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MEASUREMENT OF THE MAGNETIC DIPOLE MOMENT OF FERROMAGNETIC MATERIAL SPECIMENS

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ABSTRACT

The intrinsic magnetic properties of ferromagnetic materials can be determined in accordance with the written IEC 60404 standards. When the material is used as a component, a demagnetisation correction may need to be applied to these intrinsic properties. This is difficult to determine since it depends not only on the geometry of the component but also the magnetic permeability. For permanent magnetic materials, it is possible to measure the magnetic dipole moment, a parameter that is dependant on the material properties and the geometry. This provides important complementary component information. In this report, measurement methods for determining magnetic dipole moment are introduced and a method resulting in uncertainties as low as 0.1% (95% confidence limit) is discussed in detail. This low level of uncertainty allows the calibration of commercial magnetic moment measuring instrumentation.

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1. Introduction

The magnetic properties of ferromagnetic materials can be determined in accordance with written IEC 60404 standards. These standards are concerned with the determination of properties using closed magnetic circuits provided by systems such as electromagnets. Suitably calibrated instrumentation is required for the measurement of magnetic flux density, B , and applied magnetic field strength, H . While the written standards provide the measurement method, care must be taken when establishing the magnetic circuit since many unwanted contributions are possible.

The properties determined using these methods are intrinsic and are used to classify and specify material properties. The magnetic flux properties of a component can be estimated using material parameters by applying a complex demagnetization correction. This correction depends on both the geometry of the material and the intrinsic permeability. Because the calculation of the required demagnetising factors is only possible within certain limits, this process is not always applicable and is always prone to large errors.

If the magnetic materials being considered have an appreciable remanence then the magnetic dipole moment can be measured. The component remanence is the magnetic flux density remaining when the applied field is reduced from that needed for magnetic saturation through zero and reaches the demagnetizing value. The dipole moment is the product of this magnetic polarization and volume. In the following sections, methods that can be used to measure this magnetic dipole moment are presented.

2. Methods of measurement

Three methods for the measurement of the magnetic dipole moment are given below:

- (i) Torsional pendulum method in which the dipole moment is oscillated in a known magnetic field;
- (ii) Magnetic flux density at a distance measured using sensitive fluxgate magnetometers;
- (iii) An electrical method in which the magnetic flux of the dipole moment is detected and from this the magnetic dipole moment calculated.

The last of these methods is discussed in detail since it provides the lowest measurement uncertainty. When the instrumentation used in method (iii) is calibrated individually an uncertainty in the measurement of magnetic dipole moment of 0.1 % is possible. By calibrating reference standards using this traceable approach, it is possible to calibrate commercial systems to a low uncertainty.

3. Torsional Pendulum

This method involves the oscillation of the dipole moment to be measured in a known magnetic field. The direction of the moment is in a plane normal to the direction of the suspension. In the presence of a magnetic field a torque exists when the angle between the dipole axis and the field is not zero, this force reaching a maximum at 90 degrees. When this angle is small, the period of oscillation depends on the magnitude of the magnetic dipole moment and the applied magnetic field strength. The magnetic dipole moment, m in Am^2 , is given by:

$$m = \frac{4\pi^2 I - RT^2}{BT^2} \quad (1)$$

Where T is the period of oscillation of the dipole moment in seconds;
 B is the applied magnetic flux density in tesla;
 R is the torsional constant; and
 I is the moment of inertia of the dipole moment.

R is determined either using a non-magnetic specimen of similar dimensions and moment of inertia or the same sample in zero field.

To obtain a uniform magnetic field of known value a calibrated Helmholtz coil pair is used. This is placed inside a cancellation system to remove contributions due to the earth's magnetic field. This cancellation system should not be a magnetic shield, unless it is sufficiently large so as not to affect the properties of the Helmholtz coils. A coil cancellation system is preferred.

From equation (1) it can be seen that the determination of magnetic dipole moment using this method consists of many measurements. Each of these has an associated uncertainty, with the overall measurement uncertainty being estimated as $\pm 1.5\%$. This uncertainty is based on a standard uncertainty multiplied by a coverage factor suitable to achieve a 95 % confidence level.

