

REPORT

Modelling Acoustic Signals Containing Reflections in the Calibration of Underwater Acoustic Transducers in Laboratory Tanks

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May 2000

Modelling Acoustic Signals Containing Reflections in the Calibration of Underwater Acoustic Transducers in Laboratory Tanks

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ABSTRACT

In the calibration of an underwater electroacoustic transducer, the stepped-sinusoidal signals that are commonly used are contaminated by transients due to the resonant behaviour of the transducer as well as noise. In addition, when calibrations are undertaken in reverberant laboratory tanks of finite size, reflections from the tank boundaries often arrive before the steady-state response of the transducer can be observed directly.

Previous work has incorporated the use of signal modelling to predict the steady-state response from the initial transient-dominated part of the waveform prior to the arrival of reflections. The approach used was to model the free-time response of the device by a function consisting of a sum of complex exponential terms which were used to describe both the steady-state and resonant behaviour of the device. However, benefits may be expected, particularly for high-Q devices, from extending the time-window for the signal modelling into the region containing reflections.

In this report, models are described for acoustic signals that account both for the transient behaviour of the devices as well as the effects of reflections. Also described are the data processing algorithms for estimating values for the parameters in these models from measurement data in the cases where, firstly, the arrival times for the reflections are known and, secondly, they are regarded as additional parameters to be determined. Results are presented of applying these algorithms to analyse simulated data representing the measured response of a device as well as data obtained from measurements of real transducers.

This work described in this report was undertaken as part of project 3.1.7 of the 1999-2002 NMS Acoustics Metrology Programme.

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ISSN 1369-6785

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Approved on behalf of the Managing Director, NPL
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1 INTRODUCTION

Calibration of underwater electroacoustic transducers at kilohertz frequencies is ideally undertaken within a free-field environment. For continuous wave fields, this requires the use of a large volume of water such as a lake, reservoir or ocean in order that reflections from the medium boundaries are sufficiently attenuated by propagation losses. The impracticality and expense of utilising such volumes has led to the use of laboratory tanks for measurements. Laboratory tanks have the advantage of providing more controlled experimental conditions, but their use introduces further measurement problems, especially if the tank dimensions are small when measured in acoustic wavelengths.

During laboratory tank calibrations of transducers, a minimum of two transducers, one acting as a transmitter or projector and the other as a receiver, are suspended in a tank of finite size filled with water. The projector is driven by a signal consisting of a discrete-frequency tone-burst of finite duration. The signal detected by the receiving hydrophone is contaminated by

- a) transients due to the resonance of the devices,
- b) reflections of the transmitted signal from the tank walls, floor and the water-surface, and
- c) noise (potentially from both acoustic and electrical sources).

The extra distance travelled by the reflected signals causes them to arrive later in time than the direct path signal. The available time to observe the direct signal before the reflections arrive is termed the echo-free time, and depends on the size of the tank and the exact position of the transducers within the tank. Figure 1 is a schematic diagram of a projector and receiver in a water tank showing the main sources of reflections (ignoring multiple reflections).

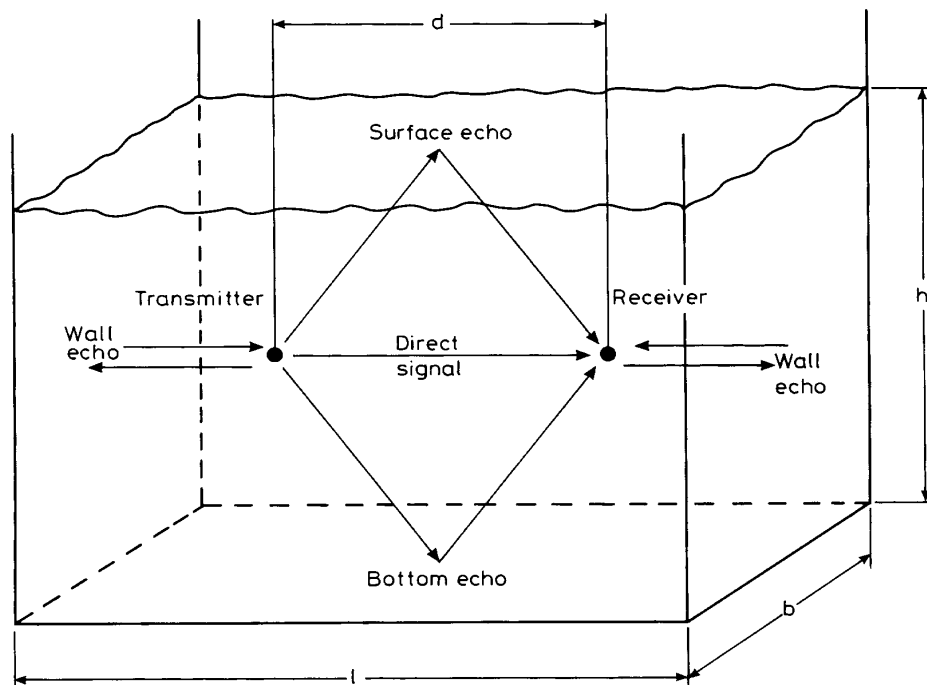


Figure 1: Schematic diagram of a projector and receiver in a water tank showing the main sources of reflections (ignoring multiple reflections).

The received signal is sampled at uniformly spaced times by use of an digitising oscilloscope or signal analyser, and measurements are made of the amplitude of the signal. Figures 2 and 3 show some example waveforms obtained in this way for a projector with a resonance frequency of (approximately) 40 kHz and corresponding to drive frequencies of 30 kHz (Figure 2) and 40 kHz (Figure 3) with the transducer positions in the tank the same in each case.

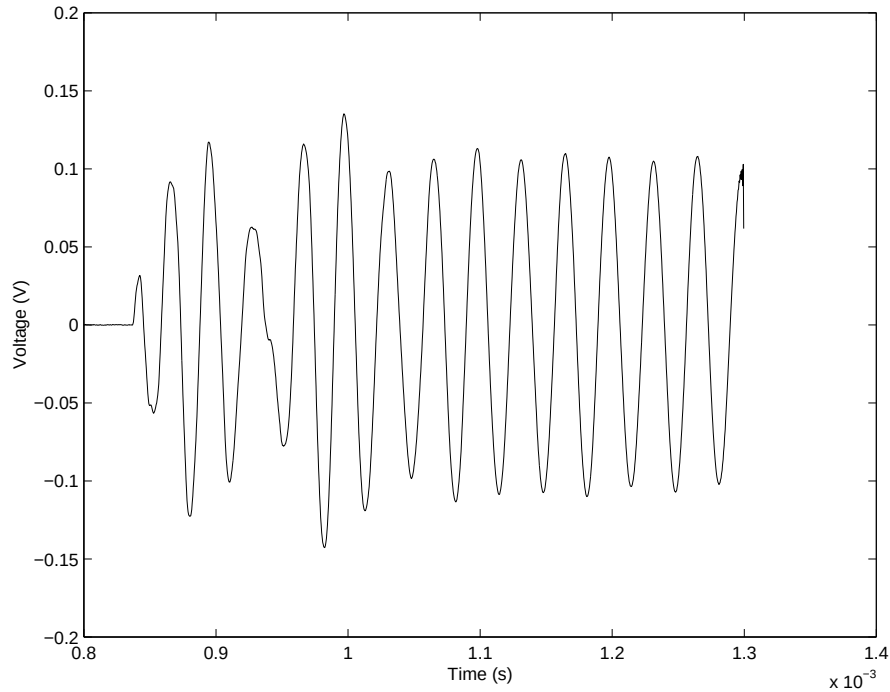


Figure 2: Measured waveform for a device with a resonance frequency of (approximately) 40 kHz driven at 30 kHz with acoustic reflection arriving at about 0.95 ms.

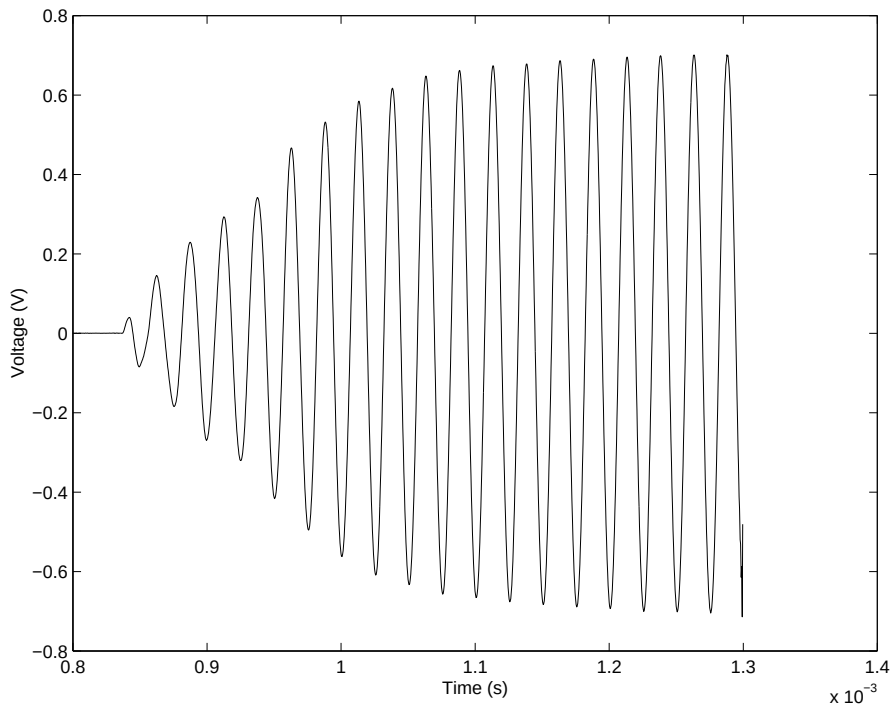


Figure 3: Measured waveform for a device with a resonance frequency of (approximately) 40 kHz driven at resonance with acoustic reflection arriving at about 0.95 ms.

