

A Cryogenic Current Comparator for Quantum Hall Resistance Ratio Measurements

Peter Kleinschmidt, Jonathan Williams, Nick Fletcher and Jan-Theodoor Janssen

National Physical Laboratory

1. Abstract

A new cryogenic current comparator (CCC) bridge for quantum Hall resistance ratio measurement has been constructed. Compared to previous CCC systems at NPL the new system differs in two aspects. Firstly, it uses a type II CCC, i.e. the ratio windings are on the outside and the detection winding is on the inside. This has the advantage that the CCC can be made very compact and fits into a standard storage Dewar. Secondly, the bridge electronics uses a digital servo and the SQUID feedback loop is disabled during current reversals. The overall relative measurement uncertainty of the system is of the order 10^{-8} .

2. Introduction

Cryogenic current comparator (CCC) bridges are used for routine calibration of resistors in the range from $10\text{ k}\Omega$ to $100\text{ }\mu\Omega$ at the National Physical Laboratory (NPL) [1,2]. This range will be extended to resistors up to $1\text{ G}\Omega$ in the near future [3]. Here, we describe a new cryogenic current comparator bridge for quantum Hall resistance ratio measurements. Unlike existing CCC bridges at NPL, the new bridge uses a digital servo, and a new CCC design was employed. The bridge is primarily designed for measurement of a $50\text{ }\Omega$ resistor, which is part of the Moving-Coil Watt experiment at NPL, in terms of the quantum Hall resistance, but it is also capable of measuring other resistance ratios.

3. Bridge Circuit

The bridge circuit (figure 1) is energized by the current sources S_1 and S_2 , with the currents set in the reciprocal ratio of the winding ratio by DAC 1 and DAC 2. The DACs are controlled by a PC via an optical fibre ring. The currents are filtered (as described in [1], not shown in diagram) and pass through resistors R_K and R_Y and windings N_X and N_Y of the CCC, respectively. A SQUID (Superconducting QUantum Interference Device) senses the resulting flux linkage in the CCC, and ADC 1 feeds its output back into the PC. The PC calculates a correction and sets DAC 3 to correct the current ratio. The two halves of the bridge are joined by a link between potential terminals of R_K and R_Y . The other two potential terminals are connected to the null detector D, which is read by ADC2. The detector reading is proportional to the difference between the deviations of the actual resistor values from their nominal values. For a complete measurement, the currents are reversed several times. While the currents are reversed, the servo, which corrects the current ratio, does not operate, and afterwards the SQUID electronics has to be reset. The SQUID therefore can only detect the flux linkage of the CCC modulo one flux quantum (F_0). This implies that when reversing the current, the flux linkage must stay the same to within one Φ_0 . Current sources and DACs of sufficient linearity are therefore required. The signal of the null detector is calibrated by injecting a current, set by DAC 4, into a separate one-turn winding. The bridge circuit also includes a Wagner circuit, which maintains the screen at the same potential as the point joining resistors and windings (see [1]).

