

**Measurement of the Distribution of Polarisation Mode Dispersion (PMD)
along an Optical Fibre System**

David J Ives

June 2000

© Crown Copyright 2000
Reproduced by permission of the Controller of HMSO

ISSN 1369-6807

National Physical Laboratory
Queens Road, Teddington, Middlesex, TW11 0LW

Extracts from this report may be reproduced provided the source is acknowledged and the extract is not taken out of context.

Approved on behalf of Managing Director, NPL
by D H Nettleton, Head of Centre for Optical and Environmental Metrology

CONTENTS

1	EXECUTIVE SUMMARY....	
2.	POLARISATION MODE DISPERSION	2
2.1	Overview	2
2.2	Definitions.....	2
2.3	Effect of PMD	4
3.	DISTRIBUTED MEASUREMENTS OVERVIEW	5
4.	FIXED WAVELENGTH POTDR.....	6
4.1	Experimental Arrangement	6
4.2	Theory	6
4.3	Measurements	7
4.4	Data Processing.....	7
4.4.1	Cycle Counting	7
4.4.2	Fourier Analysis.....	9
4.5	Summary	11
5.	TUNABLE WAVELENGTH POTDR.....	13
5.1	Experimental Arrangement.....	13
5.2	Measurement of 26 km of DS Fibre.....	13
5.3	Data Analysis	14
5.4	Summary	16
	APPENDIX A PULSE WIDTH REQUIREMENTS FOR POTDR.....	17
	APPENDIX B BI-DIRECTIONAL PMD.....	20
	APPENDIX C REFERENCES	

1. EXECUTIVE SUMMARY

This report describes the measurement of the distribution of polarisation mode dispersion (PMD) along fibre systems. The OTDR technique was used to allow single ended measurements of the distribution within an optical fibre system. Fixed wavelength polarised OTDR was found to give repeatable and sensitive measurements of beat length but proved unreliable in obtaining estimates of the polarisation coupling length and local PMD coefficient. Tunable wavelength polarised OTDR with a fixed wavelength type data analysis was able to provide a reasonable estimate of the accumulation of PMD along an optical fibre system.

2. POLARISATION MODE DISPERSION

2.1 Overview

The solution of the electromagnetic wave equation for single-spatial-mode optical fibre produces two orthogonal polarised modes. In ideal circularly symmetric optical fibre these modes are degenerate with identical propagation constants. In real fibres asymmetries in the core geometry and internal stress removes the degeneracy leading to different propagation constants for the two modes. The fibre layout and external stresses also destroy the degeneracy in ideal circularly symmetric optical fibres. In general the fibre becomes birefringent with different refractive indices for the two polarisation modes.

The asymmetries of the core and any internal and external stress are not generally constant along the length of the fibre. They vary in strength and direction such that the birefringence varies randomly along the length of the fibre. This leads to the splitting of the two polarisation modes into a multiple of modes and is considered a high polarisation mode coupled regime. In such a regime a short depolarised optical input pulse, in the absence of chromatic dispersion, will lead to an optical output that is a Gaussian pulse with an RMS width equal to the polarisation mode dispersion (PMD).

The transmission of a polychromatic source through a high polarisation mode coupled fibre leads to a range of output states and in general a depolarising of the light. Over a limited source bandwidth there exist two orthogonal input polarisation states which lead to two orthogonal output states which are independent of the source frequency (to first order). These are known as the principle states of polarisation (PSP)^[1]. To first order there is a phase difference between the states which is proportional to the source frequency (i.e. a time delay between the states independent of frequency) and this leads to a differential group delay (DGD).

2.2 Definitions

Throughout this report I will work with the Stokes vector description of polarised light and in particular the three element Stokes vector for fully polarised light. Thus the polarisation state is described by:

$$\mathbf{s} = \begin{pmatrix} s_1 \\ s_2 \\ s_3 \end{pmatrix} \quad (1)$$

where the vector is normalised such that $|\mathbf{s}|^2 = s_1^2 + s_2^2 + s_3^2 = 1$. In general I will use this to describe the polarisation state of light exiting or after an optical fibre system. The polarised Stokes vector, $\mathbf{s}$, lies on the surface of the Poincaré sphere see Figure 1.

$$s_1 = \frac{I_V - I_H}{I_V + I_H}, \quad s_2 = \frac{I_{+45} - I_{-45}}{I_{+45} + I_{-45}} \quad \& \quad s_3 = \frac{I_R - I_L}{I_R + I_L}. \quad (2)$$

Where I_V is the intensity measured through a vertical linear polariser, I_H through a horizontal linear polariser, I_{+45} through a polariser at 45° , I_{-45} through a linear polariser at -45° , I_R through a right hand circular polariser and I_L through a left handed circular polariser.

The polarisation mode dispersion is described by the vector Ω , such that

$$\frac{\partial \mathbf{s}}{\partial \omega} = \mathbf{\Omega} \times \mathbf{s}. \quad (3)$$

The vector $\mathbf{\Omega}$ is a function of length and optical frequency and has a magnitude equal to the differential group delay (DGD= $\delta\tau$) and a direction along the principle state of polarisation (PSP).

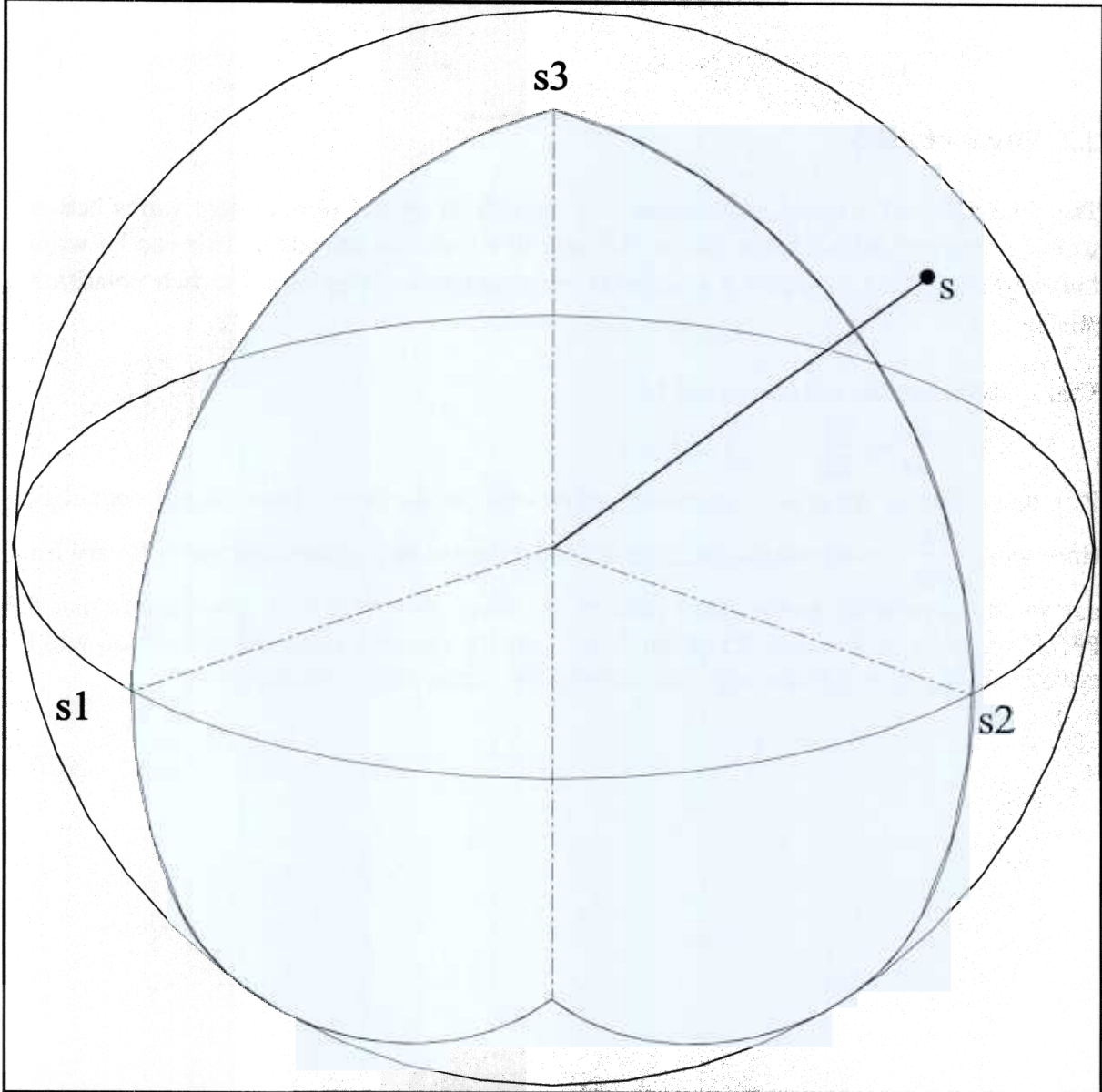


Figure 1 Poincaré Sphere.

Throughout this report I will define the polarisation mode dispersion (PMD= $\Delta\tau$) as the root mean square DGD with the average taken over either wavelength or ensemble of similar fibres.

$$\Delta\tau = \sqrt{\frac{\sum_{i=1}^N \delta\tau_i^2}{N}} \quad (4)$$

It is possible to expand $\mathbf{\Omega}(\omega)$ around ω_0 to get the polarisation effects over larger frequency bandwidths^[2].

$$\Omega(\omega) = \Omega(\omega_0) + (\omega - \omega_0) \cdot \left. \frac{\partial \Omega(\omega)}{\partial \omega} \right|_{\omega_0} + \dots = \Omega(\omega_0) + (\omega - \omega_0) \Omega_\omega(\omega_0) + \dots \quad (5)$$

where Ω_ω is referred to as the second order PMD vector. For highly mode coupled fibres the first and second order PMD are related through:

$$|\Omega_\omega|^2 = \frac{|\Omega^4|}{3}$$

2.3 Effect of PMD

The time delay of a signal, at frequency ω , through an optical fibre system varies between $\tau(\omega) + \frac{1}{2}\delta\tau(\omega)$ and $\tau(\omega) - \frac{1}{2}\delta\tau(\omega)$ for the fast and slow polarisation axis^[3]. This can be written formally as $\tau + \frac{1}{2}\Omega \cdot s$, where τ represents the mean time delay over the two polarisation modes.

