

REPORT

Uncertainty Reduction and Decorrelation of Mode-Stirred Reverberation Chamber Data Using Transformation and Expansion Techniques

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**Uncertainty Reduction and Decorrelation of
Mode-Stirred Reverberation Chamber Data Using
Transformation and Expansion Techniques**

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Abstract

This paper addresses methods for the reduction of autocorrelation between sampled data collected in mode-tuned or mode-stirred reverberation chambers. This leads to improved accuracy of the estimation of susceptibility or emission characteristics of an EUT. Ensemble decimation and orthogonal expansion are proposed as alternatives to conventional sample decimation. Orthogonal expansion also allows for a rigorous statistical characterization of such chambers, in particular for operation at relatively low mode densities.

Keywords: mode-stirred reverberation chamber, ensemble decimation, orthogonal expansion, Fredholm integral equation, Töplitz matrix, autocorrelation function, wide-sense stationary stochastic process, statistical uncertainty.

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1. Introduction

The statistical modelling of electromagnetic reverberation chambers is a powerful and robust analysis technique that assists in the EMC compliance and pre-compliance testing in such chambers [1]. However, the technique has uncertainties of various origins associated with it. Under proper reverberation conditions it enables uncertainties for the estimated maximum field of $\pm 1.5\text{dB}$ or better to be attained, at the 95%-confidence level, at a single location of the EUT [2]. For certain EMC applications, the uncertainty levels may be required to be further reduced still, in particular for small sample sizes aimed at shortening test cycles or for operation at relatively low frequencies.

In this report, we address the reduction of the uncertainty caused by the tuner data autocorrelation and the test variability. Data correlation makes the statistical characterization of the stirring process considerably more complicated.

2. Uncorrelated sampled data

A typical statistical analysis of measured reverberation chamber data starts with the calculation of the autocorrelation function (acf) $\rho(\tau)$ of the mode stirring or mode tuning process with stir variable τ . On this basis, an ‘equivalent’ number of ‘independent’ samples N_{eq} is estimated from M data points associated with M stirrer positions or sample locations. This then allows one to estimate statistics of the local magnitude or power density of the total field or a rectangular field component (e.g. maximum-to-mean ratio).

It has been recognized that a commonly used criterion for ‘un’correlatedness, viz. the correlation distance τ_{eq} in the stir domain as defined by:

$$\tau_{eq} = \rho^{-1}(e^{-1}) \quad (1)$$

is heuristic and is being used ad hoc [2]-[5]. Its lack of a rigorous foundation does not allow for accurate estimates of N_{eq} . Indeed, alternative criteria exist [2,5], defined in the stir domain or in the spectral stir domain, which yield substantially different estimates for N_{eq} but which are nevertheless of similar appeal. Moreover, the sample correlation distance has been found to exhibit variation as a function of sample size [5], again affecting the estimation of N_{eq} . Thus, defining criteria for uncorrelatedness with proper foundation is important in its own right.

In addition, field statistics have an inherent uncertainty associated with them. Although for realistic values of N_{eq} the standard deviation and confidence intervals of the field statistics are often within acceptable ranges, this presumes the value of N_{eq} to be known with sufficient accuracy. In view of the arbitrariness of the above or other criterion of uncorrelatedness, this accuracy is questionable when using one or other definition for the correlation distance. Consequently, the uncertainty on N_{eq} adds to the uncertainty on the field statistics themselves (as applicable for a known value of N_{eq} under adequate reverberation conditions) and to the classical measurement uncertainty of deterministic fields, thus giving rise to an expanded uncertainty. The effect becomes of particular concern in the trend towards using smaller values of M and N_{eq} for chamber operation and EUT testing to shorten test cycles and reduce

cost. Fig 1 illustrates the expanded uncertainty for ideal reverberation chamber data due to the uncertainty on N_{eq} which is estimated to range between $N_{eq} = 40 \dots 60$ by way of example. As can be seen, the resulting uncertainty on the bounds of the 95%-confidence interval for the maximum-to-mean ratio can be considerable, in this case about 10%.

In this paper, we present two transformation methods that enable the uncertainty on N_{eq} to be reduced and, for large M , to be asymptotically eliminated. The first method is relatively simple, but does not address the ambiguity of the criteria for uncorrelatedness. The second method is rigorous, but requires greater computational effort. For the latter, a distinction is also made between sample decorrelation and ensemble decorrelation.

