

Finite Element Analysis of Miniaturised Electro Thermo Mechanical Testing System

T E Quested
Centre for Materials Measurement and Technology
National Physical Laboratory
Teddington, Middlesex
United Kingdom, TW11 OLW

SUMMARY

The aim of this work was to provide an understanding of the temperature distributions of specimens heated within the miniaturised electro thermo mechanical testing (ETMT) system. Finite element (FE) modelling was used and its results compared with analytical solutions for simple cases.

Platinum strip was initially modelled. Temperature profiles along the strip were found as a function of current. The sensitivity of the temperature distribution to input parameters was explored; the effects of emissivity, coefficient of convective heat transfer and the free length of strip, and emissivity were all studied. The effect of the presence of a thermocouple was also investigated.

The temperature distribution between the water-cooled clamps was also modelled. The value of interfacial heat transfer between the clamps and bar was varied and the change in temperature distribution observed. This was compared to models in which material between the clamps was assumed to be held isothermally. This allowed an 'effective length' for a particular geometry and material to be found.

Studies of iron with an oxide scale were conducted to complement practical pyrometer measurements of temperature. Thermal profiles of cross-sections were determined including the temperature drop across the oxide. Different thicknesses, thermal conductivities and surface emissivity values of the oxide were modelled. These results were compared with analytical solutions.

© Crown copyright 2000
Reproduced by permission of the Controller of HMSO

ISSN 1361-4061

National Physical Laboratory
Teddington, Middlesex, UK, TW11 0LW

Extracts from this report may be reproduced provided
that the source is acknowledged.

Approved on behalf of Managing Director, NPL, by Dr C Lea,
Head of Centre for Materials Measurement & Technology

CONTENTS

INTRODUCTION	1
2 THEORY OF THERMAL MODELLING	1
3 MODELLING OF PLATINUM STRIP	2
3.1 METHOD	3
3.2 RESULTS	4
3.3 REFINEMENTS	4
3.4 CONCLUDING MODELLING	6
4 INVESTIGATION OF CLAMPED REGION OF PLATINUM	11
4.1 METHOD	11
4.2 RESULTS	12
4.3 CONCLUSIONS	13
5 MODELLING OF IRON WITH AN OXIDE SCALE	17
5.1 METHOD	17
5.2 PROBLEMS ENCOUNTERS	18
5.3 RESULTS	19
5.4 CONCLUSIONS	20
6 DISCUSSION AND CONCLUSIONS	24
7 ACKNOWLEDGEMENTS	24
8 REFERENCES	25
9 NOMENCLATURE	26
10 APPENDICES	27
APPENDIX 1: MATERIAL PROPERTIES	27
APPENDIX 2: EXAMPLES OF MODEL GEOMETRIES	31
APPENDIX 3: EMPIRICAL DETERMINATION OF CONVECTIVE HEAT LOSS	33

1 INTRODUCTION

This work forms an integral part of a project to investigate the emissivity of oxide scales formed on steels within the ETMT system [1]. The emissivities of scales will be experimentally measured by pyrometry and related to their composition and thickness. These can both be determined using electron back-scattered diffraction (EBSD) [2]. This work assesses the viability of modelling metal-oxide systems. It evaluates the data available for modelling and highlights weaknesses of the modelling.

This modelling work comprises a series of small investigations, allowing a thorough investigation by incrementally increasing the complexity of the models. Initial investigations into platinum strips were designed to see if modelling was feasible on a simple material. Refinements to the model were made to explore increasingly subtle phenomena, e.g. clamping conditions and metal-oxide systems.

A greater understanding of the radiative properties in steel-oxides systems should be possible using thermal modelling to complement experimental work. Experimental work allows the pyrometer output to be measured for a range of oxide thicknesses and compositions. Modelling work allows this to be related to the thermal distribution within the oxide.

The thermal modelling consisted of coupled thermal-electric finite element analysis using *ABAQUS Standard*, version 5.8. Verification of this modelling came from analytical solutions of the Fourier equation in one dimension.

2 THEORY OF THERMAL MODELLING

Temperature distributions within a continuous solid can be described by the Fourier equation (see equation 2.1). This relates the flow of heat in a solid to the change in internal energy due to temperature changes. It also includes a term for heat generation, e.g. by an electric current flowing.

$$\rho c_p \frac{\partial T}{\partial t} = \frac{\partial}{\partial x} \left(k \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left(k \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z} \left(k \frac{\partial T}{\partial z} \right) + q$$

Equation 2.1: *Fourier equation in three dimensions [3]; symbols are defined in section 9.*

The Fourier equation can be simplified: the left hand side is equal to zero if steady state has been reached; a partial differential of temperature with respect to a direction can be ignored if its magnitude is negligible throughout the body. These simplifications can allow analytical solutions to be produced. Numerical solutions must be sought for more complex situations.

In addition, a solution to the electric field equations must be found in order to model the temperature distribution within ETMT specimens. These two fields are dependent on one another due to the temperature dependence of the electrical conductivity. This requires a coupled thermal-electrical analysis that solves the two equations simultaneously. Maxwell's equation of conservation of electric charge describes the electric field present. Most FE packages use Maxwell's equation with a substitution of Ohm's law for the current density.

$$\oint_S \mathbf{J} \cdot d\mathbf{S} = \int_V b_c dV$$

Equation 2.2: Maxwell's equation of conservation of electric charge. $\mathbf{J}$ is the current density vector, $\mathbf{n}$ the outward normal to outward surface S of volume V and b_c internal volumetric current source per unit volume.

$$\mathbf{J} = \sigma_E \mathbf{E}$$

Equation 2.3: Ohm's Law. σ_E is the electric conductivity matrix, and $\mathbf{E}$ the electric field intensity, defined as the negative of the gradient of the electric potential.

In these investigations, the ETMT specimen was represented by a two- or three-dimensional array of nodes. These nodes defined elements: homogeneous volumes of material that exchange heat with their neighbours according to discrete versions of the Fourier equation and the Maxwell equation.

Boundary conditions were applied along the edges, surfaces and vertices of elements. These included:

- Fixed temperature;
- Prescribed heat flow;
- Fixed electrical potential;
- Constant electrical current density.

The coupled electrical-thermal analysis within *ABAQUS Standard* used an implicit iterative technique. This involves forming equations that describe the temperature at a node in terms of the temperatures of its neighbouring nodes in both the current and previous time levels. This leads to a large number of simultaneous equations, which are solved by matrix inversion.

One-dimensional analytical solutions to the steady-state form of the Fourier equation were found. The assumptions made to form these solutions were:

1. Thermal properties are constant over the relevant temperature range;
2. Heat flow down thermal gradients perpendicular to the line of the analysis is negligible.

These assumptions approximate to the physical world for analyses with small temperature ranges and for solutions along lines of symmetry in which the thermal gradient is zero in all other directions. The second assumption prohibits the modelling of radiation and convection for analyses down the length of a bar.

3 MODELLING OF PLATINUM STRIP

Platinum was initially modelled because its properties are well defined within the literature. In addition, it has no phase change in the temperature range under investigation and does not grow an oxide scale of significant thickness. Simple models were used initially to discover the relative importance of some variables. These included:

- Length of bar;
- Radiative and convective heat losses.

Analytical models also explored the sensitivity of the model to:

- Current;
- Thermal conductivity;
- Electrical Conductivity.

The basic model underwent a number of refinements in an attempt to replicate experimental results more closely. The magnitude of the change on temperature distributions due to these refinements was measured. This allowed the validity of assumptions to be evaluated. Material properties are given in appendix 1.

3.1 METHOD

The modelling work attempted to reproduce the measurement of central temperatures for five ETMT experiments. These experiments tested platinum strip 24 mm x 2 mm x 0.24 mm. Five different currents were passed through the strip to achieve prescribed temperatures in the centre of the strip. These currents and temperatures are given in table 3.1.

Current / A	Maximum Temp. / K
18.4	473
23.5	673
26.5	873
29	1073
31.5	1273

Table 3.1: Experimental currents and resultant temperatures.

Initial modelling used a two dimensional mesh which is shown in appendix 2. This assumed that heat flow in the plane of the model was negligible and that heat losses from the two smaller surfaces (0.24 mm x 24 mm) was also insignificant. The larger surfaces (2 mm x 24 mm) had radiative and convective heat loss modelled. Transient modelling was used; the end point for this modelling was steady state, defined to be where the rate of change of temperature at all nodes was less than 0.01 K s⁻¹.

The models were run with four different conditions of heat loss from the surfaces: with both radiation and convection, each heat loss separately or no surface heat loss. This allowed the contributions from both heat loss mechanisms to be observed. The determination of the coefficient of convective heat loss is explored in appendix 3. The value of the coefficient used varied as $\Delta T^{0.25}$ with a magnitude of 15 W m⁻² K⁻¹ for ΔT equal to 1000 K.

An analytical solution was found for the temperature distribution along the length of the bar ignoring surface heat losses and the temperature dependence of thermal properties. The form of this equation is shown in equation 3.1.

$$T - 294 = \frac{j^2}{2\sigma \cdot k} \left[\left(\frac{a}{2} \right)^2 - x^2 \right]$$

Equation 3.1: Temperature distribution in Kelvin along the length of a bar, length equal to 2a. The origin of the x-axis is defined as the centre of the bar. The temperature profile described is an inverted parabola, which is plotted in graph 3.1.

Equation 3.1 was used to check the modelling work. Models with the same simplifying assumptions as the analytical solution were run and the answers compared. There was very good agreement (± 1 K) between the two.

An example of the form of equation 3.1 can be seen in graph 3.1 and this is compared to the FE modelled temperature distribution of platinum. The significant difference between the two is in the centre of the bar. Here, the modelling predicts a higher temperature than the analytical solution. This is probably due to the temperature dependence of the electrical conductivity that is ignored within the analytical solution.

3.2 RESULTS

The results in table 3.2 compare the experimental results with the modelling results for the four different heat loss conditions. These results can be seen graphically in graph 3.2.

Current / A	Experimental Maximum Temp. / K	Modelling – Maximum Temp / K			
		No Radiation or convection	Radiation only	Convection only	Both Radiation and convection
18.4	473	555	552	505	503
23.5	673	988	932	825	792
26.5	873	1542	1277	1211	1084
29	1073	2084	1534	1703	1375
31.5	1273	2480	1743	2205	1619

Table 3.2: Comparison of experimental and modelling work for platinum strip heated in the ETMT.

The results show that the central temperature was over-estimated in the model by up to 350 K (35 %). This error was too large to model the temperature distribution accurately. Hence, further modelling was carried out to ascertain the reasons for this systematic error. Possible causes are listed below and the relative magnitude of each was ascertained.

- Inaccuracies in the model data
 - i. Material properties,
 - ii. Surface heat loss data,
 - iii. Dimensions of the strip.
- Additional heat losses occurring that were not originally modelled:
 - iv. From the surfaces of the thermocouple,
 - v. From the smaller surfaces ignored.

3.3 REFINEMENTS

Material Properties

All material properties found were for high purity platinum. The platinum used is unlikely to have differed significantly. Discrepancies may have occurred due to the condensing of this information into data tables for ABAQUS. ABAQUS linearly interpolates between data points and a systematic error can occur if the second derivative of a property with respect to temperature is significant (i.e. the data is highly curved). Any ensuing error can be minimised by increasing the number of data points used. This was done until no further

change occurred in the thermal profile.

The electrical conductivity data was highly curved and by increasing the number of points from 6 to 33, the temperature increased by 12 K. Increasing the surface heat loss data set from 6 points to 32 points increased the temperature by 30 K. There appeared to be no additional change in temperature on increasing the number of data points further (i.e. greater than one point every 50 K).

Surface Heat Loss

The data for emissivity were obtained from measurements on polished platinum surfaces. The values of emissivity could have been greater in the experimental work if the surfaces were not optically flat due to poor polishing or if tarnishing had occurred. This effect would be significant at high temperature because radiative heat losses increase as the fourth power of temperature. Graph 3.3 shows the relationship between emissivity and temperature distribution.

There are many problems associated with using empirical equations to calculate the coefficient of convective heat transfer in this particular problem and these are outlined in appendix 3. The values used are therefore estimates with a large uncertainty. The relationship between this coefficient and the maximum temperature of the strip is shown in graph 3.4. The temperature decreases roughly linearly with increasing coefficient of convective heat transfer.

Dimensions of the Strip

The thickness and width of the strip could be measured very accurately using micrometers and so it was assumed that the error in these measurements was small. Hence, no investigation of their effect was carried out.

The length of the strip between the water-cooled clamps (the free length) was measured using callipers. The error in this measurement was around 0.1 mm. This error could still lead to an error in the central temperature of 10 K because the length of the strip is a critical parameter. This can be seen in graph 3.5. The model assumes that the platinum within the clamps is all held at 294 K. This will not be the case as the removal of heat by the clamps is not perfect. This could lead to an increase in the effective length of the bar and a systematic error in the temperature distribution. This is investigated further in section 4.

