

Analysis of Plate-Twist Test for In-Plane Shear Modulus of Composite Materials

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SUMMARY

In this report an improved analysis of the plate-twist test for the measurement of in-plane shear modulus of composite materials and the shear modulus of isotropic materials is described and experimentally validated. The analysis removes errors connected with the position of the loading points and allows greater freedom in their positioning. In particular it is no longer necessary to position the loading points exactly on the corners of the plate. An error of 1.0 mm in the positioning of the loading points on a 150 mm square plate results in an error of 3.4% in the measured shear modulus.

The main advantages of this test method are the simple flexure mode of loading, the simple specimen preparation, the large displacements at low loads (allowing use of crosshead movement to measure displacement), the ease of use at elevated or reduced temperatures and the uniform shear stress distribution. The method is not suitable for the measurement of shear strength. The test has been applied to a range of materials, with recorded shear modulus values ranging from 0.3 GPa to 88 GPa.

The method also has advantages in its use of a plate specimen for both injection moulded material, where it is more representative of a moulded product, and for advanced prepreg laminates, where a test panel is frequently fabricated for preparing the test specimens. In both cases the low applied strains suggest the plate can be re-used in further tests. The test method is also shown to be suitable for isotropic sheet materials.

A draft standard has been produced for this test method, which will be used in a round-robin exercise to obtain precision data. The existing standards for the use of this method to measure the shear modulus of wood products, will be checked, as they may have similar sources of error to those evaluated in this report.

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1 INTRODUCTION

The use of composite materials in more highly stressed applications requires a fully comprehensive property database, together with supporting test methods. An elastic property not well supported by international standard test methods is the in-plane shear modulus, G_{12} . This property is important for composite materials as it can be relatively low, compared to the Young's modulus along the fibre direction for a unidirectionally reinforced composite, leading to poor torsional stiffness and buckling performance. Unless specifically optimised by orientating the fibre directions within the laminate to increase the shear stiffness, the shear modulus would be lower than expected if an "isotropic" material value was assumed (ie based on E_{11} and ν_{12}). Although the plate-twist test is not suitable for the measurement of shear strengths, many design situations result in buckling failures of components, controlled by the shear modulus, rather than by an in-plane shear failure.

In reference [1] results were reported for G_{12} from the plate-twist method using the analysis given in reference [2]. This analysis assumes that the loading points are exactly on the plate corners. In practice this is difficult to achieve, at best it is time consuming and at worst the loading points are actually in-board of the corners. In this current report the refinement of the plate-twist test analysis to remove errors connected with the positioning of the loading points is reported. The method is suitable for both anisotropic and isotropic materials. In the latter case it provides a check against the value derived from the Young's Modulus and Poisson's ratio.

There are advantages in the use of a plate specimen, particularly for moulded plastics and reinforced plastics as these specimens more closely represent the properties achieved in actual products. This is in contrast to the optimised properties often obtained in moulded-to-size dumbbells and flexure beams due to molecular or fibre alignment along the specimen axis, although this structure may also be possible in plate specimens when suitably gated. These plates can also be cut into coupons in perpendicular directions to assess the anisotropy present using conventional axial tests. Plate specimens are well-suited to tests on prepreg materials as a test panel will normally be prepared as the first step in fabricating coupon specimens.

Shear test methods are reviewed in Section 2 with detailed discussion on the plate-twist test given in Section 3. Section 4 presents an improved analysis to allow for the actual loading configuration and calculates correction factors for the case when the loading position is not at the corners as assumed in the existing analysis^[2]. In Section 5 the results of an extensive validation of the new analysis are reported using a range of both loading point positions and materials. The concluding comments are given in Section 6.

2 TEST METHODS FOR IN-PLANE SHEAR MODULUS

A wide range of techniques has been proposed^[3] for the measurement of the in-plane shear modulus of composites. Several of the composites methods need special specimen preparation (eg Iosipescu^[4], rail^[5], scissor shear^[6]) or have a non-uniform shear stress distribution (eg scissor shear, Iosipescu). Other methods are only applicable to particular fibre arrangements (eg $\pm 45^\circ$ tension^[7], 10° off-axis tension). The often recommended torsion test of thin-walled tubes or solid rods requires an axial twisting facility, and is only suitable for that form of specimen; and although a torsion gearbox is available for attachment to the universal test frame of a major manufacturer it is a rarely supplied option.