4. Field at a distance

Representing a magnetic moment as two poles separated by a distance l the magnetic field of the dipole can be easily calculated for points at a distance greater than $5l$. Considering a line drawn from the origin (centre) of the magnetic moment to a point P and denoting the field component along that direction as H_r and normal to this as H_ϕ then:

$$H_r = \frac{\mu_0}{4\pi} \frac{2m \cos \alpha}{x^3} \quad (2)$$

$$H_\varphi = \frac{\mu_0}{4\pi} \frac{m \sin \alpha}{x^3} \quad (3)$$

Where H is the magnetic field strength in A/m

m is the magnetic moment in Am^2 ;

μ_0 is the magnetic constant with value $4\pi \times 10^{-7}$ H/m;

α is the angle between the direction of the moment and the line drawn from the origin to point P; and

x is the distance from the origin to point P in m.

Considering a magnetic moment of 1 Am^2 of suitable dimensions such that 1 m from the origin is in the far field, then the magnetic flux density along the axis of the moment at 1 m is 200 nT. Magnetic flux densities as low as this are measured using fluxgate magnetometers. The calibration of these magnetometers to the required uncertainty requires a coil cancellation system and calibrated Helmholtz coils to generate the magnetic field.

Equations (2) and (3) depend on the third power of distance. The uncertainty in the measurement of magnetic moment using this method is therefore dominated by the measurement of the distance from the centre of the moment to the effective centre of the fluxgate. Using this method to measure the magnetic moment the estimated uncertainty was $\pm 2.7 \%$ for a confidence level of 95%.

5. Helmholtz coil method

5.1. Principles

The magnetic moment is an open circuit parameter. A magnetic field exists in the space surrounding the moment and it is possible to determine the magnetic moment by measuring the magnetic flux using a search coil.

A search coil could be placed directly on the surface at the geometrical centre of a permanent magnet dipole moment and the magnetic flux linking the coil measured. However, It is difficult to avoid flux returning from the magnet ends through the space between the coil and the specimen surface. In addition, the detected flux depends crucially on the position of the coil along the magnet length. The magnetic field close to a dipole moment depends on the

moment dimensions, which can be difficult to determine for certain geometries and magnetization states. In the far field regime the magnitude of the magnetic field of a magnetic dipole moment is given by equations (2) and (3) and is independent of the moment dimensions. A search coil placed in this field region measures a flux change when the moment is removed or reversed which depends on the magnetic moment. Positioning of the magnetic moment for this arrangement would still be significant. Using two nominally identical search coils with the magnetic moment in the middle would relax the positioning requirement.

Helmholtz coils consist of two nominally identical coils placed coaxially and separated by a distance equal to their radius. With the coils connected in series and a current flowing a magnetic field is produced at the centre with a value of

$$H = \frac{5\sqrt{5}}{8} \frac{NI}{r} \quad (4)$$

Where H is the magnetic field strength in A/m;

N is the number of turns per coil;

I is the current in A;

r is the coil radius and coil separation in m.

A property of this coil arrangement is that the magnetic field is uniform in the central region. An important parameter for such a coil is the ratio of magnetic field strength to current, H/I , which is known as the coil constant. From equation (4) this parameter depends on the physical properties of the coil and the change in this ratio along the axis of the coils is given by the equation

$$\frac{H}{I} = \left(\frac{H}{I} \right)_0 \left(1 - \frac{144}{125} \frac{x^4}{r^4} + \dots \right) \quad (5)$$

