

**Determination of the
uncertainties in
transient extensional
viscosity testing**

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November 2000

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ABSTRACT

An analysis of the uncertainties in the measurement of the transient extensional viscosity of polymer melts has been presented. Illustration of these uncertainties is made by reference to the results for a test on a high density polyethylene. This example clearly shows the rapid growth in uncertainty at strains above a value of 4 due to the exponential decrease in the specimen's cross-sectional area with time during the test. Below a strain of 4 the expanded relative uncertainty values, representing 95% confidence levels, are approximately $\pm 30\%$.

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ISSN 1361-4061

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Prepared as a deliverable of milestone 15 of the project
MPM1.2: Viscoelastic measurement techniques for polymer melts,
a project of the Department of Trade and Industry EID programme
on measurements related to the processability of materials

Approved on behalf of Managing Director, NPL
by Dr C Lea, Director, NPL Materials Centre

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1. INTRODUCTION

In the measurement of the transient extensional viscosity of polymer melts the uncertainties can be significant. The function used to calculate viscosity values contains an exponential term: physically, the specimen's cross-sectional area decreases exponentially as the test progresses. Consequently the force also decreases rapidly to a small value. Eventually, the uncertainties in the measurement will be greater than the measured values. An approximate analysis of the uncertainty of the measurement of transient extensional viscosity was previously presented [1]. However, it was considered desirable to complete a more rigorous analysis of the uncertainties of the measurement. That analysis is now presented.

The equations presented below can be used to calculate the uncertainties in the derived transient extensional viscosity values (also known as tensile stress growth coefficient) given the uncertainties in each component of the measurement system. They can be used to identify the critical parameters that generate the uncertainties in the measurements and can thus be used to improve on the measurement method. A comparison with the previous, approximate estimate of uncertainties has also been made.

The analysis has been developed for the experimental configuration used at the NPL. Where necessary it can be developed, following the same principles, for other configurations.

2. ANALYSIS OF THE UNCERTAINTIES OF MEASUREMENT OF TRANSIENT EXTENSIONAL VISCOSITY

2.1 GENERAL

Following a more rigorous approach, used for example by Kandil (2), the combined uncertainty $u_c(y)$ of the measurand y (the quantity to be measured) can be determined from the partial derivatives of the function and the uncertainties in the parameters. Assuming that individual uncertainty sources are uncorrelated, the combined uncertainty $u_c(y)$ can be computed using the root sum squares:

$$u_c(y) = \sqrt{\sum_{i=1}^m [c_i u(x_i)]^2} \quad (1)$$

where c_i is the sensitivity coefficient (partial derivative) associated with an input quantity x_i and $u(x_i)$ is the uncertainty in that quantity.

The combined uncertainty $u_c(y)$ corresponds to one standard deviation and therefore has an associated confidence level of approximately 68%. Assuming a normal distribution then it is assumed that an expanded uncertainty U for 95% confidence level can be determined using a coverage factor of 2 (i.e. equivalent to 2 standard deviations). The relative uncertainty is the ratio of the uncertainty in the parameter to the value of the parameter.

The following uncertainty analyses are based on the use of an extensional rheometer having a single rotating clamp and a fixed clamp between which the specimen is mounted [1]. For definitions and equations defining the measurement method see Appendices 1 and 2. Similar

analyses can be carried out for other instrument configurations by following the principles presented.

2.2 UNCERTAINTY IN STRAIN RATE

The strain rate $\dot{\epsilon}$ in the specimen is given (Appendix 2) by

$$\dot{\epsilon} = \frac{\omega r}{\ell_o} \quad (2)$$

where ω is the angular speed of the winding drum, r is the radius of the winding drum and ℓ_o is the original specimen length. The sensitivity coefficients, or partial derivatives, of the strain rate are thus

$$\frac{\partial \dot{\epsilon}}{\partial \omega} = \frac{r}{\ell_o} \quad (3)$$

$$\frac{\partial \dot{\epsilon}}{\partial r} = \frac{\omega}{\ell_o} \quad (4)$$

and

$$\frac{\partial \dot{\epsilon}}{\partial \ell_o} = \frac{-\omega r}{\ell_o^2} \quad (5)$$

