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Deformation of a Centrally Loaded Circular Plate of a Thermoplastic Material: Comparison of Measurements with Predictions using Finite Element Analysis

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SUMMARY

Finite element analyses have been carried out on specimens of an ABS and a propylene-ethylene copolymer in the form of a circular plate that is supported on a circular annulus and loaded at its centre by a hemispherical surface. Predictions have been made of force vs central deflection under different loading speeds using elastic-plastic materials models in Abaqus. Two models have been considered based on the von Mises and the linear Drucker-Prager yield criteria. The materials parameters for these analyses were determined from tensile and shear tests on the polymers. Calculations have also been made of surface strain distributions in the deformed specimens.

Measurements of force/deflection curves and surface displacements have been made and compared with predicted results from the two models. Force/deflection curves compare closely with measured values but predictions of surface displacements differ widely between the two models and also with experiment.

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Approved on behalf of Managing Director, NPL, by Dr C Lea,
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1 INTRODUCTION

Many thermoplastics are tough materials that can sustain large strains before failure. These materials are commonly used in applications where accidental impact loading is possible and the material must either withstand this without failure or must limit the force level sustained by other objects in the impact event. For these applications, design methods are needed to enable predictions to be made of force, deformation, stresses and strains in a component. These calculations need to take accurate account of the non-linear and rate-dependent properties of the thermoplastic. They can be made using finite element analyses if suitable materials data are available. The materials models that are available for carrying out such calculations are based on rate-dependent plasticity and have been developed to describe the behaviour of metals and some other materials. However, the validity of the models for plastics and the accuracy of calculations based on them are uncertain.

The purpose of the work reported here is to evaluate some of the available materials models through comparisons of the predicted behaviour of a component under load with experimental measurements. The component is a circular plate that is simply supported near its circumference and centrally loaded by means of a hemispherical surface (figure 1). Finite element analyses have been used to calculate force/deflection curves and surface strain distributions under different loading speeds. These calculations have used elastic-plastic materials models based on the von Mises and the linear Drucker-Prager yield criteria. The materials studied are a propylene-ethylene copolymer and an acrylonitrile-butadiene-styrene (ABS) polymer. Materials properties for these materials have been determined from tensile and shear tests carried out over a wide range of strain rate.

Apparatus has been constructed to allow measurements to be made of force vs deflection for the plate specimens under loading speeds ranging from 0.1 mm/s to 1 m/s. Tests have also been carried out using falling weight impact at an incident speed of the impactor of 4 m/s. The design of the apparatus also allows measurements of displacement of reference marks on the lower surface of the plate to be made at the lower testing speeds. These have also been compared with predictions of displacements made by the finite element analyses.

A new materials model is being developed as part of this work and will be implemented in an FE package. A later report will be concerned with comparisons of predictions of plate deformation made using the new model with the calculations and results reported here.

2 MATERIALS

Two materials have been studied for this work. One is a propylene-ethylene copolymer supplied by BASF under the trade name Novolen 2300 LL. The other is an acrylonitrile-butadiene-styrene (ABS) supplied by BASF under the trade name Terluran. These materials were obtained as compression moulded sheets from which test specimens were machined. The nominal thickness of the copolymer sheet was 3 mm and of the ABS sheet 3.6 mm.

3 APPARATUS

The apparatus developed for loading the plate specimens is shown in figure 1. The specimen has a diameter of 120 mm and rests on a circular support of diameter 100 mm which is the upper edge of a cylindrical tube. The edge has a radius of 5 mm. The cylinder is fixed to the moving cross-head of a universal testing machine. The specimen is loaded through a hemispherical steel surface that is bolted to the upper part of the loading assembly. Clamped to this part is a bracket which passes through a slot in the wall of the cylinder and supports a mirror at an angle of 45° to the vertical. This allows the lower surface of the plate to be viewed by a camera positioned remote from the test assembly. The specimen is loaded by raising the supporting cylinder. In this way, the centre of the specimen remains fixed with respect to the photographic plane of the camera. The central deflection of the plate is determined from measurement of the cross-head movement.

Measurements of surface displacement were made at a single speed of 0.1 mm/s. Load against central deflection measurements were made over a range of loading speeds. For speeds up to 1 mm/s, the apparatus was mounted in a screw driven tensile testing machine. For higher speeds up to 1 m/s, a servohydraulic machine was used. Some results at 4 m/s

were obtained using falling weight impact apparatus in which the hemispherical part was bolted to the falling carriage.

4 MEASUREMENT OF SPECIMEN DEFORMATION AND SURFACE DISPLACEMENTS

Measurements of the force on plate specimens with increasing central deflection have been made at selected loading speeds between 0.1 mm/s and 4 m/s. Results for the propylene-ethylene copolymer are shown in figure 15 and for the ABS in figure 16 where they are compared with predictions made using finite element analyses. These results are discussed in section 7.

These comparisons are made to evaluate the predictive accuracy of different materials models. A more critical evaluation of predictive accuracy is likely to be made by comparing measured and predicted levels of strain in the lower surface of the plate. Strain gauges cannot be used for the measurement of surface strain because their strain range is limited and their stiffness is too high for obtaining accurate results. Optical methods are available but their use in a confined space and for measuring strains over a curved surface is not routine. The strain distribution in the lower surface of the specimen has therefore been determined by a photographic method which involves measuring the movement of reference marks on this surface with increasing deflection of the plate.

Reference marks were positioned along 2 perpendicular lines through the centre of the face of the specimen. These lines were drawn with a black felt-tip pen. A very fine line was scribed through the centre of these lines using a sharp stylus. The stylus was also used to define a cross at radial distances of 5, 10, 15, 20, 25 and 30 mm from the centre of the plate. The specimen surface was photographed using a Nikon D1 digital camera with a Sigma 180 mm macro lens and a dedicated flash. The light sensing element in this camera consists of an array of 1320 x 2010 pixels. Simple image processing software was used to display a photographic image on a screen. Each fine cross appeared lighter than the surrounding area of black ink, and its centre could be located by eye to a single pixel enabling the coordinates to be identified.

The camera was set up so that its axis was at right angles to the image of the specimen in the mirror. This was achieved by replacing the specimen by a plane mirror and adjusting the camera orientation until it looked directly at its own reflection. The camera was positioned at a distance of 930 mm from the plate surface at which point the reference marks on the specimen filled the field of view. The photographic system was calibrated to enable horizontal displacements of the reference points during plate deformation to be determined from the changes in pixel coordinates. As the plate is deformed, the central region stays at a constant distance from the camera whereas the working distances of reference marks away from the centre increase with specimen deformation. This causes a change in the magnification of the system and hence in the calibration.

Consideration of the optics of the system shows that an increase of Δu in the undeformed working distance u leads to a reduction in the magnification by a factor $\frac{u}{u + \Delta u}$. The change in distance Δu increases with the radial position of the reference point and the deflection of the plate. The finite element analyses reported in section 6 show that at a radial position of 30 mm and a plate deflection of 30 mm, $\Delta u = 15$ mm. Taking $u = 930$ mm reveals that the largest correction that would need to be applied for the change in working distance is 1.6%. This is less than the precision and reproducibility of measurements so a correction was unnecessary.

Figure 2a shows measured values for the horizontal displacement of each reference point at specific deflections of the propylene-ethylene copolymer specimen, and figure 2b shows the corresponding results for the ABS specimen. The results for the copolymer were average values for 4 tests. The average values from 2 tests are shown for the ABS specimen. The precision in measurements is $\pm 5\%$.

5 DETERMINATION OF MATERIALS PROPERTIES

5.1 DATA REQUIREMENTS FOR FINITE ELEMENT ANALYSES

At small strains, behaviour is assumed to be linear elastic and is characterised by a Young's modulus E and Poisson's ratio ν_e . For the propylene copolymer, Young's modulus was observed to be rate-dependent and the appropriate value for E in analyses for different speeds of loading was chosen based on the mean strain rate in the specimen for each loading speed (see section 6.1).

The onset of non-linearity in a stress/strain curve is ascribed to plastic deformation and occurs at a stress level referred to as the first yield stress. The subsequent increase of stress with strain is associated with the effects of strain hardening. In this non-linear region, the total strain ϵ is considered as the sum of an elastic component ϵ_e and a plastic component ϵ_p so

$$\epsilon = \epsilon_e + \epsilon_p \quad (1)$$

Descriptions of plastic deformation are based on a yield criterion. Two criteria are considered here. The simplest is that proposed by von Mises which assumes that yielding occurs at a critical value of the shear stress. This can be expressed as

$$\sigma_T = \sqrt{3} J_{2D}^{1/2} \quad (2)$$

where σ_T is a yield stress in tension and is a material property that varies with plastic strain ϵ_p , which, from equation (1), is calculated using

$$\epsilon_p = \epsilon - \frac{\sigma_T}{E} \quad (3)$$

The term $J_{2D}^{1/2}$ is an invariant of the applied stress tensor and is related to principal stress components σ_1 , σ_2 and σ_3 by

$$J_{2D} = \frac{1}{6} \left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 \right]$$

Equation (2) therefore defines a state of plastic strain hardening characterised by the function $\sigma_T(\epsilon_p)$. This function is required by the analysis. For analyses that are based on rate-dependent plasticity, it is necessary to determine the hardening function over a range of plastic strain rate.

For the majority of plastics, it is known that equation (2) is an approximation since yielding is also sensitive to the hydrostatic stress. A more realistic criterion for plastics is derived from equation (2) and takes the form

$$\sigma_T = \frac{\sqrt{3}(\lambda + 1)}{2\lambda} J_{2D}^{1/2} + \frac{(\lambda - 1)}{2\lambda} J_1$$

where λ is a material property which is a measure of the sensitivity of yielding to the hydrostatic component of stress J_1 given by

$$J_1 = \sigma_1 + \sigma_2 + \sigma_3$$

Equation (5) is implemented in Abaqus where it is referred to as the linear Drucker-Prager model. Here it takes the form

$$d = q - p \tan \beta \quad (7)$$

where

$$\begin{aligned} q &= \sqrt{3} J_{2D}^{1/2}, & p &= -\frac{J_1}{3} \\ d &= \frac{2\lambda\sigma_T}{(\lambda + 1)}, & \tan \beta &= \frac{3(\lambda - 1)}{(\lambda + 1)} \end{aligned} \quad (8)$$

A flow parameter $\tan \psi$ is also required for the Drucker-Prager model. It can be determined from measurements of the plastic component of Poissons ratio ν_p in uniaxial tensile tests using the expression

$$\tan \psi = \frac{3(1 - 2\nu_p)}{2(1 + \nu_p)} \quad (9)$$

In the absence of data for ν_p , $\tan \psi$ can be equated to $\tan \beta$ which invokes an assumption of associated flow. The sensitivity of calculations to this assumption is explored in Section 6.

5.2 TEST METHODS

The property data introduced in the previous section were measured using tension and shear test methods described in earlier reports. Figures 3 and 4 show true stress/true strain curves for the propylene copolymer and the ABS materials measured in tension over a wide range of strain rate. At strain rates between 0.0003 s^{-1} and 0.06 s^{-1} , measurements were obtained using the ISO multipurpose test specimen and cross-head speeds of 1, 10 and 100 mm/min. Strain measurements at low values (≤ 0.01) were made using contacting extensometers to measure both the longitudinal and transverse strains. At higher strains, a contacting lateral extensometer was used for transverse strain measurements, and a video extensometer was used to obtain longitudinal strains. Results showing the variation of Poisson's ratio with strain for each material are shown at a test speed of 10 mm/min in figure 5. No significant variation of Poisson's ratio with strain rate was discernable from these tests at other speeds for either material. These results were therefore used to calculate values for true tensile stress shown in figures 3 and 4 and for the flow parameter (see equation (9)).

Data at strain rates between 0.2 and 30 were obtained using tensile specimens having the same geometry as the standard multipurpose specimen but scaled to half the size. A servohydraulic test machine was used to achieve cross-head speeds in the range 10 to 1000 mm/s. Contacting extensometers were used to measure longitudinal strain up to a strain of about 0.1 in tests carried out at speeds of 10 and 100 mm/s. At higher strains, and in tests at the higher speed of 1 m/s, longitudinal strains were derived from measurements of cross-

head movement. Data at the highest strain rate were obtained from falling-weight impact tests at a speed at contact of 4 m/s.

Shear stress/strain measurements were made using an Arcan notched-plate shear test with extensometers developed at NPL. Small corrections were made to the measurements to allow for bending displacements and the non-uniformity in stress in the gauge section of the specimen. Results for the two materials are shown with tensile data in figure 5. The cross-head speed was chosen to give a shear plastic strain rate of about 0.007 s^{-1} which corresponds to an effective strain rate of 0.004 s^{-1} , based on von Mises yielding, which is comparable with the rate for the tensile data.

5.3 DETERMINATION OF PROPERTIES FOR FINITE ELEMENT ANALYSES

Elastic properties

Values for Young's modulus were obtained from the tensile data in figures 3 and 4 at strains below 0.01. They are listed in table 1 and show a significant dependence upon strain rate for the propylene copolymer. These data demonstrate that plastics are viscoelastic but in the finite element analyses that follow this behaviour is described by rate-dependent elasticity.

Strain hardening functions

Strain hardening curves were obtained from the curves in figures 3 and 4 by subtracting the elastic component of strain at each stress using equation (3). This gives plots of true yield stress against plastic strain which are shown as a function of strain rate for each material in figures 7 and 8.