This leads to a chromatic dispersion, CD,

$$CD = \frac{\partial \tau}{\partial \omega} + \frac{1}{2} \frac{\partial \Omega}{\partial \omega} \cdot s = \frac{\partial \tau}{\partial \omega} + \frac{1}{2} \Omega_\omega \cdot s. \quad (7)$$

It is this effective chromatic dispersion which leads to the most serious signal degradation since while $\frac{\partial \tau}{\partial \omega}$ can be compensated the second term can have either sign and will vary from day to day depending on the exact state of the fibre. This leads to a specification that the PMD, $\Delta\tau$, must be less than $0.5 \text{ ps.km}^{-1/2}$ such that the effective chromatic dispersion will be statistically less than 3 ps.nm.km^{-1} , for a high polarisation mode coupled fibre.

3. DISTRIBUTED MEASUREMENTS OVERVIEW

In order to measure the distribution of PMD within an optical fibre system it is necessary to measure either the accumulation of PMD from the origin to each point or measure the polarisation properties at each point and to calculate the accumulated PMD.

One of the principle techniques for measuring the distribution of properties along an optical fibre system is Optical Time Domain Reflectometry. This generally gives spatial resolutions of a few tens of meters with overall system lengths up to 200km. In order to measure polarisation effects it is necessary to adapt the OTDR technique to POTDR^[4] where the launch and receive optics are polarisation dependent.

It is possible to adapt the OTDR technique to measure the Brillouin scattering (BOTDR) rather than the Rayleigh scattering. This is then sensitive to the polarisation state overlap between the forward pump pulse and backward signal beam.

For shorter optical systems the optical frequency domain reflectometry (OFDR) and optical coherence domain reflectometry (OCWR) allow small spatial resolution over shorter system lengths. These techniques are also inherently polarisation sensitive.

These technique gives insight into the evolution of the polarisation state along the optical fibre system.

The following techniques have been assessed in detail:

i) Fixed wavelength POTDR

This is the simplest adaptation of the normal OTDR technique. A polariser is placed on the output of the OTDR to both polarise the launch pulse and to make the receiver polarisation sensitive. Changes in the returned polarisation state are translated into amplitude changes. The trace can be analysed in terms of local birefringence and polarisation mode coupling length.

ii) Tunable wavelength POTDR

A tunable version of fixed wavelength POTDR above. This overcomes the problem that PMD does not depend on the birefringence but on the group birefringence which depends on the birefringence as a function of optical frequency. It allows for a data analysis technique similar to the traditional fixed analyser PMD measurement method and measures the accumulation of the round trip PMD from the origin to each point within the fibre system.

4. FIXED WAVELENGTH POTDR

4.1 Experimental Arrangement

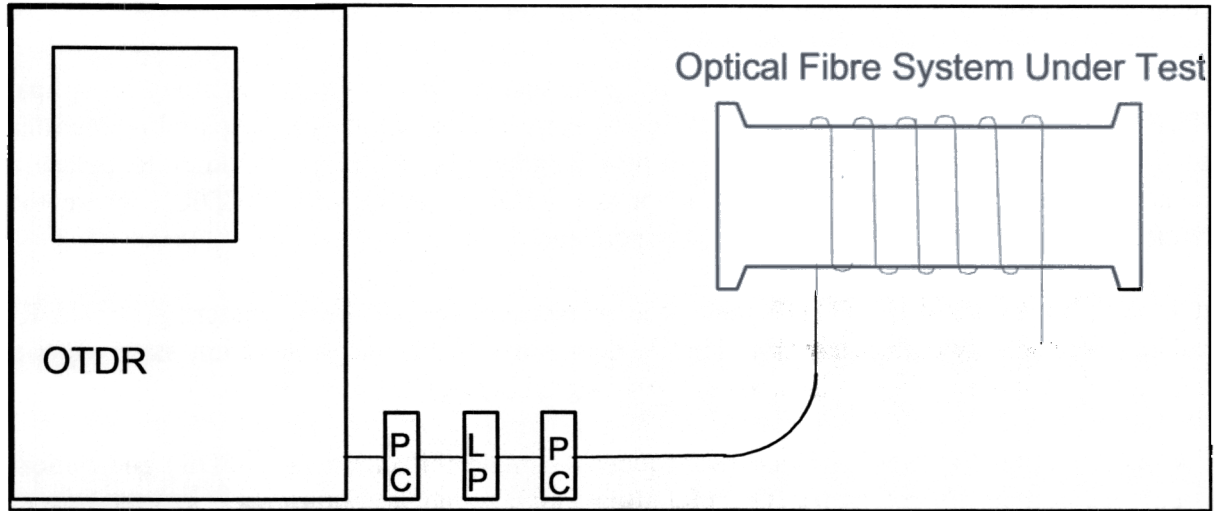


Figure 2 POTDR, Adaptation of commercial OTDR.

Figure 2 shows the simplest arrangement for a fixed wavelength POTDR. A commercial OTDR is adapted by including two polarisation controllers (PC) sandwiching a linear polariser (LP). The first polarisation controller allows the polarisation of the OTDR probe pulse to be aligned with the linear polariser, for transmission on the outward journey. The second polarisation controller allows the launch/received polarisation state to be adjusted.

4.2 Theory

The effect on the polarisation state of a signal that passes down the fibre is described

$$\frac{\partial s}{\partial z} = b(z, \omega) \times s \quad (8)$$

where s describes the Stokes vector of the polarisation state of the signal and $b(z, \omega)$ is the local fibre birefringence at position z for optical frequency, ω . For a finite element the output polarisation state, s_o , is related to the input polarisation state, s_i , through

$$s_o = R(z, \omega) s_i \quad (9)$$

where $R(z, \omega)$ is a rotation Mueller matrix for the element at z and for the optical frequency, ω . A length of fibre is made up of many elements such that for n elements we have, dropping the frequency dependence,

$$s_o = R_n \dots R_3 R_2 R_1 s_i \quad (10)$$

The effect of the backscatter process is the same as a Fresnel reflection and it reverses the handedness of the circular component of the polarisation state. This process is described by the matrix M being the diagonal matrix $(1, 1, -1)$. The polarisation state of the backscatter signal received from a point just after n segments is thus given by

$$s_o = M R_1^T R_2^T R_3^T \dots R_n^T M R_n \dots R_3 R_2 R_1 s_i. \quad (11)$$

For the simplest case of a length of HiBi fibre then $b(z, \omega)$ is a constant and the polarisation state of the signal propagation on the Poincaré sphere rotates around the birefringence axis as it travels down the fibre. The distance the signal travels to rotate once around the birefringence axis is known as the beat length. The POTDR signal will be a single periodic

sinewave with amplitude and phase which depend on the launch and received polarisation states. The beat length will be simply related to the period of this signal. For highly polarisation mode coupled fibre the situation is more complex as the birefringence axis varies randomly along the length, these situations have been assessed using Monte Carlo simulations^[5].

4.3 Measurements

A loose wound MC fibre was measured as shown in Figure 3. This was measured with POTDR1 using a nominal 20 ns pulse width, this gave a effective pulse width of 125 ns see Appendix A. The launch and received polarisation states are aligned by virtue of passing through the same linear polariser. A 1.5km lead in fibre was connected before the linear polariser and then the fibre under test. The POTDR section of the trace shows a peak to peak amplitude of approximately only 4 dB which suggests the trace is not fully resolved. Figure 7 shows a trace from the tunable POTDR of Section 5. This was measured with an effective pulse width of 40 ns and with a receive polarisation state which was orthogonal to the launch polarisation state, this has given a peak to peak amplitude between 7 and 9 dB better resolving the polarisation signal.

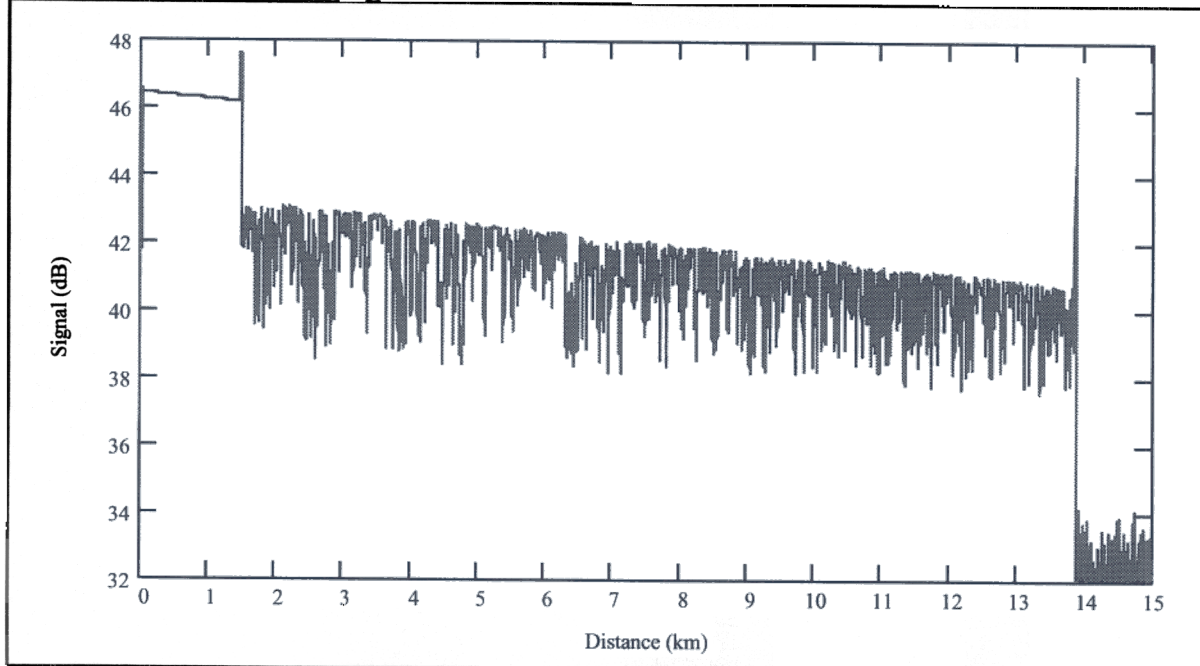


Figure 3 POTDR trace of 12.73 km MC fibre.