The arrival of a reflection at (approximately) $t = 0.95$ ms is clearly visible in Figure 2, but is much less evident in the waveform of Figure 3. This illustrates how difficult it can be to identify reflections in the received signal. Even if the length of the burst is reduced to one or two cycles this may not allow satisfactory separation of the signals since a high- Q transducer may oscillate for such a long time that the direct and reflected signals may still overlap.

Conventional signal analysis techniques, that rely on there being enough echo-free time before the arrival of the first reflection for the steady-state to be observed directly, cannot be applied in these examples because the first reflection arrives *before* steady-state is reached. In NPL Report CMAM 029 [1], an approach for overcoming this difficulty is described in which the part of the waveform in the echo-free time is modelled by a function that is assumed to describe the response of an electroacoustic transducer at frequencies close to its resonance frequency. The results reported there suggest that for moderately resonant transducers errors of less than 0.5 dB may be obtained if data representing approximately one cycle of the resonance frequency is available. However, the approach becomes progressively less accurate as the data becomes more noisy, or as the Q of the transducer is increased, or as the echo-free time is reduced. For the example waveforms of Figures 3 and 4, although the echo-free time corresponds to nearly two-and-a-half cycles of the main resonance frequency, the Q -factor is about ten and so we begin to observe the difficulties in obtaining accurate results when using the above approach.

A way to improve the results for higher Q devices is to extend the time-window for modelling into the region containing reflections from tank boundaries. To do this requires that the model used to represent the response of the device is modified to account for the effects of these reflections. The purpose of this report is to describe this extension to the approach presented in [1], and to demonstrate its application to simulated data and to measurement data obtained from real devices.

The report is organised as follows. In Section 2, a model for describing the response of an electroacoustic transducer to a direct signal and to reflections of that signal is presented. In Sections 3 and 4 a discussion is given of algorithms based on the data processing algorithm given in [1] for obtaining estimates of the values of the parameters in the model from measurement data. Two cases are considered: the arrival times are assumed to be known (Section 3), and the arrival times are regarded as additional parameters in the model whose values are to be estimated from the data (Section 4). Results of applying the algorithms to simulated data and to measurement data obtained from real devices are given in Sections 5 and 6, respectively. Here, one of the real devices considered is the ITC1001 transducer that was used in [1] to demonstrate the modelling approach restricted to the echo-free time. Finally, some conclusions are given in Section 7.

2 MODELS OF THE RESPONSE EXTENDED TO INCLUDE REFLECTIONS

In [1] the modelled data is restricted to lie within a time-window containing the free-time response. This time-window is delimited by the turn-on of the device and the arrival of the first echo or reflection. However, benefits might be expected for high- Q transducers from extending this time-window into the region containing reflections. To do this requires that the signal model be modified to include terms that represent the response of the transducer to the arrival of reflections.

Let τ_0 denote the time of arrival of the direct signal or, equivalently, the time of the turn-on of the device and τ_j , $j = 1, \dots, R$, the time of arrival of the j th reflection resulting from the reflection of the transmitted signal from a boundary of the tank or the water-surface (see Figure 1). Then, a model for the measured signal is given by

$$y(t) = \sum_{j=0}^R y_j(t), \quad (1)$$

where $y_0(t)$ models the response to the direct signal and $y_j(t)$, $j = 1, \dots, R$, the response to the j th reflection, with

$$y_j(t) = A_{j,0} \sin\{2\pi f_0(t - \tau_j) + \phi_{j,0}\} H(t - \tau_j) + \sum_{k=1}^{n_r} A_{j,k} e^{d_k(t - \tau_j)} \sin\{2\pi f_k(t - \tau_j) + \phi_{j,k}\} H(t - \tau_j), \quad (2)$$

and $H(t)$ defined by

$$H(t) = \begin{cases} 0, & t < 0, \\ 1, & t \geq 0. \end{cases}$$

Here, the first term in (2) is used to describe the steady-state response of the device with f_0 being the drive frequency. The remaining terms in (2) describe the resonant behaviour with f_k denoting the resonance frequencies and d_k are the damping factors for the device that are related to its Q -factors according to

$$Q_k = -\frac{\omega_k}{2d_k} = -\frac{\pi f_k}{d_k},$$

where $\omega_k = 2\pi f_k$ is the angular frequency.

Note that $y(t)$ reduces to the model

$$y(t) = A_0 \sin\{2\pi f_0 t + \phi_0\} + \sum_{k=1}^{n_r} A_k e^{d_k t} \sin\{2\pi f_k t + \phi_k\},$$

for the free-time response given [1] in the case that there are no reflections ($R = 0$) and the arrival of the direct signal is zero ($\tau_0 = 0$). In practice, the arrival of the direct signal is unknown, and τ_0 denotes the start of the waveform to be modelled and is set to exceed, but to be close to, this arrival time.

It is assumed that no dispersive or nonlinear effects are present within the system such that the reflected signals have the same frequency content as the direct signal. Consequently, the frequency f_0 of the steady-state response, and the frequencies f_k , $k = 1, \dots, n_r$, and damping factors d_k , $k = 1, \dots, n_r$, defining the resonances of the device, are the same for each signal arrival $y_j(t)$. However, the reflected signals may be expected to have attenuated amplitudes and be delayed in time with respect to the direct signal. Hence, the amplitudes $A_{j,k}$, $k = 1, \dots, n_r$, and phase angles $\phi_{j,k}$, $k = 1, \dots, n_r$, may be different for each response $y_j(t)$, and so these parameters are indexed using j as well as k . Models of the above form have been presented, although using different notations, in [2, 3, 4, 5].

As in [1], estimates of the values of the parameters defining the model $y(t)$ are obtained by solving the following nonlinear least-squares problem: given measurement data $\{(t_i, y_i): i = 1, \dots, m\}$,

$$\text{minimise } \sum_{i=1}^m \{y_i - y(t_i)\}^2 \quad (3)$$

with respect to the parameters of $y(t)$. We distinguish between the cases for which (a) the arrival times τ_j , $j = 1, \dots, R$, are *known*, and (b) the arrival times are *unknown* parameters to be determined as part of the minimisation process. These cases are considered in Sections 3 and 4, respectively.

3 DATA PROCESSING ALGORITHM FOR KNOWN ARRIVAL TIMES

If the arrival times τ_j , $j = 1, \dots, R$, for the reflections of the transmitted signal are known sufficiently accurately, these parameters may be fixed within the model defined by (1) and (2). Writing (2) in the equivalent form

$$y_j(t) = \left[S_{j,0} \cos\{2\pi f_0(t - \tau_j)\} + C_{j,0} \sin\{2\pi f_0(t - \tau_j)\} \right] H(t - \tau_j) \\ + \sum_{k=1}^{n_r} \left[S_{j,k} e^{d_k(t - \tau_j)} \cos\{2\pi f_k(t - \tau_j)\} + C_{j,k} e^{d_k(t - \tau_j)} \sin\{2\pi f_k(t - \tau_j)\} \right] H(t - \tau_j),$$

where

$$S_{j,k} = A_{j,k} \sin \phi_{j,k}, \quad C_{j,k} = A_{j,k} \cos \phi_{j,k},$$

the minimisation (3) is performed with respect to the parameters f_k , $k = 1, \dots, n_r$, d_k , $k = 1, \dots, n_r$, $S_{j,k}$, $k = 1, \dots, n_r$, $j = 0, \dots, R$, and $C_{j,k}$, $k = 1, \dots, n_r$, $j = 0, \dots, R$.

One way to determine the arrival times for the reflections is from geometrical considerations. If we measure the positions of the transmitting and receiving devices in relation to each other and we know the speed of the transmitted signal in the medium containing the devices, we can calculate the time it takes for the direct signal to travel between the devices. Similarly, if we measure the positions of the devices in relation to the reflecting surfaces, we can calculate the times it takes for the signal to travel between the devices following reflections at these surfaces. The minimum of these times defines the time of arrival of the first reflection.

Other approaches to determining the arrival times include the application of data processing techniques, such as wavelet analysis, to identify “features” in the measured signal. These approaches are currently being investigated at NPL.

Whichever approach is used, the quality of the final results will depend on the accuracy to which the arrival times are determined. However, it is shown in [2] (and see Section 4) that the resolution of the arrival time estimates obtained by solving the least-squares problem (3) with respect to *all* the parameters of $y(t)$, including the arrival times, is limited to the sampling points t_i . Consequently, the length δt of the sampling interval, where

$$\delta t = t_{i+1} - t_i,$$

provides a lower bound on the accuracy to which the arrival times are required to be specified.

The steps in the data processing algorithm used to solve this problem are described below: this is essentially the same data processing algorithm presented in Section 6 of [1] with modifications indicated.

STEP 1: Acquire data [1, Section 6.1].

STEP 2: Remove any non-zero dc level in the acquired data [1, Section 6.2].

STEP 3: Identify free-time response [1, Section 6.3].

STEP 4: Apply linear prediction methods to give initial estimates of the model parameters [1, Section 6.4].

The drive frequency f_0 is specified, together with the frequency f_r and Q -factor Q_r defining any known resonant behaviour of the device. Using this information, the data defining the *free-time* response of the device is filtered to remove the specified drive and resonance components [6]. The forward and backward prediction versions of the least-squares Prony-SVD method are applied to the filtered data to give estimates of the unknown poles [6, 7, 8, 9, 10]. These poles, together with those defined by the known resonances, determine initial estimates of the frequencies f_k , $k = 1, \dots, n_r$, and damping factors d_k , $k = 1, \dots, n_r$. The residues are then estimated as in the standard application of Prony's method, and these determine initial estimates of the amplitudes $A_{0,k}$, $k = 1, \dots, n_r$, and phase angles $\phi_{0,k}$, $k = 1, \dots, n_r$, and hence the parameters $S_{0,k}$, $k = 1, \dots, n_r$, and $C_{0,k}$, $k = 1, \dots, n_r$, corresponding to $y_0(t)$.