Heat Losses from the Thermocouple

These heat losses were investigated as a possible source of error because the Pt/Pt-13 %Rh thermocouples were visibly radiating heat during experiments. This causes the temperature at the centre of the strip to be lower than that without a thermocouple attached.

To find the magnitude of this temperature drop, a new model was constructed with a thermocouple included in the geometry, assumed to be made of pure platinum. It was found that the maximum temperature drop at the centre of the strip due to the presence of a thermocouple was 27 K. Graph 3.6 shows a comparison of the temperature distributions with and without a thermocouple.

Heat Losses from the Smaller Surfaces

Two-dimensional modelling was used to minimise the computational time required to run the models and allow models to be constructed and modified more easily. However, it did not allow surface heat to be modelled from the smaller surfaces. These surfaces were roughly eight times smaller than the larger surfaces and hence the magnitude of the heat losses could be estimated.

The heat losses from the larger surfaces could be multiplied by a conversion factor to account for the smaller faces. This approach was compared with 3D dimensional modelling; discrepancies of less than 5 K were present. The temperature drop due to the heat losses from the smaller surfaces was found to be a maximum of 40 K.

3.4 CONCLUDING MODELLING

To complete this work, a model that took into account all the above refinements was constructed. Material properties were defined every 50 K, the exact length of the strip was found in the records (23 mm) and the heat losses from the larger surfaces used a conversion factor to account for the smaller faces. The literature emissivity data was assumed to be correct. The thermocouple was ignored leading to a systematic error of about 3 %. The only unknown was the convective heat transfer.

The coefficient of convective heat transfer was assumed to increase as $\Delta T^{0.25}$. Its magnitude was varied until the maximum temperature of the model agreed with that of the experimental work for a total current of 31.5 A. The value of h_c was $55 \text{ W m}^{-2} \text{ K}^{-1}$, for a temperature drop of 1000 K.

The current was varied and the agreement between model and experiment was ascertained. These results are shown in table 3.3.

Current / A	Maximum Temperature / K		Temperature difference / K	Error / %
	Experimenta l	Modelling		
18.4	473	465	-8	-4
23.5	673	654	-19	-5
26.5	873	841	-32	-6
29	1073	1046	-27	-3
31.5	1273	1271	-2	0
33.0	1373	1403	+30	+3

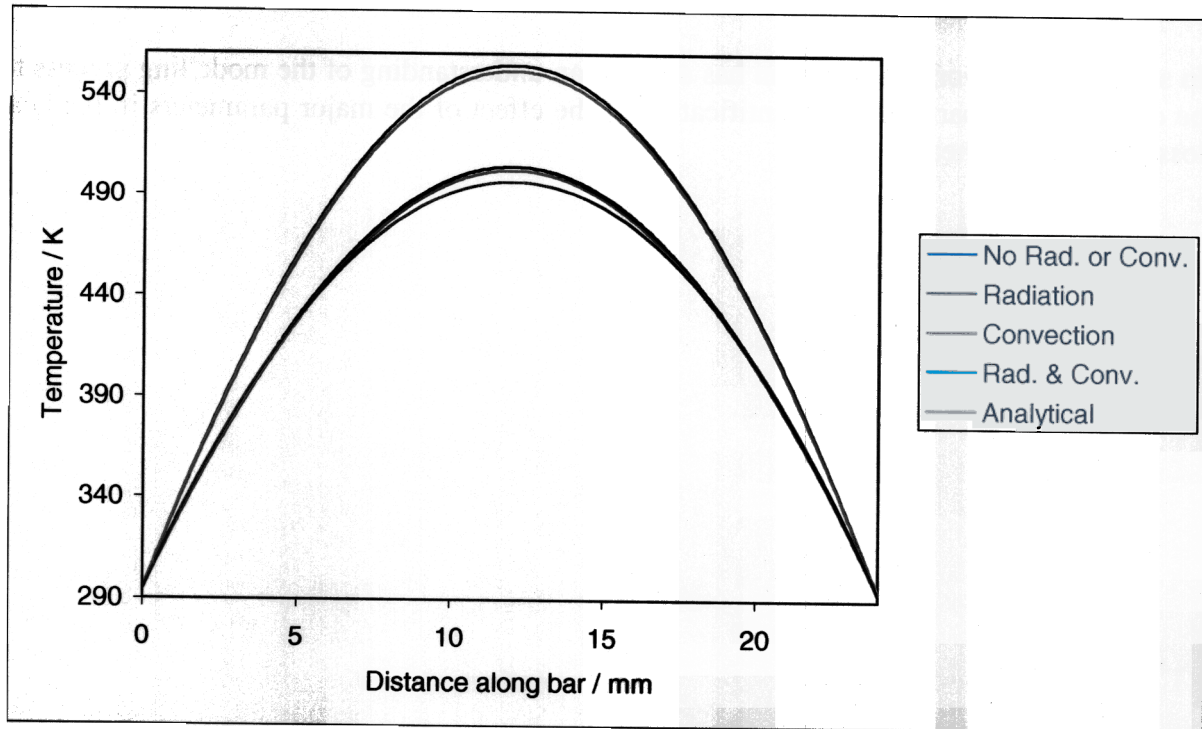
Table 3.3: Results of refined modelling of platinum strip. Error calculated from temperature difference and temperature range within the experiment.

The results deviate from experimental by up to 6 %. These errors will be due to the cooling effect of the thermocouple and errors in emissivity data.

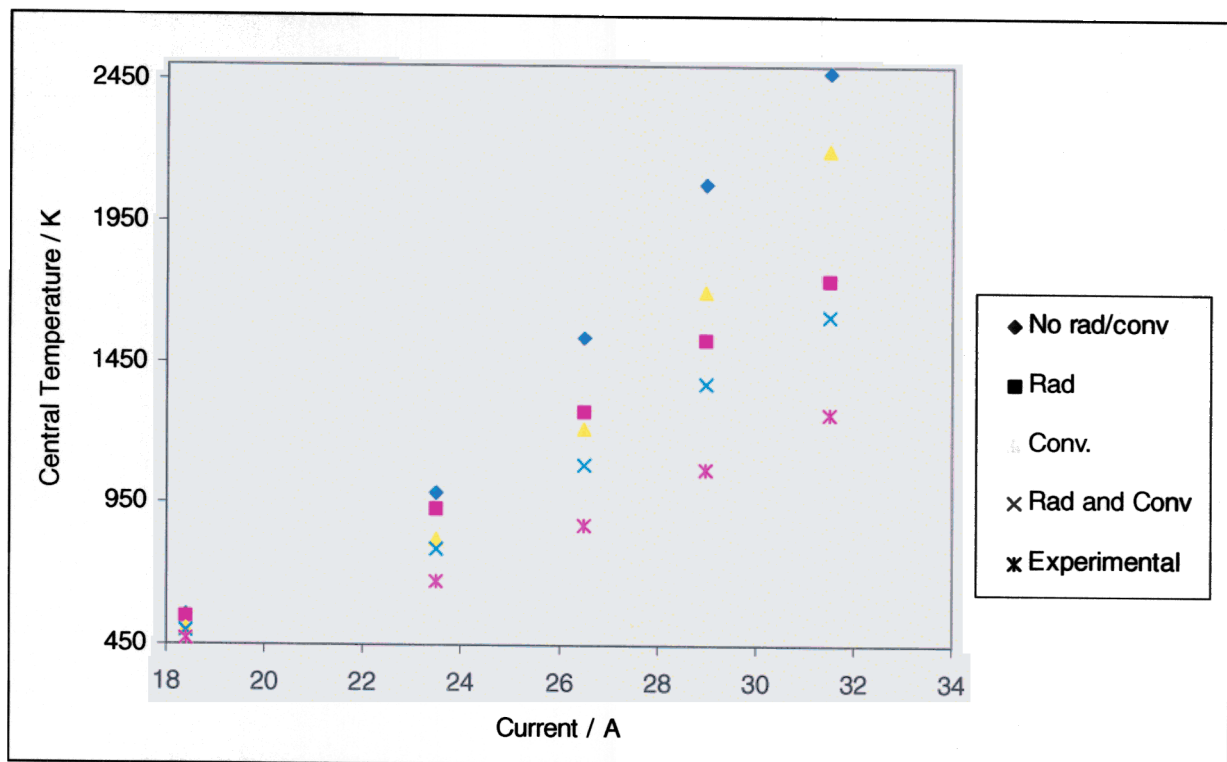
To proceed with this analysis further, one could use a model that includes the thermocouple in the geometry. The radiative and convective heat transfer coefficients could be fitted to experimental data using an iterative approach to give a better fit. This has not been undertaken because the aim of this modelling was to accurately model ferrous alloys with an oxide scale, rather than platinum. An iterative approach would probably obtain a very close

fit between experimental and modelling work; this does not, however, mean that the value obtained for heat losses are necessarily correct.

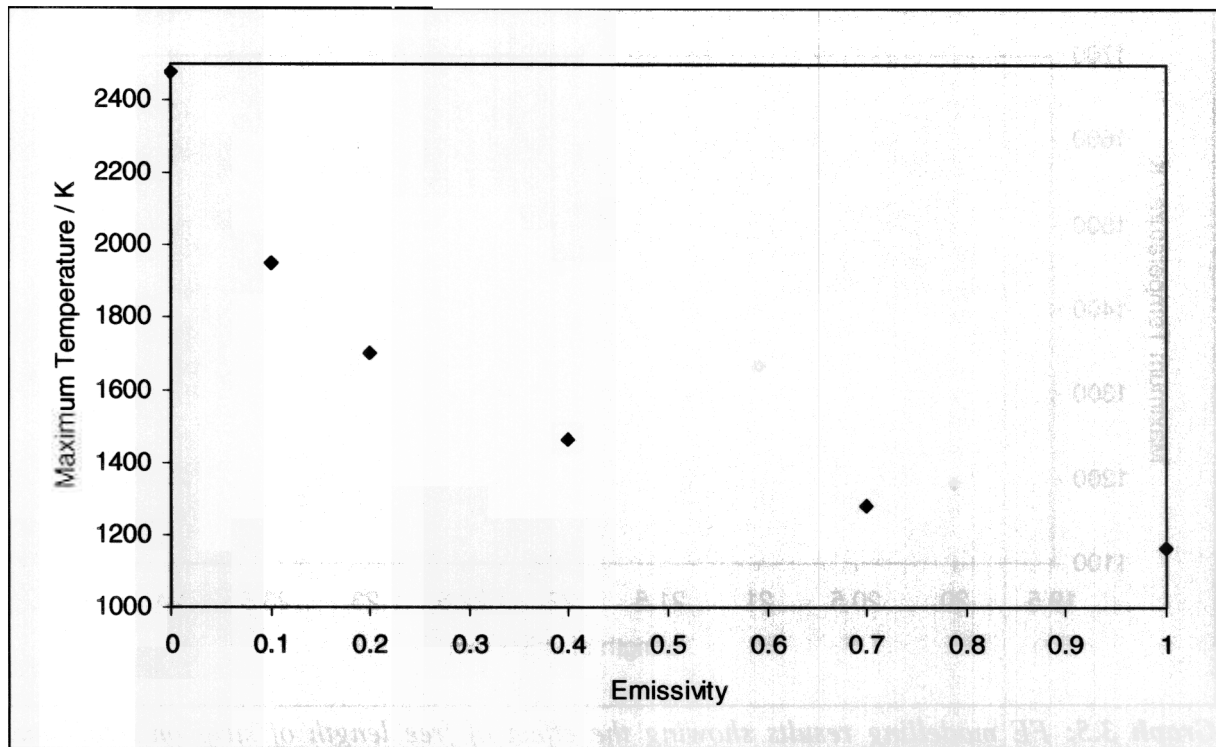
In summary, the work on platinum has allowed an understanding of the modelling process to be developed, in particular a quantification of the effect of the major parameters in the heat loss from the d.c.-heated strip.