4.4 Data Processing

The measured polarisation signal was analysed using both cycle counting and Fourier analysis.

4.4.1 Cycle Counting

Following the analysis suggested by Corsi et al[5]. Each section of the fibre was analysed by counting the number of mean level crossings, N_m , and the number of extrema, N_e . The number of extrema was found to be significantly affected by noise on the OTDR trace and so the POTDR signal was smoothed by filtering the signal to an upper frequency with a period of λ_s .

The factor $F = N_e / N_m$ is calculated and thence the beat length, L_B , is given by :

$$\text{for } F \leq 1.3, \text{ then } L_B \sim 4 * L / N_e \quad (12)$$

$$\text{for } F > 1.3, \text{ then } L_B \sim 3.25 * L / N_m \quad (13)$$

where L is the length of the fibre section.

From F and the graph in Figure 4 of reference [5], a value for L_R/L_B can be obtained where L_R is the rotation length, the length after which the birefringent axis has rotated. From this the coupling length L_C is given by:

$$L_C = - \frac{L_R}{\ln \left| \cos \frac{\pi L_R}{L_B} \right|}. \quad (14)$$

Which leads to an average PMD given by:

$$\Delta\tau(z)^2 = \frac{L_C^2}{2} \left[\frac{\lambda}{L_B c} \right]^2 \left[e^{\frac{-2z}{L_C}} + \frac{2z}{L_C} - 1 \right]$$

where z is the fibre length, λ the POTDR wavelength and assuming that the birefringence is a linear function of the optical frequency, giving a non dispersive group delay. In the long length regime this simplifies to

$$\Delta\tau(z)^2 = \left[\frac{\lambda}{L_B c} \right]^2 z L_C$$

Parameter	POTDR1			POTDR2
Nominal pulse width (ns)	20	20	20	10
Effective pulse width (ns)	125	125	125	42
Fibre Section	A	B	C	A
Start Position (m)	500	4460	8420	500
Stop Position (m)	4708	8668	12628	4708
N_m	118	136	143	156
N_e	353	310	292	366
F	3.0	2.3	2.1	2.35
L_B (m)	116	101	96	88
L_R/L_B	0.13	0.22	0.26	0.21
L_C (m)	176	85	66	78
PMD (fs.km ^{-1/2})	19	15	14	17

Table 1 Results on loose wound MC fibre.

Table 1 shows the data obtained using POTDR1 and POTDR2 for the loose wound MC fibre and a smoothing filter with a period of 14 m. This fibre has a total PMD of 100 fs and thus an average PMD coefficient of 28 fs.km^{-1/2} measured using the NPL Poincaré sphere bulk PMD

measurement system. The fibre appears to show a beat length variation of 10% and a coupling length variation of more than 50%. This large variation in coupling length seems unlikely to be a real effect and is probably an artefact of the data analysis technique.

The beat length calculated from POTDR1 and POTDR2 are in moderate agreement given the variation of optical wavelength likely between the two POTDRs.

Parameter	POTDR3 (Effective pulse width 40 ns)					
Section	A	B	C	D	E	F
Position (m)	384	4589	8795	13000	17205	21411
Position (m)	4589	8795	13000	17205	21411	25616
N_m	867	821	751	782	791	846
N_e	1121	1016	980	992	994	1078
F	1.29	1.24	1.3	1.27	1.26	1.27
L_B (m)	15.0	16.6	17.2	17.0	16.9	15.6
L_R/L_B	5.9	12.0	4.7	8.3	9.5	8.3
L_C (m)	299	∞	32	32	1	32
PMD (fs.km ^{-1/2})	174	328	53	54	10	59

Table 2 Results on dispersion shifted fibre from POTDR3.

Table 2 shows the results of measurements taken using the tunable POTDR (POTDR3) described in Section 5 at 1550 nm on a 26km length of DS fibre. The length was formed from two 13 km lengths where the outside ends of the reel were joined. The fibre has a total PMD of 570 ± 52 fs giving an average PMD coefficient of 112 ± 11 fs.km^{-1/2}. Again the beat length varies by only 6% while the coupling length varies by several orders of magnitude. The variation in the calculated coupling length is more likely due to the highly non-linear form of equation 14, and is unlikely to be a real physical effect. The variation in PMD coefficient is dominated by the variation of calculated coupling length. It can be seen that the beat length varies from short to long and back to short, this is consistent with the two reels of fibre being joined at the outside ends of the reel and the beat length being longer where the fibre is wound over a larger radius.

4.4.2 Fourier Analysis

The POTDR signal is first normalized. For POTDR where the launch and received polarisation states are aligned the local average signal is equal to 2/3 of the peak signal. The local average is found by least squares fitting the logarithm of the trace to a straight line, this will eliminate the effect of the normal attenuation. The linear POTDR trace is then normalized to lie between -1 and +1 with an average of 1/3 using this result. For POTDR where the launch and receive states are orthogonal the local average signal is equal to 1/3 and

the normalized trace will lie between -1 and $+1$ with an average of $-1/3$. This normalized linear trace is Fourier transformed to obtain the power spectral density.

From the power spectral density the second moment is calculated using

$$\langle v^2 \rangle = \frac{\int_{-\infty}^{\infty} v^2 P(v) dv}{\int_{-\infty}^{\infty} P(v) dv}$$

Following the analysis of Corsi et al ^[6], the beat length L_B has been shown to be

$$L_B = \sqrt{\frac{12}{\pi}} \frac{1}{\sqrt{\langle v^2 \rangle}} \frac{c}{2 n_g} \quad (16)$$

where n_g is the group index of the fibre, usually taken as 1.46 and $\sqrt{\langle v^2 \rangle}$ is the second moment of the power spectral density.

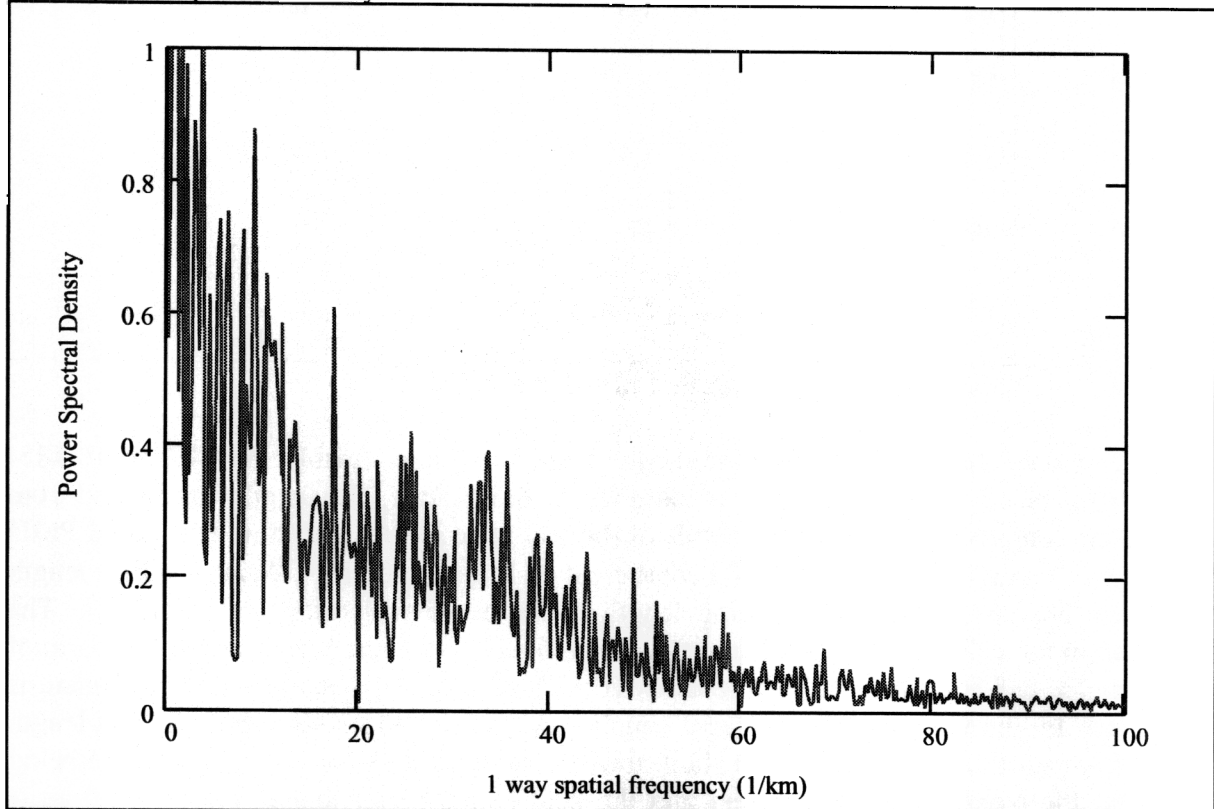
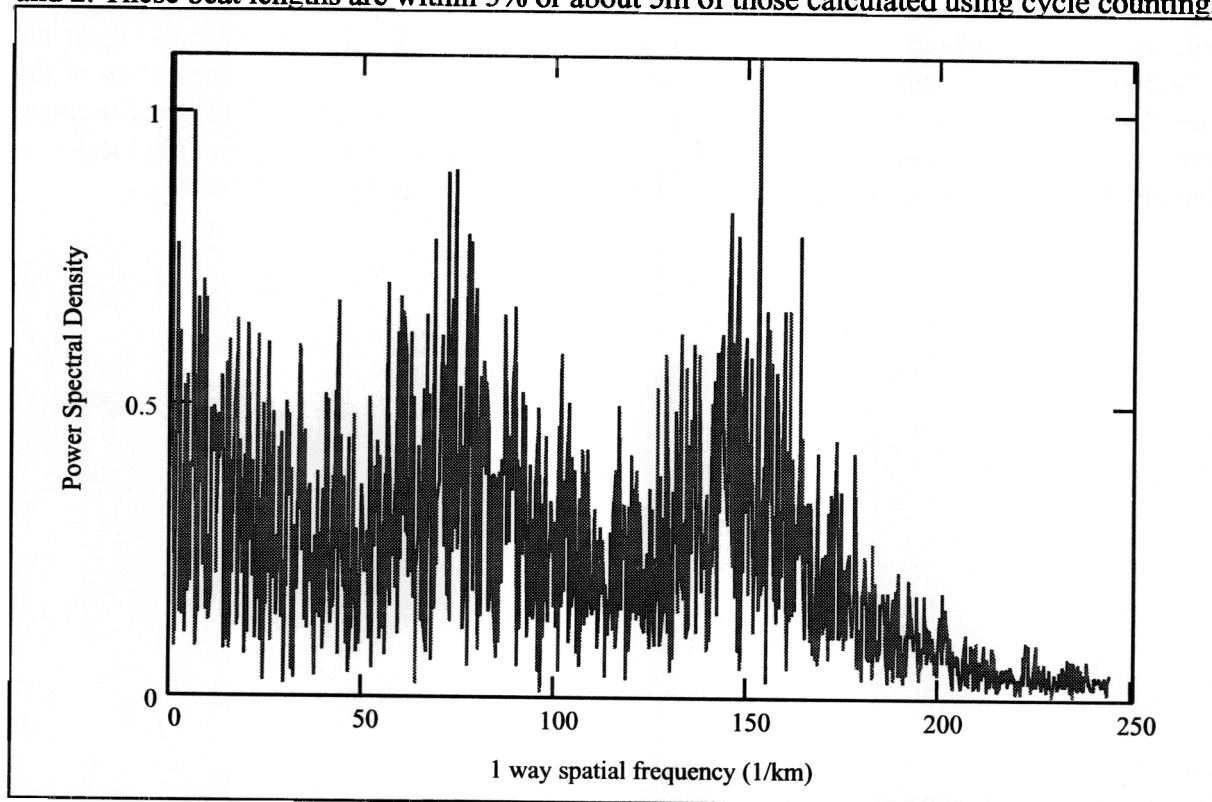