Finally, we require to set initial estimates for the remaining parameters $S_{j,k}$ and $C_{j,k}$, $k = 1, \dots, n_r$, $j = 1, \dots, R$. A somewhat crude approach would be to set these parameters to be zero or equal to the values of $S_{0,k}$ and $C_{0,k}$, $k = 1, \dots, n_r$. A better strategy is as follows.

Firstly, subtract values of the model $y_0(t)$ estimated as above from the data to give a new set of data $\{(t_i, z_i): i = 1, \dots, m\}$, where

$$z_i = y_i - y_0(t_i).$$

(If $y_0(t)$ is a good estimate of the response of the device to the direct signal, we should be left with noise in the free-time window and the response of the device to the first reflection in a time-window defined by the arrival times of the first and second reflections.)

Secondly, apply linear prediction to the data within this time-window to determine initial estimates of the amplitudes $A_{1,k}$, $k = 1, \dots, n_r$, and phase angles $\phi_{1,k}$, $k = 1, \dots, n_r$, and hence the parameters $S_{1,k}$, $k = 1, \dots, n_r$, and $C_{1,k}$, $k = 1, \dots, n_r$, corresponding to $y_1(t)$.

Finally, repeat the process for the j th reflection with

$$z_i = y_i - \sum_{l=0}^{j-1} y_l(t_i)$$

to determine initial estimates of the amplitudes $A_{j,k}$, $k = 1, \dots, n_r$, and phase angles $\phi_{j,k}$, $k = 1, \dots, n_r$, and hence the parameters $S_{j,k}$, $k = 1, \dots, n_r$, and $C_{j,k}$, $k = 1, \dots, n_r$, corresponding to $y_j(t)$.

STEP 5: Apply nonlinear methods to give final estimates of the model parameters [1, Section 6.5].

The Gauss-Newton algorithm [11], safeguarded with a line-search algorithm, is used to solve the nonlinear least-squares estimation problem defined by (3) for data within a time-window containing reflections. In the implementation of this algorithm, *analytic* expressions for the first partial derivatives of the model with respect to the unknown parameters are used to evaluate the Jacobian matrix J at each step of the algorithm (see [1, Appendix A.1]). The number R of reflections, and the times of arrival τ_j , $j = 1, \dots, R$, of the reflections, are specified. The results obtained from the linear prediction undertaken in Step 4 provide initial values for the Gauss-Newton algorithm. Unlike the algorithm described in [1], we suppose that all parameters, including those describing the resonant behaviour of the device but excluding the drive frequency f_0 , are free. Furthermore, instead of using a model with a large

number of resonance components and regularisation to deal with the resulting ill-conditioning of the estimation problem, we choose a number of resonance components (typically one or two) that properly reflects the underlying behaviour of the device.

STEP 6: Validate estimates of the model parameters [1, Section 6.6].

In addition to the approaches to validation described in [1], we can compare the results obtained by applying the data processing algorithm to windows of data containing different numbers of reflections.

4 DATA PROCESSING ALGORITHM FOR UNKNOWN ARRIVAL TIMES

If the arrival times τ_j , $j = 1, \dots, R$, for the reflections of the transmitted signal are *not* known sufficiently accurately, they may be regarded as parameters within the model defined by (1) and (2). The minimisation (3) is then performed with respect to the parameters f_k , $k = 1, \dots, n_r$, d_k , $k = 1, \dots, n_r$, $S_{j,k}$, $k = 1, \dots, n_r$, $j = 0, \dots, R$, $C_{j,k}$, $k = 1, \dots, n_r$, $j = 0, \dots, R$, and τ_j , $j = 1, \dots, R$.

However, this minimisation problem is not well-posed because the resolution of the arrival time estimates obtained by solving this problem is limited to the sampling points t_i [2, 3, 4, 5]. To see this, suppose that $\tau_j = t_K$ for some integer K and $y_j(t)$ is defined by initial conditions at $t = \tau_j$ sufficient to define the parameters $S_{j,k}$, $k = 1, \dots, n_r$, and $C_{j,k}$, $k = 1, \dots, n_r$. Now, since $y_j(t) = 0$ for $t < \tau_j$, it follows that for *any* choice of τ_j satisfying

$$t_K < \tau_j \leq t_{K+1}. \quad (4)$$

we have

$$y_j(t) = 0, \quad t \leq t_K.$$

Furthermore, for *any* choice of τ_j satisfying (4) we can use the original function $y_j(t)$ to set initial conditions at $t = \tau_j$ to define modified parameters $S_{j,k}$, $k = 1, \dots, n_r$, and $C_{j,k}$, $k = 1, \dots, n_r$, so that $y_j(t)$ is unchanged for $t \geq \tau_j$ and, in particular, $y_j(t)$ is unchanged for $t \geq t_{K+1}$. In other words, for *any* choice of τ_j satisfying (4) there is a set of initial conditions at $t = \tau_j$ defining corresponding functions $y_j(t)$ that are identical for $t \leq t_K$ and $t \geq t_{K+1}$. This means that the least-squares error

$$S_j = \sum_{i=1}^m \{y_i - y_j(t_i)\}^2$$

is also identical for these functions which, therefore, are “equally good” in their ability to fit the measurement data.

The consequence of this property for the Gauss-Newton algorithm is that for each reflection included in the signal model the Jacobian matrix J is rank deficient by one, and a unique Gauss-Newton direction (see [1, Appendix A.1]) cannot be determined by the algorithm. In order to resolve this it is suggested in [2] that amplitude constraints based on setting initial conditions at the start of each response are introduced. For example, the initial conditions

$$y_j(\tau_j) = 0, \quad j = 1, \dots, R,$$

lead to the constraint equations

$$S_{j,0} + \sum_{k=1}^{n_r} S_{j,k} = 0, \quad j = 1, \dots, R. \quad (5)$$

The constraint equations, which are linear in the parameters $S_{j,k}$, can be used to eliminate R of the minimisation parameters. For example, we can use (5) to make an explicit substitution for each $S_{j,0}$ in terms of the parameters $S_{j,k}$, $k = 1, \dots, n_r$: this is the approach described in [2]. Alternatively, we can add the constraint equations as additional observation equations with a

sufficiently large weight: this is the approach implemented in this work. In either case the rank deficiency of J is resolved, and a unique Gauss-Newton direction can be determined.

The steps in the data processing algorithm used to solve the estimation problem are essentially as described in Section 3. The Gauss-Newton algorithm, safeguarded with a line-search algorithm, is used to solve the nonlinear least-squares estimation problem defined by (3) subject to the constraint equations (5) as described above. However, in the implementation of this algorithm, the first partial derivatives of the model with respect to the unknown parameters are evaluated *numerically* using the approach based on complex variables described in [12, 13]. This avoids the need to provide analytic expressions for the derivatives with respect to the parameters τ_j , $j = 1, \dots, R$, whilst providing derivative values to high accuracy.

5 RESULTS FOR SIMULATED DATA

In this section we present results obtained from applying the data processing algorithms described in Sections 3 and 4 to data simulated to represent the measured response of a device. The purpose of these simulations is to demonstrate the advantages of extending the signal model to include reflections in cases where modelling using only the free-time response yields unsatisfactory results. In addition, we illustrate the issues discussed regarding the resolution of the arrival-time parameters associated with the reflections.

Figure 4 shows simulated data representing the measured response of a device to a direct signal and two reflections of that signal where we have chosen:

- a) the resonant behaviour to be defined by a frequency of 5 Hz and a Q -factor of 5,
- b) the steady-state behaviour to be defined by a frequency of 3 Hz,
- c) the arrival of the direct signal to be at $t = 0$ s and the steady-state amplitude for the response to the direct signal to be 1 V,
- d) the arrival of the first reflection to be at $t = 0.17$ s,
- e) the arrival of the second reflection to be at $t = 0.62$ s,
- f) the time-window for the data to be from $t = 0$ s to $t = 1$ s and to contain 500 sample points, and
- g) the signal-to-noise ratio ($1/\sigma$) to be 0.01 V.

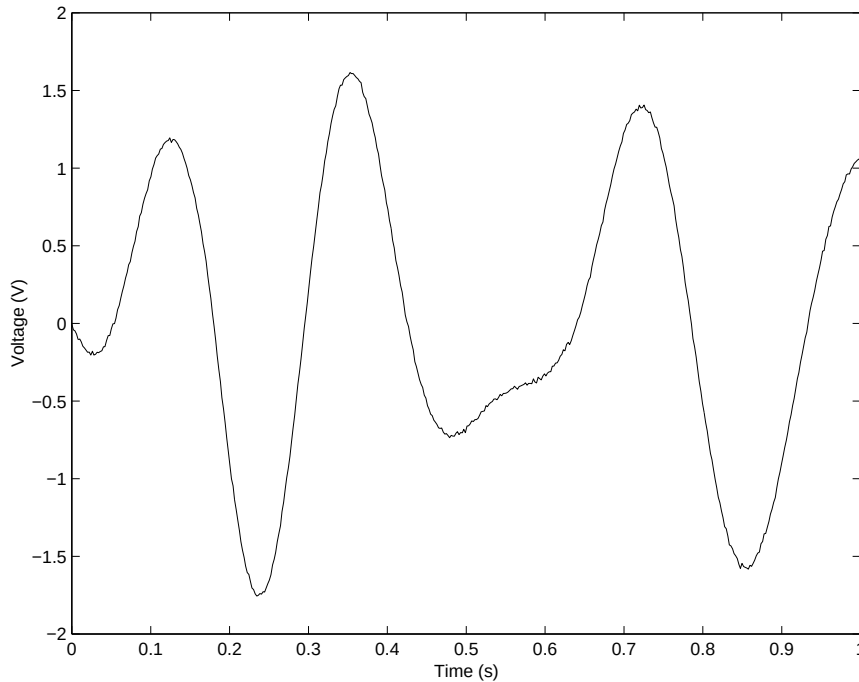


Figure 4: Simulated data representing the measured response of a device to a direct signal (arriving at $t = 0$ s) and two reflections of that signal (arriving at, respectively, $t = 0.17$ s and $t = 0.62$ s).

