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The removal of the effects of multiple reflections in antenna extrapolation data by digital filtering.

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ABSTRACT

The oscillations in the antenna extrapolation measurement data due to multiple reflections between the antennas have proved difficult to fit accurately by the least squares procedure. This report describes a method for digitally filtering the data prior to fitting to remove these oscillations. A Pascal program has been written to perform the filtering and the process has been shown to be effective.

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1 Introduction.

The three antenna extrapolation technique (Newell *et al.*, 1973) is used at NPL to measure the gain of microwave antennas. The method involves measuring the power transmission between pairs of antennas as a function of distance and extrapolating the measurements to infinite distance. Not all of the radiation incident on the receiving antenna is accepted by it; some is scattered by the antenna and its mount and is then further scattered by the transmitting antenna and its mount. These multiple reflections between the antennas give rise to oscillations in the measurement data as a function of the separation (figure 1), which must be modelled or removed.

A least squares fitting procedure, which is described in two DES reports (Borland, 1990), is used to obtain the best fit to the measured data. Until recently, the expression which was fitted included the first order multiple reflections terms (one reflection by each of the receiving and transmitting antennas) in the following form

$$Pd^2 = A_1 + A_2x + A_3x^2 + B_1x^2 \sin(2kd) + B_2x^2 \cos(2kd) + \delta \quad (1.1)$$

where P is the ratio of the received power to the transmitted power, d is the antenna separation, $x = 1/d$, $k = 2\pi/\lambda$ and δ is the fitting error. The terms with coefficients B_1 and B_2 were used to account for the multiple reflections between the antennas. We found that the expression did not fit the data very well in two respects: i) the phase of the oscillations in the measured data drifted from an exact dependence on $2kd$ and ii) the amplitude of the oscillations decayed less rapidly than $x = 1/d^2$. These shortcomings are due to the approximate expression (1.1) being used to represent the infinite series obtained in a theoretical analysis by P. F. Wacker (1972), which accurately describes the physical situation.

A method for removing the effects of the first order (and to some extent higher order) multiple reflections by digital filtering has been developed, which is the subject of this report. It should be stressed that neither the digital filtering nor the least squares process takes account of the multipath effect which is due to radiation reflected from the surroundings. It would be very difficult to model this effect mathematically. However, it is minimised by performing the measurements inside an anechoic chamber and restricting the maximum separation between the antennas.

2 Outline of the filtering process.

Although there is a phase drift over the complete range, the first order multiple reflection terms from equation (1.1) may be closely fitted locally by the function $F(d)$ where

$$F(d) = g(d) \Re [\exp i(2kd + \epsilon)]$$

$\Re$ indicates the real part, $g(d)$ is a real amplitude function which gives the amplitude of the oscillation for antenna separation, d , and ϵ is an arbitrary constant phase offset. For a particular measurement point, j , we may rewrite this as

$$F_j = g_j \Re \exp(i\phi_j)$$

where $\phi_j = 2kd_j + \varepsilon$. Each F_j in this local region, which we normally take to extend over approximately one cycle, is multiplied by a real coefficient, β_j , chosen so that

$$\sum_{j=1}^n \beta_j F_j = 0 \quad (2.3)$$

where n is the number of measurements in the cycle. To perform the filtering, the fitting function for each point in the cycle is multiplied by the appropriate coefficient, β_j ,

$$\beta_j P_j d_j^2 = A_1 \beta_j + A_2 \beta_j x_j + A_3 \beta_j x_j^2 + \beta_j F_j + \beta_j \delta_j \quad (2.4)$$

The equations for one cycle are then added together so that the oscillatory terms are annihilated, giving

$$\sum_{j=1}^n \beta_j P_j d_j^2 = A_1 \sum_{j=1}^n \beta_j + A_2 \sum_{j=1}^n \beta_j x_j + A_3 \sum_{j=1}^n \beta_j x_j^2 + \sum_{j=1}^n \beta_j \delta_j \quad (2.5)$$

The above process is repeated for every cycle in the data, each one producing an equation of the above form. The least squares process is then used to find the optimum values of A_1 , A_2 and A_3 by minimising the sum of the squares of $\sum_{j=1}^n \beta_j \delta_j$

