

Influence of Strain Rate and Temperature on the Stress/Strain Behaviour of Rubber-Toughened Plastics

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SUMMARY

A proposed model for the multiaxial stress/strain behaviour of rubber-toughened plastics has been extended to include the effects of strain rate and temperature. The model assumes that the response at low strains is elastic and that the onset of non-linearity in a stress/strain curve is due to plastic deformation which, under dilatational stress states, is enhanced by the generation and growth of cavities in the rubber particles. A power law function has been employed to model the small but significant variation in elastic modulus E with strain rate and temperature for an acrylonitrile-butadiene-styrene (ABS) plastic containing 38% by volume of rubber.

The principal effect of increasing strain rate or decreasing temperature is to increase the yield stress of matrix material between cavities. This is characterised by a strain hardening function $\sigma_o(\epsilon_o^p)$ determined from shear data for the unvoided material. Values of the effective stress σ_o increase with the effective plastic strain ϵ_o^p toward limiting values σ_{of} (the flow stress). These increases have been modelled for the ABS material at plastic strain rates $\dot{\epsilon}_o^p$ between 0.0004 and 0.2 s⁻¹ and temperatures of 0, 23 and 50 °C using a stretched exponential function. Eyring plots of σ_{of}/T versus $\log \dot{\epsilon}_o^p$ have been constructed from the shear data at each (absolute) temperature T , and extended to higher strain rates (100 s⁻¹) using measured tensile yield stresses σ_T . This procedure assumes that the ratio σ_{of}/σ_T remains constant at all strain rates and temperatures, implying that the rubber particles are fully cavitaded under tension. Over a wide strain rate range, two ($\alpha + \beta$) Eyring functions were required to obtain optimum fits to the σ_{of}/T versus $\log \dot{\epsilon}_o^p$ plots. Dynamic mechanical (DMTA) data on the ABS material

provided no clear evidence of a secondary β -loss process, however, further shear tests at higher strain rates and lower temperatures are proposed to confirm this result.

Using a numerical method for solving the elasticity, yield and flow equations of the model, the parameters obtained from modelling the modulus and strain hardening data were employed to predict the tensile stress/strain behaviour of the ABS material. Void nucleation parameters were chosen to optimise the agreement between the calculated curves and observed curves for strain rates between 0.0004 and 100 s^{-1} and temperatures of 0 , 23 and $50 \text{ }^{\circ}\text{C}$ respectively. The model successfully captures the strain softening produced by cavitation and the progressive decrease in magnitude of this effect and its elimination with increasing strain rate and decreasing temperature. A small apparent decrease in the critical volumetric strain for void nucleation with increasing strain rate and decreasing temperature is consistent with the nucleation occurring at a critical volumetric strain on the rubber particles rather than an overall volumetric strain. Discrepancies between the predicted and measured tensile stresses at higher strains are ascribed to the neglect of orientation hardening which is considered in a previous report.

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CONTENTS

1	INTRODUCTION.....	1
2	OUTLINE OF THE MODEL.....	2
2	ADDITIVITY OF ELASTIC AND PLASTIC STRAINS	2
	ELASTICITY EQUATIONS	2
	THE YIELD CRITERION....	..3
2.4	THE FLOW RULE	4
2.5	VOID NUCLEATION AND GROWTH.....	...5
2.5.1	Nucleation	5
2.5.2	Growth	6
2.6	EVOLUTION OF THE MATRIX COMPOSITION AND YIELD STRESS σ_M ...	7
2.7	SUMMARY OF BASIC MODEL PARAMETERS FOR ABS	8
3	DEPENDENCE OF ELASTIC PROPERTIES ON STRAIN RATE AND TEMPERATURE	8
4	STRAIN RATE AND TEMPERATURE DEPENDENCE OF THE STRAIN HARDENING FUNCTION	10
5	PREDICTING THE TENSILE STRESS/STRAIN BEHAVIOUR AS A FUNCTION OF STRAIN RATE AND TEMPERATURE.....	13
6	CONCLUSIONS	15
7	ACKNOWLEDGEMENT.....	16
	REFERENCES.....	17
	FIGURE CAPTIONS	19

1 INTRODUCTION

Details have previously been reported¹⁻³ of a model for describing the non-linear, multiaxial stress/strain behaviour of rubber-toughened plastics. Following a linear elastic response at low strains, the onset of non-linearity in a stress-strain curve is attributed to plastic deformation which may be enhanced by the nucleation and growth of cavities in the rubber particles. Since cavitation is assumed to require a dilatational stress component it will occur, for example, under tension but not under compression or shear. A strain-hardening function for the unvoided material may then be determined from shear data and, with the aid of a void nucleation criterion, used to interrelate the stress/strain behaviour observed for different stress states.

Predicted stress/strain curves under uniaxial tension and compression, respectively, were in good agreement with measured curves for an acrylonitrile-butadiene-styrene (ABS) copolymer containing 38% by volume of rubber particles^{1,2}. These comparisons were made at room temperature (23 °C) and at an effective strain rate of 0.004 s⁻¹. The calculations made allowance for the growth of cavities under tension, including the effects of a possible expansion of the matrix material between cavities². They also capture the observed decrease in Poisson's ratio and associated increase in volumetric strain arising from the cavitation under tension. The effects of orientation hardening were also considered² but have yet to be incorporated into the model for the general case of multiaxial loading.

In this report, extensions to the model are described that enable accurate calculations to be made of the multiaxial stress-strain behaviour over wide ranges of strain rate and temperature. When implemented into a finite element (FE) package, the extended materials model will provide an improved basis for predicting the impact performance of toughened plastic components. An outline of the proposed model will first be presented in Section 2. This will be followed by an analysis of the strain rate and temperature dependence of the modulus (Section 3) and of the strain hardening function (Section 4) for the ABS material. In Section 5 the measured tensile stress/strain curves for ABS at different strain rates and temperatures will be compared with predicted curves using parameters presented in Sections 2-4.

2 OUTLINE OF THE MODEL

2. ADDITIVITY OF ELASTIC AND PLASTIC STRAINS

In the non-linear region of a stress/strain curve each component of the (true) strain tensor ε_{ij} ($i, j = 1, 2$ or 3) is considered as the sum of a recoverable elastic component ε_{ij}^e and a plastic component ε_{ij}^p which is non-recoverable. Thus

$$\varepsilon_{ij} = \varepsilon_{ij}^e + \varepsilon_{ij}^p \quad (1)$$

2.2 ELASTICITY EQUATIONS

Assuming linear-elastic behaviour, the components σ_{ij} of the (true) stress tensor are related to the elastic strain components ε_{kl}^e ($k, l = 1, 2, 3$) by

$$\sigma_{ij} = D_{ijkl} \varepsilon_{kl}^e \quad (2)$$

where D_{ijkl} is a component of the fourth-order elasticity tensor and repeated indices indicate summation. For isotropic elasticity

$$D_{ijkl} = \frac{E}{(1+\nu_e)} \delta_{ik} \delta_{jl} + \frac{E \nu_e}{(1+\nu_e)(1-2\nu_e)} \delta_{ij} \delta_{kl} \quad (3)$$

where E and ν_e are the elastic Young's modulus and the elastic Poisson's ratio, respectively, and δ_{mn} ($m, n = i, j, k, l$) is the Kronecker delta.

2.3 THE YIELD CRITERION

The stress condition for yielding is represented by

$$\Phi = \frac{\sigma_e^2}{\sigma_M^2} - (q_1 f)^2 - 2q_1 f \cosh\left(\frac{3\sigma_k}{2\sigma_M}\right) \left(\frac{\mu\sigma_k}{\sigma_M}\right)^2 = 0 \quad (4)$$

where the yield function Φ is defined such that whenever $\Phi < 0$ the response is purely elastic. In this equation, which is based on Gurson's analysis⁴ of plasticity in porous materials, the effective (Mises) stress σ_e is given by

$$\sigma_e = \left(\frac{3}{2} S_{ij} S_{ij}\right)^{1/2} \quad (5)$$

where S_{ij} is a component of the deviatoric stress tensor

$$S_{ij} = \sigma_{ij} - \sigma_k \delta_{ij} \quad (6)$$

and σ_k is the mean normal stress

$$\sigma_k = \frac{\sigma_{ij} \delta_{ij}}{3} = \frac{\sigma_{11} + \sigma_{22} + \sigma_{33}}{3} \quad (7)$$

In equation (4), f is the effective volume fraction of cavities which increases with strain as discussed in Section 2.5. The parameter q_1 was introduced by Tvergaard⁵ and accounts for the effect of void interactions on the stress distribution in the matrix between cavities. The parameter μ was introduced by Lazzeri and Bucknall⁶ to allow for the dependence of yield stress on σ_k in the absence of voids ($f = 0$). The yield stress σ_M is $\sqrt{3}$ x the shear stress for the matrix material between voids and is discussed further in Section 2.6.