In what follows, a subscripted u denotes an uncertainty in the subscripted quantity. Thus, using equation 1, the combined uncertainty in strain rate $u_{\dot{\epsilon}}$ is given by the root sum of the squares of the product of the partial derivatives and the corresponding uncertainties

$$(u_{\dot{\epsilon}})^2 = \left(\frac{r}{\ell_o} u_{\omega} \right)^2 + \left(\frac{\omega}{\ell_o} u_r \right)^2 + \left(\frac{-\omega r}{\ell_o^2} u_{\ell_o} \right)^2 \quad (6)$$

and the combined relative uncertainty in strain rate is given, by dividing through by $\dot{\epsilon}$, by

$$\left(\frac{u_{\dot{\epsilon}}}{\dot{\epsilon}} \right)^2 = \left(\frac{u_{\omega}}{\omega} \right)^2 + \left(\frac{u_r}{r} \right)^2 + \left(\frac{u_{\ell_o}}{\ell_o} \right)^2 \quad (7)$$

2.3 UNCERTAINTY IN STRAIN

The strain ϵ is given (Appendix 2) by

$$\epsilon = \frac{\omega r t}{\ell_o} \quad (8)$$

where t is time. Thus, in a similar fashion to that for strain rate, the combined relative uncertainty in strain is given by

$$\left(\frac{u_\varepsilon}{\varepsilon}\right)^2 = \left(\frac{u_\omega}{\omega}\right)^2 + \left(\frac{u_r}{r}\right)^2 + \left(\frac{u_t}{t}\right)^2 + \left(\frac{u_{\ell_o}}{\ell_o}\right)^2 \quad (9)$$

2.4 UNCERTAINTY IN TENSILE STRESS

The tensile stress σ in the specimen is given (Appendix 2) by

$$\sigma = \frac{F e^{\left(\frac{\omega r t}{\ell_o}\right)}}{A_o} \quad (10)$$

where F is the tensile force and A_o is the original specimen cross-sectional area. Thus the sensitivity coefficients are given by

$$\frac{\partial \sigma}{\partial F} = \frac{e^{\left(\frac{\omega r t}{\ell_o}\right)}}{A_o} = \frac{\sigma}{F} \quad (11)$$

$$\frac{\partial \sigma}{\partial A_o} = \frac{-F}{A_o^2} e^{\left(\frac{\omega r t}{\ell_o}\right)} = \frac{-\sigma}{A_o} \quad (12)$$

$$\frac{\partial \sigma}{\partial \omega} = \frac{F}{A_o} \left(\frac{rt}{\ell_o}\right) e^{\left(\frac{\omega r t}{\ell_o}\right)} = \sigma \left(\frac{rt}{\ell_o}\right) \quad (13)$$

$$\frac{\partial \sigma}{\partial r} = \frac{F}{A_o} \left(\frac{\omega t}{\ell_o}\right) e^{\left(\frac{\omega r t}{\ell_o}\right)} = \sigma \left(\frac{\omega t}{\ell_o}\right) \quad (14)$$

$$\frac{\partial \sigma}{\partial t} = \frac{F}{A_o} \left(\frac{\omega r}{\ell_o}\right) e^{\left(\frac{\omega r t}{\ell_o}\right)} = \sigma \left(\frac{\omega r}{\ell_o}\right) \quad (15)$$

and

$$\frac{\partial \sigma}{\partial \ell_o} = \frac{F}{A_o} \left(\frac{-\omega r t}{\ell_o^2}\right) e^{\left(\frac{\omega r t}{\ell_o}\right)} = \sigma \left(\frac{-\omega r t}{\ell_o^2}\right) \quad (16)$$

Note: see Appendix 3 for differentiation of exponential function.