3. Ensemble decimation

In conventional sample decimation, the tuner data is subsampled at a rate corresponding to the correlation distance in the original measured sample, on the basis of an a priori chosen criterion for uncorrelatedness such as (1). For improved decimation accuracy, *ensemble* decimation factors can be calculated based on the ensemble acf. This eliminates sample variability in the associated decimation factor, yielding a more reliable value for chambers with variable boundary conditions between tests, e.g. with different EUTs in place or with EUTs in different chamber locations and orientations.

The method is based on the application of the Wiener-Khinchine theorem for the power spectral density function (psdf) [5,6]. A 3rd-order exponential interpolation was used to obtain a filtered psdf. It thus appears that the normalized one-sided psdf can be modelled to good approximation as $g(\omega) = \beta \exp(-\beta\omega)$ for $\omega > 0$, where β is a spectral parameter characterizing the ensemble stirring process. Such an ensemble characterization achieves minimum bias in the estimation. The inverse Fourier cosine transformation of $g(\omega)$ then yields the ensemble acf:

$$\rho(\tau) = (\pi / 2) F_C^{-1}[g(\omega)](\tau) = [1 + (\tau / \beta)^2]^{-1} \quad (2)$$

from which the ensemble correlation length and decimation factor follow, e.g. with (1) this yields $\tau_{eq} = \beta \sqrt{e-1}$. The merit of the technique is in the fact that the ensemble decimation factor is obtained from a *single* representative sample acf.

Fig 2 shows a typical single-sample acf and associated psdf, as measured in the *NPL* stadium reverberation chamber at 8.2 GHz in mode-tuned operation for a stirrer step $\Delta\tau$. The corresponding ensemble acf and psdf are also shown. More refined models for the psdf and stirring process often lead to nonlinear estimation problems for the spectral parameter(s).

4. Orthogonal expansion

4.1 Sample orthogonalization

Orthogonal expansions are possible for stationary as well as non-stationary processes, which may have a Gauss normal distribution or other. In this method, the sample data are maximally decorrelated, for a given sample size M with arbitrary N_{eq} , using an eigenvalue decomposition. A 1-dimensional sequence of discrete stirrer data x_k is expanded as:

$$x_k = \sum_{m=0}^{M-1} a_m \phi_{mk}(\tau) \delta(\tau + T/2 - k\Delta\tau), \quad -T/2 \leq \tau \leq T/2 \quad (3)$$

$$a_m = \int_{-T/2}^{T/2} x_k \phi_{mk}^*(\tau) \delta(\tau + T/2 - k\Delta\tau) d\tau \quad (4)$$

where T is the length of the tuner (stirrer) sweep interval in units $\Delta\tau$ and where the $M \times 1$ -vectors $\phi_m = [\phi_{mk}(\tau) \delta(\tau + T/2 - k\Delta\tau)]$ form a complete orthonormal set, chosen such that $\langle (a_m - \langle a_m \rangle)^* (a_n - \langle a_n \rangle) \rangle = \lambda_m^2 \delta_{mn}$. Upon substituting (4), the λ_m^2 and ϕ_m for standardized x_k are found as the eigenvalues and associated eigenvectors, being solutions of the matrix eigenvalue equation:

$$(R - \lambda_m^2 I) \phi_m = 0 \quad (5)$$

in which R is the $M \times M$ data autocorrelation matrix and I is the $M \times M$ unit matrix. For a wide-sense stationary stirring process, R is Hermitian so that all λ_m^2 are real positive scalars. Furthermore, all ϕ_m are then real vectors, hence if the data x_k have a complex Gauss normal distribution (e.g. for measured complex ideal reverberant fields), then the random expansion coefficients a_m also exhibit a complex Gauss normal distribution, by virtue of (3). In this case the uncorrelated a_m , generally defining a white process, are furthermore statistically independent, so that the $|a_m|^2$ exhibit a χ^2 distribution:

$$f_{|a_m|^2}(|a_m|^2) = \exp(-|a_m|^2 / \lambda_m^2) / \lambda_m^2 \quad (6)$$

For the 3-dimensional (vector) stirring process of interest, the $\phi_{mk}(\tau) \delta(\tau + T/2 - k\Delta\tau)$ are themselves 1×3 -vectors, but the a_m remain scalar. For mode-stirring, as opposed to mode-tuning operation, each 1×3 -vector x_k is itself an average across the width of the sampling window. In general, this averaging affects their distribution as compared to the point values. For multiple measurements, the probability density functions of the mean field magnitude $|E|$ or mean power density P associated with a Cartesian field component ($\alpha = x, y, z$) or with the total field (t) then follow upon inverse Fourier transformation of the respective characteristic functions associated with the $\Sigma |a_m|^2$ as:

$$f_{p_\alpha}(p_\alpha) = \int_{-\infty}^{+\infty} \left[\prod_m (1 + j\omega \lambda_m)^{-1} \right] \exp(j\omega p_\alpha) d\omega \quad (7)$$

$$f_{p_t}(p_t) = \int_{-\infty}^{+\infty} \left[\prod_m (1 + j\omega \lambda_m)^{-3} \right] \exp(j\omega p_t) d\omega \quad (8)$$

$$f_{|E_\alpha|}(|e_\alpha|) = \int_{-\infty}^{+\infty} \left[\prod_m \psi_m(\omega) \right] \exp(j\omega |e_\alpha|) d\omega \quad (9)$$

$$f_{|E_t|}(|e_t|) = \int_{-\infty}^{+\infty} \left[\prod_m \zeta_m(\omega) \right] \exp(j\omega |e_t|) d\omega \quad (10)$$

with

$$\psi_m(\omega) = 1 + \frac{\sqrt{\pi}}{2} \left(\frac{j\omega\lambda_m}{2} \right) \left[\operatorname{erf} \left(\frac{j\omega\lambda_m}{2} \right) - 1 \right] \exp \left[\left(\frac{j\omega\lambda_m}{2} \right)^2 \right]$$

$$\zeta_m(\omega) = \int_0^\infty \frac{z^5}{\lambda_m^6} \exp \left[- \left(\frac{z}{\lambda_m} + \frac{j\omega\lambda_m}{2} \right)^2 \right] dz \exp \left[\left(\frac{j\omega\lambda_m}{2} \right)^2 \right]$$

The corresponding cumulative distributions functions (cdfs) are obtained by replacing the factor $\exp(j\omega p_\alpha)$ etc. in (7)-(10) by $[\exp(j\omega p_\alpha)-1]/(j\omega)$ etc. These expressions now allow for a comparison with ideal $\chi_{2pN}^{(2)}$ distributions ($p = 1,2,3$) for the field magnitude or power density of statistically independent data [8], as is usually assumed ad hoc. Note that the existence of the pdfs (7)-(10) presume R to be positive definite, whereas the existence of the cdfs only require R to be positive semi-definite [10].

We have applied the technique to the mode-tuned operation of our chamber by solving the eigenvalue problem (5) numerically. In general, this requires the inversion of a large $M \times M$ -matrix. However, this is not required for the problem at hand if $M \rightarrow \infty$ or if the acf is sufficiently simple, as will be demonstrated next.

If the *sample* acf $\rho(\Delta\tau)$ and associated sample autocorrelation matrix $R(\Delta\tau)$ are used, and if a decomposition (decorrelation) is required only for the wide-sense stationary process associated with a large set of sample stirrer data, then the eigenvalue problem can be solved semi-analytically because of the circulant nature of the Töplitz matrix R under stationary conditions [2,7]. In the limit $M \rightarrow \infty$ and arbitrary nonzero $\Delta\tau$, the eigenvalues are found as:

$$\lambda_m = \sum_{k=0}^{M-1} R_{1k} \exp(jk\omega_m)$$

where R_1 , the first row of R , is the sampled, i.e. discrete acf; $\omega_m = 2m/M$ with $m = 0,1,\dots$ and with associated eigenvectors which are independent of R_{1k} :

$$\phi_m = [1; \exp(-j\omega_m); \exp(-j2\omega_m); \dots; \exp(-j(M-1)\omega_m)] / \sqrt{M}$$

The vector of transformed measured stirrer data, $[x_k'] = [\phi_{km}]^+ \cdot [x_m]$, then follows as [2]:

$$\begin{bmatrix} x_0' \\ x_1' \\ \vdots \\ x_{M-1}' \end{bmatrix} = M^{-1/2} \begin{bmatrix} \sum_m [\exp(-jm2\pi\omega_0 / M) x_m] \\ \sum_m [\exp(-jm2\pi\omega_1 / M) x_m] \\ \dots \\ \sum_m [\exp(-jm2\pi\omega_{M-1} / M) x_m] \end{bmatrix}$$

which become asymptotically uncorrelated when $M \rightarrow \infty$. Thus, as a result of R being circulant, the orthogonal transformation is asymptotically equivalent to a unitary discrete

Fourier transformation, for which a fast implementation is readily available. Consequently, the uncorrelated field is asymptotically obtained as the unitary Fourier transformed field.

Sample orthogonalization is recommended if the measurement or test set-up changes very little or not at all between tests (viz. for identical stepping or rotational speed of the tuner per test cycle, identical stirrers and antennas with fixed orientations, fixed cable layout, identical type, position and orientation of EUTs etc.), or if only a single assessment is required. These conditions may be difficult to meet in practical EMC tests. In this case, ensemble orthogonalization may be more appropriate.