2.4 THE FLOW RULE

Increments of plastic strain are given by the flow rule

$$d\epsilon_{ij}^p = d\lambda \frac{\partial F}{\partial \sigma_{ij}} \quad (8)$$

where F is the flow potential and $d\lambda$ is a scalar multiplier. In the initial stages in the development of the model^{1,2}, it was assumed that the flow potential could be associated with the yield potential so that $F = \Phi$. Since then, a detailed analysis of axial and transverse strain measurements in ABS specimens under uniaxial tension and compression³ revealed that flow in this polymer is non-associated. The flow potential was then assumed to be

$$F = \frac{\sigma_e^2}{\sigma_M^2} - (q_1 f)^2 - 2q_1 f \cosh\left(\frac{3\sigma_k}{2\sigma_M}\right) \left(\frac{\mu' \sigma_k}{\sigma_M} \right)$$

where μ' is the flow parameter and replaces μ under associated flow. In order to accurately model the volumetric strain in the above tests, an expression was developed that allowed μ' to be dependent on the stress state and the volume fraction of cavities. This took the form

$$\mu' = \mu'_1 (1 - v_R^*) + \Delta\mu'_R v_R^* \sinh \left[\frac{3\sigma_k (1 - K v_R^*)}{2\sigma_M (1 - f)} \right]$$

Values for the material parameters μ'_1 and $\Delta\mu'_R$ are given in table

Ignoring the effects on F of varying σ_M and f we obtain from (8) and (9)

$$d\epsilon_{ij}^p = d\lambda \left\{ \frac{3S_{ij}}{\sigma_M^2} + \left[\frac{q_1 f}{\sigma_M} \sinh \frac{3\sigma_k}{2\sigma_M} \right] + \frac{2\mu'}{3\sigma_M} \left(1 - \frac{\mu' \sigma_k}{\sigma_M} \right) \right\} \delta_{ij}$$

With the aid of the plastic work equivalence

$$\sigma_{ij} d\epsilon_{ij}^p = (1-f) \sigma_M d\epsilon_o^p$$

it follows that

$$d\lambda = \frac{(1-f) \sigma_M d\epsilon_o^p}{\sigma_{ij} \partial F / \partial \sigma_{ij}}$$

$$\frac{(1-f) \sigma_M d\epsilon_o^p}{\sigma_{ij} \left\{ \frac{3S_{ij}}{\sigma_M^2} + \left[\frac{\mu'}{\sigma_M} \sinh \frac{3\sigma_k}{2\sigma_M} \right] + \frac{2\mu'}{3\sigma_M} \left(1 - \frac{\mu'\sigma_k}{\sigma_M} \right) \right\} \delta_{ij} \right\}}$$

In these equations the effective matrix plastic strain ϵ_o^p is assumed to be independent of rubber content in the matrix, and hence of the void fraction (Section 2.6). From equations (12), (4) and (21) below with $f = 0$ it follows that $\epsilon_o^p = \gamma^p / \sqrt{3}$ where γ^p is the plastic strain determined in a shear test on the unvoided, rubber toughened material.

2.5 VOID NUCLEATION AND GROWTH

During an increment of applied strain, the change in void volume fraction df is considered as the sum of a contribution df_{nucl} from nucleation and a contribution df_{gr} from void growth, so that

$$df = df_{\text{nucl}} + df_{\text{gr}}$$

2.5.1 Nucleation

It is assumed that cavities are generated only in the rubber particles and that each particle offers negligible resistance to deformation once a small cavity has formed. The nucleation process is then regarded as involving the progressive replacement of rubber particles by an equal volume of effective voids. From the work of Bucknall and co-workers^{6,7}, we assume that the nucleation of a cavity in a given rubber particle occurs at some critical volumetric

strain in the particle that increases with the rubber shear modulus and surface energy and decreases with increasing particle diameter. For a distribution of particle sizes, the nucleation should then occur over a range of total volumetric strain (ϵ_v) related to the critical volumetric strain range for the rubber particles.

It was proposed¹ that f_{nucl} is zero for volumetric strains up to some initial value ϵ_{1v} and subsequently increases with ϵ_v according to a stretched-exponential function

$$f_{\text{nucl}} = 0 \quad \text{for } \epsilon_v \leq \epsilon_{1v} \quad (15a)$$

$$f_{\text{nucl}} = v_{R0} \left\{ 1 - \exp \left[- \left(\frac{\epsilon_v - \epsilon_{1v}}{\epsilon_{2v}} \right)^{\beta_v} \right] \right\} \quad \text{for } \epsilon_v > \epsilon_{1v} \quad (15b)$$

In equation (15b) the parameters ϵ_{2v} and β_v ($0 < \beta_v \leq 1$) determine the location and breadth of the volumetric strain range over which void nucleation occurs.

2.5.2 Growth

The growth of voids following their nucleation occurs through plastic deformation within the matrix material. On this basis it was shown² that

$$df_{\text{gr}} = (1-f) \frac{dV^p}{V} - \frac{dV_M^p}{V} \quad (16)$$

where V is the total volume, and dV^p and dV_M^p are incremental changes in the total volume and the matrix volume, respectively, due to plastic deformation in the matrix. Writing equation (16) in the form

$$df_{\text{gr}} = (1-f) d\epsilon_{ij}^p \delta_{ij} - (d\epsilon_{ij}^p \delta_{ij})_{f=0} \quad (17)$$

we obtain using (11)

$$df_{gr} = \frac{d\lambda}{\sigma_M} \left(\left(\frac{\sigma_M}{\sigma_0} \right)^2 - 2\mu'f \left(-\frac{\mu'\sigma_k}{\sigma_M} \right) \right)$$

2.6 EVOLUTION OF THE MATRIX COMPOSITION AND YIELD STRESS σ_M

Since void nucleation is assumed to involve the replacement of rubber particles with an equal effective volume of cavities, the instantaneous volume fraction v_R^* of unvoided rubber in the “matrix” surrounding the voids will decrease as the nucleation proceeds. If v_{R0} is the initial volume fraction of rubber particles it follows that¹

$$v_R^* = \frac{v_{R0} - f_{nucl}}{1 - f_{nucl}}$$

The decrease in v_R^* can give rise to a marked increase in the matrix yield stress σ_M during the nucleation. Assuming that σ_M increases linearly with v_R^* we have obtained

$$\begin{aligned} \sigma_M &= \sigma_{o1} (1 - k v_R^*) \\ &= \frac{\sigma_o (\epsilon_o^p)}{1 - k v_{R0}} \left(1 - k \frac{v_{R0} - f_{nucl}}{1 - f_{nucl}} \right) \end{aligned}$$

Here σ_o and σ_{o1} are the values of $\sqrt{3}\sigma_s$ for the unvoided material with and without rubber particles, respectively, where σ_s is the shear yield stress. The constant k specifies the dependence of shear yield stress on the volume fraction of rubber particles for unvoided material. The variation of σ_o with ϵ_o^p can be determined from shear tests on the unvoided material and represents the basic hardening data required in applications of the model.

2.7 SUMMARY OF BASIC MODEL PARAMETERS FOR ABS

Table 1 presents the values of v_{R0} , v_e , k , μ and q_1 for the ABS material used in this investigation. The methods employed in evaluating v_e , k , μ and q_1 were described in detail previously¹. They involved analyses of stress/strain data at 23 °C using a combination of uniaxial tension, shear and uniaxial compression tests. The strain rates used in the tensile and compressive tests were close to the effective strain rate ($\dot{\epsilon}_0^P = \dot{\gamma}^P / \sqrt{3}$) in the shear tests of 0.004 s^{-1} . In subsequent Sections, the parameters in Table are taken to be independent of strain rate and temperature.

