

REPORT

Experimental Investigations into Four-Wave Mixing in Optical Fibre

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Summary

This report follows on from NPL Report COEM 24 and describes the results of proof-of-principle experimental investigations into the use of four-wave mixing (FWM) to measure distributed dispersion in single mode optical fibres. The results are promising. The distributed dispersion investigation was successful in mapping the variation in chromatic dispersion along a 26 km length of fibre. Further work is ongoing to increase the accuracy of this measurement through improved data analysis.

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1. Introduction

Four-wave mixing arises in optical fibres when the power in a number of waves becomes sufficient to generate a non-linear relationship between the electric field strength and the polarisation induced in the medium. Signals are generated or amplified at a number of wavelengths due to the transfer of power from the original waves through parametric amplification. The frequencies of the generated signals are well-defined and follow from simple combinations of the incident frequencies. The efficiency of the four-wave mixing process is determined by the fibre parameters as well as the incident frequencies and powers. In this work, the dependence of the efficiency on the fibre parameters is investigated as a potential metrological tool. The theory of four-wave mixing is described in detail in [1]. The key points are briefly summarised in Appendix A.

2. Distributed Measurement of Dispersion Using Four-Wave Mixing

2.1 Theory

The efficiency of partially degenerate four-wave mixing between a tuneable pump wavelength and a fixed probe signal displays a periodic dependence on the pump wavelength. For a fibre with uniform zero dispersion wavelength, λ_0 , the main central peak of the efficiency occurs when the pump is at λ_0 and the period of the peaks is related to the dispersion slope of the fibre, $dD/d\lambda$. Experiments on a 1.1 km length of dispersion shifted fibre showed that the zero dispersion wavelength and dispersion slope of the fibre could be determined from a curve fitted to the periodic experimental data [2]. However, the same approach, when applied to a 23 km length of fibre was unsuccessful, as the efficiency curve did not exhibit any discernible periodic feature or a distinct central peak. This was attributed to variations in the zero dispersion wavelength along the fibre.

Observation of the periodic efficiency curve requires that the zero dispersion wavelength remain reasonably constant over the interaction length of the fibre. This is not normally the case in real fibres and FWM effects in long fibres tend to have complicated characteristics that depend on the variation in phase mismatch along the full length of the fibre. Consequently, continuous wave measurements are only capable of measuring the aggregate properties of an entire fibre. However, pulsed methods may be used to restrict the interaction length so that FWM mixing occurs over restricted regions of the fibre with reduced phase mismatch variation.

Two similar methods for performing non-destructive distributed measurements of the dispersion of optical fibres have recently been reported [3,4]. Both techniques use optical time domain reflectometry (OTDR) measurements to monitor FWM efficiency as two high power optical pulses co-propagate along the fibre. The two pulses are simultaneously launched into the fibre, partially degenerate FWM occurs and generates forward-propagating signals at two new wavelengths. An example of a measured spectrum due to partially degenerate FWM is shown in Figure 1.