The plate-twist method, also reviewed in^[3], has some advantages. These include the use of a simple "flexural" type loading jig, simple specimen preparation and a uniform shear stress distribution over the area of the plate except at the loading points where the local effects should be minimal as relatively large displacements are obtained for small applied loads.

Providing the plate length to thickness ratio is sufficiently large (eg greater than 35:1) then the through-thickness shear modulus, G_{13} , has negligible influence on the measured in-plane shear modulus. The plate-twist method is not suitable for the measurement of shear strength. Equally the test is not suitable for materials that are not transversely isotropic or homogeneous through the section (eg multidirectional laminates) because the in-plane shear modulus developed under the flexural loading is no longer equal to the value obtained from pure in-plane shear loading^[8]. However, the result is of value for checking the prediction from laminate analysis packages for an applied bending load.

Plate specimens are increasingly available from test panels as prepared using ISO 1268^[9] or as moulded manufacturers samples or plaques. It also appears likely that because the applied strains are very low that the plate can be re-used either in a further plate test or cut into test coupons.

3 PLATE-TWIST TEST METHOD

The test is conducted by deflecting the diagonal corners (AA') and supporting two diagonally remaining opposite corners (BB') as shown below in Figure 1. The total load is recorded as a function of the applied displacement. The plate should be square or rectangular with diagonals AA' and BB' are equal in length.

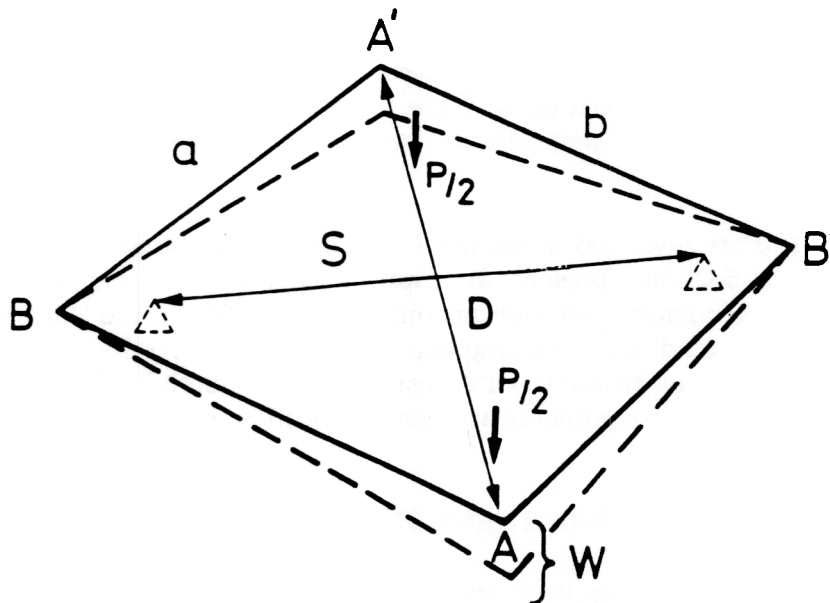


Figure 1 Plate-twist test geometry.

(D = plate diagonal length, S = test span diagonal length)

The plate deflection can be measured via the movement of the upper crosshead (defined as the loading point displacement, w_p) or at the plate centre using a LVDT or alternative displacement transducer (defined as the plate centre displacement, w_c). This loading arrangement is shown in Figure 2 for an actual specimen under a large deflection using a modified Instron flexure jig. The loading rollers on the flexure jig were replaced by points, with approximately 2 mm diameter flats. A new top loading arm was produced supporting two further loading points.

One experimental difficulty for the plate twist test is that the existing equations, see Section 4, for calculating the shear modulus assume the loading points are positioned precisely at the corners of the plate. In practice this is difficult, and time consuming to achieve, but errors are introduced if the loading points are moved in-board from the actual corner, even by a small amount. The uncertainty in the point position generates the greatest error when the loading points are near the corner of the plate (see Figure 5). In Section 4 an analysis is described which allows other loading point positions to be used and a correction factor to be applied based on the accurately known position of the loading points. An additional benefit of this new analysis is that the loading jig can be set at a standard size and differences in the dimensions of plate specimens allowed for in the analysis.



Figure 2 Plate under load on universal test machine.