Table 1 Values of Model Parameters for ABS

v_{R0}	v_e	k	μ	q_1	μ'_1	$\Delta\mu'_R$
0.38	0.39	1.35	0.22	1.57	0.16	0.39

Parameters characterising the strain rate and temperature dependence of the modulus E and strain hardening function $\sigma_0(\epsilon_0^P)$ are given in Sections 3 and 4 respectively. In Section 5 values are given for the void nucleation parameters at different strain rates and temperatures.

3 DEPENDENCE OF ELASTIC PROPERTIES ON STRAIN RATE AND TEMPERATURE

Since polymers are viscoelastic materials, the “elastic” modulus E and Poisson’s ratio v_e could each vary significantly with strain rate and temperature. Figure 1 shows the E values obtained for the ABS material (using contact extensometers for the strain measurements^{8,9}) at strain rates between 10^{-5} and 10^{-2} s^{-1} and at temperatures of 0, 23 and 50 °C. The continuous lines illustrate that the data can be modelled to within 1% using the power law function of strain rate ($\dot{\epsilon}$)

$$E = E_{uo} \frac{K_o}{(a_T \dot{\epsilon})^n} [1 + K_2 (296 - T)] \quad (22)$$

where

$$a_T = \exp \left[\frac{Q}{R} \left(\frac{1}{T} - \frac{1}{296} \right) \right] \quad (23)$$

T is the absolute temperature (°K) and the parameters E_{uo} , K_o , n , Q and K_2 have the values listed in Table 2.

Table 2 Values of Parameters Characterising the Strain Rate and Temperature Dependence of the Modulus for ABS

E_{uo} (MPa)	K_o (s ⁻ⁿ)	n	Q (J/mole)	K_2 (°K ⁻¹)
2945	0.2	0.023	1.53×10^5	0.001

The unrelaxed modulus E_{uo} in (22) represents the value of E at 23 °C in the limit $\dot{\epsilon} \rightarrow \infty$ constant K_o may be regarded as being inversely proportional to τ^n where τ is some mean relaxation time at 23 °C and the small value of n is indicative of a broad spectrum of relaxation times. This relaxation time distribution is considered to characterise the onset of the glass-rubber α -process of the styrene-acrylonitrile (SAN) plastic matrix. The principal effect of varying temperature is to produce a horizontal shift of the curves in Figure 1. The magnitude of this shift is represented by the value of the factor a_T which is related to the activation energy Q by equation (23). A small vertical shift of the curves with varying temperature is also determined by the magnitude of the constant K_2 . Note that equations (22) and (23), together with the values of parameters in Table 2, enable reliable values of E to be obtained by interpolation or extrapolation over wide strain rate and temperature ranges.

Relaxation effects will generally give rise to a decrease in v_e with increasing strain rate and decreasing temperature. However no systematic variations were observed for the ABS material over the strain rate and temperature range of the modulus measurements. An average value of $v_e = 0.39$ was obtained within the reproducibility of $\pm 3\%$.

4 STRAIN RATE AND TEMPERATURE DEPENDENCE OF THE STRAIN HARDENING FUNCTION

The principal effect of increasing strain rate and decreasing temperature is to increase the effective matrix yield stress σ_o . This, in turn, gives rise to an increase in σ_M (equation (21)) and to variations in the macroscopic components of stress and plastic strain (equations (4) and (9)). Figures 2, 3 and 4 show plots of σ_o against ϵ_o^p obtained from shear data for the ABS material at the specified effective strain rates ($\dot{\epsilon}_o^p = d\epsilon_o^p / dt$) and temperatures of 0, 23 and 50 °C respectively. At each strain-rate and temperature, σ_o increases from the initial yield stress σ_{oi} at $\epsilon_o^p = 0$ to a plateau level or flow stress σ_{of} which is attained at $\epsilon_o^p \approx 0.03$. At larger strains σ_o tends at first to decrease slightly (possibly reflecting a structural deageing) and then exhibits a gradual increase with ϵ_o^p which we ascribe to orientation hardening. Ignoring these small post-yield effects, which are discussed elsewhere², the matrix strain hardening function $\sigma_o(\epsilon_o^p)$ can be represented by

$$\sigma_o(\epsilon_o^p) = \sigma_{oi} + (\sigma_{of} - \sigma_{oi}) \left\{ 1 - \exp \left[- (\epsilon_o^p / \epsilon_{os})^{\beta_o} \right] \right\} \quad (24)$$

where the parameters ϵ_{os} and β_o ($0 < \beta_o \leq 1$) determine the range of ϵ_o^p over which the increase in σ_o occurs.

The variations in σ_o with strain rate and temperature are due largely to variations in σ_{oi} and σ_{of} , the values of $\epsilon_{os} = 0.0027$ and $\beta_o = 0.7$ being essentially unchanged. Variations in σ_{of} are conveniently analysed on the basis of the Eyring model for polymer relaxation processes. According to this model^{9,10}, an increase in stress increases the net rate of molecular rearrangements in the material and a plateau in the stress/strain curve is obtained when the strain rate produced by these rearrangements equals the applied strain rate. The molecular motions are represented by thermally activated transitions across a potential energy barrier separating two minima. It is then proposed that the levels of the minima are respectively raised and lowered by an amount σv where σ is the applied stress and v the local volume of moving molecular segments (the so-called activation volume). The barrier height, or

activation energy Q , is thus effectively lowered and raised in the forward and reverse directions respectively. The strain rate is then calculated from the net flow in the forward direction. For large stresses ($\sigma v/RT \gg 1$) the flow stress σ_{of} is given by

$$\frac{\sigma_{of}}{T} = A \left[\ln 2C \dot{\epsilon}_0^p + (Q/RT) \right]$$

where $A = R/v$, R being the gas constant, and C is a constant inversely proportional to the molecular vibration frequency.

Various studies of the yield behaviour of glassy polymers have shown that the yield stress increases more rapidly with increasing strain rate and decreasing temperature at low temperatures and high strain rates than at high temperatures and low strain rates. An extension to equation (25) has therefore been proposed^{11,12} based on the assumption that the yield stress comprises additive contributions from the glass-rubber α relaxation and from the secondary β process. The extended equation is

$$\begin{aligned} \frac{\sigma_{of}}{T} = & A_\alpha \left[\ln 2C_\alpha \dot{\epsilon}_0^p + (Q_\alpha/RT) \right] \\ & + A_\beta \sinh^{-1} \left[C_\beta \dot{\epsilon}_0^p \exp (Q_\beta/RT) \right] \end{aligned}$$

It will be noted that the β contribution to σ_{of}/T is represented by an inverse sinh function. This follows from the fact that $\sigma_0 v/RT$ for the β process will be small or zero at certain intermediate rates and temperatures.

Figure 5 shows plots of σ_{of}/T against $\log \dot{\epsilon}_0^p$ for ABS at each of the temperatures investigated. For strain rates below about 0.2 s^{-1} , the filled symbols in these plots were derived from the shear data (Figs 2-4) at $\epsilon_0^p = 0.03$ and thus characterise the yield behaviour for uncavitated material. The open symbols are values of σ_{of}/T estimated from tensile data using $\sigma_{of} = 0.53 \sigma_T$. Here σ_T is the yield stress for cavitated material in the plateau region of a tensile stress/strain curve (see Figs 6-8 below) at the equivalent plastic strain $\epsilon_T^p = 0.05$ and equivalent plastic strain rate $\dot{\epsilon}_T^p \approx (0.05/0.03)\dot{\epsilon}_0^p \approx 1.67\dot{\epsilon}_0^p$. The excellent agreement between the σ_{of} values from the shear and tensile data reflects the constancy of the ratio

$\sigma_{of}/\sigma_T = .53$. This is consistent with the assumption that voids occur under tension only in the rubber particles and that all particles have cavitated at $\epsilon_T^p = 0.05$ for strain rates up to 0.2 s^{-1} .

Since no shear data have yet been obtained at strain rates above 0.2 s^{-1} , we have sought to extend the Eyring plots using tensile data, assuming that the relation $\sigma_{of} = 1.53 \sigma_T$ holds for strain rates up to the highest levels (100 s^{-1}) investigated. The validity of this assumption requires that the dispersed particles remain rubberlike (with a very low shear modulus) and hence become fully cavitated under tension at all strain rates and temperatures investigated. This condition would appear to hold from the results of dynamic mechanical measurements on the ABS material. These data show the existence of a loss peak associated with the glass transition of the rubbery (polybutadiene) phase at -80°C ($\sim 1 \text{ Hz}$).