The hydrostatic stress sensitivity parameter λ or $\tan \beta$ in the linear Drucker-Prager yield criterion (see equation (5) or (7)) is determined from stress/strain data in shear and tension measured at the same effective plastic strain rate. A value for $\tan \beta$ was determined using the equation

$$\tan \beta = 3 \left(\frac{\sqrt{3} \sigma_s}{\sigma_T} - 1 \right)$$

where σ_s and σ_T are shear and tensile yield stresses respectively obtained at the same effective plastic strain $\bar{\epsilon}_p$. The definition of effective plastic strain used here for the determination of model parameters is based on von Mises yielding. Thus

$$\bar{\epsilon}_p = \frac{2 I_{2D}^{1/2}}{\sqrt{3}}$$

where

$$I_{2D}^{1/2} = \left\{ \frac{1}{6} [(\epsilon_{p1} - \epsilon_{p2})^2 + (\epsilon_{p2} - \epsilon_{p3})^2 + (\epsilon_{p3} - \epsilon_{p1})^2] \right\}^{1/2}$$

and ϵ_{p1} , ϵ_{p2} and ϵ_{p3} are components of principal plastic strain. It follows that under uniaxial tension

$$\bar{\epsilon}_p = \epsilon_{pT}, \text{ the plastic tensile strain}$$

and in shear

$$\bar{\epsilon}_p = \frac{\gamma_p}{\sqrt{3}}$$

where γ_p is the plastic component of the engineering shear strain and is equal to 2x the plastic component of the tensor shear strain. Similarly, the effective yield stress $\bar{\sigma}$ is given by the expression

$$\bar{\sigma} = \sqrt{3} J_{2D}^{1/2} = q$$

so that in tension

$$\bar{\sigma} = \sigma_T, \text{ the tensile yield stress}$$

and in shear

$$\bar{\sigma} = \sqrt{3} \sigma_s$$

Using equations (10) to (14), the measured stress/strain curves in tension and shear shown in figure 5 have been plotted on axes of effective stress against effective plastic strain in figure 6. In this way, values for $\sqrt{3} \sigma_s$ and σ_T can be readily selected at the same effective strain. Values were taken at an effective plastic strain of 0.03 and were used with equation 10 to calculate the values for $\tan \beta$ recorded in table 2. At smaller strains, the calculated $\tan \beta$ is dependent on the strain chosen. This arises because the shape of the tensile and shear hardening curves are different and is an indication of the limited validity of the linear Drucker-Prager model for these plastics materials.

The flow parameter ψ was determined using equation (9) with values for the plastic component of Poisson's ratio ν_p derived from the Poissons ratio measurements shown in figure 5. Values for $\tan \psi$ are given in table 2 for each material.

Table 1 - Values for Young's modulus E at different plastic strain rates $\dot{\epsilon}_p$ for the propylene copolymer and the ABS determined from experimental data in figures 3 and 4

Propylene-ethylene copolymer		ABS	
E(GPa)	$\dot{\epsilon}_p$	E(GPa)	$\dot{\epsilon}_p$
1.43	0.00035	2.27	0.0006
1.60	0.004	2.30	0.0045
1.66	0.027	2.34	0.058
1.75	0.2	2.36	0.22
1.90	2.1	2.40	2.3
2.05	29	2.44	33
2.15	91	2.47	104

Table 2 - Values for some of the parameters required for FE analyses using the linear Drucker-Prager materials models

	$\tan \beta$	ν_e	ν_p	$\tan \psi$	$\sigma(\epsilon_p)$
Propylene copolymer	0.88	0.39	0.2	0.75	see fig 7
ABS	1.55	0.38	0.15	0.9	see fig 8

6 PREDICTIONS OF DEFORMATION OF THE PLATE SPECIMEN

6.1 FINITE ELEMENT ANALYSIS

The mesh used for the analysis of deformation of the plate specimen is shown in figure 9. Only a section of the full geometry is required owing to the rotational symmetry of the problem. The supporting cylinder and hemispherical surfaces are modeled as rigid surfaces since their deformation is unlikely to be significant and it makes the contact algorithm converge faster. Boundary conditions were imposed by holding the outer support ring fixed and defining a ramped displacement on the hemisphere at different speeds.

Force levels were taken from the node controlling the hemisphere location. Since only a section of the problem is modeled, the analysis predicts a radial force which is not present in an experiment. When the model results are integrated around the hemisphere, it can be shown that the force recorded by the transducer is equivalent to the vertical component of the calculated force.

Dynamic analyses were carried out in order to simulate the variation of deformation behaviour with loading speed. Solutions were obtained using the explicit solver in Abaqus. The rate dependence of plastic deformation was modeled using the rate dependent hardening curves shown in figures 7 and 8. Numerical values of stress were entered at strain intervals of 0.01. The analysis would not run if all the curves were entered, solutions could only be obtained with a maximum of 4 curves. These were chosen from the curves in figure 7 or 8 by

first calculating a nominal strain rate in the specimen for a selected loading speed. Preliminary analyses had indicated a typical strain level in the specimen at a maximum central deflection and this was divided by the time to reach this deflection at the speed selected. A hardening curve was identified close to this nominal strain rate and two additional curves were selected at strain rates above and below this rate. The curve at the next lower rate was also chosen and assigned a strain rate of zero in accordance with the characterisation of rate-dependent hardening in Abaqus. The Young's modulus value was chosen from table 1 at the strain rate closest to the value derived for the nominal strain rate. Poissons ratio ν_e was assumed independent of rate.

6.2 RESULTS FOR THE PROPYLENE COPOLYMER

Figure 10 shows a comparison of predictions of force against central deflection for the propylene copolymer specimen obtained using two models. The linear Drucker-Prager analysis was obtained using the data in table 2 except that associated flow was assumed with $\tan \beta = \tan \psi = 0.88$. For the von Mises analysis, $\tan \beta$ and $\tan \psi$ were taken as zero. Although there is very little difference between these results, the predictions of strain distributions obtained with these models are significantly different as discussed later. The results in figure 10 were obtained with a coefficient of friction between the plate and the surfaces of the support ring of zero. The influence of friction was investigated by repeat calculations with a coefficient of 0.2. No significant difference was observed.

Predictions of force against deflection at different loading speeds are shown in figure 11. Hardening curves at appropriate strain rates were selected according to the loading speed as explained in section 6.1. Because of the similarity between results predicted using both models, the results shown in figure 11 were obtained using the linear Drucker-Prager model only. These are compared with experimental data in figure 15 and discussed in section 7.

As explained in section 4, in order to make comparisons of predictions of the strain distribution in the specimen with experimental measurements, the horizontal displacements have been calculated of reference points along a radius on the lower surface of the specimen. These calculations have been made at points at radial positions of 5, 10, 15, 20, 25 and

30 mm and are plotted against the central deflection in figure 12. Here comparisons are made between predictions obtained using the von Mises and linear Drucker-Prager models. Comparisons with experiment are discussed in section 7. Repeat calculations where the coefficient of friction at the contact with the support ring was raised from zero to 0.2 gave a small reduction in displacements which increased with the plate deflection but decreased with radial position.

6.3 RESULTS FOR THE ABS POLYMER

The parameters used for finite element analyses of the ABS using the linear Drucker-Prager materials model are given in tables 1 and 2. Analyses were carried out for non-associated flow as indicated by the difference between $\tan\beta$ and $\tan\psi$ values in table 2. A comparison of force/deflection curves for ABS predicted using the von Mises and linear Drucker-Prager models is shown in figure 10 at a loading speed of 0.1 mm/s. As with the propylene copolymer, there is little difference between force predictions below a deflection of 20 mm after which the von Mises results fall below the Drucker-Prager values.

Figure 13 shows force/deflection curves calculated at different loading speeds using the linear Drucker-Prager model. The horizontal displacements of points on a radius on the lower surface of the specimen have been calculated using both models and are plotted against the central deflection of the plate in figure 14. The results in figures 13 and 14 are compared with experimental measurements in section 7.

Predictions of force and surface displacement with increasing deflection have also been made for ABS using the linear Drucker-Prager model with the assumption of associated flow. For these calculations, $\tan\psi = \tan\beta = 1.55$. Comparison with the results for non-associated flow in figures 13 and 14 showed no significant difference over the whole deflection range. This observation was unexpected for the surface displacement results since calculations of strain are expected to be sensitive to the magnitude of $\tan\psi$.

7 COMPARISON OF FE PREDICTIONS WITH MEASUREMENTS

7.1 FORCE/DEFLECTION RESULTS

Measured force/deflection curves for the propylene copolymer are compared with predicted values at different test speeds in figure 15. The linear Drucker-Prager results are shown but the agreement with experiment will be equally satisfactory with the von Mises predictions. It can be noted that the deformation up to a central deflection of 10 mm is determined primarily by elastic deformation. In order to achieve the close agreement between measurements and predictions shown in figure 15, it was therefore necessary to consider rate-dependent elasticity in the model and include Young's modulus values that increased with loading speed.

The experimental results at 4 m/s were obtained using falling weight impact apparatus in which the hemispherical loading surface was attached to the free-falling carriage. The mirror assembly was removed and the cylinder support was fixed to the base of the machine. The mass of the falling carriage was insufficient to maintain a constant velocity which was observed to decrease to nearly zero by the end of the test.

Comparisons of measured and predicted force/deflection curves for the ABS specimens are shown in figure 16 at loading speeds of 0.1 mm/s, 10 mm/s and 1 m/s. The unequal spacing of the measured curves appears unreasonable and gives rise to a larger departure of measured and predicted results at the lowest speed of 0.1 mm/s. Measurements at each speed were repeated and the results found to be reproducible. Furthermore, the thicknesses of the plate specimens were the same within ± 0.1 mm.

7.2 SURFACE DISPLACEMENTS RESULTS

Measurements of surface displacement for the propylene copolymer at specific values of the central deflection are compared with predictions obtained using the von Mises and linear Drucker-Prager models in figure 17. It can be seen that for deflections up to 20 mm, the measurements lie, in general, closer to the von Mises predictions. Above this deflection, measured values rise more rapidly with increasing deflection than predicted by the von Mises

analysis and approach the linear Drucker-Prager results. The results for ABS are shown in figure 18 and the observations are broadly the same as those for the copolymer. The displacements predicted by the two models are however closer together for ABS than for the copolymer.

The reason for the better agreement between the von Mises predictions of surface displacement and measured values is unexpected. The results of shear and tensile tests on both polymers (shown in figure 5) reveal that yielding in these materials is sensitive to the hydrostatic component of stress. The linear Drucker-Prager predictions of surface displacement should therefore be the more accurate. It may be however that the accuracy of predictions of strain, and hence surface displacements, is influenced more by the validity of the flow law and the assumed flow potential than the yield criterion. To explore this further, the surface displacement calculations were repeated for the copolymer using the linear Drucker-Prager analysis with a value for the flow parameter $\tan \Psi$ of zero. This is the value in the von Mises analysis. Up to a central deflection of the plate of 25 mm there was very little difference in the predicted displacements and those recorded in figures 12 and 17 with $\tan \Psi = 0.88$. Above a deflection of 25 mm, the results using $\tan \Psi = 0$ were slightly higher and thus further from the measured data.

8 CONCLUSIONS AND FURTHER WORK

Finite element predictions of the variation of force with central deflection of plate specimens made using the von Mises model compare closely with those made using the linear Drucker-Prager model for both the propylene-ethylene copolymer and the ABS. Predictions using both models also give good agreement with experimental results over a wide range of loading speed. Since the properties of plastics are viscoelastic, especially the propylene copolymer, the agreement between prediction and measurement at moderate deflections is noticeably improved by including in the analysis, values for the Young's modulus that increase with the rate of loading.

Predictions of surface displacement with increasing deflection using the two models are significantly different, the Drucker-Prager results being consistently higher for both polymers.

These predictions have little sensitivity to the value used for the flow parameter $\tan \Psi$ and hence the assumption of associated flow.

Measured values of the surface displacement lie generally between the results predicted by the two models.

As part of future work in this project, measurements of force and surface displacement with central deflection will be compared with predictions made using a new elastic-plastic materials model. This model is currently under development and takes account of the effect of the nucleation of cavities on plastic deformation under stress states generating a significant component of tensile hydrostatic stress.

Future studies will also explore a suitable failure criterion to enable predictions to be made of the onset of fracture or rupture of the specimen. For this purpose, tests will be carried out at temperatures below ambient in order to initiate failure at moderate levels of the central deflection.

9 ACKNOWLEDGEMENTS

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FIGURE CAPTIONS

Apparatus for testing plate specimens under central loading. The mirror beneath the specimen enables measurement to be made of surface displacements in the deformed specimen.

Measured horizontal displacements of reference points on the lower surface of the copolymer (fig 2a) and the ABS (fig 2b) specimens plotted against the central deflection.

Tensile true stress/true strain curves for a propylene/ethylene copolymer measured at different test speeds. The plastic strain rate for each test is shown with each curve.

Tensile true stress/true strain curves for the ABS measured at different test speeds. The plastic strain rate is shown with each curve.

True stress/true strain curves measured in tension and shear and Poissons ratio for the propylene copolymer and ABS. The plastic strain rates shown with each data set correspond to the same effective plastic strain rate of approximately 0.004 s^{-1} .

Effective stress/effective plastic strain curves derived from the tension and shear test data in figure 5 using equations (12), (13), (15) and (16).

Tensile hardening curves for the propylene copolymer over a range of strain rate derived from the data in figure 3.

Tensile hardening curves for the ABS over a range of strain rate derived from the data in figure 4.

Diagram of the mesh used for finite element analyses of deformation of the centrally loaded plate.

