

**EVALUATION OF HYPERELASTIC FINITE
ELEMENT MODELS FOR FLEXIBLE ADHESIVE JOINTS**

by

Bruce C Duncan, Louise E Crocker and Jeannie M Urquhart

Project PAJex2 - Flexible Adhesives

PAJex2 Report No 2

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Summary

A comprehensive evaluation, covering a range of strain rates and temperatures, has been undertaken into the applicability of hyperelastic material models in Finite Element (FE) packages for predicting the force-extension response of lap joints bonded using a flexible adhesive. Comparisons between measured and predicted lap joint responses have indicated that, for the adhesive studied, the FE models have poor accuracy. Many modelling and experimental factors that were thought to influence the accuracy of the predictions were investigated. The findings suggested that the models do not properly describe the adhesive material. However, there is a suspicion that the adhesive studied may not be representative of all flexible adhesives. A number of tools and statistical procedures have been developed in this study that can be used to validate further adhesives or models.

Some key findings were made regarding the provision of input material properties data and the performance of the FE modelling. It was found that using uniaxial tension and volumetric tests only to obtain the input data gave the best general quality of predictions. It would seem that the expensive planar and biaxial tension tests could be omitted without compromising accuracy. A method for obtaining volumetric properties from uniaxial tension tests has been described. Generalised plane strain elements were found to be better for representing the adhesive layer than standard plane strain elements.

The results point towards simpler, lower-order models such as Neo Hooke and Arruda-Boyce being better at representing the adhesive than more complex, higher-order models. The simpler models have the additional advantages that trends in the strain rate and temperature dependence of the fitted model coefficients can be predicted. Analysis of the stress distributions at the maximum loads sustained by the joints shows that peak values of peel stress and von Mises stress are comparable with the ultimate tensile stresses obtained for bulk test specimens. Thus, stress components may be suitable as failure criteria.

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National Physical Laboratory
Teddington, Middlesex, UK, TW11 0LW

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Head of Centre for Materials Measurement and Technology

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1. INTRODUCTION

Flexible adhesives are characterised by low glass transition temperatures (T_g), low moduli and large strains to failure in comparison with structural adhesives such as epoxies. These adhesives are used extensively in non-structural applications such as sealing. However, there is a growing trend for them to be used in structural, load bearing applications. Suitable design tools are required for this to happen most effectively. Finite Element Analysis (FEA) is a powerful stress analysis tool. For accurate design predictions appropriate materials models and reliable materials properties data are required. Investigating these is the focus of study of project PAJex2 - *Flexible Adhesives* - of the DTI funded *Performance of Adhesive Joints* programme.

Hyper elastic material models offer, perhaps, the most realistic means of representing flexible adhesives in Finite Element (FE) stress analysis design packages. There are numerous hyperelastic models available in the material model libraries of FE software packages such as ABAQUS^[1]. Hyperelastic models are described in more detail in Appendix I. One of the aims of project PAJex2 was to establish the most effective models for representing flexible adhesives and efficient means of generating the required input data. A further aim of this work was to identify models that could accommodate the rate and temperature dependence of the material properties. Since the hyperelastic models are phenomenological this is most likely to be achieved through correlations between the fitted coefficients and rate/temperature rather than through any fundamental description of the physical properties of the material.

The elastomeric adhesive, M70, supplied by Evode Ltd, is used in the automotive industry for sealing panels and attaching trim. This one-part adhesive is described as a heat curing elastosol and was supplied in 300g cartridges. The uncured adhesive is a high viscosity paste and is black in both its cured and uncured states. The specific chemistry of the adhesive is not known but it is known that it contains calcium carbonate filler. It was chosen as a representative flexible adhesive for the project. This material has been characterised by bulk specimen tests (to obtain FEA input data) and lap joint shear tests over a range of strain rates and temperatures^[2].

The FEA predictions of the joint response would be expected to depend on the choice of material model and input data supplied for the determination of model coefficients. The ABAQUS manuals^[1] suggest that data obtained under several stress states are preferred to optimise hyperelastic model predictions under multi-axial stress states. Tests under uniaxial tension, biaxial tension and planar tension were performed (referred to as UBP tests). Test speeds were selected to give the same actual strain rate in all of the specimens. Obtaining the full range of UBP tests is resource intensive, requiring large quantities of adhesive, extensive preparation time and a variety of testing fixtures. The FE manuals also recommend that volumetric compressibility data be supplied if the material is likely to be compressible. Should it be possible to reduce the number of input data sets required then the cost of testing could be reduced. The uniaxial tension test is the most straightforward to perform. Therefore, the predictions obtained using data obtained from this type of data have been compared with those obtained using the full suite of tests.

This report evaluates the use of various material models and input data for predicting the force-extension response of lap joints bonded using a flexible adhesive. The lap joint specimen has been modelled by FEA and the predicted force-extension response compared with the experimental measurements.

2. FEA MODEL AND RESULTS

2.1 LAP JOINT FE MODEL

2.1.1 Mesh Geometry

The lap joint was modelled as a two-dimensional structure to simplify the mesh generation and reduce computing time. This approach is normal for the analysis of most symmetrical adhesive joints. The specimen was meshed to give the same geometry as the test specimen except that the fillets at the ends of the bondline were modelled as being square whilst they were not precisely shaped in the real specimens.

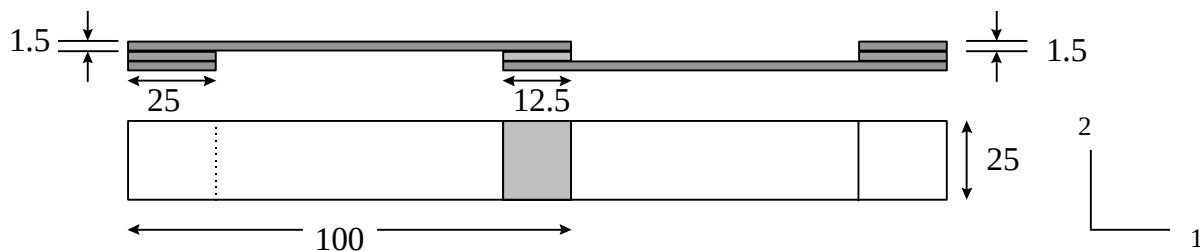


Figure 1: Lap Joint Test Specimen

The lap joint specimen is shown in Figure 1. Tabs were bonded at each end to ensure alignment when the specimen is gripped. The steel adherends were 100 mm long by 25 mm wide by 1.5 mm thick. The mild steel was modelled as an elastic-plastic material. However, it was thought unlikely that loads high enough to exceed the steel's yield stress would ever be applied. The properties of the steel were assumed to be independent of the temperatures and strain rates used. The bonded overlap was 12.5 mm long with a thickness of 0.25 mm. This was modelled as a block of 32 long by 8 deep, equally sized, cubic elements. The adhesive material in this region was modelled using different material models and input data as discussed in Appendix I of this report. In the standard model, both the steel adherends and the flexible adhesive were CPE8RH elements - quadrilateral, 8 noded, reduced integration, plane strain hybrid elements that incorporate hourglass stiffness control. Available element types are described below.

One end of the joint was constrained from movement in either the 1-direction (axial) or the 2-direction (off axis). The other end was constrained from movement in the 2-direction only. This end was displaced by 0.35mm in the 1-direction to apply load to the specimen. The reaction force at this end of the model is predicted. Two gauge nodes on the upper surfaces of the adherends, 25 mm part, corresponding to the contact points of an extensometer straddling the overlap were identified. The relative movement of these nodes in the 1-direction was used to calculate the shear extension of the adhesive.

2.1.2 ABAQUS Element Types

The elements used in the lap joint mesh were solid (continuum) elements. The solid elements in ABAQUS are suitable for linear analysis and also for complex non-linear analyses involving plasticity and large deformations. There are a number of continuum elements available within the ABAQUS element library^[1]. A brief description of the attributes of the elements used for the lap joint analysis is given below.

Quadrilateral elements were chosen over triangular elements as the quadrilateral elements have a better convergence rate and sensitivity to mesh orientation in regular meshes is not an issue. Quadrilateral elements perform best if their shape is approximately rectangular. The elements become much less accurate when they are initially distorted.

The second order form of the quadrilateral elements was used as this provides higher accuracy than first order elements for 'smooth' problems that do not involve complex contact conditions or impact. Second order elements have more nodes per element than first order elements (i.e. they have a midside node). They capture stress concentrations more effectively and are better for modelling geometric features.

Reduced integration is available for quadratic elements and was utilised in the lap joint analysis. It uses a lower-order integration to form the element stiffness. Reduced integration reduced the running time of an analysis. Second order reduced integration elements generally yield more accurate results than the corresponding first order fully integrated elements.

A further element option is the hybrid element. Hybrid elements are intended mainly for use with incompressible and almost incompressible materials. For a near incompressible material a very small change in displacement produces extremely large changes in pressure. Therefore, a purely displacement-based solution is too sensitive to be useful numerically. This singular behaviour is removed by treating the pressure stress as an independently interpolated basic solution variable, coupled to the displacement. This independent interpolation of pressure stress is the basis of the hybrid elements. Hybrid elements have more internal variables than non-hybrid elements and this increases running time. Hybrid elements are recommended for hyperelastic materials.

For structural applications, the ABAQUS element library includes plane stress, plane strain and generalised plane strain elements. Plane stress elements can be used when the thickness of a body is small relative to its lateral (in-plane) dimensions. Modelling with this element generally applies to thin, flat bodies. In contrast, plane strain elements are generally used for modelling bodies that are very thick relative to their lateral dimensions. In these elements it is assumed that the strains in the loaded body are functions of the planar coordinates only and out-of-plane normal and shear strains are equal to zero. An alternative type of plane strain element is the generalised plane strain element. In this case, the formulation places the model between two rigid planes that can only move closer or further apart. Thus, a lap joint mesh would have the ability to respond to Poisson's contractions. It is assumed that the deformation of the model is independent of the axial position so the relative motion of the two planes causes a direct strain in the axial direction only. There are no transverse shear strains. Both plane strain and generalised plane strain elements have been considered in the analysis of the lap joint.

2.2 ANALYSIS OF FEA RESULTS

FEA is a powerful tool for generating predictions of stress and strain distributions within a component, enabling the predictions of stress concentrations that may lead to premature failure of the component. These predictions are difficult to validate experimentally and one approach to validating FEA predictions is comparison with quantities that can be measured readily such as force-extension behaviour. This approach has been undertaken here. In order

to normalise for variations in the length and thickness of the bonded overlap the forces are divided by the actual overlap length and the extensions divided by the bond thickness.

Initially, some 10 hyperelastic models were investigated for a single set of test conditions (strain rate and temperature). None of these models gave any acceptable fit to the measured data (Figure 2). Most of the predictions show little curvature in the force-extension response. The exceptions where curvature is present tend to be the more complex models with more coefficients (e.g. Ogden or Polynomial with $N > 1$). These predictions were the least accurate, as the shapes of the curves do not resemble the measured data. The higher order models tended to be the least stable. In many cases the FEA failed to converge at solutions before the full extension had been applied.

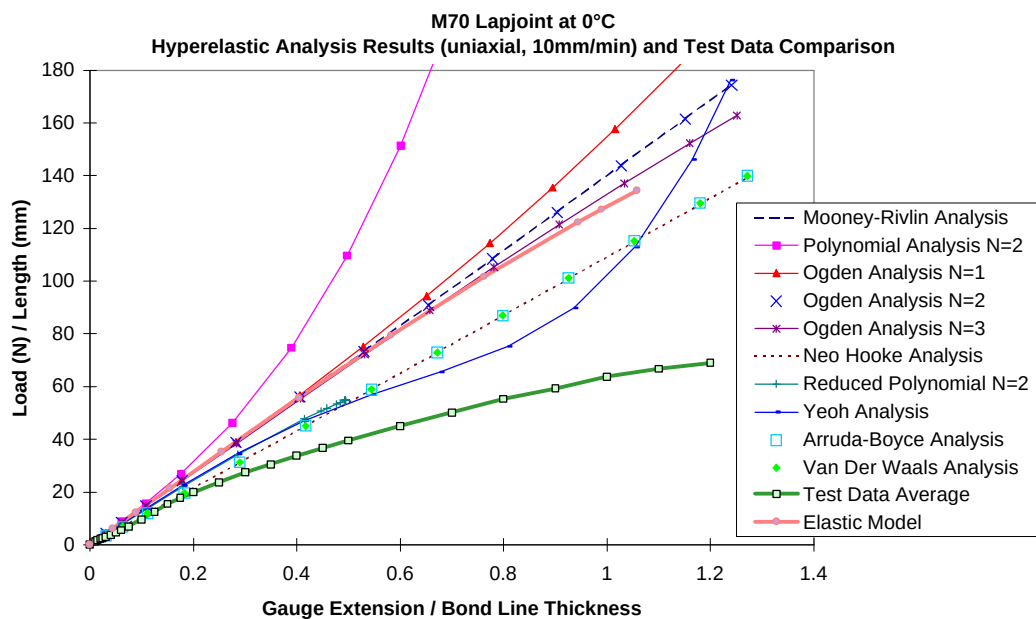


Figure 2: Comparison between measured and FEA predicted force-extension results using uniaxial tensile input data

Five material models were selected for a wider study involving a range of strain rates, temperatures and input data cases. A procedure for determining a quantitative value for the degree of agreement with the experimental data was produced. These values were used to evaluate the effects of the test factors (principally material model and input data type). The results of this study are presented in Appendix II. The full range of conditions in this study required that a minimum of 180 FEA ‘experiments’ were run. This involved significant resources to generate the required input data and perform the FEA runs. Statistical Design of Experiment (DOE) methods^[3] were investigated to devise smaller scale experimental matrices. These are outlined in Appendix II. The conclusions made from the experimental matrix requiring the least experiments (25 in total) were compared with these made using the full experimental matrix. These were very similar and this suggests that the smaller matrix may be a useful tool for evaluating the accuracy of the FEA method.

It was obvious from the force-extension plots (such as shown in Figure 2) that the FEA results significantly over-estimate the stiffness of the lap joints at larger extensions. These were confirmed in analyses covering a more extensive range of temperatures and strain rates. There are significant discrepancies between the performances of lap joint specimens predicted

from bulk specimen data and those actually measured. The causes of these discrepancies have been the subject of further investigation. This is discussed in Section 3.

The analysis of the FEA results in Appendix II indicates that, from the simple elastic and the five hyperelastic models (Mooney-Rivlin, Arruda-Boyce, Neo Hooke and Ogden (N=1 and N=3)) studied, the Arruda-Boyce and the Neo Hooke models consistently give the lowest discrepancies. The predictions made using the Mooney-Rivlin and Ogden models tend to be little better than those made using the simple Elastic model. The Ogden (N=3) model gives the greatest discrepancies and is not helped by being the least stable of the models (many FE runs using this model crashed through non-convergence well before a strain of 1.0 was reached).

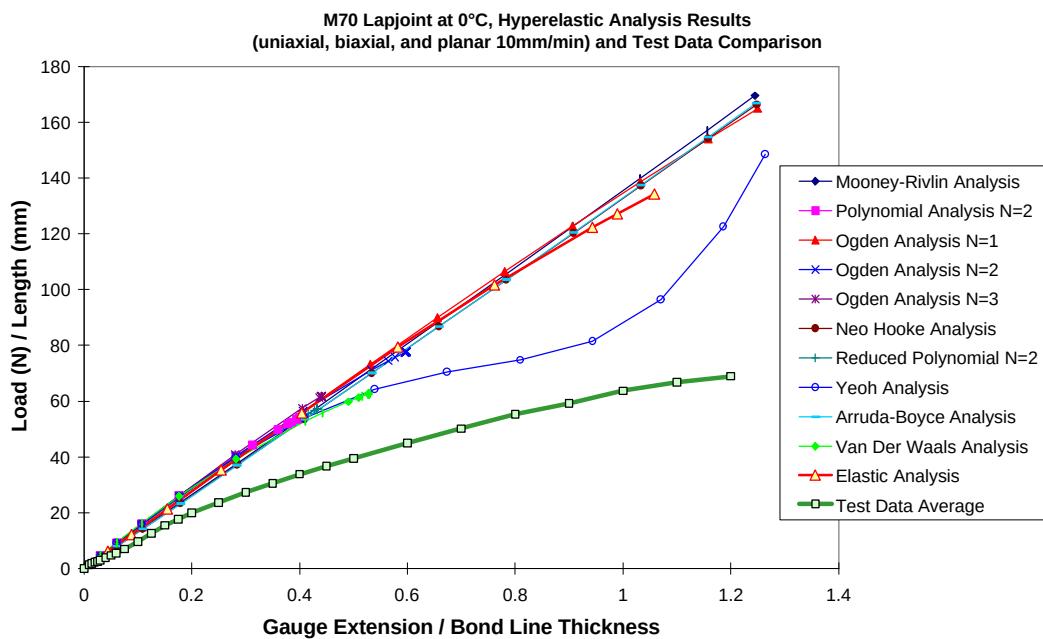


Figure 3: Comparison between measured and FEA predicted force-extension results using UBP data

The analyses were performed using four different combinations of input test data (uniaxial only (U); uniaxial and volumetric (U+V); uniaxial, biaxial and planar (UBP); and uniaxial, biaxial, planar and volumetric (UBP+V)) to derive the model coefficients. The tables in Appendix II indicate that using UBP data rather than U data, with or without volumetric data, tends to improve the agreement between the models and narrow the range of the discrepancies between the measured and predicted results (e.g. Figure 3). However, it should be noted that the discrepancies between the measured results and those predicted using the Arruda-Boyce and the Neo Hooke models actually increase when UBP data rather than U data were used. The net effect of this is that there is little overall difference between the predictions made using uniaxial data only and those made using data determined from all three tests. The effect of including the volumetric test data is to slightly reduce the discrepancies between measured and predicted results. The analysis indicates that using uniaxial and volumetric data as input give the best overall performance of the four input data types investigated.

The analysis of the FEA results suggest that, in the main, the agreement between measured and predicted results is better at 20°C than at 0°C or 40°C. However, there are insufficient

results to confirm any trends with temperature. Similarly, there are no definite patterns to be noted on the effects of strain rate on the accuracy of predictions.

The analysis of the FE results indicate that the Ogden (N=3) model is clearly unsuitable for predicting the performance of this adhesive since the models seem to be neither stable nor accurate. In terms of improving the accuracy of the FE predictions, there seemed to be little benefit to be gained through supplying planar and biaxial tension data in addition to uniaxial data. The extra costs and difficulties in generating these data do not appear to be justified. The inclusion of volumetric data brings some improvement to the FE predictions. Volumetric data can be calculated from lateral strain measurements made during uniaxial tension tests. Therefore, provided suitable extensometry is available, these data can be obtained at little additional cost.

3. EXAMINATION OF CAUSES OF POOR FEA ACCURACY

The preceding discussion shows that the FEA predictions of the lap joint performance fail to satisfactorily predict the experimental force-extension behaviour. There are large discrepancies between the measured and predicted values at large strains. A number of factors were identified as having some potential to reduce the accuracy of the FE modelling. These are grouped into the following areas.