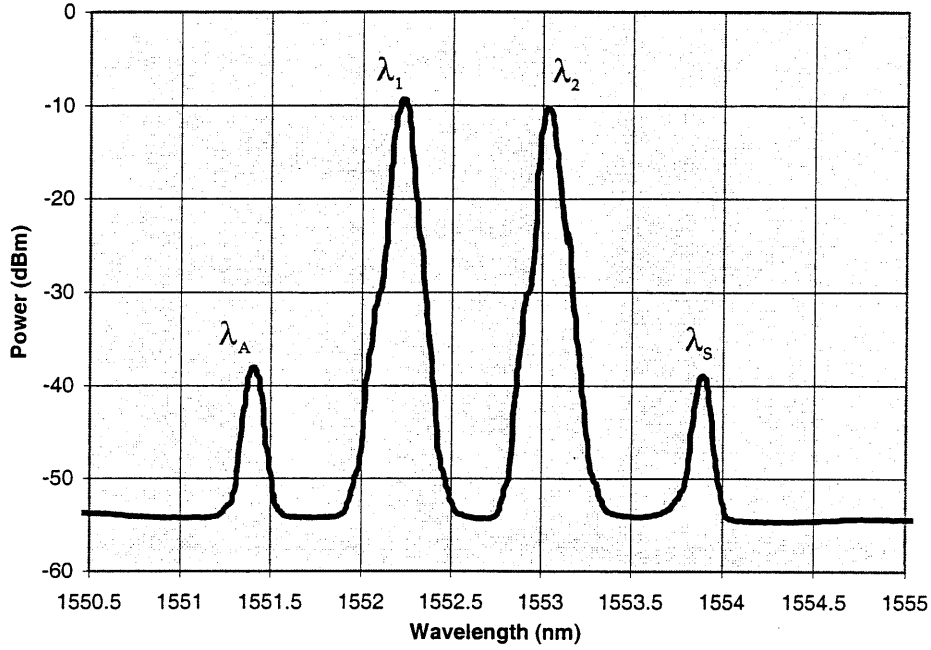


Figure 1. Example of a measured PDFWM spectrum showing two input signals (λ_1 and λ_2) and the Stokes (λ_S) and antiStokes (λ_A) products. Resolution bandwidth of the OSA is 0.08 nm.

The intensities of the FWM products vary periodically along the fibre and the period of oscillation at any point in the fibre is related to the local value of the fibre dispersion, D . The spatial frequencies of the Stokes and antiStokes waves are given by: [3,4]:

$$F_S = c \left| D(\lambda_2) \right| \left(\frac{\lambda_2 - \lambda_1}{\lambda_2} \right)^2 \quad 2-1a$$

and

$$F_A = c \left| D(\lambda_1) \right| \left(\frac{\lambda_2 - \lambda_1}{\lambda_1} \right)^2 \quad 2-1b$$

OTDR techniques measure the optical power scattered through 180° by Rayleigh scattering and joints/defects when a short pulse of light is launched into one end of a fibre. The backscattered optical power from a particular point in the fibre can be identified from the temporal delay between launching the pulse and arrival of the scattered signal. The magnitude of the detected backscattered power is proportional to the forward-propagating optical power at the point of scattering. Spatial variations in the local intensity of the FWM products along the fibre can therefore be measured from the temporal variations in the backscattered power received at the OTDR. In this way, the spatial period in equation 2-1 can be measured at points along the fibre and used to calculate the absolute value of the dispersion at these points.

A compromise is required between increasing the spatial resolution of the measurement by using a large value of $\delta\lambda$ and maintaining a high enough

level of FWM efficiency to give an acceptable SNR at the receiver. The spatial resolution of the measurement is limited by the ability to accurately determine the period of the intensity fluctuations in a given region of fibre. A spatial resolution of 500 m with a relative uncertainty of 2 % on the measured dispersion has been reported by using a pulse wavelength separation of 3.6 nm on a 25 km long dispersion shifted fibre [3]. Limits on the maximum length of fibre that can be measured are set by the sensitivity of the detector and therefore the power in the backscattered FWM products.

2.2 Experimental Results

Distributed zero-dispersion wavelength measurements were performed using the experimental system shown in Figure 2. An OTDR was used to modulate two tuneable lasers via a Mach-Zehnder electro-optic modulator (EOM) and measure the backreflected signal from the fibre under test (FUT). The wavelengths of the lasers were set to between 1545 nm and 1550 nm with λ_2 at approximately 1550 nm and the Stokes wave, F_s , measured. The wavelength separation between the lasers, $\Delta\lambda = \lambda_2 - \lambda_1$, was varied depending on the fibre type to achieve a compromise between spatial resolution and increased FWM efficiency. The polarisation of each source was adjusted with a polarisation controller (PC) to maximise the power in the FWM products. Band-pass filters (BPF) were used to cut out ASE from the amplifier and select only the backscattered Stokes wave.