4.2 Ensemble orthogonalization

Here the parameters of the stirring process $x(\tau)$ have been changed, due to differences in stirring step or speed, chamber reconfigurations including stirrer alterations, etc., or the EUT is left unspecified or is repositioned in between tests. Hence *sample* orthogonalization is no longer appropriate. In this case, the method used in the ensemble decimation procedure can be applied, prior to performing the orthogonal expansion. The ensemble psdf then yields the *ensemble* acf for constructing the kernel of the eigenvalue problem. When choosing a rational function to model the psdf, viz. $g(\omega) = 4\gamma / (\omega^2 + \gamma^2)$ with spectral parameter γ , the ensemble acf can be expressed in closed form and yields an integrable kernel for the eigenvalue equation. The orthogonal expansion now reads:

$$x(\tau) = \sum_{m=0}^{M-1} a_m \phi_m(\tau); \quad a_m = \int_{-T/2}^{T/2} x(\tau) \phi_m^*(\tau) d\tau \quad (16)$$

where the continuous eigenfunctions $\phi_m(\tau)$ are now solutions of a linear homogeneous Fredholm integral equation of the second kind:

$$\int_{-T/2}^{T/2} \rho(\tau, \nu) \phi_m(\nu) d\nu = \lambda_m^2 \phi_m(\tau) \quad (17)$$

With the associated acf $\rho(\tau, \nu) = (\pi/2) F_c^{-1}[g(\omega)] = \exp(-\gamma|\tau - \nu|)$, the transformed stirrer data can be obtained analytically, which is advantageous in applying the orthogonalization procedure in practice. The eigenfunctions are found upon twice differentiating (17) and solving the resulting 2nd-order differential equation as:

$$\phi_{2m+1}(\tau) = \left\{ \frac{T}{2} \left[1 + \frac{\sin(\beta T \sqrt{2\sigma^2 / (\beta \lambda_{2m+1}^2) - 1})}{\beta T \sqrt{2\sigma^2 / (\beta \lambda_{2m+1}^2) - 1}} \right] \right\}^{-1/2} \cos(\beta \tau \sqrt{2\sigma^2 / (\beta \lambda_{2m+1}^2) - 1}) \quad (18)$$

$$\phi_{2m}(\tau) = \left\{ \frac{T}{2} \left[1 - \frac{\sin(\beta T \sqrt{2\sigma^2 / (\beta \lambda_{2m}^2) - 1})}{\beta T \sqrt{2\sigma^2 / (\beta \lambda_{2m}^2) - 1}} \right] \right\}^{-1/2} \sin(\beta \tau \sqrt{2\sigma^2 / (\beta \lambda_{2m}^2) - 1}) \quad (19)$$

Both subsets of eigenfunctions (18)-(19) become asymptotically harmonically related in the limit $\beta T \sqrt{[2\sigma^2 / (\beta \lambda_m^2) - 1]} \rightarrow \infty$. The associated eigenvalues λ_m^2 are found from the iterative solution of the characteristic equation:

$$\tan\left(\beta \frac{T}{2} \sqrt{\frac{2\sigma^2}{\beta\lambda_m^2} - 1}\right) \pm \left[\sqrt{\frac{2\sigma^2}{\beta\lambda_m^2} - 1}\right]^{\pm 1} = 0 \quad (20)$$

For general $\rho(\tau, \nu)$, the λ_m^2 are monotonically increasing functions of T . In the limit $T \rightarrow \infty$, they form a continuum $\lambda^2(\omega)$ that coincides with $g(\omega)$ and the $\phi_m(\tau)$ then tend to $\exp(j\omega\tau)$. For the full 3-dimensional stirring process, the $\phi_m(\tau)$ are 1×3-vector eigenfunctions.

The results (18)-(20) apply to the analytical modelling of the continuous stirring process $x(\tau)$, based on measurements in our chamber, and again avoid the need for solving the integral equation (17) numerically. For more general acfs, the integration can still be performed provided (17) has a separable Pincherle-Goursat kernel of finite order, in which case the equation can always be reduced to a system of linear algebraic equations. For a wide-sense stationary stirring process, this kernel is Hermitian i.e. $\rho(\tau, \nu) = \rho^*(\nu, \tau)$, so that the λ_m^2 are real positive and the $\phi_m(\tau)$ are real, as noted in Sec 4.1.