- UNREALISTIC FEA MODELS
 - Premature crack formation and growth
 - Fillets not modelled
 - Element type
 - Misaligned test machine
- MATERIAL MODELS AND INPUT DATA
 - Hyperelastic vs. elastic-plastic
 - Shear or. tensile properties
 - Geometrical considerations
 - Data required under several stress-states
 - Volumetric properties
 - Differences in strain rate/temperature
- SAMPLE VARIABILITY
 - Material variations caused by different batches of material
 - Material variations caused by different thermal histories during cure
 - Material properties are sensitive to thickness
 - Material properties are sensitive to the age of the adhesive before sample preparation and after cure.
- ALTERNATIVE JOINT TESTS
 - Alternative shear measurements
 - Scarf joints

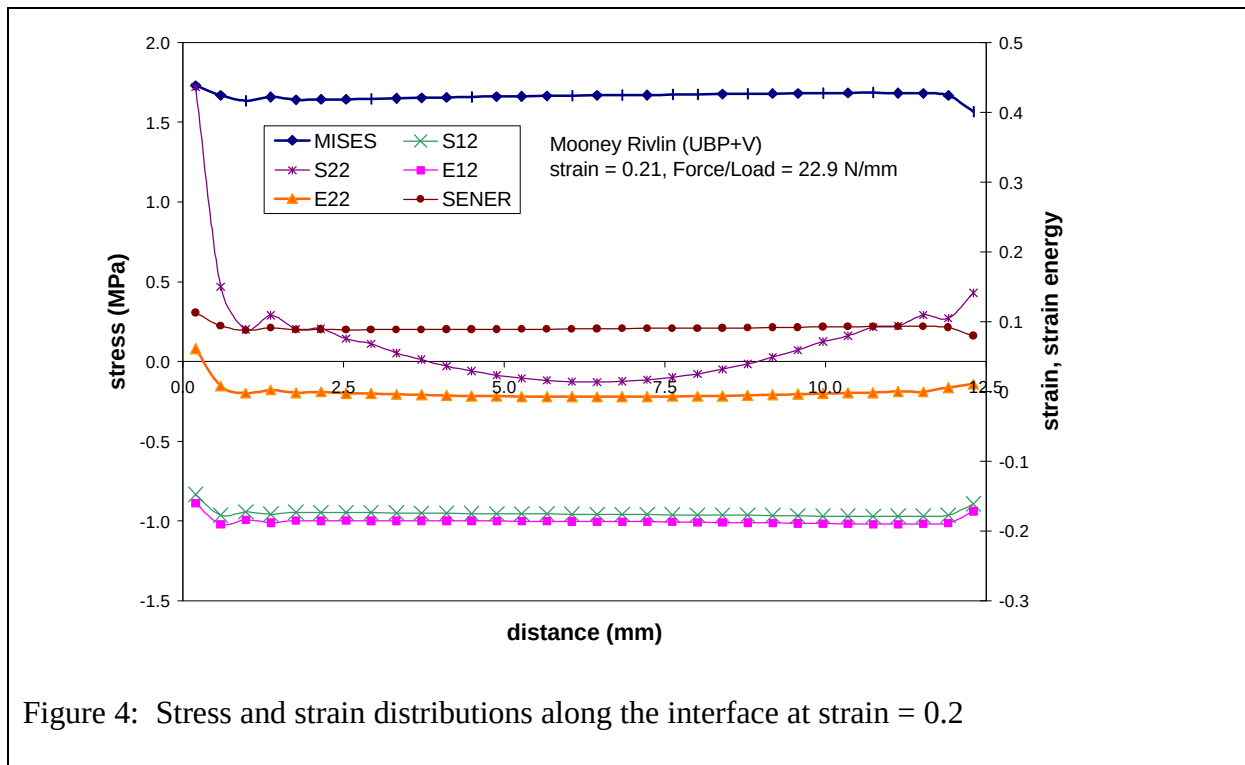
It was also considered that there might be some factor associated with shear loading that was not accounted for in the hyperelastic FEA model. An alternative joint geometry - the scarf joint - with a mixed tensile/shear stress state was studied.

3.1 UNREALISTIC FEA MODELS

3.1.1 Premature Crack Formation and Growth

Comparisons between the predicted and measured responses (such as Figures 2 and 3) suggested that the measured response broke away from the predictions around 0.2 strain with the discrepancy growing as the strain increased. This was typical for most of the FE experiments. One possible explanation for this behaviour could be the formation of a crack at a stress concentration and slow growth of this crack through the load-bearing region thus reducing the load-bearing area. Although the average shear strain of 0.2 is well within the envelope of sustainable extensions as determined for the bulk material, it is recognised that failure will be initiated at the stress concentrations that occur at the ends of the overlap.

Stress and strain components were predicted by FEA in the adhesive elements in contact with the top adherend for the loading case of 20°C, $3 \times 10^{-2} \text{ s}^{-1}$ strain rate. The different models and input data sets seemed to cause very little variation in the stress and strain distributions predicted. Figure 4 shows typical predictions around 0.2 strain. These were made using the Mooney-Rivlin model and UB+V input data. The maximum values of the stress components plotted (MISES = von Mises stress; S12 = shear stress and S22 = peel stress) are all significantly less than the average failure stress values determined from uniaxial tension tests ($3.5 \pm 0.3 \text{ MPa}$). Similarly, the strain components (E12 = shear strain and E22 = peel strain) are insignificant in comparison with the tensile failure strain ($48 \pm 9 \%$). All stress and strain values peak in the first element with the peel stress and peel strain values showing the largest concentrations. The total strain energy (SENER) value is relatively evenly distributed. No material rupture or crack formation would be expected from these values.



Similar analyses were performed for load/bond length values around the average measured lap joint failure value (55 N/mm). In common with the strain = 0.2 predictions, the material

model selected and input data used made little difference to the predicted distributions. Figure 5 shows the predictions made using the Mooney-Rivlin model and UBP+V input data. The peak values of von Mises stress and peel stress are comparable with the tensile strength of the adhesive. This may cause rupture of the adhesive and initiate failure.

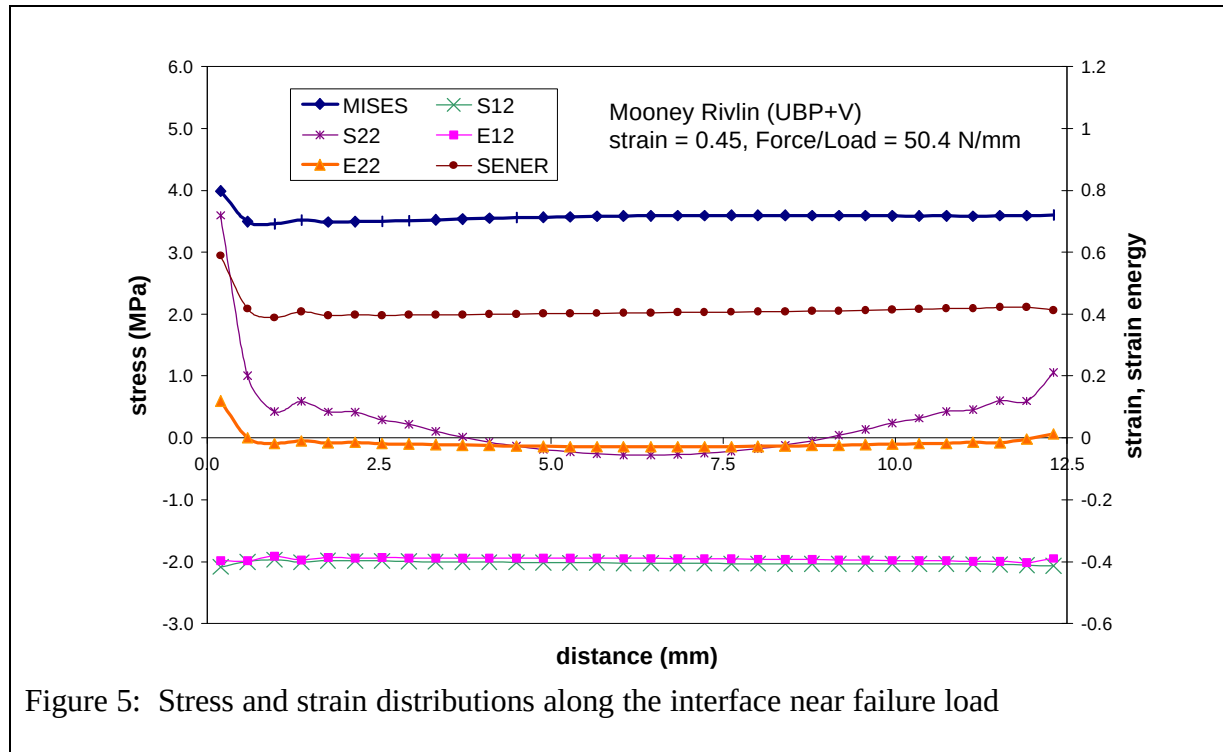


Figure 5: Stress and strain distributions along the interface near failure load

Further analyses were undertaken to predict what length of crack would be necessary to cause the loss in stiffness in comparison with predicted behaviour that was seen in the lap joint tests. A simple analytical model of the lap joint described by Owens and Lee-Sullivan^[4, 5] considering the adhesive and the adherends as elastic materials was used to predict the stiffness of the test specimen. This model is described in Appendix III. The results were in broad agreement with the FEA. One advantage of the analytical model over FEA was that it allowed easy investigation of the effects of material properties and specimen geometry (allowing for the presence of cracks along the interface) on the specimen stiffness. Thus, it was possible to estimate the required length of crack present in the adhesive layer that would reduce the predicted loads to the measured values. Typically, to achieve the maximum loads obtained in the experimental tests would have required continuous cracks in the adhesive layer of 4 to 5 mm length or roughly one third of the bonded length.

Tests were performed on lap joint specimens where the experiments were terminated at predetermined fractions of the expected maximum load. The specimens were removed and examined under a microscope for the presence of cracks. The black colour of the adhesive makes it difficult to detect cracks but the observations detected no obvious cracks even at the maximum strength of the joint. The crack lengths estimated from the analytical model would have been obvious even to the naked eye. The only point where the presence of cracks was obvious was where the specimen had been loaded well beyond the point of maximum load (Figure 6). These studies tend to suggest that macroscopic cracks initiate around the point of maximum load and grow subsequently.

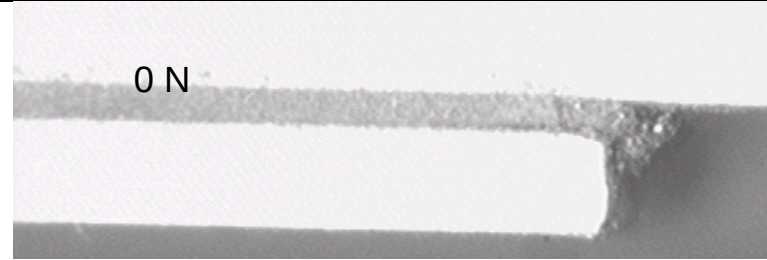
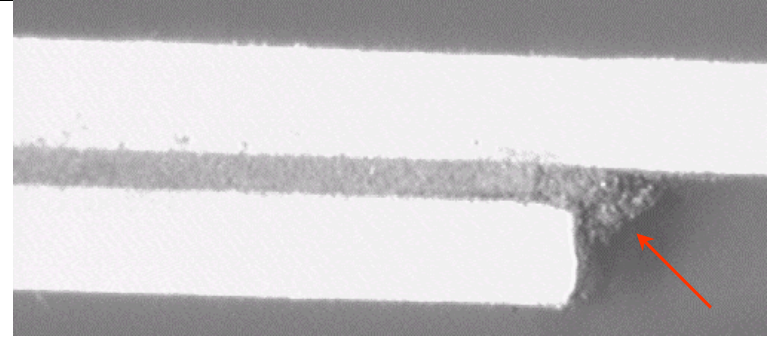
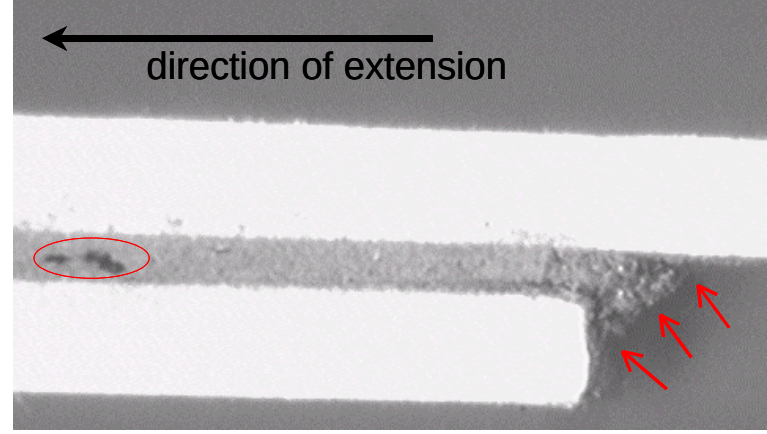
	(a) Specimen before load was applied.
	(b) Specimen at the maximum recorded load (560 N). Looks little different to the unloaded specimen. Suggestion of a crack formed in the fillet.
	(c) Specimen after extension well beyond the maximum load. A crack has formed and grown around the bottom corner into the main adhesive overlap.

Figure 6: Adhesive specimens examined for crack growth.

3.1.2 Fillets

The high viscosity of the adhesive paste made it difficult to control the shape and size of spew fillets during lap joint specimen manufacture. Therefore, the meshed model used in the FEA assumed the worst case for stress concentrations - a square end to the bond line with no adhesive fillet. The actual joints tested had fillets owing to displacement of excess adhesive from the bond during manufacture. Most of the excess adhesive was removed from specimens prior to testing. However, the adhesive was not completely squared off at the overlap in order to avoid damaging the material actually within the overlap during this process (e.g. by nicking the adhesive with a scalpel and initiating a crack). The specimen shown in Figure 6 illustrates a typical fillet size.

The general effect of fillets is to reduce the stress concentrations at the end of the overlap relative to what is experienced with a square overlap. Hence, it is believed that the fillets are beneficial to the performance of the joint, as they should inhibit the formation of fractures. Visual observations suggest that fractures are not present until after the joint has reached its

maximum load bearing capacity. The absence of fillets in the FEA model was not thought to be likely to cause significant increases in the predicted stiffness of the joint.

3.1.3 Element Type

The flexible adhesive when bonded to rigid adherends is highly constrained by the adherends. Therefore, plane strain elements need to be used in the 2-dimensional FE model. Thus, FE analyses were run using CPE8RH elements - quadrilateral, 8 noded, reduced integration, plane strain hybrid elements that incorporate hourglass stiffness control. However, the FE software offers many different types of element. Comparison analyses were run using plane stress (CPS8R, CPS8), generalised plane strain (CGPE10H) and non-hybrid plane strain (CPE8R and CPE8) elements to model the adhesive layer.

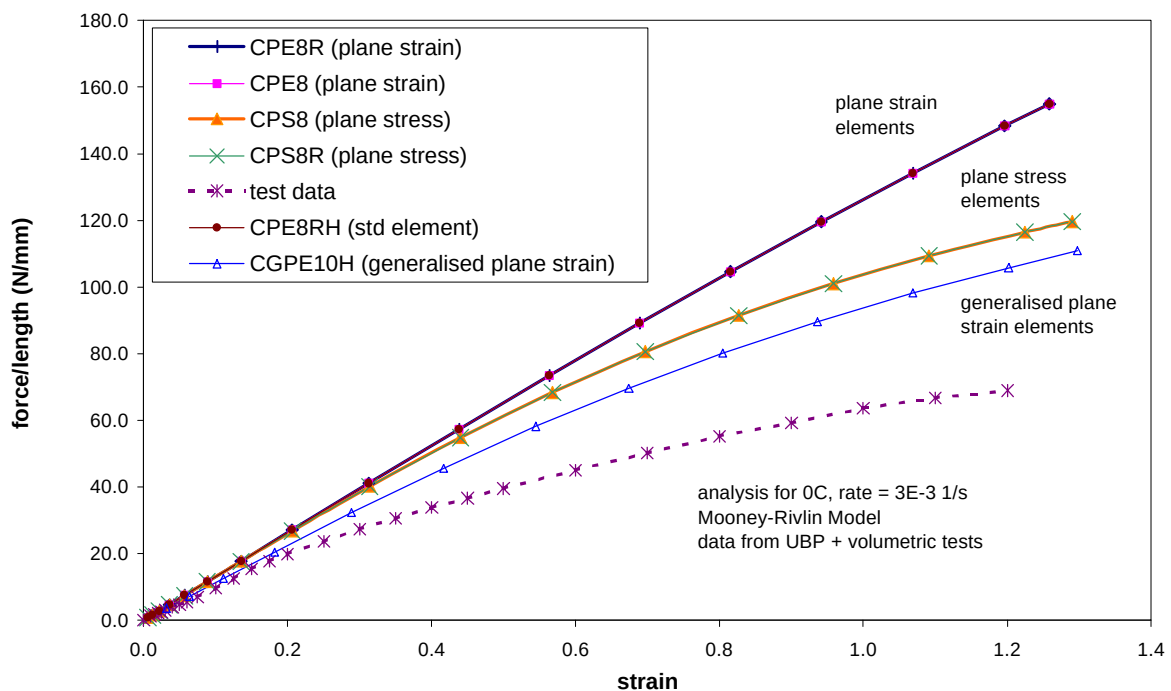


Figure 7: Effect of different element types on FE predictions.

Figure 7 shows that there are no differences between force-extension predictions made using the hybrid and non-hybrid plane strain elements. Similarly, the reduced integration elements give comparable predictions to the full integration elements. However, the plane stress and generalised plain strain element predictions show greater curvature than the plane strain models. This is more like the measured joint response. However, the stiffness of the joint is still significantly overestimated.

The generalised plain strain elements are theoretically more applicable to constrained materials than the plane stress elements. Therefore, additional analyses were performed using generalised plane strain elements in order to assess their wider applicability to other material models. These results (Figure 8) indicate that, for some of the material models, using this element gives better predictions of the lap joint response. However, predictions from some of the models are hardly affected. The results of an analysis of the predictions made using the reduced test matrix of 25 experiments are shown in Appendix II. These results indicate that

the use of the generalised plane strain element improves the predictions made using the Mooney-Rivlin and Ogden models but has little effect on the Neo Hooke or Arruda-Boyce models. Overall, the use of generalised plane strain elements improves the quality of the predictions.

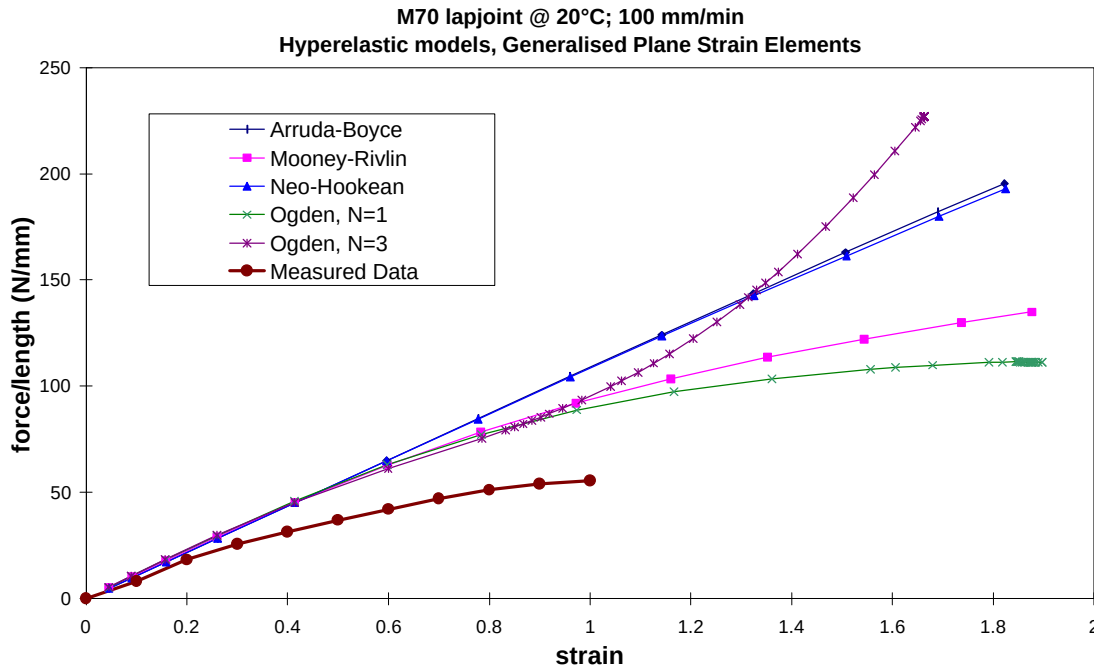


Figure 8: FEA predictions made using generalised plane strain element

3.1.4 Test Machine Alignment

The FE model applies displacements in the axial direction of the specimen but is constrained from lateral movements. This may not reflect the reality of the lap joint tests where there is a possibility that the grips of the testing machine may not be perfectly aligned. Earlier FEA^[6] showed that slight misalignments would have negligible effects on the force-extension behaviour but would alter the stress distribution in the specimen. This may lead to premature failure.