At each temperature in Figure 5 the Eyring plots are seen to exhibit an upward curvature with increasing strain rate and can be accurately modelled using two Eyring functions. The parameters obtained from optimising the fit of equation (24) to the data are included in Table 3. Also listed in Table 3 is a value estimated for the ratio $\kappa = \sigma_{oi}/\sigma_{of}$ and the values of ϵ_{os} and β_o . Figures 2-4 illustrate the fits obtained to the experimental σ_o versus ϵ_o^p data using equations (24) and (26) together with the values of parameters in Table 3. Bearing in mind the wide range of strain rate and temperature covered by the data, the agreement between the observed and calculated curves is good.

Table 3 Values of Parameters Characterising the Strain Rate and Temperature Dependence of the Matrix Strain Hardening Function for ABS (equations (24) and (26))

A_α (MPa/ $^\circ\text{K}$)	5.5×10^{-3}	$\kappa = \sigma_{oi}/\sigma_{of}$	0.313
C_α (s)	3.0×10^{-30}	ϵ_{os}	0.0027
Q_α (J/mole)	2.5×10^5	β_o	0.7
A_β (MPa/ $^\circ\text{K}$)	7.2×10^{-3}		
C_β (s)	3.0×10^{-9}		
Q_β (J/mole)	5.8×10^4		

It should be emphasised that whereas two Eyring processes were invoked to provide a fit to the σ_{of}/T versus $\log \dot{\epsilon}_0^P$ curves, the modulus data could be modelled using parameters characteristic of the single glass-rubber α -process for the SAN glassy matrix material. Furthermore there is no clear evidence from the DMTA results for the existence of a distinct secondary β -process for the SAN component. However, the loss modulus (E'') versus temperature plot exhibits a broadening on the low temperature side of the α -peak and a broad region of moderate loss between -50°C and $+50^\circ\text{C}$ which is accompanied by a fairly large increase in storage modulus (E') with decreasing temperature. Plots of shear loss modulus (G'') versus temperature for ABS have also been reported by Morbitzer et al¹³. These data indicate the presence of a shoulder on the low temperature side of the α -peak and a broad low intensity peak centred around 0°C similar to the β -peak observed for polystyrene¹⁴. It is possible that the yield behaviour could be more sensitive than modulus data to the presence of a secondary relaxation process owing to the relatively small activation volume and hence relatively high slope (A_β) of the Eyring plot compared with the α -process (see Table 3). In this context, according to the generalised theory of Ree and Eyring^{12,15} the yield stress may be expressed as the sum of terms, each similar in form to the second term in equation (26), characterising contributions from a range of flow units of different size or activation volume. It may not be necessary, therefore, to associate the two terms in equation (26) with contributions from discrete and different relaxation processes. In view of the uncertainty, however, as to whether two processes contribute to the yield behaviour, there is a need for further shear data at high strain rates and low temperatures for ABS to confirm the apparent slope changes in the σ_{of}/T versus $\log \dot{\epsilon}_0^P$ curves. The acquisition of these data would avoid the need to estimate the σ_{of} values from tensile data on cavitating specimens.

5 PREDICTING THE TENSILE STRESS/STRAIN BEHAVIOUR AS A FUNCTION OF STRAIN RATE AND TEMPERATURE

Using the values of parameters presented in Tables 1-3, a numerical method was employed for solving the elasticity, yield and flow equations in Section 2. This enabled the tensile stresses to be calculated as a function of applied uniaxial strain for ABS at different strain rates and temperatures. In these calculations, the parameters in Table 1 were assumed to be independent of strain rate and temperature and the void nucleation parameters in equation (15b) were

chosen to optimise the fit between the calculated and observed stress/ strain curves. Table 4 lists the derived values of the void nucleation parameters.

The experimental and predicted curves are compared in Figures 6-8 for temperatures of 0, 23 and 50 °C respectively. At low strain rates and high temperatures, cavitation gives rise to a decrease in tensile stress at strains between about 0.02 and 0.05. Following this strain softening effect, and a subsequent plateau in the experimental curves, a gradual increase in stress is observed at higher strains up to failure. This stress increase is ascribed to orientation hardening and has been considered elsewhere². With increasing strain rate and decreasing temperature the softening due to cavitation is reduced and the orientation hardening is evident at progressively lower strains.

Table 4 – Values of Void Nucleation Parameters Obtained for ABS

Temperature (°C)	Strain Rate (s ⁻¹)	ϵ_{1V}	ϵ_{2V}	β_V
0	0.00043	0.0035	0.0022	0.62
	0.0042	0.0035	0.0022	0.62
	0.030	0.0034	0.0021	0.62
	0.21	0.0031	0.0021	0.62
	2.2	0.0030	0.0017	0.62
	20	0.0030	0.0017	0.62
23	0.00042	0.0033	0.0020	0.62
	0.0045	0.0036	0.0021	0.62
	0.041	0.0035	0.0020	0.62
	0.20	0.0035	0.0018	0.62
	2.3	0.0031	0.0020	0.62
	33	0.0031	0.0016	0.62
	104	0.0032	0.0016	0.62
50	0.00042	0.0035	0.0023	0.62
	0.0044	0.0036	0.0022	0.62
	0.046	0.0037	0.0023	0.62
	0.30	0.0039	0.0022	0.62
	2.4	0.0037	0.0022	0.62
	33	0.0035	0.0020	0.62

Orientation hardening and cavity growth have been neglected in these calculations since their effects on yield stresses essentially cancel in the initial part of the flow region. Good agreement between the predicted and observed curves is obtained in the yield and immediate post-yield regions. In particular, the model accurately captures the strain softening due to cavitation, including the decrease in its magnitude and subsequent elimination with increasing strain rate and decreasing temperature. It should be emphasised that these effects are qualitatively predicted by the model for constant values of the void nucleation parameters ϵ_{1V} , ϵ_{2V} and β_V . However to obtain the optimum fits shown in Figures 6-8, ϵ_{1V} and ϵ_{2V} tended to decrease slightly with increasing strain rate and decreasing temperature (see Table 4). This result could reflect the fact that void nucleation occurs at a critical volumetric strain ϵ_{Vr} on the rubber particles³ rather than a critical total volumetric strain ϵ_V as assumed in the model. In this context it is informative to consider variations in the ratio $\epsilon_{1V}/\epsilon_{Vr}$, for a toughened material subjected to a hydrostatic stress, as the matrix modulus varies with strain rate and temperature. Using elastic relations for a spherical rubber particle in a spherical shell of rigid plastic⁷ together with realistic shear and bulk modulus values for the components, we estimate that ϵ_V/ϵ_{Vr} should decrease from about 0.97 to 0.84 as the calculated E for the composite element increases from 2000 MPa to 2,500 MPa. For a constant ϵ_{Vr} , the value of ϵ_V should therefore decrease by about 15% from the lowest strain rate and highest temperature to the highest strain rate and lowest temperature investigated (see Figure 1). This estimate compares reasonably well with the variations in ϵ_{1V} and ϵ_{2V} in Table 4 allowing for experimental scatter and calculation errors.

6 CONCLUSIONS

- For rubber-toughened ABS, accurate predictions of tensile stress/strain curves have been made, for strains up to the immediate post-yield region, from shear hardening data $\sigma_o(\epsilon_o^p)$ over wide ranges of strain rate and temperature.

The modelling accounts for small but significant modulus variations and for apparent contributions to the effective matrix yield stress σ_o from α and β relaxation processes.

- The critical volumetric strain for void nucleation shows a slight decrease with increasing strain rate and decreasing temperature. This could reflect a constant critical volumetric strain on the rubber particles with increasing matrix modulus.
- Further shear data at higher strain rates and lower temperatures should be obtained to confirm the involvement of contributions to the yield stress from a high-rate β -process.
- With increasing strain in the post-yield region, stress contributions from orientation hardening should be included in the model.

7 ACKNOWLEDGEMENT

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FIGURE CAPTIONS

Fig Dependence of Young's modulus E on strain rate for ABS at the temperatures indicated. The continuous lines were calculated using equations (22) and (23) with values for the parameters in Table 2

Effective stress σ_o versus effective plastic strain ϵ_o^p for ABS from shear data at 0 °C and at the strain rates $\dot{\epsilon}_o^p$ (s^{-1}) shown for each curve. The continuous lines were calculated using equations (24) and (26) with values for the parameters in Table 3.