Figure 4 Power spectral density of Section A of the loose wound MC fibre, POTDR1

Parameter	POTDR1			POTDR2
Fibre Section	A	B	C	A
Start Position (m)	500	4460	8420	500
Stop Position (m)	4708	8668	12628	4708
$\sqrt{\langle v^2 \rangle}$ (MHz)	1.74	1.87	2.00	2.41
L_B (m)	115	107	100	83

Table 3 Results on loose wound MC fibre.

Table 3 shows the results of Fourier analysis on the loose wound MC fibre using POTDRs 1 and 2. These beat lengths are within 5% or about 5m of those calculated using cycle counting.

**Figure 5** Power spectral density of Section A of the DS fibre, POTDR3.

Parameter	POTDR3					
Section	A	B	C	D	E	F
Start (m)	384	4589	8795	13000	17205	21411
Stop (m)	4589	8795	13000	17205	21411	25616
$\sqrt{\langle v^2 \rangle}$ (MHz)	10.4	9.8	9.3	9.6	9.6	10.2
L_B (m)	19.2	20.4	21.5	20.9	20.9	19.7

Table 4 Results on 26 km of DS fibre.

Table 4 shows the results of Fourier analysis on the dispersion shifted fibre using the tunable POTDR3 of Section 5. These beat lengths are within 5m of those calculated using cycle counting. The beat length calculated using Fourier analysis tend to be longer than those by cycle counting this may be due the limited resolution of the POTDR weighting lower frequencies in the power spectrum.

4.5 Summary

The POTDR technique is readily performed with an adapted commercial OTDR. The highest effective resolution is required and may be considerably worse than that suggested by the pulse width alone, see Appendix A.

The technique allows the measurement of beat length with a reasonable repeatability however no independent means of verify the actual beat length were available for comparison. The analysis of the POTDR trace to determine the coupling length appeared to be flawed giving coupling lengths which varied wildly. In order to calculate the local PMD coefficient the coupling length is required and also an assumption about the wavelength dependence of the beat length is required. The technique is unlikely to give a high accuracy PMD measurement however an indication of wether the PMD is high or low can be obtained from the local beat length. The maximum measurable PMD is limited by the resolution of the POTDR.

5. TUNABLE WAVELENGTH POTDR

5.1 Experimental Arrangement

In the absence of a commercial tunable OTDR, an experimental tunable POTDR was constructed as shown in Figure 6. A number of polarisation controllers have been omitted from the diagram for clarity.

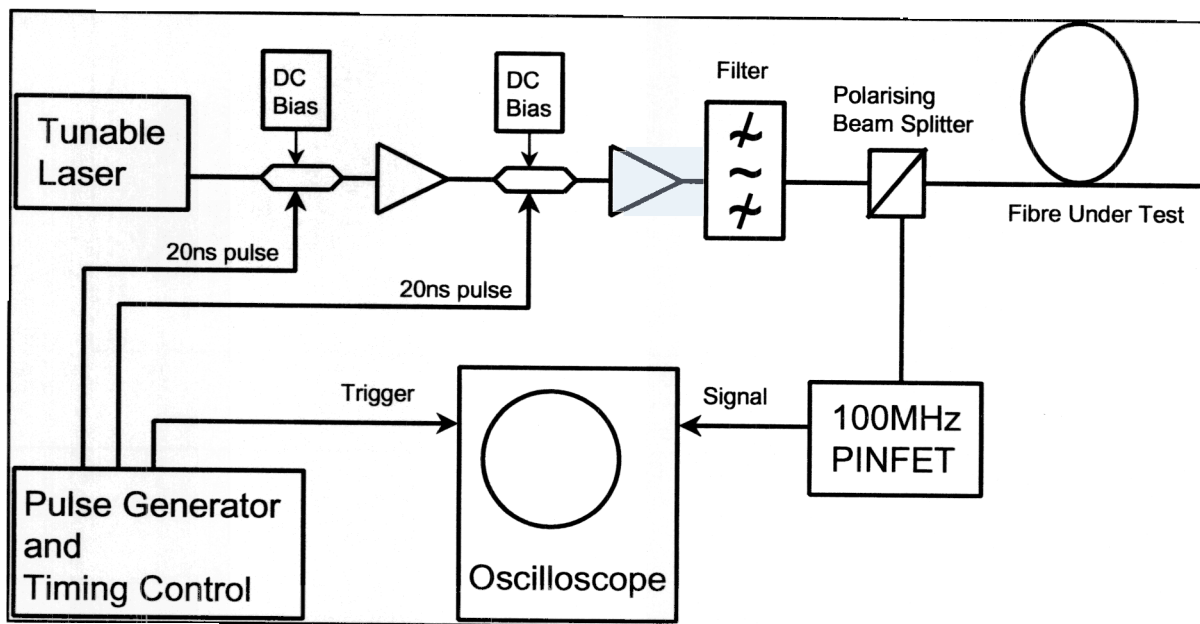


Figure 6 Tunable POTDR.

5.2 Measurement of 26 km of DS Fibre

The fibre under test was 26 km of dispersion shifted fibre. An assessment was made to identify the pulse width and detector bandwidth requirements. These resulted in the use of a 30 ns optical launch pulse with the oscilloscope sampling at 20 ns with a restricted 30 MHz bandwidth. This gave an effective resolution of 40 ns more than enough to fully resolve the structure within the POTDR trace in this instance. A launch peak power of 1 to 5 W was achieved, above this level non-linear effects began to alter the trace.

POTDR traces were recorded for 24 wavelengths between 1530 nm and 1566 nm. Each trace required the re-adjustment of the polarisation controllers, modulator DC bias and optical filter. 3000 traces were averaged in 2½ minutes. The POTDR trace recorded at a wavelength of 1550 nm is shown in Figure 7.

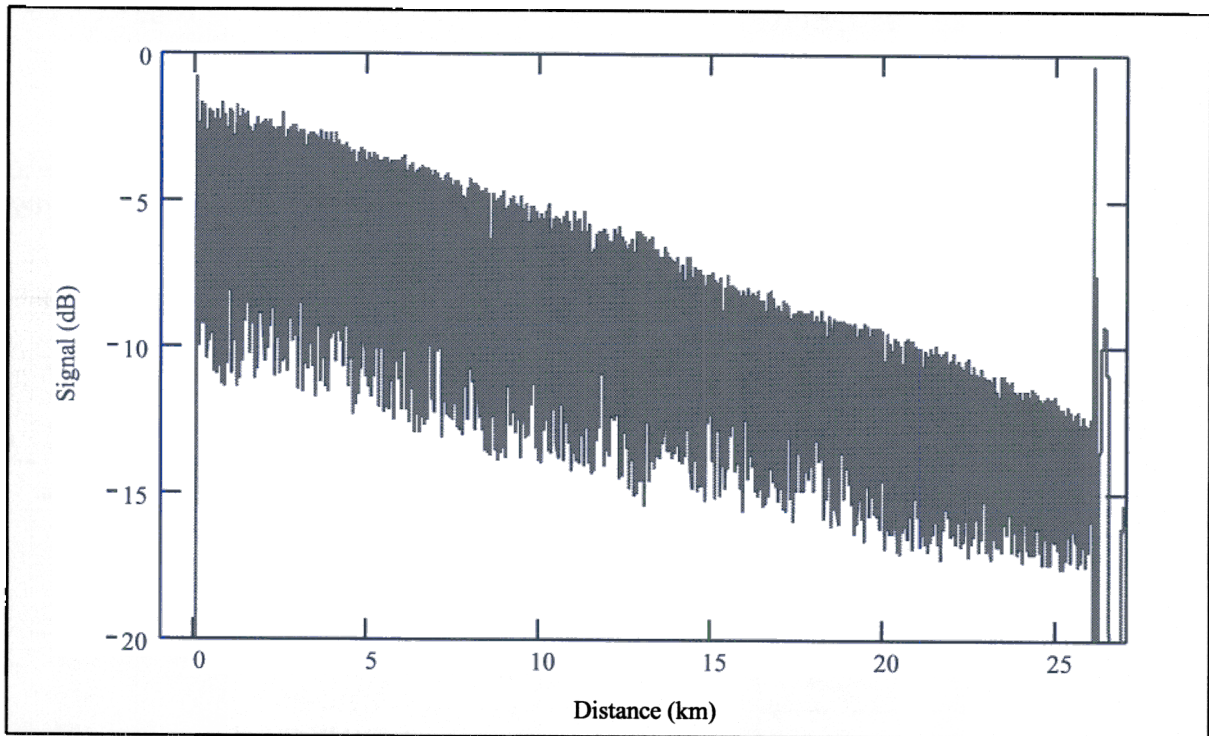


Figure 7 POTDR Trace of 26 km of DSF at 1550 nm.