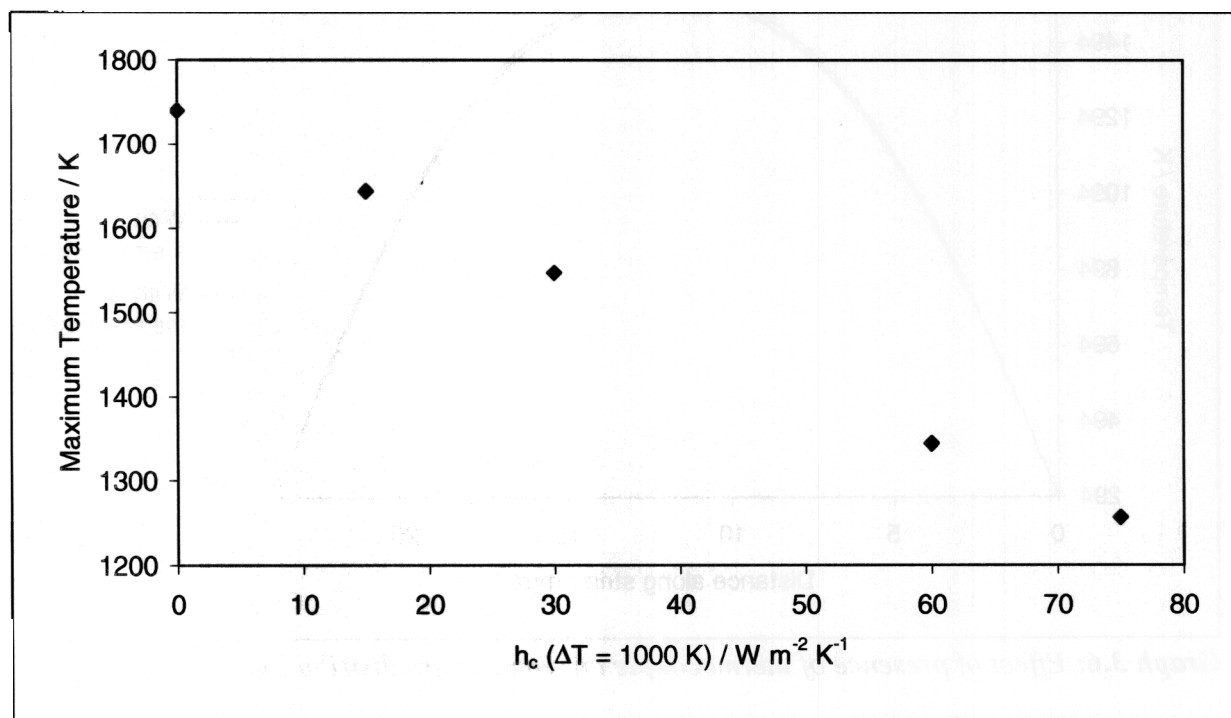
Graph 3.1: Temperature distribution obtained from FE modelling (current 18.4) and fitted analytical solution.



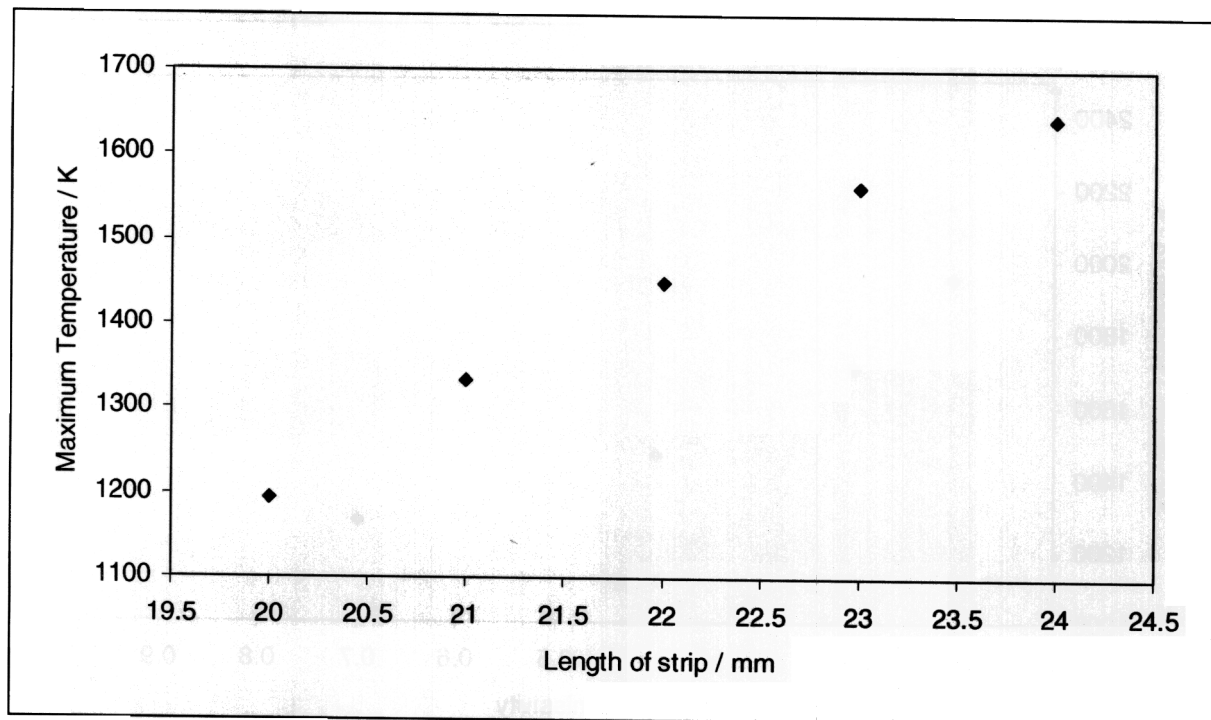
Graph 3.2: Comparison of maximum temperatures for experimental and initial FE modelling work. Four different surface heat loss conditions are shown.



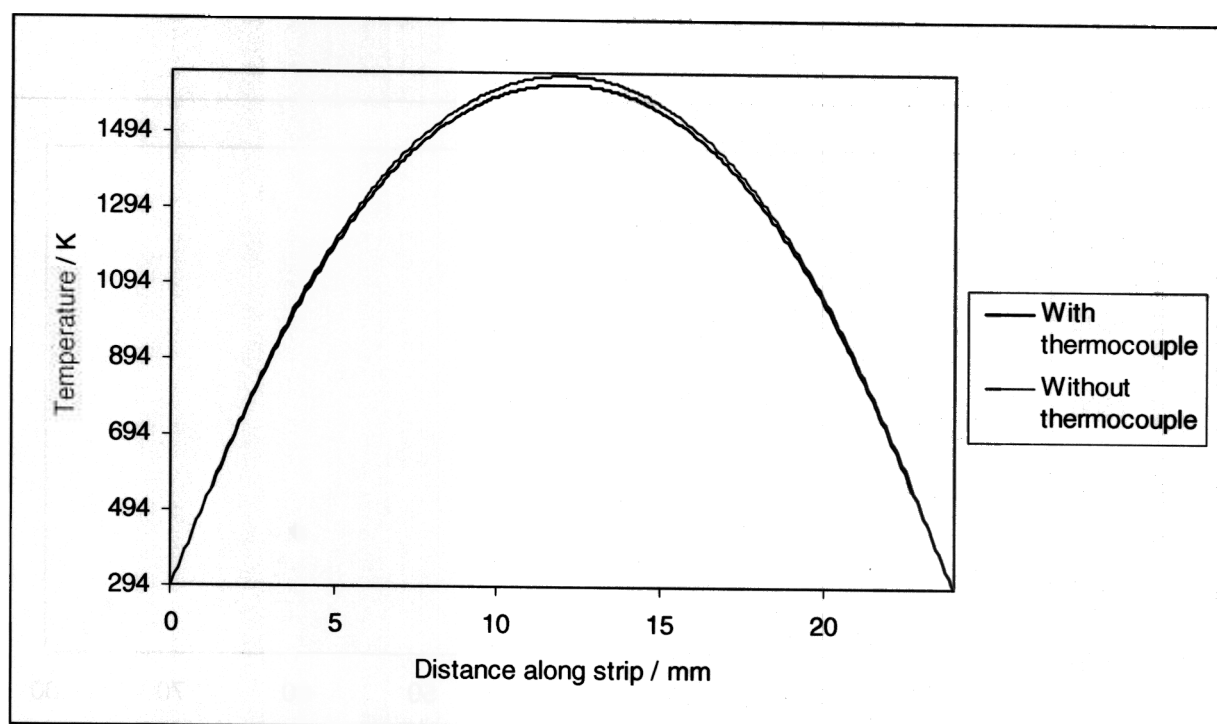
Graph 3.3: FE modelling solutions showing the effect of emissivity of surfaces on maximum temperature within strip. Convection was not modelled, no temperature dependence of emissivity was included and the current flowing was 31.5 A.



Graph 3.4: FE modelling showing the effect of coefficient of convective heat transfer (h_c) of surfaces on maximum temperature of strip. Values of h_c are given for a temperature difference between surface and atmosphere of 1000 K. Current was 31.5 A and radiation was modelled with literature values of emissivity.



Graph 3.5: FE modelling results showing the effect of free length of strip on maximum temperature within strip.



Graph 3.6: Effect of presence of thermocouple on temperature distributions of strip.

4 INVESTIGATION OF CLAMPED REGION OF PLATINUM

The aim of this work was to assess the suitability of different boundary conditions within the region of material between the water-cooled clamps. The three conditions examined for the interface between the clamps and the bar were:

1. The whole area between the clamps was at the temperature of the cooling water (294 K). This was also used for modelling in section 3.
2. The clamps were held at 294 K with an interfacial heat transfer coefficient between the clamp and the strip.
3. The edges of the bar in contact with the clamps were held at 294 K; i.e. there is perfect heat transfer between the clamp and the bar.

The effect on the central temperature was assessed and the effective free length of the bar was calculated.

4.1 METHOD

Two-dimensional FE geometries were constructed that modelled both platinum strips and iron bars, including the material between the brass clamps. The distance between the clamps was 23 mm with a further 2 mm of strip in contact with the clamps. An example of the geometry can be seen in appendix 2.

Three values of the coefficient of interfacial heat transfer (h_i) were used, 10^4 , 10^5 and 10^6 W m⁻² K⁻¹. These values were chosen because they were thought to be close to the experimental value. (10^5 W m⁻² K⁻¹ is a typical value for smooth surfaces in contact with a normal stress [4].) To model the third boundary condition, all nodes of the bar in contact with the clamp were held at 294 K. This can be thought of as an infinite interfacial heat transfer coefficient.

For both thermal boundary conditions, the current was assumed to pass into the specimen with a constant current density over the clamp-bar interface. All other properties were the same as in section 3, both within the platinum strip and the surface properties. This analysis was done for three currents: 18.4 A, 26.5 A and 31.5 A. The convection data was fitted such that the central temperature for a current of 31.5 A was 1271 K.

One of the aims of this work was to calculate the additional effective length of the strip for these clamping conditions. To allow this analysis, models of different lengths were constructed in which the nodes on the plane normal to the length of the bar were clamped at 294 K. The results from this analysis allowed a best-fit equation to be obtained relating the free length of bar to its centre temperature. The centre temperatures for the various clamping conditions were compared to the equations produced and hence the effective lengths of the strips were obtained.

Further work was undertaken to determine the effect of thermal conductivity on effective length. Simple geometries were used to ascertain the current required to generate a central temperature of 1273 K for 4 different thermal conductivities of material. A thermal contact of 10^5 W m⁻² K⁻¹ was then assumed for the interface within the clamped geometry and the effective length determined. Table 4.3 displays these results. A plot of the temperature distributions is shown in graph 4.5.

4.2 RESULTS

Length / mm	Maximum Temperature / K		
	I = 18.4 A	I = 26.5 A	I = 31.5 A
22.6	458.1	816.4	1237
22.8	461.3	828.8	1254
23.0	464.4	841.4	1271
23.2	467.8	854.2	1287
23.4	471.1	866.8	1303
23.6	474.4	879.6	1319
23.8	477.8	892.5	1335
24.0	481.2	905.5	1350

Table 4.1: Effect of length of strip on central temperature for three currents. Further investigations were carried out on strips with 23 mm between clamps. These results are displayed in graphs 4.1, 4.2 and 4.3.

$h_i / \text{W m}^{-2} \text{K}^{-1}$	Central Temp. / K	Effective Length of Strip / mm
10^4	497.6	25.0
10^5	474.5	23.6
10^6	467.9	23.2
∞	465.9	23.1

Table 4.2a: Central temperature and effective length as a function of coefficient of interfacial heat transfer for a current of 18.4 A. The effective lengths are shown in graph 4.4.