- Fig 10 Comparison of predictions of force against central deflection of specimens of the propylene copolymer and ABS obtained using the von Mises and the linear Drucker-Prager models. The loading speed is 0.1 mm/s.
- Fig 11 Predictions of force against central deflection for the propylene copolymer using the linear Drucker-Prager model for a range of loading speeds. The specimen thickness is 2.8 mm.
- Fig 12 Comparison of predictions of the horizontal displacement of points on the lower surface of the copolymer specimen at different central deflections obtained using the von Mises and the linear Drucker-Prager models. The loading speed is 0.1 mm/s.
- Fig 13 Predictions of force against deflection for ABS specimens using the linear Drucker-Prager model at different loading speeds. The specimen thickness is 3.6 mm.
- Fig 14 Predicted horizontal displacements of reference points at different radial positions on the lower surface of the ABS specimen plotted against the central deflection. Comparison of von Mises and linear Drucker-Prager model results at a loading speed of 0.1 mm/s.
- Fig 15 Comparison of predicted curves of force against central deflection at different loading speeds with experimental data for the propylene copolymer. The linear Drucker-Prager model was used for the predictions. The specimen thickness is 2.8 mm.
- Fig 16 Comparison of predicted curves of force against central deflection at different loading speeds with experimental data for ABS. The specimen thickness is 3.6 mm.

- Fig 17 Comparison of the predicted horizontal displacements of reference points on the plate surface with experimental values for the propylene copolymer.
- Fig 18 Comparison of predicted horizontal displacements of reference points on the surface of an ABS specimen with experimental values.

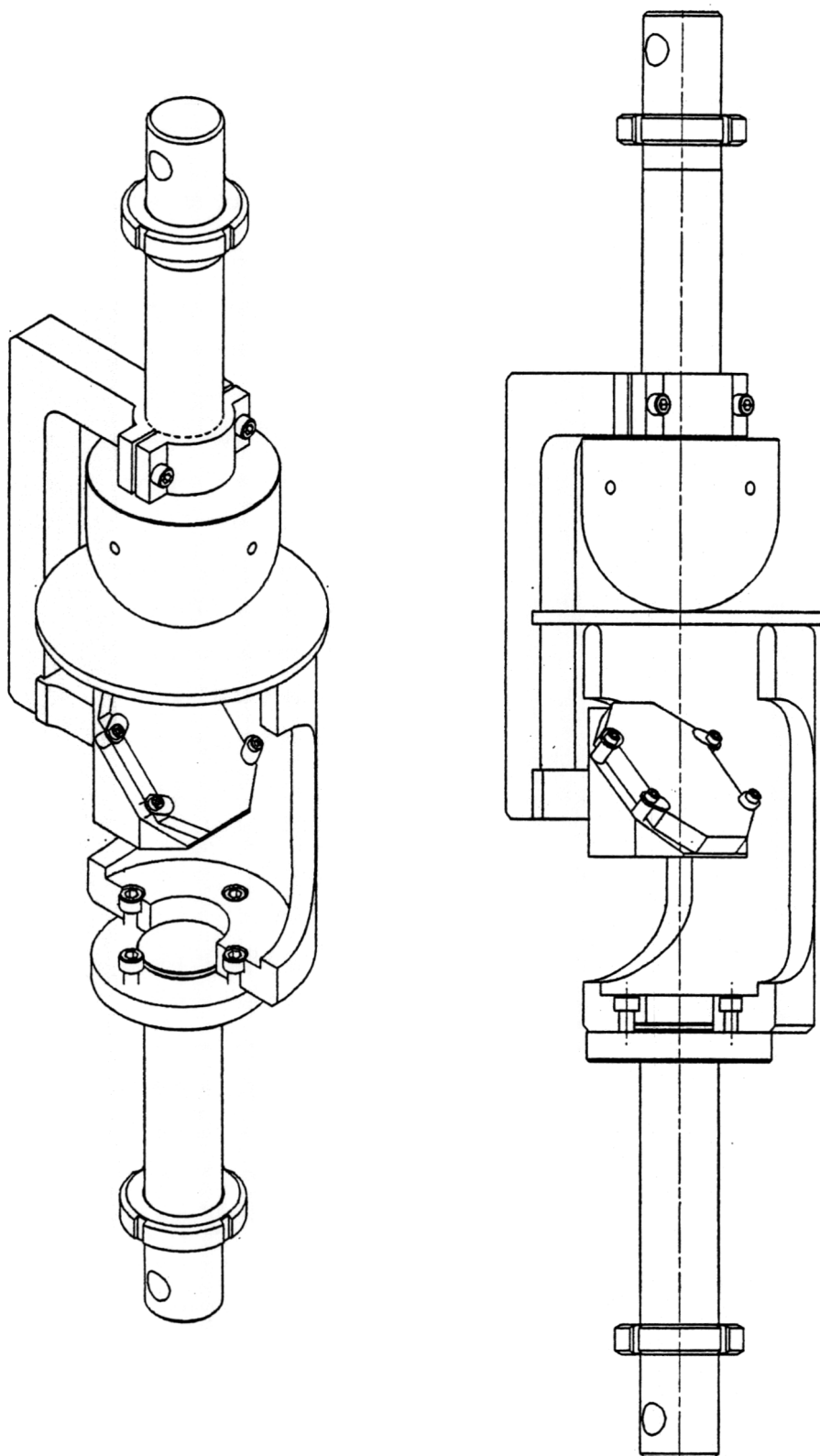


Fig 1 Apparatus for testing plate specimens under central loading. The mirror beneath the specimen enables measurement to be made of surface displacements in the deformed specimen.

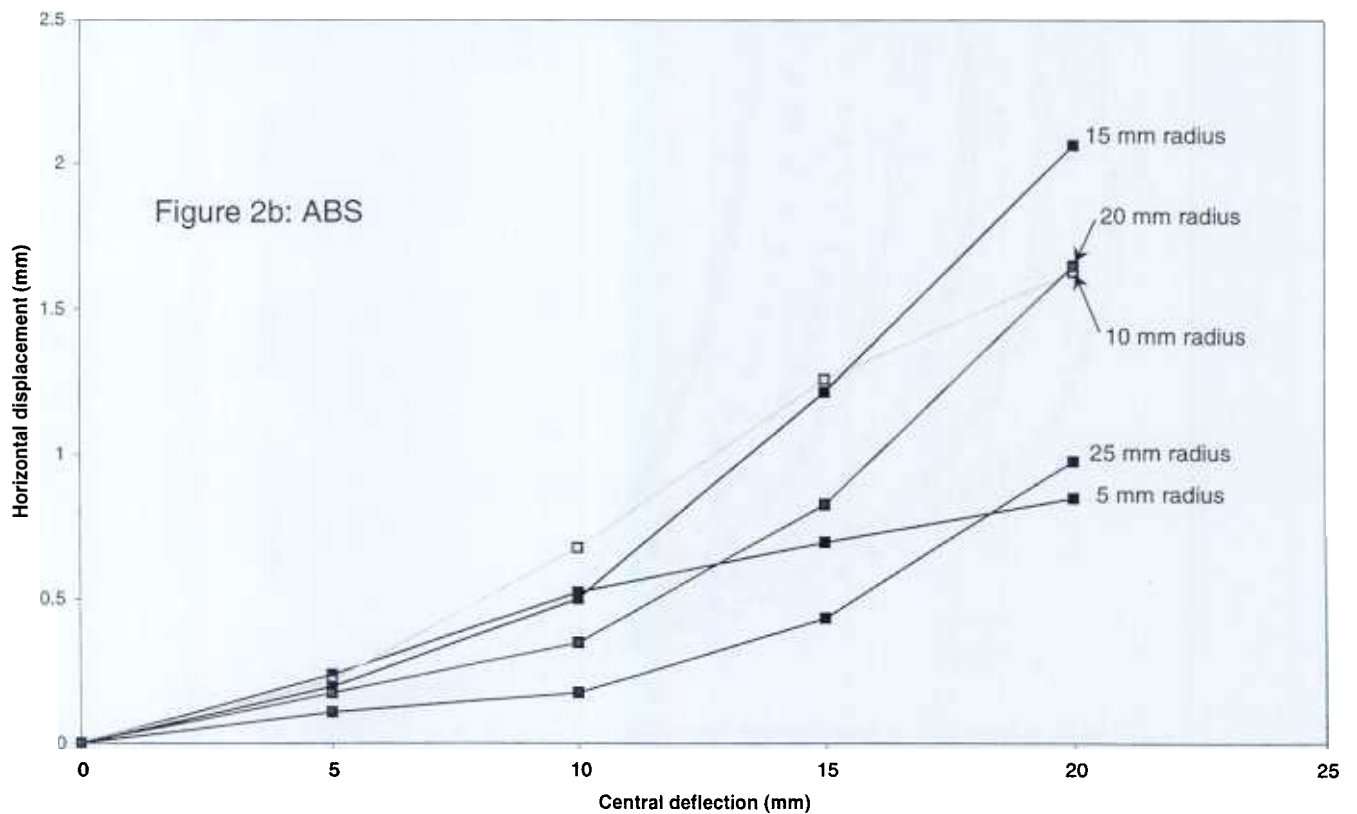
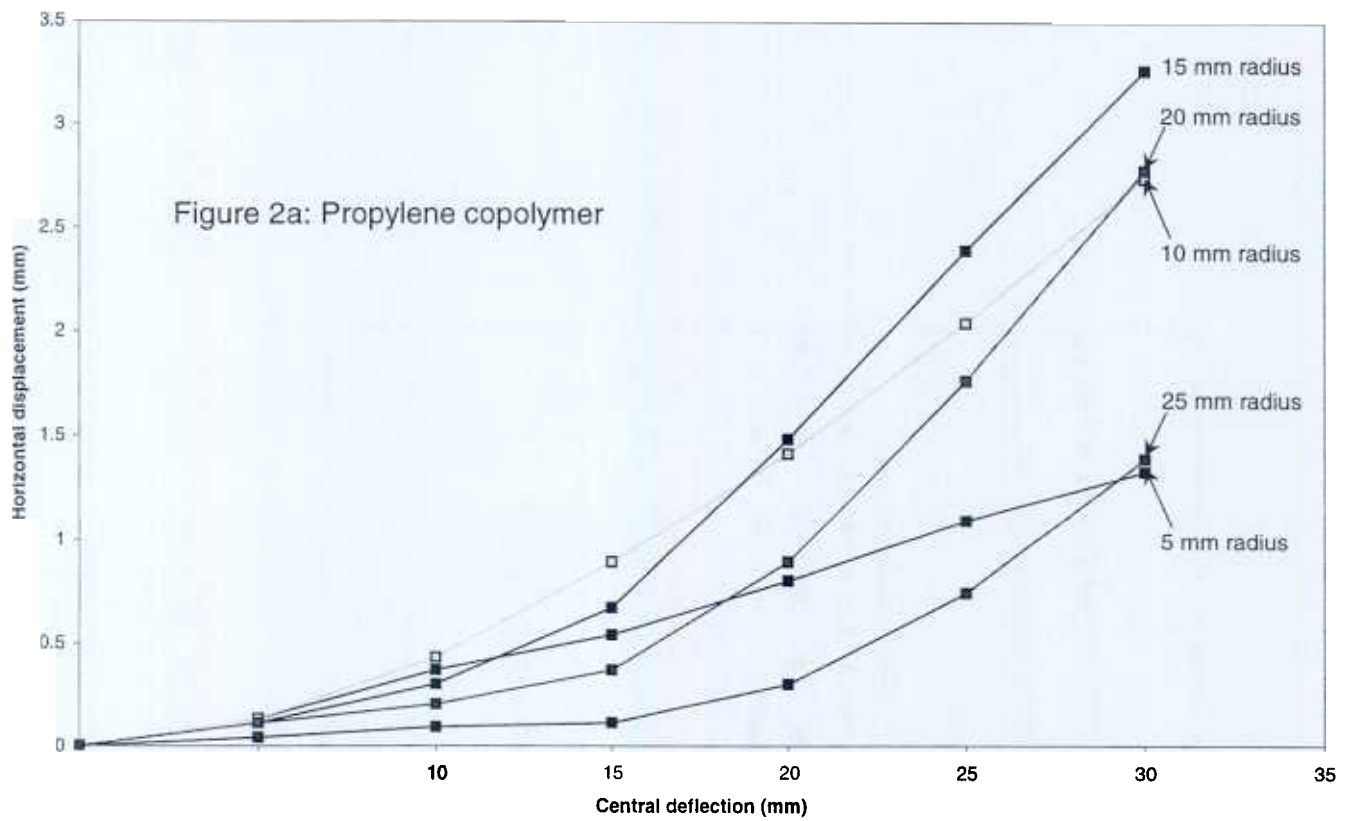


Figure 2: Measured horizontal displacements of reference points on the lower surface of propylene copolymer (figure 2a) and ABS (figure 2b) specimens plotted against central deflection. The radial positions of the reference points are indicated on each plot.