3 Determining the coefficients.

In general, the absolute phase of a point in a cycle will not be known, because the value of ε will not be known and therefore nor will the value of F_j . However, by applying the following more general condition, equation (2.3) is automatically satisfied regardless of the value of ε .

$$\sum_{j=1}^n \beta_j g_j \exp(i\phi_j) = 0 \quad (3.1)$$

Since $\exp(i\varepsilon)$ is a non-zero constant, this can be satisfied only if

$$\sum_{j=1}^n \beta_j g_j \exp(i2kd_j) = 0 \quad (3.2)$$

Now let $g_j \exp(i2kd_j) = U_j + iV_j$. Then, if $g_j = x_j^2$

$$U_j = x_j^2 \cos(2kd_j) \quad (3.3)$$

$$V_j = x_j^2 \sin(2kd_j) \quad (3.4)$$

Note that the values of U_j and V_j are completely determined by the wavelength and the

antenna separation. Equation (3.2) provides two conditions for the coefficients, β_j , to fulfil.

$$\sum_{j=1}^n \beta_j u_j = 0$$

$$\sum_{j=1}^n \beta_j v_j = 0$$

These conditions are not sufficient to determine the β_j s uniquely. There could, for example, be a constant scaling factor and also some of the coefficients could be negative while others are positive. This would be very undesirable because, when the equations represented by (2.4) are added together, there would be some cancellation of the wanted terms in addition to the annihilation of the oscillatory terms. This would produce an increase in the percentage error in the data as the random errors would not be reduced by the cancellation. The coefficients must therefore be chosen so as to maximise the wanted term, A_1 . This can be achieved by maximising the sum of the β_j s. This must be carried out subject to the conditions of equations (3.5) and (3.6) and also the following normalisation condition.

$$\sum_{j=1}^n \beta_j^2 = 1$$

This normalisation condition not only sets the scaling factor, but also ensures that the variance for each filtered cycle is the same so that an unweighted least squares process can be used to fit the filtered data.

To maximise the sum of the β_j s for each cycle subject to these three constraints, the method of Lagrange Multipliers may be used, which is described in the book by Stephenson (1973). The function to be maximised becomes

$$H = \sum_{j=1}^n \beta_j - \lambda_1 \sum_{j=1}^n \beta_j u_j - \lambda_2 \sum_{j=1}^n \beta_j v_j - \lambda_3 \left\{ \sum_{j=1}^n \beta_j^2 - 1 \right\}$$

where λ_1 , λ_2 and λ_3 are the Lagrange multipliers corresponding to the three constraints. This function is then partially differentiated with respect to each β_j and the differentials are set equal to zero to find the maximum.

$$\frac{\partial H}{\partial \beta_j} = 1 - \lambda_1 u_j - \lambda_2 v_j - 2\lambda_3 \beta_j = 0 \quad \text{for } j = 1 \dots n$$

This gives n equations of the form

$$\beta_j = \frac{1}{2\lambda_3} (1 - \lambda_1 u_j - \lambda_2 v_j) \quad \text{for } j = 1 \dots n \quad (3.10)$$

These n equations together with the three constraining equations are sufficient to determine the β_j s uniquely. Expressions for λ_1 , λ_2 and λ_3 are derived in appendix A.

4 A program to filter the data.

A Pascal program has been written to perform the filtering of the data which consists of the power ratio, P , and corresponding antenna separation, d , at each frequency. Each cycle of the data for each frequency is filtered sequentially and the following sums are recorded in an output file for least squares fitting.

$$\sum_{j=1}^n \beta_j \quad \sum_{j=1}^n \beta_j x_j \quad \sum_{j=1}^n \beta_j x_j^2 \quad \sum_{j=1}^n \beta_j P_j d_j^2$$