A similar exercise has been performed for the current geometry and material properties. This time the axial extension was 0.35 mm whilst two magnitudes of off-axis deflections (0.35 mm and 3.5 mm) were used. The larger off-axis deflections were assumed to be greater than any likely misalignments. The FEA predictions illustrated in Figure 9 show that there is minimal impact of misalignment on the shape of the force-extension response. The lateral forces required to 'bend' the specimen into the grips are predicted at around 20N. The test operator would have noticed this level of force. Once more the conclusions are that the effect of misalignment on the measured force-extension response would be insignificant.

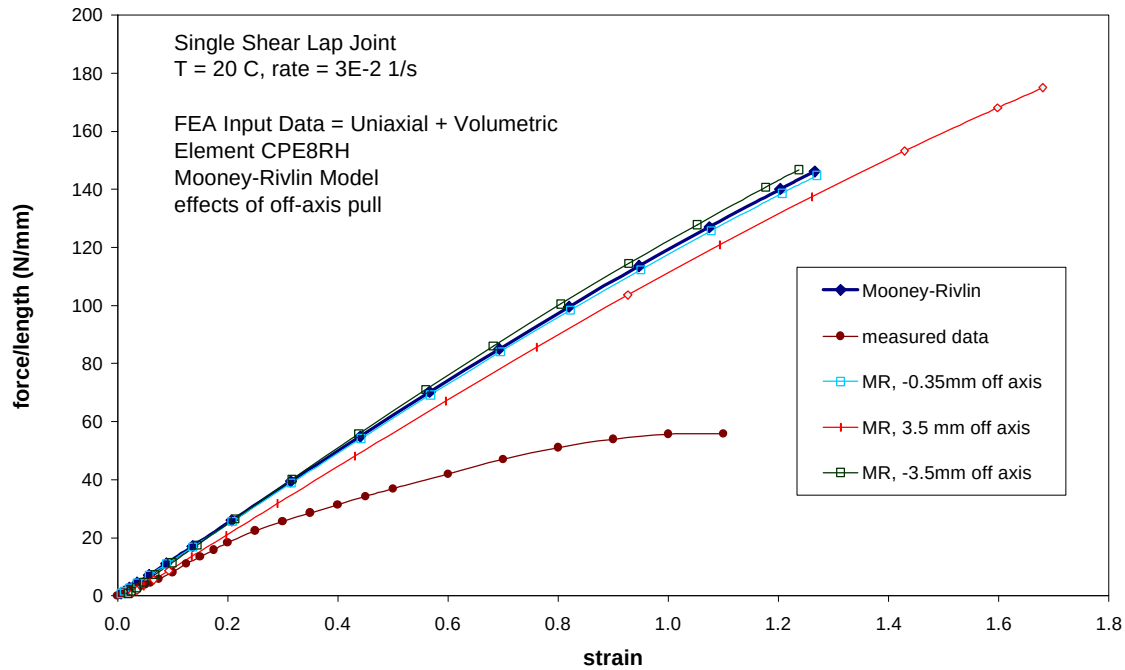


Figure 9: FEA predictions of the effects of specimen alignment

3.2 MATERIAL MODELS AND INPUT DATA

3.2.1 Hyperelastic or Elastic-Plastic Properties

The aim of this work was primarily to identify suitable material models to describe the behaviour of the flexible adhesive under multi-axial stress states. Earlier work suggested that elastic models were unsuitable for flexible adhesives and the concept of plasticity was unlikely to apply. Elastic-Plastic models require data in the form of true-stress vs. true-strain data. When the measured uniaxial tension curves were converted into this format they were more or less linear^[2]. There was very little ‘plastic yielding’ that would lead to the type of non-linear response seen in the lap joint results. Thus it seems unlikely that Elastic-Plastic models and the various yield criteria that have been applied to stiff structural will apply to these materials.

3.2.2 Shear and. Tensile Properties

The test data used to obtain the material coefficients for the modelling were determined under tension loading conditions. The main deformation mode in the lap joint specimen is shear although any bending of the adherends may introduce tensile deformation of the adhesive. However, the low stiffness of the adhesive minimises the bending of the adherends and, hence, the majority of the deformation is shear extension.

Estimates of the initial values of Young’s and shear modulus can be calculated from the coefficients of the hyperelastic models^[1]. Tables 2 and 3 show the measured and predicted Young’s (tension) and shear modulus values. The Young’s modulus predictions appear in reasonable agreement with the measured values. The sum of squares of the differences shows that the Mooney-Rivlin model appears to be the most accurate. The shear modulus predictions are less accurate. All of the predicted values are greater than the measured values.

This may be because the coefficients are calculated assuming incompressibility, which requires a Poisson's ratio of 0.5. In turn, this means that the shear modulus is exactly 1/3 of the tensile modulus. For Poisson's ratios less than 0.5 the expected shear modulus would be greater than 1/3 of the tensile modulus. Comparing the measured tensile and shear modulus values shows that the shear modulus values are less than 1/3 of the tensile values. This is not expected.

Temp	Rate	Measured	Mooney-Rivlin	Ogden	Ogden	Arruda-Boyce	Neo-Hooke
(C)	(1/s)	(MPa)	N=1	N=1	N=3		
0	3.E-04	12.80	14.40	14.82	15.30	13.81	13.99
0	3.E-03	17.30	18.03	18.90	20.41	17.39	17.62
0	3.E-02	27.10	24.47	24.80	25.67	23.13	23.43
20	3.E-04	10.90	11.22	11.67	12.18	10.96	11.10
20	3.E-03	11.80	13.12	13.24	13.54	11.96	12.12
20	3.E-02	15.70	15.35	15.73	16.02	14.35	14.54
40	3.E-04	10.20	10.09	10.26	10.31	9.84	9.97
40	3.E-03	10.80	12.46	12.64	13.23	11.73	11.88
40	3.E-02	12.50	13.18	13.00	13.21	12.18	12.34
80	3.E-04	9.00	9.99	9.66	9.28	9.25	9.37
80	3.E-03	10.50	10.60	9.21	8.31	8.98	9.09
80	3.E-02	12.60	11.54	11.52	11.51	11.37	11.52
Sum of (predicted-measured) ² s			17.3	21.5	35.2	23.6	21.0

Table 2: Measured and fitted Young's modulus values

If differences between the material properties in bulk and joint form are discounted then this discrepancy could be due to either the material model or inaccuracies in the lap joint measurements. Considering the lap joint measurements it is thought unlikely that there are significant errors in either the force measurement or the bond length measurement.

Therefore, it more likely that errors could arise in the strain measurement. There are two components in the strain measurement - bond line thickness and shear extension across the bond line. Bond line thickness was determined by calculating the difference between the thickness of the joint at the bonded region and the thickness of the individual adherends. These dimensions could be measured to an accuracy of 0.01 mm. This is a reasonably significant fraction of the 0.15 to 0.25 mm bond line thickness obtained and introduces uncertainties into the calculated shear strain (extension divided by bondline thickness) particularly at small extensions. Thus, errors may be introduced to the measured shear modulus values. At larger strains the significance of uncertainties in bondline thickness will decline.

Shear extensions were calculated from the movement of two points on the surface of the adherends on either side of the bond line, separated by approximately 25 mm. The assumption has been made, supported by analytical and FE modelling, that the deformation of the adherends is negligible compared with the deformation of the adhesive. Most extension measurements were made using extensometers attached to straddle the bond line. Some extension measurements were made over the same region using a video extensometer. Both methods showed that the measured extension across the bond line was around 80-85 % of the crosshead movement. FEA predictions indicate that approximately 93 % of the movement of the end of the lap joint will occur in this 25 mm bonded region. The FEA prediction ignores

the compliance of the test machine load train that would reduce the gauge extension as a proportion of the total crosshead movement. Hence, the gauge extensions seem reasonable measurements. The uncertainties in the extension measurement appear insignificant in comparison with the discrepancies with the predicted extensions.

Temp	Rate	Measured	Mooney-Rivlin	Ogden	Ogden	Arruda-Boyce	Neo-Hooke
(C)	(1/s)	(MPa)	N=1	N=1	N=3		
0	3.E-04	3.72	4.80	4.94	5.10	4.60	4.66
0	3.E-03	4.49	6.01	6.30	6.80	5.80	5.87
0	3.E-02	5.88	8.16	8.27	8.56	7.71	7.81
20	3.E-04	3.74	3.74	3.89	4.06	3.65	3.70
20	3.E-03	3.28	4.37	4.41	4.51	3.99	4.04
20	3.E-02	3.36	5.12	5.24	5.34	4.78	4.85
40	3.E-04	2.83	3.36	3.42	3.44	3.28	3.32
40	3.E-03	2.96	4.15	4.21	4.41	3.91	3.96
40	3.E-02	3.04	4.39	4.33	4.40	4.06	4.11
80	3.E-04	2.15	3.33	3.22	3.09	3.08	3.12
80	3.E-03	2.04	3.53	3.07	2.77	2.99	3.03
80	3.E-02	2.15	3.85	3.84	3.84	3.79	3.84
Sum of (predicted-measured) ²			22.99	23.97	28.57	14.99	16.50

Table 3: Measured and fitted shear modulus values

3.2.3 Accuracy of Data Fits

The accuracy of the hyperelastic model coefficient sets derived from the test data was checked using single 3-D continuum element FE tests under different states of stress^[7]. The checks were not made for models where volumetric properties were used. The model coefficients derived from uniaxial data only were compared with the uniaxial test data whilst those models generated using uniaxial, biaxial and planar data were compared with all three sets of test data. Comparisons were made for coefficients generated for different strain rates and temperatures. The analysis concentrated on the five models used in the analyses reported earlier. The quality of the fits were assessed visually and assigned to one of three categories - A = good agreement with test data, B = acceptable fit and C = poor fit to test data. Thus, a high number of A's suggests that the data are generally fitted well by the models whilst mainly C's suggest poor fits. These observations are summarised in Table 4.

The analysis of the fits produced from model coefficients derived from only uniaxial tension data indicates that:

- The most complex models produce the best fits (i.e. higher order models such as Polynomial or Ogden with N>1).
- The Arruda-Boyce and Neo Hooke models, which give the least inaccurate predictions of the lap joint response, do not fit the tensile data well.
- There is little correlation between achieving a good correlation with the input test data and obtaining a reasonable prediction of the lap joint response.

Analysis of the plots derived from uniaxial, biaxial and planar test data shows that:

- The quality of the fits to the uniaxial test data are substantially inferior to those where only uniaxial data are supplied;

- There are very few cases of good fits to the biaxial data and in many cases the fits are very poor;
- The planar test data appears to be consistently the best fitted of the test data;
- Only the Mooney-Rivlin and Ogden, N=1 models consistently fit the uniaxial data acceptably.

	Uniaxial Only	Uniaxial, Biaxial and Planar			Number of Fitted Coefficients	
Model	Uniaxial	Uniaxial	Biaxial	Planar	Deviatoric	Volumetric
Mooney-Rivlin	A = 9 B = 7 C = 0	A = 5 B = 8 C = 0	A = 0 B = 6 C = 6	A = 2 B = 7 C = 3	2	1
Polynomial, N=2	A = 16 B = 0 C = 0	A = 1 B = 0 C = 2	A = 1 B = 0 C = 1	A = 0 B = 2 C = 0	5	2
Ogden, N=1	A = 6 B = 10 C = 0	A = 4 B = 8 C = 1	A = 0 B = 6 C = 7	A = 0 B = 10 C = 2	2	1
Ogden, N=2	A = 14 B = 2 C = 0	A = 0 B = 1 C = 1	A = 0 B = 1 C = 1	A = 0 B = 2 C = 0	4	2
Ogden, N=3	A = 15 B = 1 C = 0	A = 1 B = 7 C = 4	A = 0 B = 6 C = 6	A = 4 B = 6 C = 2	6	3
Neo Hooke	A = 0 B = 5 C = 6	A = 0 B = 5 C = 8	A = 0 B = 1 C = 10	A = 5 B = 6 C = 1	1	1
Reduced Polynomial, N=2	A = 5 B = 2 C = 4	A = 0 B = 1 C = 1	A = 0 B = 0 C = 2	A = 1 B = 1 C = 0	2	2
Yeoh	A = 8 B = 1 C = 2	A = 0 B = 0 C = 2	A = 0 B = 0 C = 2	A = 0 B = 2 C = 0	3	3
Arruda Boyce	A = 0 B = 4 C = 7	A = 0 B = 5 C = 9	A = 0 B = 1 C = 11	A = 3 B = 8 C = 1	2	1
Van der Waals	A = 3 B = 1 C = 0	A = 0 B = 2 C = 0	A = 0 B = 0 C = 2	A = 1 B = 1 C = 0	4	1
Fit Quality	A = good	B = acceptable	C = poor			

Table 4: Quality of single element model fits to input data

Even when satisfactory fits to single element models were obtained there was still some doubt over extending that to more complicated specimen geometries. Therefore, in order to further verify the models and fitted coefficients, the bulk test specimen was modelled and results compared against the actual data measured. Figure 9 shows that the tension test is accurately modelled by the FEA using the Mooney-Rivlin. One point to note is that the volumetric properties should be included in order to obtain accurate predictions of true stress and strain in

the elements in the gauge section of the specimen. The lack of significant differences between predictions made using Mooney-Rivlin coefficients derived from only uniaxial tension data (U) and those obtained using uniaxial, planar and biaxial (UBP) tension data suggest that for this model the planar and biaxial data may be omitted without seriously affecting the accuracy of the predictions.

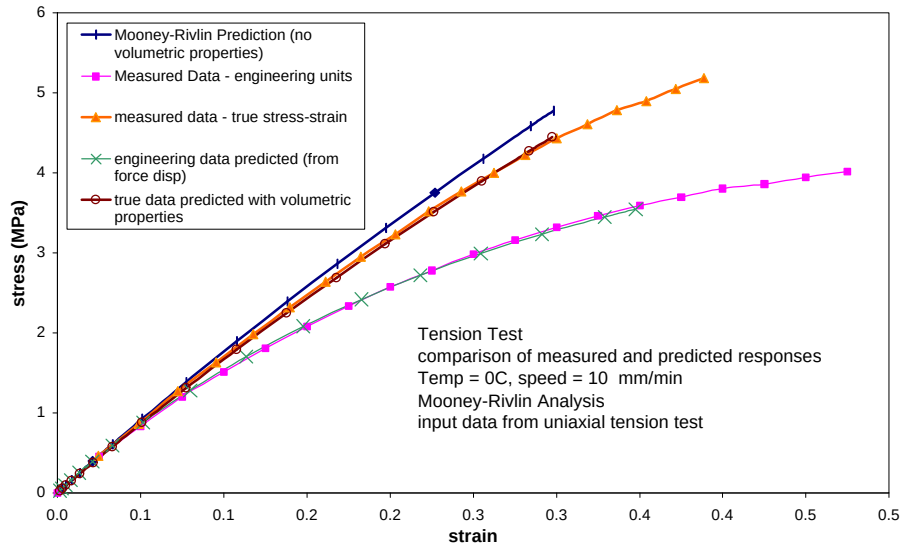


Figure 9: FEA of uniaxial tension test

Figure 10 shows the predicted and measured stress-strain behaviour of the planar test. The agreement is not as good as for the uniaxial tension test but the discrepancies are not too large (category B fit). Single element fits to the planar data were also acceptable. Interestingly, omitting the volumetric data improves the accuracy of the prediction. There is little difference between the predicted curves obtained with input data sourced from uniaxial tension and those where the input data was obtained from UBP data. This may suggest that more weight is given to the uniaxial data when the model coefficients are fitted.

The predicted responses for the biaxial test specimen are shown in Figure 11. In contrast to the uniaxial and planar test configurations the FEA is unable to predict the behaviour of the test specimen. Even when using only biaxial data as input data for the determination of the model coefficients the predictions are significantly in error. Examination of single element predictions for biaxial behaviour showed that none of the hyperelastic material models gave accurate fits to the input biaxial test data. Some of the higher order models look to give better fits but the presence of maxima, minima and inflection points in the model response suggest that any extrapolation to higher strains would be extremely inaccurate.

Previous investigations of the biaxial test discussed in Appendix I showed much better agreement between FEA predictions and test measurements. This was also true of an evaluation performed in the earlier Performance of Adhesive Joints programme^[6]. However, the materials used in these investigations were true rubbers and these might have been more ideal for hyperelastic behaviour than the adhesive in the current study.

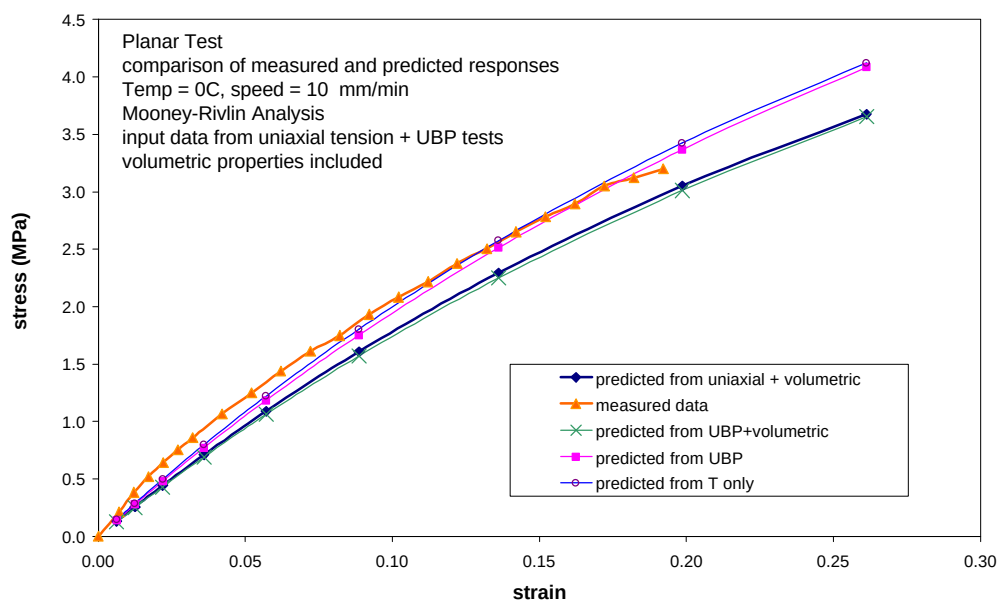


Figure 10: FEA of planar test specimen

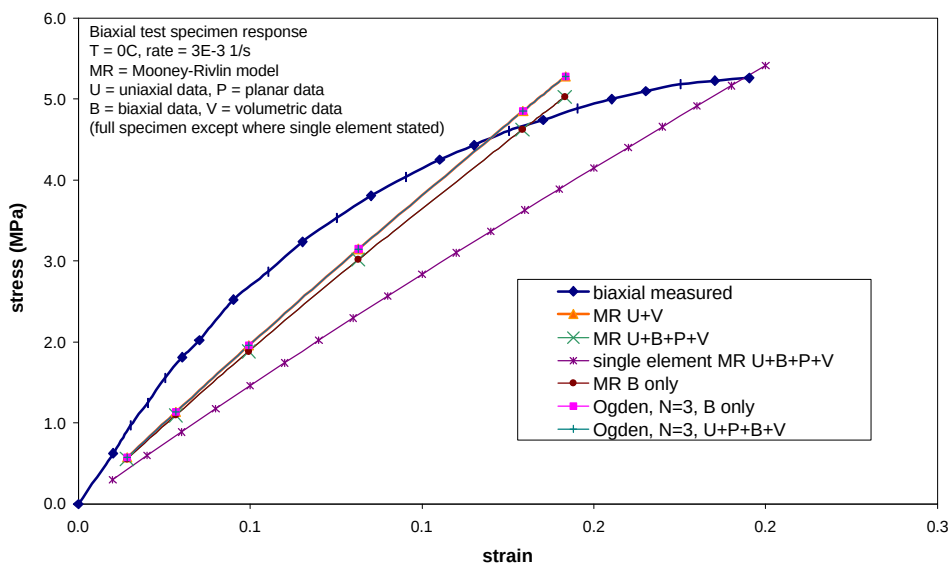


Figure 11: FEA of biaxial test specimen

3.2.4 Volumetric Properties

The hyperelastic models separate into two parts describing the deviatoric and volumetric behaviour. For example the Polynomial model (of which the Mooney-Rivlin is a special form with $N=1$) is written:

$$U = \sum_{i+j=1}^N C_{ij} (\bar{I}_1 - 3)^i (\bar{I}_2 - 3)^j + \sum_{i=1}^N \frac{1}{D_i} (J^{el} - 1)^{2i}$$

where I_1 and I_2 are the first and second deviatoric strain invariants respectively and J^{el} is the elastic volume ratio. The C_{ij} coefficients are determined from fits to combinations of the uniaxial, biaxial and planar properties. D_i coefficients are determined independently from volumetric stress (pressure)-volumetric strain data.

