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GENERAL PRINCIPLES FOR EVALUATING AND EXPRESSING UNCERTAINTY IN MEASUREMENT

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Abstract

In recent years, the process of assessing the uncertainty associated with a measurement has itself become thwarted by uncertainty! This has been due partly to the apparently incompatible practices prescribed to evaluate uncertainty, reported in guidance documents, standards and published papers. This paper discusses general principles common to most of these documents and attempts to unify at least some of the basic concepts. The discussion is accompanied throughout by a simple example.

Introduction

Measurement attempts to determine the true value of a characteristic for a device. Since all measurement processes are affected by errors, some degree of error will inevitably be present in the measurement result, making it an approximation of the true value. An important part of an overall measurement process is the identification of all the contributing errors and, where possible, making the necessary corrections. Where corrections are not possible, the effect the remaining errors have on the measurement result needs to be evaluated. This is the uncertainty evaluation process for the measurement. The resulting value for the measurement uncertainty indicates quantitatively the doubt about the accuracy of the result. This allows the user of the result to assess its reliability and enables meaningful comparison with other results.

This paper discusses the general principles involved in evaluating and expressing uncertainty in measurement. Each stage of the process is discussed, including identifying, evaluating and combining the uncertainty contributions, and expressing the overall uncertainty. An example is used to illustrate each step of the process. This example concerns a measurement of a nominal 100 Ω resistor made using an impedance bridge operating at 250 MHz. The example is typical and kept simple for clarity.

Understanding the measurement system and identifying uncertainty contributions

One of the most important aspects of uncertainty evaluation is the need for a detailed understanding of the overall measurement process. Such understanding is required to ensure

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all potential contributions to the measurement uncertainty are identified. This means the design engineer or skilled operator of the measurement system is often best suited to perform the evaluation exercise.

The exercise begins by examining in detail the measurement process and identifying potential sources of error. This often involves representing the measurement system using a variety of means, including flow diagrams, computer simulations etc. In our example, the bridge was represented as a block diagram which revealed the following four sources of error; (i) the connection repeatability of the resistor to the bridge, (ii) the electrical noise present on the signals detected by the bridge, (iii) the uncertainty accompanying the transfer standard used to calibrate the bridge, and (iv) the accuracy of the bridge's frequency source.

Identified errors should be corrected, where possible, *e.g.*, by performing an instrument calibration. Where correction is either not possible or incomplete, the remaining error should be treated as an uncertainty contribution. In our example, all the identified errors remain after calibration, so they are treated as uncertainty contributions.

Quantifying uncertainty contributions and their effect on the final result

Having identified all the contributions to the uncertainty in the measurement process, each contribution requires evaluation. This means either estimating the limits within which the value for a contribution will be, or estimating the range over which it might vary. The most common statistic used to estimate this range is standard deviation. In our example, the uncertainty associated with the transfer standard (a coaxial capacitor) used to calibrate the bridge can be taken from its calibration certificate, giving us a value of ± 0.28 pF. This range, as with the ranges for the other contributions, corresponds to one standard deviation. The frequency source accuracy was found in the manufacturer's specification for the source, giving us a value of ± 10 Hz.

The next step is to determine the effect each contribution has on the final measurement result. This can be a difficult process involving a variety of methods, often involving experimentation or based on experience. In our example, the contribution from the transfer standard can be assessed by calibrating the bridge under two conditions; firstly using a value for the transfer standard given on its calibration certificate, then calibrating again using this value *plus* its uncertainty. The difference between the two subsequent resistance measurements made by the bridge is the uncertainty contribution, which was found to be ± 0.26 Ω . The same technique can be used for the contribution from the accuracy of the frequency source - it was found to be ± 0.019 Ω . Note that these contributions are in the same units as the final measurement result, *i.e.*, ohms.

Sometimes the two steps given above can be combined, *i.e.*, variations in the contribution can be monitored simultaneously with the effects the variations have on the measurement result. In our example, this can be done for the contribution due to electrical noise by measuring the item repeatedly under essentially the same conditions. Alternatively, the electrical noise and connection repeatability contributions can be evaluated simultaneously using a similar method, but disconnecting and re-connecting the resistor between repeat measurements. The contribution due to connector repeatability and electrical noise, using this technique, was found to be ± 0.097 Ω .

Combining the contributions and expressing overall uncertainty

Having identified each contribution and its effect on the measurement result, we need to combine the contributions to give an overall uncertainty figure. In most cases, this can be accomplished by; (i) summing the squared standard deviations of the contributions, (ii) taking the square-root of this sum, and (iii) multiplying the square-root by a coverage factor. The coverage factor is usually chosen to give the required confidence in the result, *e.g.*, for normal distributions, a value of 2 corresponds approximately to a probability of 95% that the uncertainty interval will include the true value (or in other words, a one-in-twenty chance that it won't!). If greater confidence is required, a larger coverage factor can be used, *e.g.*, a value of 3 corresponds approximately to a probability of 99% for the same distribution.

In our example we have;

$$k [(0.26)^2 + (0.019)^2 + (0.097)^2]^{1/2} \Omega.$$

where k is the coverage factor. If we use $k = 2$, then the overall uncertainty for the resistance measurement is $\pm 0.56 \Omega$.

Discussion

An uncertainty evaluation process can be extremely complicated or very simple. For measurement systems whose accuracy is governed predominantly by the transfer standard used for its calibration, this can be the only contribution requiring consideration. Our example illustrates this. If we re-calculate the overall uncertainty omitting the uncertainty due to source frequency accuracy, the answer is still $\pm 0.56 \Omega$. Clearly, this contribution can be ignored. Indeed, if we were to leave out both the frequency accuracy contribution *and* the electrical noise/connection repeatability contributions, the calculated overall uncertainty would only be reduced to $\pm 0.52 \Omega$. This is equivalent to assuming all uncertainty in the measurement is due to the uncertainty accompanying the transfer standard used to calibrate the bridge.

The reduced uncertainty produced by omitting contributions can often be compensated for by reducing the number of significant figures used to express the overall uncertainty and rounding the number pessimistically, *i.e.*, by rounding *up*. This would cause our value of $\pm 0.52 \Omega$ to be rounded up to $\pm 0.6 \Omega$. Successful identification of components whose contribution to the overall uncertainty is negligible can help reduce considerably the complexity of the uncertainty evaluation process. It is important however, to demonstrate that a given contribution has a negligible effect on a result before omitting it from the calculations.

Summary

This paper discussed some of the more general principles one might encounter during a typical uncertainty evaluation process. Most of the guidelines currently available conform, broadly speaking, to these general principles. Departures will inevitably occur, both in

written standards and practical aspects of uncertainty evaluation, but these departures are a consequence of the wide variety of situations to which these principles are being applied.

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