The simulation is such that during the echo-free time, i.e., from $t = 0$ s to $t = 0.17$ s, we observe only about one half of a cycle of the steady-state response to the direct signal. Furthermore, for the complete observation time of 1 s, the simulated signal is contaminated by the resonant behaviour of the device, i.e., there is never a time for which the resonant behaviour is sufficiently damped for us to observe only the steady-state behaviour of the device.

In our first simulated experiment (*Experiment A*) we use the data processing algorithm described in [1] to model a part of the measured signal restricted to the echo-free time between the arrival of the direct signal and that of the first reflection. The model comprises an undamped sinusoidal component (to represent the steady-state behaviour) and a single damped sinusoidal component (to represent the resonant behaviour). The data and the resulting fitted model are shown in Figure 5, and estimates of the key parameters defining the fitted model are given in Table 1. The figure shows the complete set of measured data, the echo-free time window (delimited by $t = 0$ s and the vertical dotted line at $t = 0.1182$ s), and the model fitted to the data within this window and then extrapolated to span the complete observation window. Although the data is represented by the model to an accuracy that reflects the data noise, it is clear from the results given in Table 1 that the estimates of the key parameters are quite inaccurate.

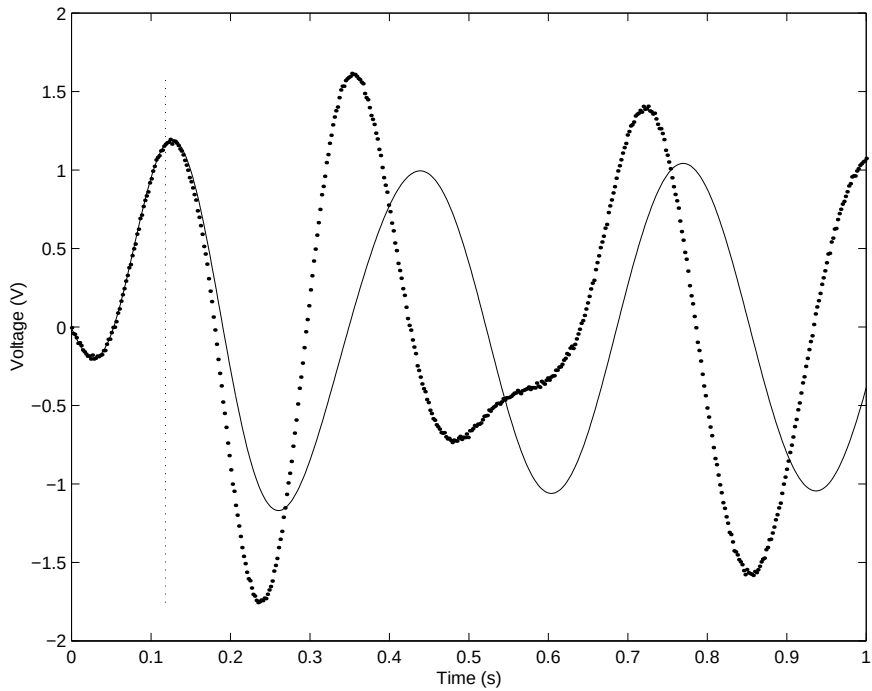


Figure 5: Data and fitted model restricted to the echo-free time for the measured waveform shown in Figure 4 (*Experiment A*).

In our second experiment (*Experiment B*) we use the data processing algorithm described in Section 3 to model the complete measured signal with the arrival-time for the reflections set equal to their correct values of 0.17 s and 0.62 s. The model comprises undamped and damped sinusoidal components as above, but now additional delayed terms to represent the response of the device to each reflection are included. The data and the resulting fitted model are shown in Figure 6, and the corresponding residual errors are plotted in Figure 7. (In these figures the arrival-times for the reflections are marked by vertical dashed lines.) Estimates of the key parameters defining the fitted model are given in Table 1. It is clear that these estimates are considerably more accurate than those obtained from modelling data restricted only to the echo-free time.

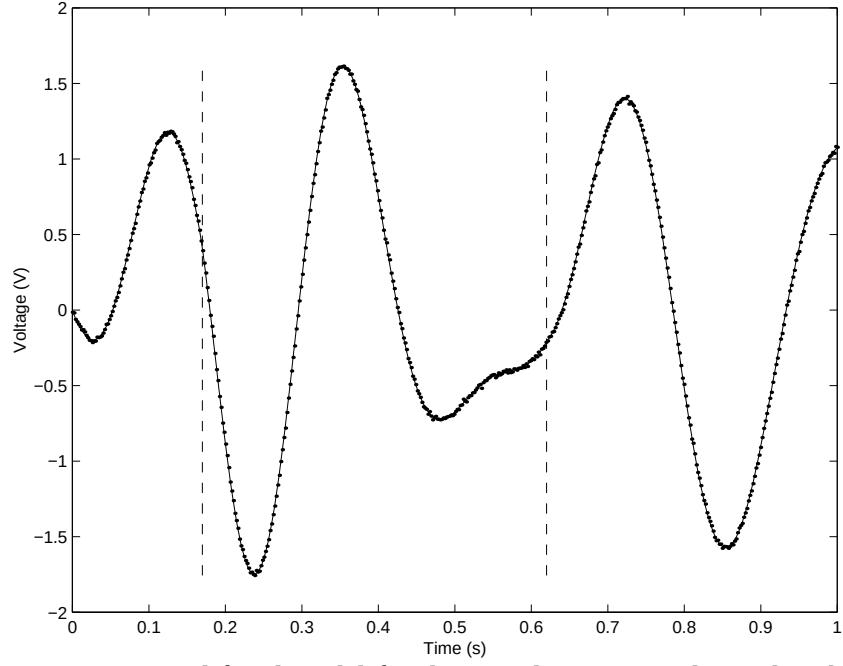


Figure 6: Data and fitted model for the complete measured signal with the arrival-times for the reflections set equal to (their correct values of) $t = 0.17$ s and $t = 0.62$ s (marked as vertical dashed lines) (*Experiment B*).

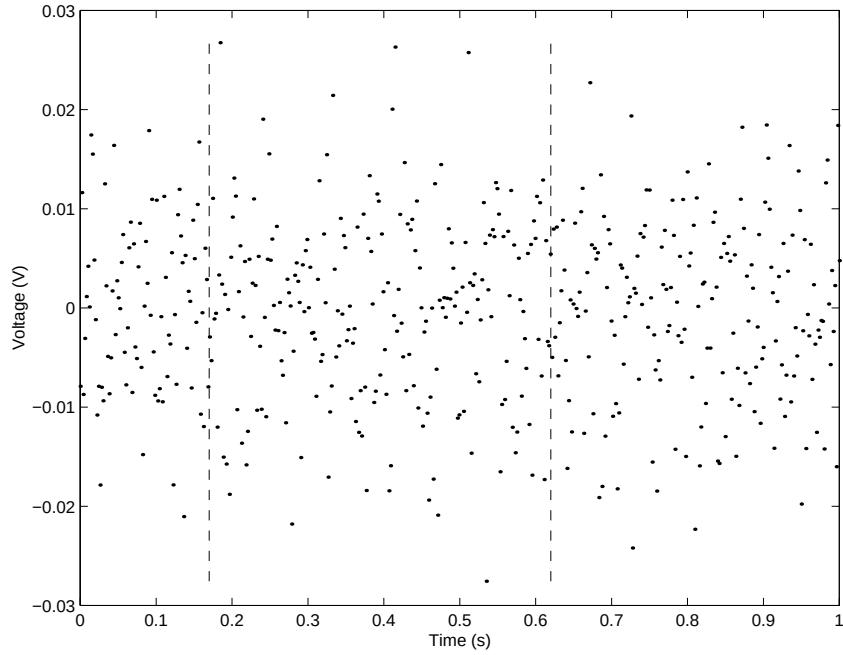


Figure 7: Residual errors corresponding to the data and fitted model shown in Figure 6 (*Experiment B*).

Experiments C, D and E are intended to illustrate the dependence of results returned by the data processing algorithm described in Section 3 on the accuracy of the arrival-times specified for the reflections. In order to make the results clearer, we use for these experiments data simulated as above, except that the data is this time *free* from measurement noise. In *Experiment C* we apply the data processing algorithm described in Section 3 with the arrival-times for the reflections set equal to their correct values of 0.17 s and 0.62 s. The fitted model reproduces the simulated signal “exactly”, i.e., to an accuracy of the order of the computational precision η (Figure 8), as well as returning accurate estimates of the model parameters (Table 1).

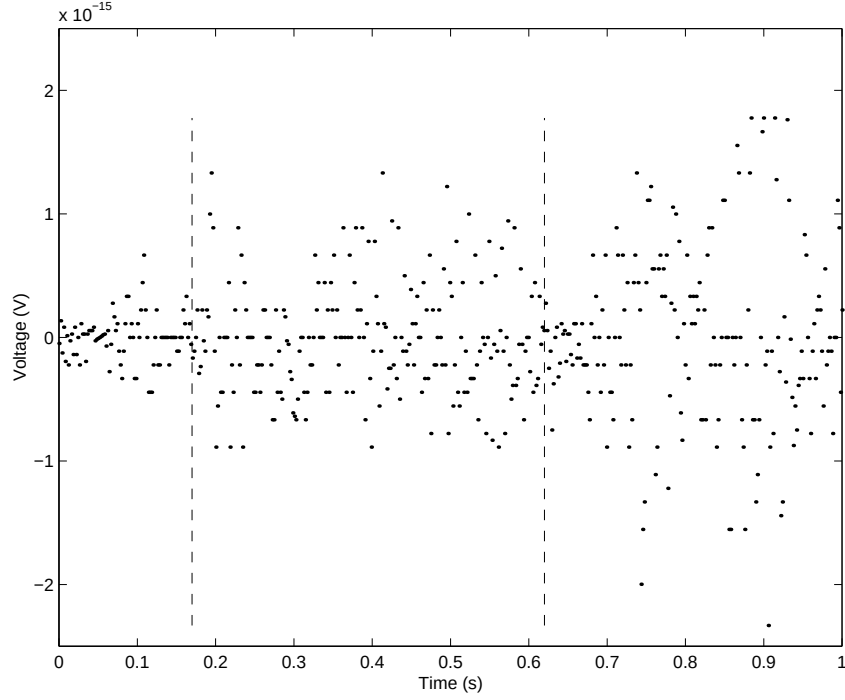


Figure 8: Residual errors associated with a fitted model for noise-free data where the arrival-times for the reflections are set equal to their correct values of $t = 0.17$ s and $t = 0.62$ s (*Experiment C*).