4 THEORETICAL ASPECTS

The equations in the conventional analysis^[2] for determining G_{12} from the plate-twist tests are:

a) G_{12} measured by loading points displacement (w_p) is given by,

$$\frac{3}{4} \frac{\Delta Pab}{\Delta w_p h^3} \quad (1)$$

which reduces for a square plate ($a = b$) to,

$$\frac{3}{4} \frac{\Delta Pa^2}{\Delta w_p h^3} \quad (2)$$

where G_{12} is equal to G for isotropic materials.

b) G_{12} measured by plate centre displacement (w_c) is given by,

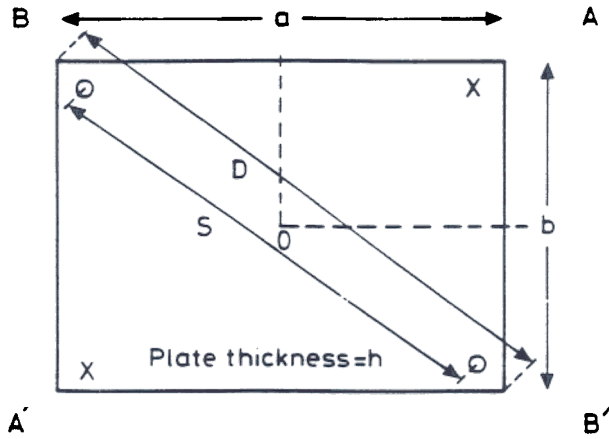
$$\frac{3}{8} \frac{\Delta Pab}{\Delta w_c h^3} \quad (3)$$

where a and b are the plate edge dimensions, h is plate thickness and ΔP is the change in load (total) for a change in displacement, Δw .

The thickness h should be one thirty-fifth or less of a or b (whichever is the smaller) in order to ensure the effect of through-thickness shear deflections are negligible. The plate geometry is shown in Figure 3.

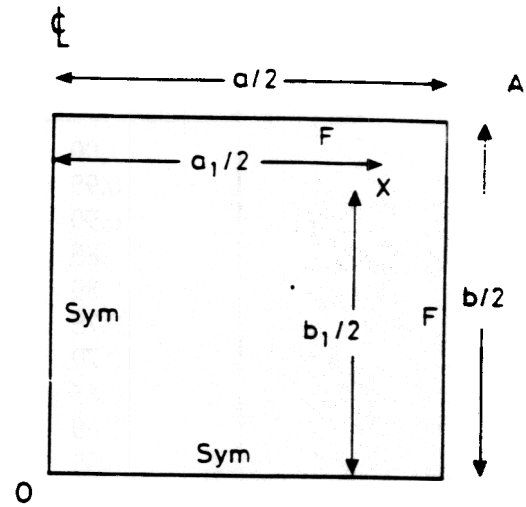
The information required to modify the basic analysis given above to allow for loading points away from the corner was obtained from the work described in^[10]. This work involved the numerical determination of the displacement of a rectangular orthotropic plate subjected to transverse loading. Both uniform and point loads were treated, together with a variety of edge loading conditions using finite difference methods to obtain the numerical solutions.

The quarter panel shown in Figure 4 was analysed using 16 mesh points per side. The correction factor K was determined for a range of values of the in-board loading position S/D , where S is the diagonal distance between loading points and D is the diagonal length of the plate specimen). The correction factor K is equal to the ratio of the deflection at the actual loading points to the deflection at the corners when loaded at the corners for a given load. Values of K determined for S/D values ranging from 0.5 to 1.0 are given in Table 1 (isotropic materials) and Table 2 (orthotropic materials).



o = lower support points
x = upper loading points

Figure 3 Full plate-twist specimen



x = loading point
Sym = Symmetry edge
F = Free edge

Figure 4 Quarter-plate element

The availability of the correction factors allows Equations (1) and (3) to be modified for the actual loading position as follows,

$$G_{12} = \frac{3}{4} \frac{\Delta P a b K}{\Delta w_p h^3}$$

$$G_{12} = \frac{3}{8} \frac{\Delta P a b K}{\Delta w_c h^3}$$

As above, G_{12} defaults to G for an isotropic material.

The correction factors predict that for loading points positioned only a small distance, eg 1.0 mm for a 150 mm square plate from the actual corner, the standard analysis significantly overestimates the shear modulus by 3.4%, and for a 2.0 mm offset by 6.8%.

