

REPORT

Some Comparisons of the Roscoe and Beckwith- Newell Analyses for the Determination of Viscosity of Liquid Metals using the Oscillating Cup Viscometer

**NMW 5.08 The Choice of
Equations for the
Measurement of Viscosity by
the Oscillating Cylinder
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SUMMARY

It is shown that a small error exists in much of the literature quoting the Roscoe equation for the determination of viscosity using an oscillating cup viscometer. Typical data from experiments with liquid metals are analysed using the correct Roscoe equation and the Beckwith-Newell formula. Comparison of the results indicate that the formulae are essentially equivalent. There is a brief discussion on the role of wetting of the crucible by the liquid under investigation and end effects.

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Approved on behalf of Managing Director, NPL, by Dr C Lea,
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1 INTRODUCTION

Experiments to determine the viscosity of liquid metals using the oscillating cup technique have been performed at the Materials Centre, NPL for some years [1]. The results have so far been analysed using Roscoe's equation [2,3], mainly for reasons of simplicity of presentation of the working formulae of that reference and following other workers in the liquid metals field [4,5]. A more complete model of oscillating cup viscometers was described by Kestin, Newell and Beckwith [7,8] at around the same time. This analysis was extended to cylinder and disk oscillating viscometers by a number of workers [e.g. 9,10,11]. The resulting working equations were not stated explicitly in the early references and required some further analysis for the derivation. The necessary analysis is described by Nieuwoudt et al [9] as 'straightforward, though tedious'. Recently, it has been suggested that the Beckwith-Newell model is more accurate and comprehensive than that of Roscoe and should therefore be adopted for future analysis of viscosity data at NPL.

It is the purpose of this report to clarify this suggestion by comparing results for the two models calculated from typical NPL data.

2 THE ROSCOE ANALYSIS

Roscoe [2] describes the derivation of the necessary formulae for determining viscosity from oscillatory spherical and cylindrical viscometers. Certain mathematical approximations are made which involve asymptotic expansion of Bessel and hyperbolic functions. These approximations are not all performed in a systematic manner, but the final operating formulae are clear; we reproduce here the most important expression for the determination of the viscosity:-

$$I\Delta = \frac{R^3 H}{2} (\pi \eta \rho T)^{\frac{1}{2}} \left[\left(1 + \frac{1}{4} \frac{R}{H} \right) a_0 - \left(\frac{3}{2} + \frac{4}{\pi} \frac{R}{H} \right) \frac{1}{p} + \left(\frac{3}{8} + \frac{9}{4} \frac{R}{H} \right) \frac{a_2}{2p^2} - \left(\frac{63}{128} - \frac{45}{64} \frac{R}{H} \right) \frac{a_4}{4p^4} \dots \right] \quad (2.1)$$

where

$$a_0 = (1 - \Delta) \left\{ \frac{\sqrt{(1 + \Delta^2)} + 1}{2} \right\}^{\frac{1}{2}} - (1 + \Delta) \left\{ \frac{\sqrt{1 + \Delta^2} - 1}{2} \right\}^{\frac{1}{2}} \quad (2.2)$$

$$= 1 - \frac{3}{2} \Delta - \frac{3}{8} \Delta^2 - \frac{1}{16} \Delta^3 \dots \quad (2.2A)$$

$$a_2 = \left\{ \frac{\sqrt{1 + \Delta^2} + 1}{2} \right\}^{\frac{1}{2}} + \left\{ \frac{\sqrt{1 + \Delta^2} - 1}{2} \right\}^{\frac{1}{2}} \\ = 1 + \frac{\Delta}{2} + \frac{1}{8} \Delta^2 - \frac{1}{16} \Delta^3 \dots \quad (2.3)$$

$$a_4 = a_2 / (1 + \Delta^2)^{\frac{1}{2}} = 1 + \frac{\Delta}{2} - \frac{3}{8}\Delta^2 - \frac{5}{16}\Delta^3 \dots \quad (2.4)$$

and
$$p = (\rho\pi/\eta T)^{\frac{1}{2}} R .$$

In (2.1), R is the cylinder radius, H the height of the liquid within it, I is the moment of inertia of the suspension, $2\pi\Delta$ the logarithmic decrement between consecutive swings, ρ is the fluid density, η the viscosity and T is the period of oscillation.

It must be pointed out here that a mis-print occurs in the original form of equation (2.2A) in reference [2]. The numerator of the second term is omitted. However in a number of subsequent publications (e.g. references [1,2,3,4,5]) the complete term appears as $(-\Delta/2)$. Expansion of (2.2) by the binomial theorem and subsequent simplification yields the term $(-3\Delta/2)$, as given above. This has been confirmed using the symbolic algebraic system REDUCE. It is likely that this has been pointed out in more recent literature, but at the time of writing the authors are unaware of any reports that do so. We show later the effect on the result that this discrepancy can have. It is possible that some workers using the Roscoe analysis have performed their calculations using the full expression for a_0 given by (2.2). In this case no algebraic errors will occur, but there will be some small loss of accuracy due to numerical cancellation.