Now, the time-points closest to the arrival-time of $t = 0.17$ s are at $t = 0.1683$ s and $t = 0.1703$ s, and those closest to the arrival time of $t = 0.62$ s are at $t = 0.6192$ s and $t = 0.6212$ s. In *Experiment D* we apply the data processing algorithm described in Section 3 with the arrival-times for the reflections set equal to $t = 0.169$ s and $t = 0.621$. Each arrival-time is perturbed by 0.001 s in such a way that the arrival-time remains within the *same* data sampling interval as defined by the time-points given above. As for *Experiment C*, the fitted model reproduces the simulated signal “exactly” (Figure 9) and returns accurate estimates of the key model parameters (Table 1). In *Experiment E*, however, the arrival-times are set equal to $t = 0.171$ s and $t = 0.619$ s. Here, each arrival-time is again perturbed by 0.001 s but in such a way that the arrival-times lie within a *different* data sampling interval. The simulated signal is no longer reproduced “exactly” (Figure 10), and the key model parameters are not estimated accurately (Table 1). The observed degradation in the results can be expected to increase for larger perturbations in the arrival-time estimates.

Finally, in *Experiment F* we apply the data processing algorithm described in Section 4 in which the arrival-times for the reflections are parameters to be determined by the estimation algorithm. Initial estimates of the arrival-times are set equal to $t = 0.2$ s and $t = 0.6$ s, respectively. The data and the resulting fitted model are shown in Figure 11, and the corresponding residual errors are plotted in Figure 12. (In these figures the initial estimates of the arrival-times for the reflections are marked by vertical dashed lines, and the computed estimates returned by the data processing algorithm by vertical dotted lines.) Estimates of the key parameters defining the fitted model are given in Table 1, and the estimates of the arrival-times returned by the algorithm are, respectively, $t = 0.1696$ s and $t = 0.6198$ s. Although these times are not equal to their simulated values, they are contained within the correct data sampling intervals, which is all we may expect from this experiment.

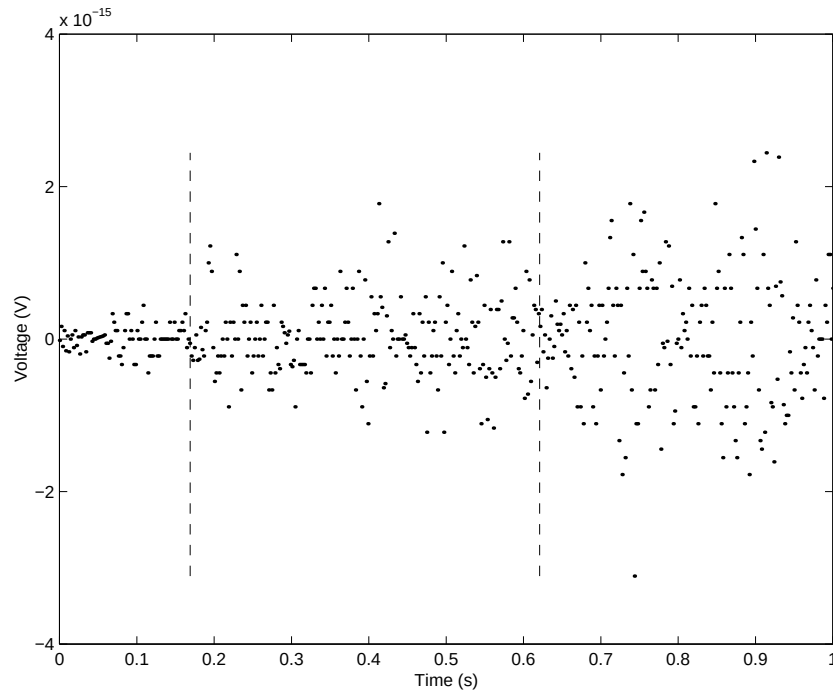


Figure 9: As Figure 8 but the arrival-times for the reflections are perturbed by 0.001 s and set equal to $t = 0.169$ s and $t = 0.621$ s (*Experiment D*).

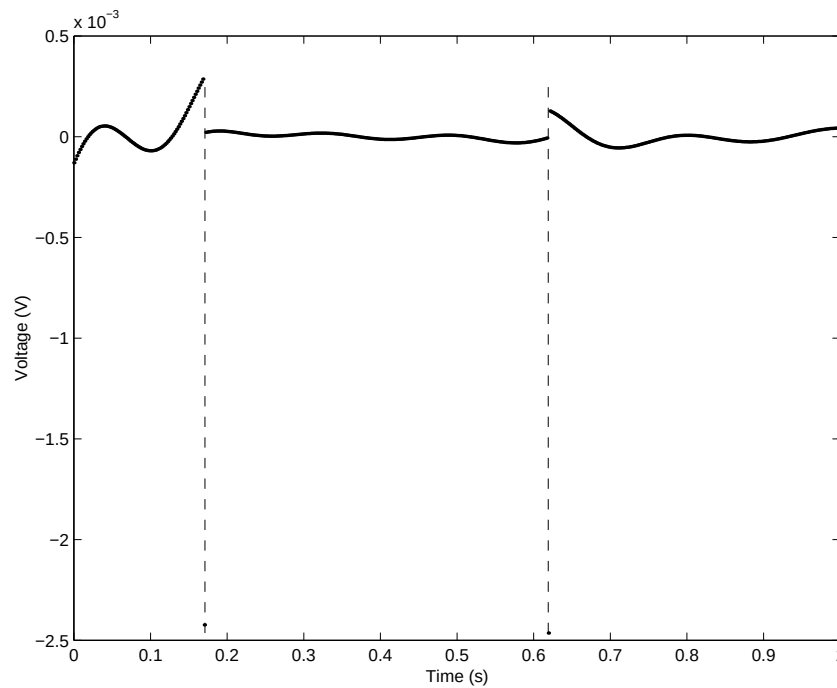


Figure 10: As Figure 8 but the arrival-times for the reflections are perturbed by 0.001 s and set equal to $t = 0.171$ s and $t = 0.619$ s (*Experiment E*).

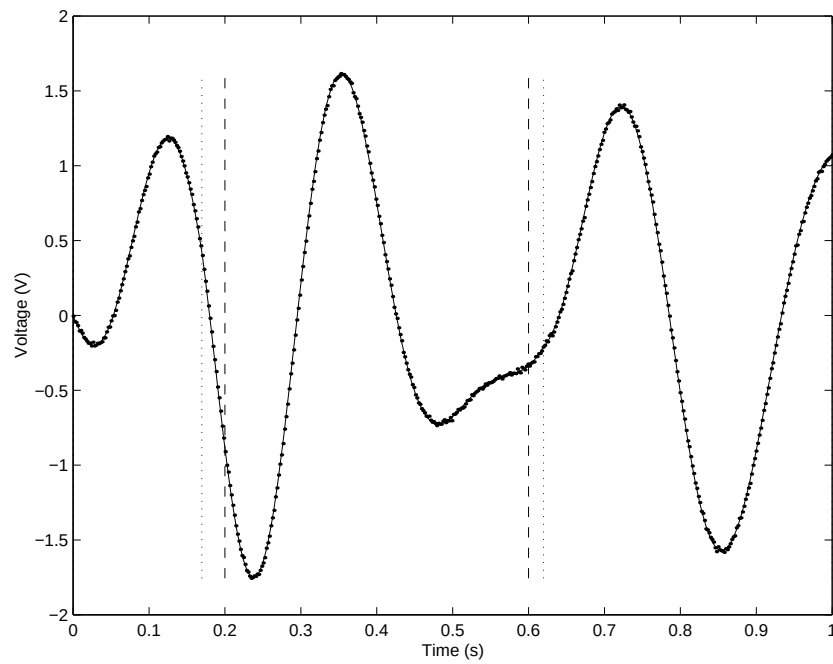


Figure 11: Data and fitted model for the complete measured signal with the arrival-times for the reflections included as parameters to be determined. The vertical dashed lines mark the initial estimates of these times, and the vertical dotted lines their computed values(*Experiment F*) .

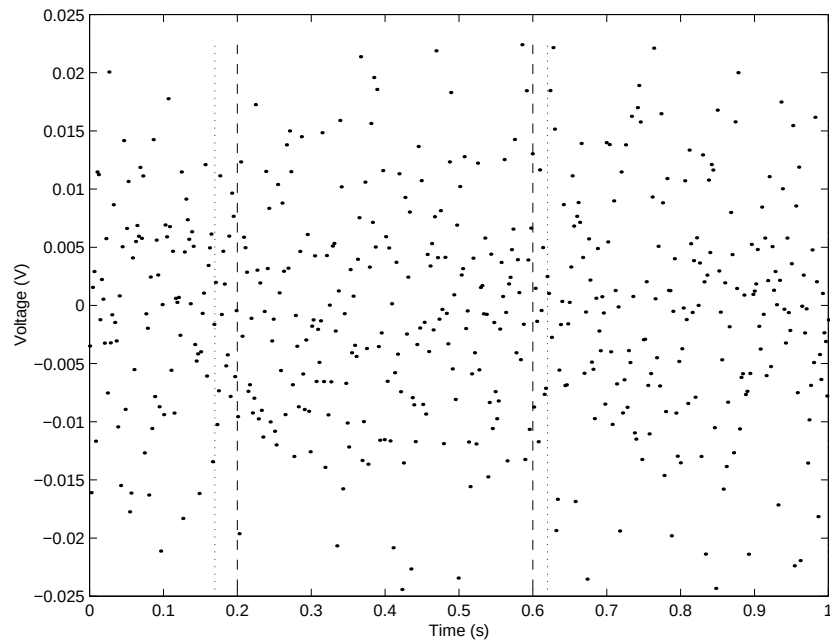


Figure 12: Residual errors corresponding to the data and fitted model shown in Figure 11 (*Experiment F*).

