

A Further Evaluation of the Cavitation Model for Rubber-Toughened Plastics

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SUMMARY

The version of the cavitation model described in earlier reports predicts that, contrary to experimental observation, rubber particles will cavitate under shear and compressive stresses. This arises because the cavity nucleation criterion is expressed in terms of the total volumetric strain in the polymer, and the volumetric strain during plastic deformation is relatively large for associated flow in shear and compression. This report investigates the assumption of associated flow and explores alternative relationships for the cavity nucleation criterion based on the total and the elastic volumetric strain on the rubber particles.

Measurements of volumetric strain in uniaxial tension and compression tests reveal that plastic flow is non-associated and imply that the flow parameter depends on the stress state. Expressions for the volumetric strain on rubber particles have been derived that take account of a predicted reduction in elastic properties with cavity nucleation. Cavity nucleation criteria based on these expressions give satisfactory predictions of stress/strain curves and Poisson's ratio under uniaxial tension and compression but a poor fit to stress strain behaviour in a butt-tension test. Based on these observations, a preferred version of the model is identified for future FE analyses.

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1 INTRODUCTION

An elastic-plastic model has previously been proposed for the multiaxial stress/strain behaviour of rubber-toughened plastics [1-3]. The model assumes that the elastic response at low strain is linear and isotropic and described by the generalised Hooke's Law with Young's modulus E and elastic Poisson's ratio ν^e . The onset of non-linearity in a stress-strain curve is then ascribed to plastic deformation which, under dilatational stress states, is enhanced by the generation and growth of cavities in the rubber particles. The yield stress criterion and flow rule are based on Gurson's theory of plasticity in porous ductile materials [4], following developments by Tvergaard [5] and by Bucknall and co-workers [6,7]. The proposed yield function allows for the effect of the hydrostatic component of stress on the yield stress of matrix material between cavities. It also accounts for the change in matrix composition and hence the matrix yield stress during void nucleation. The nucleation is assumed to occur over a critical range of volumetric strain ϵ_v and to involve the replacement of rubber particles by an volume of effective cavities. Different nucleation functions have been studied for describing the dependence of effective void fraction f_{nucl} on ϵ_v .

For an acrylonitrile-butadiene-styrene (ABS) copolymer containing 38 volume percent rubber, close agreement was obtained between predicted and measured stress-strain curves under uniaxial tension at 23 °C and a strain rate of 0.004 s⁻¹. This is illustrated in Figure 1 where ν_{R0} is the volume fraction of rubber in the uncavitated material, k is a constant specifying the dependence of the matrix yield stress on the volume fraction of uncavitated rubber particles, and parameters q_1 and μ are defined by equation (1) below. The predicted curve was derived from shear hardening data for the uncavitated material, the dependence of f_{nucl} on ϵ_v being represented by a stretched-exponential function with parameters ϵ_{1v} , ϵ_{2v} and β_v^{1-3} . The model also captured the marked decrease in Poisson's ratio (see figure 2) and associated increase in volumetric strain resulting from the cavitation during tensile deformation. However, according to the proposed flow rule, a substantial expansion during plastic deformation was predicted in the absence of cavitation for all stress states, including shear and compression. This plastic expansion produces a volumetric strain ϵ_v under shear or compression which, according to the proposed nucleation function, is sufficient to induce cavitation (see figure 3). Since no cavitation was observed in the shear and compression tests, an extension was made

to the void nucleation criterion suggesting that cavitation will occur at a critical volumetric strain ε_v providing also that the mean normal stress exceeds some critical positive value. With this added condition, good agreement was obtained between the predicted and observed stress-strain curves for the ABS material under uniaxial compression (see figure 3).

The need for this condition implies a limitation in the physical basis of the model. This report describes modifications to the flow rule and void nucleation criterion aimed at providing accurate predictions of the behaviour under different stress states without invoking the above condition. A modified flow rule allows small or zero plastic volume changes to occur in the absence of cavitation, or in the matrix material between cavities, whilst still capturing the large plastic expansion that occurs for cavitated material under dilatational stress states. The void nucleation criterion now assumes that cavitation will occur over a critical positive range of volumetric strain on the rubber particles in the matrix. This is consistent with the calculations of Bucknall and co-workers and requires a consideration of the evolution of elastic, as well as plastic, properties during the void nucleation and growth.

2 MODIFICATION TO THE FLOW RULE

2.1 NON-ASSOCIATED FLOW

The plastic component of the volumetric strain developed under any stress state is derived from the flow rule. In order to investigate the factors that influence this strain level, it is necessary to consider the most general form for the flow potential, namely that for non-associated flow.

In previous versions of the model, the flow behaviour was assumed to be associated with the yield surface implying that the flow potential F could be identified with the yield function Φ . Thus

$$F = \Phi = \frac{\sigma_e^2}{\sigma_M^2} - (q_1 f)^2 + 2q_1 f \cosh\left(\frac{3\sigma_k}{2\sigma_M}\right) \left(1 - \frac{\mu\sigma_k}{\sigma_M}\right)^2 = 0 \quad (1)$$

where σ_k and σ_e are the hydrostatic and effective shear components of the applied stress, σ_M is an effective yield stress of the matrix material around cavities, f is the volume fraction of cavities, q_1 is a cavity interaction parameter and μ is a material parameter that is a measure of the sensitivity of plastic deformation of the uncavitated material to the hydrostatic component of stress. The situation of non-associated flow is now considered under which the flow potential takes the form

$$f = \frac{\sigma_e^2}{\sigma_M^2} - (q_1 f)^2 + 2q_1 f \cosh\left(\frac{3\sigma_k}{2\sigma_M} - \left(\frac{\mu' \sigma_k}{\sigma_M}\right)^2\right) = 0$$

Here, the influence of hydrostatic stress on flow differs from its influence on yielding through the replacement of μ by μ' .

2.2 DEFORMATION UNDER UNIAXIAL COMPRESSION

Plastic strain increments are determined from the flow rule

$$d\varepsilon_{ij}^P = d\lambda \frac{\partial F}{\partial \sigma_{ij}}$$

Under uniaxial compression, the volumetric plastic strain increment is

$$d\varepsilon_V^P = d\varepsilon_{11}^P + 2d\varepsilon_{22}^P$$

where ε_{22}^P and ε_{11}^P are plastic components of the transverse and axial strain. For uncavitated material ($f = 0$), it follows that

$$d\varepsilon_V^P = d\lambda \cdot \frac{2\mu'}{\sigma_o} \left(1 + \frac{\mu'}{3} \frac{\sigma_c}{\sigma_o}\right)$$

which gives

$$\epsilon_V^P = \frac{\mu' \epsilon_o^P \left(1 + \frac{\mu'}{3} \frac{\sigma_c}{\sigma_o} \right)}{\sigma_c \left(\frac{2\sigma_c}{\sigma_o^2} + \frac{2\mu'}{3\sigma_o} \left(1 + \frac{\mu' \sigma_c}{3\sigma_o} \right) \right)}$$

where σ_c is a compressive yield stress and σ_o is the value of σ_M for $f = 0$. σ_o is determined using $\sigma_o = \sqrt{3} \sigma_s$ where σ_s is the shear yield stress for unvoided material at an effective plastic strain $\epsilon_o^P = \gamma^P / \sqrt{3}$ where γ^P is the plastic component of shear strain.

It is now clear why, under associated flow with $\mu' = \mu = 0.22$, (the situation in previous calculations using the model) the total volumetric strain under uniaxial compression is significant and positive for only moderate values of compressive strain ϵ_c where

$$\epsilon_c^P = \frac{(3-\mu)}{3} \epsilon_o^P \quad (7)$$

Thus for a cavity nucleation criterion based on the total volumetric strain, cavities are predicted to occur under uniaxial compression unless the flow parameter μ' for uncavitated material is zero or very small.

2.3 MEASUREMENT OF THE FLOW PARAMETER μ'

An experimental value for μ' has been determined from simultaneous longitudinal and lateral strain measurements under a uniaxial compressive stress. The test conditions were the same as those used to obtain experimental stress/strain curves under compression [3] and employed specimens that were 12 mm $\times$ 12 mm $\times$ 3.6 mm thick loaded between lubricated parallel surfaces. A plot of the Poisson's ratio against the compressive strain is shown in figure 4. From these data, values for the plastic component of Poisson's ratio were derived and are also plotted in the figure. The plastic component of Poisson's ratio is given by

$$v^P = \frac{d\epsilon_{22}^P}{d\epsilon_{11}^P}$$

From equation (3), under a uniaxial compressive strain

$$v^P = \frac{9 + 2\mu'(3-\mu) + 2\mu'^2}{18 - 2\mu'(3-\mu) - 2\mu'^2}$$

Taking a value for $v^P = 0.53$ from figure 4 and for $\mu = 0.22$, it follows from equation (9) that $\mu' = 0.06$.

2.4 AN EXPRESSION FOR THE FLOW PARAMETER μ'

Even allowing for some experimental uncertainty in the true magnitude of μ' , it is evident that this parameter is smaller than the value ($= \mu = 0.22$) used in earlier calculations. There are two implications to this evidence. Firstly, a non-associated flow potential given by equation (2) with a small value for μ' means that volumetric strains under uniaxial tension are also small and able to nucleate cavities in the rubber only at high strains. A small positive value of $\mu' = 0.06$ will also give rise to volumetric strains under shear and compression sufficient to generate cavities at high strains, contrary to experiment. These observations motivated a search for an alternative to the total volumetric strain as the quantity controlling cavity nucleation. Alternative nucleation criteria are considered in section 4.

Secondly, a small value for $\mu' = 0.06$ is unable to explain the large fall in Poisson's ratio measured in uniaxial tension and shown in figure 2. To explain this result, a flow parameter that increases with cavity volume fraction was invoked. The following expression was used in early studies of alternative nucleation criteria discussed in Section 4.

$$\mu' = \mu'_2 + (\mu'_1 - \mu'_2) f / v_{R0}$$

A value of $\mu'_2 = 0.06$ and $\mu'_1 = 0.16$ allowed Poisson's ratio changes with strain to be accurately modelled under both uniaxial compression and uniaxial tension. This expression for μ' was later replaced by an alternative version that is introduced in section 4.4.

3 A BUTT-JOINT TEST FOR PLASTICS

As part of previous work on the development of the new model, the validity of the model was assessed through comparisons of predictions of behaviour under uniaxial tension and uniaxial compression with experimental results. Recently, measurements have been made on a butt-joint specimen of ABS loaded in tension [8]. With this test, the stress state in the polymer has a higher component of hydrostatic stress than occurs in the uniaxial tension test, so the test data provide a useful additional evaluation of the model. Results from this test have been used in the work reported here to assess the validity of alternative cavity nucleation criteria discussed in the next section.

The butt-joint specimen is shown in figure 5 together with an extensometer for the measurement of axial strain in the polymer. The polymer sample consists of a disc of diameter 25 mm whose surfaces have been machined to give a thickness of 2 mm. The surface machining was carried out to obtain surfaces that were accurately plane and parallel and also to establish an increased diameter to thickness ratio so as to achieve a more uniform strain distribution in the polymer. The polymer surfaces were bonded to two 25 mm diameter, cylindrical, steel adherends. The polymer samples had been surface treated by the Institute of Surface Science and Technology, Loughborough University to increase the strength of the bond to the steel adherends. Different surface treatments were explored and the one giving optimum strength was selected. This was chromic acid immersion for a duration of 5 minutes at 60 °C. Samples were bonded to the adherends using the Ciba, two-part adhesive Araldite 2011 and a cure of 60 minutes at 50 °C. The bonding process was carried out within 48 hours of surface treatment. During this process, the adherends were supported by a rigid, precision frame that ensured accurate alignment of the axes of the two adherends.

The v-shape grooves in each adherend serve to locate an extensometer consisting of two rings with knife edges on their inner surfaces. Each ring locates in one of the grooves. One of the rings supports three precision displacement transducers that are equally spaced around the ring. The core of each transducer contacts the surface of the second ring. A value for the nominal axial strain in the polymer sample under tension is determined from the average extension divided by the thickness of the sample disc. A small correction is applied for the

contribution from the elastic deformation of the adherends to the average displacement recorded by the extensometers.

The specimen is loaded through collet grips that are rigidly mounted in the test assembly to minimise any lateral deformation of the specimen during loading. Without this constraint to lateral movement, the specimen is observed to bend during plastic deformation causing a non-uniform strain in the polymer. An alignment fixture is used to position one grip and enable the specimen to be mounted precisely in the test assembly. The three displacement transducers provide a check on the axially of the loading during the test. The readings will diverge if the specimen is misaligned.

Figure 10 shows a plot of the average stress against average strain in an ABS sample in a butt-joint specimen. Average values are recorded that do not take account of the small non-uniformity of stress and strain distributions near the circumference of the polymer. The effective plastic strain rate in this test is 1.10^{-4} s^{-1} .

4 EVALUATION OF ALTERNATIVE CAVITY NUCLEATION CRITERIA

4. GENERAL

The aim of the work reported in this section is to explore the use of cavity nucleation criteria based on alternative quantities to the total volumetric strain that will avoid cavitation under shear or uniaxial compression. The investigation has concentrated on criteria involving the volumetric strain acting on the particles of rubber. The validity of these criteria has then been assessed through comparisons of predicted stress/strain curves under uniaxial tension and butt-tension and Poisson's ratio vs strain curves with experimental results. The derivation of the volumetric strain acting on the rubber requires that proper account should be taken of the influence of cavitation on elastic properties, a feature of the model that has been ignored until now. This is considered next.