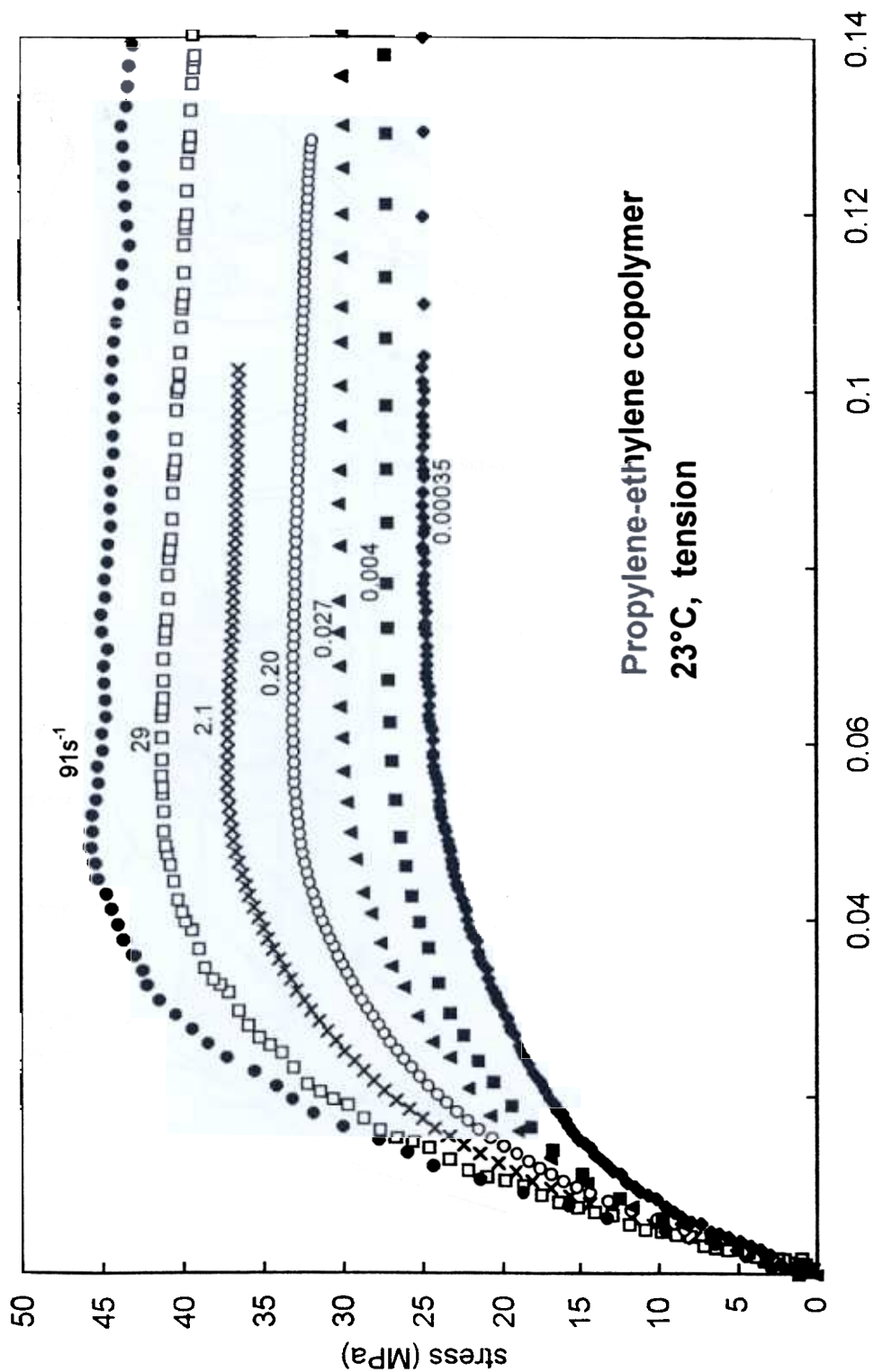


Fig 3 Tensile true stress/true strain curves for a propylene/ethylene copolymer measured at different test speeds. The plastic strain rate for each test is shown with each curve.

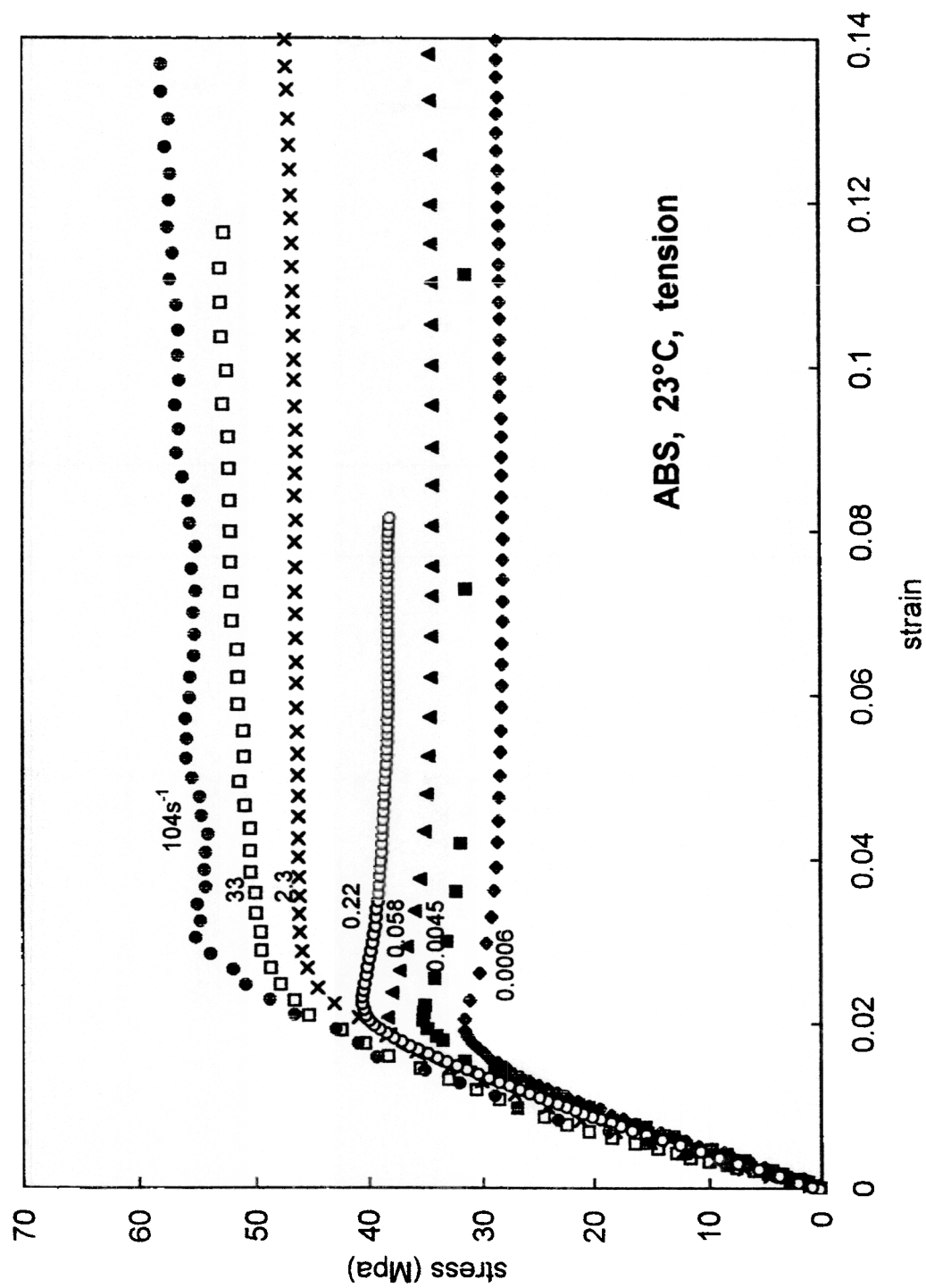


Fig 4 Tensile true stress/true strain curves for the ABS measured at different test speeds. The plastic strain rate is shown with each curve.

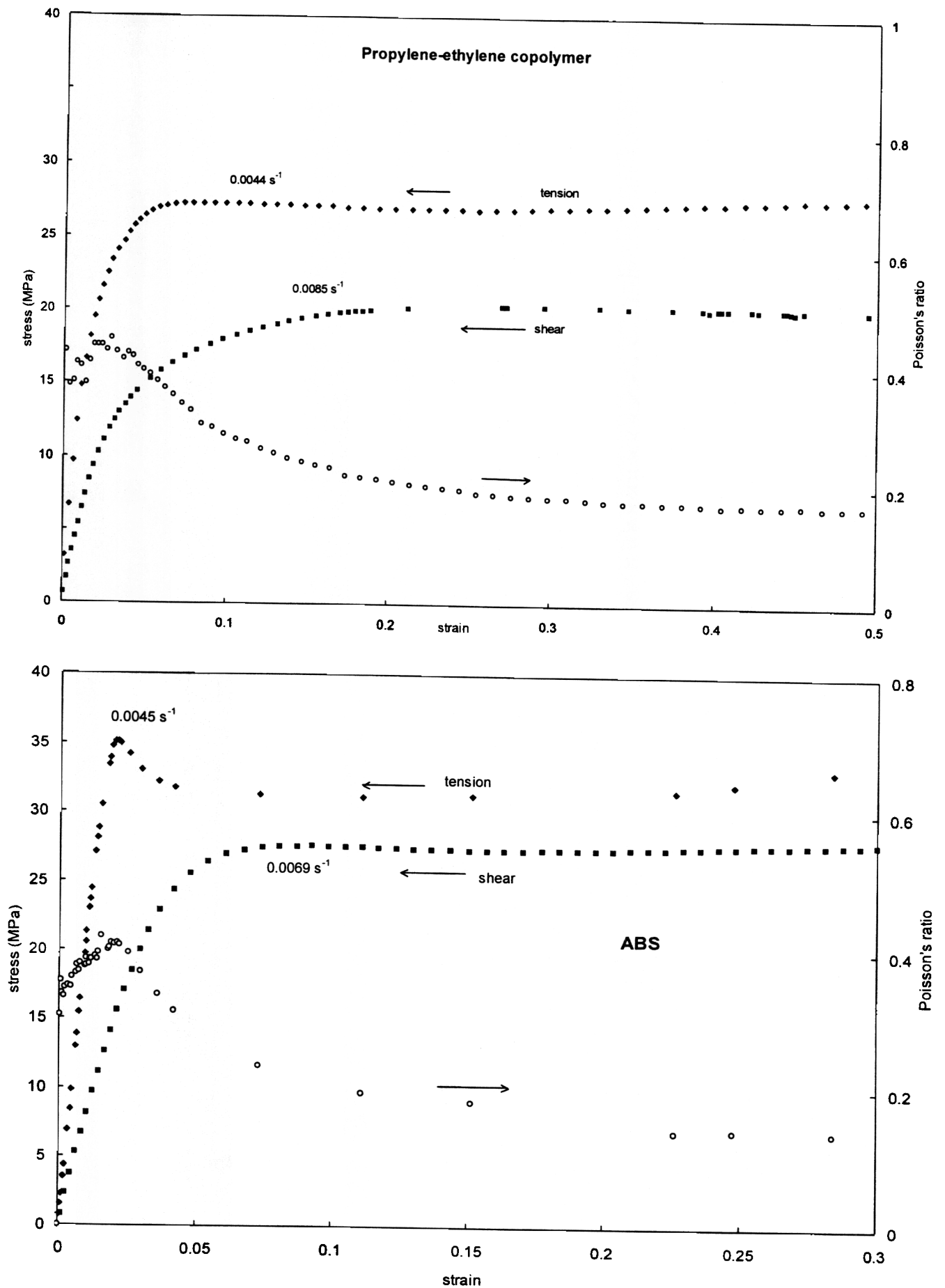


Figure 5: True stress/true strain curves measured in tension and shear and Poisson's ratio for the propylene copolymer and ABS. The plastic strain rates shown with each data set correspond to the same effective plastic strain rate of approximately 0.004 s⁻¹.