Thus the combined relative uncertainty in stress is given by the expression

$$\left(\frac{u_\sigma}{\sigma}\right)^2 = \left(\frac{u_F}{F}\right)^2 + \left(\frac{-u_A}{A_o}\right)^2 + \left(\frac{rtu_\omega}{\ell_o}\right)^2 + \left(\frac{\omega tu_r}{\ell_o}\right)^2 + \left(\frac{\omega ru_t}{\ell_o}\right)^2 + \left(\frac{-\omega rtu_{\ell_o}}{\ell_o^2}\right)^2 \quad (17)$$

2.5 UNCERTAINTY IN TRANSIENT EXTENSIONAL VISCOSITY

Given the expression for transient extensional viscosity η_E (Appendix 2):

$$\eta_E = \frac{F \ell_o e^{\left(\frac{\omega r t}{\ell_o}\right)}}{A_o \omega r} \quad (18)$$

then the sensitivity coefficients c_i are given by

$$\frac{\partial \eta_E}{\partial F} = \frac{\ell_o e^{\left(\frac{\omega r t}{\ell_o}\right)}}{A_o \omega r} \quad (19)$$

$$\frac{\partial \eta_E}{\partial A_o} = \frac{-F \ell_o e^{\left(\frac{\omega r t}{\ell_o}\right)}}{A_o^2 \omega r} \quad (20)$$

$$\frac{\partial \eta_E}{\partial r} = \frac{F \ell_o}{A_o \omega} \left(\frac{-e^{\left(\frac{\omega r t}{\ell_o}\right)}}{r^2} + \frac{\omega t e^{\left(\frac{\omega r t}{\ell_o}\right)}}{\ell_o r} \right) \quad (21)$$

$$\frac{\partial \eta_E}{\partial \omega} = \frac{F \ell_o}{A_o r} \left(\frac{-e^{\left(\frac{\omega r t}{\ell_o}\right)}}{\omega^2} + \frac{r t e^{\left(\frac{\omega r t}{\ell_o}\right)}}{\ell_o \omega} \right) \quad (22)$$

$$\frac{\partial \eta_E}{\partial \ell_o} = \frac{F}{A_o \omega r} \left(e^{\left(\frac{\omega r t}{\ell_o}\right)} + \frac{\ell_o \omega r t e^{\left(\frac{\omega r t}{\ell_o}\right)}}{-\ell_o^2} \right) \quad (23)$$

$$= \frac{F}{A_o \omega r} \left(e^{\left(\frac{\omega r t}{\ell_o}\right)} - \frac{\omega r t e^{\left(\frac{\omega r t}{\ell_o}\right)}}{\ell_o} \right)$$

and

$$\frac{\partial \eta_E}{\partial t} = \frac{F e^{\left(\frac{\omega r t}{\ell_o}\right)}}{A_o} \quad (24)$$

The combined relative uncertainty in the tensile stress growth coefficient values is then evaluated as

$$\left(\frac{u_{\eta_E}}{\eta_E}\right)^2 = \left(\frac{\partial \eta_E}{\partial F} \frac{u_F}{\eta_E}\right)^2 + \left(\frac{\partial \eta_E}{\partial A_o} \frac{u_{A_o}}{\eta_E}\right)^2 + \left(\frac{\partial \eta_E}{\partial r} \frac{u_r}{\eta_E}\right)^2 + \left(\frac{\partial \eta_E}{\partial \omega} \frac{u_\omega}{\eta_E}\right)^2 + \left(\frac{\partial \eta_E}{\partial \ell_o} \frac{u_{\ell_o}}{\eta_E}\right)^2 + \left(\frac{\partial \eta_E}{\partial t} \frac{u_t}{\eta_E}\right)^2 \quad (25)$$

where the partial derivatives are given in Equations 19-24.

3. EVALUATION OF UNCERTAINTY IN TRANSIENT EXTENSIONAL VISCOSITY MEASUREMENTS

The values of the terms in the above uncertainty budgets, except for the force terms, are relatively easy to determine. The difficulty with the force term is that the force decreases considerably at high strain values and thus the relative uncertainty in the force will increase significantly as the test progresses. To determine the effect of its contribution to the combined uncertainty in the determination of the transient extensional viscosity values further analysis, presented below, is required.