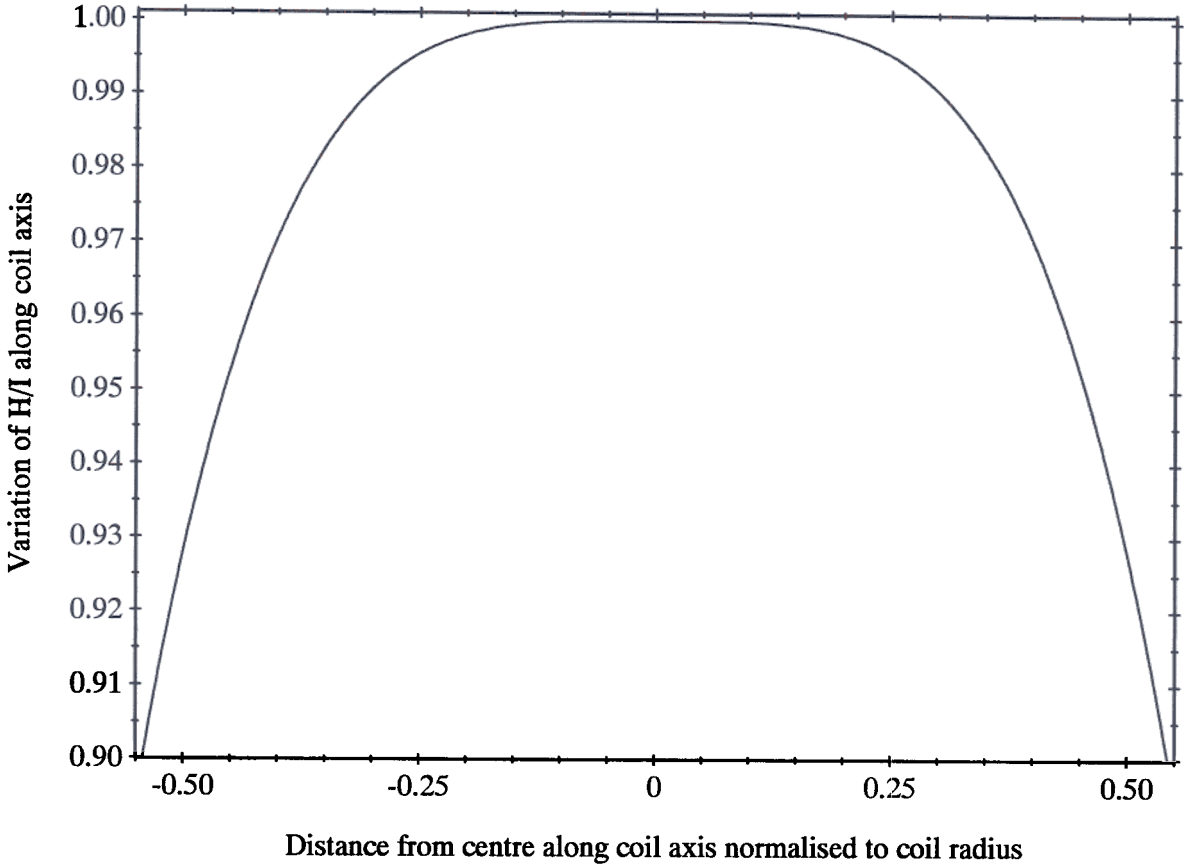
Where the subscript 0 refers to the value at the centre,

x is the distance from the centre and

r the coil radius.

In equation (5) higher order terms have been neglected.

Equation 5 is plotted in the figure below. In this figure, the y-axis is normalised to the central value and the x-axis is the distance along the coil axis normalised to the coil radius.



From the figure, it can be seen that the coil ratio H/I is uniform to 1% along the coil axis for a distance equal to 0.543 the coil radius. The variation of the coil constant across the central plane parallel to the coils is smaller.

Because of this good H/I uniformity, Helmholtz coils used in reverse to detect magnetic flux changes are suitable for measuring magnetic moment. The open access to this uniform region further aids the measurement through ease of test specimen alignment. It is possible however to use other coil systems, such as solenoids, for measuring magnetic moment.

To calculate a magnetic moment from a change in flux determined using Helmholtz coils it is necessary to integrate the far field over the turns of the coils. See reference [1] for details. The change in flux, $\Delta\vartheta$, resulting from removing the magnetic dipole moment is given by

$$\Delta\vartheta = m \times \frac{H}{I}$$

Where m is the magnetic moment in Wbm and H/I the coil constant in A/m/A.

Rearranging equation (6)

$$m = \Delta\vartheta \times \frac{I}{H}$$

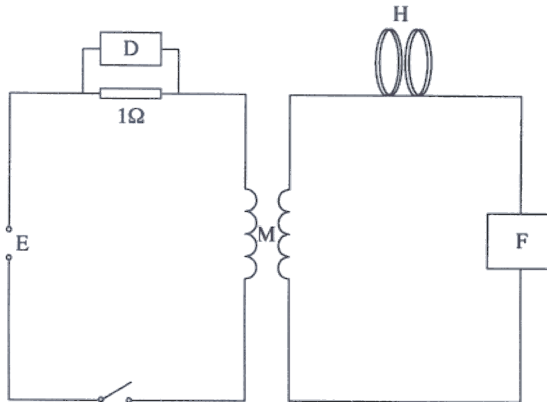
Using Helmholtz coils it is therefore possible to measure a magnetic moment by measuring the change in flux when the moment is removed from the coil. When removing the moment from the coil care must be taken to ensure that the moment no longer couples with the Helmholtz coils. This can be achieved by removing the moment to a suitably large distance. A more convenient approach is to rotate the moment in the coils about an axis that produces a change in signal. This also doubles the detected signal. For this rotation method the calculation of magnetic moment from the measured flux change is

$$m = \frac{\Delta\Phi}{2} \times \frac{I}{H} \quad (8)$$

5.2. System calibrations and measurement procedure

Calibration of integrator

The integrator is used to measure the change in flux in the coil. In order to establish the region in which the integrator needs to be calibrated a preliminary measurement is made. The circuit is set up as shown in circuit 1. The Helmholtz coils are incorporated as a passive component so that the resistance of the calibration circuit is the same as for the measurement circuit. Care must be taken to ensure that the mutual inductor and Helmholtz coils do not couple, by careful separation and orientation.