<i>Experiment</i>	<i>Estimated steady-state amplitude (V)</i>	<i>Estimated resonance frequency (Hz)</i>	<i>Estimated Q-factor</i>	<i>Root-mean-square residual σ (V)</i>
A	1.044800	5.377760	2.594114	0.009642
B	0.993803	5.003759	5.041233	0.009265
C	1.000000	5.000000	5.000000	0
D	1.000000	5.000000	5.000000	0
E	1.000022	4.999953	5.000084	0.000164
F	1.001312	4.999156	4.978083	0.009497

Table 1: Estimates of steady-state amplitude and resonant behaviour for the simulations described in Section 9.3. The simulated signal has a steady-state amplitude for the response to the direct signal of 1 V, a resonance frequency of 5 Hz and Q-factor of 5.

6 RESULTS FOR EXPERIMENTAL DATA

In this section we present results obtained from applying the data processing algorithms described before to data obtained from measurements of real devices. In Section 6.1 we present results for the ITC1001 transducer considered in [1]. Then, we present in Section 6.2 results for a higher- Q device for which the benefits of extending the time-window and model to include a reflection are more apparent.

For each device, measurements are made corresponding to two experimental set-ups. In the first, the device under test (which acts as a transmitter or projector) and a second transducer (which acts as a receiver of known sensitivity M_H) are suspended a known distance d apart and close to the centre of the tank in order that the steady-state response of the device may be observed directly and before the arrival of any reflections. Measurements are made for a number of frequencies, and the data is analysed to give an estimate of the (free-field) transmitting voltage response S_p for the device under test given by

$$S_p = \frac{V_H d}{V_p M_H},$$

where V_H is the voltage output of the receiving hydrophone and V_p is the voltage driving the projector. V_H is set equal to the value $A_{0,0}$ (equation (2)) obtained from modelling the waveform measured by the receiving hydrophone.

In the second experiment, the devices are raised in the tank towards the water surface, keeping their separation constant, and so that a reflection arrives before the steady-state response of the device may be observed. Measurements are made at a number of frequencies and used to give estimates of the transmitting voltage response in two ways:

- a) by modelling data restricted to the echo-free time-window using the algorithm described in [1], and
- b) by modelling data contained in a time-window extended to include a single reflection using the algorithm described in Section 5.

The estimates of the transmitting voltage response so-obtained are comparing against the (free-field) response established from the first experiment.

6.1 ITC1001 TRANSDUCER

Figures 13 and 14 show for the two experimental set-ups, respectively, the measured responses for the ITC1001 transducer when driven close to its resonance frequency of approximately 18 kHz. In Figure 13, the steady-state response of the device is clearly visible, and consequently it is possible to make a direct measurement of the steady-state amplitude from which the (free-field) transmitting voltage response may be obtained. In this work, we have chosen to best-fit the data contained within the indicated time-window by a model composed of an undamped sinusoid of the drive frequency and a single damped sinusoid, and predict the steady-state amplitude from the fitted model.

Figure 14 shows the response of the device at the same frequency but in the presence of a reflection that arrives approximately 55 μ s after the arrival of the direct signal. This time the steady-state response cannot be observed directly. Two time-windows indicated in the figure for modelling. The first time-window, delimited by the first and second dotted line, defines the echo-free time and lasts for approximately two-thirds of a cycle of the device's resonance. An estimate of the steady-state amplitude is obtained by modelling this data by a function of

the form (2) with $R = 0$ and $n_r = 1$ (as above). The second time-window, delimited by the first and third dotted lines, defines an extended time-window that includes the reflection. An estimate of the steady-state amplitude is obtained by modelling this data by a function of the form (2) with $R = 1$ and $n_r = 1$.

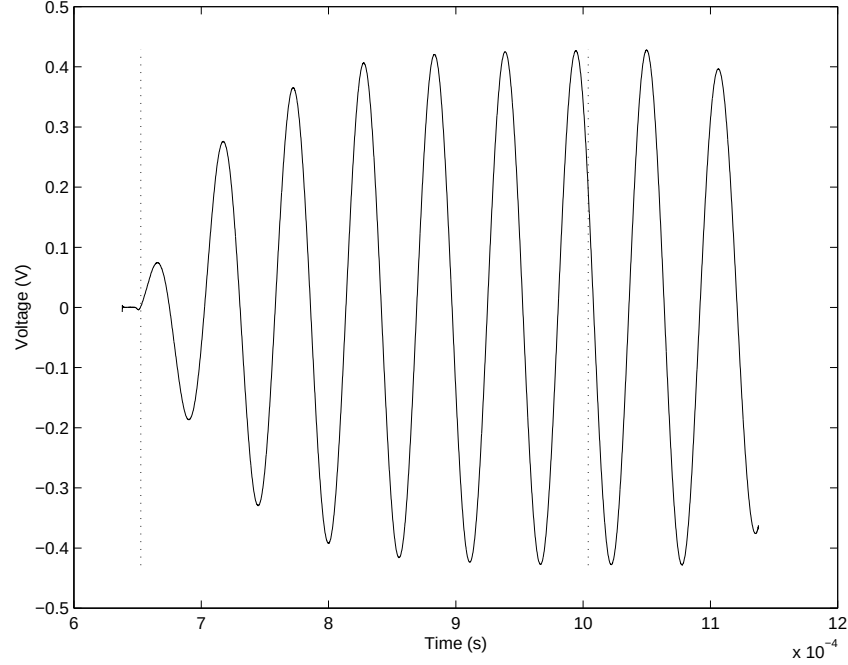


Figure 13: Data for the ITC1001 transducer corresponding to a 18 kHz drive voltage and measured close to the centre of the tank. The dotted lines delimit the time-window used to establish the (free-field) transmitting voltage response for the device.

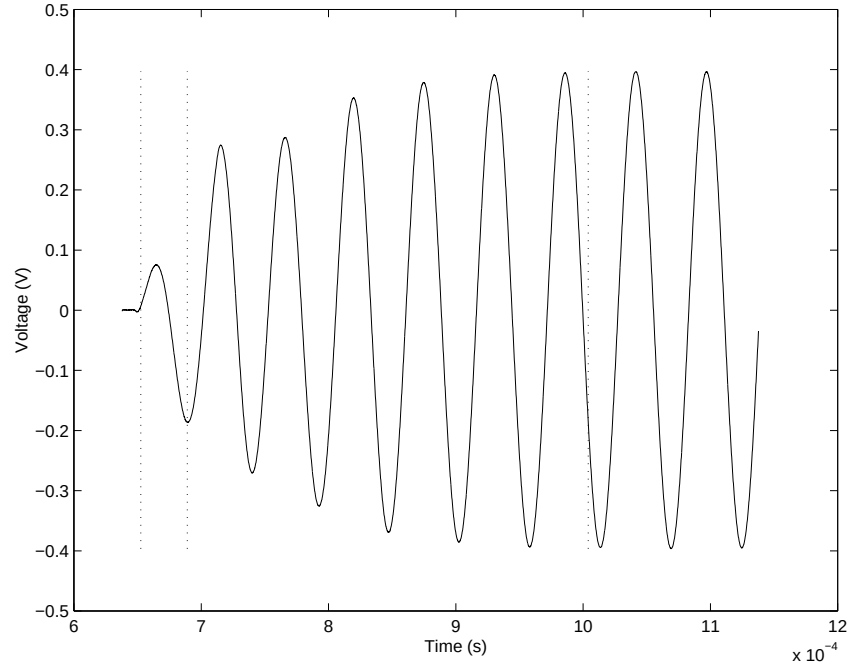


Figure 14: Data for the ITC1001 transducer corresponding a 18 kHz drive voltage and measured close to the water surface. The first and second dotted lines delimit a time-window defining the echo-free time; the first and third dotted lines delimit an extended time-window that includes a single reflection

The estimates of the transmitting voltage response obtained from (i) free-field measurements, (ii) measurements made during the echo-free time, and (iii) measurements made during the extended time-window are shown in Figure 15. The differences between (i) and (ii), and between (i) and (iii), are then presented in Figure 16.

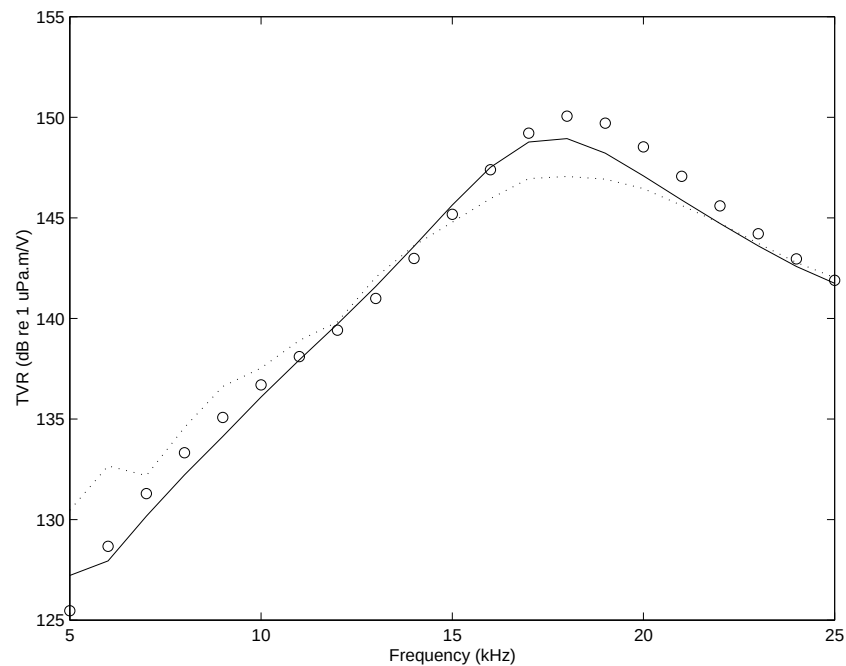


Figure 15: Transmitting voltage responses from (i) free-field measurements (small circles), (ii) measurements made during the echo-free time (dotted line), and (iii) measurements made during an extended time-window that includes a single reflection (solid line).