The numerical treatment of (2.1) is as follows. We re-write this as

$$K = x[b_0 - b_1x + b_2x^2 - b_3x^4] \quad (2.5)$$

where

$$K = \frac{2I\Delta}{\pi\rho HR^4} , \quad x = \left(\frac{\eta T}{\pi\rho} \right)^{\frac{1}{2}} \cdot \frac{1}{R}$$

$$b_0 = \left(1 + \frac{R}{4H} \right) a_0 ,$$

$$b_1 = \left(\frac{3}{2} + \frac{4R}{\pi H} \right) ,$$

$$b_2 = \left(\frac{3}{8} + \frac{9R}{4H} \right) \frac{a_2}{2} ,$$

$$b_3 = \left(\frac{63}{128} + \frac{45R}{64H} \right) \frac{a_4}{4} ,$$

Equation (2.5) is then solved iteratively for x using the Newton-Raphson technique. The viscosity is then obtained from the definition of x .

3 THE BECKWITH, KESTIN AND NEWELL ANALYSIS

Kestin and Newell [7] and Beckwith and Newell [8] describe an alternative analysis to that of Roscoe for the oscillating cup viscometer. These reports carefully describe the applicability of the analysis and [8] identifies the 'small', 'intermediate' and 'large' cup regimes according to the magnitude of certain dimensionless parameters. Table 1 to this report shows a selection of typical dimensions and other data from the NPL viscosity experimental programs. The seventh and eighth columns of the table show the parameter H/δ and R/δ which determine the relevant cup regime for the analysis as described in [8]. The quantity δ is defined in Table 1. In reference 8 these parameters are denoted by η_0 and ξ_0 , respectively. The Table indicates that for all entries

$$68 < \eta_0 < 200 \quad \text{and} \quad 8 < \xi_0 < 23.$$

Reference to Figure 1 of this report (which is reproduced from [8]) shows that these values correspond to the 'large cup' regime.

The corresponding operating equation is not given explicitly in [7,8], so to avoid repeating the 'straightforward but tedious' derivation of these we follow the form due to Brockner et al [12] which were obtained from [8]. From equation (6) of [12] we have

$$\frac{\pi \rho H R^4}{2I} \left[A(p - \Delta q) \frac{1}{x} - \frac{B}{x^2} + \frac{Cq}{x^3} \right] = 2(\Delta - \Delta_0 / \omega) \quad (3.1)$$

where

$$A = 4 + \frac{R}{H}$$

$$B = 6 + \frac{16}{\pi} \cdot \frac{R}{H}$$

$$C = \frac{3}{2} + \frac{9R}{H}$$

$$\omega = T_0 / T$$

$$p = 1/(2(\Delta + (1 + \Delta^2)^{\frac{1}{2}}))^{\frac{1}{2}}$$

$$q = 1/(2p)$$

$$x = R(2\pi\rho/(\eta T))^{\frac{1}{2}}$$

where T_0 is the oscillation period and $2\pi\Delta_0$ is the logarithmic decrement for the empty cup. The other quantities are as defined in the previous section.

We re-write (3.1) as

$$K = zb_0 - z^2b_1 + z^3b_2 \quad (3.2)$$

where
$$K = \frac{4I}{\pi \rho H R^4} \left(\Delta - \frac{\Delta_0}{\omega} \right)$$

$$b_0 = A(p - \Delta q)$$

$$b_1 = B$$

$$b_2 = C_p$$

and
$$z = 1/x.$$

Equation (3.2) is solved iteratively for z , which enables the calculation for η through the expression for x .

4 NUMERICAL COMPARISONS

Appendix 1 to this report shows two sets of oscillating cup viscometry data extracted from NPL experimental programs. One set is for a eutectoid steel and the other for aluminium.

Substitution of the steel data into the Roscoe equation (2.5) and subsequent numerical solution for the viscosity gives 6.047 mPa. The corresponding value calculated using the Beckwith-Newell analysis (3.2) yields 6.048 mPa. For the aluminium data, the viscosity results for the Roscoe and Beckwith-Newell analyses are 0.949 mPa and 0.954 mPa respectively. The difference between the results for aluminium is about 0.5% and that for steel very much less.

The effect of the error in the expansion for a_0 mentioned in Section 2 is as follows. With the error re-introduced the steel value becomes 5.762 mPa (a discrepancy of 4.7%) and that for aluminium becomes 0.937 mPa (a discrepancy of 1.3%). It is perhaps debatable whether these differences would be considered serious, although they may be significant in some circumstances.

5 ADDITIONAL FACTORS WHICH INFLUENCE VISCOSITY DETERMINATION

Besides the challenges of choosing the correct way to extract the viscosity from the experimental results, there are other factors which contribute to the uncertainty of the results. Amongst the more difficult to treat are wetting effects and inadequate analysis of the end effects.