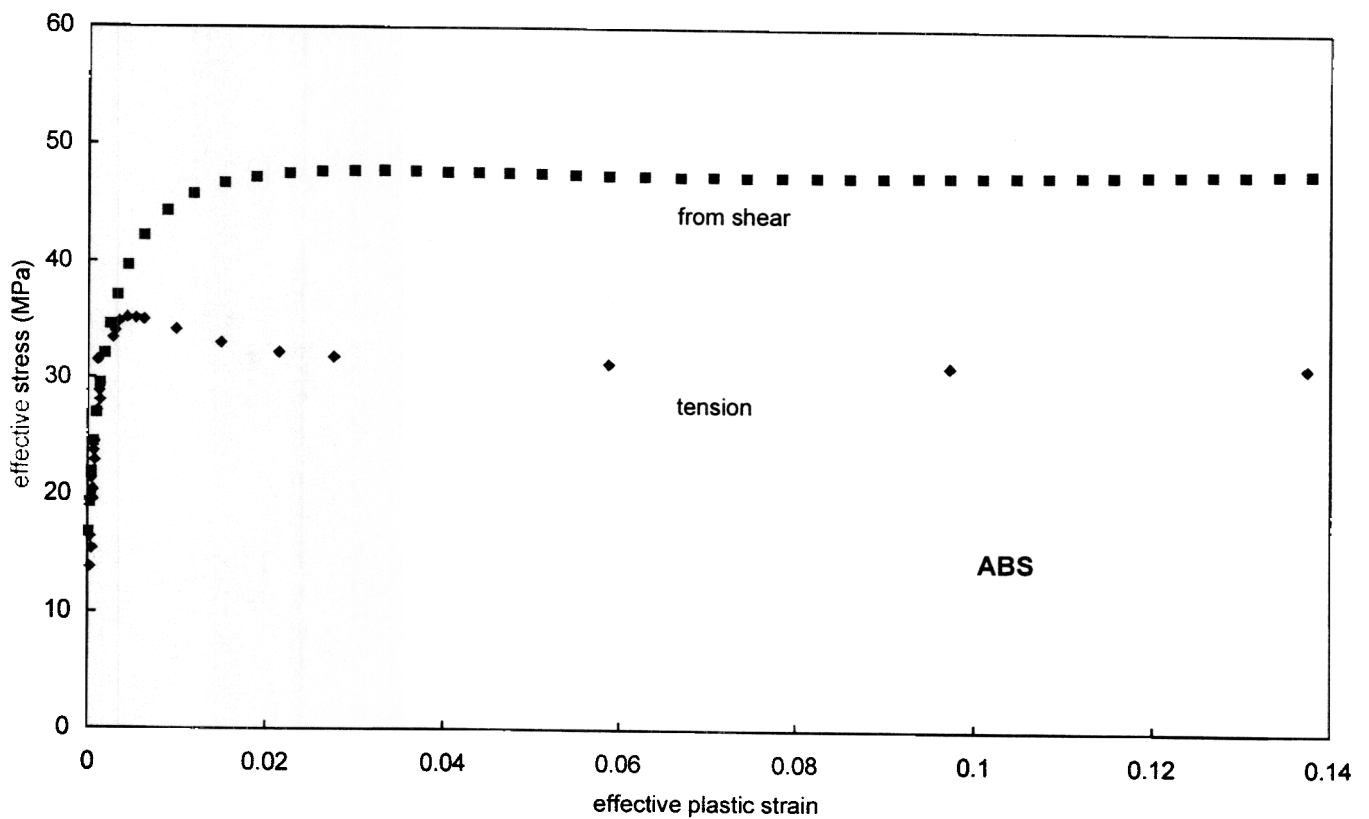
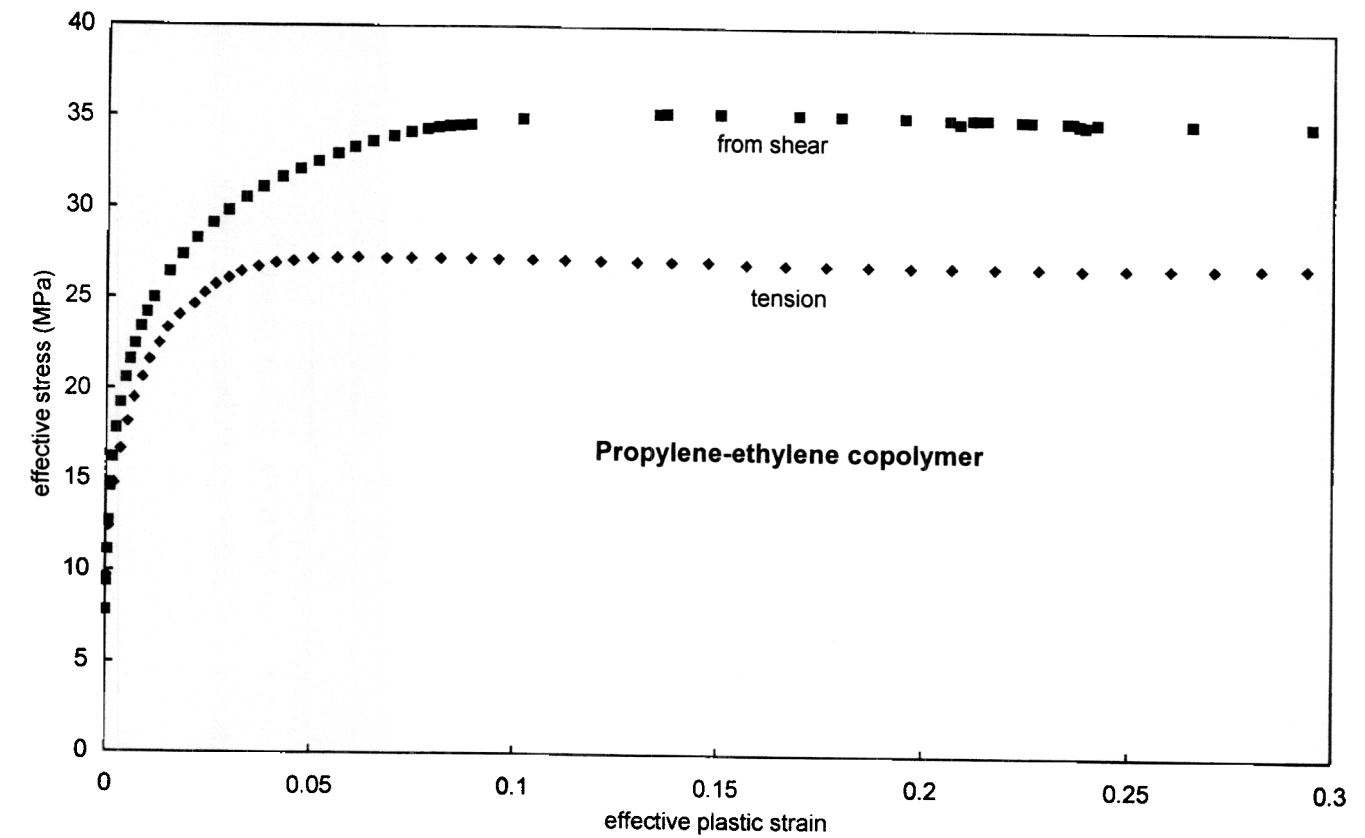


Figure 6: Effective stress/effective plastic strain curves derived from the tension and shear data in figure 5 using equations (12), (13), (15) and (16).

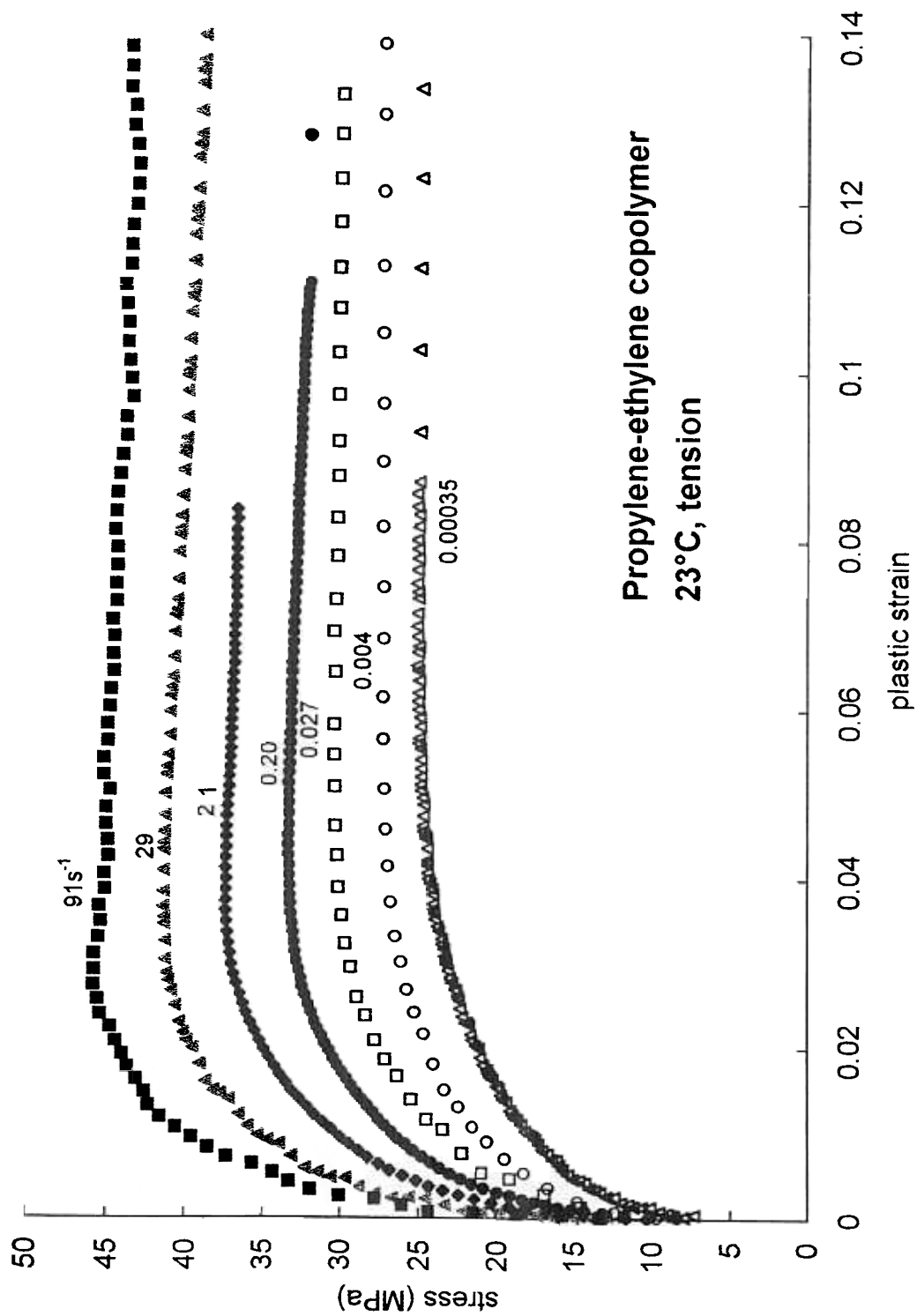


Figure 7 Tensile hardening curves for the propylene copolymer over a range of strain rates derived from the data in figure 3.

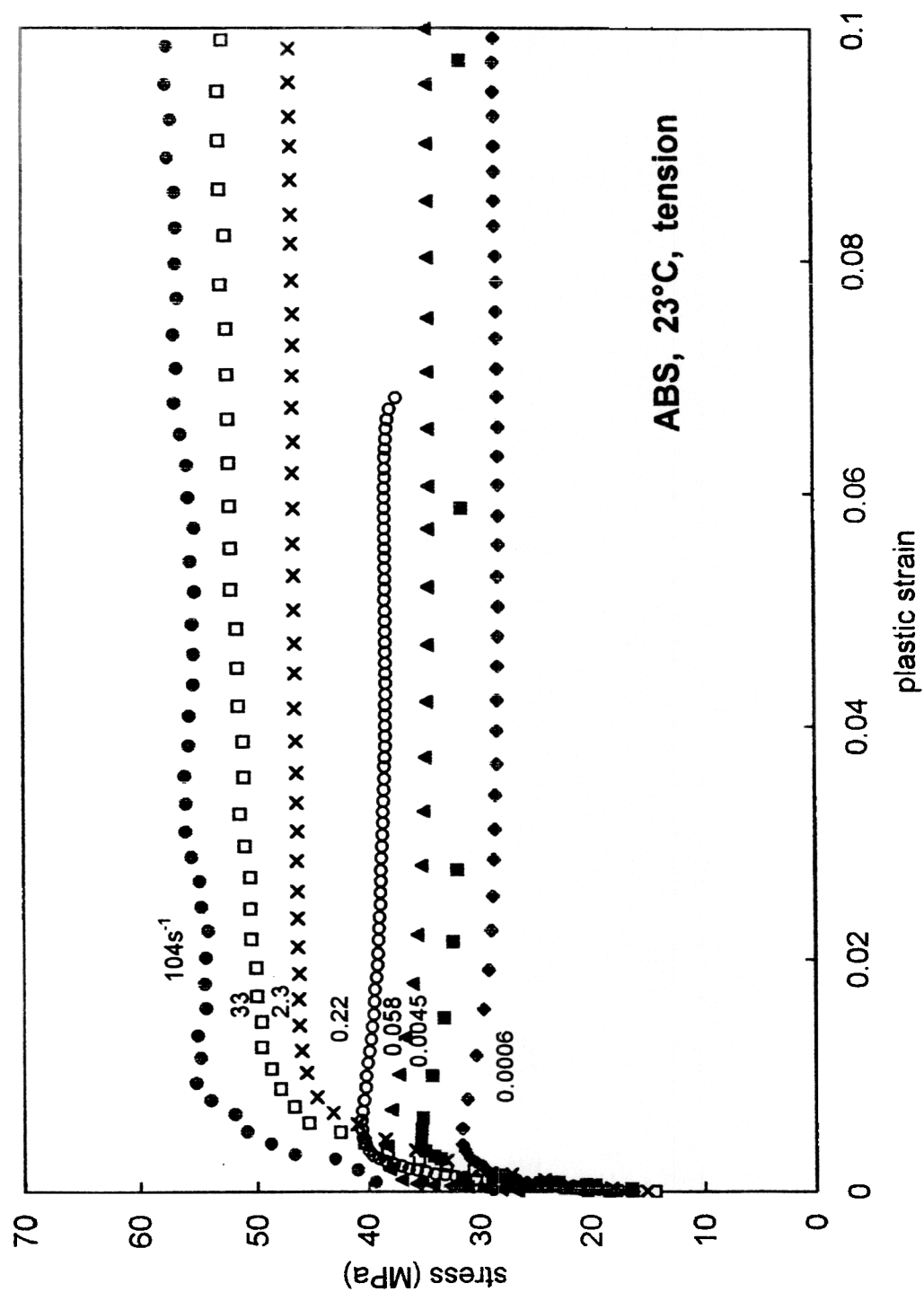


Figure 8 Tensile hardening curves for the ABS over a range of strain rate derived from the data in figure 4.