Circuit 1.

KEY:	D	digital voltmeter
	E	DC supply
	F	flux integrator
	H	Helmholtz coils
	M	mutual inductor

The integrator is calibrated at appropriate points of the selected range using the following equation:

$$\Delta\Phi = M\Delta I \quad (9)$$

Where M is the mutual inductance for the calibration (in H)
 ΔI is the change in primary current in the mutual inductor (in A)
 $\Delta\Phi$ is the required change in flux (in Wb)

The current from a DC supply is passed through the primary winding of the mutual inductor and measured by connecting a calibrated DMM across a suitable calibrated resistor. The current is set and then switched off and the integrator reading noted.

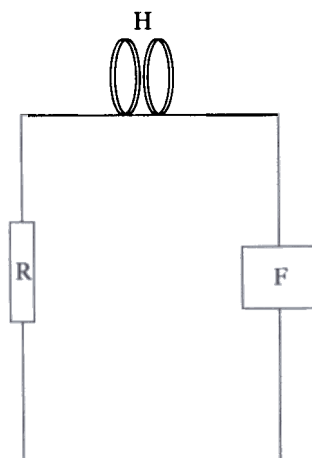
Calibration of Helmholtz coil H/I constant.

From equation (4) the H/I constant of Helmholtz coils can be determined from the dimensions and number of turns per coil. The uncertainty in this determination of the coil constant is therefore dependant on how well the dimensions can be measured. For coils with a high H/I resulting from a large number of turns, the effective diameter of the coil and effective spacing need to be considered and the dimensional uncertainty will become large.

To achieve a low measurement uncertainty for the magnetic moment the contribution from the calibration of the Helmholtz coil constant should be as small as possible. To achieve this low uncertainty the uniform field region of such a coil can be used. Nuclear magnetic resonance requires a uniform magnetic field over the active volume of the sensor. This uniformity is available with Helmholtz coils and resonance techniques are therefore used to determine the magnetic field strength. The current in the coil is determined using a calibrated resistor and calibrated digital voltmeter. By using a cancellation system to limit uncertainties due to the earth's magnetic field, it is possible to calibrate the H/I constant of Helmholtz coils with an estimated uncertainty of 0.03%.

Measurement of magnetic dipole moment.

The circuit is set up as shown in circuit 2. The mutual inductor is replaced by a resistor of the same value as the secondary winding resistance. This eliminates the possibility of coupling between the permanent magnet and the mutual inductor.



KEY:	F	flux integrator
	H	Helmholtz coils
	R	Resistor to replace secondary of mutual inductor

Circuit 2.

The test specimen is placed longitudinally and coaxially at the centre of the Helmholtz coils, the integrator zeroed and the specimen rotated through approximately 180° and aligned for the maximum integrator reading. The integrator is then zeroed again and the specimen rotated through approximately 180° and aligned for the maximum integrator reading. The integrator reading is then recorded.

It is not always possible to rotate some specimens within the Helmholtz coils. In this case the specimen is aligned for the maximum integrator reading, and the integrator is zeroed. The specimen is then removed to a distance sufficient that the integrator is no longer affected by movement of the specimen. This technique gives an integrator reading that does not need to be halved before the moment is calculated.

From the integrator reading, corrected for the calibration, the flux change, $\Delta\Phi$, can be calculated. From this it is possible to calculate the magnetic moment, m , using equation (7) for extraction and equation (8) for rotation.

Note on units of magnetic moment.

In equation (6) the unit of magnetic dipole moment was given as Wbm. When discussing equations (2) and (3) (field at a distance) the unit was given as Am^2 . These are both units of the same quantity with the difference arising because the intrinsic magnetic flux density of materials can be given as *magnetization* or *polarization*. These differ by the magnetic constant μ_0 . A magnetic dipole moment in Wbm is converted to Am^2 by dividing by μ_0 . The dimensions of the equation should be checked if there is doubt as to which unit is appropriate.

6. Measurement uncertainty

In the previous sections, three methods for measuring magnetic dipole moment were introduced. In sections 3 and 4, the estimated uncertainty for the measurement of magnetic dipole moment was given as $\pm 1.5\%$ for the torsional pendulum and $\pm 2.7\%$ for the field at a distance method. For the torsional pendulum, the number of contributions to the uncertainty resulted in this value. For the field at a distance uncertainty it is the sensitivity of the measurement to distance that results in the high uncertainty.