5. Examples

We applied the above techniques to measured mode-tuned reverberation data as collected in the *NPL* untuned stadium reverberation chamber (diameter $d = 70$ cm) [2,3,5,6] at 2 GHz and 8.2 GHz, corresponding to $d/\lambda = 4.67$ and 19.13, thus yielding a relatively low and high mode density, respectively.

In Fig 3, the dashed line shows the measured cdf for the mean-normalized amplitude of a mean-normalized rectangular field component $|E_z|/\langle|E_z|\rangle$ at 8.2 GHz, after ensemble decimation with decimation factor $m_z = 4$, obtained using the $\rho^{-1}(e^{-1})$ -criterion. For comparison, the dotted line shows the theoretical χ_2 cdf for ideal reverberation chambers. The good agreement between both cdfs indicates adequate reverberation performance at this frequency, with decimation being sufficient for obtaining approximately independent samples. The solid line shows the measured cdf at 2 GHz, obtained as an average over 25 measurement locations with $M = 1200$ per location. This distribution is to be compared with the theoretical cdf (dot-dashed line), obtained using the sample orthogonalization as $[E_z'] = [\phi_{km}]^+ \cdot [E_z]$ and based on the measured acf kernel. The transformed fields fully justify a comparison with a theoretical cdf based on independent samples. This, then, enables various field statistics to be estimated more accurately.

Fig 4 compares the magnitude of the sample acf ρ_z (crosses), after sample decimation, with the one for sample orthogonalization, ρ_z' (diamonds). As can be seen from a comparison of the values of the acf for the first two stirrer positions, the correlation distance after orthogonalization is only about a quarter of the corresponding value for the decimated data, regardless of the choice of the threshold for uncorrelatedness. The Figure effectively indicates that the value of N_{eq} , estimated using sample decimation as $N_{eq} = M/m_z = 300$, is being overestimated and that orthogonalization gives rise to superior decorrelation.

6. Conclusions

We have shown how ensemble decimation and orthogonal expansion of stirrer data can lead to improved estimates of the equivalent number of independent samples and sample statistics of the field magnitude and power density.

Ensemble decimation uses the psdf rather than the measured psdf to improve on the estimated level of data correlation, hence it serves in reducing the test variability and in improving the test reproducibility. Recall that uncorrelated data are merely *linearly* independent, unless they obey a Gauss normal distribution in which case uncorrelatedness guarantees statistical (i.e. linear as well as higher-order nonlinear) independence. Decimation or orthogonal expansion of stirrer data representing field magnitude or power density, obeying non-Gaussian distributions as opposed to complex field data, may therefore leave residual nonlinear dependencies between them.

Orthogonal expansion was found to yield superior data decorrelation. For arbitrarily large sample sizes, the orthogonal expansion asymptotically reduces to a discrete Fourier transformation of the acf for stationary data, i.e. a transformation of the stirrer data into the Fourier spectral stir domain. In practice, the accuracy of such approximation depends on the length $T = M\Delta\tau$ of the stir interval relative to the calculated scale of fluctuation θ_r of the stirring process [5]. The orthogonal expansion is furthermore useful in studying the distribution function and statistics of the actual field magnitude or power density, especially for moderate levels of mode density. The accuracy of the estimated distribution function rests on the accuracy of the modelling of the autocorrelation function or the power spectral density of the stirring process. Furthermore, the technique enables one to determine the effect of local stir averaging and, by extension, local spatial averaging on the change in the distribution function of the stirrer data. This is of considerable importance in determining level crossing statistics for EMC and EUT failure testing [2,5,6].

The above analysis has concentrated on data autocorrelation, ignoring the cross-correlation that may exist between in-phase and quadrature or between Cartesian field components. These cross-correlations can be eliminated along similar lines, by requiring $\langle (a_m - \langle a_m \rangle)^* (b_n - \langle b_n \rangle) \rangle = \lambda_m^2 \delta_{mn}$ for the expansion coefficients a_m and b_n of the components. In an alternative approach, the cross-correlation between the field components can be taken into account explicitly via a correlation coefficient [9]. This method requires an estimate for the thus introduced additional statistical parameter.