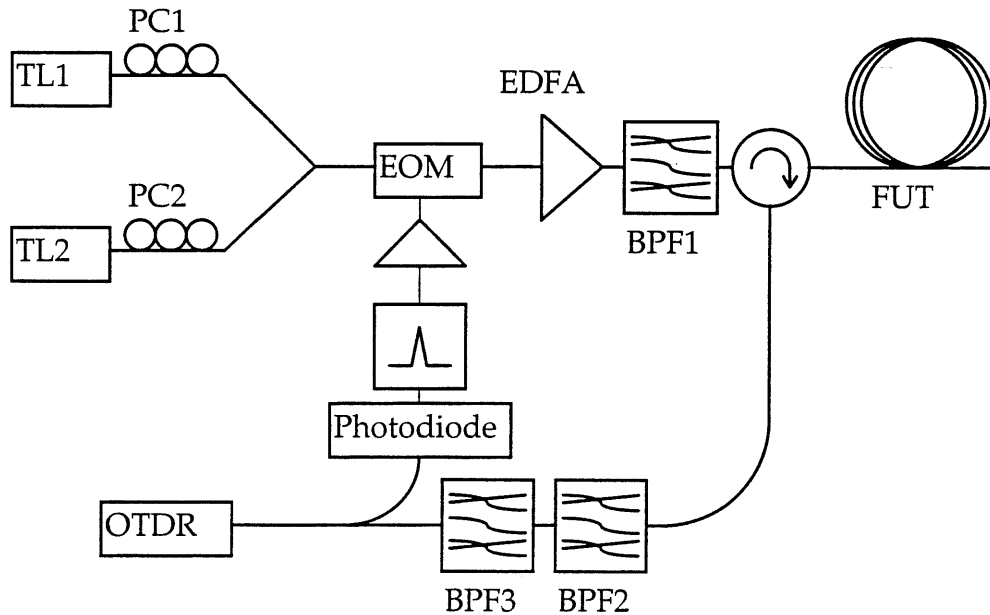


Figure 2. Experimental system used to measure zero-dispersion wavelength distribution along an optical fibre.

The local period of the oscillations in the FWM product intensity was calculated manually from the separations of the peaks and troughs in the OTDR traces. These values were then used in equation to calculate the absolute value of the dispersion at the mid-points between each pair of

neighbouring peaks and troughs. For illustrative purposes, the absolute dispersion was given the correct sign for the type of fibre in the wavelength region used.

Measurements were performed on two lengths of fibre: 12 km of a non-zero dispersion-shifted large effective area fibre (LEAF) and a 25.6 km sample consisting of two 12.8 km dispersion-shifted fibres spliced together. The chromatic dispersion of these fibres had previously been measured using the phase shift technique. Chromatic dispersion values calculated from a 5-term Sellmeier curve fitted to the experimental data are plotted in Figure 3.