In general, antennas are calibrated at a number of frequencies, sometimes as many as 120. To maximise the efficiency of the measurement process, the power ratio is measured at each frequency for a given antenna separation before moving to the next position. Thus it is not possible to arrange the measurement positions so that there are an integral number of points per cycle of the oscillations at all frequencies. This creates a problem when deciding the number of points to filter in any particular cycle. Figure 2 shows one cycle of the oscillatory term plotted in an Argand diagram (no attempt has been made to represent the variation in amplitude with d as this is likely to be very small for one cycle). Note that the points need not be equally spaced, provided their positions are accurately known, although the uncertainty is improved if they are. The data points are read from the input file up to, but not including, the point for which the phase, $2kd$, has advanced by 2π . The data are filtered twice, once using all the points and on the second occasion the final point is omitted. The aim of the filtering is to annihilate the oscillatory terms while maximising the wanted terms which is achieved by maximising the sum of the coefficients, β_j . To determine the optimum number of points, the value of the following sum is compared for the two cases.

$$\sum_{j=1}^n \frac{\beta_j}{\sqrt{n}}$$

The number of points, n , which gives the greatest value for the above sum is the number of points used for that cycle. The justification for this criterion is given in appendix B. If the final point is not required, it is used as the first point for the next cycle, otherwise the next point in the data file is used. Thus, in Figure 2, the point corresponding to $j = 10$ may be the final point of the cycle shown or the first point of the next cycle. There may not be sufficient data points to complete the final cycle for a particular frequency. However, such partial cycles may still be filtered, but the coefficients are checked to ensure they are all positive - if not, the corresponding data points are discarded.

In principle, it is necessary for there to be only three points per cycle to perform the filtering (two points per cycle could be used provided they are precisely in antiphase). In practice, at least six points per cycle are probably desirable, as a certain degree of filtering of the higher order multiple reflection terms will then be achieved. One might believe, mistakenly, that by filtering a number of data points to produce a single equation, some of the data is "lost", but because the variance of the sample is unchanged as a result of the normalisation condition used and the wanted part of the equation increases as $\sqrt{n}$, the percentage error in the filtered equation is reduced by a factor $1/\sqrt{n}$. In fact, it would be a mistake to re-use data points by performing a rolling filtering in an attempt to preserve the number of data points as this would produce correlated errors.

5 Graphical representation of the filtered equations.

A fundamental aspect of the filtering process which has been described is that the filtering is performed on the fitting equations that the data points represent in the least squares procedure, not on the data points themselves. This is essential, because the fitting function is not linear, but it poses a problem if one wishes to represent the filtered data in graphical form. Equation (2.5) gives the form of the filtered equations. Dividing equation (2.5) by the sum of the coefficients, β_j , one obtains

$$\frac{\sum \beta_j P_j d_j^2}{\sum \beta_j} = A_1 + A_2 \frac{\sum \beta_j x_j}{\sum \beta_j} + A_3 \frac{\sum \beta_j x_j^2}{\sum \beta_j} + \frac{\sum \beta_j \delta_j}{\sum \beta_j} \quad (5.1)$$

The left hand side of the equation poses no problem; it is a weighted average over the cycle of Pd^2 , which we shall denote by $\langle Pd^2 \rangle$. However, there is a difficulty in deciding on a corresponding value of x , which we shall denote by $\bar{x}$. It would not be satisfactory, for example, to take the average value of x , which is the coefficient of A_2 in equation (5.1). The method we have used for graphical representation is as follows:-

After obtaining the values of A_1 , A_2 and A_3 by least squares fitting over the whole range, we can determine the error, $\langle \delta \rangle$, in $\langle Pd^2 \rangle$ for each cycle. From equation (5.1), it is given by

$$\langle \delta \rangle = \frac{\sum \beta_j P_j d_j^2}{\sum \beta_j} - \langle Pd^2 \rangle - A_1 - A_2 \frac{\sum \beta_j x_j}{\sum \beta_j} - A_3 \frac{\sum \beta_j x_j^2}{\sum \beta_j}$$

Thus an "error-free" value of Pd^2 is given by $\langle Pd^2 \rangle - \langle \delta \rangle$ and from this we can determine the value of $\bar{x}$ by solving the quadratic equation

$$\langle Pd^2 \rangle - \langle \delta \rangle = A_1 + A_2 \bar{x} + A_3 \bar{x}^2$$

In the absence of any errors, the set of data pairs ($\langle Pd^2 \rangle, \bar{x}$) would yield the same A_1 , A_2 and A_3 and thus give the correct curve. The effect of the error, $\langle \delta \rangle$, would cause some departure from the correct curve. Purely random errors will be reduced by the weighted averaging, whereas correlated errors extending over more than one cycle, such as from floor reflections, would still be apparent in the graphical representation. Figure 3 shows the result of applying this procedure to the data of Figure 1.