4.2 THE INFLUENCE OF CAVITATION ON ELASTIC PROPERTIES

Nomenclature

ϵ_V	average total volumetric strain in the polymer
ϵ_V^e	elastic component of ϵ_V
ϵ_{VM}^e	elastic volumetric strain on the matrix material between voids
$\epsilon_{VR}^e, \epsilon_{VR}^p$	elastic and plastic volumetric strains on the rubber particles, respectively
ϵ_{VR}	total volumetric strain on the rubber
K_R	bulk modulus of the rubber
K_I, G_I, v_I^e	bulk and shear moduli and elastic component of Poisson's ratio, respectively of the poly(styrene-acrylonitrile) component of the matrix material (see fig 6a)
K_M, G_M, v_M^e	bulk and shear moduli and elastic component of Poisson's ratio of the matrix material between cavities. They depend on f
K, G, v^e	bulk and shear moduli and elastic component of Poisson's ratio of the polymer taking account of the influence of rubber-particle cavitation. They depend on f
v_R^*	volume fraction of rubber in the matrix material between cavities

Theoretical relationships

The polymer is considered to consist of cavities in a matrix of poly(styrene-acrylonitrile) and rubber as illustrated in figure 6. The elastic properties of the cavities are considered to be

As cavities form by the removal of rubber particles, the matrix properties evolve from those for the fully rubber-toughened material with $v_R^* = v_{R0}$ and $f = 0$ to pure poly(styrene-acrylonitrile) when $v_R^* = 0$ and $f = v_{R0}$. The changes in matrix properties K_M , G_M and v_M^e as cavitation proceeds are given by (see appendix A)

$$K_M = \frac{4G_I K_I (1 - v_R^*)}{4G_I (1 - \zeta v_R^*) + 3K_I v_R^* (1 - \zeta)} \quad (11)$$

$$G_M = G_1 + \frac{v_R^*}{(1-v_R^*)} \frac{15(1-v_1^e)}{7-5v_1^e}$$

$$v_M^e = \frac{3K_M - 2G_M}{2(G_M + 3K_M)}$$

The changes in the elastic properties of the polymer with void volume fraction f during the cavitation of rubber particles are then given by (see appendix B)

$$K = \frac{4G_M K_M (1-f)}{4G_M + 3K_M f}$$

$$G = G_M \left[\frac{f}{(1-f)} \left\{ \frac{15(1-v_M^e)}{7-5v_M^e} \right\} \right]^{-1}$$

These expressions then enable new values for E and v^e to be derived as follows for use in FE calculations

$$v^e = \frac{3K - 2G}{2(G + 3K)}$$

$$E = 2G (1 + v^e)$$

4.2.3 Determination of elastic material parameters

For the calculation of elastic properties K , G and hence E and v^e in cavitated material (equations (14) to (17)), values are needed for the input properties K_1 , G_1 , v_1^e and K_R . These were calculated from measured values for the elastic properties E_0 and v_0^e for the uncavitated rubber-toughened ABS and relationships between component and composite properties for particle filled plastics (see appendix C). Derived values are shown in table 1.

Table 1 Values for the material property parameters used in calculations of the dependence of elastic properties on the volume fraction of cavities

K_1 (MPa)	G_1 (MPa)	v_1^e	K_R (MPa)
5300	1770	0.35	1850

Calculations of changes in elastic properties as cavities form in a uniaxial tension test are shown in figure 7 using equations (14) to (17) with the parameters in table 1. They demonstrate that the main changes in elastic properties occur in the bulk modulus K and v^e whilst changes to E and G are relatively small.

A NUCLEATION CRITERION BASED ON THE ELASTIC VOLUMETRIC STRAIN ON THE RUBBER PARTICLES, ϵ_{VR}^e

An expression for the elastic volumetric strain on the rubber is derived in Appendix D and is given by

$$\epsilon_{VR}^e = \frac{\sigma_k \zeta}{(1-f) K_R} \quad (18)$$

It is now assumed that cavities form in rubber particles at a critical value for ϵ_{VR}^e that depends upon the size of the rubber particle. Owing to the distribution of particle sizes in ABS, a cavity nucleation criterion is needed that relates the volume fraction of cavities to the range of critical values for ϵ_{VR}^e responsible for cavitation. A suitable form for this function was identified through comparisons of predictions of tensile stress/strain curves with experimental data. The function chosen is

$$\begin{aligned} f_{\text{nucI}} &= 0 \text{ for } \epsilon_{VR}^e < \epsilon_{1VR}^e \\ f_{\text{nucI}} &= v_{Ro} \left(\frac{\epsilon_{VR}^e - \epsilon_{1VR}^e}{\epsilon_{2VR}^e - \epsilon_{1VR}^e} \right)^{\beta_{VR}} \\ \text{and } f_{\text{nucI}} &= v_{Ro} \text{ for } \epsilon_{VR}^e > \epsilon_{2VR}^e \end{aligned} \quad (19)$$

where f_{nucl} is the effective volume fraction of voids caused by the nucleation of cavities in rubber particles. The total effective volume fraction of voids f will generally be larger than this as a result of cavity growth.

A plot of f_{nucl} against ϵ_{VR}^e is shown in figure 8 obtained with values for the void nucleation parameters recorded in the figure. These values were chosen to give the best fit of the predicted tensile stress/strain curve to experimental data. The comparison of predicted and experimental curves is shown in figure 9 and demonstrates that the range of ϵ_{VR}^e ($\epsilon_{1\text{VR}}^e$ to $\epsilon_{2\text{VR}}^e$ obtained using equation (18) is too narrow to satisfactorily explain cavity nucleation under uniaxial tension

A comparison of predicted and measured stress/strain curves for the butt-tension test is shown in figure 10 and confirms that equation (18) underpredicts the strain range over which cavity nucleation is observed. This conclusion is emphasised in this test because of the sensitivity of results to the bulk modulus which varies significantly with cavity volume fraction in this version of the model.

4.4 A NUCLEATION CRITERION BASED ON THE TOTAL VOLUMETRIC STRAIN ON THE RUBBER ϵ_{VR}

Cavity nucleation is now considered to be controlled by the total volumetric strain on the rubber, $\epsilon_{\text{VR}} = \epsilon_{\text{VR}}^e + \epsilon_{\text{VR}}^p$. This extends the strain range over which cavity formation is able to occur compared with a criterion in terms of ϵ_{VR}^e alone. An expression for ϵ_{VR} is derived in appendix E. The background theory to this derivation leads to an alternative expression for the dependence of flow parameter μ' on normal stress and cavity volume fraction and takes the form

$$\mu' = \mu'_1 (1 - v_R^*) + \Delta\mu'_R v_R^* \sinh \left[\frac{3\sigma_k (1 - k v_R^*)}{2\sigma_M (1 - f)} \right] \quad (20)$$

It follows that

$$d\varepsilon_{VR}^P = \frac{d\lambda}{(1-f) \sigma_M} 2\Delta\mu'_R \sinh \left[\frac{3\sigma_k (1-k v_R^*)}{2\sigma_M (1-f)} \right] \quad (21)$$

Because of the dependence of ε_{VR}^P on the hydrostatic stress σ_k , equation (21) ensures that a nucleation criterion based on ε_{VR} will not create cavities under shear or compression.

A suitable form for this criterion that gives good predictions of tensile behaviour is the stretched-exponential used in previous work [1]. The criterion is thus

$$f_{\text{nucl}} = 0 \text{ for } \varepsilon_{VR} < \varepsilon_{1VR}$$

$$f_{\text{nucl}} = v_{Ro} \left\{ -\exp \left[-\frac{\varepsilon_{VR} - \varepsilon_{1VR}}{\varepsilon_{2VR}} \right]^{\beta_{VR}} \right\} \text{ for } \varepsilon_{VR} \geq \varepsilon_{1VR} \quad (22)$$

Values for μ'_1 and $\Delta\mu'_R$ in the expression for the flow parameter, equation (20), were found by obtaining the best agreement between predicted and measured Poisson's ratio vs strain results under tension and compression shown in figure (see Appendix E). The values obtained are given in table 2. Values for the parameters in the nucleation criterion, equation (22), are also given in table 2 and were derived by obtaining the best fit of predicted uniaxial tensile stress/strain results shown in figure 12 to experimental data.

Table 2 Values for the parameters in the nucleation criterion equation (22) and in the expression for the flow parameter equation (20)

μ'_1	$\Delta\mu'_R$	ε_{1VR}	ε_{2VR}	β_{VR}
0.16	0.39	0.005	0.0015	0.8

Whilst the fit to uniaxial tension stress/strain data in figure 12 using this version of the model is very good, the predicted stress/strain curve under butt-tension, shown in figure 13, again shows too large a reduction in stress in the early stages of cavity nucleation.

The sharp discontinuity in the predicted curve at the onset of cavity nucleation is attributable to the reduction in the bulk modulus which has been introduced in the versions of the model considered in this report (see section 4.2). The additional increase in volumetric strain associated with this reduction in K with cavitation is also responsible for the predicted sharp decrease in Poisson's ratio with strain seen in figure 11. This is more rapid than is seen experimentally and is further evidence that the predicted changes to elastic properties resulting from cavity nucleation, are not consistent with experimental data. The predicted reductions in elastic properties result from a reduction in the bulk modulus of the rubber particles following cavitation. This is responsible for the reduction in the applied stress required for plastic deformation of the matrix material between cavitated rubber particles. It may be that the reduction in the bulk modulus of the rubber particle takes place gradually following the nucleation of a very small cavity at the onset of cavitation. With further strain, the cavity may grow in the rubber thereby reducing the bulk modulus progressively.

5 A PREFERRED VERSION OF THE MODEL

It is apparent that attempts described in this report to explore further developments of the model to overcome perceived shortcomings in earlier versions have been only partially successful. The new expression for the flow parameter μ' , given by equation (20), is able to account for the small changes in volumetric strain observed under uniaxial compression as well as the large changes under uniaxial tension that are needed to explain the fall in Poisson's ratio under tension with strain. Changes to the model that attempt to account for decreases in elastic properties with cavity nucleation require additional calculations and input data and lead to less accurate predictions of butt-tension and Poisson's ratio results than were obtained with earlier versions of the model. These changes therefore seem unnecessary.

A cavity nucleation criterion based on the total volumetric strain on the rubber particles ϵ_{VR} given by equations (22), (21) and (18) is physically attractive since it dictates that cavities will nucleate under tension but not under shear or compression. However, determination of the elastic component of ϵ_{VR} requires additional data for the properties of the rubber and the matrix polymer given in equation (18). A cavity nucleation criterion, developed in earlier work [1], expressed in terms of the total volumetric strain ϵ_V in the material gives predictions

of results under uniaxial and butt tension that agree rather better with experiment and avoids the need for the additional data.

The preferred version of the model that will be used for further FE analyses of the deformation of the centrally loaded plate will therefore employ the flow potential for non-associated flow given by equation (2) with the flow parameter μ' given by equation (20). The cavity nucleation function from reference [1] is

$$f_{\text{nucl}} = 0 \text{ for } \epsilon_v < \epsilon_{1V} \text{ and } \sigma_k < 5 \text{ MPa}$$

$$f_{\text{nucl}} = v_{\text{Ro}} \left\{ 1 - \exp \left(- \left(\frac{\epsilon_v - \epsilon_{1V}}{\epsilon_{2V}} \right)^\beta \right) \right\} \text{ for } \epsilon_v \geq \epsilon_{1V}$$

The condition on the hydrostatic stress in equation (23a) is included, as explained in the Introduction, to avoid cavitation in shear or compression that will occur if the nucleation criterion is expressed in terms of the total volumetric strain. Although use of the revised flow parameter with non-associated flow in place of associated flow will shift cavity nucleation with this criterion to higher strains under these stress states, the stress condition is needed to avoid cavitation at any strain level.

Values for the parameters in equation (23b) that give the best agreement with experimental uniaxial tension data are given in table 3.

Table 3 Values for the parameters in the nucleation criterion equation (23) and the expression for the flow parameter equation (20)

μ'_1	$\Delta\mu'_R$	ϵ_{1V}	ϵ_{2V}	β_v
0.16	0.39	0.0036	0.002	0.62

Figures 14 to 17 show comparisons of experimental data measured under uniaxial tension, uniaxial compression and butt tension with predictions using the preferred version of the model and the parameters in table 3. In order to illustrate the influence that rubber particle

cavitation has on predicted behaviour, stress-strain curves in uniaxial and butt tension are also shown for the case of no cavitation $f = 0$.

6 CONCLUSIONS

For the case of associated flow with a measured value for the flow parameter $\mu' = \mu = 0.22$ for ABS, relatively large volume expansions of the polymer are predicted during plastic deformation. These are predicted under uniaxial compression and shear as well as uniaxial tension. With a cavity nucleation criterion based on the total volumetric strain in the polymer, this leads to the prediction of cavitation under compression and shear.

Measurements of transverse strain under compression have revealed that the volume changes are much less than predicted assuming associated flow. The cavitation model has therefore been modified to account for non-associated flow characterised by a flow parameter μ' that is less than μ . In order to model volume changes measured in compression and tension, it is necessary for μ' to vary with stress state. An appropriate expression for μ' has been derived in this report.