$h_i / \text{W m}^{-2} \text{K}^{-1}$	Central Temp. / K	Effective Length of Strip / mm
10^4	964.6	24.9
10^5	879.8	23.6
10^6	854.6	23.2
∞	846.7	23.1

Table 4.2b: As above except current is 26.5 A

$h_i / \text{W m}^{-2} \text{K}^{-1}$	Central Temp. / K	Effective Length of Strip / mm
10^4	1413	24.8
10^5	1319	23.6
10^6	1288	23.2
∞	1278	23.1

Table 4.2c: As above except current is 31.5 A

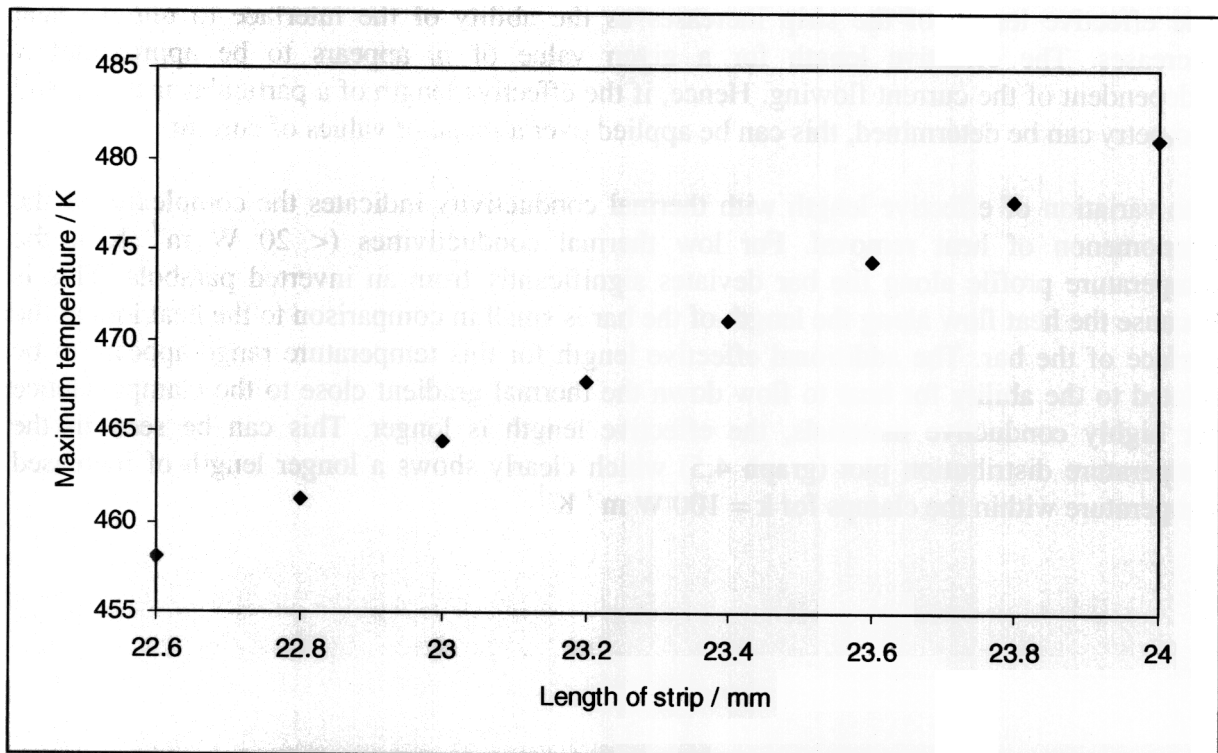
$k / \text{W m}^{-1} \text{K}^{-1}$	Effective length / mm
10	23.3
20	23.4
50	23.5
100	23.7

Table 4.3: Effect of thermal conductivity on effective length

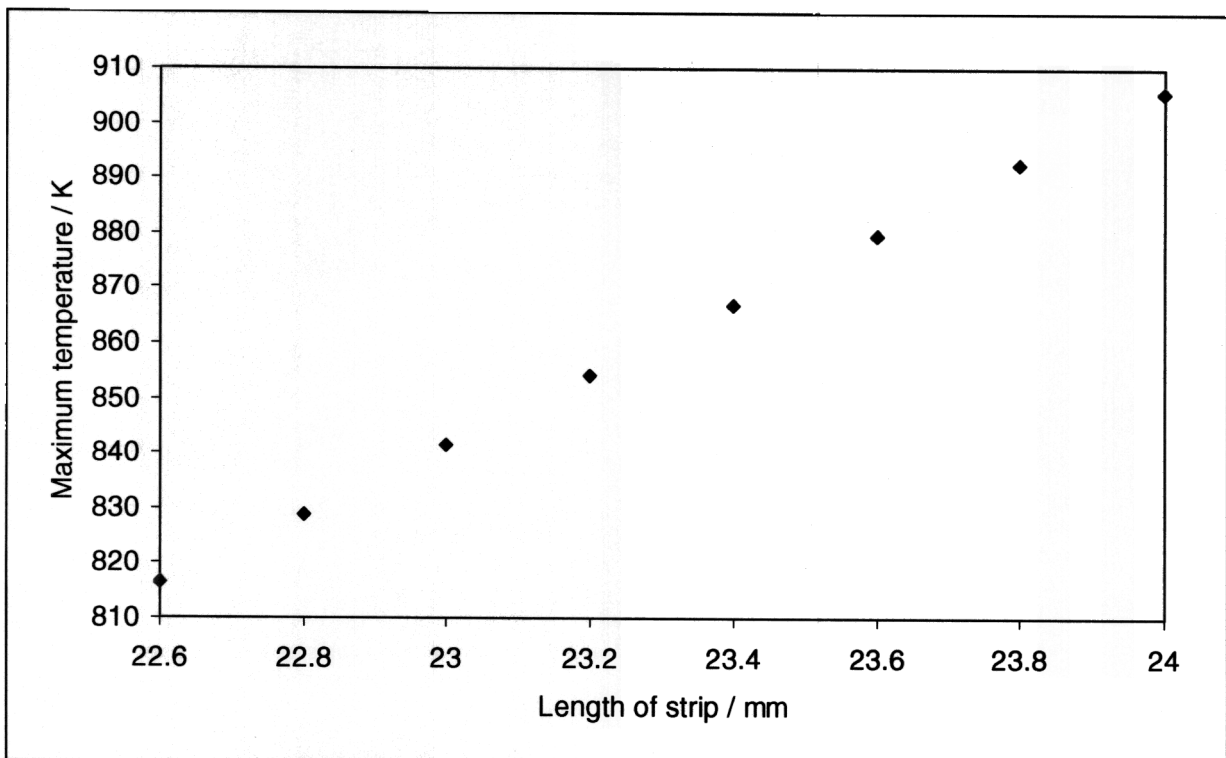
4.3 CONCLUSIONS

The effective length of the strip increases as the ability of the interface to remove heat decreases. The effective length for a given value of h_i appears to be approximately independent of the current flowing. Hence, if the effective length of a particular material and geometry can be determined, this can be applied over a range of values of current.

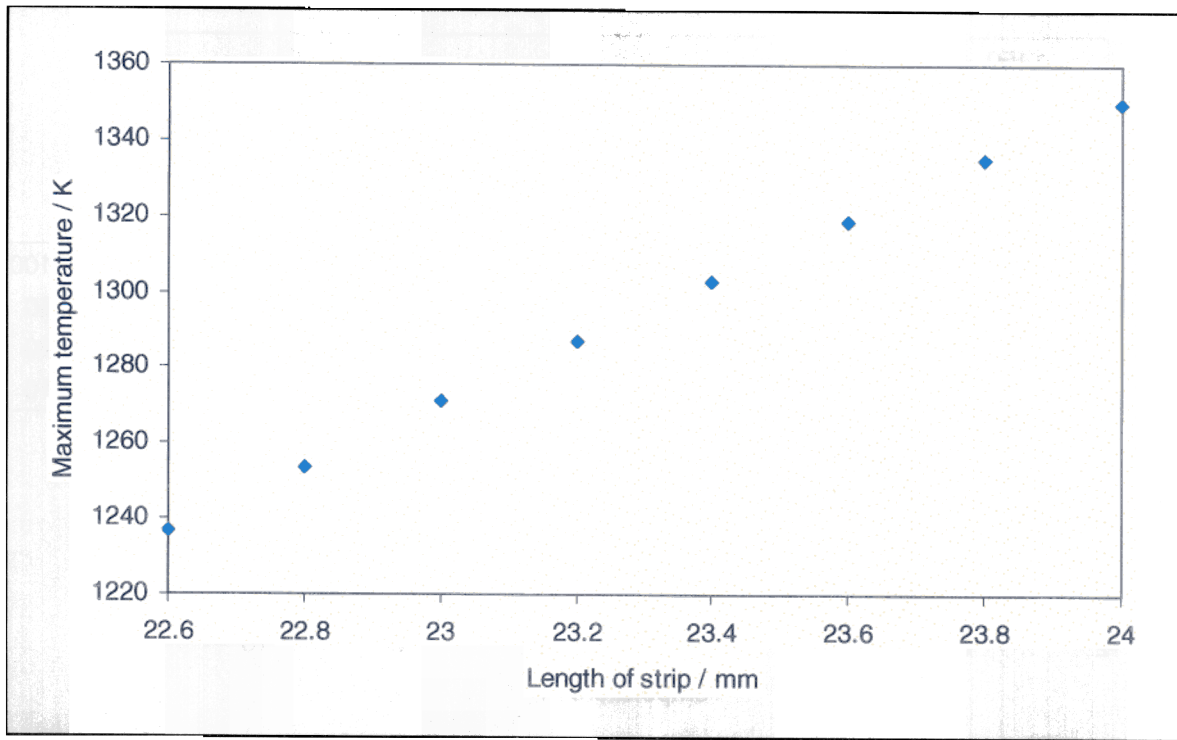
The variation of effective length with thermal conductivity indicates the complexity of the phenomenon of heat removal. For low thermal conductivities ($< 20 \text{ W m}^{-1} \text{ K}^{-1}$), the temperature profile along the bar deviates significantly from an inverted parabola. This is because the heat flow along the length of the bar is small in comparison to the heat loss at the surface of the bar. The additional effective length for this temperature range appears to be related to the ability for heat to flow down the thermal gradient close to the clamps. Hence for highly conductive materials, the effective length is longer. This can be seen in the temperature distribution plot (graph 4.5) which clearly shows a longer length of increased temperature within the clamps for $k = 100 \text{ W m}^{-1} \text{ K}^{-1}$.



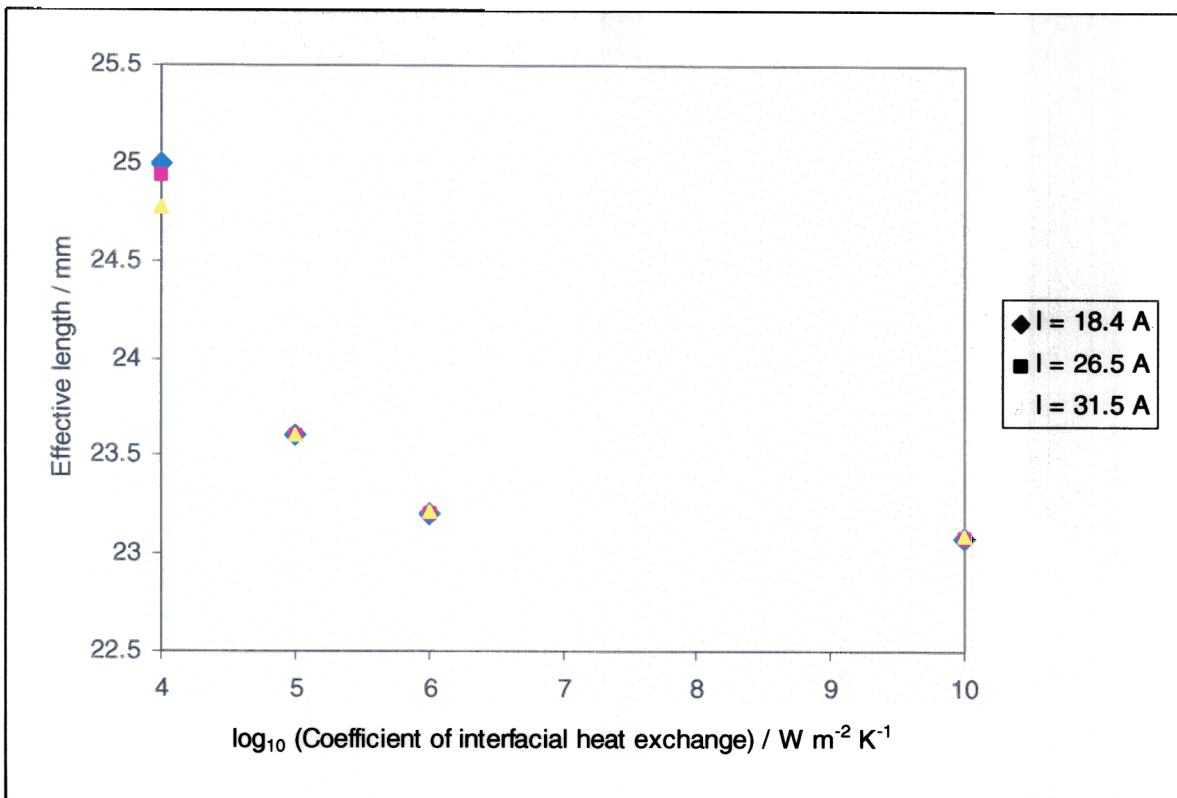
Graph 4.1: Central temperature as a function of strip length for current = 18.4 A. Used to calculate effective length.