Table 1 Correction factors K for isotropic materials

Loading Position S/D	Poisson's Ratio (ν)		
	0.25	0.30	0.40
1.00	1.000	1.000	1.000
0.95	0.822	0.822	0.821
0.90	0.675	0.674	0.672
0.85	0.551	0.549	0.546
0.80	0.446	0.443	0.440
0.75	0.357	0.354	0.350
0.70	0.284	0.280	0.276
0.65	0.222	0.219	0.214
0.60	0.171	0.168	0.163
0.55	0.130	0.127	0.122
0.50	0.096	0.094	0.090

Table 2 Correction factors K for orthotropic materials

[D_1 and D_2 are the in-plane flexural stiffnesses and D_3 relates to the plate shear stiffness^[12].]

In-board loading position S/D	Poisson's ratio $\nu_{12} = 0.15$			Poisson's Ratio $\nu_{12} = 0.30$			
	Stiffness ratio (D_3/D_2)			Stiffness ratio (D_1/D_2)			
	1	0.75	0.5	1	2	4	10
1.00	1	1	1	1	1	1	1
0.95	0.823	0.823	0.818	0.822	0.819	0.819	0.819
0.90	0.675	0.674	0.672	0.675	0.672	0.669	0.668
0.85	-	-	-	-	-	-	-
0.80	0.449	0.449	0.431	0.446	0.438	0.434	0.431

The data for a material with a Poisson's ratio ν equal to 0.3 are plotted in Figure 5 against the in-board ratio (S/D). The calculated data is shown as crosses against a fitted line which is given by,

$$K = \frac{1}{2.136} \left[\left(\frac{S}{D} \right)^{4.13} + \left(\frac{S}{D} \right)^{3.6} + 0.136 \left(\frac{S}{D} \right) \right] \quad (4)$$

where $S \leq D$.

Although this equation is ungainly, it can be included easily in modern computer processing software^[11] to enable both uncorrected and corrected results to be calculated automatically.

Alternatively, a correction curve can be plotted from the data in Tables 1 or 2 and the K value obtained from this graph for intermediate values.

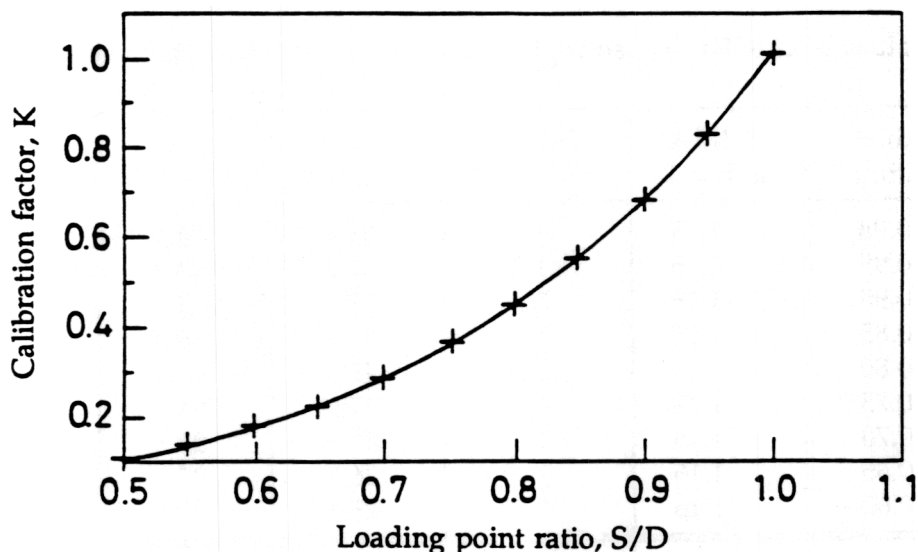


Figure 5 Comparison of finite difference results (+) and fitted line (equ. 4)(-).

It should be noted that there are slight dependencies of the correction factor on both Poisson's ratio and the anisotropy. Figure 5 data refer to a Poisson's ratio (ν) of 0.3 which is typical of metals and discontinuous fibre reinforced plastics. In Table 1 are shown additional data for $\nu = 0.25$ and 0.4 and in Table 2 for orthotropic materials of different Poisson's ratios and stiffness ratios^[12]. The differences are in all cases small and for normal loading conditions of in-board position ≥ 0.75 , the differences are 1%.