5.1 THE WETTING OF THE CRUCIBLE BY THE MOLTEN METAL

Iida and Guthrie [3] discuss the phenomena and suggest for the measurement of the viscosity of lead in a graphite crucible that the liquid does not completely wet the side walls of the

vessel. This results in slippage between the liquid/wall interfaces, thus providing smaller damping than anticipated and in consequence a smaller viscosity is measured.

If liquids react with the crucible, (for example no crucible material is completely inert to liquid iron at the high temperatures) the interaction between the wall and the metal will depend on the time and the temperature of the experiment.

Kimura et al [13,14,15] reported the viscosity values of silicon obtained using "wetting" SiC crucibles and showed results 20% higher than measurements in "non wetting" BN crucibles. The crucibles in the study had a relatively small aspect ratio (height/radius) and it may be that a fuller analysis of the data for measuring the viscosity is required.

Sato [16] carried out viscosity measurements on silicon using crucibles fabricated from a variety of materials. The results showed some variation in the viscosity values and these were lower than those reported by other workers. It could be conjectured that the Roscoe equation used was in error.

5.2 THE INSUFFICIENT WEIGHTING OF THE END CORRECTION

Iida and Guthrie state that although they consider that the Roscoe equations provide the most accurate method for determining the viscosity using an oscillating cup viscometer, the agreement with results by the capillary method are often not good. They suggest the end effect is inadequately weighted in the Roscoe treatment and should also include the effect of the liquid meniscus on the height of liquid.

It was suggested that a correction factor, ξ , is introduced into the formula such that when $\xi = 1$, the formula is identical to the unmodified Roscoe equation. They give a factor of 1.04 for a set of five viscometer-calibration liquids and 0.96 ± 0.04 for iron based upon tin, bismuth, lead and copper calibration.

The correction factor will depend on the construction of the viscosity apparatus but more importantly on the materials (wetting) and dimensions (end effects) of the vessel.

The authors of this report would suggest that until the analysis of the extraction of the viscosity from the experiment is clarified, the effects arising from the two discussed effects are difficult to identify.

6 CONCLUSION

It has been shown that the experimental NPL program for the determination of viscosity falls within the 'large cup' regime as described by Beckwith-Newell [8]. For a limited data set, it has been shown here that the relevant Beckwith-Newell and Roscoe operating equations for the oscillating cup viscometer give almost identical results. Present resources do not allow more extensive comparisons involving additional data.

In Section 2, attention has been drawn to an omission or mis-print in the original Roscoe report [2], which can lead to errors of a few percent. This error appears to have persisted in the subsequent literature.

Other complicating factors which may affect viscosity results, such as wetting and end effects, have been briefly discussed.

TABLE 1 Typical Data Obtained with the NPL Oscillating Cup Viscometer

Material	Viscosity η mPa.s	Density ρ kgm ⁻³	Period T (s)	Cup height H (m)	Cup radius R (m)	$\delta = (\eta T / 2\pi\rho)^{1/2}$ mm	H/ δ	R/ δ	Temp. °C
Aluminium	0.967	2346	4.51	0.068	0.0078	0.544	125	14.3	750
Al Alloy LM25	0.985	2399	4.04	0.066	0.0075	0.514	128	14.6	700
Copper	4.3	7987	4.12	0.077	0.0076	0.594	130	12.8	1140
Cu Alloy CuSn	3.12	7760	5.61	0.062	0.0076	0.599	104	12.7	1201
Zinc	3.39	6513	4.07	0.074	0.0075	0.581	127	12.9	475
Zn Alloy Zn/Al	2.33	6020	4.06	0.080	0.0075	0.500	160	15.0	550
Nickel	4.24	7816	5.63	0.076	0.0076	0.697	109	10.9	1525
Ni Alloy IN718	5.81	6935	5.65	0.076	0.0076	0.868	87.6	8.8	1455
Co Alloy x45	6.73	7860	5.64	0.060	0.0076	0.877	68.4	8.7	1504
Iron	5.35	6988	5.64	0.070	0.0076	0.829	84.4	9.2	1592
Fe Alloy Eutectoid	5.96	6905	5.68	0.076	0.0076	0.883	86.1	8.6	1492
Mercury	1.73	13546	5.44	0.064	0.0075	0.333	192	22.5	20

APPENDIX 1 Typical Experimental Data from Oscillating Viscometer

Material	Eutectoid Steel	Aluminium
Density ρ (Kg/m ³)	6896	2330
Cup Radius R (m)	0.00761	0.00782
Cup Height h (m)	0.0760	0.0681
Inertia I (Kg m ²)	$2.86_{10^{-5}}$	$1.86_{10^{-5}}$
Log Decrement (Empty)	0.00428	0.0099
Log Decrement (Full)	0.0791	0.0365
Period T_0 (Empty) (s)	5.554	4.497
Period T (Full) (s)	5.678	4.507

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