5.3 Data Analysis

The POTDR traces as a function of wavelength allow for a data analysis technique which is based on the fixed analyser PMD measurement method^[7]. The traces are not analysed individually at fixed wavelengths as a function of distance but collectively at fixed distances as a function of wavelength.

The analysis of the set of 24 POTDR traces at wavelengths between 1530 and 1566 nm began with a normalisation to remove the effect of the fibre attenuation. The linear traces were scan with a rolling window of ~800m to find the local signal maximum and minimum. The trace was then normalised using these local values to obtain a POTDR signal which varied between 1 and -1. At each distance in the fibre system the POTDR signal as a function of wavelength was extracted. The data for 24 km is shown in Figure 8.

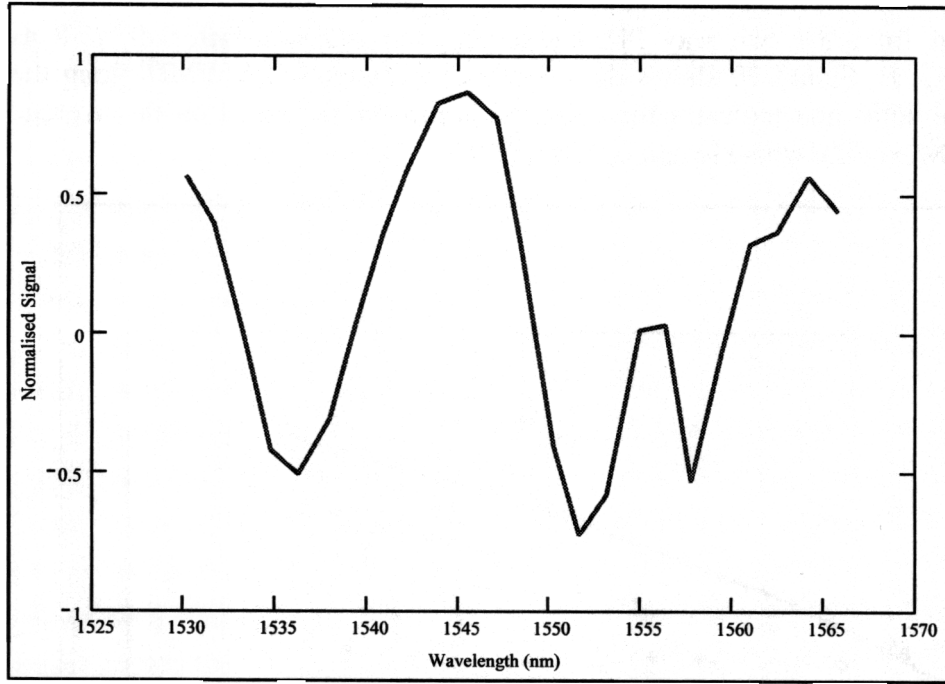


Figure 8 Equivalent fixed analyser scan at 24 km.

This data was then rescaled to give POTDR signal as a function of optical frequency, with equal steps in optical frequency being provided by cubic spline interpolation. The Fourier transform of this POTDR signal versus optical frequency gives a probability density of the DGD^[8]. For the highly mode coupled DS fibre this was analysed by fitting a Gaussian function the 2nd moment of which is the accumulated two way PMD of the fibre system from the POTDR to the distance analysed. Figure 9 shows the Fourier transform of the POTDR signal versus optical frequency and the fitted Gaussian function for data at a distance of 24 km from the POTDR.

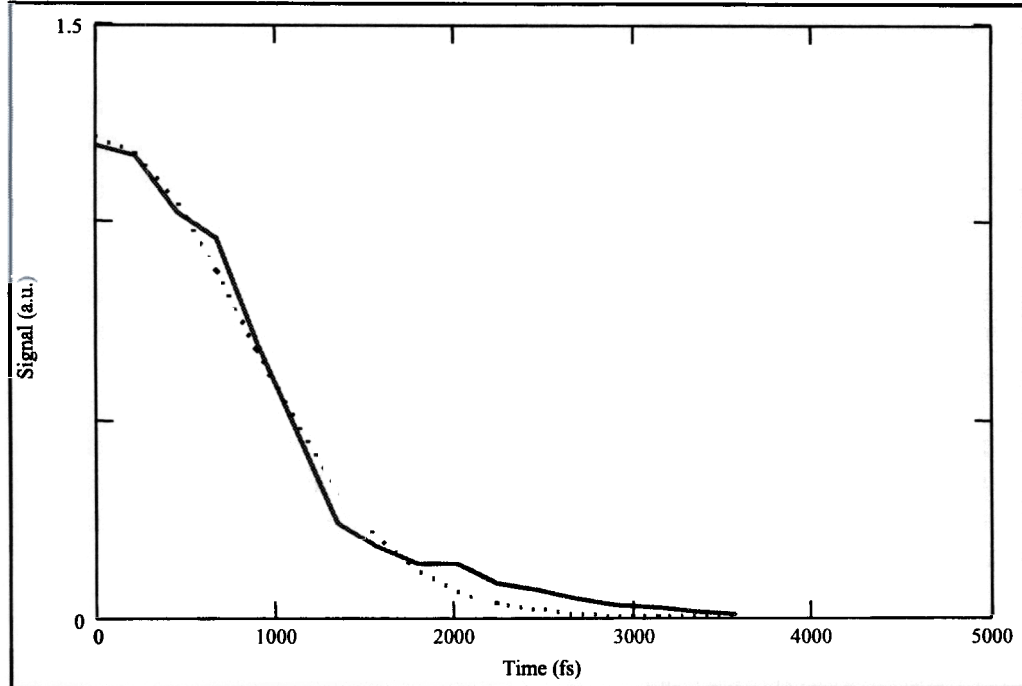


Figure 9 Solid line shows measured data, dashed line is fitted Gaussian, at 24 km.

In order to improve the noise the PMD was averaged over a 200 m window. The one way

PMD was calculated from the two way PMD assuming the constant factor of $\sqrt{3}/8$ as described in Appendix B. Figure 10 shows the measured accumulation of PMD along the 26 km DS fibre. The solid line indicates the expected accumulation based on the average measured two way PMD of the whole length of 570 ± 52 fs.

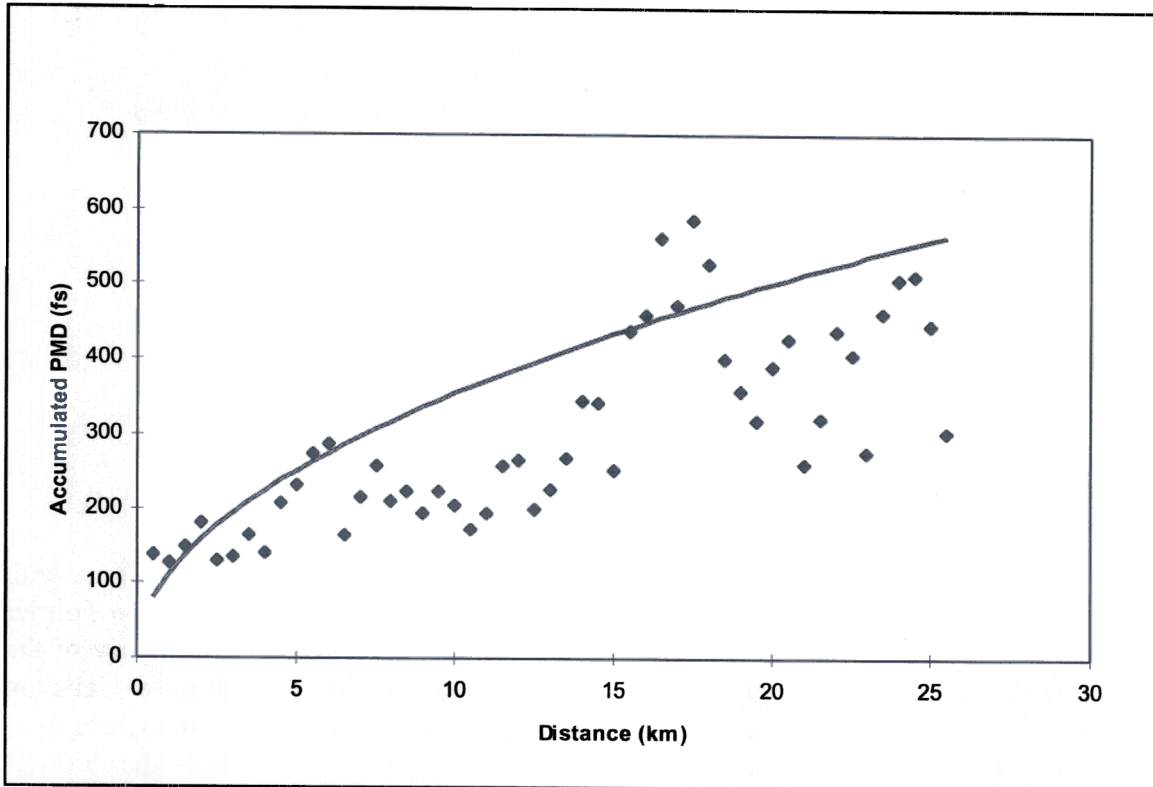


Figure 10 Accumulation of PMD in 26 km of DS fibre.