Effective stress σ_o versus effective plastic strain ϵ_o^p from shear data at 23 °C and strain rates $\dot{\epsilon}_o^p$ (s^{-1}) shown for each curve. The continuous lines were calculated using equations (24) and (26) with values for the parameters in Table 3

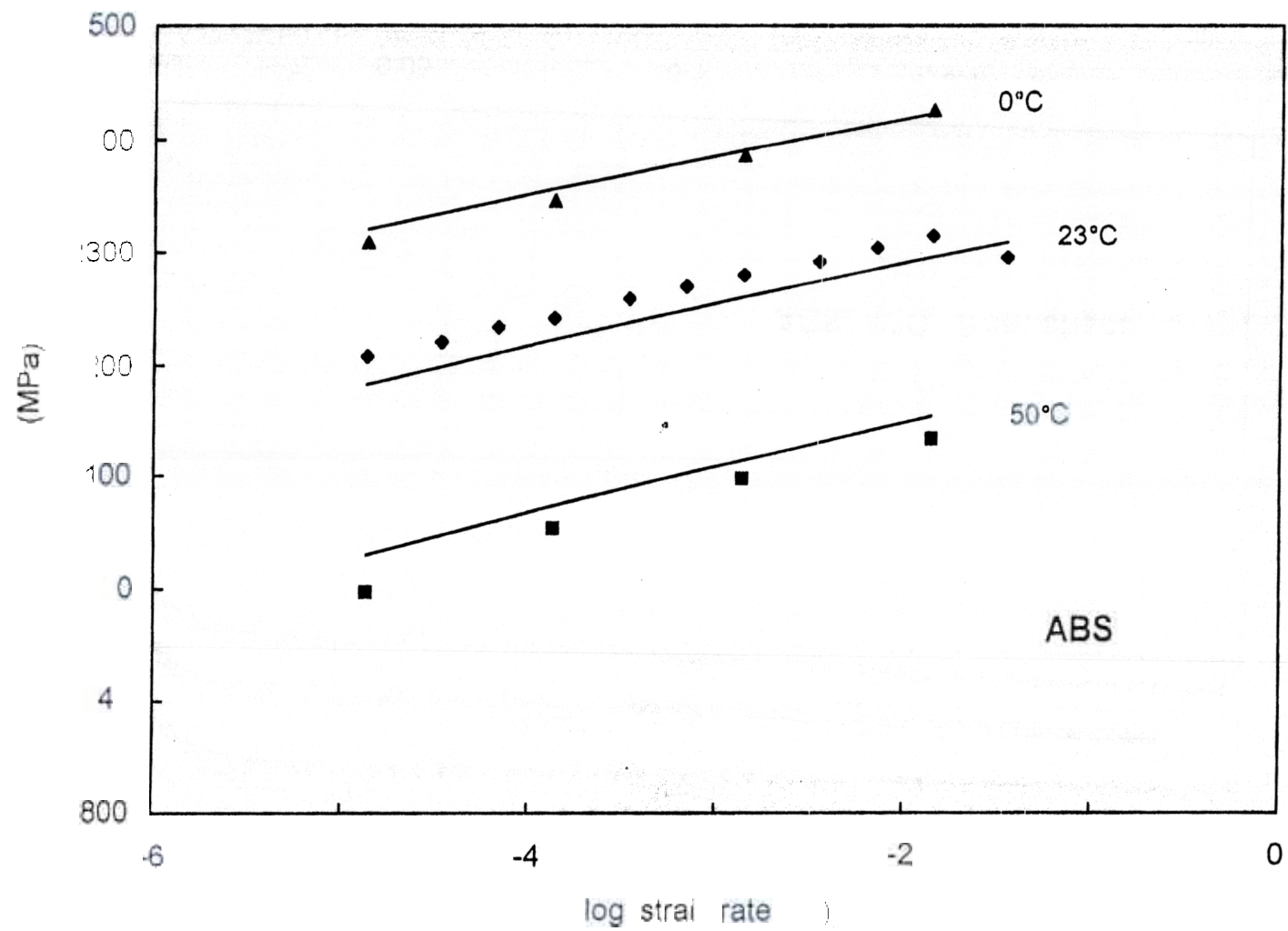
Effective stress σ_o versus effective plastic strain ϵ_o^p from shear data at 50 °C and strain rates $\dot{\epsilon}_o^p$ (s^{-1}) shown for each curve. The continuous lines were calculated using equations (24) and (26) with values for the parameters in Table 3

Dependence of σ_{of}/T on $\log \dot{\epsilon}_o^p$ at the temperatures indicated. Filled symbols ($\blacktriangle$, $\blacklozenge$, $\blacksquare$) were obtained from the shear data in Figs 2-4 where σ_{of} is the value of σ_o at $\epsilon_o^p = 0.03$. Open symbols (Δ , $\diamond$, $\square$) were estimated from tensile yield stresses in Figs 6-8 at an equivalent plastic strain and at equivalent plastic strain rates as described in the text. Continuous lines were calculated by fitting equation (26) to the data giving the values of parameters shown in Table 3

Tensile stress/strain data at 0 °C at the specified strain rates (s^{-1}). The continuous lines were calculated from a numerical solution of the model equations using the parameters listed in Tables 1-3. Values of void nucleation parameters giving an optimum fit to the experimental data are presented in Table 4.

Fig 7 Tensile stress/strain curves at 23 °C at the specified strain rates (s^{-1}). The continuous lines were calculated from a numerical solution of the model equations using the parameters in Tables 1-3. Values of the void nucleation parameters giving an optimum fit to the data are included in Table 4.

Tensile stress/strain curves at 50 °C at the specified strain rates (s^{-1}). The continuous lines were calculated from a numerical solution of the model equations using the parameters listed in Tables 1-3. The values of void nucleation parameters giving an optimum fit to the data are included in Table 4.