A common assumption is to assume that since the ratio of bulk modulus to shear modulus is high the hyperelastic material is incompressible and that D_1 is 0. In which case Poisson's ratio (ν) must be 0.5. Uniaxial tensile measurements on the M70 adhesive show that Poisson's ratio is less than 0.5 and decreases with strain^[2]. Therefore, the assumption of incompressibility is untrue. What is not clear was how the volumetric properties influence the FEA predictions.

The recommended method for determining volumetric data is compression^[1]. A piston and cylinder test fitting has been manufactured for this purpose and is described in a further report^[9]. Adhesive specimens are prepared and cured in the cylinder. Compressive displacements are measured using three sensitive displacement transducers equally spaced around the circumference of the fixture. These measurements also act to check the axiality of the compression.

Alternatively, the Poisson's ratio data can be used to calculate volumetric strains in the material during the tensile test. Stretching a unit cube with side length of 1 unit (volume $V_0=1$) of a material with Poisson's ratio ν to a strain of ϵ results in the three dimensions of the material becoming $1+\epsilon$, $1-\nu\epsilon$ and $1-\nu\epsilon$. The volume V is $(1+\epsilon)(1-\nu\epsilon)(1-\nu\epsilon)$. The volumetric strain is V/V_0 and the pressure is $-1 \times$ true stress. This analysis has been applied to the tensile results to obtain data for evaluating the volumetric properties of the adhesive.

Figure 12 shows the effect of including volumetric properties in the models. These particular predictions were made using the Mooney-Rivlin model with input coefficients calculated from uniaxial tension data only. There is a small difference between predictions calculated assuming incompressibility ($D_1=0$) and when tensile volumetric data were input (fitted $D_1 = 4.48 \times 10^{-2}$). Appendix II shows that generally there is a small improvement in the predictions if volumetric data are included. However, the discrepancies are still unacceptably large. Further analyses were performed using volumetric coefficients derived from compression testing. However, these coefficients were an order of magnitude smaller than those obtained from uniaxial tension. There were negligible differences between predictions including this volumetric coefficient and those omitting volumetric properties. Therefore, the volumetric properties used in all further analyses were derived from tensile measurements. These results suggest that volumetric properties differ between tension (dilatational) and compressive loading. Where stresses are compressive the assumption that the material is incompressible may be reasonable. However, under other states of stress it is likely that volumetric properties would need to be included in the model.

In order to further explore the influence of the volumetric coefficient on the predicted force-extension of the lap joint analyses were run with the volumetric coefficient D_1 set equal to 100 and 1000 times the original fitted value ($D_0 = 4.48 \times 10^{-2}$). The results of these analyses are also shown in Figure 12. The inflated D_1 values reduce the differences between the measured and predicted behaviour significantly and approximately half the discrepancies at large strains. The predicted force-extension curves also show a significant degree of curvature. There is little difference between the predictions for 100 times D_0 and those for 1000 times D_0 suggesting that even higher D_1 values would have little effect on the accuracy of the predictions. The model becomes less stable with increasing D_1 and the FEA with $D_1=1000D_0$ failed to converge before the full extension was applied.

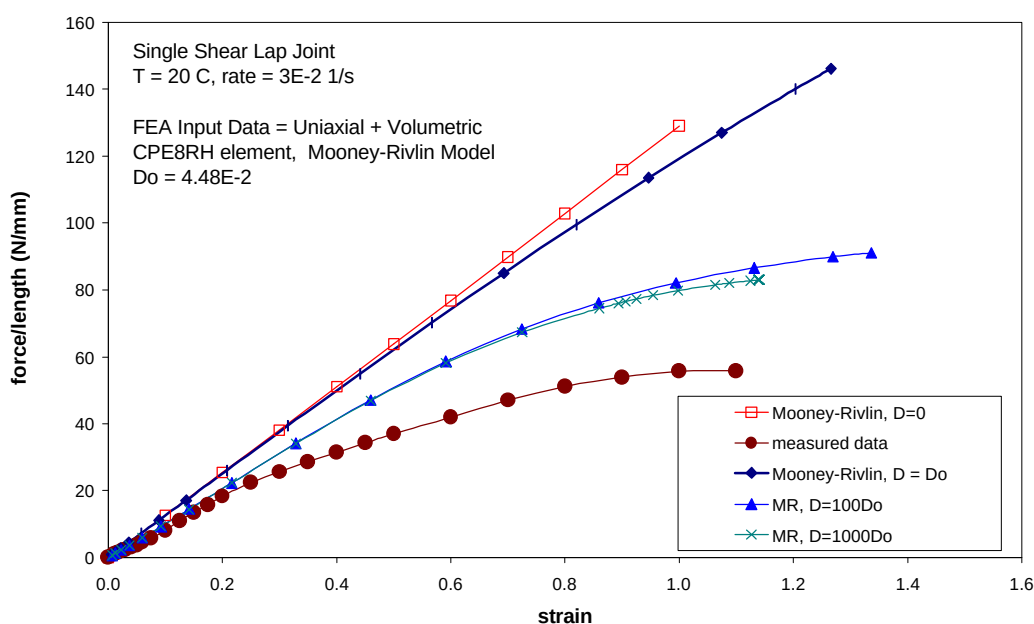


Figure 12: Effects of including volumetric properties

3.3. SAMPLE VARIABILITY

3.3.1 Batch-to-Batch Material Variations

The adhesive was packed in 300 g cartridges and delivered in three batches of four cartridges. The bulk test specimens were prepared first followed by the lap joint samples. Hence, the lap joint specimens were prepared using adhesive from the final batch whilst bulk test specimens were prepared using adhesives from all three batches. This may be a source of variation between the bulk material and the lap joint specimens.

The results of the uniaxial tension tests were re-analysed statistically using Nutec Inc's Qualitek-4 analysis software to investigate the effects of material batch (and specimen thickness) in addition to the effects of temperature and strain rate. The sheets of adhesive prepared were numbered from 001 to 0150 in the order that they were manufactured. Specimens from sheets 001 to 090 were assigned to the 'early' batch of specimens (level 1) and specimens 091 to 150 to the 'later' batch (level 2). The statistical analysis of the results (ratio of stress to strain at failure) indicated that the specimen batch had little effect on the

properties. As Figure 13 shows even that small effect acts towards a higher material stiffness. That is opposite to any effects expected from the FEA results of material batch on the lap joint properties.

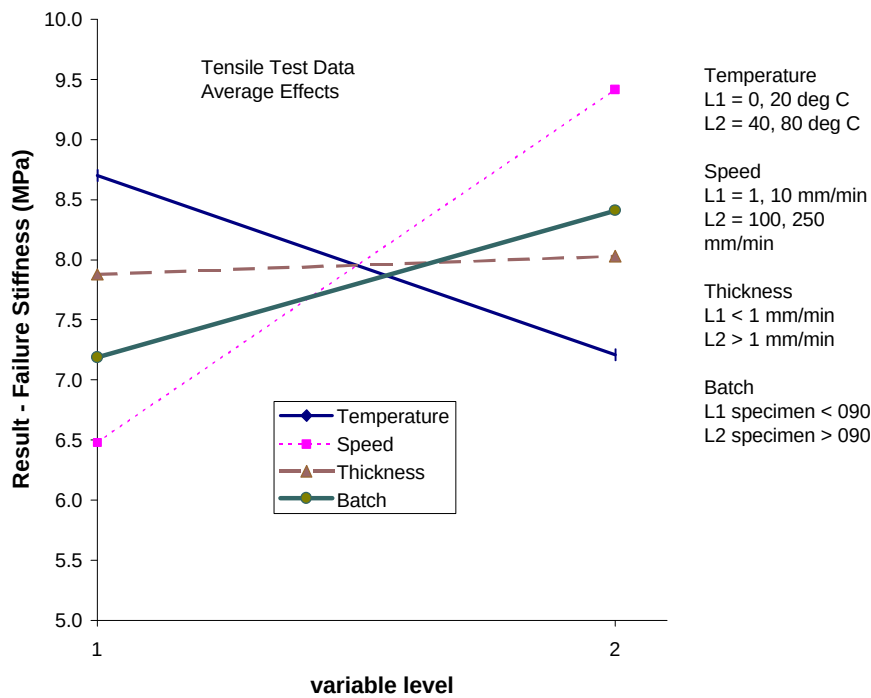


Figure 13: Statistical analysis of uniaxial tension data

The adhesive M70 was also the subject of study in an earlier project (PAJ1). The specimens tested in this project were made from completely different batches of material manufactured around 3 years before the material tested in the current project (PAJex2). As Table 5 shows there were considerable differences between the two batches of samples. Whether this was due to material variability or differences in specimen manufacturing procedures is not known.

Temp (°C)	Failure Stress (MPa)		Failure Strain (%)	
	PAJ1	PAJex2	PAJ1	PAJex2
0	1.79	3.11	58	42
20	1.61	2.19	48	39
40	1.21	1.92	31	29
80	0.97	1.46	32	22

Table 5: Comparison between specimens from different projects

3.3.2 Effects of Specimen Thickness

The thicknesses of the lap joint specimens (nominally 0.25 mm) are significantly less than the thicknesses of the bulk test specimens (around 1 mm) used to obtain the material properties. If the properties of the M70 adhesive are sensitive to thickness then this may help account for discrepancies between the predicted and measured data. However, as Figure 13 shows the material properties of the bulk specimens appear to have little dependence on specimen thickness.

A similar analysis was performed on the lap joint test results. Two levels of adhesive layer thickness L1 and L2 were chosen as ≤ 0.2 mm and > 0.2 mm respectively. The ratio of force/bond length to strain at failure appeared sensitive to the thickness of the adhesive layer. Further investigation indicated that the load/bond length at failure was relatively insensitive to bond line thickness. However, failure strain decreases with decreasing bond line thickness. Thus, thin joints appear stiffer at failure. The reasons for this are not known but may be linked to the increasing measurement uncertainty in the strain with decreasing bond line thickness. FEA predicts little difference in normalised joint stiffness between thickness values of 0.15 mm and 0.25 mm.

3.3.3 Effect of Thermal Histories During Cure

The possibility finishing with different states of cure in bulk and joint materials was recognised as a potential problem at the start of the work. The bulk and joint material are cured in different preparation jigs. The differing thermal capacities of the jigs could lead to differences in the temperature profile in the adhesive during cure^[9]. To overcome this difficulty, temperature histories of the adhesive were measured during cure. This helped to establish cure schedules for the bulk material and the lap joint specimens that gave the adhesive equivalent thermal histories. The cure schedules in Table 6 gave approximately the same material temperature profiles during the time spent curing in the oven.

Specimen Type	Oven Temp	Duration in Oven	Completion
Bulk adhesive	200 °C	45 minutes	remove from oven and cool to room temperature
Lap Joints Arcan Joints Scarf Joints	215 °C	45 minutes	remove from oven and cool to room temperature

Table 6: Cure schedules

Temperature profiles during the cooling stage were not monitored. However, since the thermal capacity of the joint specimen preparation jig appeared to be the higher (as a higher oven temperature was needed to heat at the same rate as the bulk material) it is likely that the lap joints will cure more slowly than bulk sheets. This suggests a possibility that the material in the lap joints may cure further than the bulk material. The result of this would be to give a stiffer material in the joint than the bulk. This was opposite to observations. Assessment of material prepared in both jigs showed that the glass transition temperatures (between -30 °C and -35 °C) were very similar indicating equivalent cure states.

3.3.4 Age of the Adhesive Samples

Two aspects of material age need to be considered - the age of the adhesive paste before manufacturing test specimens and the time between making and testing specimens. The bulk test specimens were made soon after receipt of the adhesive but tested, in some cases, many months after manufacture. The specimens were stored in desiccators between manufacture and storage. The joint specimens were prepared after the bulk specimens using the remainder of the adhesive. Thus, the adhesive in the cartridge had aged by a few weeks to months before the joints were manufactured. These joint specimens were tested relatively soon after preparation.

To investigate whether material age has a significant effect some additional test specimens have been manufactured and tested. Test specimens were prepared from a cartridge left over from the manufacture of the original batch of specimens. The cartridge had been stored on a fridge during this period (4-6 months). Bulk tensile and lap joint specimens were manufactured and tested within a short period of time. The ‘ages’ of the bulk and joint specimens were therefore the same. The temperatures profiles logged during cure were approximately the same. DMTA measurements showed that the glass transition temperature was similar to that found for the original samples manufactured (between -30 °C and -35 °C).

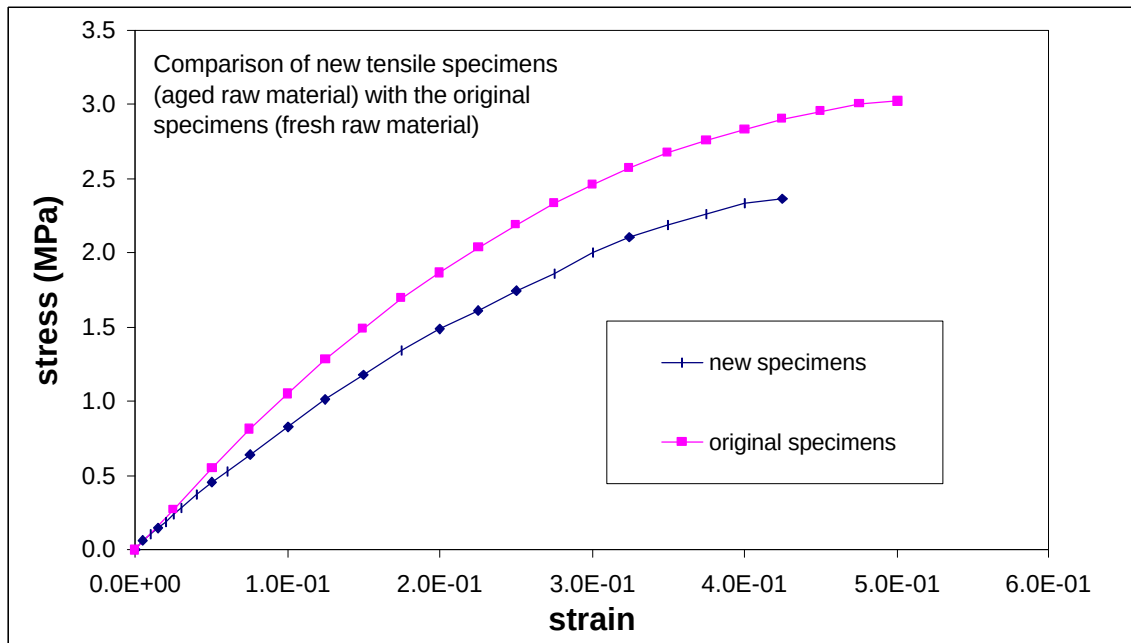


Figure 14: Comparison of the tensile properties of bulk specimens manufactured at different times

Figure 14 shows a comparison between the uniaxial tensile properties of the original and the new material. The tests were performed at 20 °C with a crosshead speed of 10 mm per min (strain rate $3 \times 10^{-3} \text{ s}^{-1}$). The new material appears to have a lower stiffness than the original material. This is reflected in the reduced Young’s modulus of 9.9 MPa compared with the original 11.8 MPa. The Poisson’s ratio data are similar to the original results. On the face of it this would seem to indicate degradation of the material properties of the adhesive. However, the new material data are not outside of the scatter in the properties of the original results.

Similarly, lap joints manufactured at the same time as the additional uniaxial specimens show reduced stiffness in comparison to the original batch of joint specimens (Figure 15). These results, together with the Arcan and uniaxial tension results, tend to indicate that the properties of the adhesive may have degraded since it was originally supplied.

3.3.5 Differences in Test Strain Rate/Temperature

The properties of the adhesive are rate and temperature sensitive as has been shown in earlier work^[2]. Therefore, test temperatures and speeds were set to give comparable sample temperatures and strain rates to those in the bulk specimen tests used to generate the model input data. Temperatures were comparable to within ± 2 °C and samples were allowed to equilibrate at temperature for at least 30 minutes before testing. Test speeds were chosen to

give strain rates in the adhesive layer within $\pm 25\%$ of the bulk specimen test rates. Measured extension rates were checked for consistency with the set strain rates. The differences between the model predictions and the measured results are too large to be due to errors in either temperature or strain rate - these would need to be 10s of $^{\circ}\text{C}$ or decades of rate to give the predicted curves obtained.

3.4. ALTERNATIVE JOINT TESTS

3.4.1 Alternative Shear Measurements

One area of further study was an alternative test method to check the properties of the adhesive in shear. Bulk specimen shear tests such as the Arcan shear test used to characterise structural adhesives were thought unsuitable due to the extremely low stiffness of the test specimens. Therefore, additional shear measurements were performed using an alternative test joint - the Arcan joint^[10]. This method uses much thicker and, therefore, more rigid steel adherends than the lap joint specimen. The measurement of the extension of gauge points either side of the adhesive bond line is used to characterise the shear extensions. The test has been used previously to accurately determine the shear properties of structural adhesives^[10]. The results for the M70 adhesive are shown in Figure 15. The stress-strain response is significantly less stiff than the original lap joint specimens but roughly comparable with the additional lap joint specimens manufactured during the same period.