Under non-associated flow, cavitation is still predicted under shear and compression although at somewhat larger strains. Since there is no experimental evidence for this, the influence of alternative relationships for the cavity nucleation criterion has been explored. This work has concentrated on relating the cavity nucleation function to the volumetric strain on the rubber particles rather than the total volumetric strain. The expressions derived take account of a predicted reduction in elastic properties caused by cavity nucleation. In addition, they predict cavitation only under stress states with a positive hydrostatic component and therefore not under shear or compression.

Predictions of behaviour under uniaxial and butt tension using these revised versions of the model are, however, not as close to experimental data as earlier versions of the model. The cause for the poorer agreement has been traced to the revised equations for elastic behaviour which predict a decrease in bulk properties with cavitation. Whilst these decreases are

probably real, it is concluded that they must occur more gradually with increasing strain than predicted.

Because of the added complexity of calculations of volumetric strain on the rubber and the requirement for additional material property data for these calculations, the preferred version of the model for future stress analyses should use the cavity nucleation function based on the total volumetric strain. With this criterion, it will however be necessary to implement the condition that cavities will only nucleate when the hydrostatic stress component exceeds a small positive value to avoid cavitation under shear and compression. Equations for non-associated flow should also be included in the future version of the model.

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APPENDIX A DERIVATION OF THE MATRIX ELASTIC PROPERTIES

In calculating the variations in K_M , G_M and v_M^e during the cavitation, each element of the matrix was represented by a sphere of the rubber R surrounded by a spherical shell of the rigid plastic component 1. With this model (Fig 6b) the varying volume fraction of rubber in the matrix v_R^* is given by

$$v_R^* = \frac{v_{Ro} - f_{nucl}}{1 - f_{nucl}}$$

Values of K_M were then calculated using

$$K_M = \frac{4G_1 K_1 (1 - v_R^*)}{4G_1 (1 - \zeta v_R^*) + 3K_1 v_R^* (1 - \zeta)}$$

where

$$\zeta = \frac{\sigma_{kR}}{\sigma_{kM}} = \frac{K_R (4G_1 + 3K_1)}{4G_1 (K_R - K_1) v_R^* + K_1 (4G_1 + 3K_R)}$$

Equations (A2) and (A3) follow from equations (16) and (15), respectively, of reference 7. In (A3), σ_{kR} and σ_{kM} are the normal stresses acting on the rubber particle and the matrix, respectively, as illustrated in Figure 6b.

The matrix shear modulus G_M was evaluated using an equation developed by Kerner [9]. Since the shear modulus of the rubber is negligible compared with G_1 , Kerner's equation gives [10]

$$G_M = G_1 \left[1 + \frac{v_R^*}{(1 - v_R^*)} \left\{ \frac{15(1 - v_1^e)}{7 - 5v_1^e} \right\} \right]^{-1}$$

The elastic Poisson's ratio of the matrix was then obtained using

$$v_M^e = \frac{3K_M - 2G_M}{2(G_M + 3K_M)}$$

APPENDIX B DERIVATION OF K, G AND ν^e

Figure 6c illustrates the model employed for calculating variations in the material elastic properties during the void nucleation and growth. It comprises a spherical void V, of varying volume fraction f , surrounded by a spherical shell of matrix material M. Since the normal stress σ_{kv} at the void-matrix interface is zero it follows that $\zeta = \sigma_{kv}/\sigma_k = 0$ and from equation (16) of reference [7] we obtain

$$K = \frac{4G_M K_M (1-f)}{4G_M + 3K_M f}$$

Noting that the shear modulus of the void is zero, Kerner's equation also gives [10]

$$G = G_M + \frac{f}{1-f} \left\{ \frac{15(1-\nu_M^e)}{7-5\nu_M^e} \right\}^{-1}$$

From the calculated values of K and G, variations in the elastic Poisson's ratio ν^e and the Young's modulus E may also be derived for the material using

$$\nu^e = \frac{3K-2G}{2(G+3K)}$$

and

$$E = 2G (1 + \nu^e)$$

APPENDIX C DERIVATION OF G_1 , v_1^e , K_1 and K_R

To obtain values for the parameters G_1 , v_1^e , K_1 and K_R , G_o and K_o are first determined for the unvoided rubber-toughened material from the measured E_o and v_o^e ;

$$G_o = \frac{E_o}{2(1+v_o^e)}$$

$$K_o = \frac{E_o}{3(1-2v_o^e)}$$

The elastic Poisson's ratio v_1^e is next measured or estimated for the untoughened plastic and G_1 and K_1 calculated with the aid of Kerner's equation.

$$G_1 = G_o \left[\frac{v_{Ro}}{1-v_{Ro}} \left\{ \frac{15(1-v_1^e)}{7-5v_1^e} \right\} \right]$$

$$K_1 = \frac{2G_1(1+v_1^e)}{3(1-2v_1^e)}$$

From the derived values of K_o , G_1 and K_1 , K_R is then calculated using the model of Figure 6b applied to the unvoided material. This gives

$$K_R = \frac{4G_1 K_1 \zeta_o (1-v_{Ro})}{4G_1 (1-\zeta_o v_{Ro}) + 3K_1 (1-\zeta_o)}$$

where

$$\zeta_o = \frac{K_o (4G_1 + 3K_1 v_{Ro}) - 4G_1 K_1 (1-v_{Ro})}{K_o v_{Ro} (4G_1 + 3K_1)}$$

Note that equations (C5) and (C6) were obtained from (A3) and (A2), respectively, after making the replacements $\zeta \rightarrow \zeta_o$, $v_R^* \rightarrow v_{Ro}$ and $K_M \rightarrow K_o$.

APPENDIX D DERIVATION OF A RELATIONSHIP FOR ϵ_{VR}^e

The elastic volumetric strain on the rubber particles is given by

$$\epsilon_{VR}^e = \frac{\sigma_{kR}}{K_R} = \frac{\sigma_{kM} \zeta}{K_R}$$

To express σ_{kM} in terms of the applied macroscopic normal stress σ_k , we first consider the relationship between the total elastic volumetric strain ϵ_V and the elastic volumetric strain on the matrix (ϵ_{VM}) and voids (ϵ_{VV}) respectively. A given elastic volume increment dV^e can be expressed as the sum of contributions from the matrix dV_M^e and voids dV_V^e

$$dV^e = dV_M^e + dV_V^e$$

Thus

$$\begin{aligned} \epsilon_V^e &= \frac{dV^e}{V} = \frac{dV_M^e}{V} + \frac{dV_V^e}{V} \\ &= \frac{V_M}{V} \frac{dV_M^e}{V_M} + \frac{V_V}{V} \frac{dV_V^e}{V_V} \\ &= (1-f) \epsilon_{VM}^e + f \epsilon_{VV}^e \end{aligned}$$

and

$$\epsilon_{VM}^e = \frac{\epsilon_V^e - f \epsilon_{VV}^e}{(1-f)}$$

From equations (12), (13) of reference 7 with $P_R = \sigma_{kR} = 0$ and $\phi = f$ we next obtain

$$\epsilon_V^e = \frac{\sigma_k}{(1-f)} \left[\frac{4G_M + 3K_M f}{4G_M K_M} \right]$$

$$\epsilon_{VV}^e = \frac{\sigma_k}{f} \left[\frac{4G_M + 3K_M}{4G_M K_M} \right]$$

Equations (D2), (D3) and (D4) give

$$\varepsilon_{VM}^e = \frac{\sigma_{kM}}{K_M} = \frac{\sigma_k}{(1-f) K_M}$$

from which it follows that

$$\sigma_{kM} = \frac{\sigma_k}{(1-f)}$$

Equations (D1) and (D6) then yield

$$\varepsilon_{VR}^e = \frac{\sigma_k \zeta}{(1-f) K_R}$$

APPENDIX E DERIVATION OF RELATIONSHIPS FOR μ' and ε_{VR}^P

In calculating the possible volumetric strain on the rubber particles produced by plastic deformation of the matrix, note first that an incremental volume increase in the matrix dV_M^P can be written as the sum of contributions from the rubber component (dV_R^P) and the rigid plastic component (dV_I^P), i.e.

$$dV_M^P = dV_R^P + dV_I^P$$

Then

$$\begin{aligned} d\varepsilon_{VM}^P &= \frac{dV_M^P}{V_M} = \frac{dV_R^P}{V_M} + \frac{dV_I^P}{V_M} \\ &= \frac{V_R}{V_M} \frac{dV_R^P}{V_R} + \frac{V_I}{V_M} \frac{dV_I^P}{V_I} \\ &= v_R^* d\varepsilon_{VR}^P + (1-v_R^*) d\varepsilon_{VI}^P \end{aligned} \quad (E1)$$

where $d\varepsilon_{VR}^P$ and $d\varepsilon_{VI}^P$ are the increments of volumetric strain on the rubber and plastic components respectively. The volume increment dV_M^P can also be related to the total volume increment dV^P and the volume increment on the voids dV_V^P

$$dV^P = dV_V^P + dV_M^P$$

Thus

$$\begin{aligned} d\varepsilon_V^P &= \frac{dV^P}{V} = \frac{dV_V^P}{V} + \frac{dV_M^P}{V} \\ &= \frac{V_V}{V} \frac{dV_V^P}{V_V} + \frac{V_M}{V} \frac{dV_M^P}{V_M} \\ &= f d\varepsilon_{VV}^P + (1-f) d\varepsilon_{VM}^P \end{aligned} \quad (E2)$$

where $d\varepsilon_{VV}^P$ is the plastic volumetric strain increment on the voids.

A relation for the total volumetric strain increment $d\varepsilon_V^P$ due to plastic deformation can also be obtained from the flow rule equation (3). Using equation (2) and

$$d\varepsilon_V^P = d\varepsilon_{11}^P + d\varepsilon_{22}^P + d\varepsilon_{33}^P$$

we obtain

$$d\varepsilon_V^P = \frac{d\lambda}{\sigma_M} \left[3q_1 f \sinh\left(\frac{3\sigma_k}{2\sigma_M}\right) + 2\mu' \left(1 - \frac{\mu' \sigma_k}{\sigma_M}\right) \right]$$

A comparison of (E2) with (E3) then gives

$$d\varepsilon_{VM}^P = \frac{d\lambda}{(1-f)} \frac{2\mu'}{\sigma_M} \left(1 - \frac{\mu' \sigma_k}{\sigma_M}\right)$$

Since $\mu' \sigma_k / \sigma_M$ is small compared with unity, equations (E1) and (E4) suggest that μ' can be expressed as a sum of contributions from the plastic (μ'_1) and rubber ($\Delta\mu'_R$) components

$$\mu' = \mu'_1 (1 - v_R^*) + \Delta\mu'_R v_R^* \sinh\left[\frac{3\sigma_k (1 - kv_R^*)}{2\sigma_M (1-f)}\right]$$

It follows that

$$d\varepsilon_{VR}^P = \frac{d\lambda}{(1-f)} \frac{2}{\sigma_M} \Delta\mu'_R \sinh\left[\frac{3\sigma_k (1 - kv_R^*)}{2\sigma_M (1-f)}\right]$$

The sinh function in (E5) and (E6) is introduced following Gurson's model so that the total volumetric strain on the rubber ε_{VR} is positive only for positive values of σ_k . This ensures that cavitation will occur only for dilatational stress states when the nucleation criterion is given by equation (22). Note that $\sigma_k / (1-f)$ is the applied normal stress component on the matrix and that $\sigma_M / (1 - kv_R^*)$ is the effective yield stress of the untoughened plastic component of the matrix.

A value of $\mu'_1 = 0.16$ was obtained for the ABS material as that μ' value required to fit the Poisson's ratio (v^P) data under uniaxial tension for fully cavitated material. This follows from equation (E5) with $v_R^* = 0$. The value of $\Delta\mu'_R$ was then evaluated from the μ'_C value of 0.06 determined as in Section 2.3 from the data under uniaxial compression. In this case $f = 0$, $v_R^* = v_{R0}$, $\sigma_M = \sigma_o$ and (E5) gives

$$\Delta\mu'_R = \frac{\mu'_C - \mu'_1(1 - v_{R0})}{v_{R0} \sinh \left[\frac{\sigma_C(1 - kv_{R0})}{2\sigma_o} \right]} \quad (E7)$$

where σ_C is of negative sign.

FIGURE CAPTIONS

Fig 1. Comparison of the measured (■) stress-strain curve in uniaxial tension with the curve (—) predicted [1-3] from shear hardening data using a stretched-exponential dependence of f_{nuc} on ϵ_V . Values of the parameters employed are included on the figure.

Comparison of the measured (■) dependence of Poisson's ratio on tensile strain with the predicted (—) dependence [1-3] assuming associated flow and a stretched-exponential dependence of f_{nuc} on ϵ_V . Values of parameters as in Figure 1

Comparison of the measured (O) and predicted (—) stress-strain curves under uniaxial compression. The predicted curves illustrate the effect of cavitation that would arise from matrix expansion during associated flow assuming a stretched-exponential dependence of f_{nuc} on ϵ_V .

Variation of the observed Poisson's ratio ν and derived plastic component of Poisson's ratio ν^P with strain during a uniaxial compression test.

Fig 5. Section detail of the extensometer for the butt-joint tensile test showing location grooves on the specimen.

Fig 6 (a) Illustration of the model employed in deriving the variations in elastic properties and volumetric strains during cavitation of a rubber-toughened plastic. R, l and V represent the rubber, the rigid plastic component and the voids (cavities) respectively. (b) model employed in deriving the elastic properties of the matrix M. (c) model used in deriving the material elastic properties.