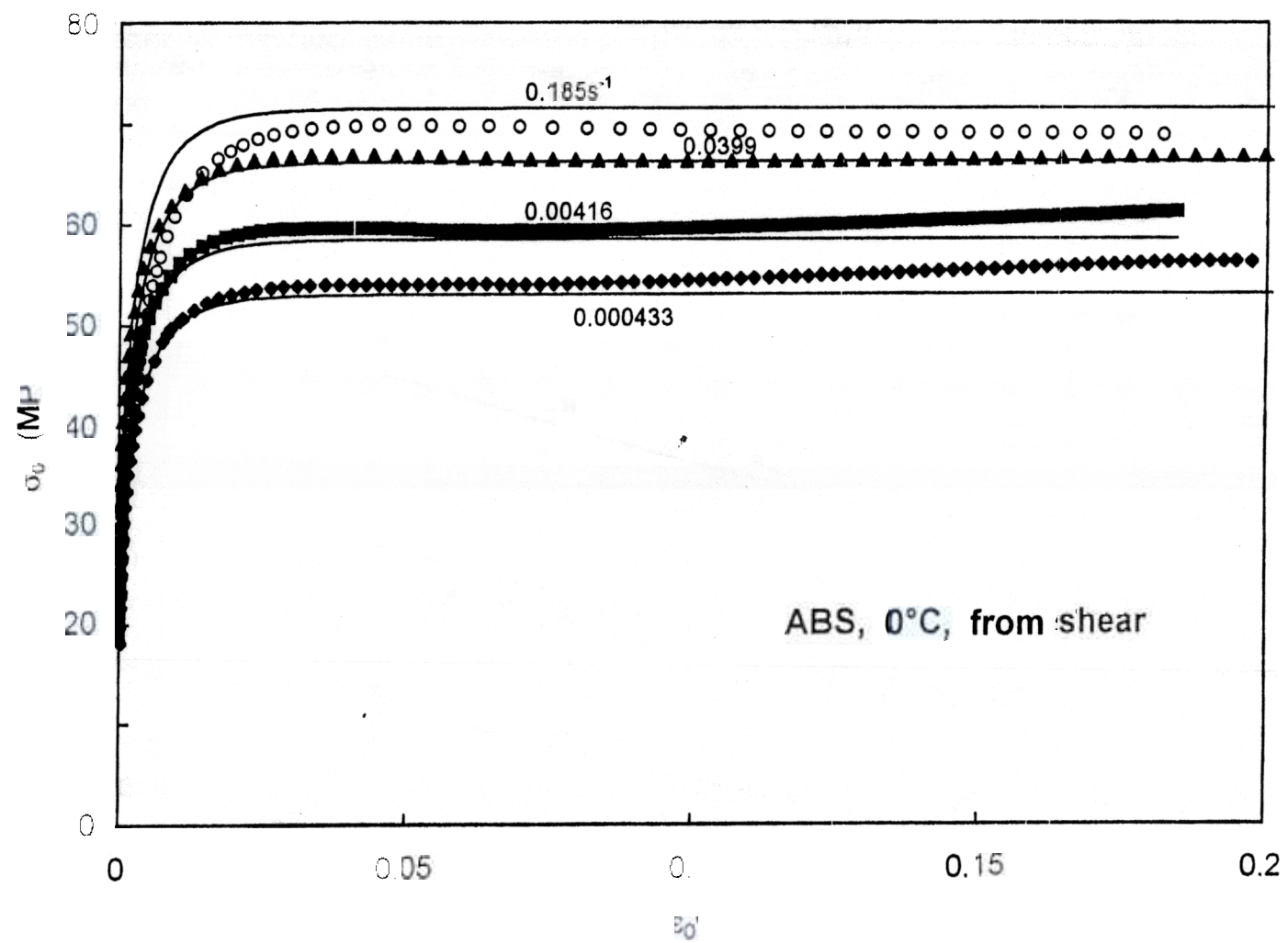


Fig 2

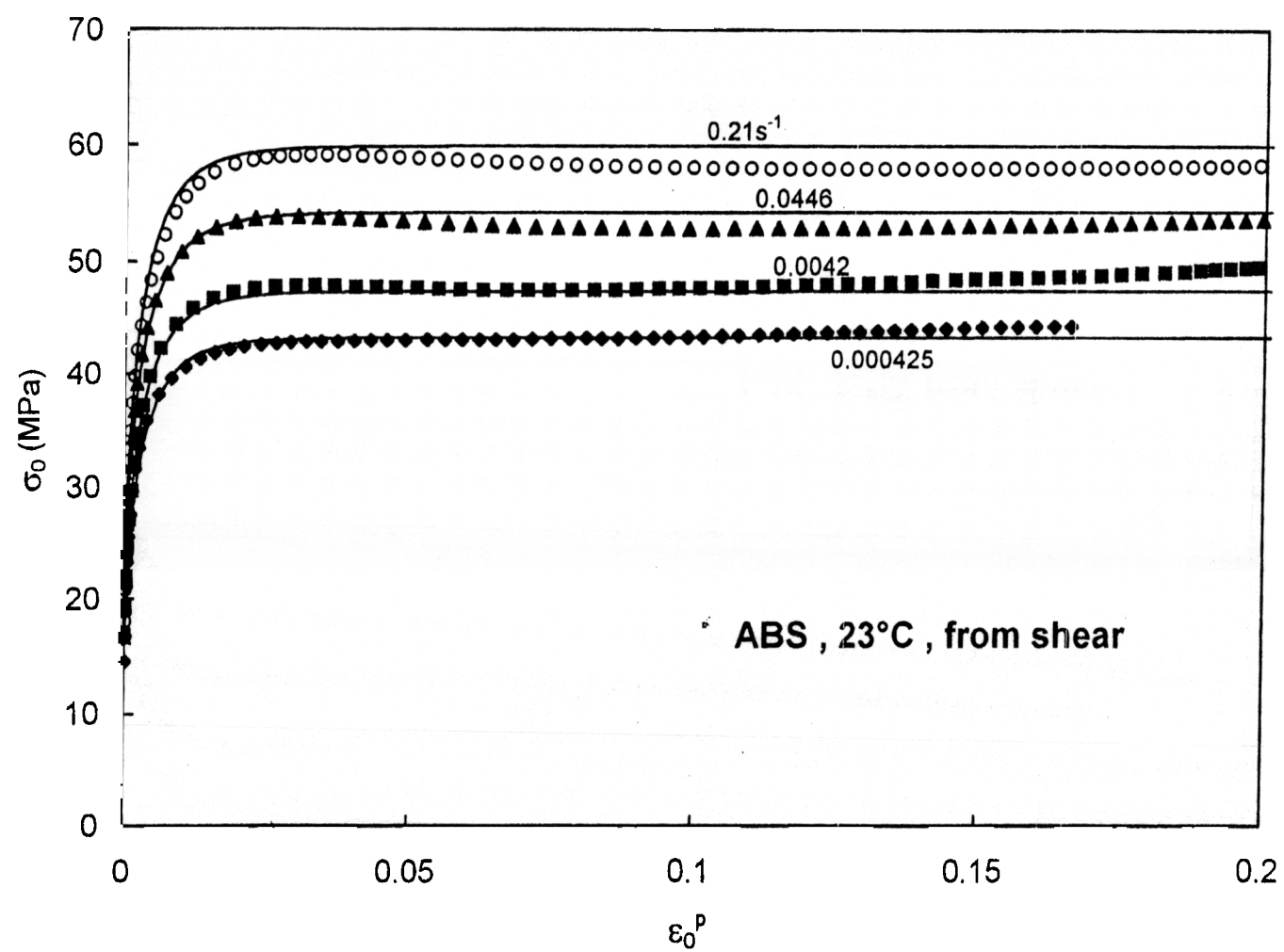