6 Concluding comments.

The digital filtering described in this report is a versatile way to remove the effects of multiple reflections from the measurement data prior to curve fitting. It can cope with the general case where there is a non-integral number of points per cycle and these points are not equally spaced. The program which performs the filtering has been tested extensively and has produced excellent results.

A $1/d^2$ form has been assumed for the amplitude function, $g(d)$, in equations (3.3) and (3.4) in accordance with the oscillatory terms in equation (1.1), although the amplitude has been found to decay less rapidly than this. However, because each cycle is filtered independently of the others and the change in amplitude over one cycle is likely to be small, the exact dependence of the amplitude on antenna separation is not critical. This is not true when the least squares method is used to fit the oscillatory terms, as in that case the amplitude taper is assumed to be continuous throughout the data. The same arguments are true of the phase variation with antenna separation; since the digital filtering treats each cycle separately, any difference between the assumed phase and the actual phase will always be small.

The description of the filtering process given here has assumed that a three term polynomial is used to fit the filtered data as this has been found to be the optimum number in most circumstances. However, it is a simple matter to extend the treatment to cater for the case of a four term polynomial when conditions dictate that this would be the optimum number to use.

Acknowledgement.

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Appendix A - Calculation of λ_1 , λ_2 and λ_3

To avoid an excessive use of subscripts and superscripts in the equations that follow, the sum $\sum_{j=1}^n$ is represented by the symbol $\sum$

If equation (3.10) is used to substitute for β_j in equations (3.5) and (3.6), one can show that

$$\lambda_1 \sum u_j^2 + \lambda_2 \sum u_j v_j = \sum u_j \quad (\text{a.1})$$

$$\lambda_1 \sum u_j v_j + \lambda_2 \sum v_j^2 = \sum v_j \quad (\text{a.2})$$

Let $A_{11} = \sum u_j^2$, $A_{12} = \sum u_j v_j$, $A_{22} = \sum v_j^2$, $B_1 = \sum u_j$ and $B_2 = \sum v_j$. Then equations (a.1) and (a.2) may be represented in matrix form by

$$\begin{pmatrix} A_{11} & A_{12} \\ A_{12} & A_{22} \end{pmatrix} \begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix} = \begin{pmatrix} B_1 \\ B_2 \end{pmatrix} \quad (\text{a.3})$$

Simple matrix methods may then be used to solve for λ_1 , λ_2 giving

$$\lambda_1 = (A_{22}B_1 - A_{12}B_2) / D \quad (\text{a.4})$$

$$\lambda_2 = (A_{11}B_2 - A_{12}B_1) / D \quad (\text{a.5})$$

where $D = (A_{11}A_{22} - A_{12}^2)$, the determinant of the first matrix in equation (a.3).

To obtain λ_3 we observe that the normalisation condition (3.7) can be rewritten, with reference to equation (3.10), as

$$1 = \sum \left| \right.$$

Using equations (3.5) and (3.6), this gives

$$1 = \frac{1}{2\lambda_3} \sum \beta_j = \frac{1}{4\lambda_3^2} \sum (1 - \lambda_1 u_j - \lambda_2 v_j) \quad (\text{a.7})$$

Hence λ_3 may be expressed in terms of λ_1 and λ_2 as

$$\lambda_3 = \frac{1}{2} (n - \lambda_1 B_1 - \lambda_2 B_2)^{1/2} \quad (\text{a.8})$$

Appendix B - Choice of the last point used in a cycle.