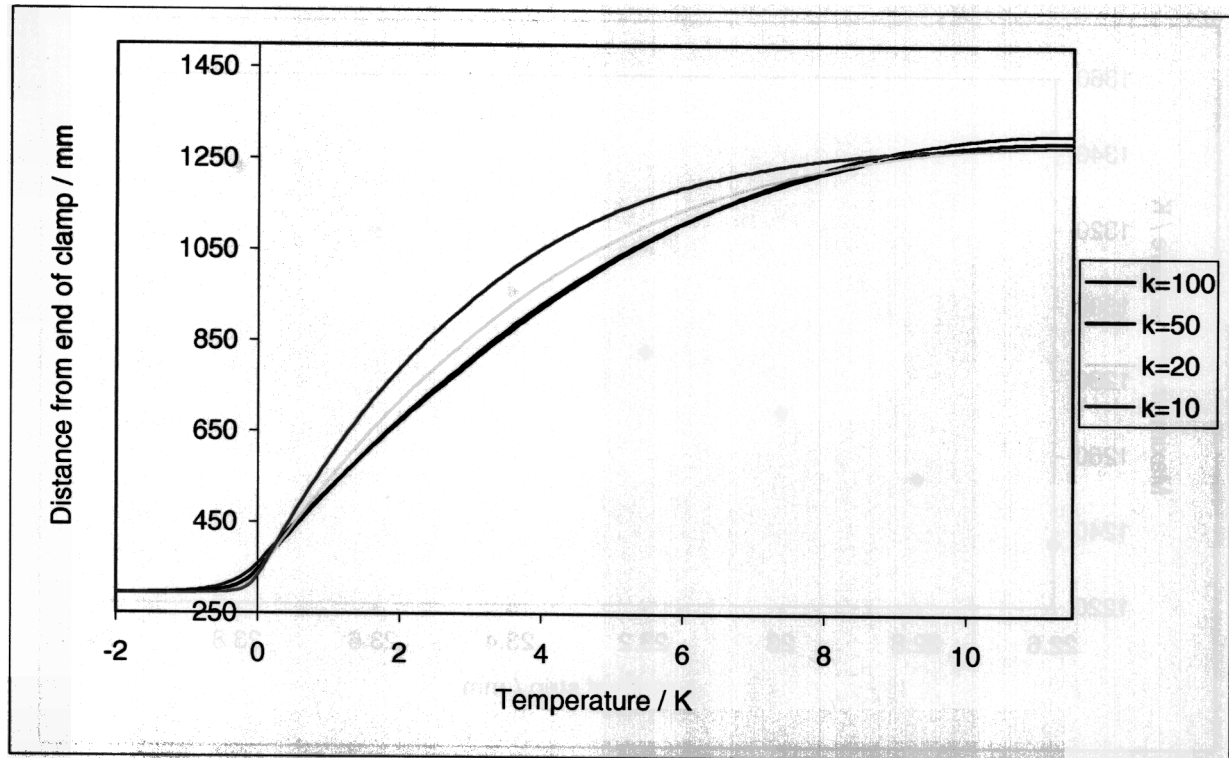
Graph 4.2: Central temperature as a function of strip length for current = 26.5 A.



Graph 4.3: Central temperature as a function of strip length for current = 31.5 A.



Graph 4.4: Effective length as function of coefficient of interfacial heat exchange. Value of $10^{10} \text{ W m}^{-2} \text{K}^{-1}$ represents an infinite coefficient. It should be noted that the effective length is independent of current for higher values of interfacial heat exchange.



Graph 4.5: Temperature distribution along bars clamped bars for 4 different thermal conductivities.

5 MODELLING OF IRON WITH AN OXIDE SCALE

The objective of this section was to find the cross-sectional temperature profile in a bar of iron surrounded by an oxide scale halfway between the clamps. This would allow an estimation of the temperature drop across the oxide to be made. Analytic and FE analyses were used to model varying thicknesses and thermal conductivities of the oxide. This data is useful because it should assist in the determination of emissivity from pyrometry data.

5.1 METHOD

FE models were constructed to represent iron bars with a free length of 22 mm with height and width equal to 1 mm and 2 mm respectively. One eighth of the bar was modelled, a rectangular block measuring roughly 0.5 mm x 1.0 mm x 11 mm, surrounded by an oxide of uniform thickness on two sides. The rest of the bar was related by symmetry to the volume modelled. A cross-section can be seen in appendix 2. The oxide layer was assumed to have a perfect thermal contact with the iron, and to be non-porous.

Four model geometries were set up, with oxide coatings of 0, 50, 75 and 100 μm . The oxide could only be one element thick due to restrictions of modelling. It was assumed that the change in dimensions of the iron consumed to form the oxide would not effect the temperature distributions significantly. It would have an effect on the current needed to achieve a particular central temperature but as the modelling work is not being directly compared to experimental results, the change in dimensions in the iron has been ignored.

Analytical solutions were derived from a one-dimensional form of the Fourier equation. The analysis was carried out on a line midway along the length of the specimen. The line ran from the centre of the specimen to the outer edge of the oxide. This analysis assumed constant thermal properties, no heat generation within the oxide and that the thermal gradient was negligible in directions perpendicular to the line being analysed.

The surface of the oxide was assumed to be at 1273 K. The heat flux over the surface was calculated and it was assumed that this heat flux was equal to that flowing down the temperature gradient within the oxide. The temperature distribution within the metal was assumed to be parabolic with the heat flux at the oxide-metal interface equal to that within the oxide.

$$j_{\text{surface}} = j_{\text{oxide}} = j_{\text{iron}}$$

Equation 5.1: Heat fluxes, j , equal at surface of oxide, across the oxide and in the iron at the metal-oxide interface.

$$h_c \Delta T + \epsilon \sigma_{\text{Boltz}} (T_s^4 - T_a^4) = k_{\text{oxide}} \left. \frac{dT}{dx} \right|_{\text{oxide}} = k_{\text{iron}} \left. \frac{dT}{dx} \right|_{\text{iron}}$$

Equation 5.2: Substituting expressions for heat fluxes in equation 5.1. The left hand side can be calculated and the thermal conductivities are known, hence the thermal gradients of the oxide and iron can be found.

$$T_{\text{drop}} = \frac{d_{\text{oxide}}}{k_{\text{oxide}}} \left[h_c \cdot \Delta T + \varepsilon \cdot \sigma_{\text{Boltz}} \cdot (T_s^4 - T_a^4) \right]$$

Equation 5.3: Substituting into equation 5.2 and rearranging to yield an expression for the temperature drop across the oxide.

A comparison was made between this analytical method and the finite element analysis. This allowed the validity of the analytical assumptions to be evaluated. An example of this comparison can be seen in graph 5.4.

5.2 PROBLEMS ENCOUNTERS

There were a number of problems of how to represent the bar adequately in the finite element models. The first problem was what thermal properties to assign to the oxide; the second problem was an apparent discontinuity in behaviour of maximum temperature as a function of current.

Thermal Conductivity of the Oxide

Optical micrographs of oxides formed from steel heated in air within the ETMT were examined and found to be heterogeneous. The oxide appeared to have 3 or 4 layers, some of them two-phase. Porosity appeared in some layers. The modelling attempted to represent this complex layer as a single coherent layer.

Hematite (Fe_2O_3) was chosen as the representative oxide on which to base the thermal properties of the oxide layer because it forms a stable coherent layer where present. Also, most of the relevant temperature-dependent thermal data were available. These data are given in appendix 1.

Thermal conductivity and electrical conductivity data for the oxide proved difficult to locate. Hematite was assumed to be an insulator so a negligible value of $1 \Omega^{-1} \text{ m}^{-1}$ was entered for the electrical conductivity. Touloukian [5] quotes a room temperature value of $0.65 \text{ W m}^{-1} \text{ K}^{-1}$ for the thermal conductivity. At higher temperatures, data was available for hematite pellets (12-14 mm diameter) formed from hematite ore ground to $44 \mu\text{m}$ particles [6]. From this, thermal conductivity could be estimated at $0.95 \pm 0.15 \text{ W m}^{-1} \text{ K}^{-1}$. This value was for a porous pellet and hence non-porous hematite will probably have a higher value of thermal conductivity. Hence this value can be thought of as an underestimate.

Although a hematite scale has been assumed, this is rarely the case. Burke and Higginson [2] have used EBSD to show that, for interstitial free steel, the prevalent oxide is wustite (FeO) above 700°C and magnetite (Fe_3O_4) below 700°C . The thermal conductivity of magnetite is $4.1 \text{ W m}^{-1} \text{ K}^{-1}$ at 180°C [5]. No data could be found for wustite.

For the modelling studies, a range of thermal conductivity values of 0.5, 1, 2, and $4 \text{ W m}^{-1} \text{ K}^{-1}$ was thus used; $0.5 \text{ W m}^{-1} \text{ K}^{-1}$ represents a porous hematite layer, $4 \text{ W m}^{-1} \text{ K}^{-1}$ a coherent magnetite layer. This allowed the temperature drop to be evaluated as a function of thermal conductivity.

Current-Temperature Discontinuity

This problem manifested itself when attempts were made to achieve a central bar temperature

of 1273 K within FE models. The temperature was seen to increase discontinuously at a certain current rendering it impossible to achieve the desired temperature distribution. This behaviour had a number of possible causes:

1. Modelling errors: the mesh was too coarse in the region where the temperature was changing rapidly.
2. It is a real phenomenon, i.e. it would be impossible to achieve some temperatures experimentally with a steady current.
3. One or more of the assumptions of the model are incorrect.

The magnitude of the discontinuity was reduced if a constant thermal conductivity was used and could not be detected if a constant electrical conductivity was used. This suggests that it is likely to have been caused by the large variation with temperature of these parameters (especially the latter). Furthermore, quadrupling the node density along the length of the bar had no effect on the results. More research needs to be carried out to be certain of the reason for this discontinuity.

To complete the FE modelling work of this section, modified thermal and electrical properties were used at low temperatures. This should not affect the results close to the centre of the bar but does mean that the currents used will be slightly in error. A typical temperature distribution map can be seen in graph 5.5 at the end of this section.

5.3 RESULTS

Oxide Thickness / μm	$T_{\text{drop}} / \text{K}$	
	Model	Analytical
50	8 ± 1	7.7
75	12 ± 1	11.5
100	16 ± 1	15.3

Table 5.1: Variation in temperature drop over oxide with oxide thickness. Errors quoted are the precision to which ABAQUS gives results. Graph 5.1 displays these results.

Thermal Conductivity / $\text{W m}^{-1} \text{K}^{-1}$	$T_{\text{drop}} / \text{K}$	
	Model	Analytical
0.5	4 ± 1	3.6
1.0	8 ± 1	7.3
2.0	15 ± 1	14.5
4.0	29 ± 1	29.1

Table 5.2: Variation in temperature drop over oxide with thermal conductivity. Graph 5.2 illustrates these results.

Emissivity	T_{drop} / K
0.2	3.0
0.4	6.0
0.6	8.9
0.8	11.9
1.0	14.9

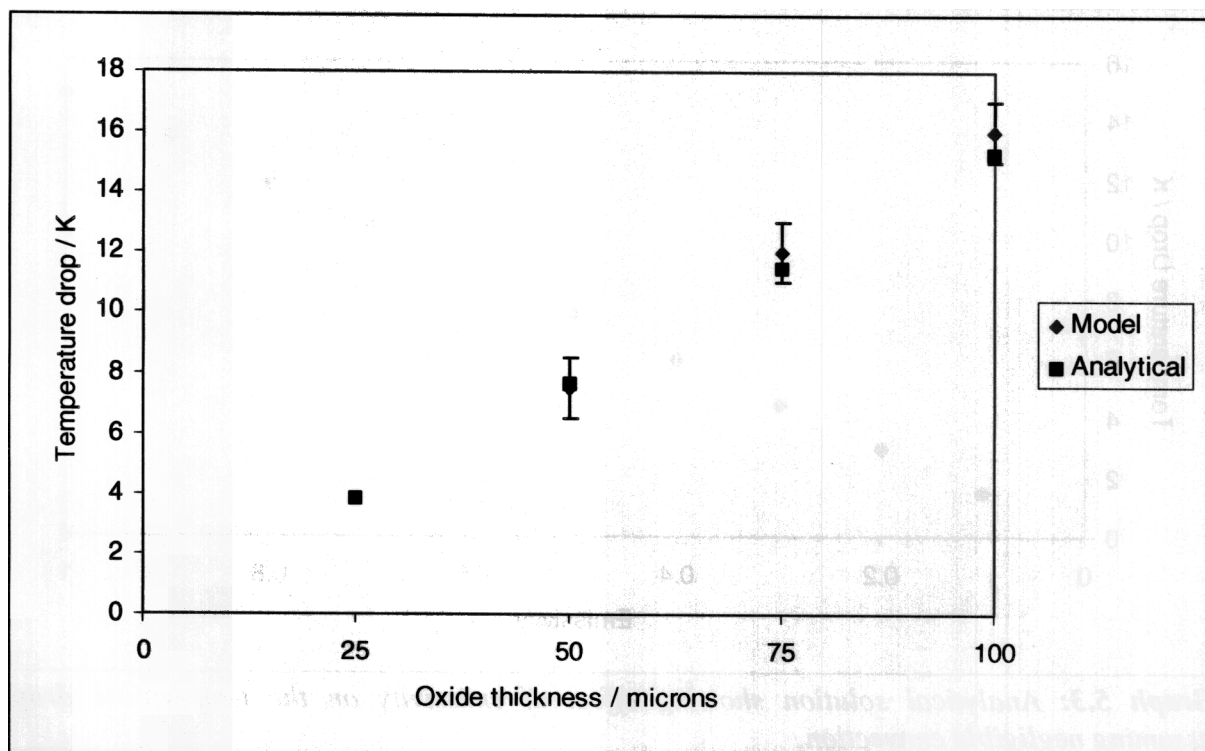
Table 5.3: Variation in temperature drop over oxide with emissivity at oxide surface. Convection was assumed to be negligible. Graph 5.3 illustrates these results.