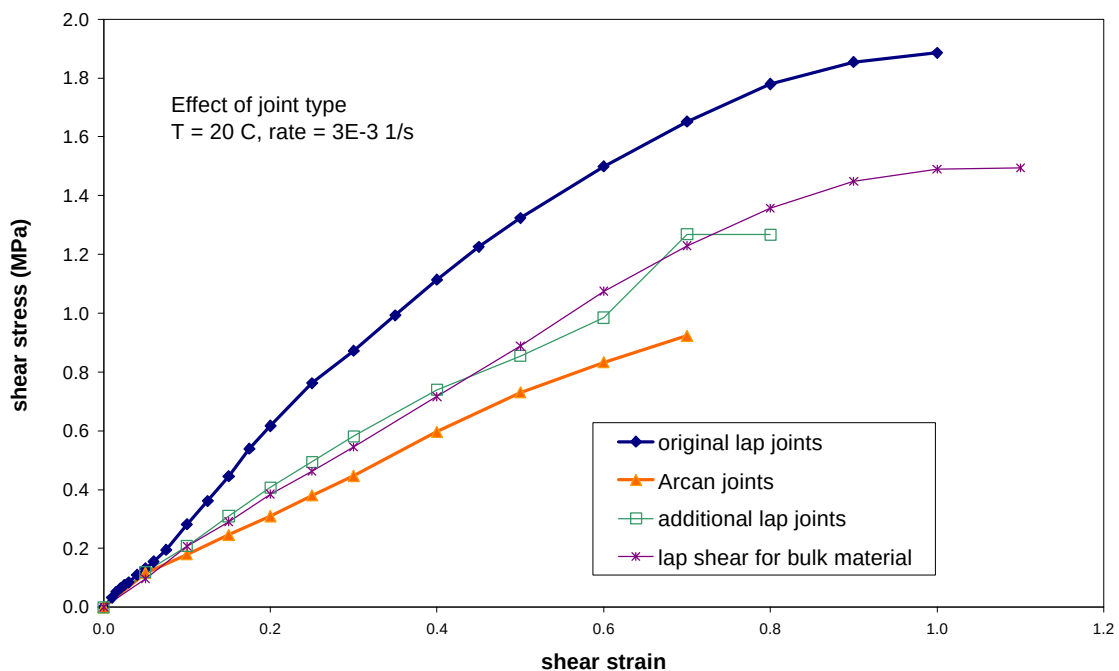


Figure 15: Shear test data determined from different joint configurations

Lap joint specimens were also manufactured by glueing thin sheets of pre-cast adhesive to the adherends. Thus, shear lap joint specimens were manufactured using bulk adhesive material allowing direct comparisons between bulk and joint material. The adhesive used to bond the M70 sheets to the steel adherends was a cyanoacrylate. It is assumed that the thin, stiff film of this adhesive deforms negligibly in comparison to the low modulus flexible material. These lap shear specimens were tested using the same test procedure as normal lap joint

specimens. The results are shown in Figure 15 and suggest that there is little difference between the shear responses of material manufactured as bulk sheet or bonded joints. This supports the assumption that there are no significant differences between the adhesive prepared as bulk material and that in the bonded joints.

3.4.2 Scarf Joints

In order to test the ability of the FEA and materials models to model joints under an alternative state of stress a number of analyses were performed for the scarf joint specimen. The specimen is illustrated in Figure 16. The stress state in the adhesive bond line for this specimen is a combination of shear and tension. The angle of the scarf joint was 30° and, for the FEA, the thickness of the bond along the length axis was 0.5 mm (0.25 mm perpendicular to the adherend edges). The adherends were 10 mm thick aluminium.

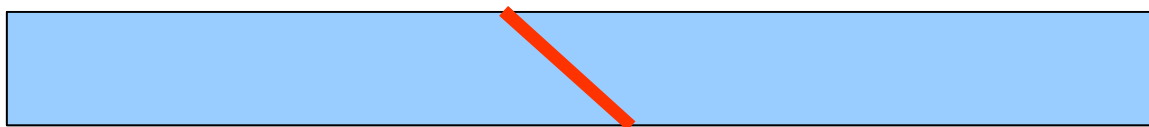


Figure 16: Scarf joint specimen

Specimens were manufactured using aluminium adherends with bond line thicknesses between 0.7 and 1.0 mm. Areas to be bonded were grit blasted and acetone wiped prior to testing. Extensions across the bond line were measured by video extensometry. For purposes of comparison the extensions were normalised by the bond line thickness to obtain a 'strain' in the length direction. All test specimens failed through cohesive failure of the adhesive.

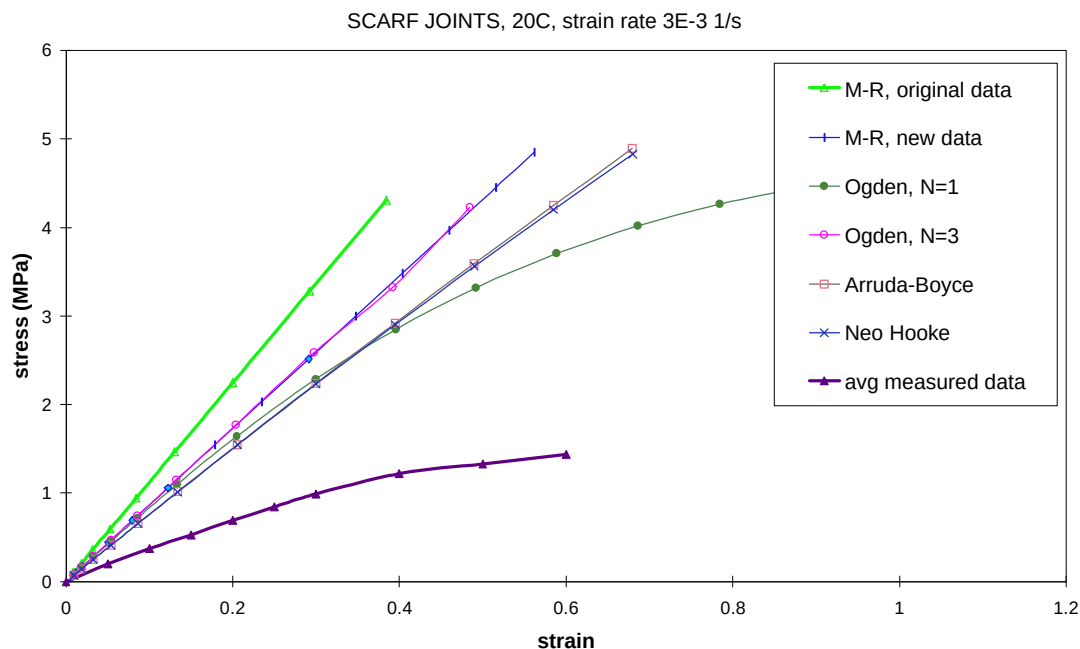


Figure 17: Comparison of measured and predicted scarf joint properties

All test measurements and input properties data were obtained at 20°C and a strain rate of $3 \times 10^{-3} \text{ s}^{-1}$. The comparisons between the FEA and test results are shown in Figure 17. In common with the lap joint results, the predicted force-extension plots are much stiffer than the measured response. As with the lap joint specimens there are very large discrepancies

between the FEA predictions and the measured properties. Further work is planned on the analysis of the scarf joint specimen and this will be reported later.

3.5. DISCUSSION

The analysis of the FEA results leads to the following conclusions regarding the suitability of the hyperelastic models for characterising the behaviour of the M70 adhesive.

- None of the FEA predictions gives an acceptable fit to the measured response of either the lap shear or the scarf joint specimens.
- No significant differences between bulk and joint material have been found that may explain the large discrepancies observed.
- Different aspects of the FEA model have been explored (element type, input data types, available materials models) but only the changing the element type brings any significant improvements in the prediction of the joint responses.
- Initiation and growth of macro-cracks has been considered as a possible source of difference between the model and the actual joints. However, no cracks have been observed to form at loads where the discrepancies between the FEA and measurements start to become significant. Stress analyses indicate that stress concentrations are considerably lower than the adhesive strength.
- The responses of the uniaxial and planar tension tests can be predicted accurately by some of the models but not by others. There is no evident improvement in terms of prediction accuracy gained from the models that fit the tensile data well. The biaxial tension data was not accurately predicted by the FEA.
- Comparison between predictions made from only uniaxial tension data and those made using the combined uniaxial, planar and biaxial tension tests indicate that for some models little additional benefit is gained from the additional test data. Thus, it seems that the planar and the biaxial tests could be omitted thus reducing the cost of obtaining input data with little compromise to the accuracy of the predictions.
- Volumetric test data seem to improve the predictions to some extent. These data can be obtained if lateral strain measurements are made during uniaxial tension tests.
- Overall the combination of uniaxial tensile and volumetric data gives the best predictions.

None of these investigations have determined the definite cause of the discrepancies between the measured and predicted responses of the flexible adhesive joint specimens. Unfortunately, the most likely conclusion must be that the hyperelastic material models do not apply to this material and these stress situations. The stress states where hydrostatic stress is significant due to constraint of the adhesive (joint specimens, biaxial tension) cannot be accurately modelled whereas those where hydrostatic stress is less significant (uniaxial, planar tension) are acceptably modelled.

A possible reason for the failure of the material models is cavitation in the rubber under dilatational, hydrostatic stress. The M70 adhesive is a mineral filled material. The presence of solid particles in the 'rubber' matrix may lead to cavitation and loss of stiffness under hydrostatic, multi-axial stress. The situation may be analogous to toughened structural adhesives where cavitation around rubber particles influences mechanical performance. Also, owing to the high viscosity of the adhesive paste, it is impossible to completely exclude air

when making bulk specimens or joints. The presence of voids is not explicitly accounted for in the FE model. These voids are another site for growth of cavities.

A recent paper demonstrated the effects of cavitation on the mechanical properties of rubbers^[11]. Cavitation was shown to degrade the mechanical properties of rubbers experiencing plane strain (hydrostatic) stress states. It was shown that cavities form at a critical hydrostatic stress. This value was observed to be approximately 5/6 of Young's modulus (E). This is significantly greater than the levels of stress seen with the M70 adhesive. However, true rubbers will differ from heterogeneous systems such as adhesives and it is likely that cavities already exist in the manufactured adhesive specimens and the observed deformations reflect growth of these cavities. It was suggested that cavitation could be modelled through the volumetric part of the hyperelastic function using a cavitated bulk modulus that is many times smaller than the initial bulk modulus of the material.

This suggests that the volumetric/hydrostatic properties of the adhesive will be important. However, the volumetric properties determined under uniaxial tension and compression do little to improve the predictions possibly because these stress states may not be severe enough to cause the cavitation occurring within highly constrained adhesive joints. The butt tension test may be a better test for obtaining the volumetric properties.

Work is underway at NPL to develop a cavitation based materials model for rubber-toughened polymers and adhesives. The model aims to predict reductions in yield stress due to the formation of cavities around the rubber particles under certain states of stress. Application of this model to flexible adhesives was considered but has been rejected at this stage owing to the following:

- The cavitation model seeks to determine the plastic yield stresses of elastic-plastic materials whereas all experimental evidence points towards little or no plastic yield in the bulk M70 material.
- The cavitation model addresses lower yield stresses in tension than would be expected from the shear behaviour. The findings of this work have been opposite to this - i.e. lower stiffness in shear in comparison to predictions from tension data.
- The microstructure of the flexible adhesive (solid particles in a flexible matrix) is completely different to structural adhesives (flexible particles in a stiff matrix). It is likely that the fundamental material behaviour will differ. Thus, empirical and theoretically derived constants for the existing cavitation model are unlikely to be valid.
- Significant amounts of additional experimental measurements and data analysis would be necessary to determine coefficients for any modified model. The value of this has to be weighed against the general applicability of the model.

It is not known whether the behaviour of M70 is typical of flexible adhesives in general. It may be the case that M70 belongs to a small class of flexible materials not described by hyperelastic models and that for many flexible adhesives the existing hyperelastic models may give acceptable design predictions. Investigation of the behaviour of other flexible adhesives would help determine if new models are required.

4. RATE AND TEMPERATURE DEPENDENCE OF MODEL COEFFICIENTS

The mechanical properties of flexible adhesives are sensitive to strain rate and temperature. Accurate design calculations may need to consider the response of components under different rates of loading and/or service temperatures. Obtaining input data for such general analyses could be expensive even if only uniaxial test data are required. An ideal material model for a strain rate and temperature dependent material would obey simple rules for determining the input coefficients under any arbitrary test conditions.

In Appendix I, the hyperelastic models were described as phenomenological and not fundamentally based on the physical properties of the materials. Therefore, it is not possible to theoretically calculate input coefficients for the various models. Instead, the coefficients are determined by fitting to measured test data. Thus, the strain rate and temperature dependence of the models will be expressed through the values of the fitted coefficients. If it is possible to describe the rate and temperature dependence of these coefficients then, by interpolation, it should be possible to derive coefficient values for any test condition. Therefore, material models where simple trends are apparent in the coefficients will offer advantages for design predictions.

Hyperelastic model coefficients were determined at several strain rates and temperatures as part of the FEA modelling studies outlined in earlier sections. These are tabulated in Appendix IV and analysed to determine the presence or otherwise of simple trends in the coefficients. It is apparent from the tables that, where models have three or more fitted deviatoric coefficients (C_{ij} or μ_i), the values show much scatter and trends are not present. This is not really surprising as the coefficients are calculated through least squares fitting routines. The higher the number of coefficients the greater is the sensitivity to the data and higher the variation in coefficients for small changes in the data.

Thus, models with fewer coefficients - such as Mooney-Rivlin, first order Ogden, Neo Hooke and Arruda-Boyce - offer greater possibilities for modelling rate and temperature sensitivities of the coefficients. Selected coefficients are plotted against test rate and temperature in Figures IV-1, IV-.2 and IV-3 of Appendix IV.

In Figure IV-1, the C_{01} and C_{10} coefficients are presented as a function of test rate for the Mooney-Rivlin model. These values were derived from both uniaxial only and combined uniaxial, biaxial and planar data. There is quite a degree of scatter in the coefficient values that, in the main, obscures any underlying trends. The scatter is smaller for the coefficients obtained from uniaxial only test data. The plots of coefficient values as functions of temperature (not shown) are similarly scattered.

Figure IV-2 shows the rate and temperature dependence of the μ_1 and α_1 coefficients for the Ogden, N=1 model. Values were obtained from combined uniaxial, biaxial and planar data. There are obvious trends in the μ_1 coefficient values - a logarithmic increase with test rate and exponential type decay with temperature. However, the α_1 coefficients are more scattered and trends cannot be determined. Unfortunately, to predict behaviour under any test condition it must be possible to calculate reliable values for all of the input coefficients. The situation with the Ogden, N=1 model, where the rate and temperature dependence of one coefficient cannot be modelled, makes application of the model to any general test condition extremely difficult.

The Neo Hooke model requires only one deviatoric coefficient, C_{10} . As Figure IV-3 shows this coefficient can be related to both test rate and temperature. The Arruda-Boyce model has two coefficients, μ and λ_m . From the Tables above, it is obvious that λ_m is very close to 7 and is independent of rate and temperature. The formulation of the Arruda-Boyce model shown in Appendix I suggests that the model predictions will be insensitive to minor errors in the value of λ_m and that assuming $\lambda_m = 7$ will not significantly affect accuracy. The rate and temperature sensitivity of the μ parameter is capable of being modelled. Like the Neo Hooke C_{10} and first order Ogden μ_1 values, the Arruda-Boyce μ coefficient increases with the logarithm of test rate and decays in an exponential fashion with temperature. The functional forms of these rate and temperature relationships are not surprising as these three coefficients should all be directly proportional to the elastic modulus of the material^[1]. These forms of relationships were also seen with measured quantities for this adhesive such as tensile and lap joint strengths.

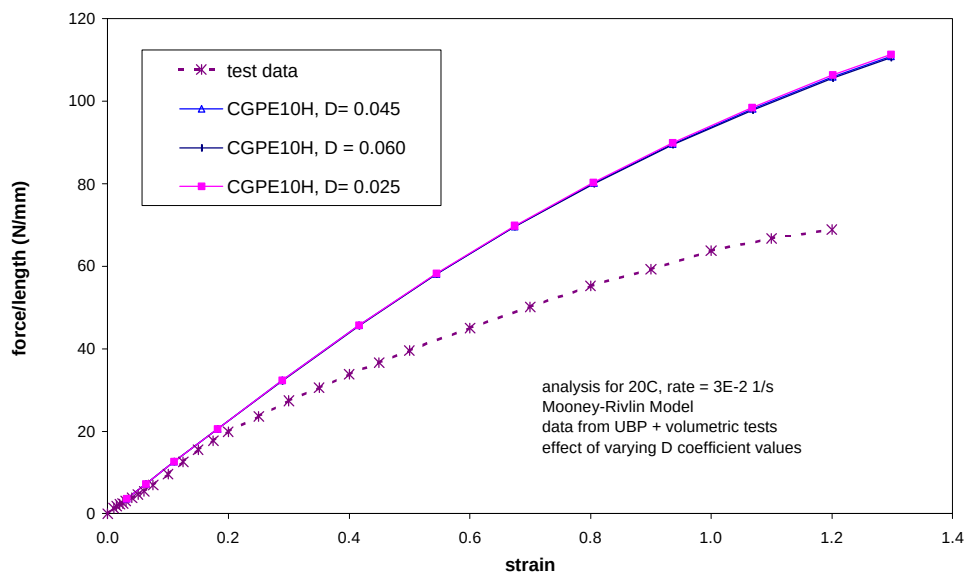


Figure 18: Analysis of the sensitivity of FEA predictions to volumetric coefficient values

In the above discussion, the rate and temperature dependence of the volumetric D_i coefficients has been ignored. Examination of the tabulated values in Appendix IV suggests that D_1 is scattered but tends to lie in the range 2×10^{-2} to 6×10^{-2} . The values for D_2 and D_3 are also scattered but tend to be close to -1×10^{-3} and 1×10^{-4} respectively.

The sensitivities of the FEA predictions to the D_i values are not known. Some analyses, that were discussed earlier, using D_1 values set to be 100 and 1000 times greater than the typical fitted D_1 values showed significant differences from the standard analyses. However, these set values were far outside the scatter in the fitted D_1 values. A sensitivity analysis for the influence of the value of D_1 in the Mooney-Rivlin model was performed. D_1 values that covered the range of values obtained from volumetric test data at different rates and temperatures were selected (0.025, 0.045 and 0.060). FEA predictions were obtained using constant C_{01} and C_{10} values (for temperature 20 °C and strain rate = $3 \times 10^{-2} \text{ s}^{-1}$) and these different D_1 values. The results (Figure 18) show that the scatter in the D_1 values has little effect on the force-extension predictions. This suggests that a single average value for D_1 could be used to represent all test rates and temperatures.

5. CONCLUDING REMARKS

The main findings of this work that force-extension response of a simple joint specimen is inadequately predicted by Finite Element modelling would seem to seem to represent a set back to the process of producing reliable tools for designing with flexible adhesives. However, it is unclear whether the adhesive studied (Evode Elastosol M70) represents typical behaviour of flexible adhesives. The applicability of the FE models to other flexible adhesives would need to be evaluated before making definitive conclusions regarding the general accuracy of the material models.

Whilst not finding a suitable suite of material models this study has provided progress in developing tools for validating FEA models.

- A quantitative procedure for evaluating the accuracy of model predictions over ranges of test conditions was presented.
- A much-reduced size of experimental matrix for evaluating the influence of four key factors was designed using statistical DOE.
- This reduced matrix of tests gave the same conclusions as the full experimental study regarding the ranking of the influence of the various factors.

This work has also improved insights into some of the effects of key elements in the FE modelling and determination of input data for flexible adhesives.

- The significance of experimental factors such adhesive batch, age of specimens and preparation procedures on the variability of the test results has been demonstrated.
- Generalised plane elements gave significant improvements over standard plane strain elements in the accuracy of the FE predictions. Hence, it is recommended that generalised plane strain elements be used to represent adhesives in 2-dimensional models of bonded structures.
- Higher order material models (with greater numbers of fitted coefficients) give superior fits to the input test data than simpler models. However, these higher order models tend to give the worst predictions of the lap joint performance.
- The combination of uniaxial tensile and (tensile) volumetric properties as input test data gave the best FE predictions.
- Apparently biaxial tension and planar tension data can be omitted without compromising the accuracy of the predictions.
- Input data for determining the coefficients that describe the volumetric behaviour in hyperelastic models can be obtained from lateral strain measurements in the uniaxial tension test. Such data may be more applicable than that obtained in compression.
- It is only feasible to model the strain rate and temperature dependence of hyperelastic coefficients for material models that have few fitted coefficients. The Neo Hooke and Arruda-Boyce models appear amenable to such an approach.
- The FEA predictions were shown to be reasonably insensitive to the values of some of the coefficients. This is the case for the D_1 volumetric coefficient in many of the models where variations of ± 50 % of the average value (roughly the scatter in the fitted values) caused minimal changes in the predictions. Thus, only a representative

value of this coefficient need be obtained. Rate and temperature dependence can be ignored.