Consider a cycle for which all the coefficients, β_j , are the same; this would be the case if the amplitude of the oscillatory terms were constant, the points were equally spaced and there were an integral number of points per cycle. In that case, using the normalisation condition (equation 3.7), the value of each coefficient would be

$$\beta_j = \frac{1}{\sqrt{n}} \quad (\text{b.1})$$

Hence

$$\sum_{j=1}^n \beta_j = \sqrt{n} \quad (\text{b.2})$$

Thus the sum of the coefficients varies as the square root of the number points in the cycle when the coefficients have equal magnitude. This will be approximately true for the general case where there is a slight amplitude taper, the points are nearly equally spaced and there is a non-integral number of points per cycle. Therefore, when comparing sums with different numbers of points, one must divide by $\sqrt{n}$.

$Pd^2 \times (4\pi/\lambda)^2$ for two WG18 antennas at 15 GHz

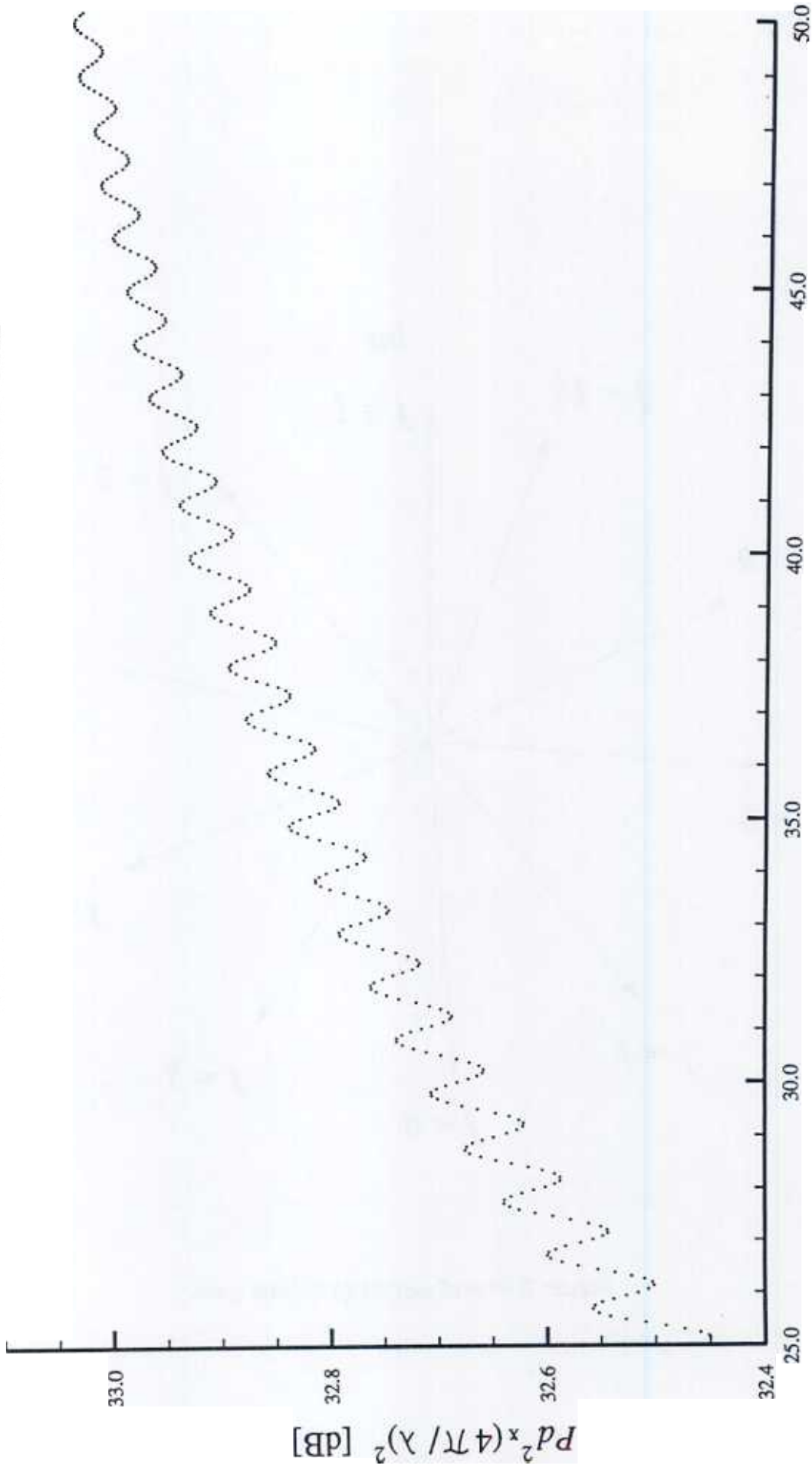
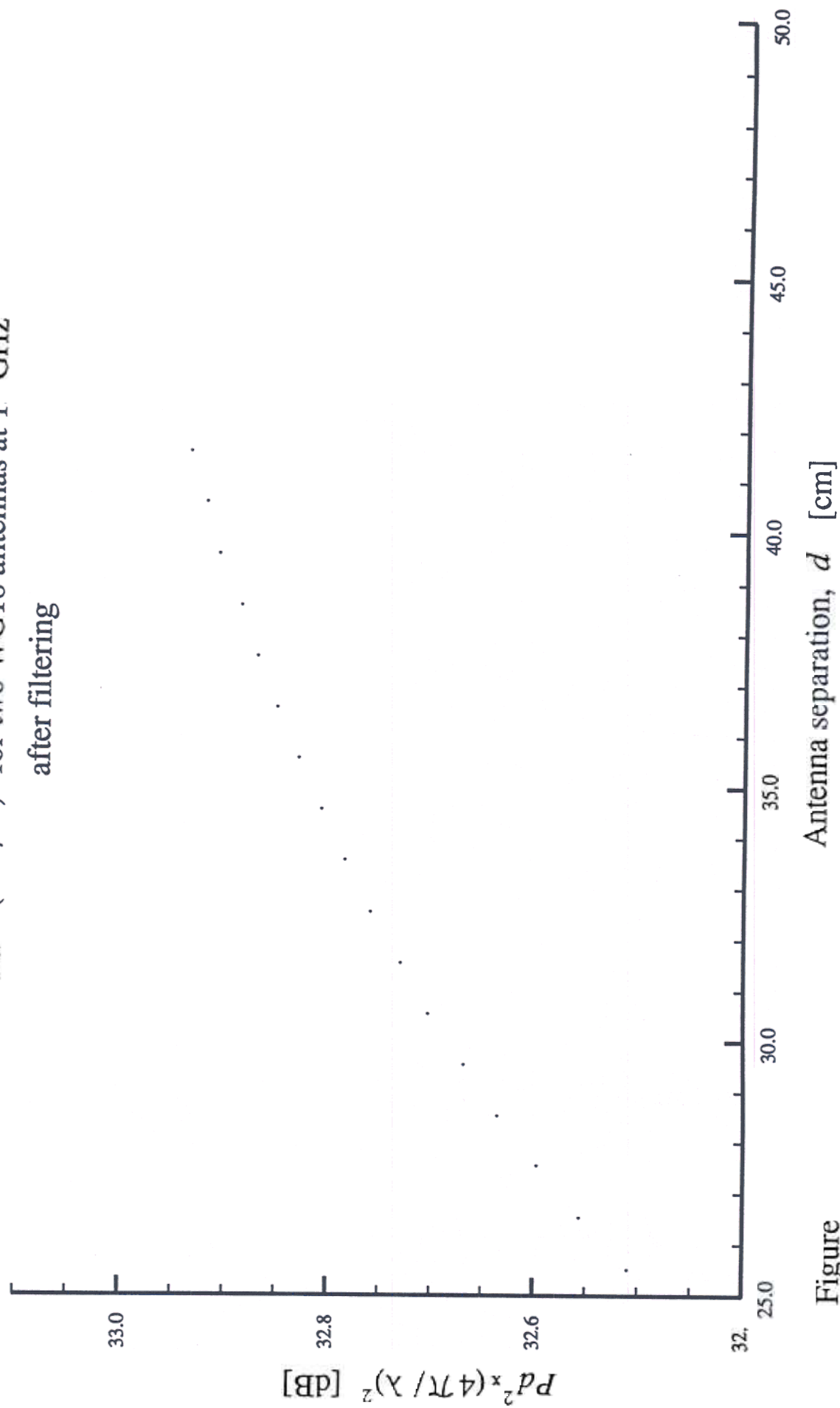


Figure 1.