5.4 CONCLUSIONS

The results of analytical modelling suggest that the temperature drop across the oxide layer is proportional to the thickness and emissivity of the oxide and inversely proportional to the thermal conductivity. The results of the FEA were in accordance with the analytical solutions, and hence backed up these relationships.

The temperature profiles created by analytical solutions also agree with the FEA within the iron bar. This suggests that the assumption that the heat flux perpendicular to the line of analysis is negligible is valid. This is likely to be due to the fact that the line of analysis was also a line of symmetry.

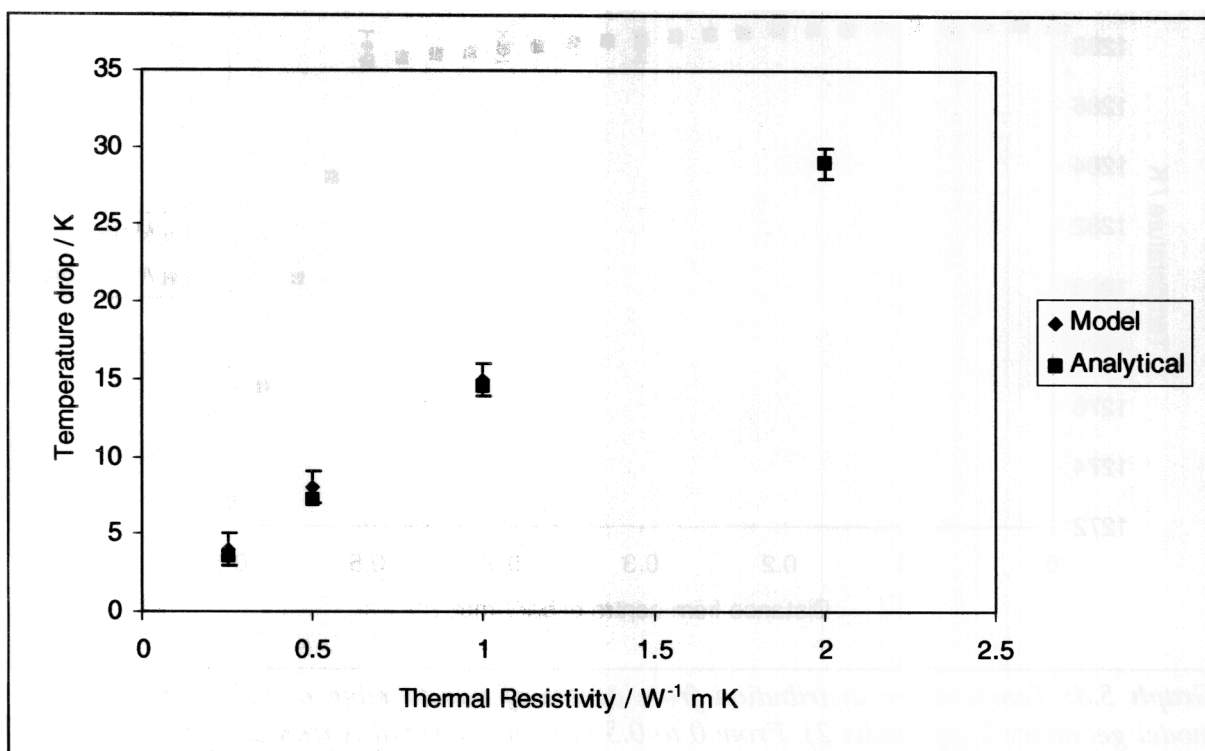
This modelling of metal-oxides systems had many simplifying assumptions. The perfect thermal contact between the metal and the oxide was the first. A thermal barrier can be modelled within the analytical solution by including a temperature drop at the interface that has a heat flow across it equal to that within the oxide. In the same way, interfaces between the various layers can be modelled. The porosity and the two-phase nature of some layers can be modelled by using an effective thermal conductivity that accounts for these more complex morphologies. The question of how to collect data for the modelling work is considered in section 6.



Graph 5.1: Temperature drop across oxide as a function of oxide thickness. Error bars show the precision of modelling results.

Thermal conductivity of oxide = $0.95 \text{ W m}^{-1} \text{ K}^{-1}$

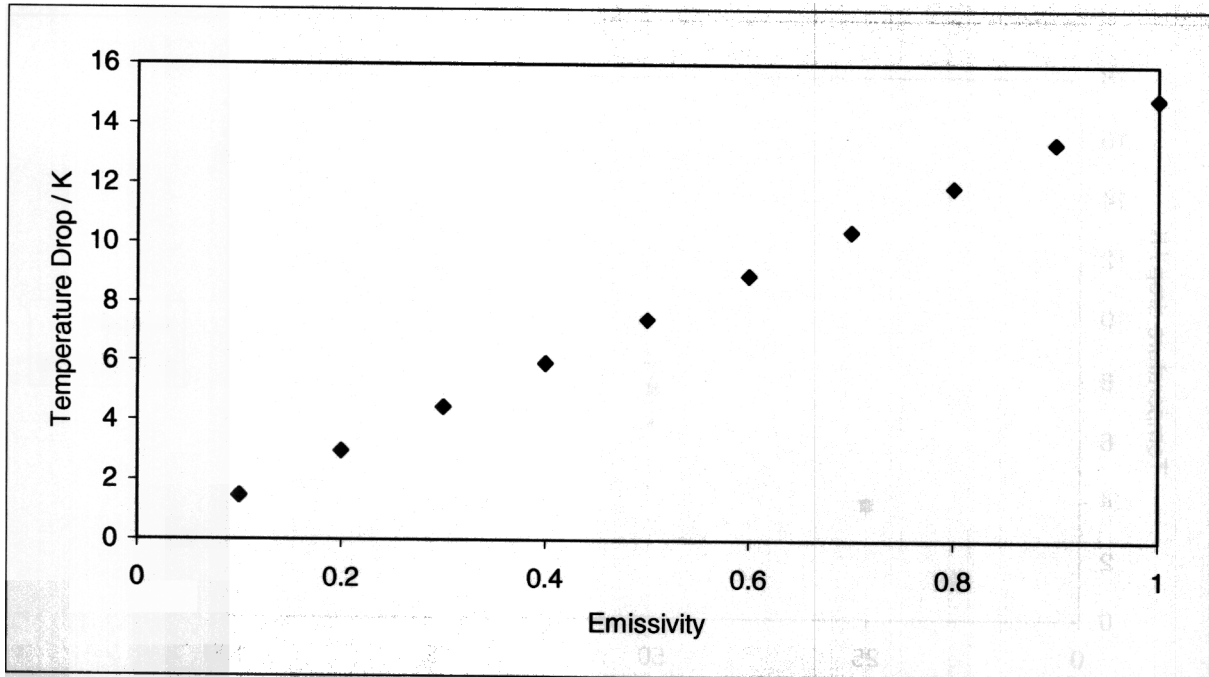
Heat flux from surface of oxide = $1.45 \times 10^5 \text{ W m}^{-2}$



Graph 5.2: Temperature drop across oxide as a function of thermal resistivity.

Oxide thickness = $100 \text{ } \mu\text{m}$

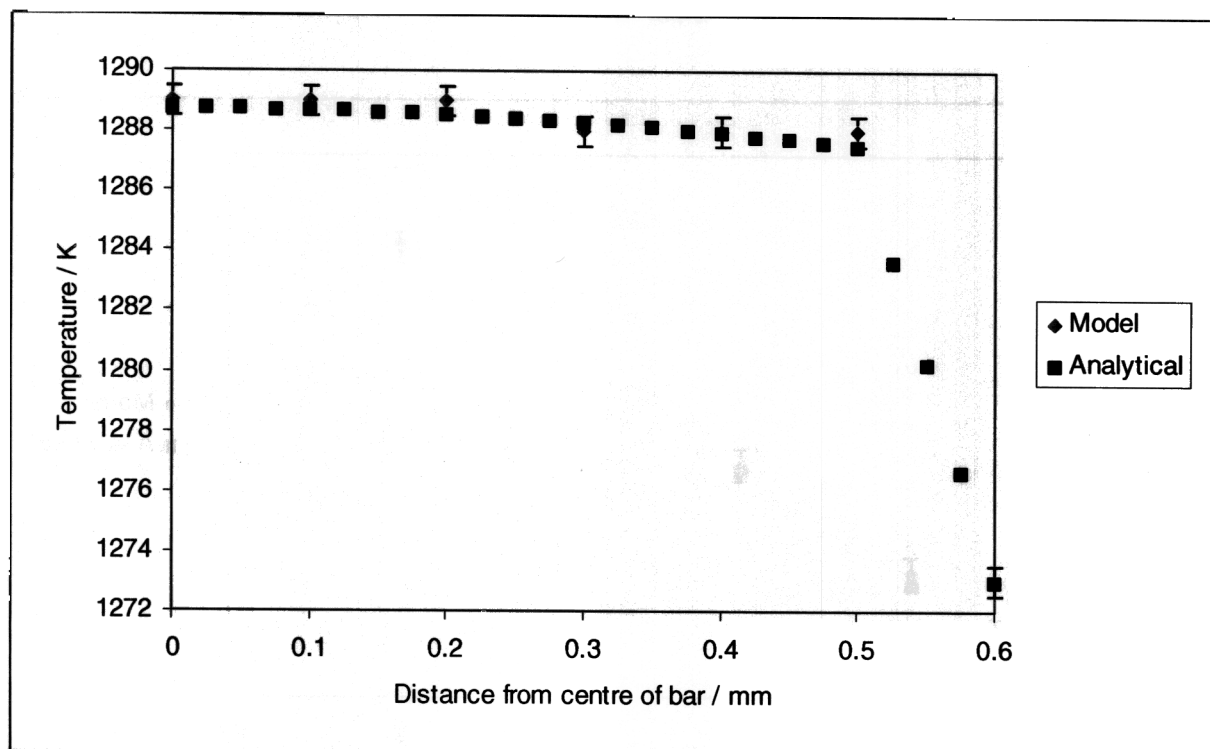
Heat flux from surface = $1.45 \times 10^5 \text{ W m}^{-2}$



Graph 5.3: Analytical solution showing effect of emissivity on the temperature drop assuming negligible convection.