5 EXPERIMENTAL VALIDATION

A series of experiments has been undertaken to validate the analysis given in Section 4 for a selection of orthotropic and isotropic materials over a range of loading positions. The experimental programme used several isotropic materials (glass, polypropylene, polymethylmethacrylate and steel) and several reinforced plastics/composites (see Table 5).

The tests were undertaken on an Instron 4507 test machine using Series IX software and a 1 kN load cell. A test method was prepared which allowed both the plate dimensions and the support dimensions to be entered in order that the software could calculate the in-board loading ratio and hence the correction factor based on equation (4). Test displacement rates were in the range 1-10 mm/min depending on the material under test, with some allowance made for the increased stiffness of the plate at lower in-board ratios by reducing the test rate.

Displacement was measured either by the movement of the crosshead carrying the moving pair of loading points or by a LVDT transducer monitoring plate centre deflection. The measured deflection was corrected for the loading train deflection when the latter was found to be significant. A set of loading curves indicating the linear response recorded in all cases, and the increased stiffness of the specimen as the loading points were moved further in-

board are shown in Figure 6. In [13] it was found that the behaviour becomes significantly non-linear when the corner deflection exceeds the plate thickness. In these tests the maximum deflections were always restricted to less than half plate thickness.

In Table 3 the measured values of the in-plane shear modulus, G_{12} , (equals G for isotropic materials) are given for three materials tested at in-board loading positions (S/D) from 0.6 to 0.98.

Table 3 Values of G (GPa) for isotropic square and rectangular specimens

Loading Position	PMMA $a/b = 1$	PMMA $a/b = 0.8$	Glass $a/b = 1$	Steel $a/b = 1$
0.98	1.13	1.21	28.4	94.4
0.95	1.15	1.23	27.2	93.8
0.90	1.15	1.24	27.6	95.7
0.85	1.17	1.19	27.4	96.0
0.80	1.13	1.20	27.6	96.8
0.75	1.17	1.18	27.0	94.8
0.70	1.15	1.18	26.8	95.8
0.65	1.16	1.12	26.8	95.3
0.60	1.16	1.18	25.8	95.1
Mean (GPa)	1.15	1.19	27.3	95.3
SD (GPa)	.02	.04	.5	.9
CV (%)	1.3	2.9	1.9	0.95

Similar data are given in Table 4 for two anisotropic materials.

Table 4 Values of G_{12} (GPa) for orthotropic square and rectangular specimens

Loading Position	Glass fibre fabric/epoxy		Aligned carbon fibre/epoxy
	$a/b = 1$	$a/b = 0.8$	$a/b = 1$
0.98	4.47	4.71	6.04
0.95	4.55	4.80	6.06
0.90	4.66	4.82	6.07
0.85	4.78	4.69	6.20
0.80	4.95	4.72	6.20
0.75	5.25	4.68	6.16
0.70	4.96	4.79	6.16
0.65	4.81	4.59	-
0.60	4.98	4.92	-
0.55	5.36	4.99	-
0.50	-	5.06	-
MEAN (GPa)	4.84	4.80	6.13
SD (GPa)	.30	.14	.07
CV (%)	6.2	3.1	1.14