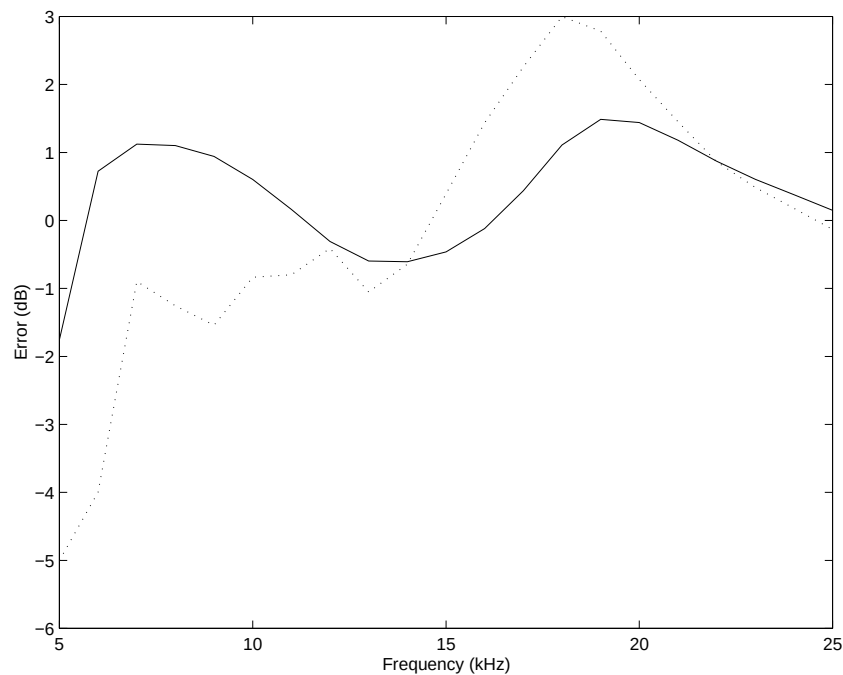


Figure 16: Differences between free-field transmitting voltage response and that estimated from (i) echo-free time (dotted line), and (ii) extended time-window that includes a reflection (solid line).

The results shown in Figure 16 indicate that the estimate of the transmitting voltage response obtained from measurements made during the echo-free time is quite poor. This is as expected given the short duration of the echo-free time. However, the transmitting voltage response estimated from the extended time-window is considerably more accurate and achieves (almost) an accuracy of 1 dB across the complete frequency range.

6.2 HIGH-Q TRANSDUCER

Figures 17 and 18 show for the two experimental set-ups respectively, the measured responses for a high-Q transducer when driven close to its (main) resonance frequency of approximately 40 kHz. The transducer is a high-power sandwich transducer commonly used in ultrasonic cleaning baths. In Figure 17, the steady-state response of the device is clearly visible. Analysis of data of this form, made essentially in a free-field environment, suggest that two resonances are present: one at 40 kHz with an associated Q of approximately 10, and the other at 100 kHz with an associated Q of the order of 25.

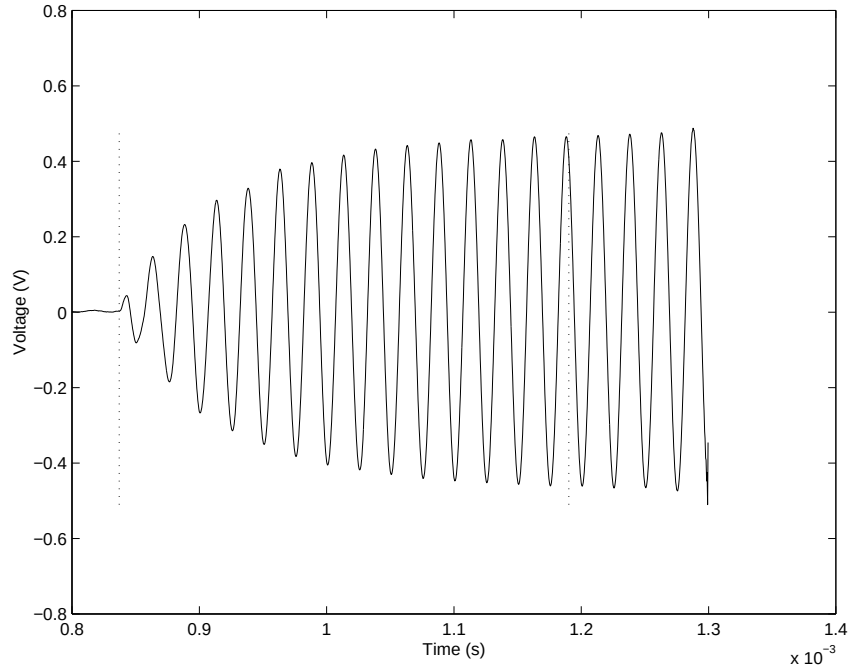


Figure 17: Data for a high-Q transducer corresponding to a 40 kHz drive voltage and measured close to the centre of the tank. The dotted lines delimit the time-window used to establish the (free-field) transmitting voltage response for the device.

Figure 18 shows the response of the device at the same frequency but in the presence of a reflection that arrives approximately 100 μ s after the arrival of the direct signal. This time the steady-state response cannot be observed directly. Indicated in the figure two time-windows for modelling. The first time-window, delimited by the first and second dotted line, defines the echo-free time and lasts for approximately two cycles of the device's (main) resonance. An estimate of the steady-state amplitude is obtained by modelling this data by a function of the form (2) with $R = 0$ and $n_r = 2$ (as above). The second time-window, delimited by the first and third dotted lines, defines an extended time-window that includes the reflection. An estimate of the steady-state amplitude is obtained by modelling this data by a function of the form (2) with $R = 1$ and $n_r = 2$.

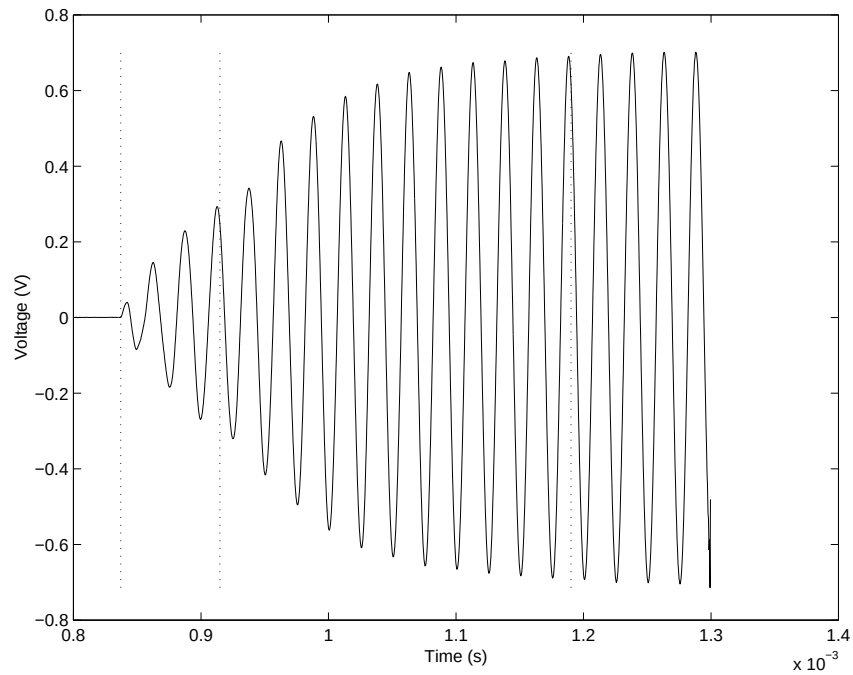


Figure 18: Data for a high-Q transducer corresponding to a 40 kHz drive voltage and measured close to the water surface. The first and second dotted lines delimit a time-window defining the echo-free time; the first and third dotted lines delimit an extended time-window that includes a single reflection

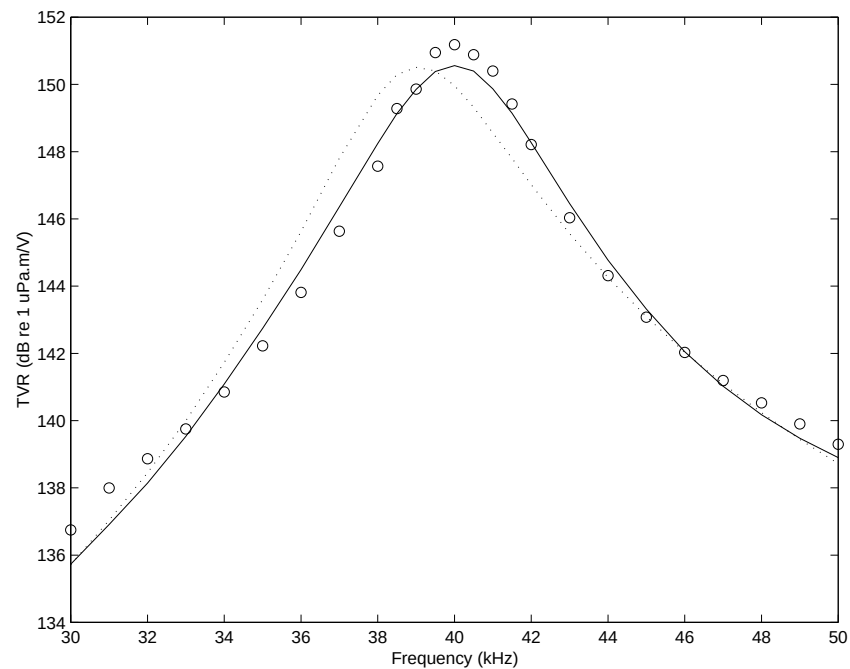


Figure 19: Transmitting voltage responses from (i) free-field measurements (small circles), (ii) measurements made during the echo-free time (dotted line), and (iii) measurements made during an extended time-window that includes a single reflection (solid line).