Calculated variations in elastic properties with strain produced by cavitation during a uniaxial tensile test. Values of the parameters employed in the calculations are given in Table 1 and the void nucleation function is given by equation (19) with parameters listed in Figure 8.

Fig 8. Void nucleation function f_{nucl} plotted against ϵ_{VR}^e according to equation (19) with the given parameters.

Fig 9. Comparison of the measured (■) stress-strain curve in uniaxial tension with the curve (—) predicted using the void nucleation function and parameters in Figure 8. Values for μ' were derived using equation (10) with $\mu'_1 = 0.18$ and $\mu'_2 = 0.06$. Other parameters given in Table and Figure 1

Fig 10. Comparison of the measured (▲) stress-strain curve in a butt-tension test with the curve (—) predicted using the void nucleation function and parameters in Figure 8. Values for μ' were derived using equation (10) with $\mu'_1 = 0.18$ and $\mu'_2 = 0.06$. Other parameters given in Table and Figure 1.

Fig 1 Strain-dependence of the measured Poisson's ratio under uniaxial tension (■) and compression (O) respectively. Also shown are the theoretical curves (—) giving the values of μ'_1 and $\Delta\mu_R$ shown in Table 2. The predicted tensile curve is based on the void nucleation function given by equation (22) with parameters $\epsilon_{1\text{VR}}$, $\epsilon_{2\text{VR}}$ and β_{VR} in Table 2.

Fig 12. Comparison of the measured (■) stress-strain curve in uniaxial tension with the curve (—) predicted using the void nucleation function given by equation (22) with the parameters $\epsilon_{1\text{VR}}$, $\epsilon_{2\text{VR}}$ and β_{VR} in Table 2. Other parameters given in Tables and 2.

Fig 13. Comparison of the measured (▲) stress-strain curve in a butt-tension test with the curve (—) predicted using the void nucleation function given by equation (22) with the parameters $\epsilon_{1\text{VR}}$, $\epsilon_{2\text{VR}}$ and β_{VR} in Table 2. Other parameters in Tables 1 and 2.

Fig 14. Comparison of the measured (■) stress-strain curve in uniaxial tension with the predicted curve (—) using the void nucleation function given by equations (23a) and (23b) with parameters in Table 3. Also shown is the curve predicted in the absence of cavitation ($f = 0$).

Fig 15. Strain-dependence of the measured Poisson's ratio in uniaxial tension (■) and compression (O). The theoretical curves (—) were derived using the values of μ'_1 and $\Delta\mu'_R$ in Table 3. The predicted tensile curve is based on the nucleation function given by equation (23a) and (23b) with parameters in Table 3.

Fig 16. Comparison of the measured (O) stress-strain curve under uniaxial compression with the curve predicted using values of μ'_1 and $\Delta\mu'_R$ in Table 3. The effects of cavitation on the predicted curve were avoided through use of the condition $\sigma_k < 5$ MPa in equation (23a) for the void nucleation function.

Fig 17. Comparison of the measured (▲) stress-strain curve in a butt-tension test with a predicted curve (—) using the void nucleation function given by equations (23a) and (23b) with parameters in Table 3. Also shown is the curve predicted in the absence of cavitation ($f = 0$).

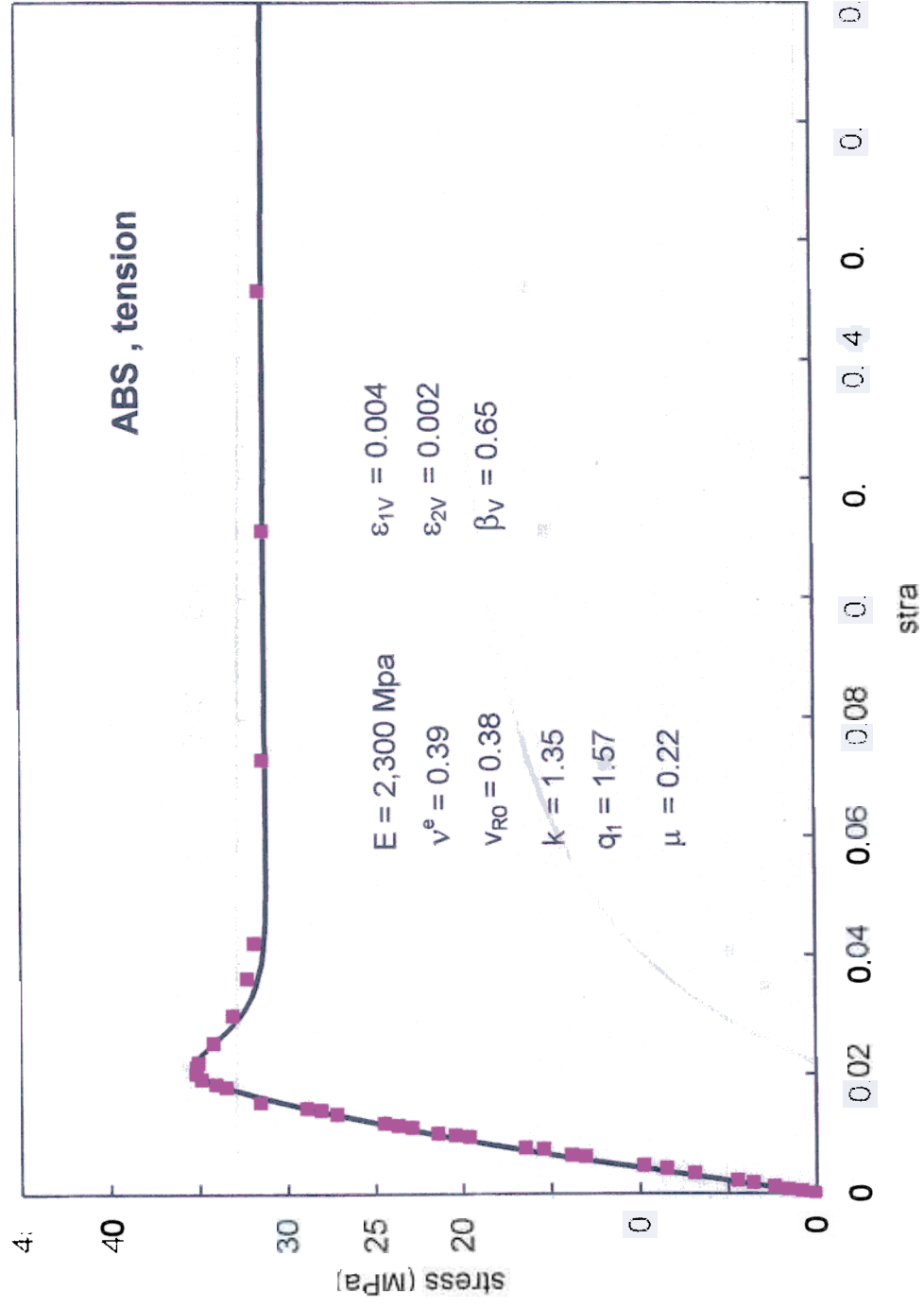


Fig.