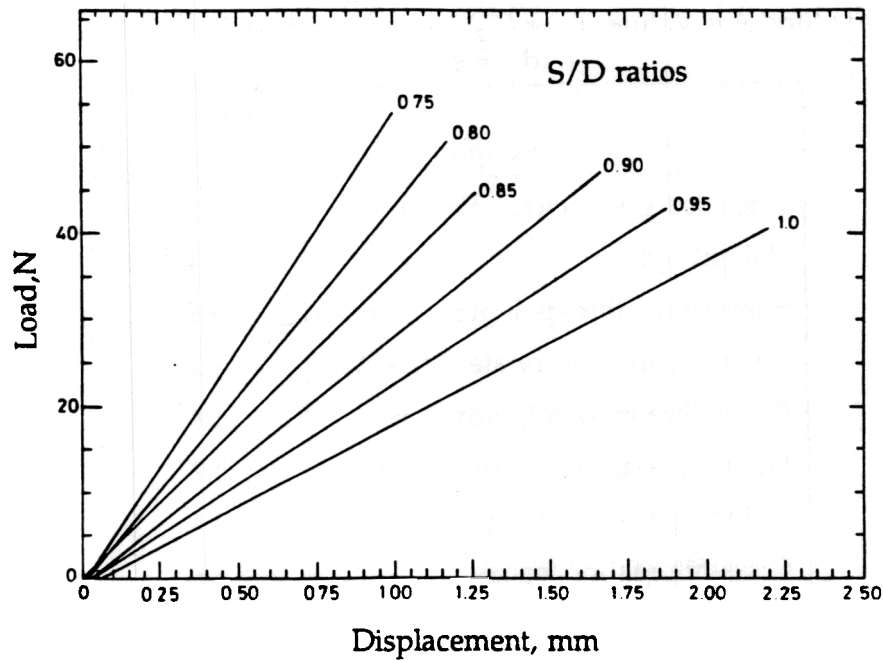


Figure 6 Load v displacement outputs for several values of S/D.

In Figure 7 are shown values of G_{12} for an aligned carbon-fibre/epoxy (CFRP) material using a square specimen. The low scatter shown in this Figure confirms the consistency of the numerical data (ie low values of CV) shown in Tables 3 and 4, and is typical of all the materials tested.

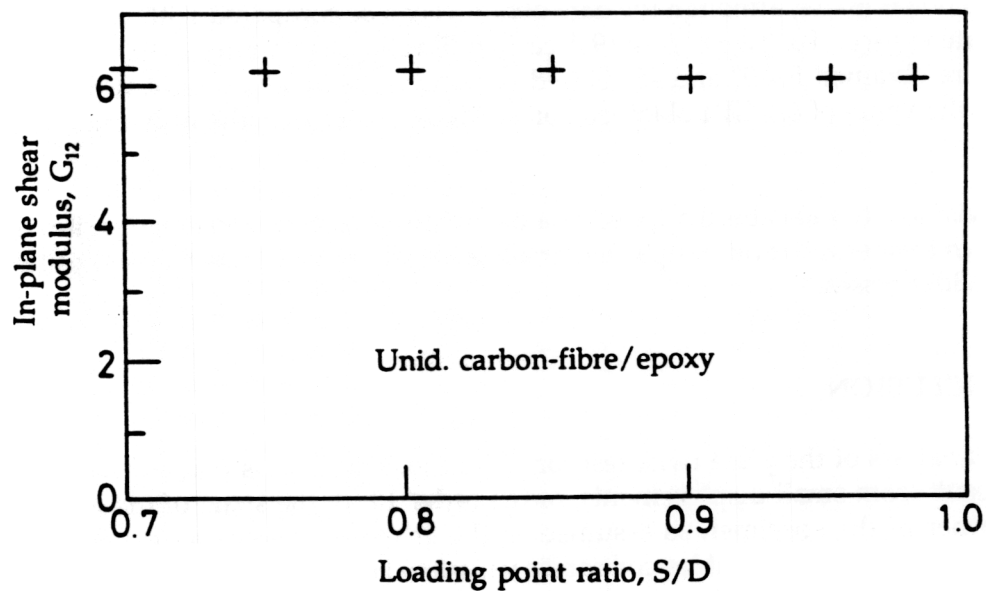


Figure 7 Variation of calculated shear modulus with in-board S/D loading position

Finally, results are shown in Table 5 for a wider range of materials to illustrate the range of materials that can be tested using this technique and the range of shear moduli measured. The results are generally in agreement with other predicted or measured values but detailed comparisons for the same sample of several materials using different techniques are planned.

Table 5 Shear Modulus Values (G or G_{12}) Measured using Plate-Twist Test Method
(NB * = see text)

Material Description	G or G_{12} (GPa)
Chopped glass-fibre/polyurethane	0.29
Polypropylene	0.45
Short glass-fibre/polypropylene	0.67
Polymethylmethacrylate (PMMA)	1.2
Glass-fibre mat/polypropylene	1.8
Glass fibre/polyester pultrusion	3.2
Swirled glass mat/polyester	3.6
Glass-fibre fabric/epoxy	4.6
Sheet moulding compound (SMC)	4.5
Unid. carbon fibre/epoxy (CFRP)	6.1
0° biased multid. CFRP*	15.3
45° biased multid. CFRP*	24.3
Aluminium alloy	24.7
Glass (sheet)	28.6
Steel	82.3

Material values (ie basic unidirectional specimen data) cannot be obtained from cross-ply or multidirectional plates, although the data obtained can be compared with values predicted from laminate theory. For two 0°/45°/90° carbon fibre/epoxy laminates values of 15.3 and 24.3 GPa were obtained for 0° and 45° biased lay-ups, respectively. These values are much higher than the value of 6.1 GPa obtained for the same material in the fully unidirectionally form.