The estimates of the transmitting voltage response obtained from (i) free-field measurements, (ii) measurements made during the echo-free time, and (iii) measurements made during the extended time-window are shown in Figure 19. The differences between (i) and (ii), and between (i) and (iii), are then presented in Figure 20.

The results shown in Figure 20 indicate as for the ITC1001 transducer (Section 6.1) the benefit of modelling measurements made during an extended time-window. Errors in the transmitting voltage response bounded by (approximately) 1 dB are achieved, compared with 2 dB for that obtained from measurements made during the echo-free time.

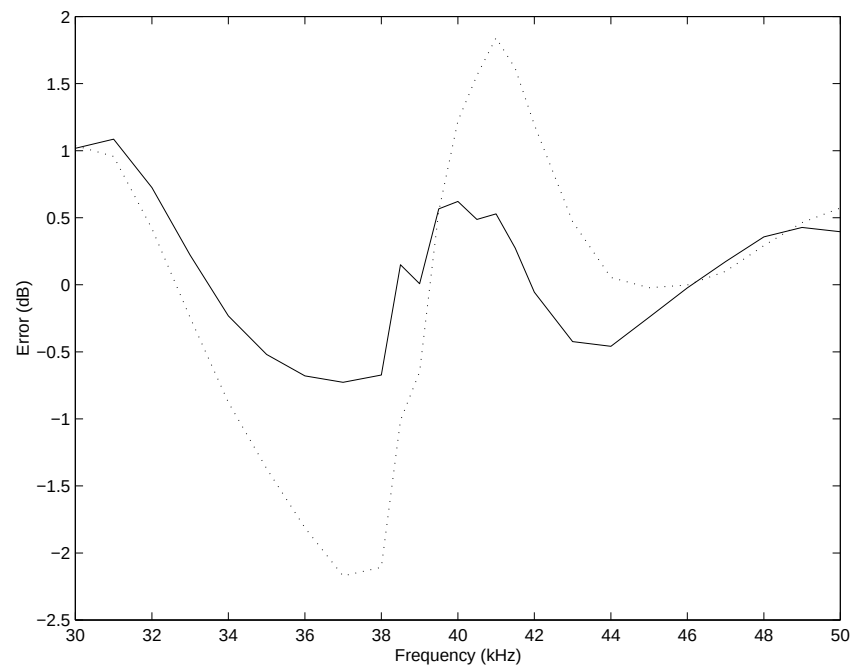


Figure 20: Differences between free-field transmitting voltage response and that estimated from (i) echo-free time (dotted line), and (ii) extended time-window that includes a reflection (solid line).

7 DISCUSSION AND CONCLUSIONS

This report has described the extension of the signal modelling method of [1] to include boundary reflections and has presented the results of applying the technique to the calibration of underwater acoustic transducers. Results demonstrate that even for relatively high- Q devices it is possible to obtain agreement with the transducer free-field response to within ± 1 dB when using the extended model, an improvement of over 1 dB on using only the echo-free signal.

The signal-modelling techniques described here can be useful in any situation where the echo-free time available for measurements is so restricted that steady-state signals cannot be observed. Examples include:

- (i) making measurements of transducer sensitivity or directional response in tanks with dimensions of only a few acoustic wavelengths - signal modelling techniques can help to extend the lower limiting frequency at which measurements may be successfully made in the tank;
- (ii) in characterisation of acoustic materials where the finite size of the panel under test can lead to multipath signals, both from the wave diffracted around the edge of the panel and from multiple passes within the panel.

There are still some difficulties with regard to this approach. Some of the difficulties with regard to the echo-free signal modelling have been described before and also apply here [1, 14]. The work described in this report has examined models which include only the first one or two reflections. In instances where many reflections are arriving almost simultaneously the technique is less likely to perform so well. The assumptions made about the reflections being scaled replicas of the direct signal may not always be valid. Since the amplitudes of the resonance frequency and drive frequency for each echo arrival in the model are independent, boundaries with reflection coefficients that depend on frequency should not pose a problem. However, any nonlinear or resonant behaviour which modifies the frequency content of the signals will cause the method to fail.

Perhaps the main difficulty is in the sensitivity to accuracy of the echo arrival time estimates. If these initial estimates are poor, the model may not recover even when arrival times are allowed to vary as parameters within the model. There are several methods which can be used to assist in identifying the echoes:

- (i) estimates may be made using *a priori* information about the problem, in particular by use of the knowledge of the geometry of the tank and the exact position of the transducers within it (careful consideration may be needed to obtain this information with sufficient accuracy);
- (ii) the arrival times may be estimated empirically by inspection of the received waveform (however, as has been shown, the position of reflections is not always easy to determine simply by observing the signal);
- (iii) the estimates may be made by performing some form of pre-processing on the signal - one particular technique that has been shown to successfully identify echoes is by the use of wavelets, a technique currently under study at NPL [15].

Finally, if the signal-modelling method is to be useful in routine calibration and measurement work, the software must allow automatic analysis to be undertaken in something like real time with a minimum of input from the operator and without the need for expert interpretation. Currently the software is written in the Matlab language and is designed with a user interface that provides numerous graphical displays allowing user input at crucial stages

of the modelling process. This was important during the development and assessment of the method, but for routine work a more streamlined version is more appropriate. There should not be great difficulty in compiling the current code into a form that may be called from routine calibration software, but further thought may be required to fully automate all of the decision making without operator interaction.

8 ACKNOWLEDGEMENTS

This report constitutes the deliverable of project 3.1.7 of the 1999-2002 NMS Acoustics Metrology Programme and has been funded by the National Measurement System Policy Unit of the UK Department of Trade and Industry.

The authors would like to thank the staff of the Underwater Sound Reference Division of the US Naval Underwater Warfare Center for informative discussions during this work, in particular Phil Ainsleigh and Steve Forsythe. They would also like to thank Maurice Cox (NPL) for his advice and comments on this report.

9 REFERENCES

- [1] ROBINSON, S.P. and HARRIS, P.M. Modelling acoustic signals in the calibration of underwater electroacoustic transducers in reverberant laboratory tanks. *NPL Report CMAM 029*, March 1999.
- [2] AINSLEIGH, P.L. and GEORGE, J.D. Signal modeling in reverberant environments with application to underwater electroacoustic transducer calibration. *J. Acoust. Soc. Am.* 1995, **98**, 270-279.
- [3] GEORGE, J.D., MUISE, R.R. and ABEL, J.S. Simultaneous estimation of time-of-arrival, pole, and amplitude parameters for transient exponential signals. *Proceedings of IEEE International Conference on Acoustics, Speech and Signal Processing*, 1992, **5**, 425-428.
- [4] AINSLEIGH, P.L. and GEORGE, J.D. Modeling exponential signals in a dispersive multipath environment. *Proceedings of IEEE International Conference on Acoustics, Speech and Signal Processing*, 1992, **5**, 457-460.
- [5] AINSLEIGH, P.L. and GEORGE, J.D. Multipath signal modeling for electroacoustic transducer calibration in reverberant underwater environments. *Proceedings of UDT*, 1994, 97-101.
- [6] MARPLE, S.L. *Digital Spectral Analysis with Applications*. Prentice Hall, Englewood Cliffs, New Jersey. 1987.
- [7] BEATTY, L.G., GEORGE, J.D. and ROBINSON, A.Z. Use of the complex exponential expansion as a signal representation for underwater acoustic calibration. *J. Acoust. Soc. Am.*, 1978, **63**, 1782-1794.
- [8] TRIVETT, D.H. and ROBINSON, A.Z. Modified Prony method approach to echo-reduction measurements. *J. Acoust. Soc. Am.*, 1981, **70**, 1166-1175.
- [9] TUFTS, D.W. and KUMARESAN, R. Estimation of frequencies of Multiple Sinusoids: Making linear prediction perform like maximum likelihood. *Proceedings of the IEEE*, 1982, **70**, 975-989.
- [10] GEORGE, J.D., JAIN, V.K. and AINSLEIGH, P.L. Estimating steady-state response of a resonant transducer in a reverberant underwater environment. *IEEE International Conference on Acoustics, Speech and Signal Processing*, 1992, **5**, 1988, 2737-2740.
- [11] GILL, P.E., MURRAY, W. and WRIGHT, M.H. *Practical Optimization*. Academic Press, 1981.
- [12] LYNESS, J.N. and MOLER, C.B. Numerical differentiation of analytic functions. *SIAM J. Numer. Anal.*, 1967, **4**, 202-210.
- [13] SQUIRE, W. and TRAPP, G. Using complex variables to estimate derivatives of real functions. *SIAM Review*, 1998, **40**, 110-112.
- [14] FORSYTHE, S. E. and AINSLEIGH, P. L. Acoustic calibration in reverberant environments: a survey of USRD measurement methodology. *Proceedings of the 16th*

International Congress on Acoustics and 135th meeting of the Acoustical Society of America, 1998, 55-56.

- [15] LORD, G. J., PARDO-IGUZQUIZA, E. and SMITH, I. M. A practical guide to wavelets for metrology. *NPL Report CMSC 02/00*, April 2000.