- Stress analyses close to the maximum loads sustained by the experimental joints tended to give peak peel stress and von Mises stress values comparable with the ultimate tensile stress values measured in uniaxial tensile tests. Macroscopic cracks in the adhesive layer are not observed prior to the maximum load. This suggests that these properties are candidates as failure criteria.

The suspicion remains that the M70 adhesive may not show typical behaviours for a flexible adhesive. The suitability of the hyperelastic FE models should be investigated for additional adhesives in order to establish general conclusions regarding their suitability. The above conclusions suggest these can be simplified considerably from the full suite of tests described in this report. Evaluation methods have been produced that quantify the quality of the predictions and reduce the number of experiments required. Future studies should concentrate on the lower-order models with fewer coefficients, such as the Neo Hooke and Arruda-Boyce, and omit higher-order models (e.g. Ogden, $N=3$) since the accuracy of higher-order models appears to be poor and the strain rate/temperature dependence of the coefficients cannot be modelled. The determination of test data can be simplified since omitting the planar tension and biaxial tension tests does not seem to compromise the FE predictions. Volumetric properties are required and these can be obtained simply from lateral strain measurements in uniaxial tension tests.

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APPENDIX I:

HYPERELASTIC MODELS

Flexible adhesives are characterised by low glass transition temperatures (T_g), low moduli and large strains to failure in comparison with structural adhesives such as epoxies. Although generally used in non-structural applications such as sealing, they are also starting to be used in applications where structural behaviour is more critical. Methods that predict the behaviour of flexible adhesives are required for designing these bonded joints. Flexible adhesives are unlikely to be modelled accurately by Elastic-Plastic models used for structural adhesives. These flexible adhesives may be closer to rubber in their properties. Hence, it may be possible to adequately model the response of the adhesive using the hyperelastic material models developed for rubbers.

Finite Element (FE) stress analysis packages are powerful tools for design. Commercial FE packages have large libraries of hyperelastic models designed for rubber materials. For instance in the HKS package ABAQUS^[1] the following models are available: Polynomial; Mooney-Rivlin; reduced polynomial; Neo Hooke; Yeoh; Arruda-Boyce; Ogden and Van der Waals. The Mooney-Rivlin and reduced polynomial models are particular cases of the polynomial model. The Neo Hooke and Yeoh models are special forms of the reduced polynomial model.

These models assume that the material behaviour is elastic and isotropic. The hyperelastic materials are described in terms of a strain energy potential, relating stresses to strains. The assumption of isotropy is fundamental to the characterisation of rubber by a strain energy potential. In the unstrained state, the initial orientation of the long chain molecules is assumed to be random. Hence, the material is initially isotropic. Stretching of the rubber causes the chains to become aligned, but as this orientation is in the direction of straining the material is assumed to be isotropic throughout the deformation history. Consequently, the strain energy potential can be formulated as a function of the strain invariants^[1]. The strain energy potential is shown below for each form of hyperelastic model.

Polynomial Models

The form of the **polynomial** strain energy potential is

$$U = \sum_{i+j=1}^N C_{ij} (\bar{I}_1 - 3)^i (\bar{I}_2 - 3)^j + \sum_{i=1}^N \frac{1}{D_i} (J^{el} - 1)^{2i}$$

where U is the strain energy potential per reference volume, N is the polynomial order, C_{ij} and D_i are temperature-dependent material parameters, $\bar{I}_1$ and $\bar{I}_2$ are the first and second deviatoric strain invariants, defined as

$$\begin{aligned}\bar{I}_1 &= \bar{I}_1^2 + \bar{I}_2^2 + \bar{I}_3^2 \\ \bar{I}_2 &= \bar{I}_1^{-2} + \bar{I}_2^{-2} + \bar{I}_3^{-2}\end{aligned}$$

where the deviatoric stretches $\bar{I}_i = J^{-1/3} I_i$, λ_i are the principle stretches, J is the total volume ratio and J^{el} is the elastic volume ratio. The D_i parameters allow for the inclusion of compressibility.

The **Mooney-Rivlin** case is obtained from the polynomial form of the hyperelastic model by setting the parameter N to one, i.e. the first order polynomial. The Mooney-Rivlin model uses only linear functions of the invariants.

$$U = C_{10}(\bar{I}_1 - 3) + C_{01}(\bar{I}_2 - 3) + \frac{1}{D_1}(J^{el} - 1)^2$$

In the **Reduced Polynomial** form, the dependence on the second invariant is suppressed. The model includes only the first strain invariant, this is equivalent to the polynomial form with $C_{ij} = 0$ for $j \neq 0$.

$$U = \sum_{i=1}^N C_{i0}(\bar{I}_1 - 3)^i + \sum_{i=1}^N \frac{1}{D_i}(J^{el} - 1)^{2i}$$

The **Neo Hooke** form is the reduced polynomial strain energy potential with $N=1$.

$$U = C_{10}(\bar{I}_1 - 3) + \frac{1}{D_1}(J^{el} - 1)^2$$

The **Yeoh** model is also obtained from the reduced polynomial form by setting $N=3$.

$$U = C_{10}(\bar{I}_1 - 3) + C_{20}(\bar{I}_1 - 3)^2 + C_{30}(\bar{I}_1 - 3)^3 + \frac{1}{D_1}(J^{el} - 1)^2 + \frac{1}{D_2}(J^{el} - 1)^4 + \frac{1}{D_3}(J^{el} - 1)^6$$

In all the above equations, the parameters are the same as those defined above for the polynomial model.

Ogden Model

The **Ogden** strain energy potential is expressed in terms of principle stretches. In ABAQUS the form of the Ogden strain energy potential is

$$U = \sum_{i=1}^N \frac{2m_i}{a_i} (\bar{I}_1^{a_i} + \bar{I}_2^{a_i} + \bar{I}_3^{a_i} - 3) + \sum_{i=1}^N \frac{1}{D_i}(J^{el} - 1)^{2i}$$

where $\bar{I}_i$ are the deviatoric principle stretches. N is the order of the polynomial, μ_i , α_i and D_i are temperature-dependent material properties. The Mooney-Rivlin and Neo Hooke forms of polynomial model can also be obtained from the general Ogden strain energy potential for special choices of μ_i and α_i .

Van der Waals Model

The form of the **Van der Waals** strain energy potential is

$$U = m \left\{ - (I_m^2 - 3) [\ln(1-h) + h] - \frac{2}{3} a \left(\frac{\tilde{I} - 3}{2} \right)^{\frac{3}{2}} \right\} + \frac{1}{D} \left(\frac{J_{el}^2 - 1}{2} - \ln J_{el} \right)$$

where $\tilde{I} = (1-b)\bar{I}_1 + b\bar{I}_2$ and $h = \sqrt{\frac{\tilde{I} - 3}{I_m^2 - 3}}$

Here, U is the strain energy per unit of reference volume; μ is the initial shear modulus; λ_m is the locking stretch; a is the global interaction parameter; β is an invariant mixture parameter and D governs compressibility. These parameters can be temperature dependent. $\bar{I}_1$ and $\bar{I}_2$ are the first and second deviatoric strain invariants, as above; J is the total volume ratio and J_{el} is the elastic volume ratio. The Van Der Waals model is sometimes known as the Killian model.

Arruda-Boyce Model

The **Arruda-Boyce** model has a strain energy potential of the form

$$U = m \left\{ \frac{1}{2} (\bar{I}_1 - 3) + \frac{1}{20 I_m^2} (\bar{I}_1^2 - 9) + \frac{11}{1050 I_m^4} (\bar{I}_1^3 - 27) + \frac{19}{7000 I_m^6} (\bar{I}_1^4 - 81) \right. \\ \left. + \frac{519}{673750 I_m^8} (\bar{I}_1^5 - 243) + \dots \right\} + \frac{1}{D} \left(\frac{J_{el}^2 - 1}{2} - \ln J_{el} \right)$$

where U is the strain energy per unit of reference volume; μ , λ_m and D are temperature dependent material parameters, $\bar{I}_1$ is the first deviatoric strain invariant, J_{el} is the elastic volume ratio; and λ_i are the principle stretches.

Discussion – Using Hyperelastic Models

The majority of the hyperelastic material models are phenomenological, that is the derived constants bear no relationship to the physical or chemical characteristics of rubbers. They consider the bulk properties of the material and attempt to describe the material as an amorphous, isotropic solid. As a result the models can become inaccurate if the material changes state during deformation. Hence, these models are best suited to application such as gum rubber where the effects of such changes are negligible^[12].

The most commonly used hyperelastic model is the Mooney-Rivlin form of the polynomial model. The Mooney-Rivlin model is known to be a simple representation of rubbers, though this simplicity also makes it convenient to use. However, it has limited accuracy – it is not a self-consistent basis for the interpretation of all types of strain^[12, 13, 14]. Treloar^[15] notes that for a single strain type, experiments usually conform to the two-term Mooney-Rivlin equation but that using this equation for general strain, i.e. a loading type that does not conform to the test, is an unwarranted extrapolation.

The derivation of phenomenological models such as the Mooney-Rivlin requires experimental data to obtain the model coefficients. It is thought best to use data from a range of deformation states to improve the accuracy of predictions – especially when trying to predict the behaviour of a material under general loading conditions. In ABAQUS, data are required from uniaxial tension, equi-biaxial tension and planar tension. Volumetric data can also be incorporated if compressibility is important^[1]. If data from only one state of strain are available, Charlton et al^[13] make the general recommendation to use the N=1 two term Mooney-Rivlin form of the polynomial model. The C_{ij} coefficients required for both the Mooney-Rivlin model and other forms of the polynomial model should really be considered as merely regression coefficients rather than attempt to give them any physical meaning^[13], such as relating them to moduli i.e. $E=6(C_{10} + C_{01}) \approx 3G$ for a Mooney-Rivlin analysis.

The Ogden model is also a phenomenological model that makes no attempt to link the adjustable parameters to any physical deformation model. The model generally requires more than one set of experimental data to obtain the number of constants required to capture the state of deformation dependence. The Ogden function has the advantage that the exponent values (α_i) can be any real number which gives the model the ability to fit non-linear material test data^[13].

The Neo Hooke model has some theoretical relevance since the mathematical representation is analogous to that of an ideal gas. The neo-Hooke potential represents the Helmholtz free energy of a molecular network with Gaussian chain-length distribution. Similarly, the Van der Waals strain energy potential is analogous to the equations of state of a real gas. This introduces two additional material parameters, the locking stretch (λ_m) which accounts for the finite extendibility of the non-Gaussian chain network, and the global interaction parameter (a) which models the interaction between chains.

The Arruda-Boyce model is a constitutive relationship for nonlinear elasticity that should better describe the state of deformation dependant response of rubber materials. The aim of the model is to represent the behaviour of the material while requiring only a small number of “physically based” parameters^[16]. This model is also known as the eight-chain model since it was developed from a representative volume element where eight springs emanate from the centre of a cube to its corners^[1]. The Arruda-Boyce strain energy potential depends on the first strain invariant only. The physical interpretation of this is that the eight chains are stretched equally under the action of a general deformation state.

When making predictions using any of these models, the stability of the model is of great importance. To obtain stability the requirement is for the energy function to obey the laws of thermodynamics i.e. energy functions that mathematically require the solid to increase its internal energy when work is done on it^[17]. The material constants are only regression coefficients. They accurately fit experimental data, but this does not assure thermodynamic stability. If stability is not considered while selecting material constants, the numerical procedures may have difficulty converging or giving meaningful results. ABAQUS^[1] performs stability checks and warns of unstable ranges and types of deformation. Unstable models may have difficulty converging to a solution during FEA.

A constrained regression analysis can be used for the selection of material constants, as demonstrated by Quigley et al^[18]. In this case, when the regression analysis is performed on the experimental data, the analysis is constrained using numerical algorithms that select only positive material constants to fit the experimental data. Although the resulting model does not

fit the experimental data as accurately as one based on an unconstrained regression analysis, it should be more thermodynamically stable.

For some models, stability is assured by following some simple rules^[1].

- For positive values of initial shear modulus, μ , and locking stretch, λ_m , the Arruda-Boyce form is always stable.
- For positive values of the coefficients C_{10} the Neo Hooke form is always stable.
- Given positive values of the initial shear modulus, μ , and locking stretch, λ_m , the stability of the Van der Waals model depends on the global interaction parameter, a .
- For the Yeoh model, stability is assured if all $C_{i0} > 0$. Typically, however, C_{20} will be negative. Thus, reducing the absolute value of C_{20} or magnifying the absolute value of C_{10} will help make the Yeoh model more stable.

In the literature^[16, 20, 22] hyperelastic models have commonly been used to predict behaviour in uniaxial, biaxial and planar tension. These predictions have generally been compared to experimental data. A brief selection of experimental findings is noted below.

The FE predictions made using hyperelastic models have frequently been compared with experimental data obtained by Treloar^[19]. Treloar tested vulcanised rubber in simple tension, pure shear (planar tension) and uniform equi-biaxial tension. All samples were cut from the same sheet to avoid batch-to-batch variation which is common in rubbers. For example, Treloar's data were compared to predictions made using the Ogden model, and the predictions were found to be in good agreement^[20]. Arruda and Boyce^[16] also showed good agreement between Treloar's data and predictions obtained using the Arruda-Boyce model. The ABAQUS manuals also use the Treloar data to illustrate the fits provided by the various hyperelastic models.

Arruda and Boyce^[16] also repeated their work with three other materials (silicon, gum and neoprene rubber) with reasonable results. Allport and Day^[12] tested tensile compression blocks of styrene-butadiene (SBR). Their work indicated that the Arruda-Boyce model could give good agreement with measured behaviour in both tension and compression. They suggested that this was a considerable advance over the Mooney-Rivlin and Ogden type models which require different coefficients in tension and compression. Comparisons with Mooney-Rivlin and Ogden models demonstrated that good agreement was possible by tuning coefficients.

Kaliske and Rothert^[21] compared experimental results from rubber samples in a non-homogeneous shear test with predictions from selected hyperelastic models. They found that the Yeoh and Arruda-Boyce models gave the best fits followed by the Van der Waals model.

Raos^[22] compared the prediction from four models (Neo Hooke, Mooney-Rivlin, Van der Waals and Ogden) with experimental tests carried out on an SBR rubber compound under conditions of uniaxial and biaxial loads. In particular, the author investigated the influence of the initial strain ranges used to obtain the coefficients, while predictions were made over the full strain range. It was found that, even if the strain range is chosen adequately, the Neo Hooke and Mooney-Rivlin models could describe experimental data only in compression and moderate extension (up to 80%). The Van der Waals and Ogden models covered the whole range of applied strains with acceptable results, though these models exhibited some instability and sensitivity to the change of strain range. It was found that the biaxial tests were very sensitive and required reliable maintenance of testing boundary conditions. Of the

four modes, the Van der Waals and Ogden models were especially sensitive to biaxial conditions. It is known^[1] that accuracy is improved if the FEA is supplied with mechanical properties data over strain ranges approximating to those in the predictions.

Briefly summarised, the Mooney-Rivlin model is the most commonly used model, its simplicity making it convenient to use. Although, if predicting a material under general loading conditions it is best to use experimental data from a range of experimental tests (uniaxial, biaxial and planar tension, and volumetric compression). The Arruda-Boyce, Neo Hooke and Van der Waals models offer a physical interpretation and provide a better description of general deformation modes when the parameters are based only on one test. In all cases, it is best to obtain experimental data over the range of strain of interest (this is especially true of the Ogden and polynomial models), and to select the model coefficients carefully to ensure stability. In the literature reviewed above, hyperelastic models have been used to predict the behaviour of ideal rubbers. No reference is made to correlations between predictions and experiment for non-idealised materials e.g. filled rubbers or blends. The current Performance of Adhesive Joints Extension Programme is attempting to apply these hyperelastic models to a non-idealised material. Based on the above discussions, in order to have a manageable study, the decision was made to restrict the initial investigations to five material models - Mooney-Rivlin, Ogden (N=1), Ogden (N=3), Neo Hooke and Arruda-Boyce.

References

The documents referenced in these Appendices are listed in Section 7 of the main report.

APPENDIX II

ANALYSIS OF FINITE ELEMENT EXPERIMENTS

The FE experiments, covering several temperatures, strain rates, material models and combinations of input test data, generated a large quantity of results. These were plotted in the form of reaction force/length versus strain (gauge extension/bond thickness) in order to qualitatively compare the results with measured data. Statistical DOE methods were considered as an option for the analysis of this data. This type of analysis would require a single value as the 'experimental' result.

In order to generate a single result that quantifies the agreement or discrepancies between the model predictions and the measured joint responses the following procedure has been used.

1. FEA of lap joint specimen to obtain force/bond length vs. strain predictions. The model coefficients determined from mechanical tests performed at set strain rates and temperatures were used. Where the FEA run crashed due to unstable material models (as in several runs using the OGDEN, N=3 model) the force/extension values for strains following the final value obtained were set as zero.
2. Measurements of lap joint specimens were made at corresponding strain rates and temperatures.
3. The predicted and measured data were sampled to obtain force/bond length values at set strains - arbitrarily chosen as 0.1 intervals between 0.1 and 1.0 strain.
4. At each strain the differences between the values of force/bond-length predicted and those measured under the corresponding test conditions were calculated for each analysis. For each FEA run the sum of the absolute differences between the measured and the predicted results was calculated.
5. Since properties are rate and temperature dependent it was thought best to normalise the sum of differences against the sum of the measured values for each case run. For a perfect fit this normalised value would be zero.

The results of these analyses undertaken using the elastic model and 5 different hyperelastic material models (Mooney-Rivlin; Ogden, N=1; Ogden, N=3; Neo Hooke and Arruda-Boyce) were determined using input data determined from four different combinations of test data:

1. Uniaxial tests only (U)
2. Uniaxial, biaxial and planar tests (UBP)
3. Uniaxial and tensile volumetric tests (U+V)
4. Uniaxial, biaxial, planar and volumetric tests (UBP+V)

In cases 1 and 2 the D_i coefficients in the hyperelastic material models are defined as zero. In cases 3 and 4 D_i values are calculated from the volumetric test data. These values are independent of any uniaxial, biaxial or planar test data used to determine the model coefficients for the deviatoric part of the hyperelastic models i.e. D_i coefficients for cases 3 and 4 are identical.

The results of these FE experiments, determined using the above procedure, are shown in Tables II-1 to II-4. Also included in Table II-1 are the results of FE experiments using the elastic model with Young's modulus and Poisson's ratio determined from tensile tests.