7. References

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8. Figures

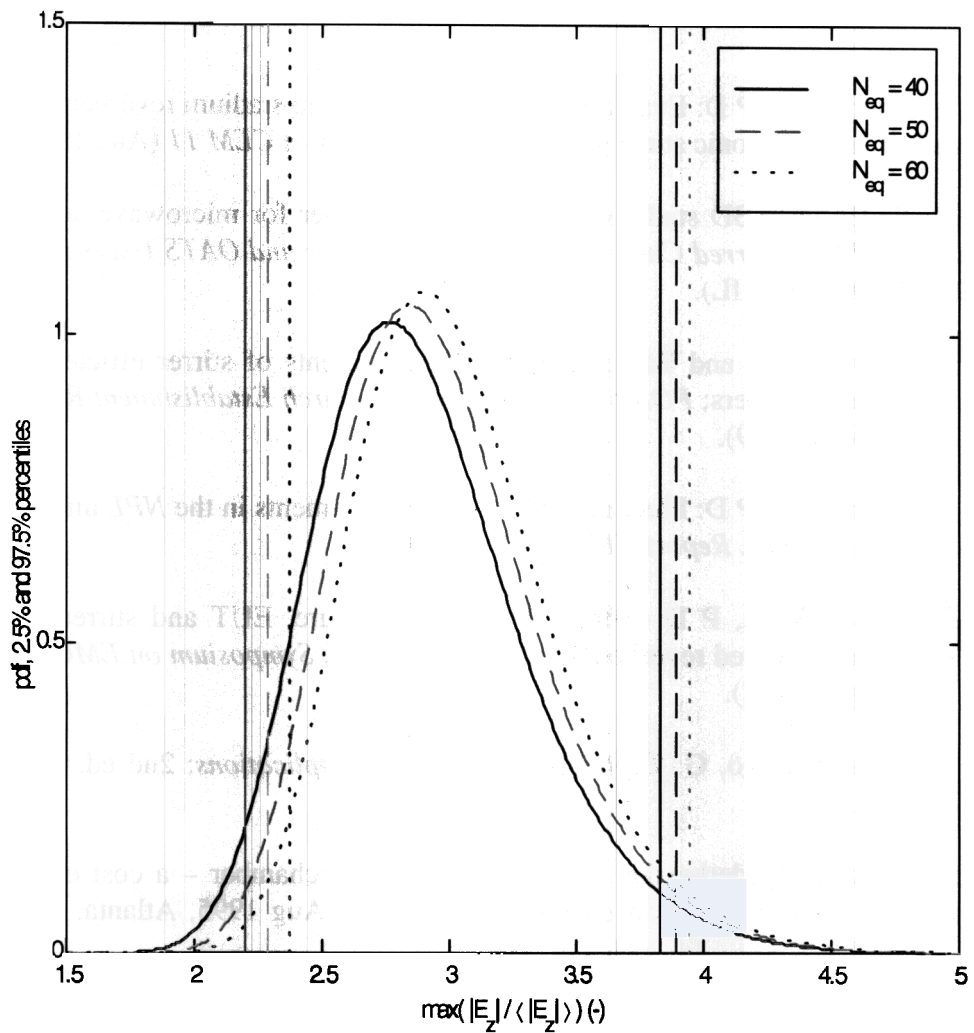


Figure 1: Theoretical probability density functions, 2.5% and 97.5% percentiles of maximum-to-mean field strength for Cartesian field components for various N_{eq} .