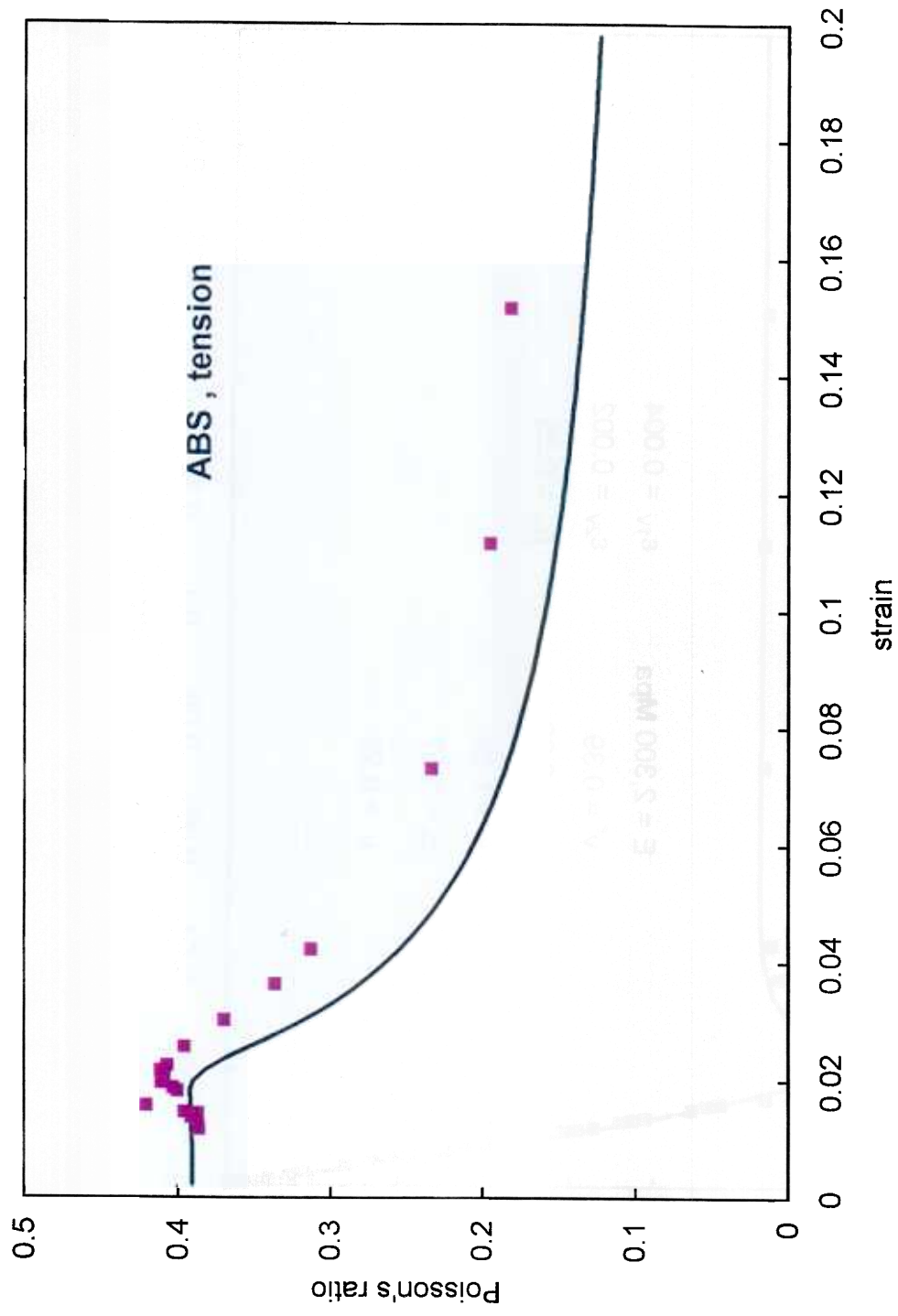


Fig. 2

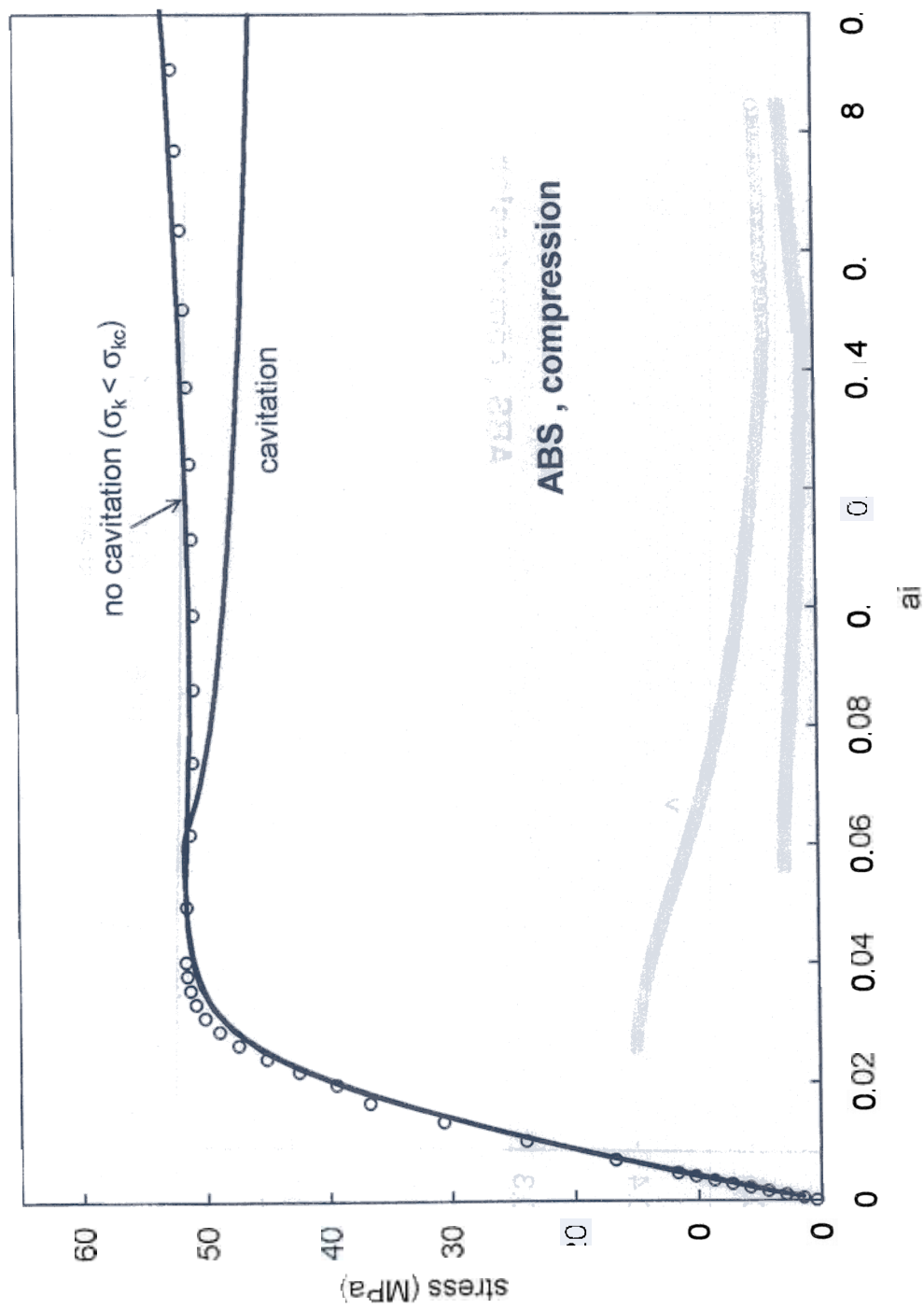


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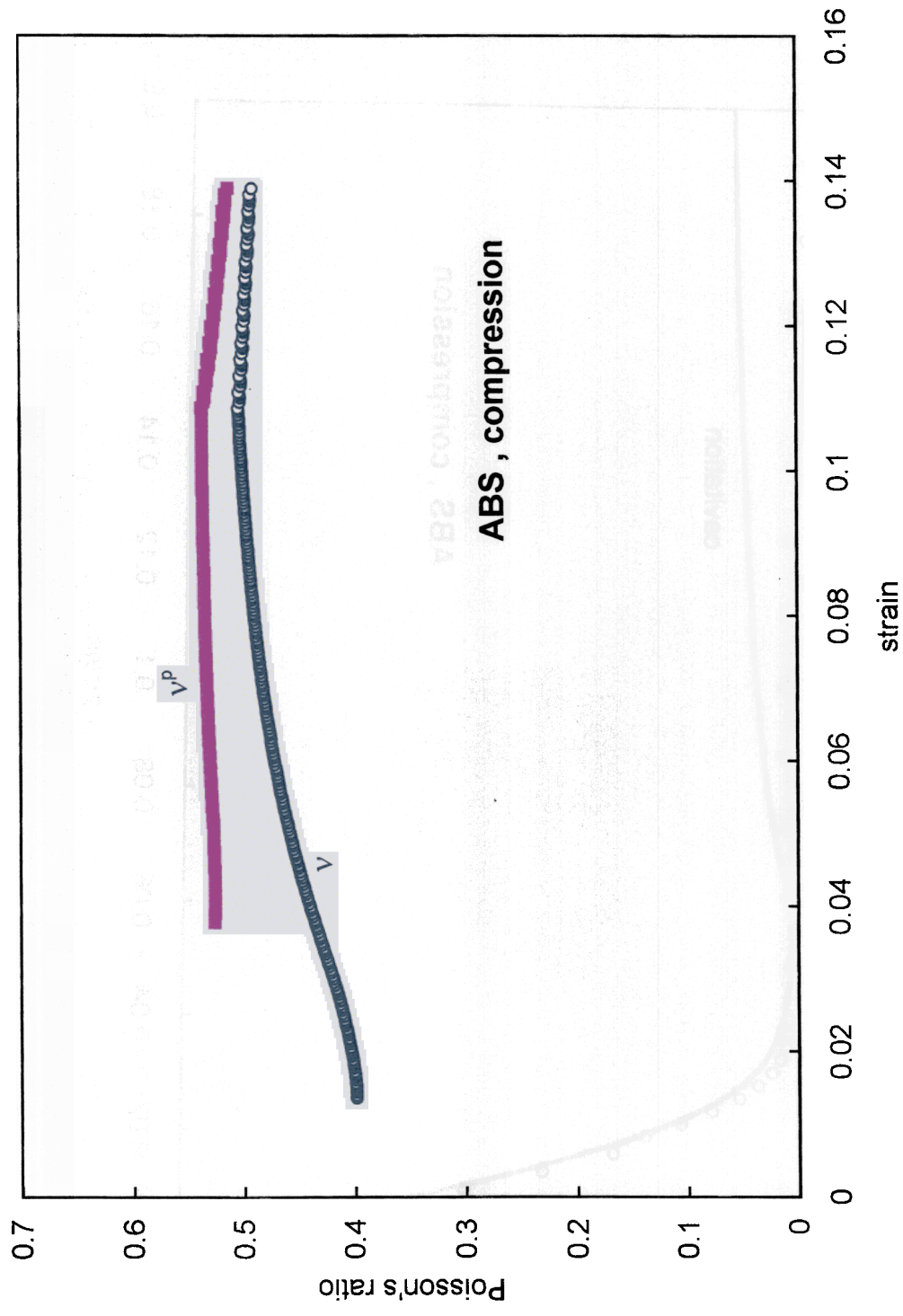


Fig. 4

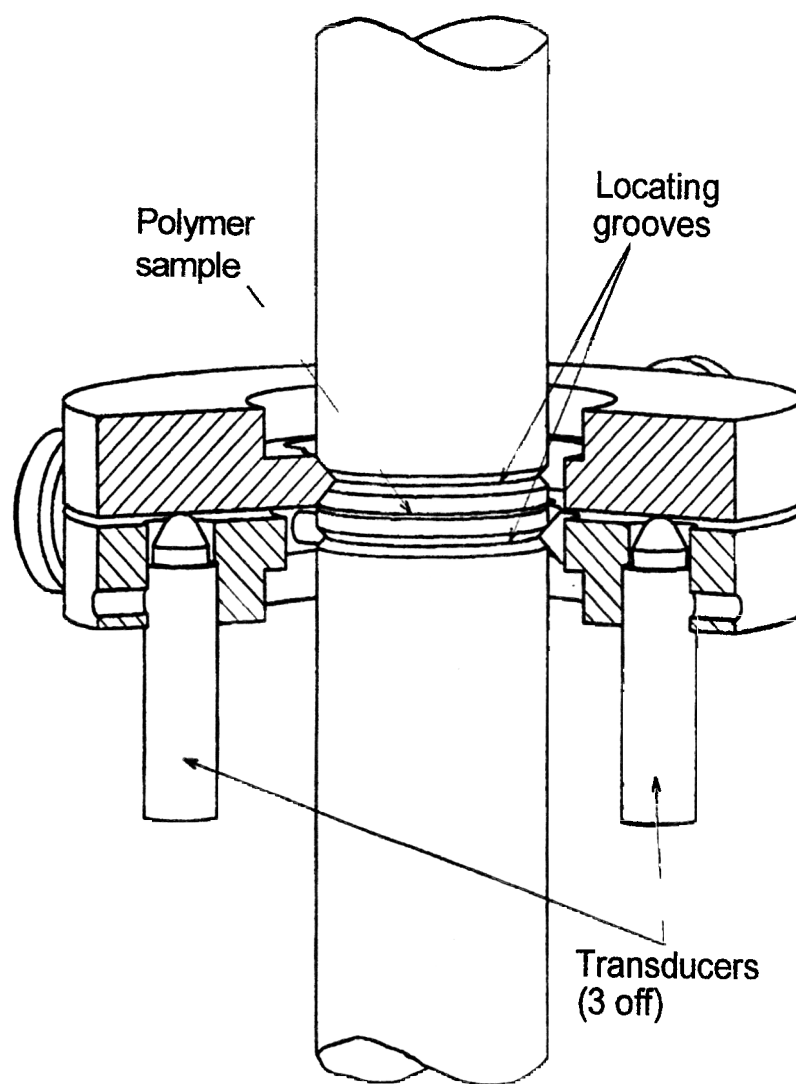
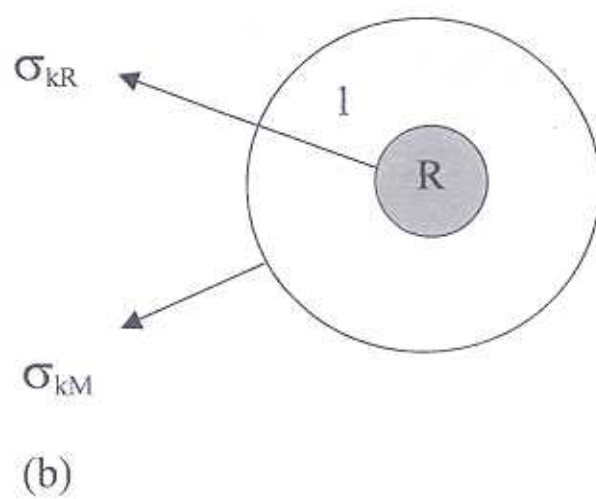
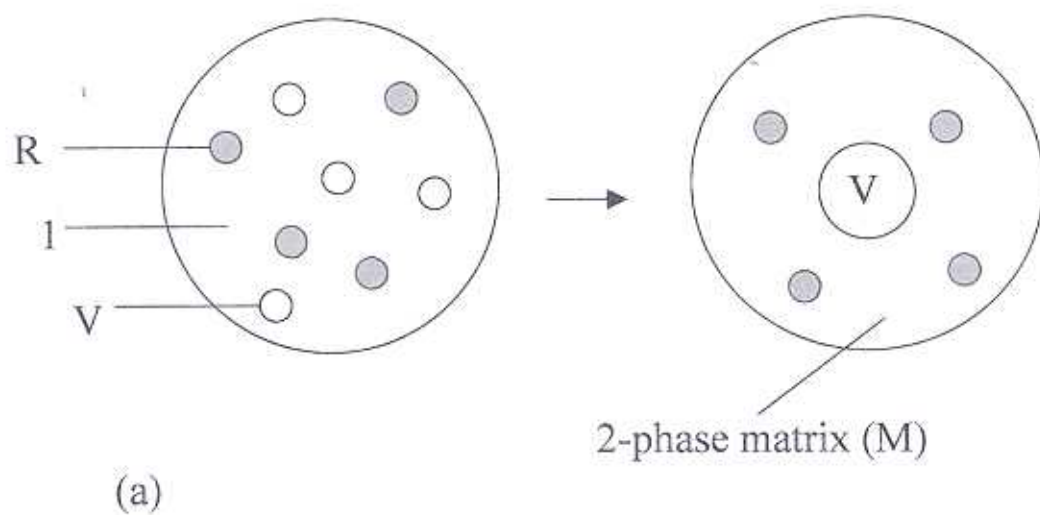