5.4 Summary

The tunable wavelength POTDR is more complex than the fixed wavelength version however this allows for a better data analysis technique in the form of the well proven fixed analyser PMD measurement method. The use of different wavelengths allows us to measure group delay effects such as PMD without assumptions about dispersion characteristics.

The PMD accuracy is limited in the short length regime to the resolution of the technique which is limited by the wavelength scan range to approximately 100 fs. The PMD accuracy in the long length regime is limited by the PMD statistics.

The maximum measurable accumulated PMD is limited by the wavelength step, a wavelength step of 0.1 nm will limit the maximum PMD to 10 ps. Also at the distance where the measurement is taken the POTDR trace must be fully resolved and this limits the maximum PMD coefficient at this distance.

APPENDIX A PULSE WIDTH REQUIREMENTS FOR POTDR

Using the experimental arrangement of Figure 2 the loose wound MC fibre was measured. In order to identify the required POTDR resolution traces were recorded for the various nominal pulse widths. The centre 8192m of each traces was normalised to remove the fibre attenuation and converted to a linear regime with zero average before being Fourier transformed to determine the Spectral power of the oscillatory components within the POTDR trace, see Figure A-2 & A-1.

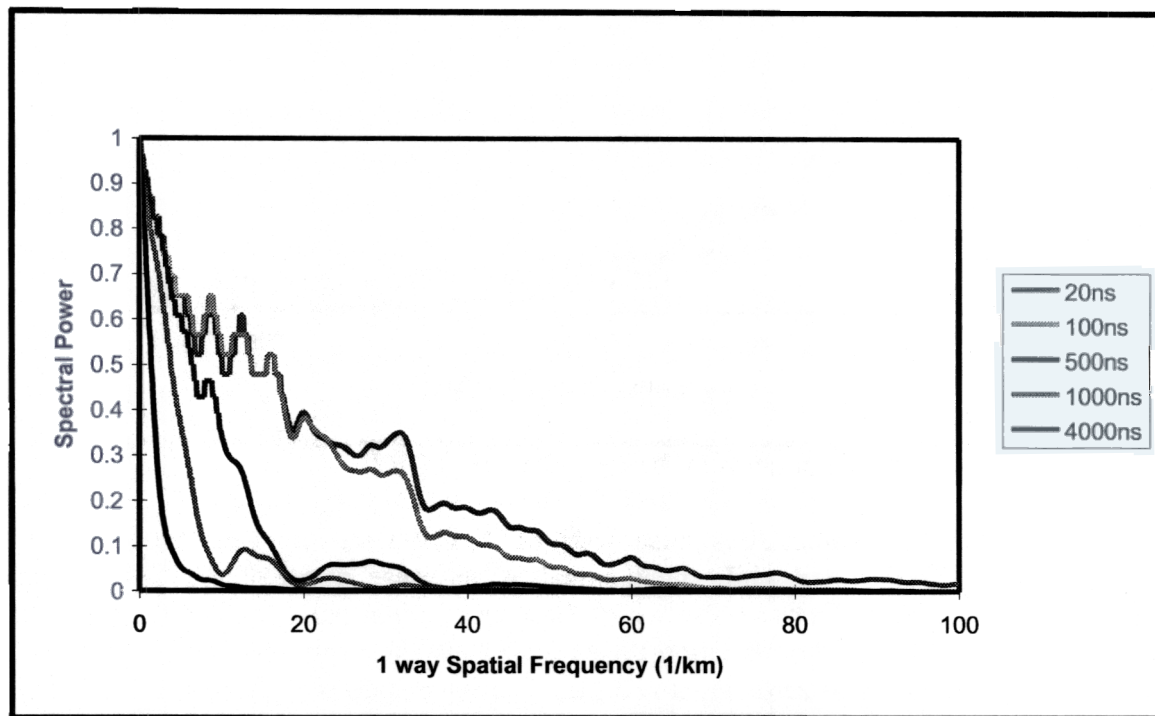


Figure A-1 Oscillatory components in the trace from POTDR1

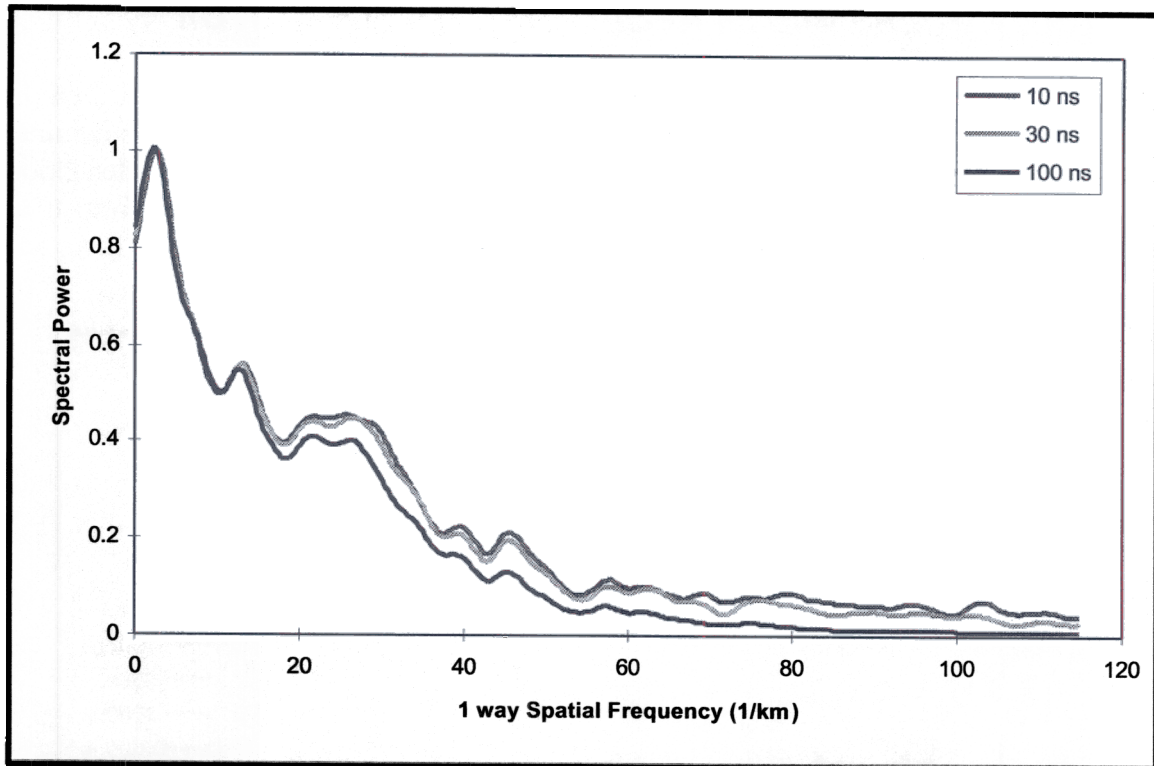


Figure A-2 Oscillatory components in the trace from POTDR2.

It would appear that the nominal 20ns pulse width of POTDR1 may not be short enough to fully resolve the POTDR trace while the nominal 30 ns pulse width of POTDR2 seems sufficient. To confirm the effective pulse width for each nominal pulse a 4% reflection was measured using the OTDR. The reflection traces were cropped to give the reflection in the centre of an 8192m section of zero signal and then Fourier transformed. The spectral power of the components within the reflection is shown in Figures A-3 & A-4.

This suggests an effective pulse widths of the launch pulse / receiver combination given in Tables A1 & A2 for the two OTDRs.

Nominal Pulse Width (ns)	Effective 1 Way Resolution (m)	Effective Pulse Width (ns)
20	13	125
100	22	215
500	58	560
1000	112	1100
4000	442	4300

Table A1 Effective pulse width for OTDR1.

Nominal Pulse Width (ns)	Effective 1 Way Resolution (m)	Effective Pulse Width (ns)
10	4.3	42
30	6.7	65
100	26	250
275	95	900
1000	140	1400

Table A2 Effective pulse width for OTDR2.

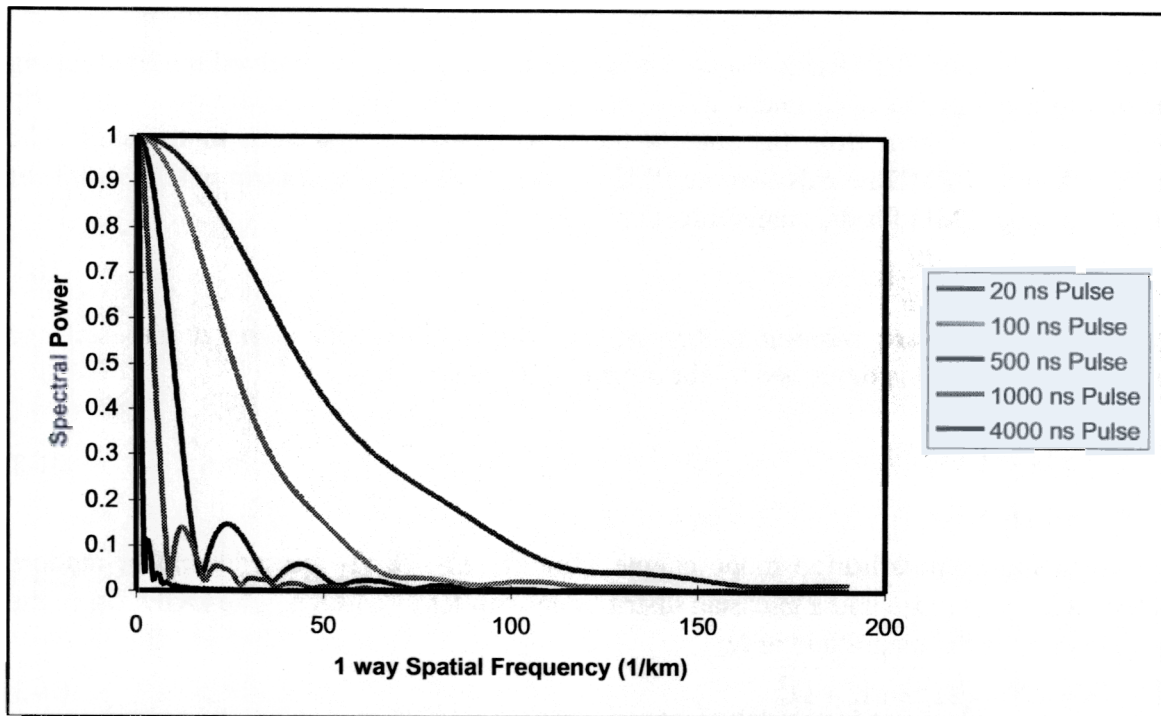


Figure A-3 Oscillatory components in the trace of a 4% reflection, OTDR1.