Temp	Rate	Mooney-Rivlin	Ogden	Ogden	Neo Hooke	Arruda-Boyce	Elastic
deg C	1/s	N=1	N=1	N=3			
0	3.E-02	1.30	1.54	4.51	0.73	0.73	1.24
0	3.E-03	0.89	1.00	0.83	0.48	0.48	0.82
0	3.E-04	0.50	0.34	0.72	0.28	0.29	0.44
20	3.E-02	0.91	0.84	0.74	0.46	0.46	0.81
20	3.E-03	0.69	0.49	0.73	0.36	0.36	0.60
20	3.E-04	0.69	0.48	0.80	0.40	0.40	0.61
40	3.E-02	1.02	0.81	0.88	0.68	0.69	0.94
40	3.E-03	0.99	0.79	0.85	0.74	0.75	0.88
40	3.E-04	1.07	0.98	0.89	0.81	0.81	1.01
Average		0.90	0.81	1.22	0.55	0.55	0.82

Table II-1: Analysis results for FEA predictions generated using uniaxial only input test data

Temp	Rate	Mooney-Rivlin	Ogden	Ogden	Neo Hooke	Arruda-Boyce
deg C	1/s	N=1	N=1	N=3		
0	3.E-02	0.98	0.98	0.85	0.91	0.91
0	3.E-03	0.83	0.86	0.91	0.79	0.81
0	3.E-04	0.56	0.56	0.59	0.51	0.52
20	3.E-02	0.72	0.69	0.61	0.64	0.64
20	3.E-03	0.71	0.73	0.64	0.59	0.59
20	3.E-04	0.59	0.56	0.90	0.59	0.62
40	3.E-02	0.98	1.00	1.20	0.86	0.87
40	3.E-03	1.17	1.34	1.61	1.08	1.08
40	3.E-04	0.94	0.86	0.87	0.91	0.92
80	3.E-02	1.13	1.06	0.98	1.12	1.13
80	3.E-03	1.06	0.70	3.61	0.78	0.79
80	3.E-04	1.21	1.15	3.57	1.08	1.09
Average	(0C, 20C and 40C only)	0.83	0.84	0.91	0.77	0.77

Table II-2: Analysis results for FEA predictions generated using uniaxial, biaxial and planar input test data

Temp deg C	Rate 1/s	Mooney-Rivlin N=1	Ogden N=1	Ogden N=3	Neo Hooke	Arruda-Boyce
0	3.E-02	1.15	1.34	3.79	0.70	0.70
0	3.E-03	0.80	0.89	0.85	0.46	0.46
0	3.E-04	0.45	0.31	0.62	0.27	0.27
20	3.E-02	0.82	0.76	0.78	0.44	0.44
20	3.E-03	0.64	0.46	0.73	0.35	0.35
20	3.E-04	0.64	0.46	0.80	0.39	0.39
40	3.E-02	0.94	0.76	0.82	0.66	0.67
40	3.E-03	0.85	0.69	0.86	0.66	0.66
40	3.E-04	0.99	0.92	0.86	0.78	0.79
Average		0.81	0.73	1.12	0.52	0.53

Table II-3: Analysis results for FEA predictions generated using uniaxial and (tensile) volumetric data.

Temp deg C	Rate 1/s	Mooney-Rivlin N=1	Ogden N=1	Ogden N=3	Neo Hooke	Arruda-Boyce
0	3.E-02	0.89	0.90	0.84	0.87	0.88
0	3.E-03	0.76	0.79	0.90	0.76	0.77
0	3.E-04	0.49	0.50	0.65	0.49	0.49
20	3.E-02	0.66	0.63	0.71	0.62	0.62
20	3.E-03	0.66	0.67	0.60	0.57	0.58
20	3.E-04	0.56	0.53	0.90	0.57	0.57
40	3.E-02	0.90	0.92	1.13	0.84	0.84
40	3.E-03	1.09	1.23	1.53	1.05	1.05
40	3.E-04	0.88	0.81	0.82	0.89	0.89
Average		0.77	0.77	0.90	0.74	0.74

Table II-4: Analysis results for FEA predictions generated using uniaxial, biaxial, planar and (tensile) volumetric data.

The full suite of FE experiments outlined above required 180 different runs to cover the 5 material models, 4 input data types, 3 temperatures and 3 strain rates investigated. This involves much effort. Therefore, statistical Design of Experiments (DOE) methods were investigated as a means of providing sufficient experimental coverage from a reduced number of conditions.

Terry Twine of Risk and Statistical Consulting performed the statistical experimental design work. Three experimental options with reduced numbers of required experiments were developed^[3]. These are summarised in Table II-5.

Option	Number of Experimental Runs	Experiment Resolution
Downgraded Graeco-Latin Square	25	3
Asymmetric Orthogonal	50	3
Downgraded Regular Fraction	125	4
Full Factorial	180	Full

Table II-5: Summary of experimental matrices

The choice of experiment involves balancing the experimental size with the experimental resolution^[3]. The resolution is important as it establishes the degree to which the fractional design can untangle the key effects. A resolution 3 design can lead to the main effect being confounded by one or more two-factor interactions. A resolution 4 design will confound the main effects with one or more 3-way interactions. The full factorial is preferred as it can evaluate each main and interaction effect. Thus there is no confounding with the full factorial design.

The lowest level option offers a resolution 3 design with only 25 experiments needed. This can only offer valid conclusions if the main effects of the factors dominate and there is little interaction between the factors. In the belief that this was the case, an analysis of the 25 experiment case was performed using the experimental matrix outlined in Table II-6. The effects of each factor are summarised for the full factorial and reduced experimental matrix in Tables II-7 and II-8 respectively.

RUN	RATE	TEMP	DATA	MODEL
1	S1	T3	D1	M1
2	S2	T2	D2	M1
3	S3	T3	D3	M1
4	S3	T2	D3	M1
5	S2	T1	D4	M1
6	S3	T3	D1	M2
7	S2	T3	D2	M2
8	S2	T1	D3	M2
9	S3	T2	D3	M2
10	S1	T2	D4	M2
11	S3	T1	D1	M3
12	S1	T3	D2	M3
13	S2	T3	D3	M3
14	S2	T2	D3	M3
15	S3	T2	D4	M3
16	S2	T2	D1	M4
17	S3	T2	D2	M4
18	S1	T1	D3	M4
19	S3	T3	D3	M4
20	S2	T3	D4	M4
21	S2	T2	D1	M5
22	S3	T1	D2	M5
23	S2	T3	D3	M5
24	S1	T2	D3	M5
25	S3	T3	D4	M5
Levels	rate (s ⁻¹)	temperature	input data	model
1	3.00E-04	0 C	U	Mooney-Rivlin
2	3.00E-03	20 C	UBP	Ogden, N=1
3	3.00E-02	40 C	U+V	Ogden, N=3
4			UBP+V	Neo Hooke
5				Arruda-Boyce

Table II-6: Experimental array for lowest resolution option (25 experiments)

Table II-7 and II-8 show that, with regard to ranking the main effects of the various factors, the reduced experimental matrix leads to almost identical conclusions as the full factorial matrix. Hence, it would seem that the assumption of little interaction between the various factors holds in this case.

Level	Model	Avg Effect	Data Type	Avg Effect	Temperature	Avg Effect	Strain Rate	Avg Effect
1	Mooney-Rivlin	0.83	U	0.80	0 C	0.84	3.00E-04	0.65
2	Ogden, N=1	0.79	UBP	0.82	20 C	0.61	3.00E-03	0.78
3	Ogden, N=3	1.04	U+V	0.74	40 C	0.92	3.00E-02	0.94
4	Neo Hooke	0.64	UBP+V	0.78				
5	Arruda-Boyce	0.65						

Table II-7: Analysis results for full experimental matrix (180 standard FEA experiments).

Level	Model	Avg Effect	Data Type	Avg Effect	Temperature	Avg Effect	Strain Rate	Avg Effect
1	Mooney-Rivlin	0.860	U	1.422	0 C	1.468	3.00E-04	0.626
2	Ogden, N=1	0.866	UBP	0.894	20 C	0.601	3.00E-03	0.755
3	Ogden, N=3	1.502	U+V	0.681	40 C	0.893	3.00E-02	1.160
4	Neo Hooke	0.596	UBP+V	0.778				
5	Arruda-Boyce	0.632						

Table II-8: Analysis results for reduced matrix (25 standard FEA experiments).

The reduced experimental matrix was also used to evaluate the effect of changing the element type used to represent the adhesive layer in the FE model. As Section 3.1.3 in the main body of the report discusses, generalised plane strain elements may offer improved performance over standard plane strain elements. Evaluating this change of element would have required 180 additional FEA experiments for full coverage of all the factors. Using the reduced experimental matrix only 25 FEA experiments were required.

The analysis indicates that, bar the Arruda-Boyce model, all predictions are improved using the generalised plane strain elements. Once again the Ogden, N=3 model is significantly worse than the other material models. The results repeat the earlier finding that the combination of uniaxial and volumetric data gives the best predictions.

Level	Model	Avg Effect	Data Type	Avg Effect	Temperature	Avg Effect	Strain Rate	Avg Effect
1	Mooney-Rivlin	0.602	U	0.698	0 C	0.728	3.00E-04	0.537
2	Ogden, N=1	0.600	UBP	0.638	20 C	0.451	3.00E-03	0.633
3	Ogden, N=3	0.850	U+V	0.599	40 C	0.778	3.00E-02	0.692
4	Neo Hooke	0.487	UBP+V	0.651				
5	Arruda-Boyce	0.647						

Table II-9: Analysis results for reduced matrix (25 FEA experiments using generalised plane strain elements).

APPENDIX III

ANALYTICAL MODEL

The model, based on the Owens and Lee-Sullivan lap joint stiffness model^[4, 5] considers the sum of deformations of adherends and adhesive in response to an applied force in order predict the stiffness of the lap joint. Although the approach treats the components as simple elastic materials and only addresses extension in the axial direction it enables investigation into the effects of parametric changes to the specimen geometry.

Four sub-deformations (δ_1 , δ_2 , δ_3 and δ_4) of the overall extension (Δ) are considered. The sub-deformations δ_1 and δ_2 are tensile extensions of the unbonded areas of the adherends 1 and 2. These deformations are calculated:

$$d_i = \frac{PL_i}{E_i b_i t_i}, \quad i = 1 \text{ or } 2$$

where P is the applied force, L_i the length of the unbonded adherend, E_i the adherend's tensile elastic modulus, b_i the width ($b_1 = b_2$) and t_i the thickness. These values can vary between the adherends to incorporate different adherend geometries and materials although.

The deformation δ_4 is the deformation of the bonded areas of the adherends. The stress distribution in this region is non-uniform as load is transferred from one adherend to the other. This shear lag analysis is performed by dividing the bonded region into a number of equally sized elements of length dx . The stress in each element is then calculated using the Adams-Pepiatt^[23] stress lag expression

$$s_i = \frac{P}{bt_1} \left[1 - \Psi(1 - \cosh ax_i) - \frac{(1 - \Psi(1 - \cosh aB) \sinh ax_i)}{\sinh aB} \right]$$

where

$$a = \sqrt{K}, \quad \Psi = \frac{E_2 t_2}{E_1 t_1 + E_2 t_2} \quad \text{and} \quad K = \frac{2G_1 G_2 G_3 (E_1 t_1 + E_2 t_2)}{E_1 t_1 E_2 t_2 (t_1 G_2 G_3 + t_2 G_1 G_3 + t_3 G_1 G_2)}$$

B is the total bond length, x_i is the distance along the bondline (assumed to be at the mid point of the element, therefore for element n , $x_n = (n+0.5)dx$), and G is the shear modulus. The subscript 1 refers to the loaded adherend, subscript 2 refers to the fixed adherend and subscript 3 refers to the adhesive layer. The total deflection of adherend 1 is assumed equal to that in adherend 2 and only one contribution to the overall deformation is needed, thus δ_4 is calculated:

$$d_4 = \frac{\sum_{i=1}^n s_i dx}{E_1}$$

δ_3 is the shear deformation of the adhesive layer.

$$d_3 = \frac{Pt_3}{GbB}$$

Therefore the total extension of the bonded structure is calculated from the sum of the sub-deformations.

$$\Delta = d_1 + d_2 + d_3 + d_4$$

Analysis of this model for the specimen geometry and materials used in this study indicates that the deformations of the adherends (δ_1 , δ_2 and δ_4) are considerably less significant than the shear extension of the adhesive, δ_3 . The overall length of the lap joint assumed in this analysis was equal to the gauge length of the extensometer used to measure the experimental joint extensions. The predicted extension (Δ) ought to be equal to the measured extension and, thus, the strain ($=\Delta/t_3$) ought to correspond to the experimentally determined strain. Figure III-1 shows that the predicted force-extension behaviour is comparable to, if slightly stiffer, than the FEA predictions.

The model allows the introduction of cracks into the adhesive layer through reducing the bond length B and increasing the lengths of the unbonded adherends L_1 and L_2 such that the total length ($B+L_1+L_2$) is constant.

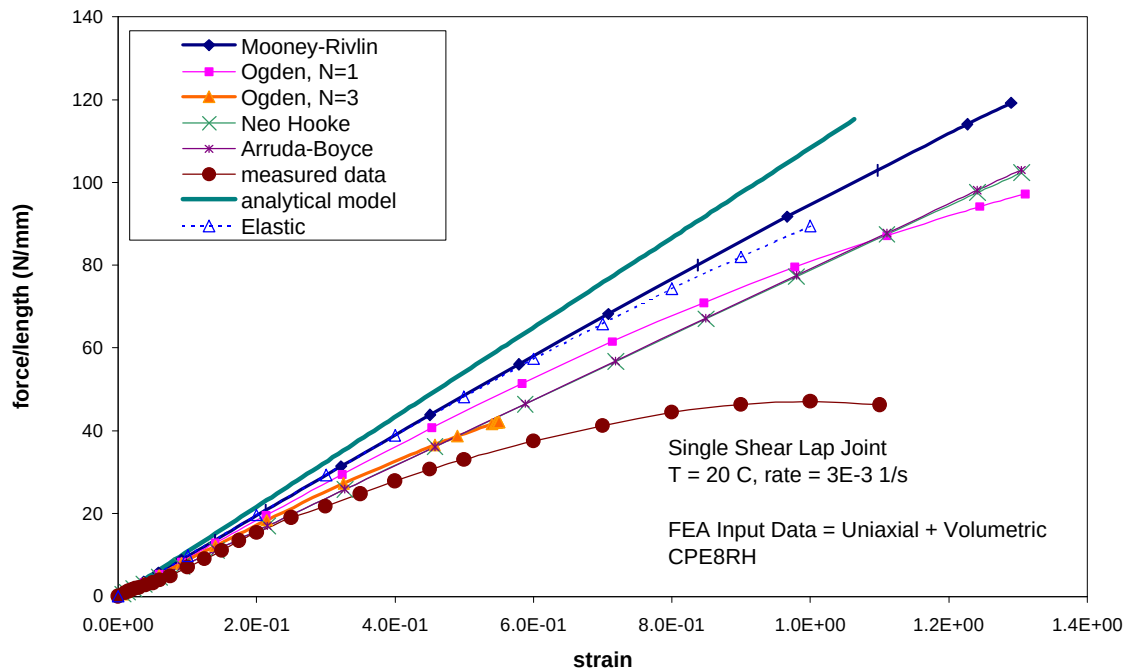


Figure III-1: Comparison between analytical and FE model predictions

APPENDIX IV

STRAIN RATE AND TEMPERATURE DEPENDENCE OF FITTED HYPERELASTIC COEFFICIENTS

Table IV-1: Hyperelastic Coefficients Derived from Uniaxial, Biaxial and Planar Tension Data

Temperature, °C	Test Rate mm/min	Mooney-Rivlin Model (Polynomial Model N=1)			Temperature, °C	Test Rate mm/min	Ogden Model N=1		
		C ₁₀	C ₀₁	D ₁			μ ₁	α ₁	D ₁
0	1	0.200349	2.19959	0.04812205	0	1	4.93856	-1.58345	0.04812205
0	10	0.220215	2.78504	0.0425855	0	10	6.30085	-1.50249	0.04258554
0	100	-0.127318	4.20562	0.03322993	0	100	8.26636	-1.78433	0.03322993
20	1	0.593886	1.27654	0.04369576	20	1	3.8907	-0.588596	0.04369576
20	10	0.04713647	2.13972	0.03830735	20	10	4.41336	-2.00846	0.03830735
20	100	0.464189	2.09402	0.04479876	20	100	5.24468	-1.11695	0.04479876
40	1	0.48492	1.19717	0.05725425	40	1	3.42073	-0.862854	0.05725425
40	10	-0.264237	2.34155	0.04339303	40	10	4.21392	-2.75528	0.04339303
40	100	-0.07200351	2.26947	0.04748503	40	100	4.3324	-2.32647	0.04748503
80	1	0.05421327	1.61015	ND	80	1	3.22006	-2.06117	ND
80	10	0.351246	1.41496	ND	80	10	3.07103	0.819332	ND
80	100	1.90552	0.01781365	ND	80	100	3.83947	1.51461	ND

ND = D values not determined

Test Rate 1 mm/min is equivalent to strain rate = $3 \times 10^{-4} \text{ s}^{-1}$

Test Rate 10 mm/min is equivalent to strain rate = $3 \times 10^{-3} \text{ s}^{-1}$

Test Rate 100 mm/min is equivalent to strain rate = $3 \times 10^{-2} \text{ s}^{-1}$

Test Rate 250 mm/min is equivalent to strain rate = $8 \times 10^{-2} \text{ s}^{-1}$

Temperature °C	Test Rate mm/min	Ogden Model N=3								
		μ_1	α_1	μ_2	α_2	μ_3	α_3	D ₁	D ₂	D ₃
0	1	-6.81404	1.09847	-12.1032	1.10738	24.0178	0.50824	0.03748	-0.0026020	0.00025480
0	10	139.511	-4.312	-63.8192	-3.39	-68.8882	-5.23078	0.02708	-0.0010080	0.00005578
0	100	19.2296	0.73456	-26.2879	1.5256	15.6146	0.731792	0.02315	-0.0010688	6.52279E-05
20	1	12.1838	-6.75013	0.422261	5.97909	-8.54671	-8.46953	0.03693	-0.0018220	0.0002863
20	10	-154.337	3.07356	64.9845	3.79058	93.8665	2.31595	0.02564	-0.0006035	0.00002112
20	100	-120.452	3.2929	49.1036	4.02554	76.6892	2.50973	0.02680	-0.0009617	0.00005486
40	1	3.75466	1.99859	-1.70592	3.99891	1.38817	-1.99863	0.04699	-0.0009047	0.00001266
40	10	-420.471	5.46762	187.862	6.15823	237.019	4.75029	0.03510	-0.0006963	0.00001115
40	100	-270.066	3.97241	119.115	4.75032	155.353	3.15496	0.03519	-0.0011760	0.00004972
80	1	-48.7902	-4.16796	26.9848	-2.51371	24.8979	-5.83137	ND	ND	ND
80	10	-146.21	-2.24077	72.9338	-0.867979	76.0465	-3.64326	ND	ND	ND
80	100	7.1246	-6.5636	1.29982	6.16433	-4.58614	-8.38323	ND	ND	ND

Temperature °C	Test Rate mm/min	Neo Hooke Model (Reduced Polynomial N=1)		Temperature, °C	Test Rate mm/min	Arruda-Boyce Model		
		C ₁₀	D ₁			μ_1	λ_m	D
0	1	2.33201	0.04812205	0	1	4.60341	7.00384	0.04374112
0	10	2.93727	0.04258554	0	10	5.79770	7.00545	0.03717262
0	100	3.90554	0.03322993	0	100	7.71047	7.00301	0.02984625
20	1	1.84973	0.04369576	20	1	3.65206	7.00267	0.04164969
20	10	2.01968	0.03830735	20	10	3.98671	7.00515	0.03431645
20	100	2.42284	0.04479876	20	100	4.78228	7.00535	0.03879469
40	1	1.66101	0.05725425	40	1	3.28030	7.00058	0.05347369
40	10	1.97944	0.04339303	40	10	3.90863	7.00173	0.04052466
40	100	2.05604	0.04748503	40	100	4.05965	7.00170	0.04322925
80	1	1.56168	ND	80	1	3.08440	7.00018	ND
80	10	1.51565	ND	80	10	2.99348	6.99961	ND
80	100	1.91957	ND	80	100	3.79096	7.00015	ND