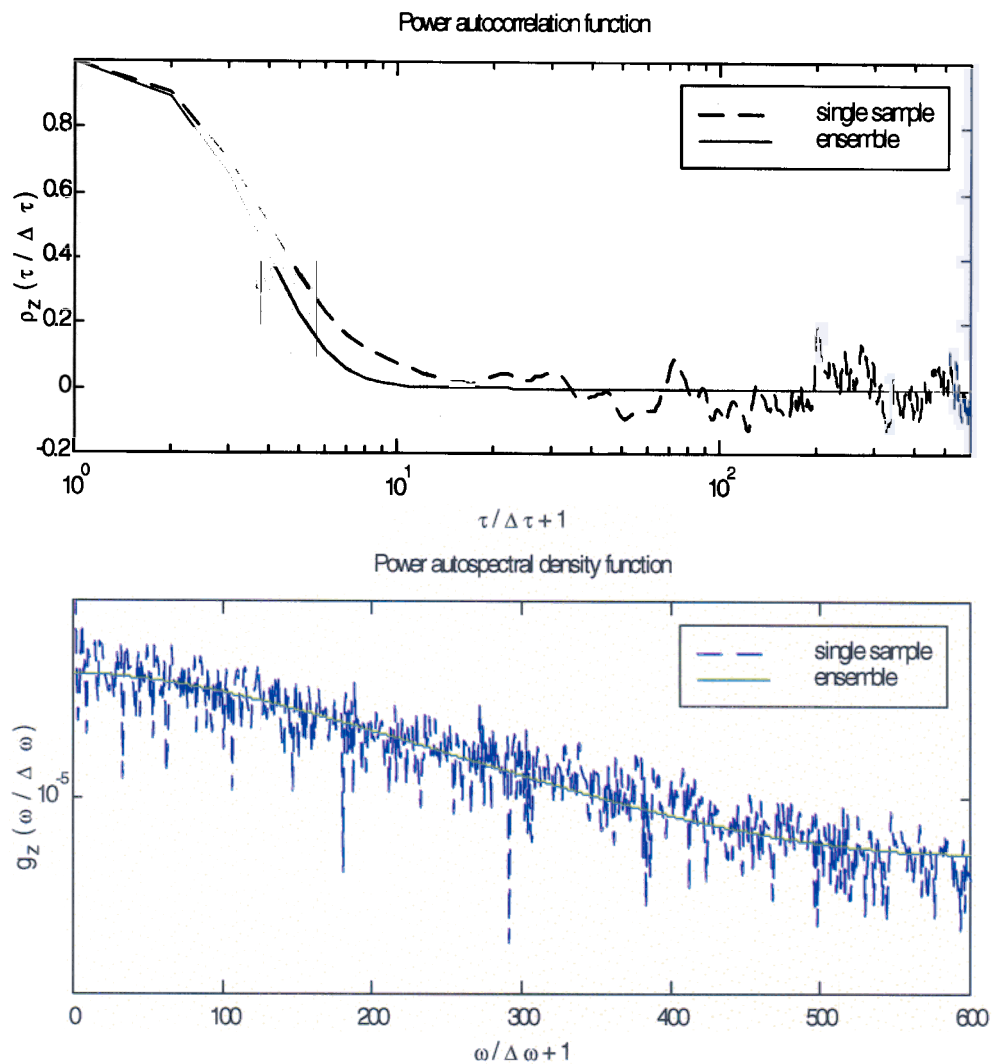


Figure 2: Single-sample and ensemble autocorrelation function and power autospectral density function for Cartesian field component @ 8.2 GHz.