Fig.5



$$V_R^* = \frac{V_{Ro} - f_{nucl}}{1 - f_{nucl}}$$

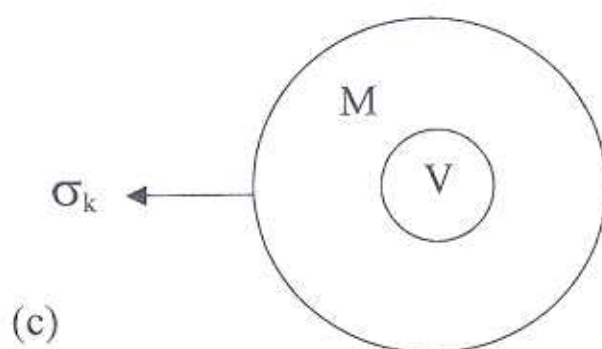


Fig.6

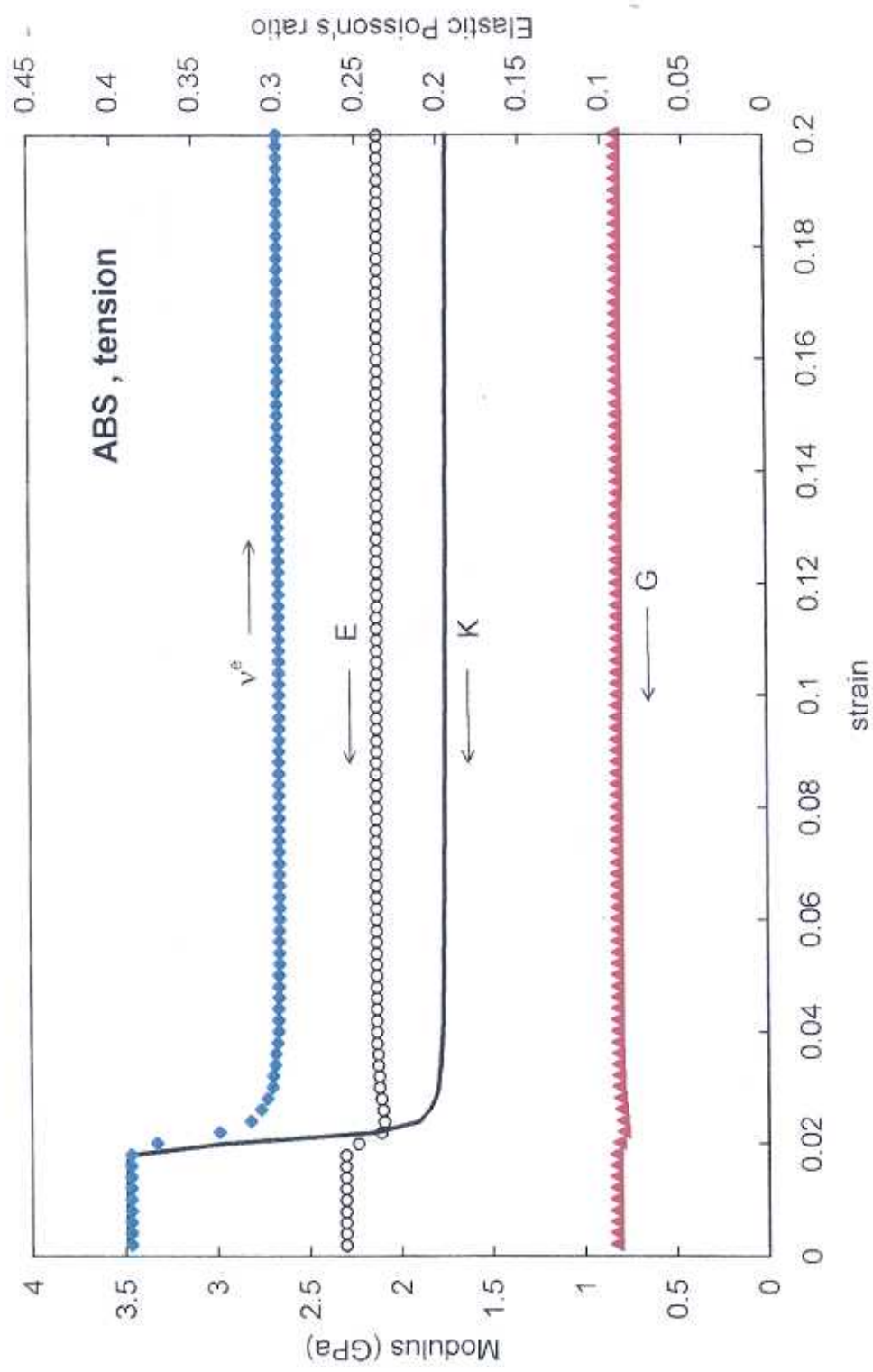


Fig. 7

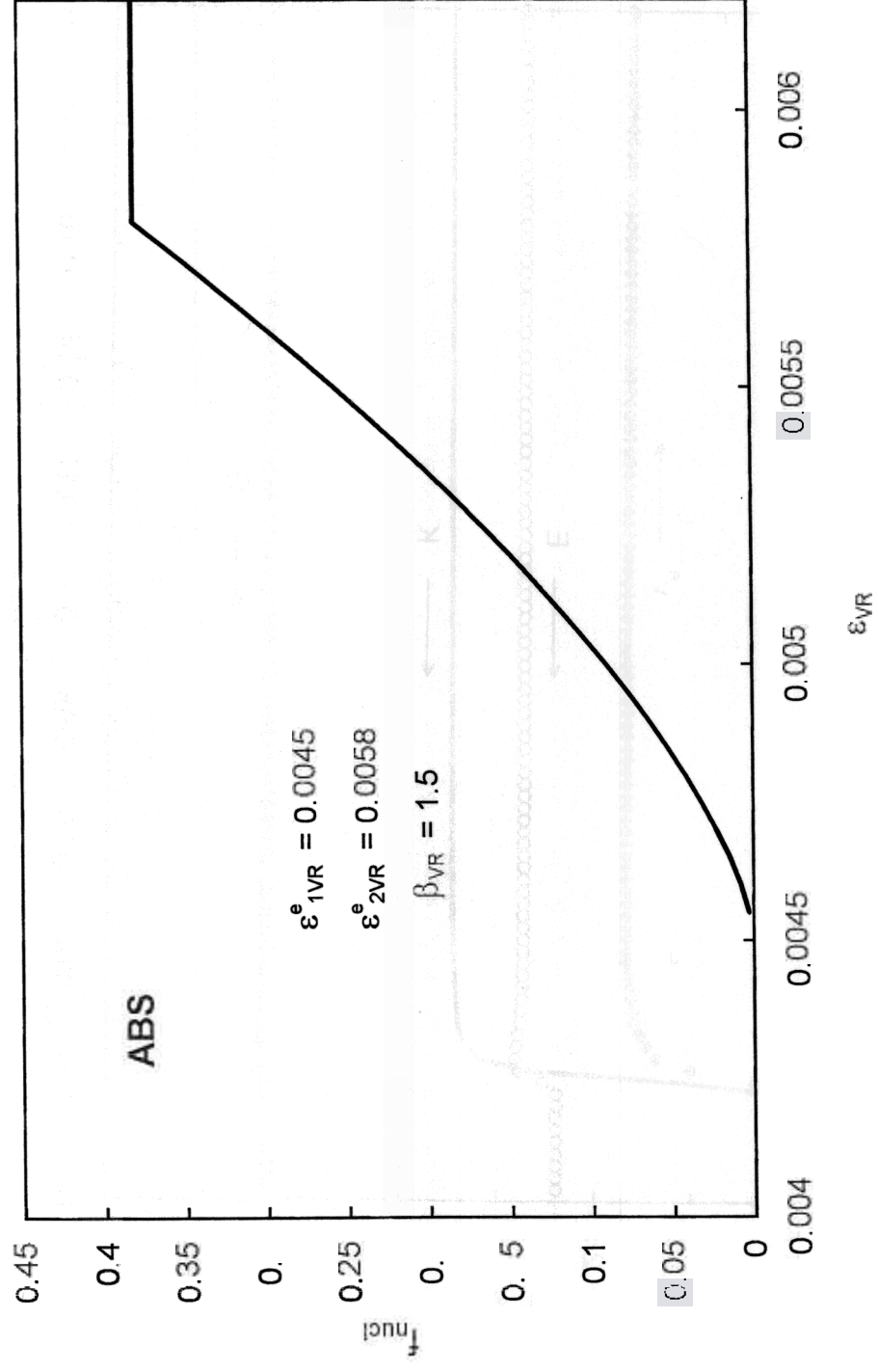
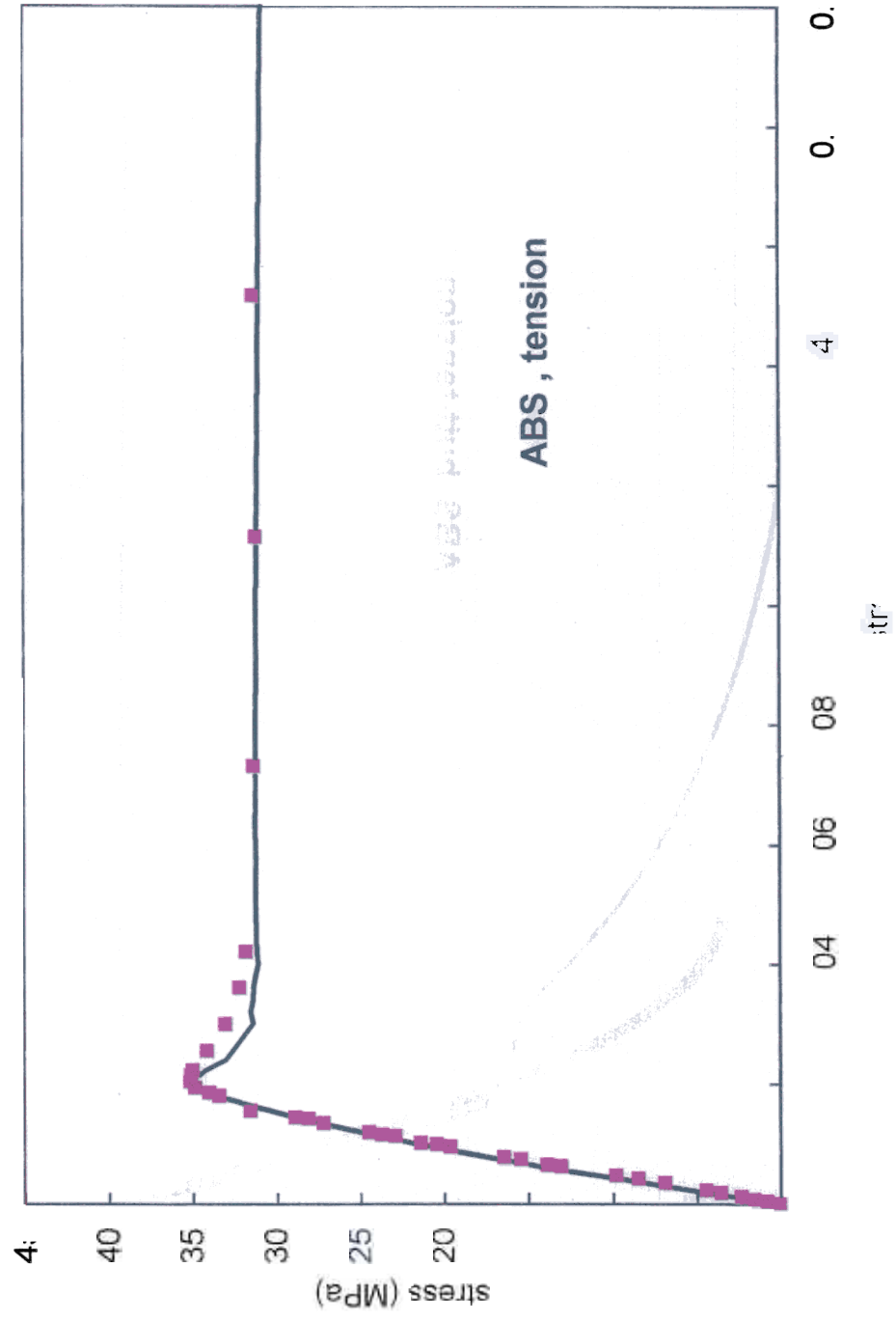