Fig. 3

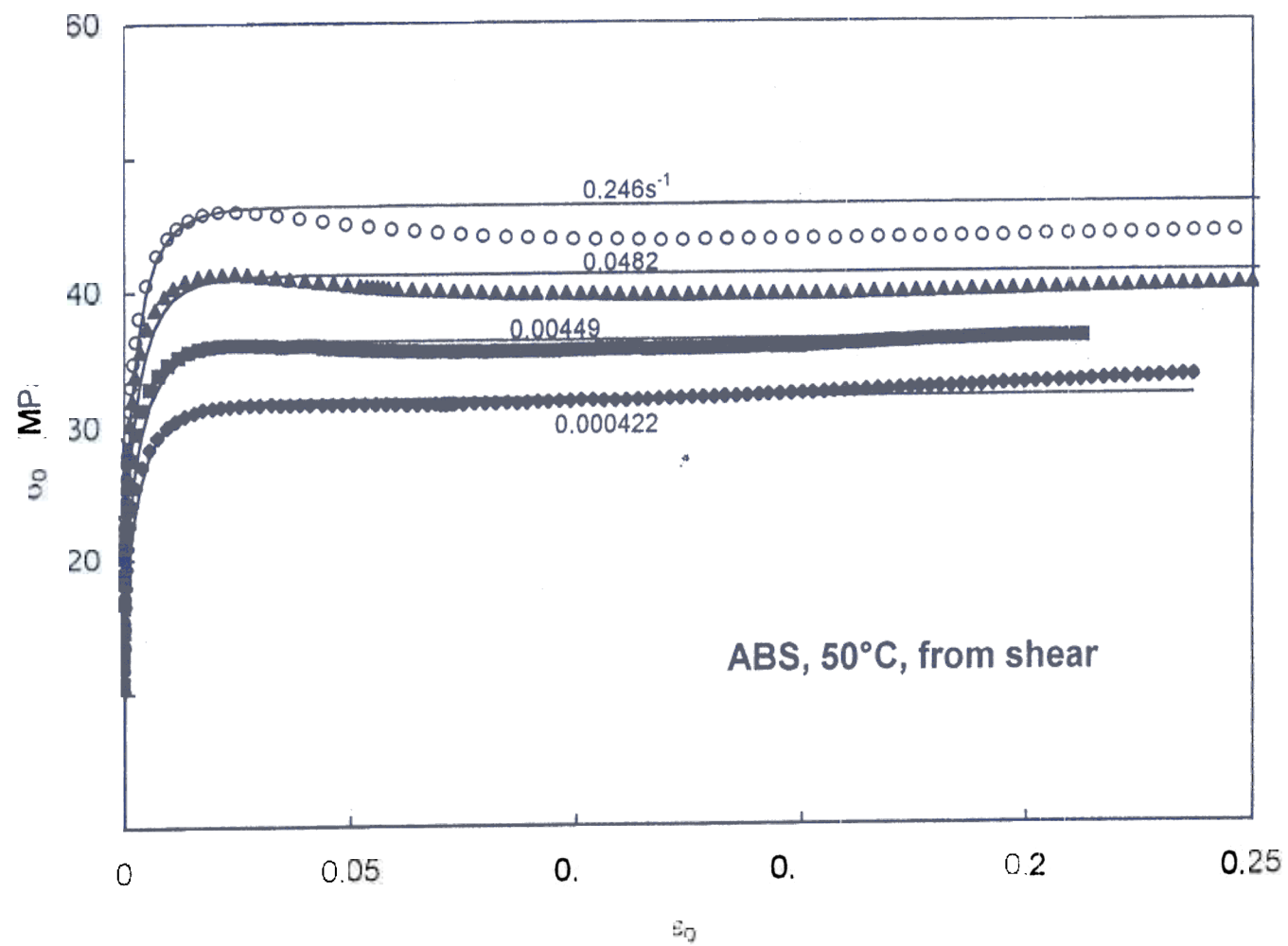
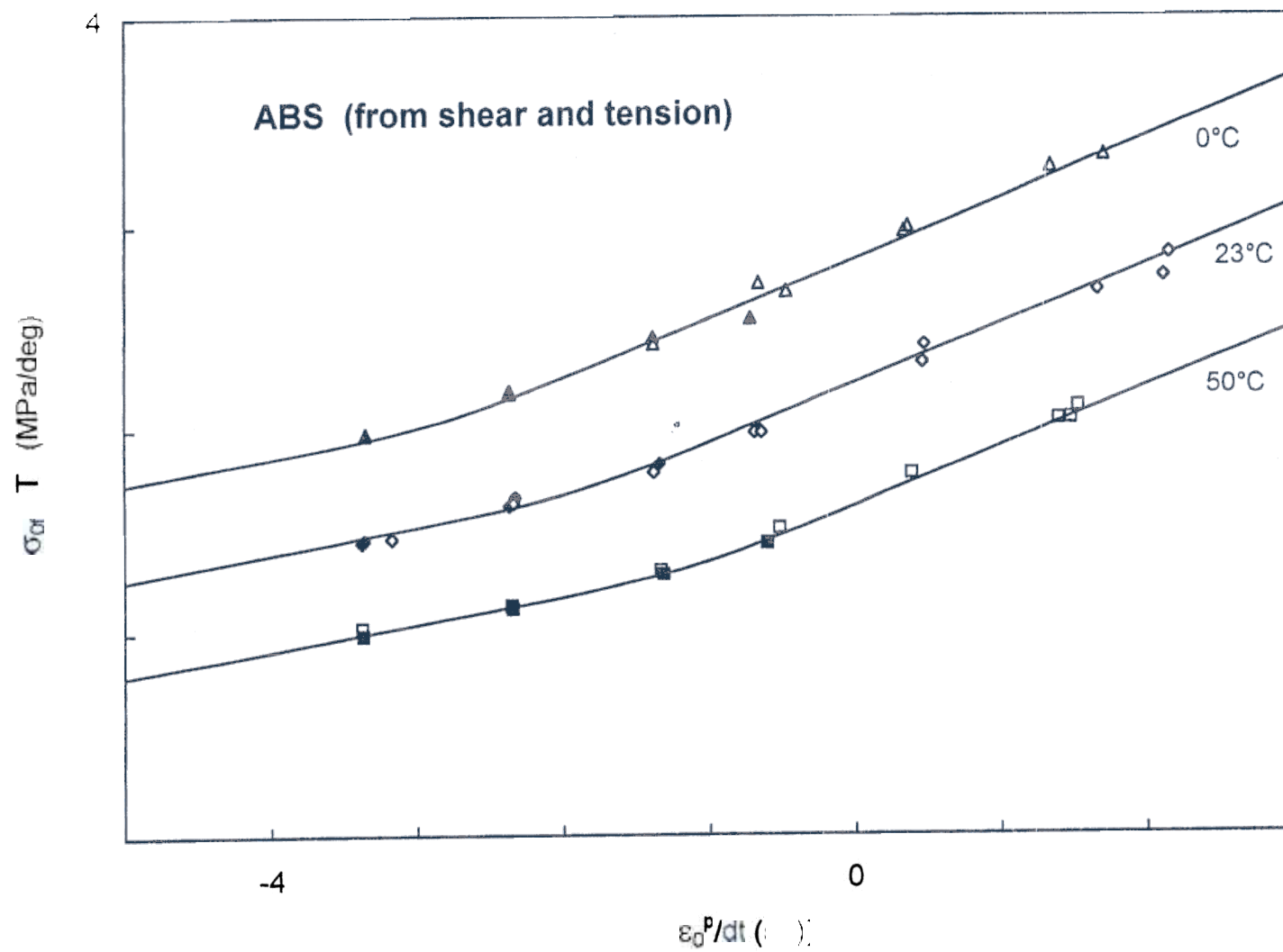