Thermal conductivity of oxide = $0.95 \text{ W m}^{-1} \text{ K}^{-1}$

Oxide thickness = $100 \text{ } \mu\text{m}$

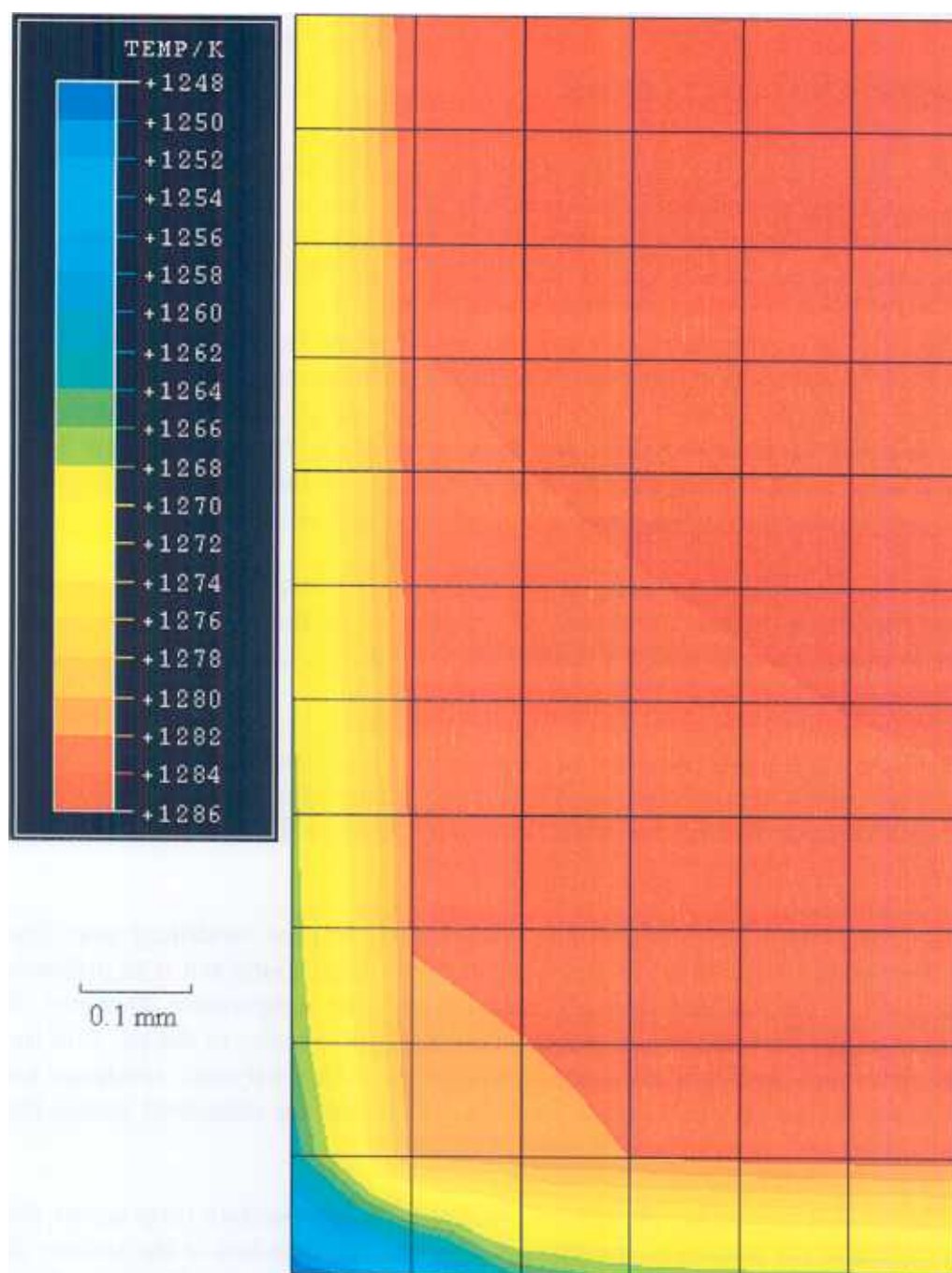


Graph 5.4: Temperature distribution from centre of bar to edge of oxide (described in model geometry 2, appendix 2). From 0 to 0.5 mm, the material is iron and from 0.5 to 0.6 mm the material is oxide.

Thermal conductivity of oxide = $0.95 \text{ W m}^{-1} \text{ K}^{-1}$

Heat flux from surface = $1.45 \times 10^5 \text{ W m}^{-2}$

Oxide thickness = $100 \text{ } \mu\text{m}$



Graph 5.5: Temperature distribution map for quadrant of cross-section halfway between clamps. Left-hand and lower elements composed of oxide, 100 μm thick.

Thermal conductivity of oxide = $0.95 \text{ W m}^{-1} \text{ K}^{-1}$

Emissivity of oxide surface = 0.85

Convective heat transfer coefficient = $15 \text{ W m}^{-2} \text{ K}^{-1}$

Thermal conductivity of iron = $29.7 \text{ W m}^{-1} \text{ K}^{-1}$

6 DISCUSSION AND CONCLUSIONS

It has been shown that temperature distributions within simple one-phase systems such as platinum can be successfully modelled. The sensitivity of the thermal distribution to input variables was determined. Thus from a measurement of the centre temperature it is possible to estimate some unknown parameters if other variables are known with confidence.

The work in section 4 is a refinement that may be useful when further modelling of the ETMT is carried out. It shows that significant variations in temperature distributions arise from the clamping conditions of the ETMT. These are only rough estimates because the geometry of the situation has been simplified and because interfacial heat transfer data is not available. Experimental work measuring the temperature of various positions along the length of an iron bar should be able to measure the effective length of the iron bar.

This problem could be minimised by using substantially longer bars and therefore smaller heating currents. The longer the bar, the flatter the temperature profile in the central region. This would lead to experiments in which a small portion of the bar could be considered as isothermal. [7]

The work on platinum strip highlighted the non-linearity of this system. 'Regimes' can be seen to operate over certain temperature ranges in which some effects dominate over the others. This modelling work allowed the important variables to be determined for relevant temperature ranges.

The modelling approach used for platinum was not so successful for modelling iron. The reasons for the discontinuity described in section 5 have not been found but it is probably due to large variation in thermal and electrical conductivity with temperature. However, it was still possible to model the temperature distribution close to the centre of the bar with the correct material properties and this allowed a comparison with analytical solutions for temperature gradients across an oxide layer. Analytical solutions are consistent within the range of data used here.

This work has shown that for the simple system modelled, the temperature drop across the oxide is simply related to the thickness, thermal conductivity and heat loss at the surface of the oxide. This work can easily be extended to model more complex systems. However, there is a lack of data to adequately describe the behaviour of the oxides and the various interfaces present in scaled ferrous bars. Without this data, increasing the complexity of the model is not advantageous because the results will be of high uncertainty.

7 ACKNOWLEDGEMENTS

Support for this work was provided by a NPL Strategic Research Project, 9SRP6030, *The Emissivity of Oxide Scales*.

8 REFERENCES

- [1] B. Roebuck, M.G. Gee, M. Brooks, D. Cox, R. Reed
Miniature Multiproperty Tests at Elevated Temperatures
Conference Paper: COMPASS 99, Swansea, UK, March 29-April 1, 1999
- [2] D.P. Burke, R.L. Higginson
Characterisation of Multicomponent Scales by Electron Back Scattered Diffraction
Scripta Mater. **42**, No.3, 2000, pp. 277-281
- [3] H. Schuh
Heat Transfer in Structures
Pergamon Press, 1965
- [4] T.W. Clyne,
Personal correspondence with the author
- [5] Y.S Touloukian,
Thermophysical Properties of Matter
IFI/ Plenum Data Corporation, 1972
ISBN 0-306-67020-8
- [6] Watts, J. K. Wright,
Thermal Diffusivities of Dried Hematite Pellets
Trans. Instn. Min. Metall. (sect. C), **91**, March 1982, pp 18-20
- [7] S.C. Jain and K.S. Krishan
The Distribution of temperature along a thin rod electrically heated in vacuo
Proc. Royal Soc., (Series A), **222**, No. 1149, 23 March 1954, pp 167-180

9 NOMENCLATURE

Dimensional Analysis

Symbol	Parameter	Units	Dimensions
h	Total heat transfer coefficient over gas film	$\text{W m}^{-2} \text{K}^{-1}$	$\text{M t}^{-3} \text{T}^{-1}$
L	Characteristic length	m	L
k	Thermal conductivity	$\text{W m}^{-1} \text{K}^{-1}$	$\text{M L t}^{-3} \text{T}^{-1}$
u_{∞}	Velocity of fluid	m s^{-1}	L t^{-1}
ν	Kinematic viscosity	m s^{-2}	$\text{L}^2 \text{t}^{-1}$
α	Thermal diffusivity	m s^{-2}	$\text{L}^2 \text{t}^{-1}$
c_p	Specific heat capacity	$\text{J kg}^{-1} \text{K}^{-1}$	$\text{L}^2 \text{t}^{-2} \text{T}^{-1}$
η	Viscosity	Pa s	$\text{M L}^{-1} \text{t}^{-1}$
g	Acceleration due to gravity	m s^{-2}	L t^{-2}
β	Thermal expansion coefficient	K^{-1}	T^{-1}
ΔT	Temperature difference between surface and surroundings	K	T
ρ	Density	kg m^{-3}	M L^{-3}

Other Symbols

Symbol	Parameter	Units
T	Temperature	K
T_a	Ambient temperature	K
T_s	Surface temperature	K
ε	Emissivity	Ratio
σ_{Boltz}	Boltzmann factor	$\text{W m}^{-2} \text{K}^{-4}$
t	Time	s
I	Current	A
x, y, z	Orthogonal directions vectors	m
d	Thickness	m
ε	Emissivity	ratio
q	Heat generation term	W m^{-3}
h_c	Convective heat transfer coefficient	$\text{W m}^{-2} \text{K}^{-1}$
h_i	Interfacial heat transfer coefficient	$\text{W m}^{-2} \text{K}^{-1}$
h_{tot}	Effective heat transfer coefficient – includes radiative and convective terms	$\text{W m}^{-2} \text{K}^{-1}$
σ	Electrical conductivity	$\Omega^{-1} \text{m}^{-1}$
I	Electric current	A
j	Current density	A m^{-2}

10 APPENDICES

APPENDIX 1: MATERIAL PROPERTIES

The tables of material properties shown here were taken from various references. They were entered as material data tables in the ABAQUS input files. ABAQUS linearly interpolates between points to obtain a value for a property at a given temperature. For temperatures in excess of the tabulated data, the analysis assumes the property has a value equal to that of the highest temperature.

Platinum

Density: [i,ii]

T / K	$\rho / \text{kg m}^{-3}$
293	21 450
800	21 137
1400	20 723
1900	20 311

Specific Heat Capacity: [i]

T / K	$c_p / \text{J kg}^{-1} \text{K}^{-1}$
290	131.5
900	148.8
1500	166.2
2000	189.0

Thermal Conductivity: [i]

T / K	$k / \text{W m}^{-1} \text{K}^{-1}$
290	73.2
500	71.9
700	72.3
1000	74.8
1300	79.1
1800	87.7

Electrical Conductivity: [iii]

T / K	$\sigma / 10^6 \Omega^{-1} \text{m}^{-1}$
290	9.51
500	5.49
800	3.52
1200	2.46
1500	2.05
2000	1.67

Emissivity: [i,ii]

T / K	ϵ (ratio)
290	0.035
500	0.064
700	0.091
1000	0.127
1300	0.157
1600	0.179
1900	0.191

Density: [i,ii]

T / K	$\rho / \text{kg m}^{-3}$
298	7 874
673	7 747
873	7 679
1043	7 622
1184	7 574
1185	7 650
1273	7 605
1394	7 448
1395	7 441
1538	7 350

Thermal Conductivity: [i]

T / K	$k / \text{W m}^{-1} \text{K}^{-1}$
300	80.3
400	69.4
500	61.3
600	54.7
700	48.7
800	43.3
900	38.0
1000	32.6
1100	29.7
1200	28.2
1300	29.9
1400	30.9
1500	31.8

Specific Heat Capacity: [i]

T / K	$c_p / \text{J kg}^{-1} \text{K}^{-1}$
293	441.0
323	453.6
373	478.8
473	520.8
573	554.4
673	604.8
773	663.6
873	756.0
923	819.0
973	911.4
1023	1134.0
1033	1335.6
1073	835.8
1123	730.8

Specific Heat Capacity: (cont.)

T / K	$c_p / \text{J kg}^{-1} \text{K}^{-1}$
1173	659.4
1223	554.4
1273	571.2
1323	588.0
1373	609.0
1423	625.8
1473	642.6
1523	655.2

Electrical Conductivity: [iii]

T / K	$K / 10^6 \Omega^{-1} \text{m}^{-1}$
273	11.7
298	10.4
400	6.21
472	4.48
548	3.42
572	3.16
651	2.48
671.5	2.34
750	1.90
771	1.81
850	1.49
871	1.41
951	1.18
971	1.13
1018	1.02
1060	0.955
1068	0.950
1109	0.921
1118	0.917
1151	0.903
1170	0.898
1213	0.889
1219	0.885
1265	0.873
1269	0.872
1321	0.857
1368	0.845
1388	0.840

Iron (cont.)**Emissivity: [i]**

T / K	ϵ (ratio)
83.2	0.01
1111	0.19

1200	887
------	-----

Hematite**Density: [i]**

T / K	ρ / kg m ⁻³
293	5040
400	5023
500	5007
600	4989
700	4971
800	4952
900	4932
1000	4913
1100	4892
1200	4871
1300	4851
1400	4830

Specific Heat Capacity: (cont.)

T / K	C_p / J kg ⁻¹ K ⁻¹
1300	891
1400	896
1500	901
1600	905
1700	909
1750	912

Electrical Conductivity

Values for electrical conductivity could not be found. A negligible value of 1 Ω^{-1} m⁻¹ was assumed.