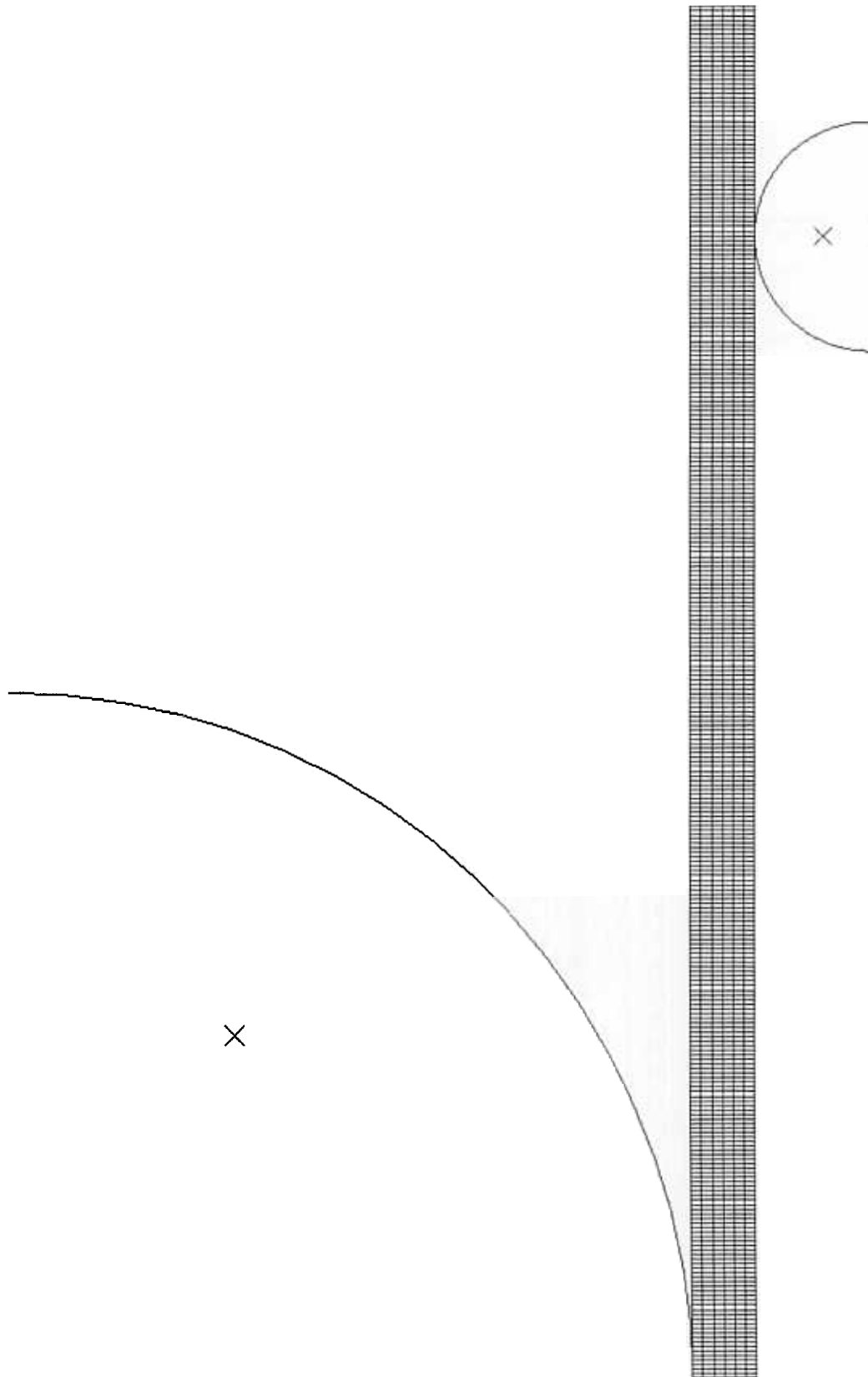


Figure 9: Diagram of the mesh used for finite element analyses of deformation of the centrally loaded plate.

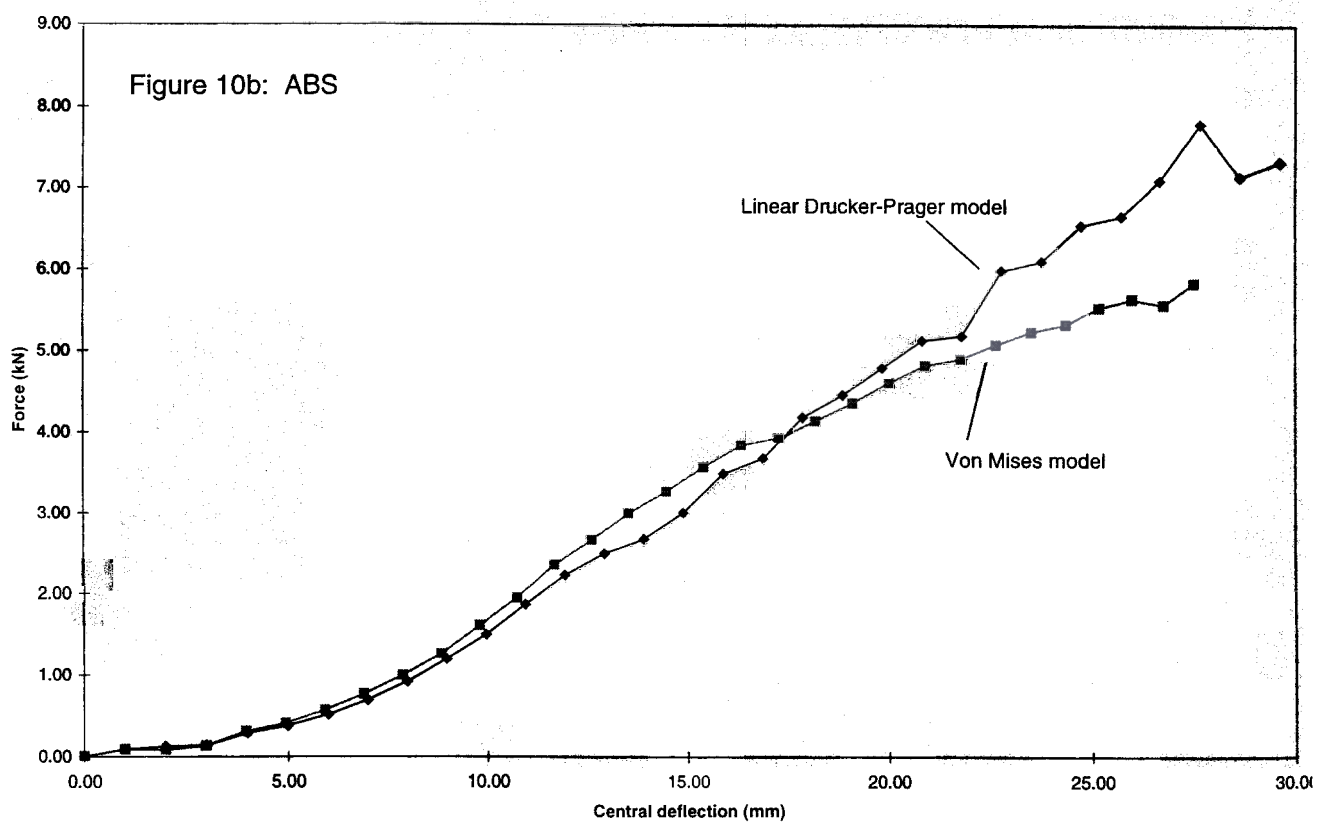
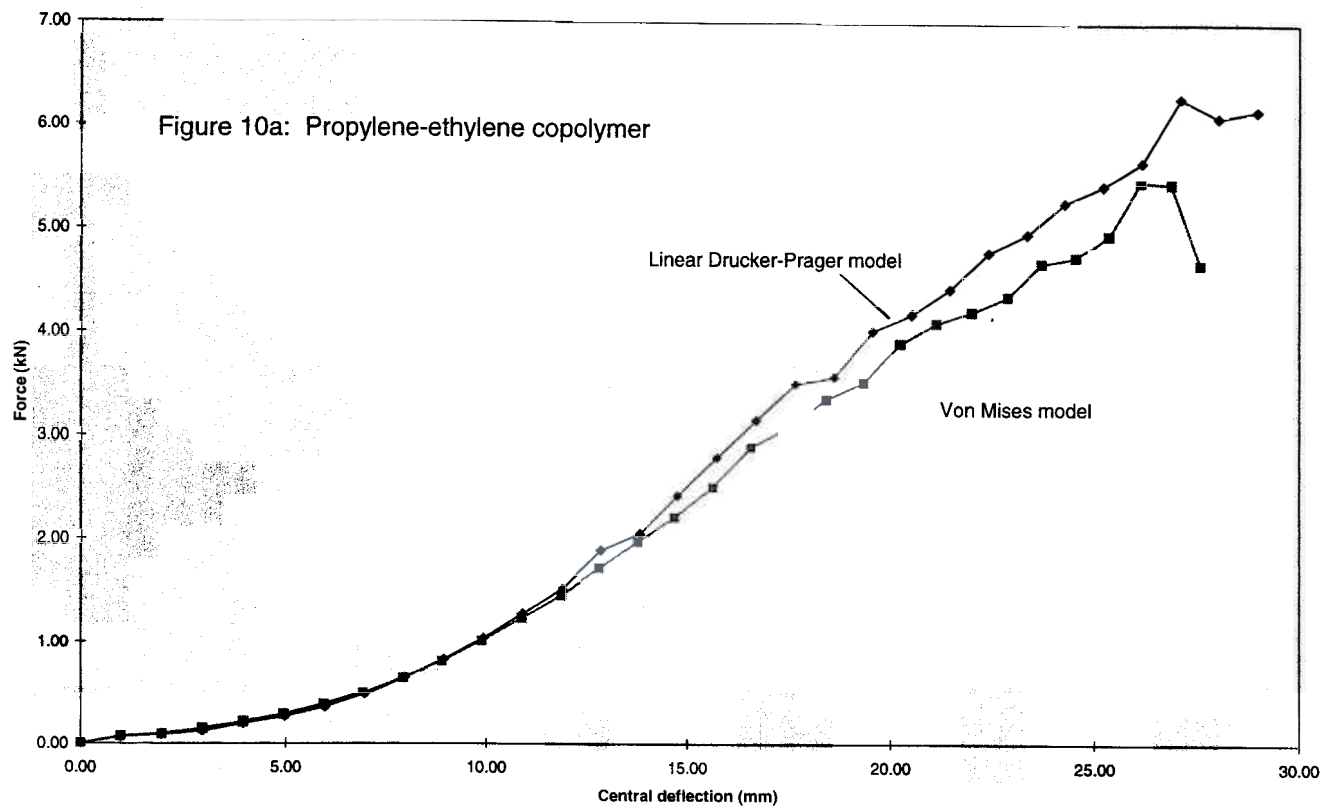


Figure 10: Comparisons of predictions of force against central deflection for specimens of the propylene copolymer (figure 10a) and ABS (figure 10b) using the von Mises and the linear Drucker-Prager models. The loading speed is 0.1 mm/s.

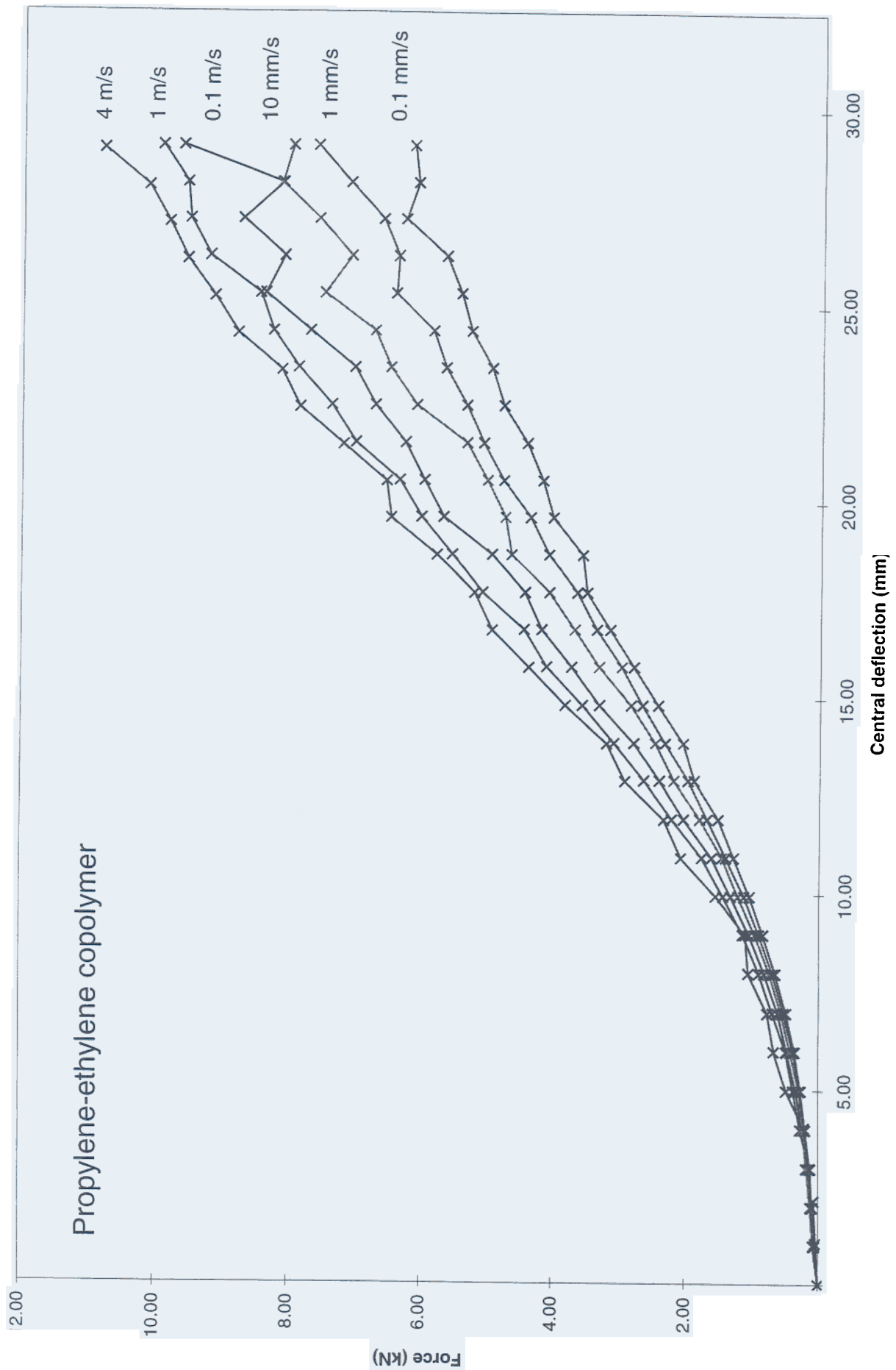


Figure 11: Predictions of force against central deflection for the propylene copolymer using the linear Drucker-Prager model for a range of loading speeds