$Pd \ (4\pi/\lambda)^2$ for two WG18 antennas at 1 GHz
after filtering



Figure

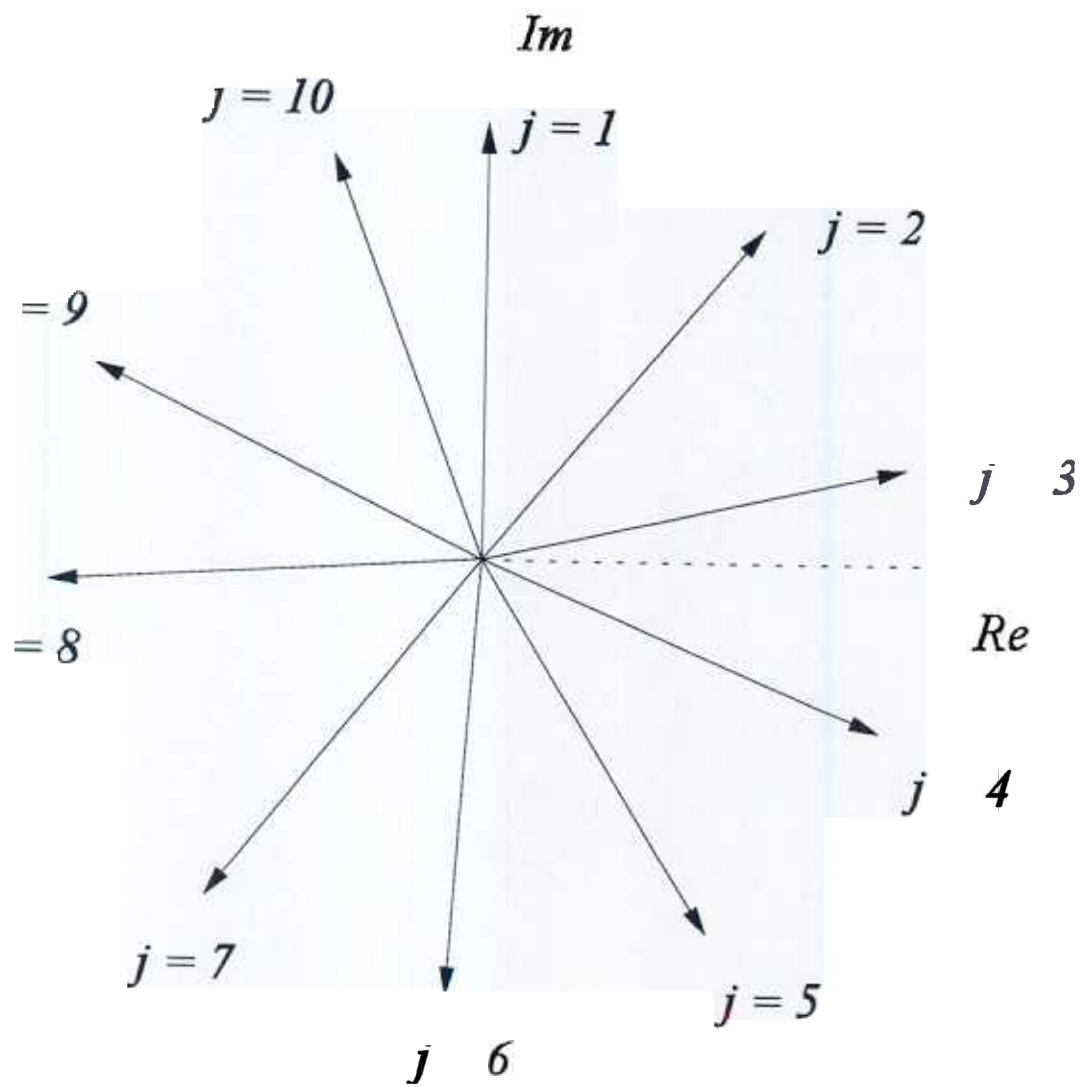


Figure 2. Plot of $\exp(i2kd_j)$ for one cycle.