The force F supported by the specimen during testing is given by

$$F = \sigma A \quad (26)$$

where A is the cross-sectional area of the specimen and σ is the tensile stress. Using the definition of transient extensional viscosity η_E^+ (or tensile stress growth coefficient), Appendix 1, then

$$\sigma = \eta_E^+ \dot{\epsilon} \quad (27)$$

Thus the force

$$F = \eta_E^+ \dot{\epsilon} A \quad (28)$$

or, using Equations A2.4, A2.2 and A2.16 of Appendix 2,

$$F = \eta_E^+ \dot{\epsilon} A_o e^{\left(\frac{-\omega r t}{\ell_o}\right)} \quad (29)$$

Assuming that the transient extensional viscosity as a function of time can be fitted reasonably well using a power law model (see Figure 8 of reference 1) then the equation of a straight line fit to the data is of the form

$$\log(\eta_E^+) = m \log(t) + \log(\eta_{E,o}^+) \quad (30)$$

where $\eta_E^+ = \eta_{E,o}^+$ at $t = 1$ s and $\eta_{E,o}^+$ and m are constants. Equation 30 can be rewritten as

$$\eta_E^+ = \eta_{E,o}^+ t^m \quad (31)$$

The force over the test duration is thus approximated by

$$F = \eta_{E,o}^+ t^m \dot{\epsilon} A_o e^{\left(\frac{-\omega t}{\ell_o}\right)} \quad (32)$$

Given the values of $\eta_{E,o}^+$ and m , the force F can be determined as a function of time and, assuming that all strain rate data fall on the same master curve, also as a function of strain if one ignores any additional strain hardening phenomenon. Thus the relative uncertainty in the force F , given the decreasing magnitude of force at high strains, can be reasonably accurately determined. However the combined uncertainty in transient extensional viscosity values must be quoted as a function of time rather than as a single value.

Analysis is presented based on the case of a high density polyethylene HDPE (HGH000) at 150 °C and a strain rate of $\approx 1 \text{ s}^{-1}$, Table 1. The transient extensional viscosity values are plotted along with 95% confidence limits derived using Equation 25 and a coverage factor of 2, Figure 1. It clearly illustrates the significant increase in measurement uncertainties at high strains due, predominantly, to the increase in the uncertainty in the measurement of force resulting from the significant reduction in the cross-sectional area of the specimen at high strain values. Expanded uncertainties representing 95% confidence limits were estimated to be of the order of $\pm 20\%$ at a strain of 3.3, $\pm 50\%$ at a strain of 4.8 and $\pm 100\%$ at a strain of 5.7. The initial decrease in uncertainty, up to a strain of 1, is due to the low forces being generated in the specimen at the start of the test resulting in a high relative uncertainty for force.

In comparing the predictions of this model with the previous approximate model [1] over the Hencky strain range of 0 – 4, the two models differ in their prediction of the relative uncertainty for the case presented by up to approximately 30% with the more rigorous model yielding lower predictions. In the strain range up to 6 there is a maximum in difference in the predictions of the two models at a strain of approximately 2.

4. CONCLUSIONS

The analysis has shown the importance of evaluating the uncertainty in transient extensional viscosity measurements particularly at high values of strain. For a test on a high density polyethylene, evaluation clearly showed the rapid growth in uncertainty at strains above a value of 4. This growth in uncertainty values arises due to the exponential decrease in specimen cross sectional area with time during the test. Below a strain of 4 the expanded relative uncertainty values, representing 95% confidence levels, are approximately $\pm 30\%$.

5. ACKNOWLEDGEMENTS

The work was carried out as part of a project on viscoelastic measurement techniques for polymer melts (MPM 1.2, milestone 15). This project is part of a programme of underpinning research financed by the Engineering Industries Directorate (EID) of the Department of Trade and Industry on measurements related to the processability of materials.