Fig 8



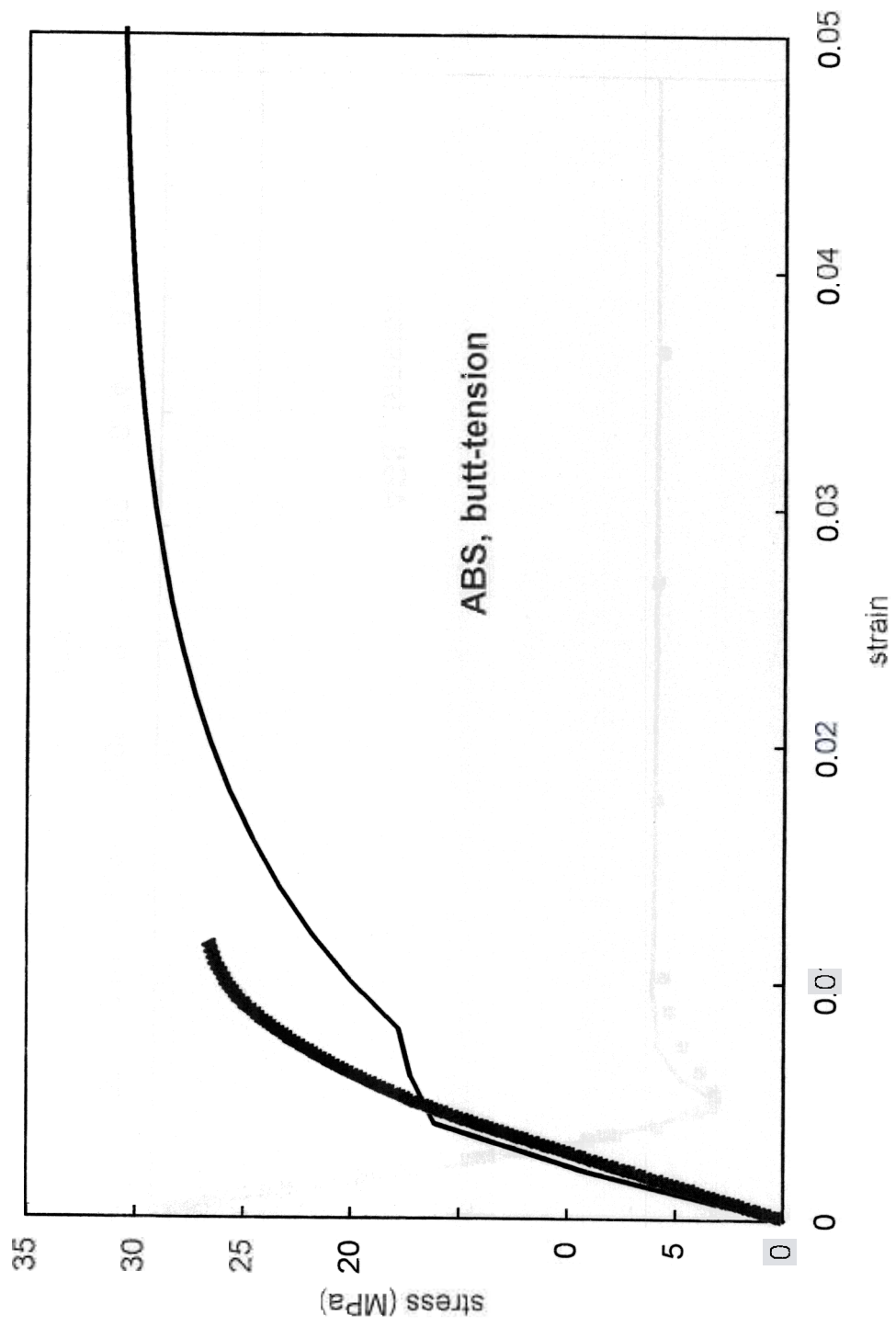


Fig 10

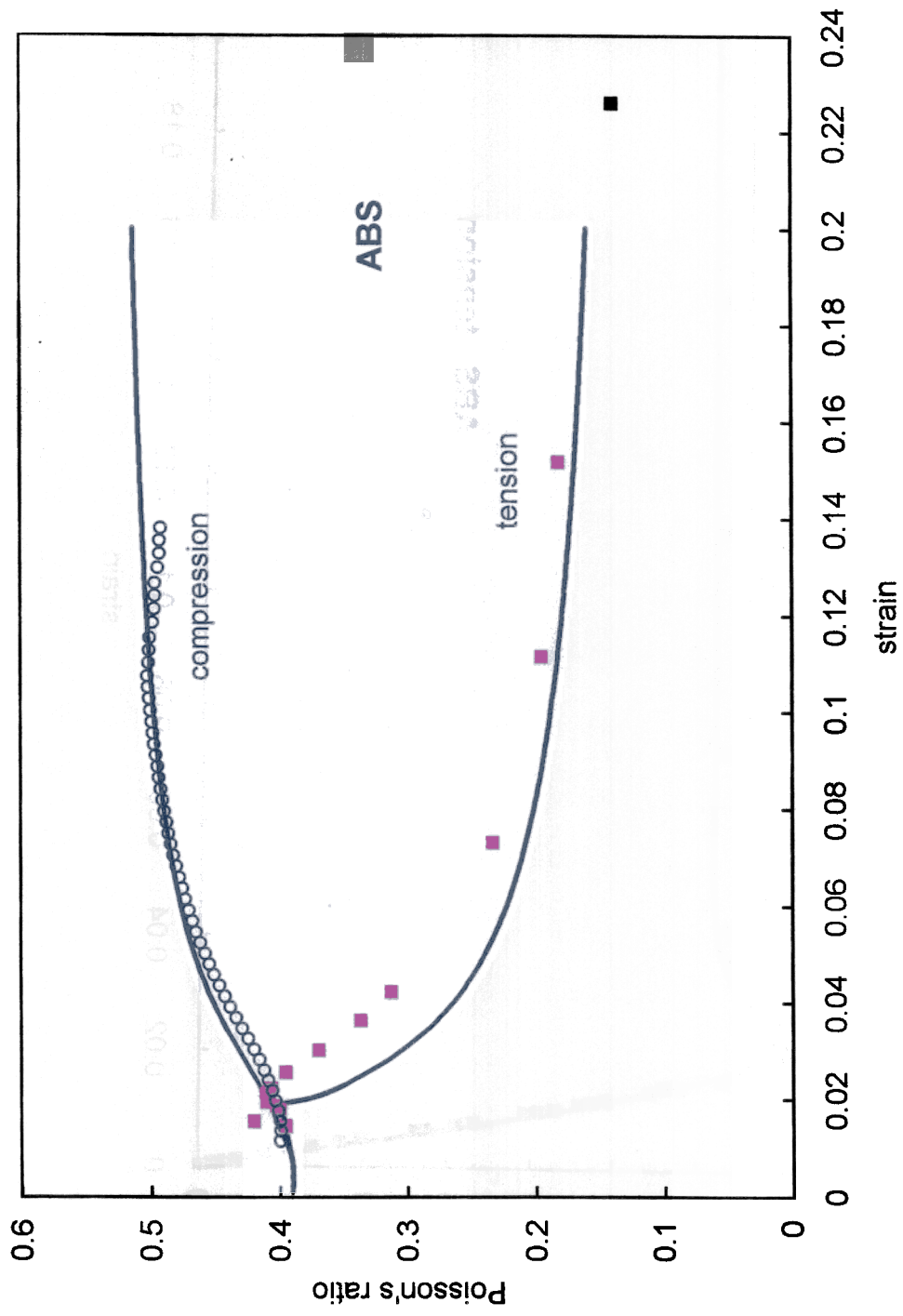
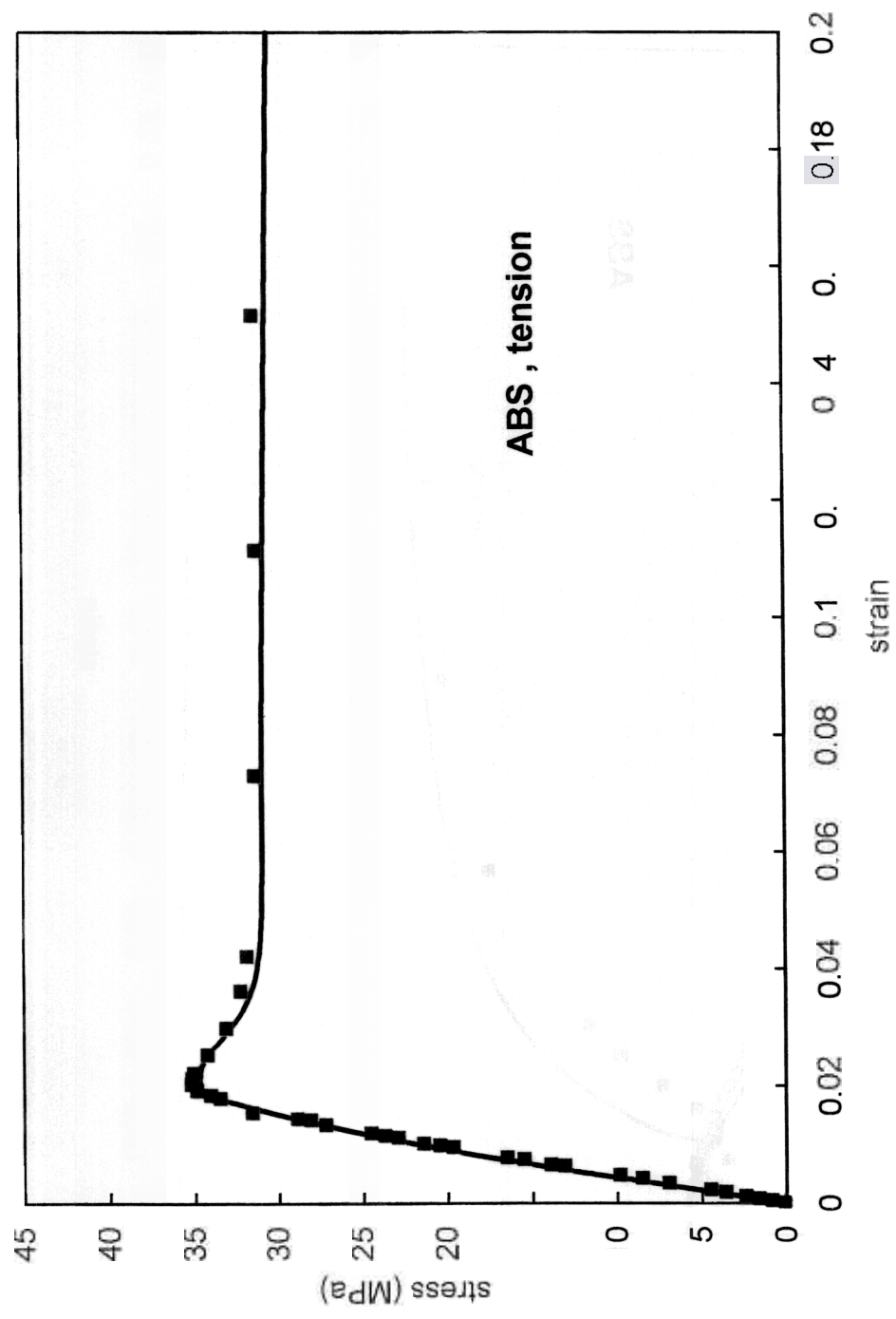


Fig. 11



Fig

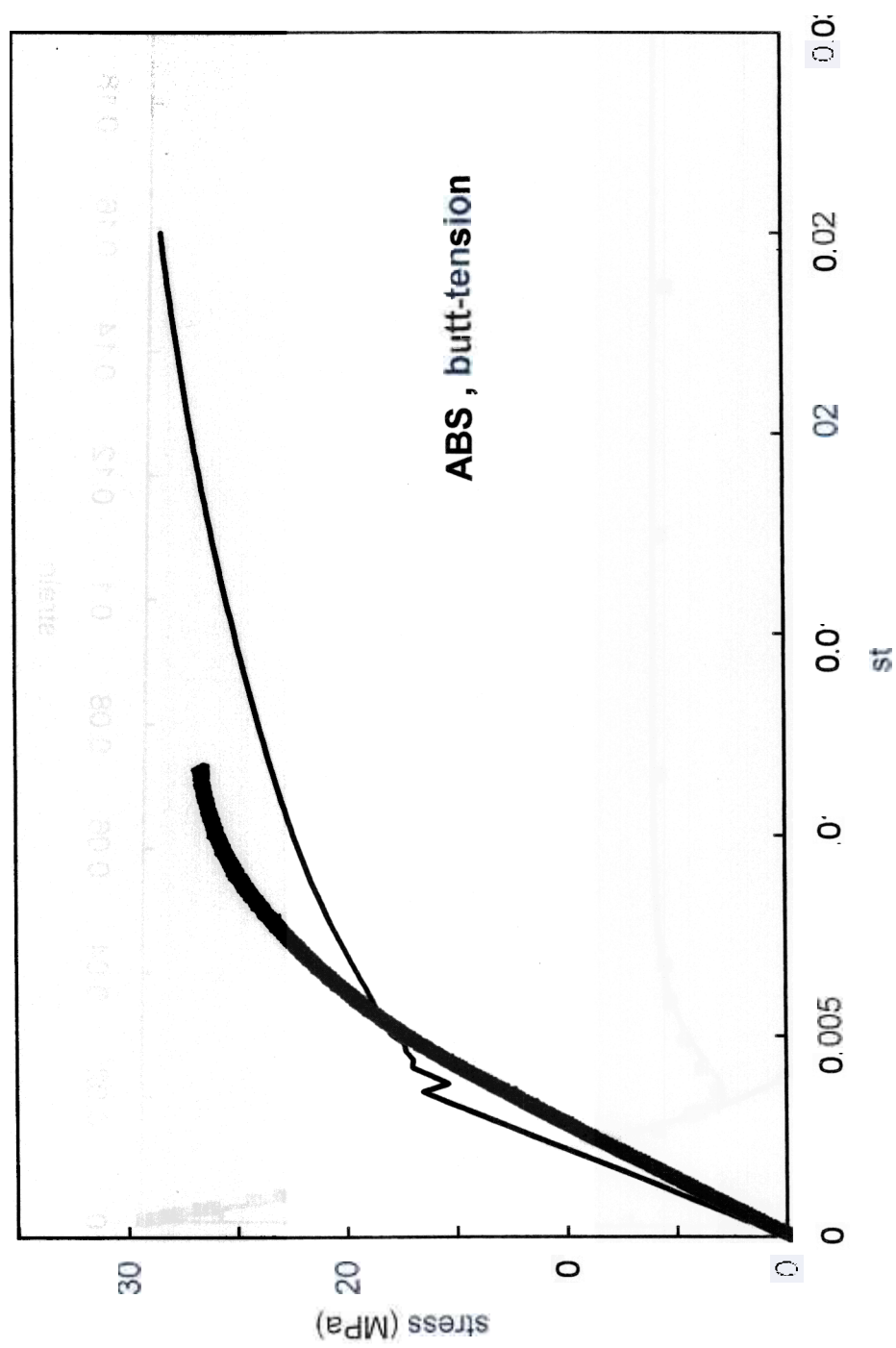


Fig.

