.000 00	0.000 39

Combined water density uncertainty at 20 °C: 0.001 44

Combined density uncertainty at 20 °C:	0.028 53
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APPENDIX 5.6

UNCERTAINTY BUDGET FOR ARTEFACT A1
Using silica sphere to determine density of water

All uncertainties calculated at $k=1$

Parameter		Nominal value		Uncertainty Contribution		Divisor	Standard uncertainty (u) (source unit)	Sensitivity coefficient (c _x)	Standard uncertainty (u)
	Symbol				Value				
Table A - contributions to air density uncertainty									
Pressure	p _{A(p)}	1 020 hPa	Stability	20.000 00 Pa	√3	11.547 01 Pa	c ₁₅	0.000 01	±(kg m ⁻³)
			Calibration	6.000 00 Pa	2	3.000 00 Pa		0.000 01	0.000 12
Temperature	p _{A(t)}	20 °C	Stability	0.050 00 °C	√3	0.028 87 °C	c ₁₆	0.004 00	-0.000 12
			Gradient	0.200 00 °C	√3	0.115 47 °C		0.004 00	-0.000 46
Dew point			Calibration	0.030 00 °C	2	0.015 00 °C		0.004 00	-0.000 06
	p _{A(d)}	7 °C	Stability	2.000 00 °C	√3	1.154 70 °C	c ₁₇	0.000 30	-0.000 35
			Gradient	2.000 00 °C	√3	1.154 70 °C		0.000 30	-0.000 35
			Calibration	0.200 00 °C	2	0.100 00 °C		0.000 30	-0.000 03
Carbon dioxide	p _{A(CO2)}	400 ppm		0.000 04 kg m ⁻³	√3	0.000 02 kg m ⁻³	c ₁₈	0.400 00	0.000 01
CIPM equation				0.000 12 kg m ⁻³	1	0.000 12 kg m ⁻³		1.000 00	0.000 12
Combined air density uncertainty:									0.000 71

Table B - contributions to volume of artefact at T °C								
Air density	ρ_A	0.001 2 g cm ⁻³	From table A	0.000 71 kg m ⁻³	1	0.000 71 kg m ⁻³	c_6 78.156 31	$\pm(\text{cm}^3)$ 0.000 06
Volume of standards	V_{S1}	78 cm ³	Calibration	0.010 00 cm ³	2	0.005 00 cm ³	c_7 0.001 20	0.000 01
	V_{S2}	0 cm ³	Calibration	0.000 00 cm ³	2	0.000 00 cm ³	c_8 -0.001 20	0.000 00
Mass of standards	M_{S2}	0 g	Calibration	0.000 00 g	2	0.000 00 g	c_9 1.002 00	0.000 00
	M_{S1}	610 g	Calibration	0.000 02 g	2	0.000 01 g	c_{10} -1.002 00	-0.000 01
Mass of artefact	M_{A1}	1 000 g	Calibration	0.000 05 g	2	0.000 02 g	c_{11} 1.002 00	0.000 02
Poising discrepancy	Δp	0.002 g	Calibration	0.000 19 g	2	0.000 10 g	c_{12} 1.002 00	0.000 10
Surface tension	Δw	0.000 2 g	Repeatability	0.000 21 g	$\sqrt{3}$	0.000 12 g	c_{13} 1.002 00	0.000 12
Density of water	ρ_w	0.998 g cm ⁻³	Calibration	0.001 77 kg m ⁻³	2	0.000 89 kg m ⁻³	c_{14} -0.391.661	-0.000 35
Combined volume uncertainty at T °C:								0.000 38

Table C - contributions to volume of artefact at 20 °C								
Volume at t °C	V_{A1}	127 cm ³	From table B	0.000 38 cm ³	1	0.000 38 cm ³	c_3 1.000 00	$\pm(\text{cm}^3)$ 0.000 38
Thermal expansion	α	4.70E-05 °C ⁻¹		0.000 00 °C ⁻¹	1	0.000 00 °C ⁻¹	c_4 12.700 00	0.000 00
Temperature difference	Δt	0.10 °C	Stability	0.005 00 °C	$\sqrt{3}$	0.002 89 °C	c_5 0.005 97	0.000 02
			Gradient	0.010 00 °C	$\sqrt{3}$	0.005 77 °C	0.005 97	0.000 03
			Calibration	0.001 00 °C	2	0.000 50 °C	0.005 97	0.000 00
Combined volume uncertainty at 20 °C:								0.000 39

Table D - contributions to density at 20 °C								
Volume of artefact at 20 °C	$V_{A1(20)}$	127 cm ³	From table C	0.000 39 cm ³	1	0.000 39 cm ³	c_1 -0.062 00	$\pm(\text{kg m}^{-3})$ -0.023 95
Mass of artefact	M_{A1}	100 0 g	Calibration	0.000 05 g	2	0.000 02 g	c_2 7.874 02	0.000 18
Standard deviation	σ		Repeatability	0.019 80 kg m ⁻³	1	0.019 80 kg m ⁻³	1.000 00	0.019 80
Combined density uncertainty at 20 °C:								0.031 08