Fig 4



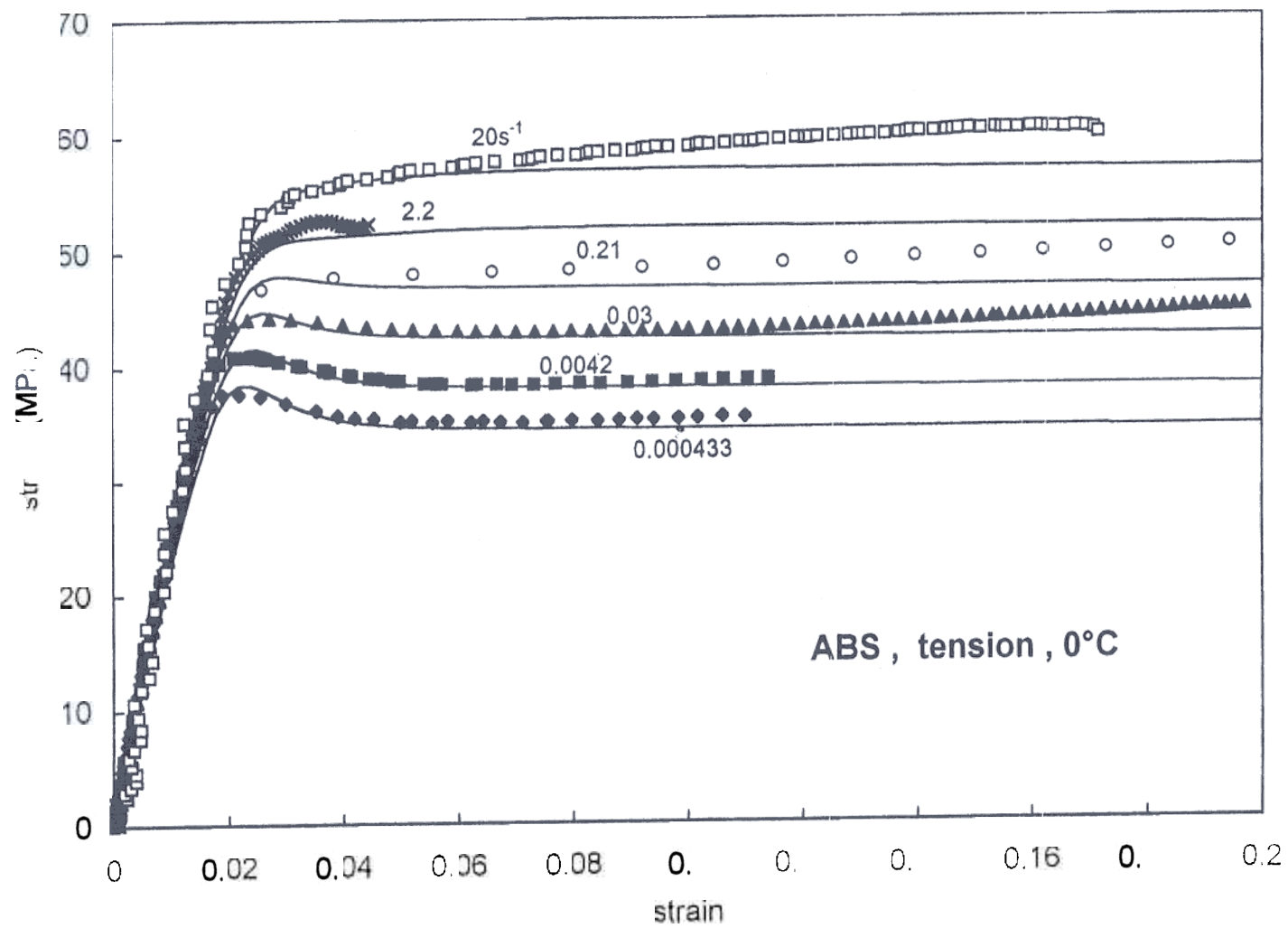


Fig 6

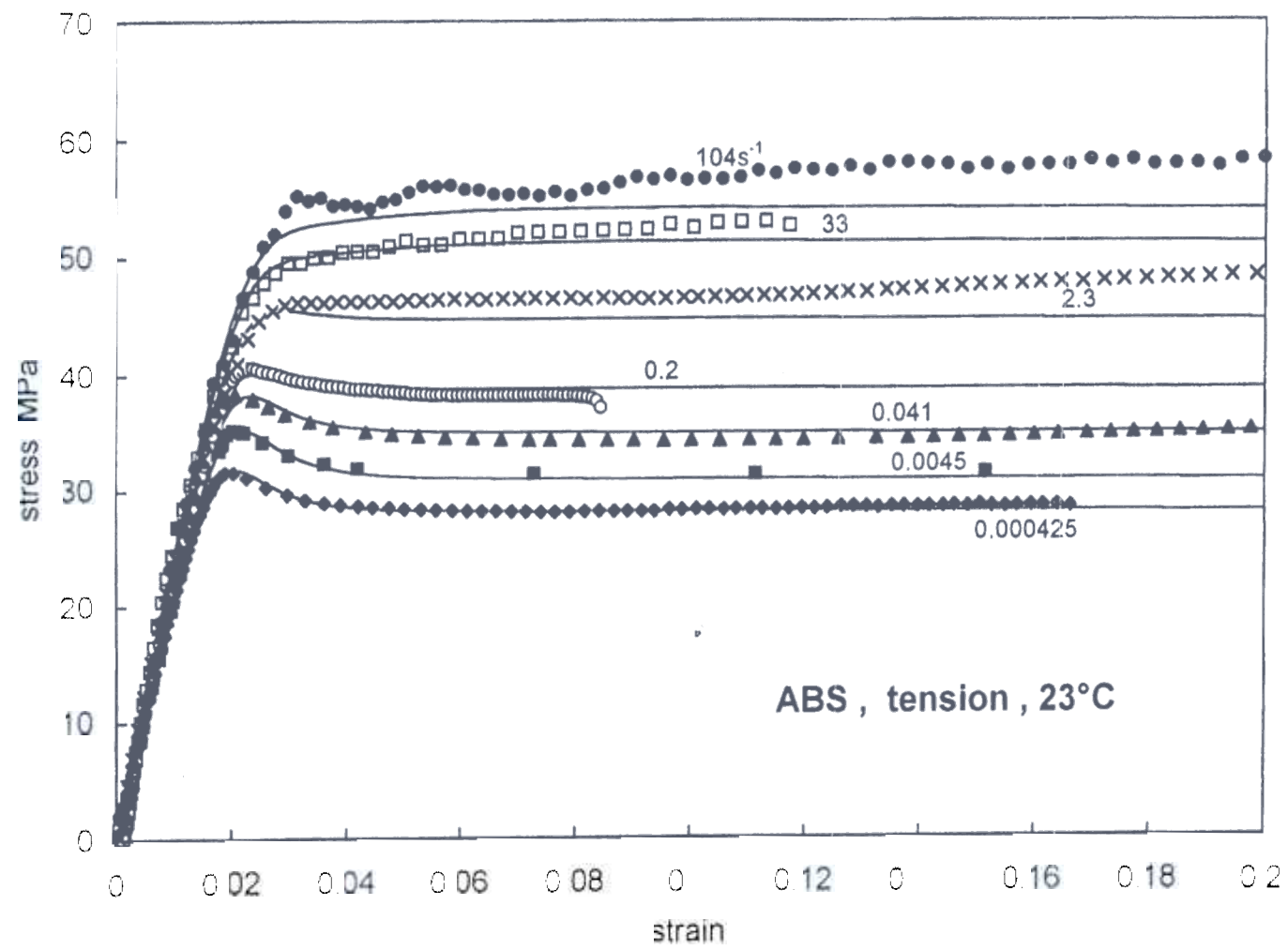
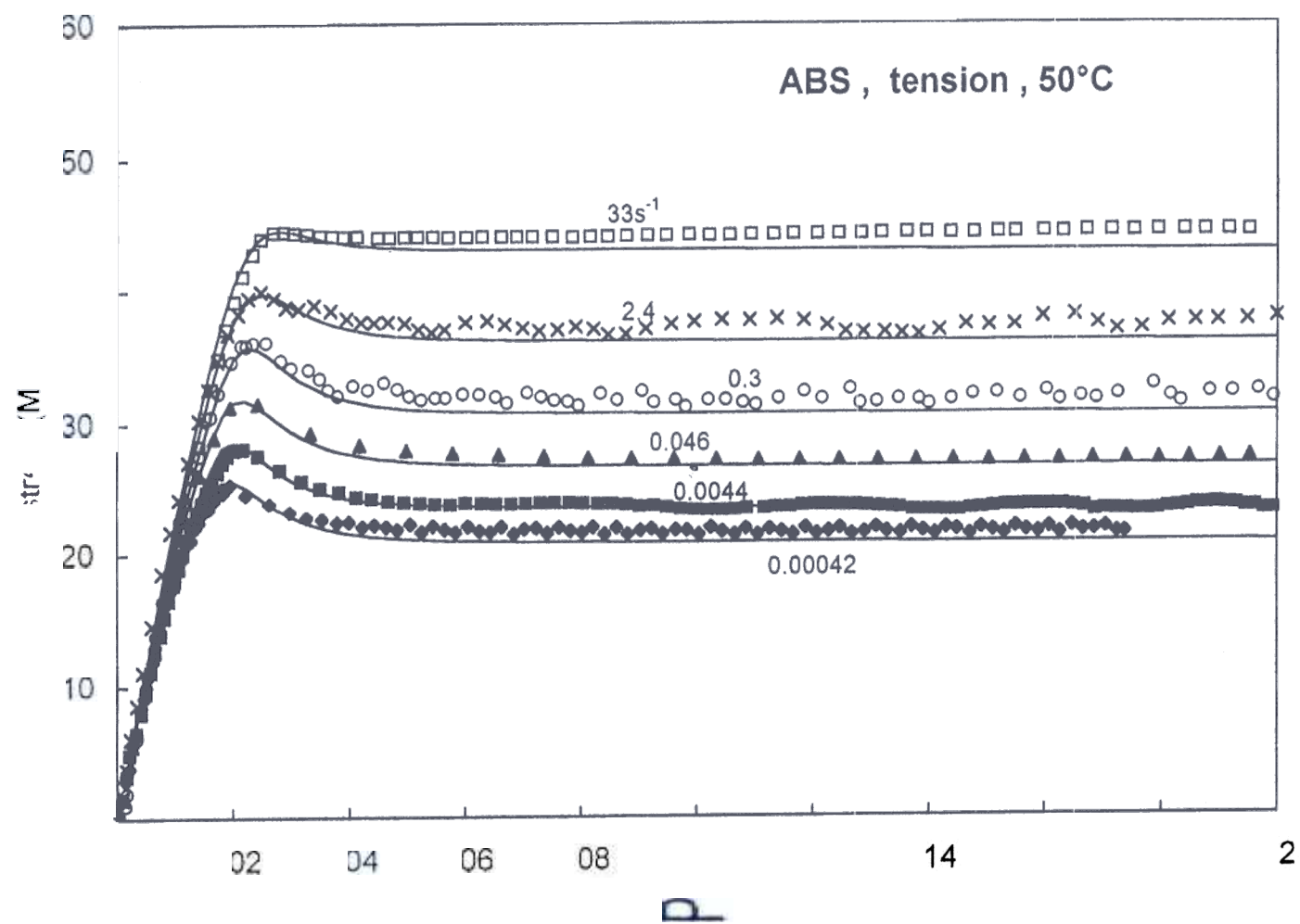


Fig.7



Fig