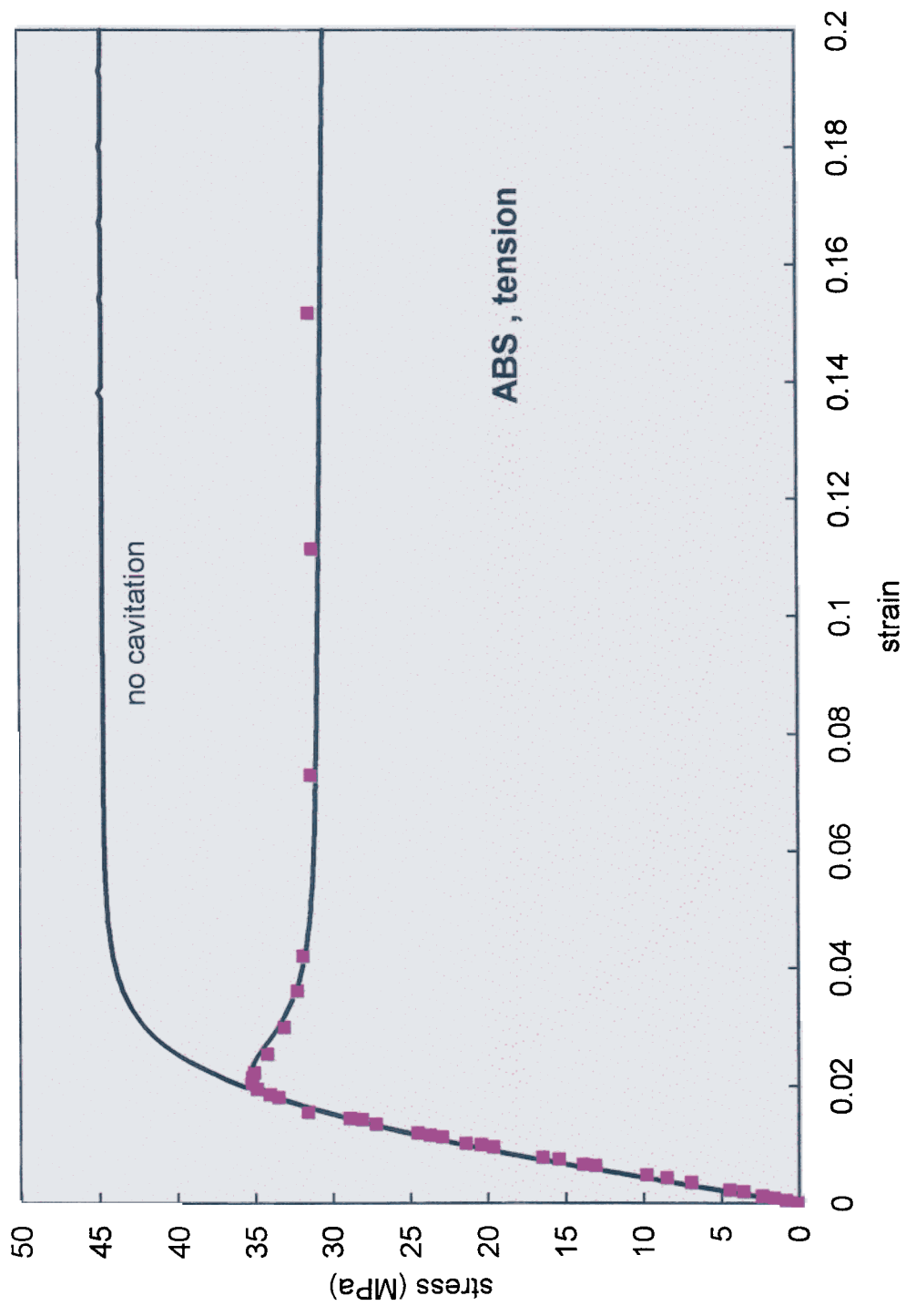
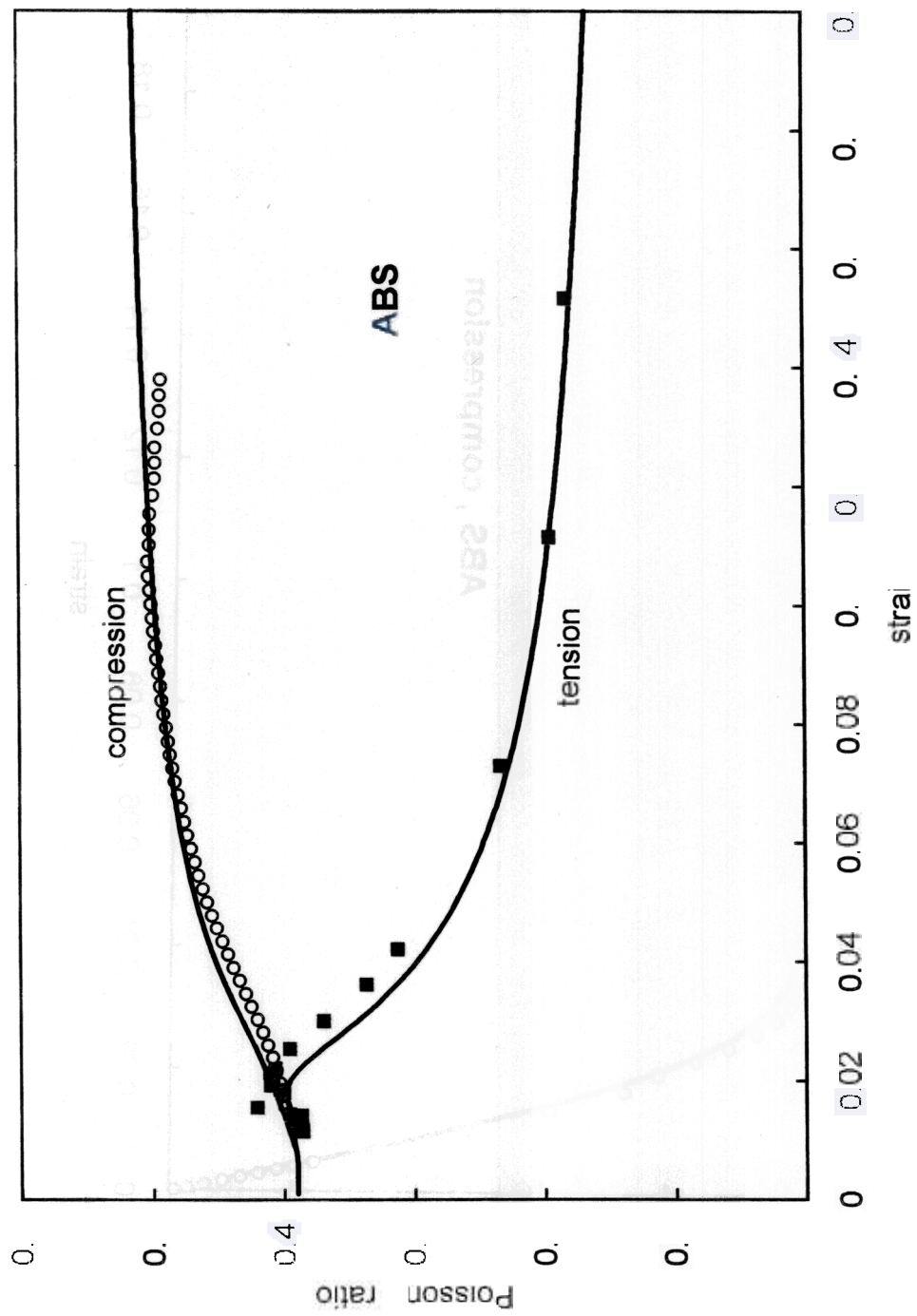


Fig. 14



Fig

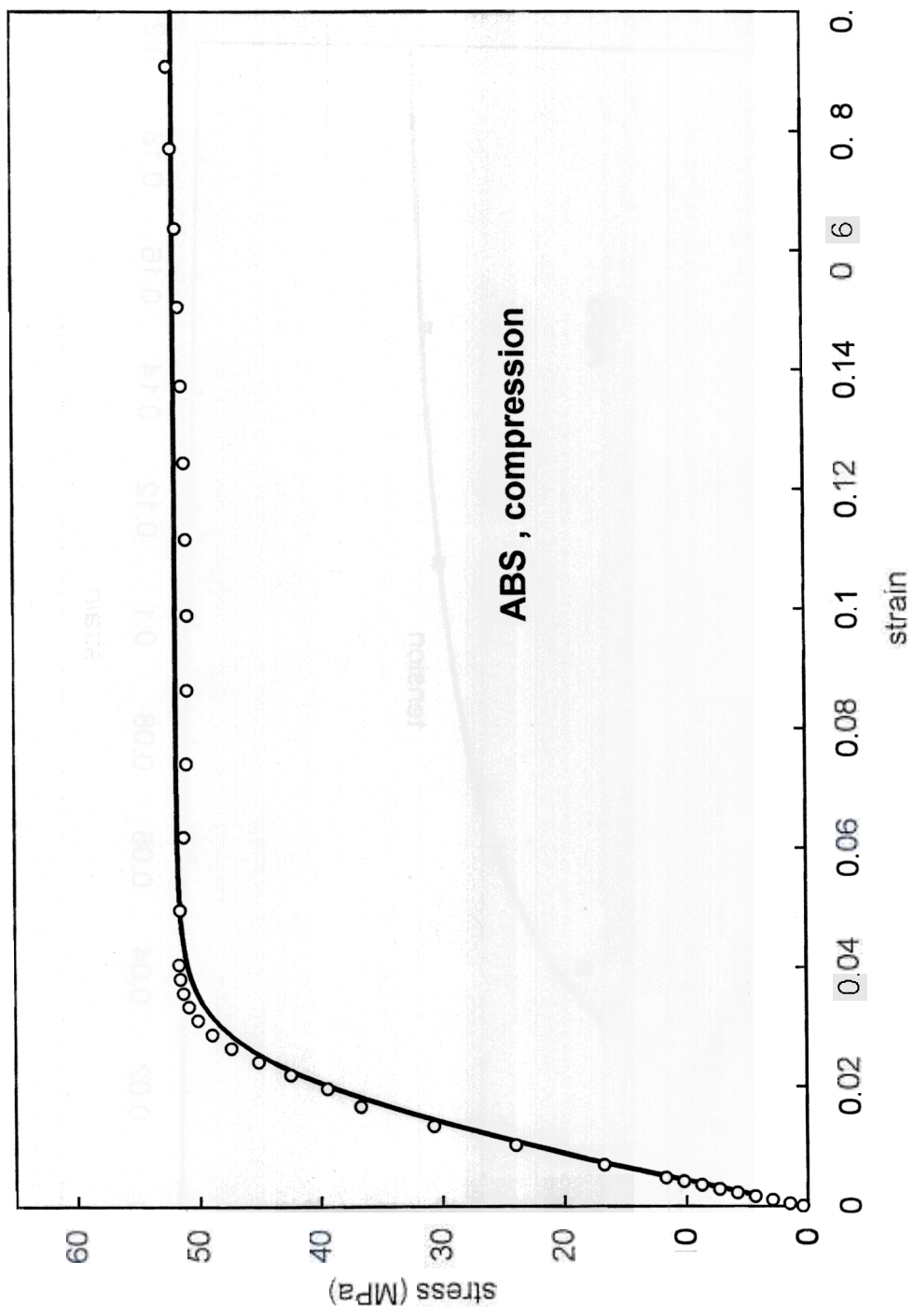


Fig 16

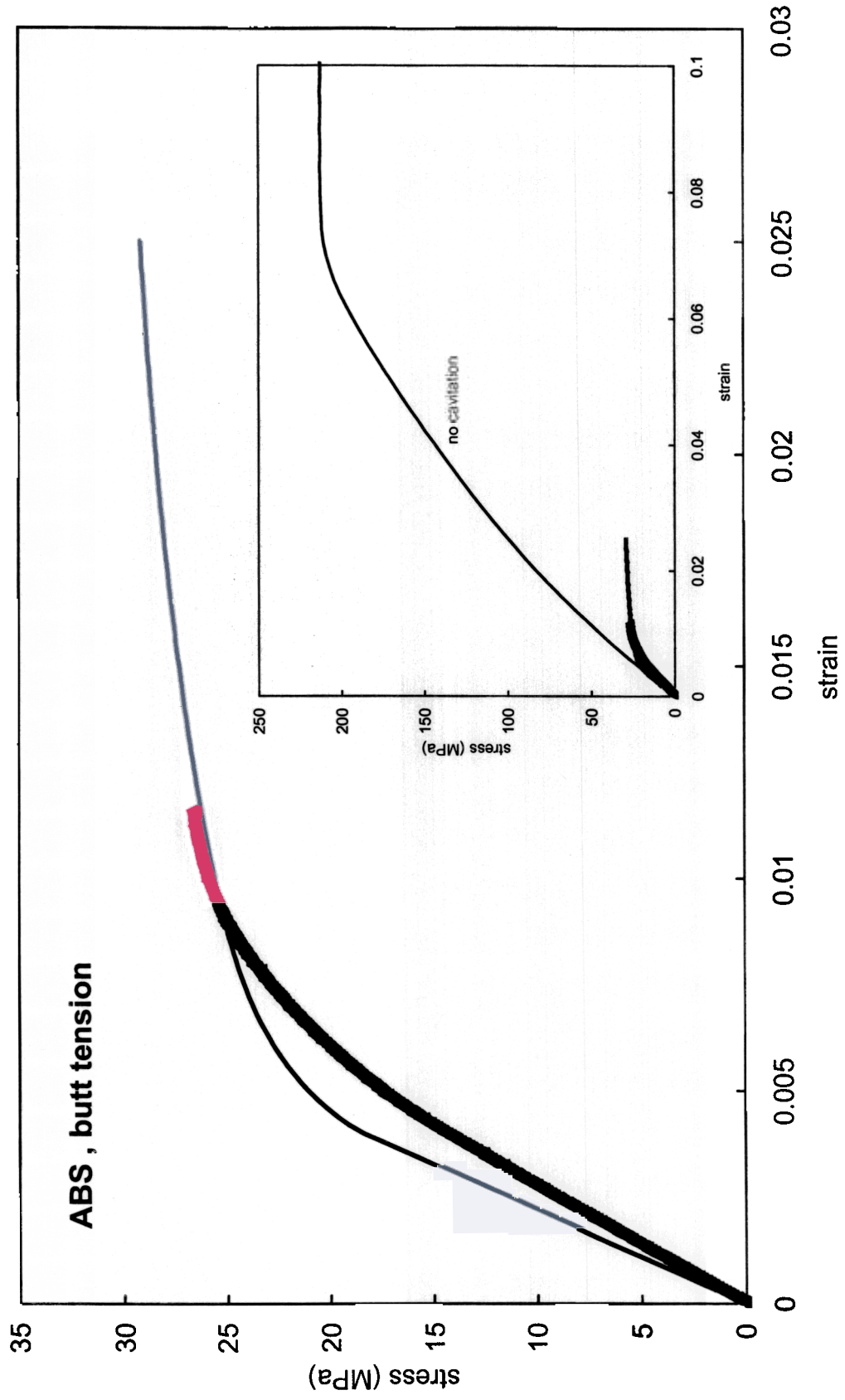


Fig. 17