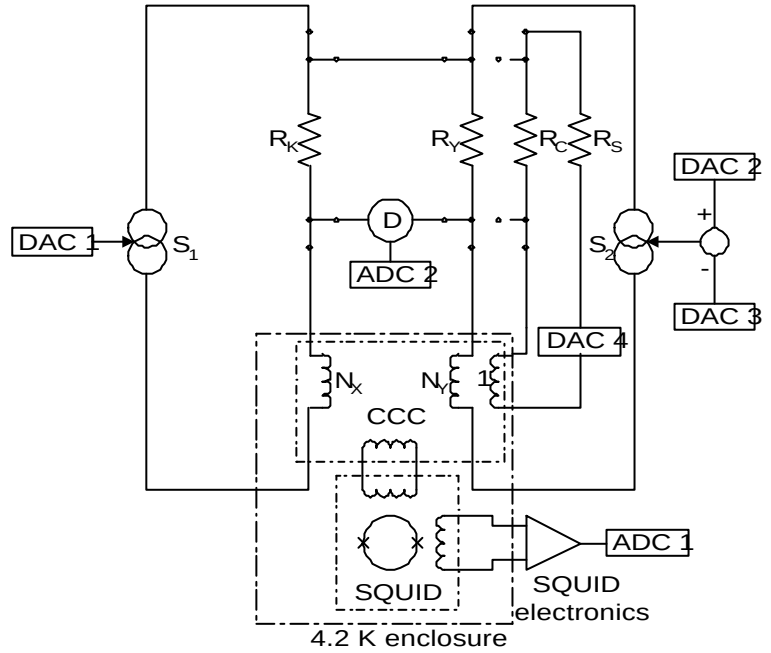


Figure 1: Schematic diagram of bridge circuit

4. Cryogenic Current Comparator

A detailed description of the cryogenic current comparator used in the new bridge will be given elsewhere [4], so we only provide a brief outline of its characteristics here. A type II design was chosen (current carrying windings on the outside, detection winding on the inside), with dimensions such that the CCC fits into the 2-inch neck of a standard storage Dewar. In combination with a DC SQUID a sensitivity of about $20 \mu\text{At}/\Phi_0$ was obtained. The specified noise level of the DC SQUID is about $3 \times 10^{-6} \Phi_0/\sqrt{\text{Hz}}$ resulting in a current noise of $60 \text{ pAt}/\sqrt{\text{Hz}}$. The windings were wound as a rope of 18 strands, wound 32 times, a rope of 20 strands, wound 100 times, and a number of separate windings, connected to give the numbers of turns listed in table 1. Table 2

Combination	Resulting Windings
$20 \times 100 \text{ t} + 2 \times 32 \text{ t} + 1 \text{ t}$	2065 t
$10 \times 32 \text{ t}$	320 t
$5 \times 32 \text{ t}$	160 t
Separate windings	32 t, 16 t, 8 t, 1 t, 1 t

Table 1: CCC windings

Measurement	Turns	Currents
$50 \Omega / \text{QHR}(4)$	16 / 2065	16 mA / 120 μA
$50 \Omega / \text{QHR}(2)$	8 / 2065	16 mA / 60 μA
$100 \Omega / \text{QHR}(4)$	32 / 2065	3 mA / 50 μA
$100 \Omega / \text{QHR}(2)$	16 / 2065	3 mA / 25 μA

Table 2: Selection of possible winding and resistance ratios

contains some of the possible resistance ratios which can be measured using these windings.

5. Current Sources

The circuit of the current source is shown in figure 2. It is a modification of the current sources used in previous CCC bridges at NPL [5], with improved linearity. The circuit consists of two halves, the output currents of which are summed to give I_{out} . The current in the top (bottom) half is sensed by resistor R_{10} (R_{11}). The operational amplifier A_1 (A_2) maintains this current such that the voltage fed back to its non-inverting input by divider R_6 and R_7 (R_8 and R_9) equals that at its inverting input set by the input voltage and divider R_1 to R_5 . The resulting output current is proportional to the input voltage, with the gain determined by $R_{10} = R_{11}$ and the ratio $R_6/R_7 = R_8/R_9$. The main difference compared to the circuit described in [5] lies in a reduction of common mode voltage at the input of A_1 and A_2 . This was achieved by using V_{R1} (ground) as positive and ground (V_{R2}) as negative supply voltages of A_1 (A_2). The reduction of common mode proved to be crucial in achieving the required linearity. Due to the change in the supply voltages, T_3 and T_4 had to be changed from a common-base configuration to a common-emitter configuration. After choosing R_{10} and R_{11} such that self-heating did not affect the performance of the current source (in case of the largest range several precision wire-wound resistors in parallel proved necessary), linearity to within ± 10 ppm was achieved.