Table IV-2: Hyperelastic Coefficients Derived from Uniaxial Tension Data Only

Temperature °C	Test Rate mm/min	Mooney-Rivlin Model (Polynomial Model N=1)			Temperature °C	Test Rate mm/min	Polynomial Model N=2, D values not determined				
		C ₁₀	C ₀₁	D ₁			C ₁₀	C ₀₁	C ₂₀	C ₁₁	C ₀₂
0	1	0.583461	1.72698	0.04812205	0	1	0.061592	2.09149	23.7168	-65.1714	45.4023
0	10	-0.348917	3.46486	0.0425855	0	10	-4.53904	7.78222	15.5906	-46.2232	36.4671
0	100	-1.860430	6.68548	0.03322993	0	100	-22.929	28.829	61.7353	-178.955	140.98
0	250	-1.517590	6.12765	ND	0	250	-9.54877	14.5578	25.0484	-72.2623	56.4649
20	1	0.145894	1.83277	0.04369576	20	1	5.34247	-3.69937	-20.6623	54.8858	-39.4374
20	10	0.140135	2.01861	0.03830735	20	10	4.179	-2.33991	-6.25965	17.609	-14.4393
20	100	-0.258126	3.11229	0.04479876	20	100	-0.853753	3.60662	7.66011	-22.9034	17.528
20	250	-0.287823	3.29897	ND	20	250	4.63122	-2.09354	-3.53395	10.2127	-9.6562
40	1	0.118036	1.68801	0.05725425	40	1	-1.94871	3.78276	49.6386	-125.491	80.9747
40	10	0.408288	1.48056	0.04339303	40	10	3.28831	-1.5559	-11.9892	31.9836	-23.1192
40	100	0.115096	2.12689	0.04748503	40	100	4.9229	-2.98527	-15.2369	40.9501	-30.3074
40	250	-0.120282	2.46347	ND	40	250	4.03829	-2.01847	-3.40254	10.3438	-9.91663
80	1	0.068770	1.62019	ND	80	1	1.81366	-0.166581	-124.89	293.54	-174.135
80	10	0.496183	1.38353	ND	80	10	-2.73146	4.42553	184.177	-466.091	298.128
80	100	0.088845	2.02046	ND	80	100	4.82526	-2.94688	194.809	-455.191	262.952
80	250	0.589446	1.50878	ND	80	250	4.52177	-2.53241	-81.3607	204.134	131.058

Temperature °C	Test Rate mm/min	Ogden Model N=1			Temperature °C	Test Rate mm/min	Ogden Model N=2, D values not determined			
		μ_1	α_1	D_1			μ_1	α_1	μ_2	α_2
0	1	4.39914	0.285304	0.04812205	0	1	11.5805	-1.00373	-7.44293	-3.78193
0	10	6.19226	-2.81551	0.04258554	0	10	-0.663618	2.00016	6.88339	-1.99928
0	100	9.20599	-3.78931	0.03322993	0	100	-1.93972	8.93795	13.5472	-11.3154
0	250	8.87921	-3.62875	ND	0	250	-0.431924	9.91223	10.425	-7.46709
20	1	3.7117	-0.239683	0.04369576	20	1	-5.02871	5.82751	8.53673	4.3817
20	10	4.05489	-0.365622	0.03830735	20	10	-5.38927	4.63175	9.25309	3.38447
20	100	5.51913	-1.89724	0.04479876	20	100	-0.143595	7.77096	5.52388	-0.310178
20	250	5.57988	-1.14142	ND	20	250	-8.56432	4.0022	13.8437	2.93788
40	1	3.58347	-1.3766	0.05725425	40	1	0.30152	1.99997	3.29683	-1.99978
40	10	3.65578	-0.082578	0.04339303	40	10	-4.71793	4.72985	8.2779	3.31225
40	100	4.26222	-0.547635	0.04748503	40	100	-6.36225	5.14997	10.4372	3.81357
40	250	4.43597	-1.11436	ND	40	250	-5.59333	4.20451	9.83429	2.85305
80	1	3.34696	-1.46645	ND	80	1	0.248248	1.99997	3.10989	-1.99978
80	10	3.51102	0.806863	ND	80	10	18.5742	-2.17172	-15.576	-5.53586
80	100	4.20244	-0.120339	ND	80	100	-7.72272	8.52895	11.7119	6.80908
80	250	4.1478	-0.225937	ND	80	250	1.2373	1.99988	2.9455	-1.99983

Temperature, °C	Test Rate mm/min	Ogden Model N=3								
		μ_1	α_1	μ_2	α_2	μ_3	α_3	D ₁	D ₂	D ₃
0	1	10.1879	1.99981	-3.64556	3.99982	-2.34149	-2.00007	0.03748	-0.002602	0.0002548
0	10	0.935026	2.00003	-0.674522	4.00004	5.90512	-2.00027	0.02708	-0.001008	0.00005578
0	100	-16.379	9.48158	17.4935	6.57557	11.2016	-22.4668	0.02315	-0.001069	6.52279E-05
0	250	-11.9545	1.99981	3.71788	3.99988	17.8362	-2.00048	ND	ND	ND
20	1	13.6162	1.99978	-5.62388	3.99977	-4.5572	-2.00015	0.03693	-0.001822	0.0002863
20	10	10.0739	1.99957	-3.85859	3.99959	-2.40054	-2.00018	0.02564	-0.000604	0.00002112
20	100	5.49832	2.00004	-2.32164	4.00003	2.19991	-2.00003	0.02680	-0.000962	0.00005486
20	250	8.12891	-9.29714	2.60533	12.8077	-6.19171	24.998	ND	ND	ND
40	1	2.33049	2.00000	-0.998274	4.00000	2.23449	-2.00001	0.04699	-0.000905	0.00001266
40	10	9.0974	1.99992	-3.74619	3.99993	-1.81199	-2.00003	0.03510	-0.000696	0.00001115
40	100	13.0293	1.99983	-5.52373	3.99983	-3.47929	-2.00009	0.03519	-0.001176	0.00004972
40	250	9.71403	1.99965	-4.00306	3.99966	-1.50066	-2.00009	ND	ND	ND
80	1	4.75171	2.00002	-2.31473	4.00001	0.868565	-2.00001	ND	ND	ND
80	10	32.438	2.00011	-14.9803	4.00011	-14.3773	-1.99988	ND	ND	ND
80	100	34.6159	1.99987	-17.0985	3.99986	-13.6571	-2.00011	ND	ND	ND
80	250	3.17207	2.00002	-0.933243	4.00001	1.91019	-2.00002	ND	ND	ND

Temperature, °C	Test Rate mm/min	Reduced Polynomial Model N=2 D values not determined		Temperature °C	Test Rate mm/min	Yeoh Model (Reduced Polynomial Model N=3) D values not determined		
		C ₁₀	C ₂₀			C ₁₀	C ₂₀	C ₃₀
0	1	2.12363	-0.390843	0	1	2.12144	-0.37418	-0.022062
0	10	2.78503	-0.836659	0	10	2.88034	-1.49454	0.846079
0	100	4.26667	-1.86237	0	100	4.58356	-4.34391	3.81886
0	250	4.08742	-1.69198	0	250	4.32211	-3.58304	2.93243
20	1	1.78877	-0.456465	20	1	1.76729	-0.258406	-0.316704
20	10	1.92965	-0.427437	20	10	1.92888	-0.422255	-0.006180
20	100	2.51578	-0.651931	20	100	2.577	-1.00929	0.384953
20	250	2.59563	-0.593318	20	250	2.61532	-0.692689	9.109084
40	1	1.69819	-0.688211	40	1	1.72679	-1.16647	1.43218
40	10	1.76748	-0.479049	40	10	1.77327	-0.554689	0.172829
80	1	1.59869	-0.795219	80	1	1.61156	-1.09423	1.24083

Temperature °C	Test Rate mm/min	Neo Hooke Model (Reduced Polynomial N=1)		Temperature, °C	Test Rate mm/min	Arruda-Boyce Model		
		C ₁₀	D ₁			μ ₁	λ _m	D
0	1	1.94834	0.04812205	0	1	3.84482	7.00089	0.04374112
0	10	2.38004	0.04258554	0	10	4.69621	7.00152	0.03717262
0	100	3.49679	0.03322993	0	100	6.90078	7.00132	0.02984625
0	250	3.39715	ND	0	250	6.70424	7.00126	ND
20	1	1.61774	0.04369576	20	1	3.19293	7.00076	0.04164969
20	10	1.71135	0.03830735	20	10	3.37662	7.00141	0.03431645
20	100	2.13626	0.04479876	20	100	4.21424	7.0021	0.03879469
20	250	2.18585	ND	20	250	4.31096	7.0029	ND
40	1	1.56243	0.05725425	40	1	3.08498	7.00023	0.05347369
40	10	1.64474	0.04339303	40	10	3.24709	7.00031	0.04052466
40	100	1.84538	0.04748503	40	100	3.64252	7.00064	0.04322925
80	1	1.48655	ND	80	1	2.93553	7.00012	ND
80	10	1.66305	ND	80	10	3.28375	7.00014	ND
80	100	1.94484	ND	80	100	3.84055	7.0001	ND

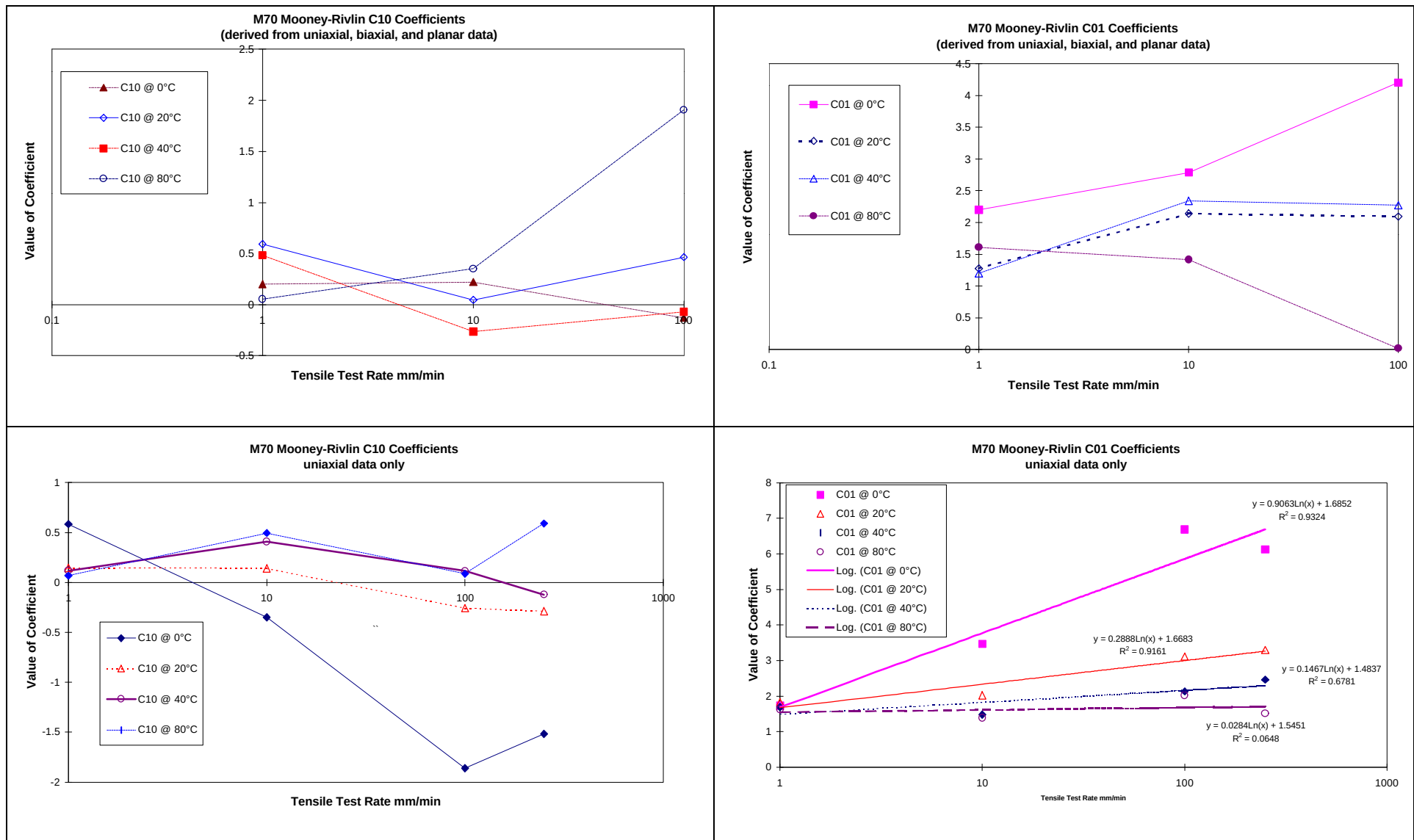


Figure IV-1: Rate and Temperature Dependence of Mooney-Rivlin Coefficients

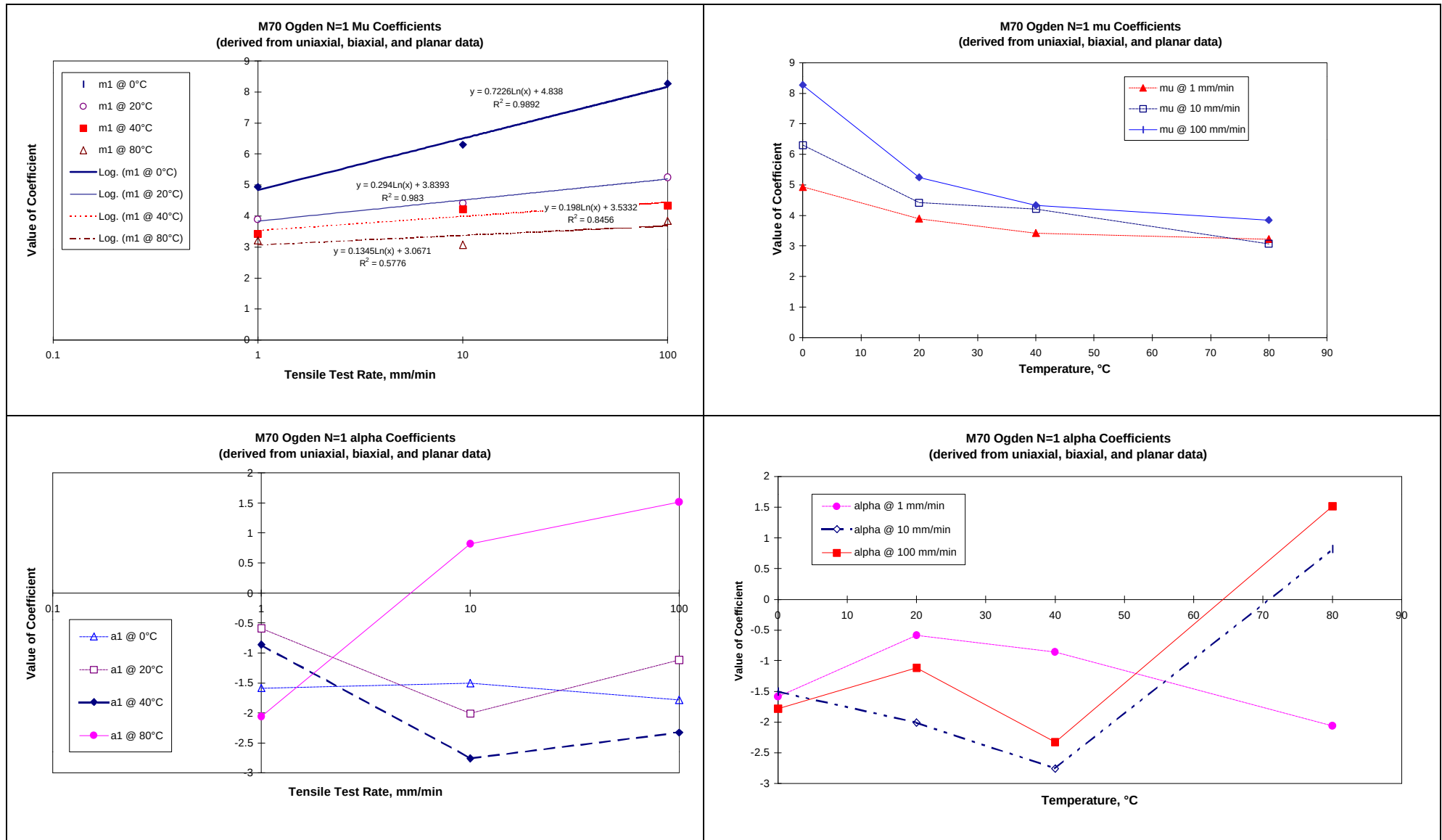


Figure IV-2: Rate and Temperature Dependence of Ogden, N=1 Coefficients

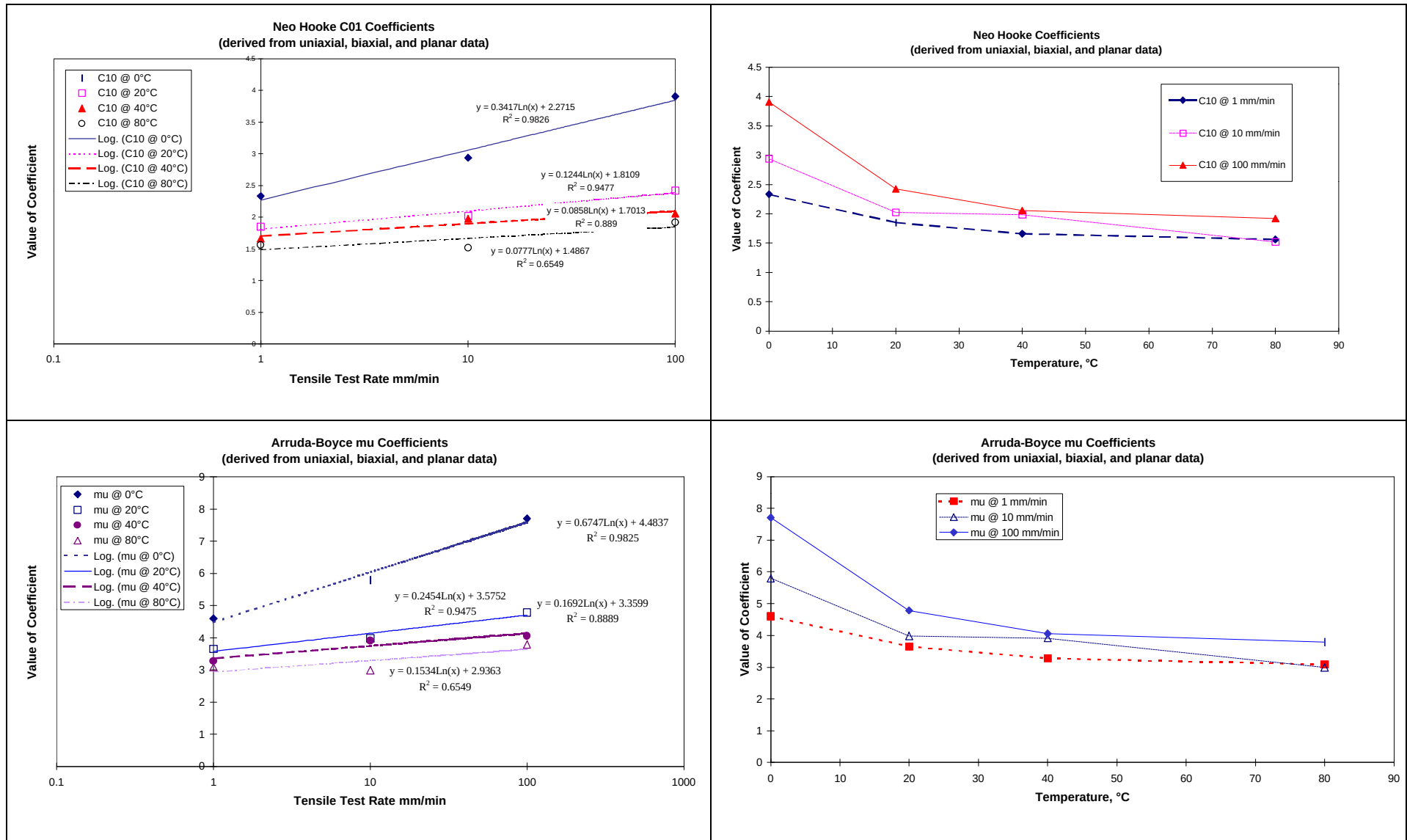


Figure IV-3: Rate and Temperature Dependence of Neo Hooke and Arruda-Boyce Coefficients