Table 1: Example values and uncertainties in the extensional testing parameters

$\eta_{E,o}^+ = 2.4 \times 10^5$ Pa.s and $m = 0.6$ estimated for an HDPE (HGH000) at 150 °C at a strain rate of $\approx 1 \text{ s}^{-1}$.

Quantity	Symbol	Units	Type	Probability distribution	Divisor	Quantity value x	Quantity range, $\pm$	Standard uncertainty u_x	Relative uncertainty u_x/x
Initial specimen diameter 1	d_1	m	A	Normal	1	0.0033	0.0001	0.0001	0.030
Initial specimen diameter 2	d_2	m	A	Normal	1	0.0033	0.0001	0.0001	0.030
Force	F	N	B	Rectangular	$\sqrt{3}$	variable	0.016	0.0092	variable
Effective specimen length	ℓ_o	m	B	Rectangular	$\sqrt{3}$	0.1	0.003	0.0017	0.017
Angular speed	ω	rad/s	B	Rectangular	$\sqrt{3}$	5.0	0.015	0.0087	0.0017
Drum radius	r	m	B	Rectangular	$\sqrt{3}$	0.02	0.00075	0.00043	0.022
Time	t	s	B	Rectangular	$\sqrt{3}$	variable	0.024	0.014	variable
Initial area*	A_o	m ²	A	Normal	1	8.6×10^{-6}	3.7×10^{-7}	3.7×10^{-7}	0.043

Note: * indicates calculated values based on the assumption of independent orthogonal measurements of the specimen.

The standard uncertainty is obtained by dividing the quantity range by the divisor.

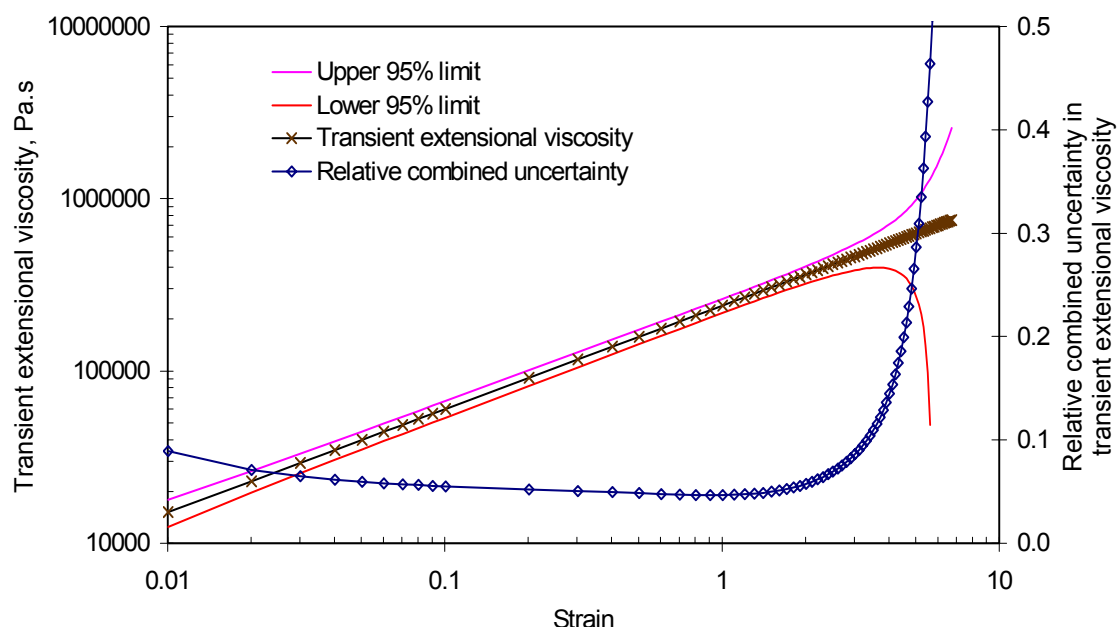


Figure 1. Analysis of the uncertainties in transient extensional viscosity values yielding 95% upper and lower bands, based on a HDPE (HGH) at 150 °C and 1 s^{-1} . The relative combined uncertainty values presented (right-hand y-axis) are for one standard deviation.