Thermal Conductivity:

Calculated from thermal diffusivity data

T/K	k / W m ⁻¹ K ⁻¹
293	0.65 [i]
1273	0.95 [iv]

Emissivity: [ii]

T / K	ϵ (ratio)
373	0.74
773	0.84
1473	0.89

Specific Heat Capacity: [i]

T / K	C_p / J kg ⁻¹ K ⁻¹
298	750
400	805
500	856
600	906
700	955
800	1004
900	1053
950	1078
951	944
1000	944
1050	944
1051	880
1100	882

References

- [i] Y.S Touloukian,
Thermophysical Properties of Matter
IFI/ Plenum Data Corporation, 1972
ISBN 0-306-67020-8

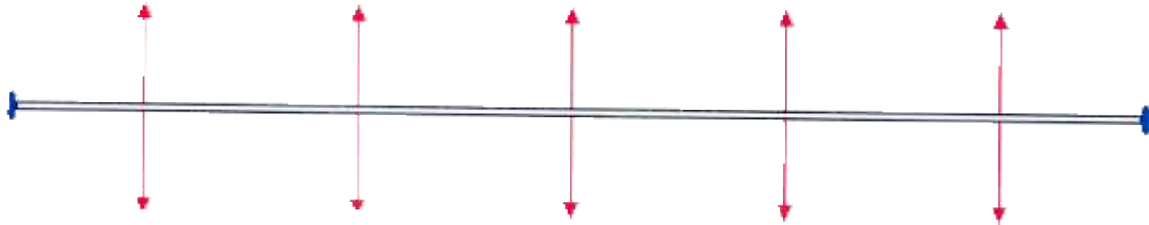
- [ii] Ed. R.C. Weast
Handbook of Chemistry and Physics,
Published CRC Press Inc, 1979
ISBN 0-8493-0460-8

- [iii] L. Binkle and M. Brunen.
Thermal Conductivity, Electrical Resistivity and Lorenz Function Data for Metallic Elements in the range 273 to 1500 K
Report no. JUEL-3006
Issued by Forschungszentrum Julich Germany, December 1994.

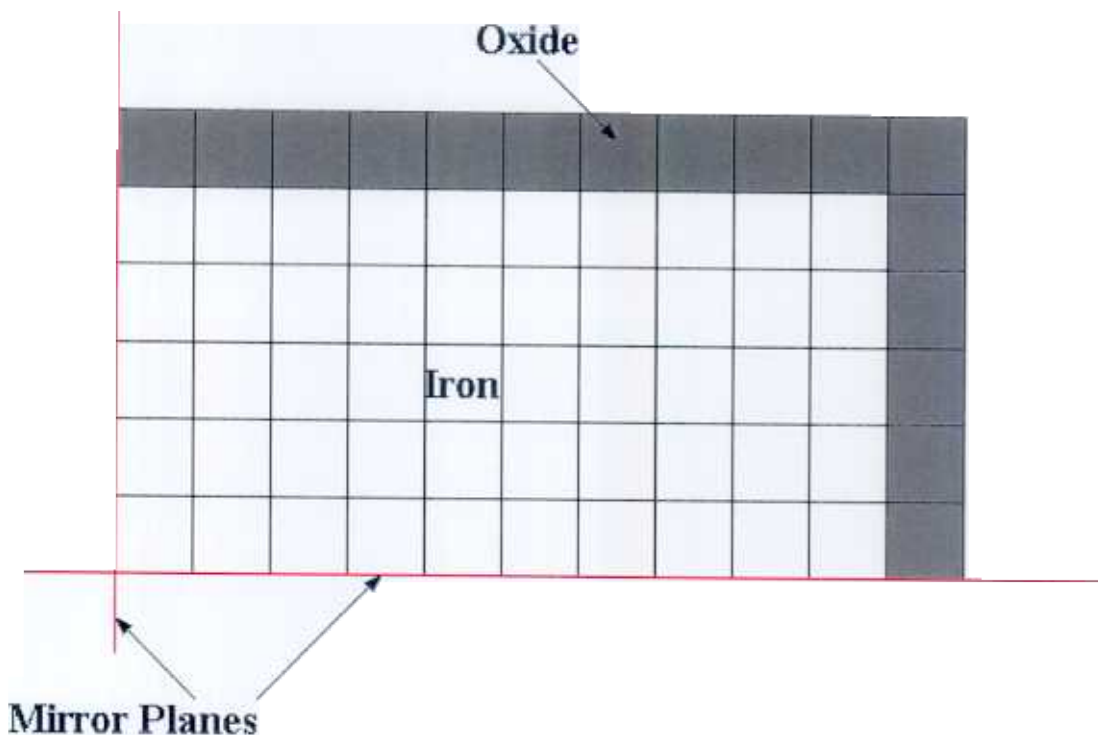
- [iv] Watts, J. K. Wright,
Thermal Diffusivities of Dried Hematite Pellets
Trans. Instn. Min. Metall. (sect. C), **91**, March 1982, pp 18-20

APPENDIX 2: EXAMPLES OF MODEL GEOMETRIES

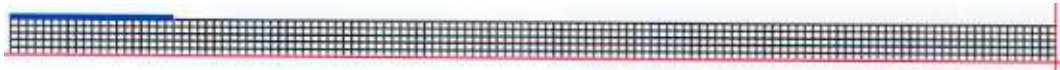
This section outlines some of the model geometries used within the investigations.



Geometry Number 1: Two-dimensional platinum bar. Blue lines represent nodes held at 294 K. Red lines show radiating and convecting surfaces. Geometry length is 24 mm (with 160 elements) and the height is 0.24 mm (4 elements; elements are not shown in diagram). Related three-dimensional models have a depth of 2 mm. The nodes of the left hand face are given a fixed potential (earthed) while a current of fixed magnitude was passed through the nodes on the right hand side.



Geometry number 2: Cross-section of iron with 0.1 mm thick oxide layer. The elements have dimensions of 0.1 mm. The dimensions of the octant of the bar modelled are 0.6 mm x 1.1 mm x 11 mm. Different thicknesses of oxide sometimes required different numbers of elements within the iron. The comparison of analytical and modelling solutions in graph 4.3 was made for the left hand edge of this cross-section.



Geometry number 3: Two-dimensional geometry for exploring the effect of clamping. This example corresponds to a quarter of an iron bar. The red lines again represent mirror planes. There is 2 mm of the bar within the clamps and a further 'free' length of 11 mm modelled. The blue region represents the thermal contact between the clamp and the bar.

Platinum strip was modelled in the same way, the only difference in geometry being the height of the strip.

APPENDIX 3: EMPIRICAL DETERMINATION OF CONVECTIVE HEAT LOSS

Modelling of convective heat losses is complicated due to the many processes that occur during convection. Calculations based on first principal are too complex for this modelling; hence an empirical approach was used in this project. This involved calculating dimensionless parameters that describe properties of the gas and the geometry of the surface. Empirical equations exist which relate the convective heat loss coefficient to these dimensionless parameters. These equations have been shown to fit extensive data. [a] Physical interpretations of the relationships do exist although they have not been derived from first principles.

The dimensionless parameters used were:

- Nusselt number (Nu) = heat transfer / conductive heat transfer (in absence of convection).

$$Nu = \frac{hL}{k}$$

- Reynolds number (Re) = inertial forces / viscous forces.

$$Re = \frac{u_{\infty}L}{\nu}$$

- Prandtl number (Pr) = momentum diffusivity / thermal diffusivity.

$$Pr = \frac{\nu}{\alpha} = \frac{c_p \eta}{k}$$

- Grashof number (Gr) = buoyant forces / viscous forces

$$Gr = \frac{g\beta L^3 \Delta T}{\nu^2}$$

Symbols are defined in section 9.

These dimensionless parameters characterise the flow of gas over the solid's surface. The exact values that determine the type of flow are dependent on the nature of the surface: curved, horizontal or vertical, and whether the convection is natural (caused by temperature difference between the solid and fluid) or forced.

Calculations for ETMT Specimens

The platinum bar has 4 faces. The relative area of the horizontal faces is much greater than the vertical faces, hence the latter were assumed to have the same convective heat loss coefficient as the former. The platinum bar is placed in an enclosed environment with argon, oxygen or air pumped through the system. This flow of gas will allow forced convection to occur. Natural convection will also occur; both forms of convection are considered here.

Parameter	Value
k	$0.018 \text{ W m}^{-1} \text{ K}^{-1}$
ν	$1.2 \times 10^{-4} \text{ m}^2 \text{ s}^{-1}$
α	$1.79 \times 10^{-4} \text{ m}^2 \text{ s}^{-1}$
c_p	$520 \text{ J Kg}^{-1} \text{ K}^{-1}$
η	$2.2 \times 10^{-4} \text{ Pa s}$
β	$3.67 \times 10^{-3} \text{ }^\circ\text{C}^{-1}$
ρ	1.78 Kg m^{-3}

Table 107.1: Data for argon at 298 K and 1 atmosphere

The data for argon in table 10.1 was used to calculate the dimensionless parameters. These gave a value 6.4 for Pr. The Grashof number is only used in free convection; the characteristic length was taken to be the shortest side of the upper face (i.e. 2 mm) and this gave Gr equal to 20.0.

The velocity of the fluid must be known to evaluate the Reynolds number in forced convection analyses. For this calculation, a generous value of 0.1 m s^{-1} was assumed. The characteristic length was the maximum distance the gas can flow over the surface. This was assumed to be the length of the bar: 24 mm. This gave a Reynolds number of 20.

Free Convection

The values of Gr and Pr are outside of the ranges for the equations given by Schuh [a] for horizontal faces. However, Schuh also provides graphs of $\log_{10}(\text{Nu})$ versus $\log_{10}(\text{Gr.Pr})$, thus allowing the Nusselt number to be estimated from Gr and Pr. For $\log_{10}(\text{Gr.Pr}) = 2.1$, $\log_{10}(\text{Nu}) \approx 0.4$, hence for the upper surface, Nu equals 2.5.

Problems arise when trying to calculate the heat transfer coefficient for surfaces from the Nusselt number. The Nusselt number, as already stated, is the ratio of the total heat transfer coefficient to heat transfer by conduction. The total heat transfer is h and the conductive heat transfer coefficient is given by K/L . Schuh states that K is the thermal conductivity of the film. If this is the case, then the distance L should be the distance across the film to the heat sink (either another surface or a fast moving fluid) or to a distance at which the temperature of the fluid is the same as the ambient temperature. Schuh also states that this distance is equal to the characteristic dimension used to calculate the Grashof number i.e. the width of the bar. Unless the two distances are comparable, then this interpretation appears to be false.

Applying the equation leads to a value of $23 \text{ W m}^{-2} \text{ K}^{-1}$ for the convective heat transfer coefficient (h_c). This equation is for an upper surface 1000 K hotter than the surrounding

medium (i.e. $\Delta T = 1000$ K). It can be shown from equations and graphs in [a] that h_c is approximately proportional to $\Delta T^{0.25}$. It can also be seen that the magnitude of convection from lower surfaces is approximately half that from the upper surfaces [a,b].

The convective heat flow would be $2.3 \times 10^4 \text{ W m}^{-2}$ compared with a radiative heat flow of $1.6 \times 10^4 \text{ W m}^{-2}$ for a surface temperature of 1293 K and a ambient temperature of 293 K. Hence the heat losses from the two mechanisms are comparable in magnitude.

By approximating the bar to a cylinder with a diameter approximately equal to 2 mm, a similar analysis can be completed. The characteristic length to calculate the Grashof and Nusselt numbers is taken to be this diameter. This analysis gives similar result to the above analysis.

Forced Convection

The average Nusselt number for non-turbulent flow over the upper horizontal face is given by equation 10.1.

$$Nu = 0.664 Pr^{0.33} Re^{0.5}$$

Equation 10.1: *Relating dimensionless parameters for forced convection. Valid for $0.6 < Pr < 50$; $Re < 5 \times 10^5$.*

Both dimensionless parameters fall into these bounds and Nu is found to be 2.6. The convective heat loss coefficient is roughly $4 \times 10^3 \text{ W m}^{-2}$ for this surface. Hence the magnitude of heat lost to forced convection is probably smaller than that of natural convection.

In conclusion, the value of heat flow cannot be calculated exactly from information available. For the modelling work, a single value of h_c was used to approximate convection from all surfaces for a given temperature. This representative value was $15 \text{ W m}^{-2} \text{ K}^{-2}$ for $\Delta T = 1000$ K with h_c proportional to $\Delta T^{0.25}$.

References

- [a] H. Schuh
Heat Transfer in Structures
Pergamon Press, 1965
- [b] M. Jakob, G. Hawkins
Elements of Heat Transfer and Insulation
John Wiley & Sons Inc. 1942