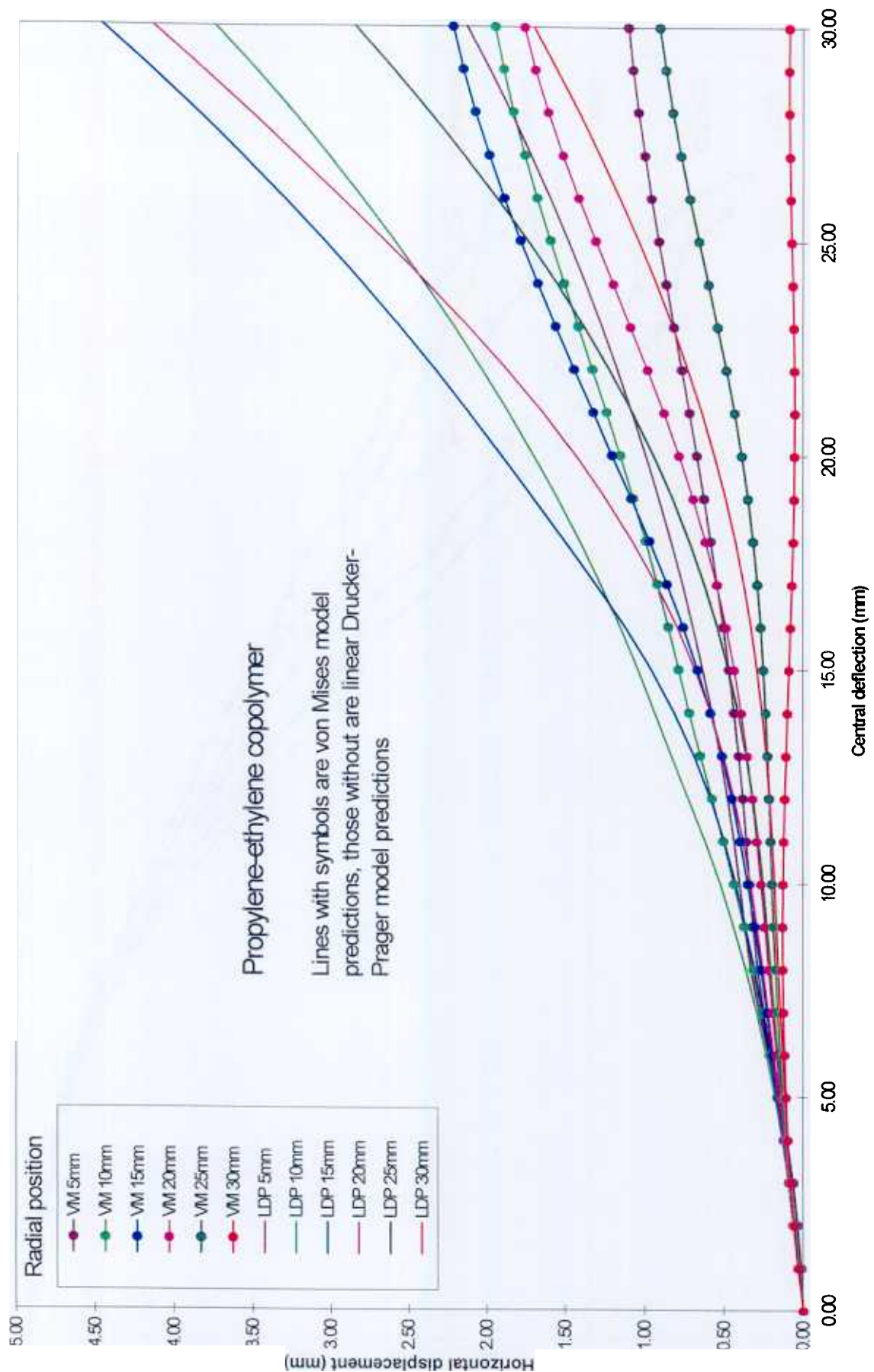


Figure 12: Comparison of predictions of the horizontal displacement of reference points on the lower surface of the ^{copolymer} specimen with increasing central deflection obtained using the von Mises and the linear Drucker-Prager models. The reference points are located at the radial positions indicated. The loading speed is 0.1 mm/s.

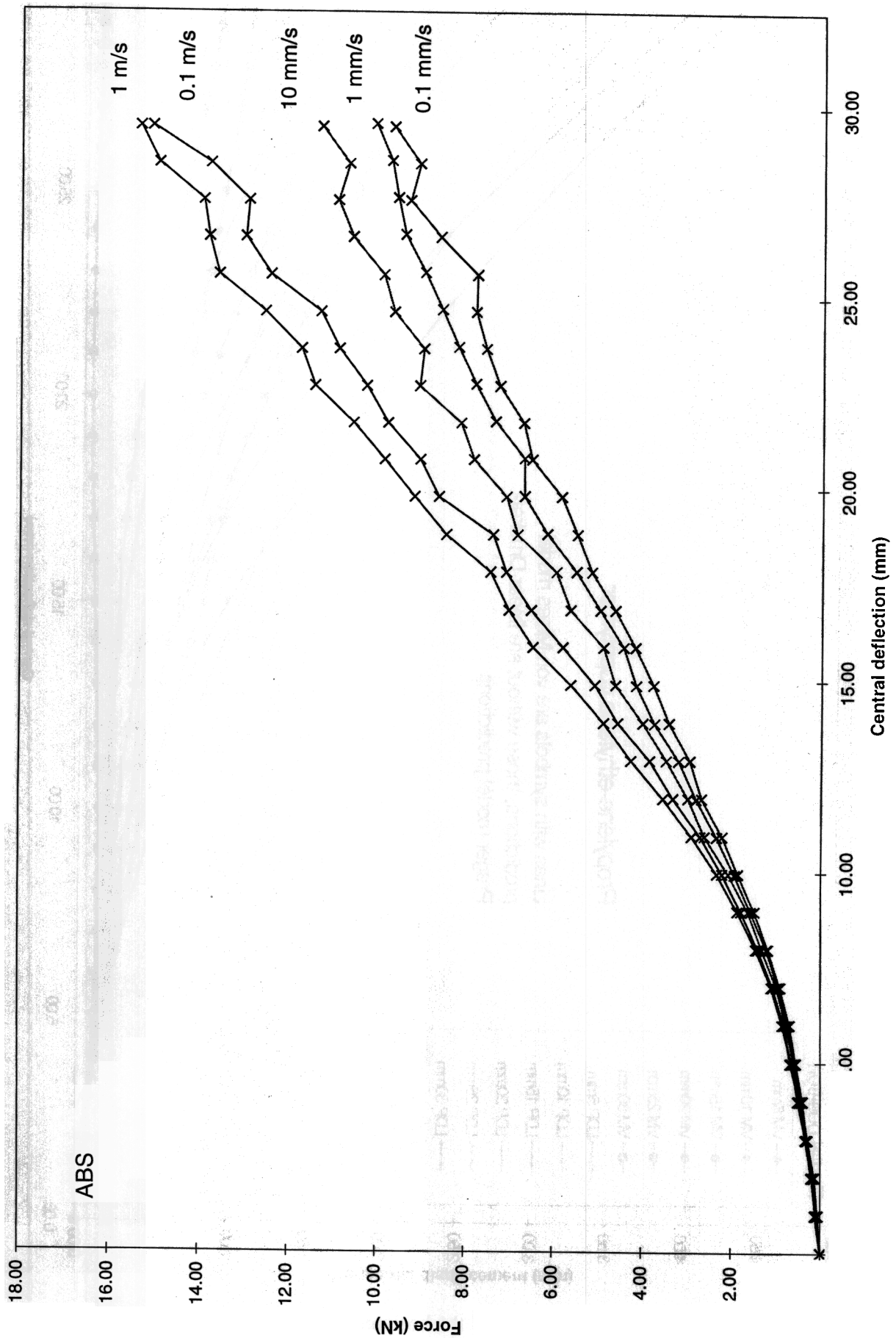


Figure 13: Predictions of force against deflection for ABS specimens using the linear Drucker-Prager model at different loading speeds

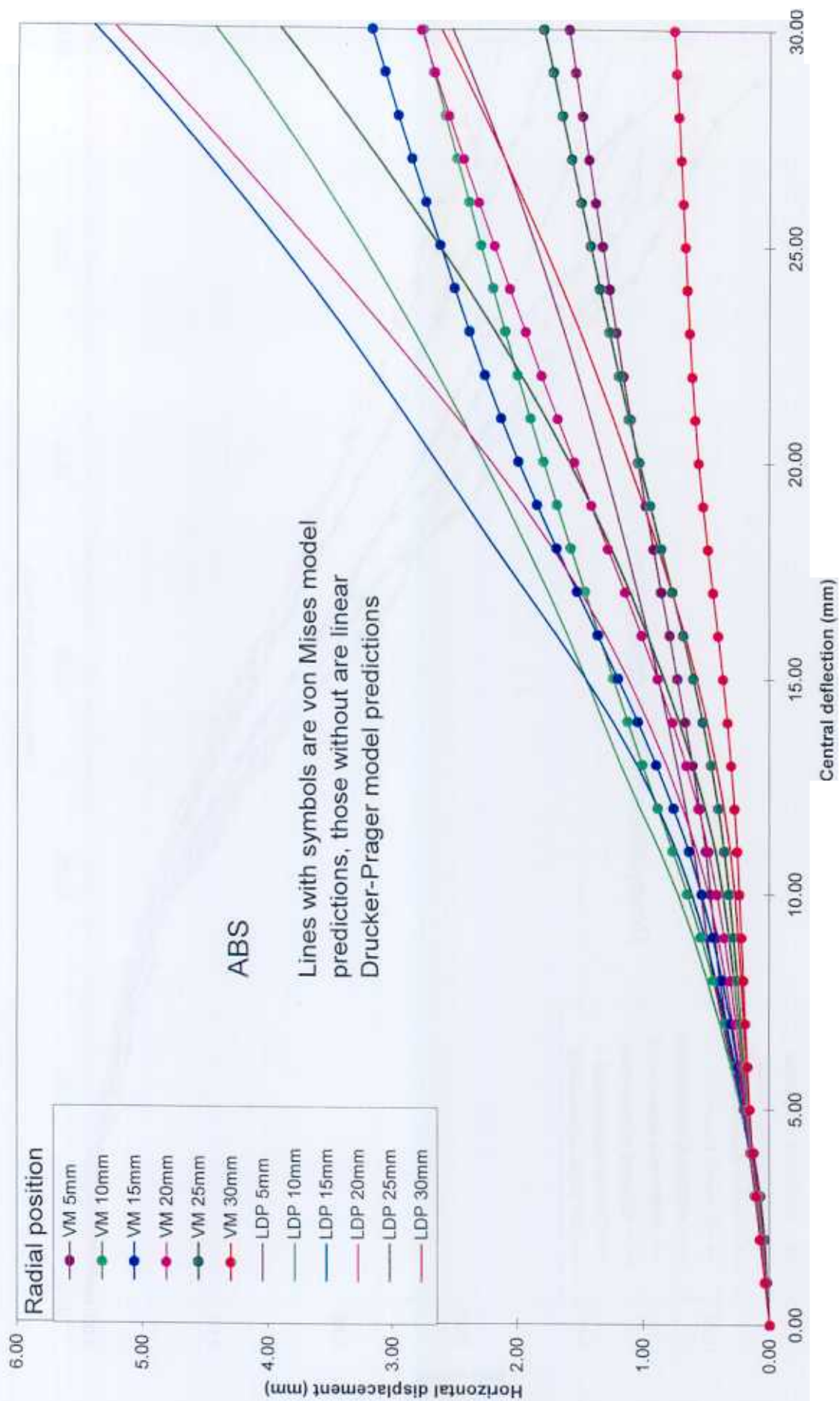


Figure 14: Predicted horizontal displacements of reference points at different radial positions on the lower surface of the ^{ABS}/specimen plotted against the central deflection.

Von Mises and linear Drucker-Prager models used, with a loading speed of 0.1 mm/s.

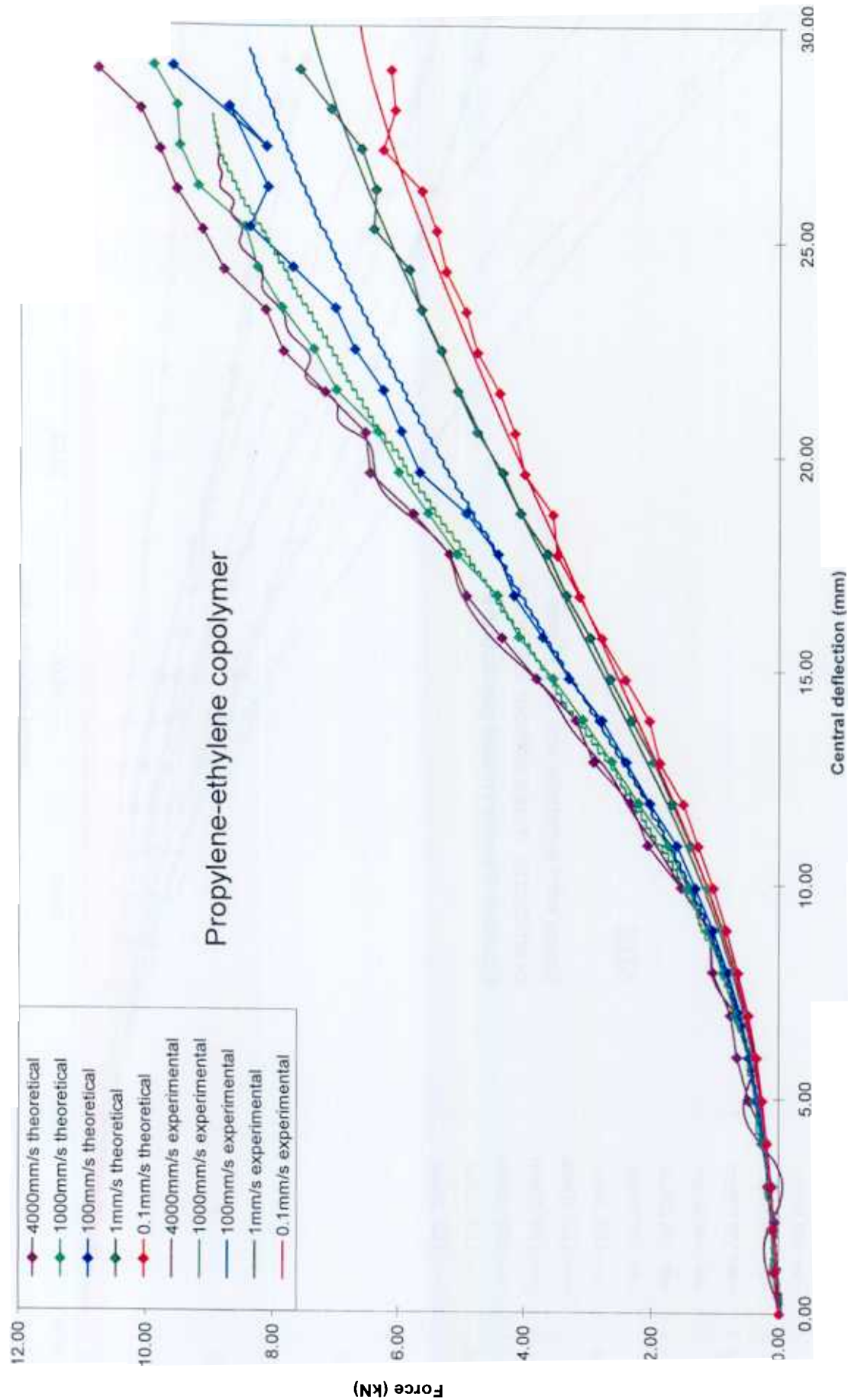


Figure 15: Comparison of predicted curves of force against central deflection at different loading speeds with experimental data for the propylene copolymer. The linear Drucker Prager model was used for predictions.