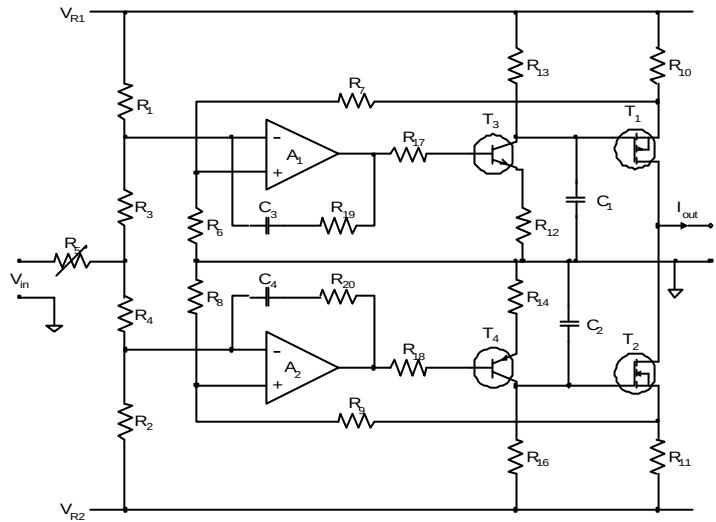


Figure 2: Schematic diagram of current source

6. Digital Servo

The objective of the digital servo is to correct current source S_2 via DAC 3 so that the ampere-turn imbalance in the CCC, as measured at the SQUID output, is as close as possible to zero. The equation for the digital servo is derived from the circuit behaviour of a simple analogue feedback loop with a 20 dB/decade roll-off in loop gain. Compensation for two time constants in the feedback loop is required arising from the CCC filter which has a bandwidth of 100 Hz and the anti-aliasing filter at the SQUID output which has a bandwidth of 10 Hz. The equation is

$$-\Delta V_2 = \frac{V_1}{t_1} \cdot \Delta t + \frac{(t_2 + t_3)}{t_1} \cdot \Delta V_1 + \frac{t_2 t_3}{t_1} \cdot \frac{\Delta V_1}{\Delta t}$$

where V_1 is the voltage at the SQUID output, V_2 is the correction signal generated by DAC 3, Δt is the time interval between samples, τ_2 and τ_3 are equal to the two time constants to be compensated and τ_1 controls the overall gain of the servo. Observing a Fourier transform of the samples of the SQUID output V_1 as a function of the time constant parameters optimised the servo. The Fourier transform clearly shows the noise spectrum from the current sources and the amount it is reduced by the servo. Under optimum conditions, the servo reduces the noise by 30 to 100 times at a frequency of 0.1 Hz (which is the region of interest for precision measurements). The SQUID output is sampled using an ADC with a 1 ms integration time and a corresponding resolution of about 15 bits. The servo operates at just under 1000 samples per second.

An important part of the operation of the servo is that it has to lock to the same Φ_0 when the servo is closed after each current reversal. It is found that, provided the current sources are adjusted to better than $\Phi_0/2$, the servo will reliably lock to the correct Φ_0 without any difficulty.

7. Calibration

The null detector is read directly by ADC 2 and to convert these voltage readings into resistance ratio deviations, the relationship between the voltage measured by ADC 2 and the voltage across the resistors in the bridge has to be determined. There are many ways of doing this and the method of placing a high resistance shunt in parallel with one of the resistors being measured was adopted in earlier bridges at NPL [1]. The method used here is a variation on this idea. Before a measurement is started, three settings of DAC 4 are found so that the voltage across R_C is equal to the voltage across R_Y for forward, reverse and zero measurement currents. A measurement sequence then consists of a number of current reversals with DAC 4 set to the zero current value, followed by a further set of reversals with DAC 4 set to the appropriate forward or reverse value. This has then the effect of generating a calibrated change in the null detector reading as if a shunt of value $N_Y R_C$ had been connected across R_Y . The advantage of this virtual shunt connection technique is that no switching relays are involved and there are no awkward lead resistance corrections to make. The voltage generated by DAC 4 only needs to be stable and reproducible during the course of a measurement. If DAC 4 were sufficiently linear, a further enhancement would be to choose output voltages so that the output from the null detector is zero, thereby eliminating the linearity of this component.

8. Results

Figure 3 shows the results of a typical measurement sequence lasting approximately 7.5 minutes. The detector signal is recorded for several reversals (8 in this case) of the bridge current sources. For the last 3 reversals the calibration is included by setting DAC 4 to the required voltages, as explained in section 7 (note that in this case the calibration makes the detector signal smaller). This data is then analysed using a least squares fitting routine. Four parameters are obtained from the fit, the offset signal caused by thermal emfs in the bridge, the linear drift caused by changing flux linkage to the SQUID, the voltage as a result of the difference in R_K and R_Y and the voltage difference as a result of the calibration. These last two parameters are used to calculate the value of the unknown resistor R_Y .