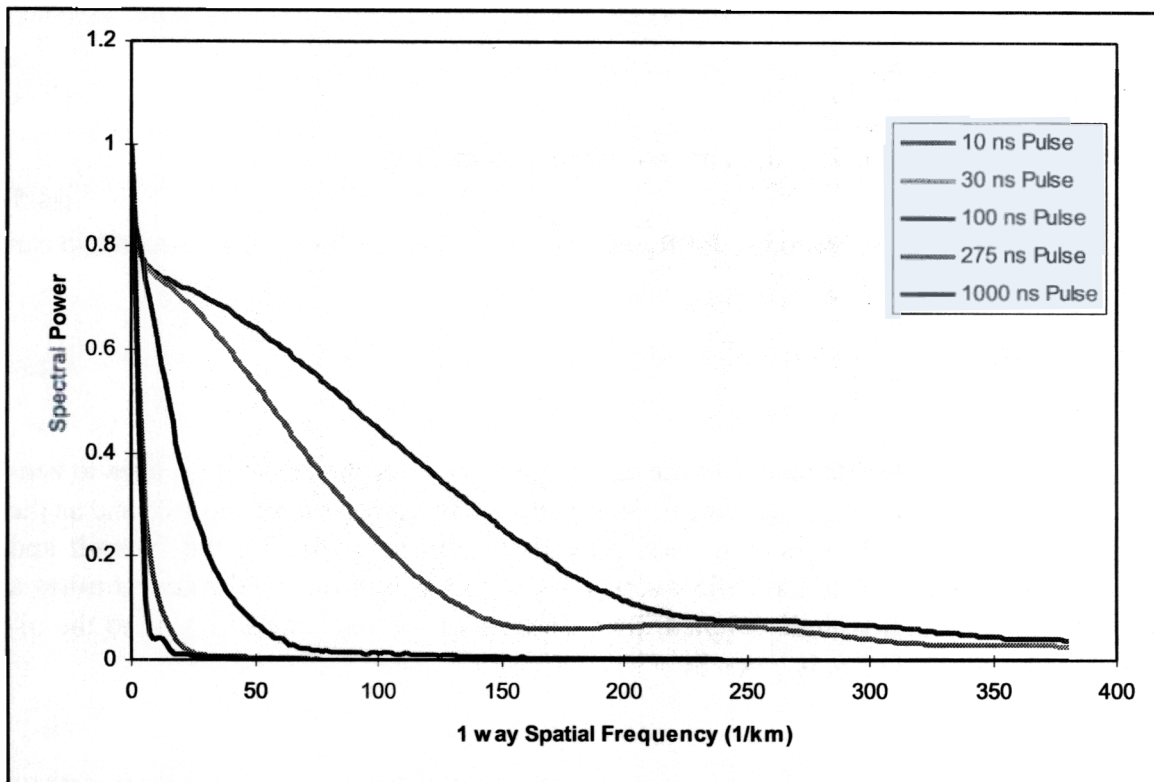


Figure A-4 Oscillatory components in the trace of a 4% reflection, OTDR2.

The loose wound MC fibre has a known PMD of $28 \text{ fs.km}^{-1/2}$.

APPENDIX B BI-DIRECTIONAL PMD

The measurements of PMD made using distributed techniques have involved the light taking a round trip through the fibre under test. Thus we are measuring the PMD of a round trip through the fibre system from the start to the scattering point and back to the start. The statistical theory of PMD reveals that the RMS average PMD of a round trip will be $\sqrt{8/3}$ of the RMS average PMD for the single direction^[5]. Now

$$\frac{\partial s}{\partial \omega} = \Omega \times s = \frac{\partial R}{\partial \omega} R^{-1} s \quad (\text{B-1})$$

where R is the forward rotation matrix of the fibre system and s the output state of polarisation. The PMD is described by the 3 element vector,

$$\Omega = \begin{pmatrix} \Omega_1 \\ \Omega_2 \\ \Omega_3 \end{pmatrix}$$

where for a highly polarisation mode couple fibre Ω_1 , Ω_2 , & Ω_3 are independent random variables with zero mean and a gaussian distribution with RMS width α . The DGD, $\delta\tau$, in the single direction is the magnitude of Ω

$$\delta\tau = |\Omega| = \sqrt{\Omega_1^2 + \Omega_2^2 + \Omega_3^2} \quad (\text{B-3})$$

which has an RMS average value of $\sqrt{3}\alpha$.

In the bi-directional round trip

$$\frac{\partial s}{\partial \omega} = \Omega_B \times s = \frac{\partial R_B}{\partial \omega} R_B^{-1} s \quad (\text{B-4})$$

where R_B is the bi-directional rotation matrix of the system. Now

$$R_B = M R^T M R \quad (\text{B-5})$$

where $M = \text{diag}(1, 1, -1)$. Substituting for R_B into equation B-4, the PMD of the round trip can be shown^[5] to be described by

$$\Omega_B = M R^T \begin{pmatrix} 2\Omega_1 \\ 2\Omega_2 \\ 0 \end{pmatrix} \quad (\text{B-6})$$

The elements Ω_1 & Ω_2 double up due to the double pass however the element Ω_3 goes to zero as the forward and backward passes cancel. Ω_3 represents the circular birefringence and as the handedness of the light changes sign upon reflection or linear scattering the forward and backward passes cancel. Thus the RMS average DGD of the bi-directional round trip has a value of $\sqrt{8}\alpha$. For a highly mode coupled fibre the ratio of the one way PMD $\Delta\tau$ to the bi-directional two way PMD $\Delta\tau_B$ is given by

$$\frac{\Delta\tau}{\Delta\tau_B} = \sqrt{\frac{3}{8}} \quad (\text{B-7})$$

It can also be shown that the statistical probability distribution of the one way DGD, $\delta\tau$, is given by a Maxwellian distribution

$$P(\delta\tau) = \sqrt{\frac{2}{\pi}} \frac{\delta\tau^2}{\alpha^3} e^{-\frac{\delta\tau^2}{2\alpha^2}} \quad (\text{B-8})$$

while the statistical probability distribution of the bi-directional round trip DGD, $\delta\tau_B$, is given by a Rayleigh distribution

$$P(\delta\tau_B) = \frac{\delta\tau_B}{4\alpha^2} e^{-\frac{\delta\tau_B^2}{8\alpha^2}}. \quad (\text{B-9})$$

To test the theory, the PMD of 26 km of dispersion shifted fibre was measured both one-way and round trip using the NPL Poincaré sphere PMD measurement system. The fibre was measured at four different temperatures between 21°C and 43°C, to give more statistically independent measurements, using a wavelength scan from 1490 nm to 1620 nm in 1 nm steps.

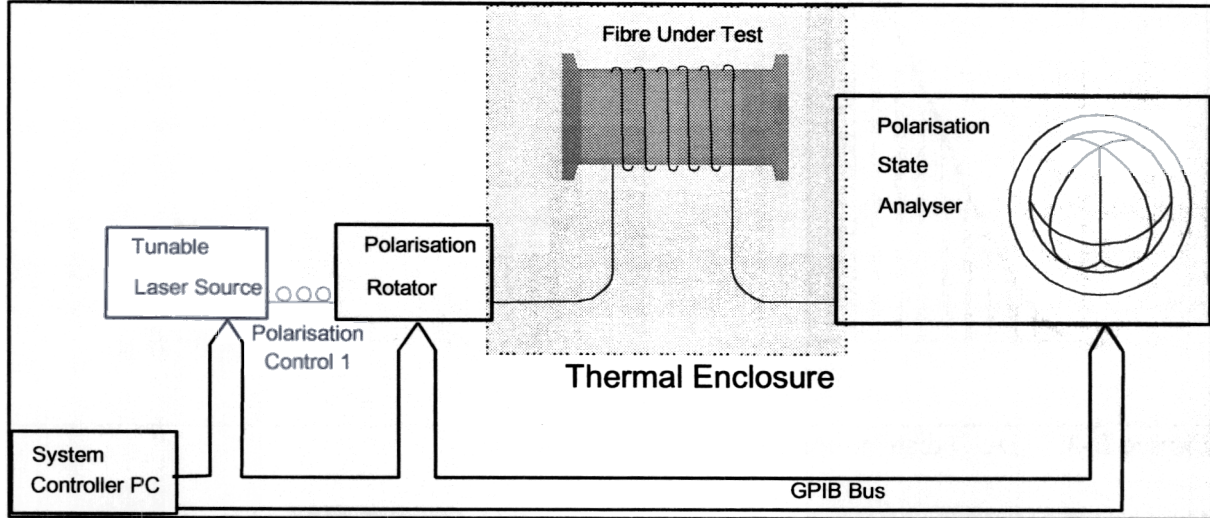


Figure B-1 Measurement setup for one-way measurement.