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Appendix A1: Definitions of strain, strain rate, stress and material properties functions in tensile (simple) extension

The following definitions are given by Whorlow [3] for strains and strain rates. Further descriptions are also given by, for example, Gupta *et al* [4] and Dealy [5, 6].

A1.1 Elongation ratio

The *Elongation ratio* (ER) is the ratio of the current length ℓ to the initial length ℓ_o of the specimen:

$$ER = \ell / \ell_o. \quad (A1.1)$$

A1.2 Cauchy ϵ_C and Hencky ϵ strains

The *Cauchy strain* ϵ_C is given by the ratio of the change in length $\delta \ell$ to the initial length ℓ_o of the specimen:

$$\epsilon_C = \delta \ell / \ell_o \quad (A1.2)$$

The *Hencky strain* ϵ (also referred to as the natural or true strain) is given by the natural logarithm of the elongation ratio:

$$\epsilon = \ln(\ell / \ell_o) \quad (A1.3)$$

Thus the elongation ratio is related to the Cauchy strain by

$$ER = 1 + \epsilon_C \quad (A1.4)$$

or

$$\ell / \ell_o = 1 + \delta \ell / \ell_o \quad (A1.5)$$

To illustrate the difference between these measures of strain a specimen that was stretched to 10 times its original length has an elongation ratio of 10, a Cauchy strain of 9 and a Hencky strain of 2.3.

A1.3 Cauchy $\dot{\epsilon}_C$ and Hencky $\dot{\epsilon}$ strain rates

The *Cauchy strain rate* is given by

$$\dot{\epsilon}_C = 1 / \ell_o \times \partial \ell / \partial t \quad (A1.6)$$

and the *Hencky strain rate* by

$$\dot{\epsilon} = 1 / \ell \times \partial \ell / \partial t \quad (A1.7)$$

In describing and modelling plastics processing the Hencky strain is preferred as the rate of strain of an element of fluid within the flow is independent of its original length and is determined only from the velocity field of that element. It is thus a more suitable characteristic of the flow.

A1.4 Extensional flows: velocity fields

Terminology and definitions for materials functions describing the response of viscoelastic fluids to various shearing and extensional deformations are presented by Dealy [7]. Mathematical descriptions of extensional flows are presented, for example, by Meissner *et al* [8] and Walters [9].

For *tensile (simple or uniaxial) extension* where $\dot{\varepsilon}$ is the true strain rate and ε is the true strain, defined as

$$\varepsilon = \ln(\ell / \ell_0) \quad (\text{A1.8})$$

then in rectangular coordinates:

$$v_1 = \dot{\varepsilon} x_1 \quad (\text{A1.9})$$

$$v_2 = -\frac{1}{2} \dot{\varepsilon} x_2 \quad (\text{A1.10})$$

$$v_3 = -\frac{1}{2} \dot{\varepsilon} x_3 \quad (\text{A1.11})$$

where $\dot{\varepsilon} \geq 0$.

Alternatively, in cylindrical coordinates:

$$v_z = \dot{\varepsilon} z \quad (\text{A1.12})$$

$$v_r = -\frac{1}{2} \dot{\varepsilon} r \quad (\text{A1.13})$$

where $\dot{\varepsilon} \geq 0$.

A1.5 Material properties

Following the notation presented by Dealy [7], and prepared by the Nomenclature Committee of the Society of Rheology, for start-up flow in tensile (simple) extension at constant (Hencky) strain rate $\dot{\varepsilon}$ the following definitions are given. Equivalent expressions for cessation of steady tensile extension, tensile creep, tensile recoil and tensile step strain are presented by Dealy [7].