The plate twist test has also been applied to a ceramic and metal matrix composite. The low shear strain in this test method has particular advantages for metal matrix composites which have low yield stresses.

6 CONCLUSION

The refined analysis of the plate-twist test for shear modulus shows that appreciable errors, 3-8%, can result from small displacements, of the order 1% of the span, of the loading point from the corner of the specimen as assumed in the analysis. The experimental validation shows that the new analysis enables consistent values to be obtained over a range of loading geometries for both square and rectangular plates, and for both isotropic and anisotropic materials. However, material values (ie basic unidirectional specimen data) cannot be obtained from cross-ply or multidirectional plates although the data can be used to confirm theoretical predictions.

The test was shown to be suitable for materials with shear moduli ranging from 0.29 GPa to 88.2 GPa, and had the advantage for moulded materials of being applicable to specimens more representative of actual products than moulded to size dumbbell and beam specimens. The test is suitable for unidirectional material and avoids the necessity of preparing a 0°/90° cross-ply test plate solely to undertake the tensile test on $\pm 45^\circ$ aligned coupons. However, the plate twist test only gives shear modulus data and the Iosipescu, $\pm 45^\circ$ or 10° off-axis tensile tests are required for measuring the shear strength properties.

Further work is being undertaken to compare the results with data obtained from other shear test methods, and to develop the method towards a standard test method by undertaking precision trials. It is planned that a standard plate specimen (say 150 mm $\times$ 150 mm) and in-board ratio (0.95) will be included in the standard, with other sizes as options, and a maximum thickness requirement of $35 h \leq a \leq b$.

The plate twist test has been standardised for wood products^[14,15], but, both methods use clamped plates to introduce the load near the corners. This introduces inaccuracies because the exact position of the loading points are not known and this affects the measured response of the plate. Also, these standards use a dial gauge and attachment to measure the relative movements of points on opposing diagonals at the mid-span point. The effect of the relationship between the position of the loading points and the position of the deflection measurement points, and the effect of the load introduction method, on the accuracy of the measurement will be investigated.

REFERENCES

- 1 JOHNSON A F and SIMS G D, "Simplified Design Procedures for Composite Plates under Flexural Loading", Proc 2nd Int Conference on Composite Structures. Paisley College of Technology, Paper 22, p.302-325, (1983).
- 2 ALLEN H G, "Analysis and design of structural sandwich panels", Published Pergamon, (1969).
- 3 ABDALLAH M G, "Review of the state of art of advanced composites interlaminar shear test methods", Proceedings 1986 SEM Meeting on Experimental Mechanics, New Orleans, (1986).
- 4 BROUGHTON W R, KUMOSA M and HULL D, "Analysis of the Iosipescu shear test as applied to unidirectional carbon-fibre reinforced composites", Composite Science and Technology, 38, p.299-325, (1990).
- 5 ASTM D 4255, "Standard guide for testing in-plane shear properties of composites laminates", (1983).
- 6 MARKHAM M F and DAWSON D, "Interlaminar shear strength of fibre-reinforced composites", Composites, p.173-175 (1975).
- 7 ASTM D3518, "In-plane stress-strain response of unidirectional reinforced plastics", (1982).
- 8 WHITNEY J M, "Experimental determination of shear modulus of laminated fiber-reinforced composites", Experimental mechanics, October 1967.
- 9 ISO 1268 - Reinforced plastics and composites - Methods for preparation of test plates (currently being revised and expanded to include similar standards, eg ISO 295).

- 10 FERRISS D H, "Numerical solution of rectangular anisotropic plates in flexure with free edges", NPL Report DMA(A)71, (1983).
- 11 Series IX Test Machine Software, Version 5.0, Instron Ltd, (1991).
- 12 JOHNSON A F and SIMS G D, "Design procedures for plastics panels", NPL Design Guide, (1987).
- 13 FOYE R L, "Deflection limits on the plate-twisting test", J Composite Materials 1, p.194-198, (1967).
- 14 BS 4512 : 1969. "Methods of test for clear plywood"
- 15 ASTM D 3044-76 "Standard Test Method for Shear Modulus of Plywood"