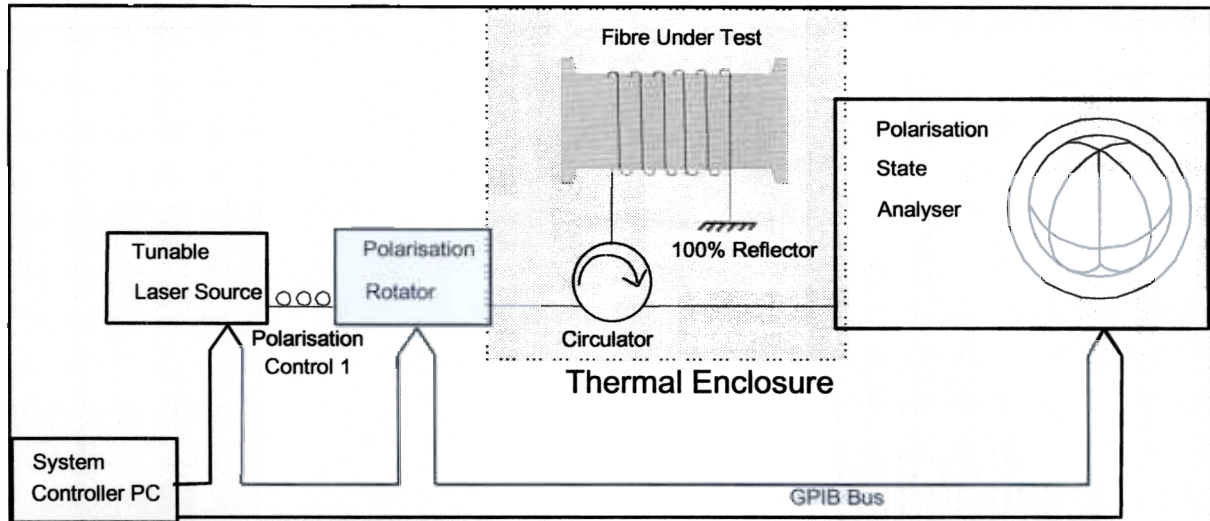


Figure B-2 PMD Measurement setup for round-trip measurement.

The measured data was analysed by calculating the principle state of polarisation (PSP) and the rate of rotation of the polarisation state around the PSP to obtain the differential group delay. The PMD was then calculated as the RMS DGD. The distribution of DGD measurements was compared using the Chi squared test against the two distribution described in equations B-8 and B-9.

The one-way PMD was calculated as 570 ± 52 fs and the Chi squared test revealed a 99.9% probability that the data came from a Maxwellian distribution but only a 32.2% probability that the data came from a Rayleigh distribution.

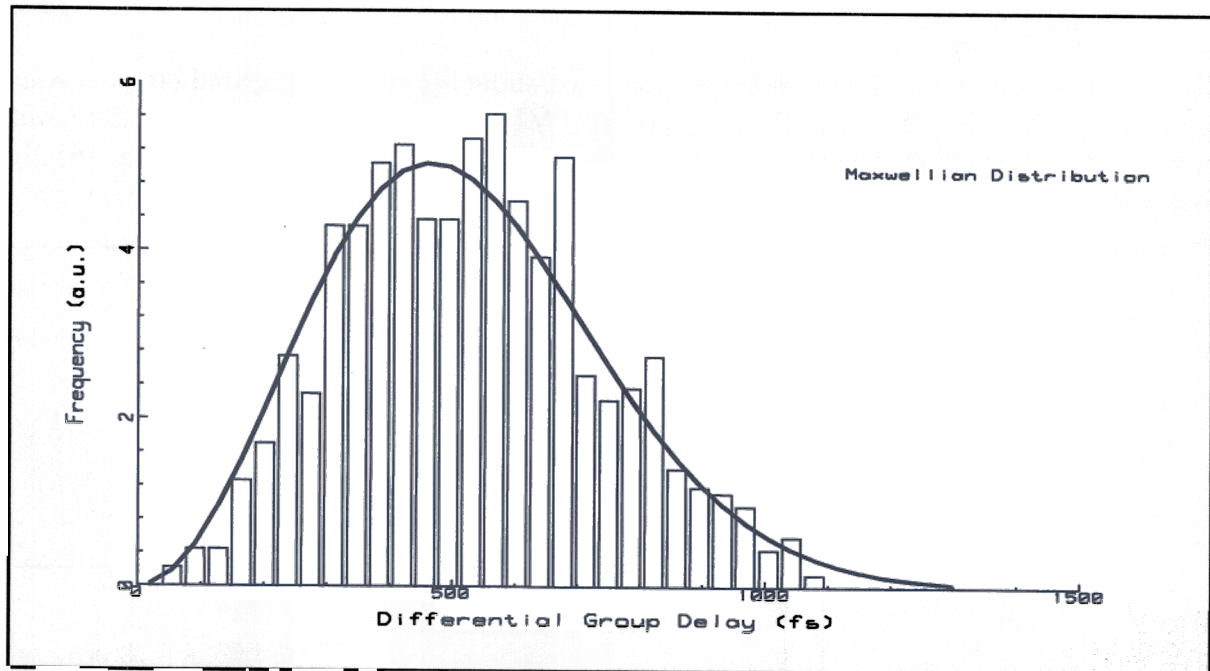


Figure B-3 DGD distrib for way 26 km of DSF

The two-way trip PMD calculated 902 fs and the Chi squared test revealed 99.9% probability that the data came from Rayleigh but 9% probability that the data came from a Maxwellian distribution

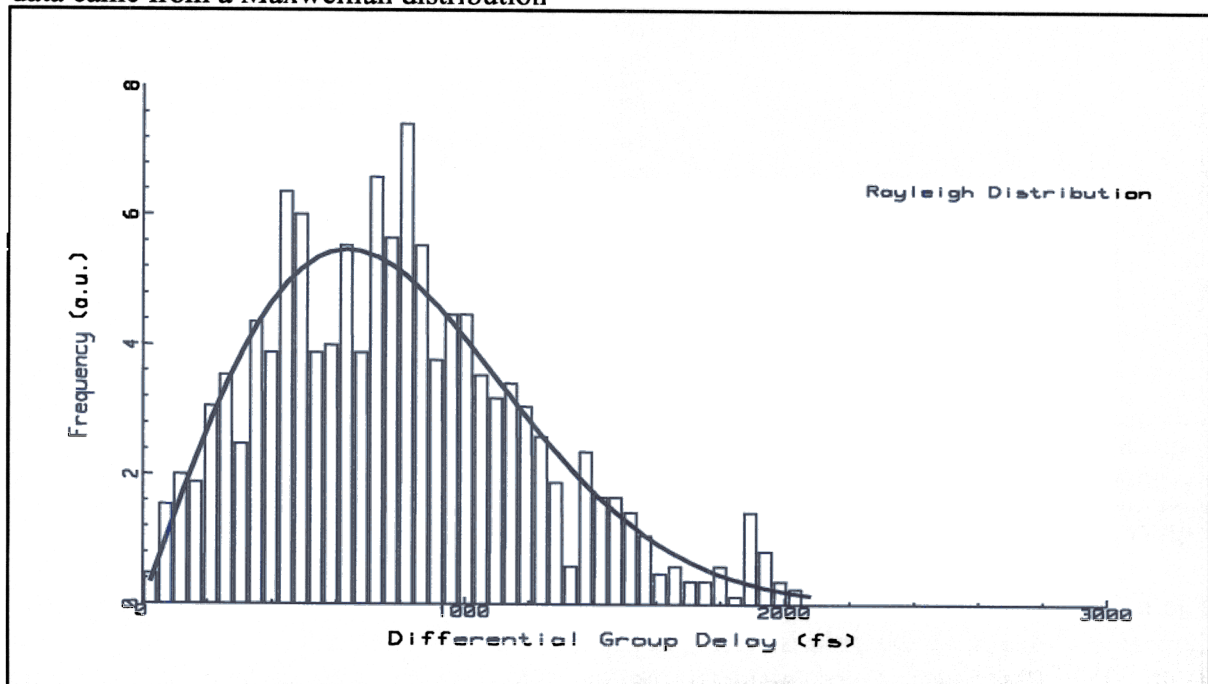


Figure B-4 DGD distrib for round-trip 26 km of DSF

The ratio of the round-trip PMD to the way PMD $\sqrt{2}$

In conclusion, the measurements and theory agree with expectation for real fibre

APPENDIX C REFERENCES

- C.D. POOLE, R.E. WAGNER, "Phenomenological approach to polarisation dispersion in long single-mode fibres", *Elect. Lett.*, Vol 22, 1986, pp 1029 – 1030.
- 2 G.J. FOSCHINI, C.D. POOLE, "Statistical Theory of Polarisation Dispersion in Single Mode Fibres", *J. of Light. Tech.*, Vol 9, 1991, pp 1439 - 1456.
- 3 P. CIPRUT, B. GISIN, N. GISIN, R. PASSEY, J.P. VON DER WEID, F. PRIETO, C.W. ZIMMER, "Second-Order Polarization Mode Dispersion: Impact on Analog and Digital Transmissions", *J. of Light. Tech.*, Vol 16, 1998, pp 757 - 771.
- 4 A.J. ROGERS, "Polarisation-optical time domain reflectometry: a technique for the measurement of field distributions", *Applied Optics*, Vol 20 No 6, March 1981, pp 1060 – 1074.
- 5 F. CORSI, A. GALTAROSA, L. PALMIERI, "Polarisation Mode Dispersion Characterisation of Single-Mode Optical Fiber Using Backscatter Technique", *J of Light Tech.*, Vol 16 No 10, October 1998, pp 1832 – 1843.
- 6 F. CORSI, A. GALTAROSSA, L. PALMIERI, "Beat Length Characterization Based on Backscattering Analysis in Randomly Perturbed Single-Mode Fibres", *J of Light Tech.*, Vol 17 No 7 July 1999, pp1172–1178.
- 7 H. SUNNERUD, B.-E. OLSSON, P.A. ANDREKSON, "Technique for characterisation of polarisation mode dispersion accumulation along optical fibres", *Electronics Lett.*, Vol 34 No 4, February 1998, pp 397 – 394.
- 8 A.J. BARLOW, "Polarisation Mode Dispersion Measurement using Fourier Analysis: Investigation of mode coupling behaviour in fibers", *SOFM 1994, NIST Special Publication 864*, 1994, pp 167 – 170.