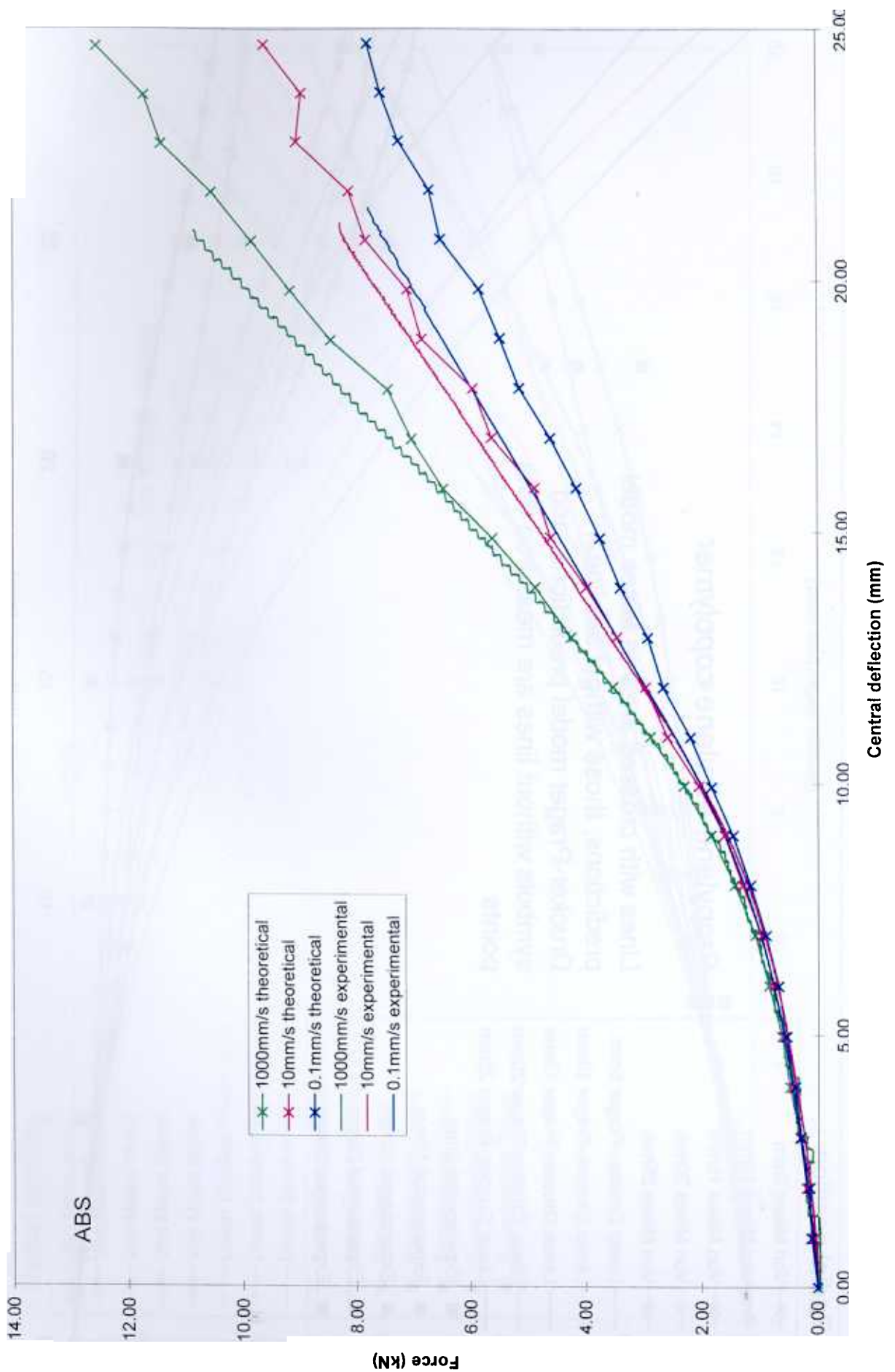


Figure 16: Comparison of predicted curves of force against central deflection at different loading speeds with experimental data for ABS. The linear Drucker-Prager model was used for predictions

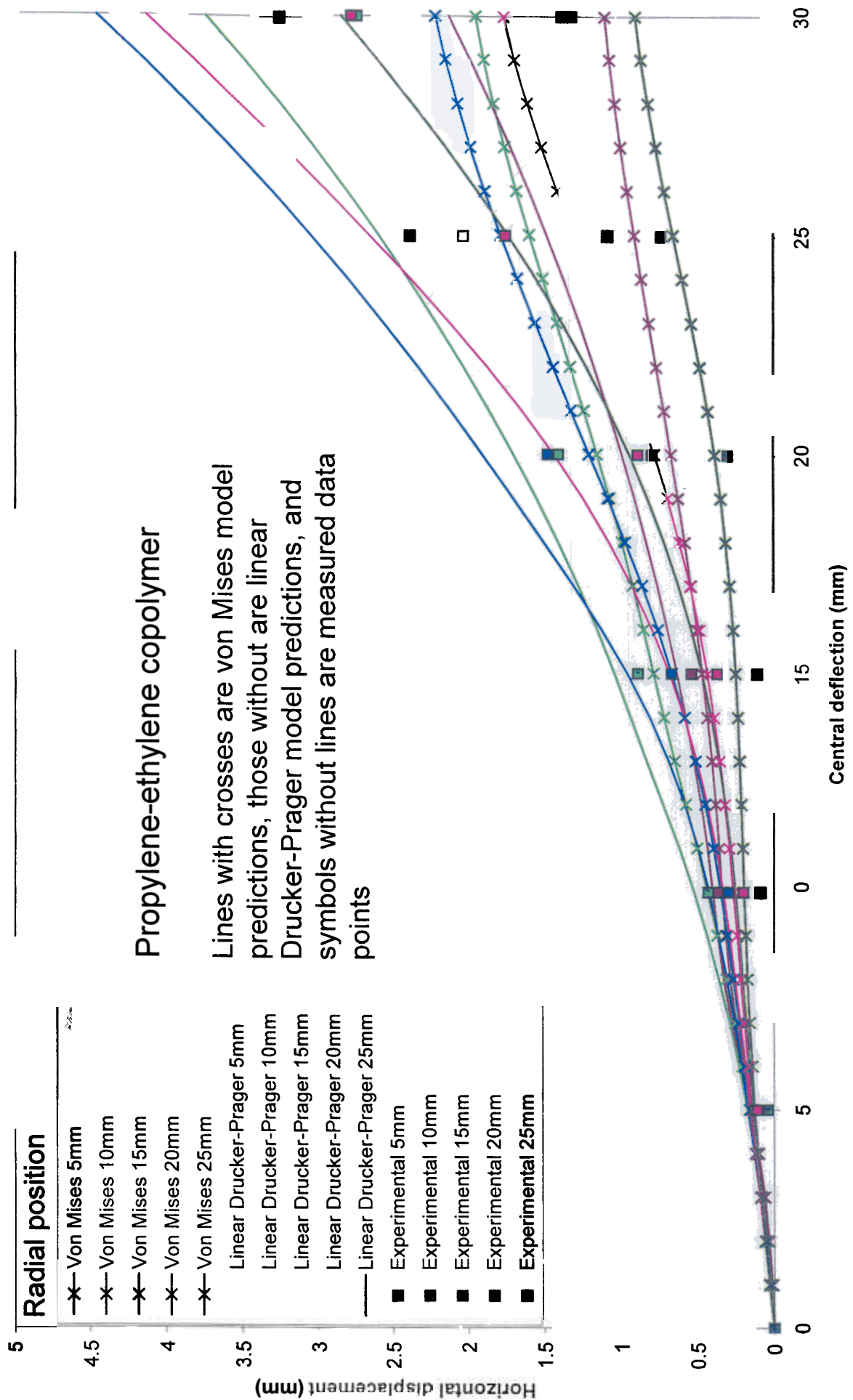


Figure 17: Comparison of the predicted horizontal displacements of reference points on the plate surface with experimental values for the propylene copolymer.

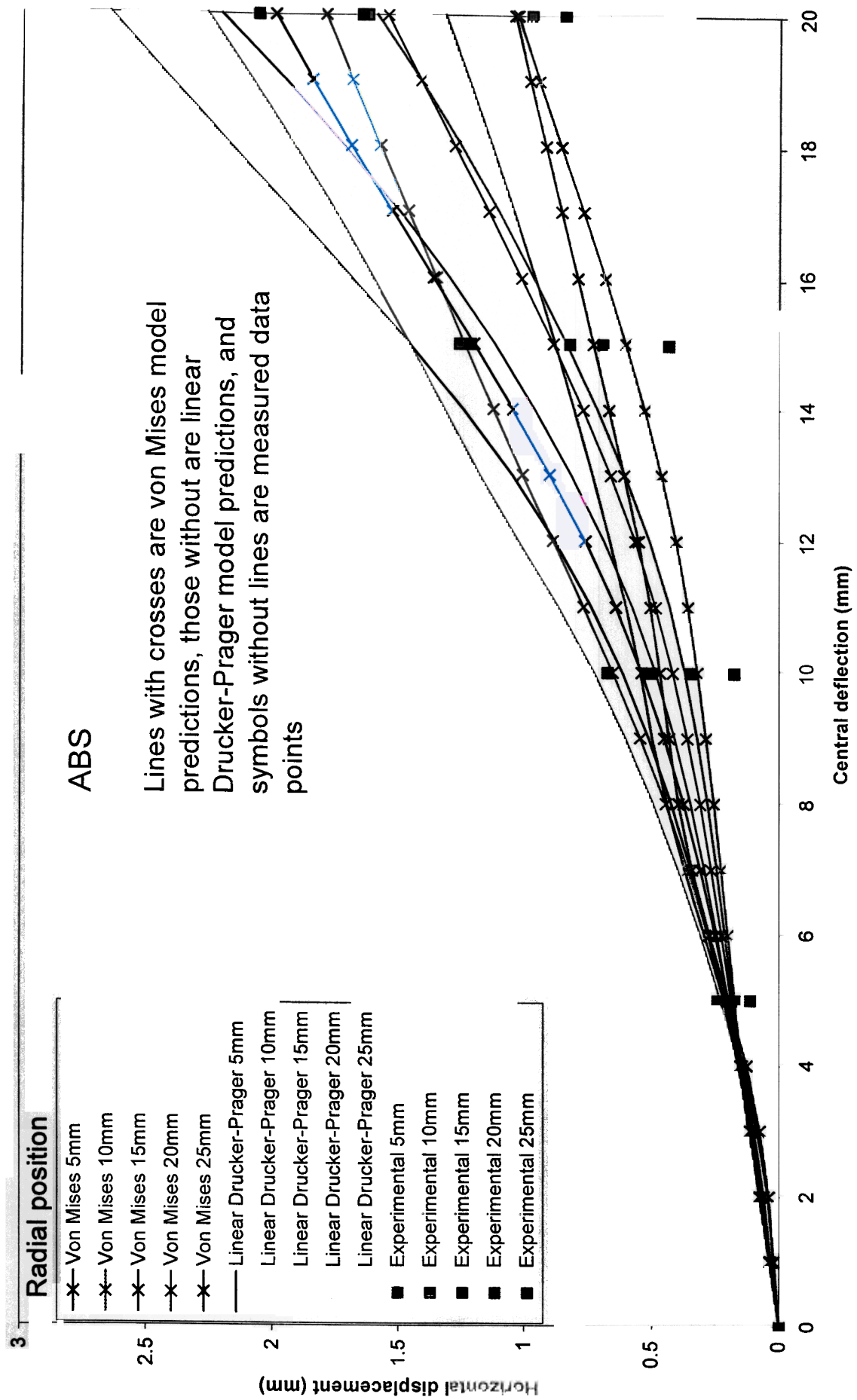


Figure 18: Comparison of the predicted horizontal displacements of reference points on the plate with experimental values for ABS.