A1.5.1 Net tensile stress

The *net tensile stress* σ_E is defined by

$$\sigma_E = \sigma_{11} - \sigma_{22} = \sigma_{11} - \sigma_{33} = \sigma_{zz} - \sigma_{rr} \quad (\text{A1.14})$$

where σ_{ii} is a stress tensor in either rectangular or axisymmetric co-ordinates. The tensile stress

growth function is indicated by σ_E^+ where the + indicates start-up rather than cessation of flow.

A1.5.2 Tensile stress growth coefficient or transient extensional viscosity

The *tensile stress growth coefficient* η_E^+ is defined by:

$$\eta_E^+(t, \dot{\epsilon}) = \sigma_E / \dot{\epsilon} \quad (\text{A1.15})$$

where t is time. It is also referred to herein as the *transient extensional viscosity*.

A1.5.3 Tensile viscosity

The *tensile viscosity* η_E is defined by:

$$\eta_E(t, \dot{\epsilon}) = \lim_{t \rightarrow \infty} [\eta_E^+(t, \dot{\epsilon})] \quad (\text{A1.16})$$

It defines the limiting extensional viscosity and as such represents an equilibrium extensional viscosity if a steady value is achieved.

Appendix A2: Analysis of extensional rheometer testing

A2.1 Definitions and terminology

v - velocity of separation of the ends of the specimen, m.s^{-1} .

ω - angular velocity of drum, rad.s^{-1} .

r - drum radius, m

t - time, s

ℓ_o - original specimen length, m

ℓ - specimen length at time t , m

A_o - original specimen cross-sectional area, m^2

A - specimen cross-sectional area at time t , m^2

F - force, N

ε - Hencky strain

$\dot{\varepsilon}$ - Hencky strain rate, s^{-1}

σ_E - net tensile stress, Pa

η_E^+ - tensile stress growth coefficient, Pa.s

u_x - standard uncertainty in x , where x represents any quantity

The Hencky strain and strain rate and the tensile stress growth coefficient are defined in Appendix A1.

A2.2 Analysis of extensional flow

Rearranging the Hencky strain

$$\varepsilon = \ln \left(\frac{\ell}{\ell_o} \right) \quad (\text{A2.1})$$

yields

$$\ell = \ell_o e^{(\varepsilon)} \quad (\text{A2.2})$$

or, for constant strain rate $\dot{\varepsilon}$,

$$\ell = \ell_o e^{(\dot{\epsilon} t)} \quad (\text{A2.3})$$

Assuming conservation of volume of the specimen then

$$\ell_o A_o = \ell A \quad (\text{A2.4})$$

The tensile stress, expressed as the ratio of the force to area,

$$\sigma_E = \frac{F}{A} \quad (\text{A2.5})$$

can thus be rewritten, using Equation A2.4, as

$$\sigma_E = \frac{F \ell}{A_o \ell_o} \quad (\text{A2.6})$$

Substitution for ℓ using Equations A2.2 of A2.3 yields

$$\sigma_E = \frac{F e^{(\epsilon)}}{A_o} \quad (\text{A2.7})$$

or

$$\sigma_E = \frac{F e^{(\dot{\epsilon} t)}}{A_o} \quad (\text{A2.8})$$

respectively. Thus the tensile stress growth coefficient, the ratio of tensile stress to Hencky strain rate, is given by

$$\eta_E^+ = \frac{F e^{(\epsilon)}}{A_o \dot{\epsilon}} \quad (\text{A2.9})$$

or

$$\eta_E^+ = \frac{F e^{(\dot{\epsilon} t)}}{A_o \dot{\epsilon}} \quad (\text{A2.10})$$