For the Helmholtz coil method the uncertainty was evaluated in accordance with M 3003 [2] and a typical example is given below.

UNCERTAINTY IN THE MEASUREMENT OF DIPOLE MOMENT OF PERMANENT MAGNETS						
Source of Uncertainty	Value %	± Probability distribution	Divisor	c_i	u_i ± %	V_i or V_{eff}
Calibration of Integrator						
Calibration of 1 ohm resistor	0.000006	normal	2	1	0.0000	inf
Calibration of DVM across resistor	0.0004	normal	2	1	0.0002	inf
Resolution of DVM for current	0.00003	rectangular	1.732	1	0.0000	inf
Calibration of Mutual Inductor	0.0150	normal	2	1	0.0075	inf
Resolution of DVM on flux integrator output	0.056	rectangular	1.732	1	0.0323	inf
Repeatability	0.0228	normal	1	1	0.0228	4
Measurement of specimen						
Calibration of Helmholtz coils	0.025	normal	2	1	0.0125	inf
Alignment of specimen	0.02	rectangular	1.732	1	0.0115	inf
Resolution of DVM on flux integrator output	0.027	rectangular	1.732	1	0.0156	inf
Repeatability	0.0231	normal	1	1	0.0231	4
Combined uncertainty					0.0518	52.1
Expanded uncertainty			k=	2.05	0.1063	

In this case, the dimensions of the test specimen were such that no correction to the H/I was needed. When H/I changes along the length of the test specimen either a correction is applied with a suitable contribution to the overall measurement uncertainty, or the full magnitude of the change is included in the uncertainty.

7. Comparison of methods

Four magnets with the dimensions given in table 1 below were measured using the methods described in sections 3 to 5.

Magnet reference	Approximate length (cm)	Approximate diameter (cm)
A	2.9	0.6
B	7.2	0.6
C	8.5	0.8
D	11.5	1.0

Table 1. Magnet dimensions.

Given in the following tables are the measured magnetic moments and estimated measurement uncertainties. For the Helmholtz coil method, a contribution has been added to the uncertainty budget in section 6 due to the non-uniformity of H/I along the magnet length.

Magnet reference	Measured magnetic moment Am^2	Estimated uncertainty ($\pm\%$)
A	0.471	1.5
B	1.540	1.5
C	3.01	1.5
D	5.94	1.5

Table 2. Torsional Pendulum method.

Magnet reference	Measured magnetic moment Am^2	Estimated uncertainty ($\pm\%$)
A	0.472	2.7
B	-	-
C	-	-
D	5.93	2.7

Table 3. Field at a distance.

Magnet reference	Measured magnetic moment Am^2	Estimated uncertainty ($\pm\%$)
A	0.4721	0.2
B	1.538	0.2
C	3.004	0.3
D	5.961	0.3

Table 4. Helmholtz coil method.

From the reported measurements and the estimated measurement uncertainty, the methods can be seen to give consistent results.

8. Calibration of commercial systems

Several manufacturers provide the instrumentation needed for the Helmholtz coil method of measuring magnetic dipole moment. In section 5.2 the necessary calibrations were discussed. Since the commercial systems are used in situations where high throughput is necessary, it is not convenient to maintain the individual instrument calibrations. In this case, the moment measurement setup is calibrated as a system. To achieve this a reference moment is needed with the magnetic dipole moment calibrated. To achieve as low a calibration uncertainty for the system as possible the uncertainty of the reference dipole moment must be low and the approach given above should be used. This includes using a Helmholtz coil system with suitable dimensions to limit any contribution due to non-uniform H/I to the overall measurement uncertainty.

9. Conclusions

Magnetic dipole moment is a quantity that provides useful information about component properties. The measurement of magnetic dipole moment with a low uncertainty is therefore required. Three methods have been introduced and the uncertainty of each measurement presented and discussed. The method using a Helmholtz coil pair to determine the magnetic moment can provide a measurement uncertainty of 0.1% with a confidence level of not less than 95%. A calibrated reference with this uncertainty provides an excellent basis for the calibration of commercial systems.

10. References

- [1] Trout, S.R., *Use of Helmholtz Coils for Magnetic Measurements*, IEE Trans. Mag. Vol. 24, No. 4, July 1988.
- [2] UKAS document M 3003, Edition 1, December 1997.