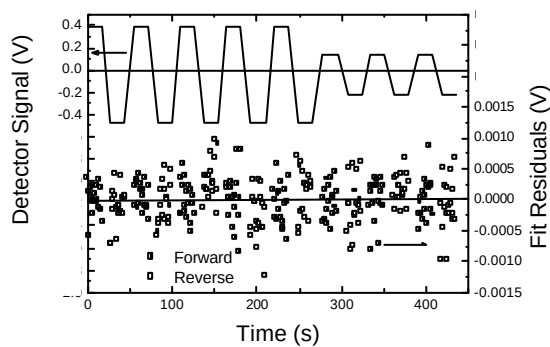


Figure 3: Detector signal (with gain of 10^4) and fitting residuals for a typical measurement. The residuals demonstrate that no systematic signal is left in the data.

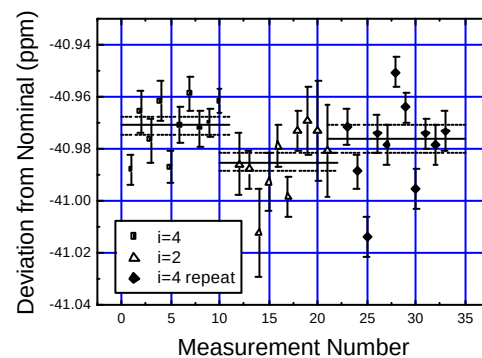
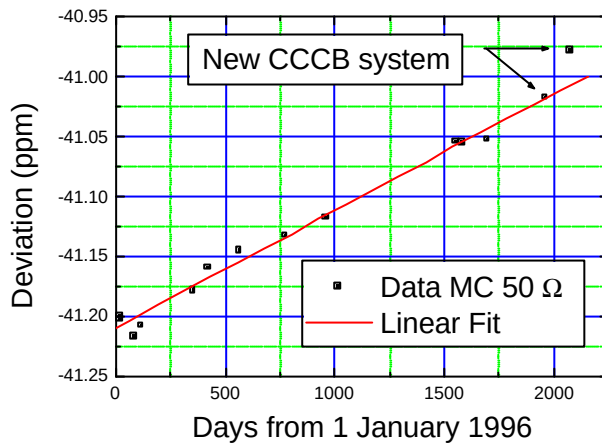


Figure 4: Results of 50Ω against $i = 4$ and $i = 2$ QHR. The solid lines represent the weighted means and the dashed lines the 1σ deviations.

Figure 4 shows the results of a series of measurements for a $50\ \Omega$ resistor against the quantum Hall resistance at the $i = 4$ and $i = 2$ plateaux. The type-A uncertainty on the individual data points is between 8-9 ppb and the type-A uncertainty on the weighted mean between 3-4 ppb. We observe that the last two data sets are in good agreement with each other. However, the first data set appears to be too high. A possible explanation could be the improper quantisation of the QHR device. The measurements of $i = 4$ against $50\ \Omega$ results in a current of $120\ \mu\text{A}$ through the device and the device temperature in our system is 1.3 K. It is well known [6] that under such conditions the plateau width reduces and the device might even become un-quantised. We therefore slightly adjusted the magnetic field closer to centre value for the second $i = 4$ measurement set. The results show that improper quantisation of the device cannot be ruled out and a more systematic analysis of the device characteristics as a function of measurement current and temperature is needed.



Finally, figure 5 shows the behaviour of the $50\ \Omega$ resistor over the last 5 years as measured using the existing CCC bridge [1]. The last two points are the results of the new CCCB system. The first new CCCB point falls exactly on the resistor drift curve, however, the second new CCCB point is significantly above this curve (0.04 ppm). In between these two data points a temperature control failure occurred in the laboratory, which caused the resistor to cool down and change its value. This result demonstrates the importance of both a quantum standard and reliable temperature control of artefact standards.

Figure 5: Resistor value versus time for the last five years.

9. Conclusion

In conclusion, we have developed an automated CCC bridge for measuring resistance standards against a quantum Hall device. The bridge uses a digital servo and the SQUID feedback loop is disabled during current reversals. Furthermore, the CCC is housed in a normal helium transport Dewar and the temperature of the quantum Hall device is obtained using a pumped ^4He bath. The system is therefore relatively cheap and easy to operate. The standard type-A uncertainty over ten measurements is approximately 3ppb. We believe that an improvement to sub-ppb level should still be possible.

10. Acknowledgement

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11. References

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