A2.3 Analysis of the deformation in extensional rheometers

The following is an analysis of testing carried out using the NPL rheometer [1]. Analysis of flow in other configurations can be similarly derived. For the specimen the effective length is given by the separation between the clamps that in the case of the NPL instrument is between the fixed clamp and the point at which a line drawn from the fixed clamp meets the rotating drum clamp at a tangent (see Figure 1 in reference 1). The effective specimen length in the test region remains unchanged during the test as the specimen is wound onto the drum. For the

extensional rheometer using a single rotating drum the instantaneous velocity of separation of the ends of the specimen, v , is given by

$$v = \omega r \quad (\text{A2.11})$$

The velocity can also be written as

$$v = \partial \ell / \partial t \quad (\text{A2.12})$$

Thus, by using the definition of Hencky strain rate

$$\dot{\varepsilon} = 1/\ell \times \partial \ell / \partial t \quad (\text{A2.13})$$

the strain rate in the specimen is given by

$$\dot{\varepsilon} = \frac{\omega r}{\ell_o} \quad (\text{A2.14})$$

This assumes that the length ℓ_o is constant. The specimen length will change with force due to the compliance of the force transducer but this was considered to be minimal for the configuration used.

Integrating Equation A2.14 with respect to time yields the strain

$$\varepsilon = \int_0^t \dot{\varepsilon} \, dt \quad (\text{A2.15})$$

Thus

$$\varepsilon = \frac{\omega r t}{\ell_o} \quad (\text{A2.16})$$

Thus the tensile stress is given, using Equation A2.7, by

$$\sigma = \frac{F e^{\left(\frac{\omega r t}{\ell_o}\right)}}{A_o} \quad (\text{A2.17})$$

and the tensile stress growth coefficient, using Equation A2.9, by

$$\eta_E^+ = \frac{F \ell_o e^{\left(\frac{\omega r t}{\ell_o}\right)}}{A_o \omega r} \quad (\text{A2.18})$$

Similar equations can be derived for experimental configurations in which there are two sets of rotating clamps, in which case the above analysis is for one half of that symmetric geometry, or in which the clamps are translating.

In the case of using two sets of rotating clamps having the same angular speed, thus producing a stagnation point at the centre of the specimen, then the above equations apply except that ℓ_o is then equal to a half of the specimen length.

In the case of translating clamps to obtain a constant strain rate $\dot{\epsilon}$ then the relative velocity of the clamps V_c , defined by

$$V_c = \frac{\partial \ell}{\partial t} \quad (\text{A2.19})$$

is given, using Equation A2.3, as

$$V_c = \ell_o \dot{\epsilon} e^{(\dot{\epsilon} t)} \quad (\text{A2.20})$$

Appendix A3: Solution to the differential of the exponential term

To solve the differential of the exponential extensional viscosity function to determine the sensitivity coefficients the method of substitution of variables was used. To illustrate this, consider the following equation

$$\eta = \ell e^{\frac{a}{\ell}} \quad (\text{A3.1})$$

To find the differential of η with respect to ℓ where a is a constant let

$$\phi = \frac{1}{\ell} \quad (\text{A3.2})$$

Thus

$$\ell = \frac{1}{\phi} \quad (\text{A3.3})$$

and

$$\frac{d\phi}{d\ell} = \frac{-1}{\ell^2} \quad (\text{A3.4})$$

By substitution of Equation 3.3 into Equation 3.1

$$\eta = \frac{e^{a\phi}}{\phi} \quad (\text{A3.5})$$

Differentiation of η with respect to ϕ yields

$$\frac{d\eta}{d\phi} = \frac{-e^{a\phi}}{\phi^2} + \frac{a e^{a\phi}}{\phi} \quad (\text{A3.6})$$

As

$$\frac{d\eta}{d\ell} = \frac{d\eta}{d\phi} \frac{d\phi}{d\ell} = \left(\frac{-e^{a\phi}}{\phi^2} + \frac{a e^{a\phi}}{\phi} \right) \left(\frac{-1}{\ell^2} \right) \quad (\text{A3.7})$$

then substitution of ℓ for ϕ into Equation A3.7 yields

$$\begin{aligned}
\frac{d\eta}{d\ell} &= \left(-\ell^2 e^{\frac{a}{\ell}} + a \ell e^{\frac{a}{\ell}} \right) \left(\frac{-1}{\ell^2} \right) \\
&= e^{\frac{a}{\ell}} - \frac{ae^{\frac{a}{\ell}}}{\ell}
\end{aligned} \tag{A3.8}$$