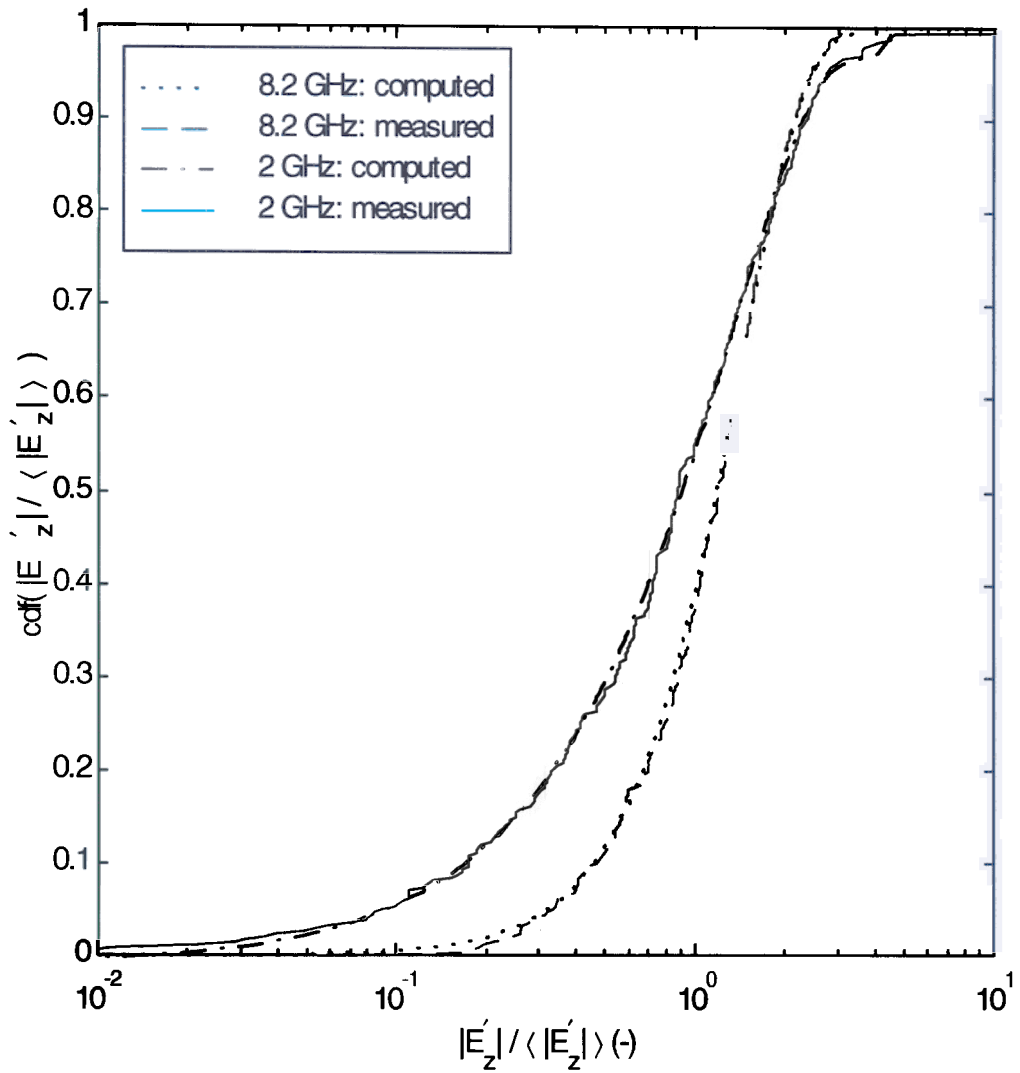


Figure 3: Theoretical and computed cumulative distribution function of transformed Cartesian field components @ 2.5 and 8.2 GHz, based on measured distributions.

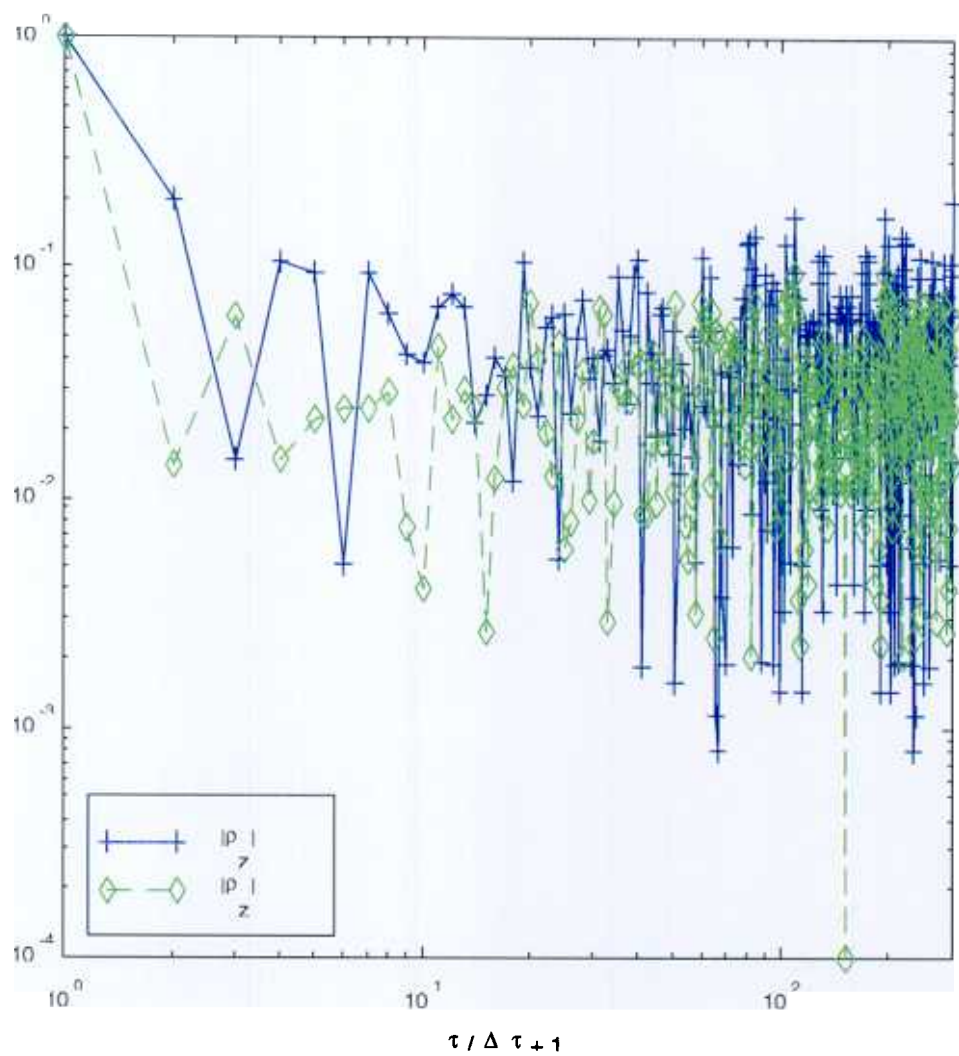


Figure 4: Measured autocorrelation function of stirrer data after ensemble decimation (ρ_z) and after orthogonal expansion (ρ_z') @ 8.2 GHz.