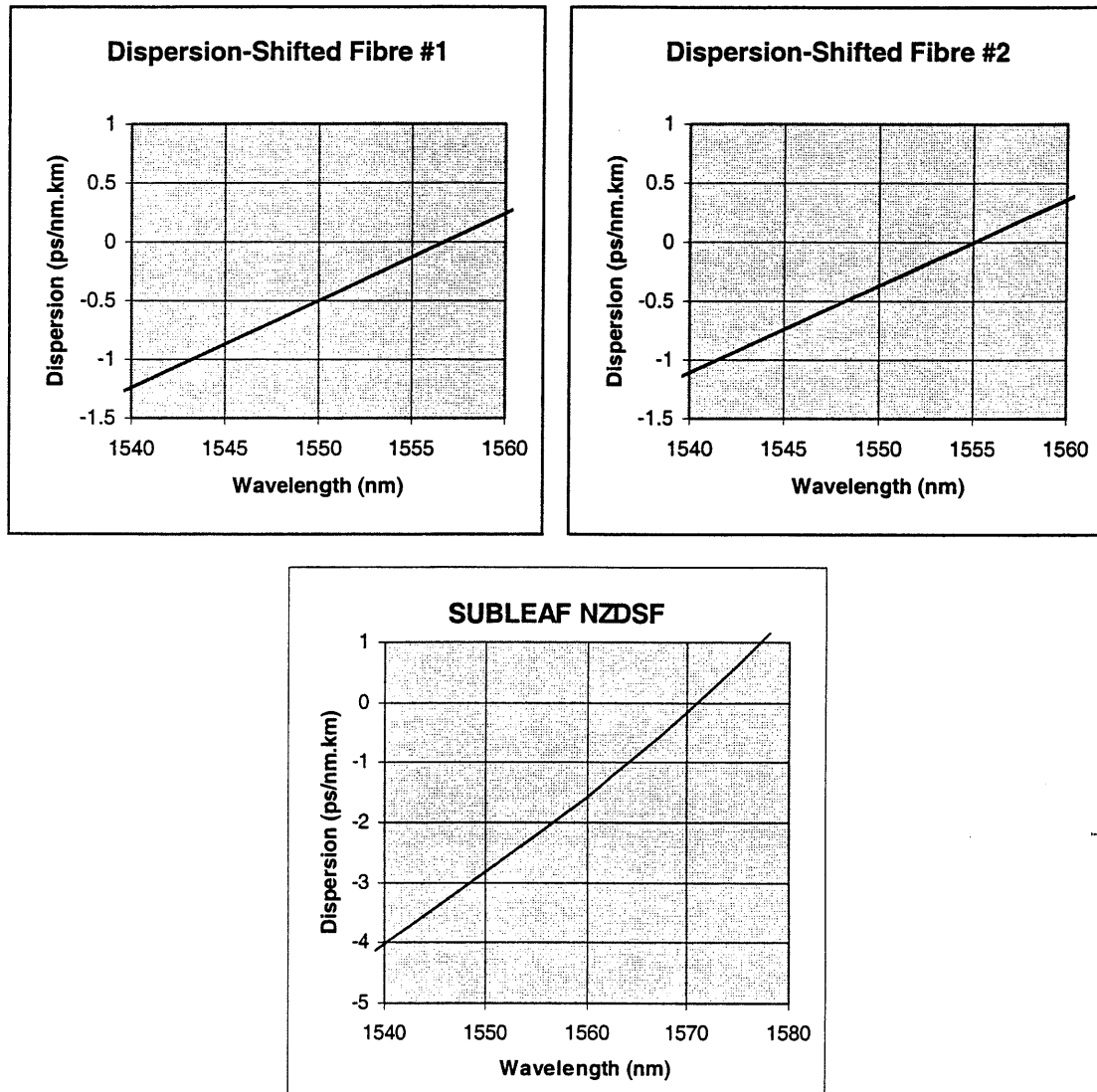


Figure 3. Chromatic dispersion measured as a function of wavelength using the phase shift technique.

Fibre	Length (km)	Zero Dispersion Wavelength (nm)	Dispersion at 1550 nm (ps/nm.km)
DSF #1	12.8	1556.6 ± 0.5	-0.49
DSF #2	12.8	1555.1 ± 0.5	-0.37
NZDSLEAF	13.2	1571 ± 1	-2.83

Table 1. Chromatic dispersion of each fibre measured by the phase shift technique.

From these results, we see that the two dispersion-shifted fibres have aggregate chromatic dispersions differing by approximately 0.1 ps/nm.km over the wavelength range of interest. An OTDR trace measured from one end of the concatenated fibre using the system in Figure 2 is shown in Figure 4. The dispersion distribution calculated from the trace is plotted in Figure 5.

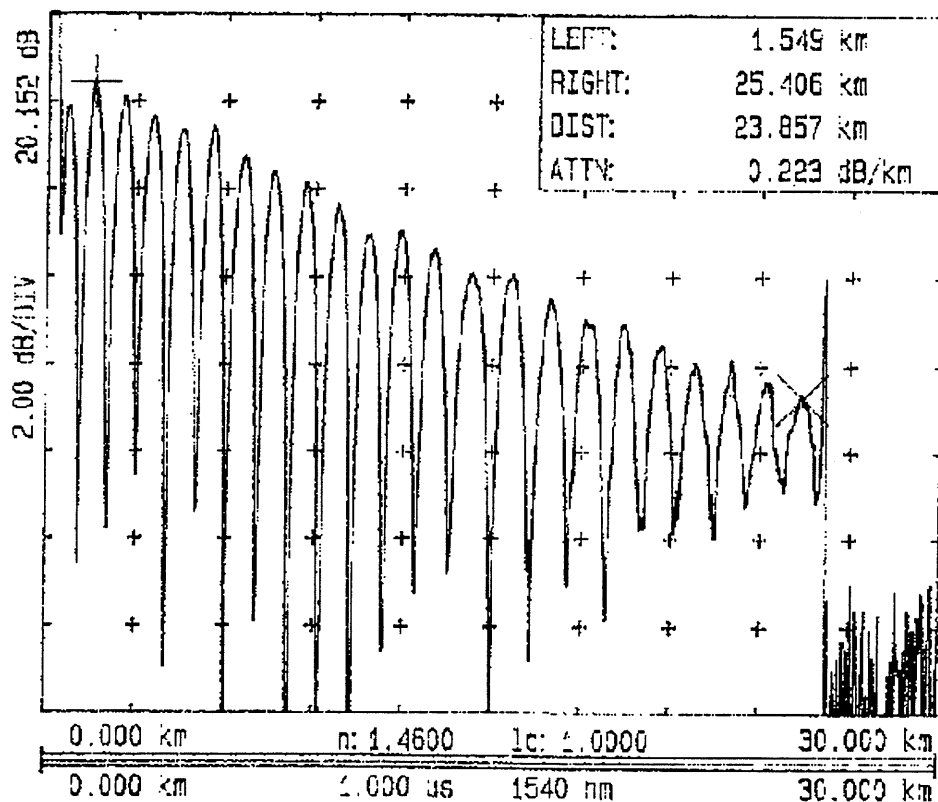


Figure 4. Measured OTDR trace of the FWM antiStokes product power in two 12.8 km sections of dispersion-shifted fibre spliced together. $\Delta\lambda = 3.5$ nm

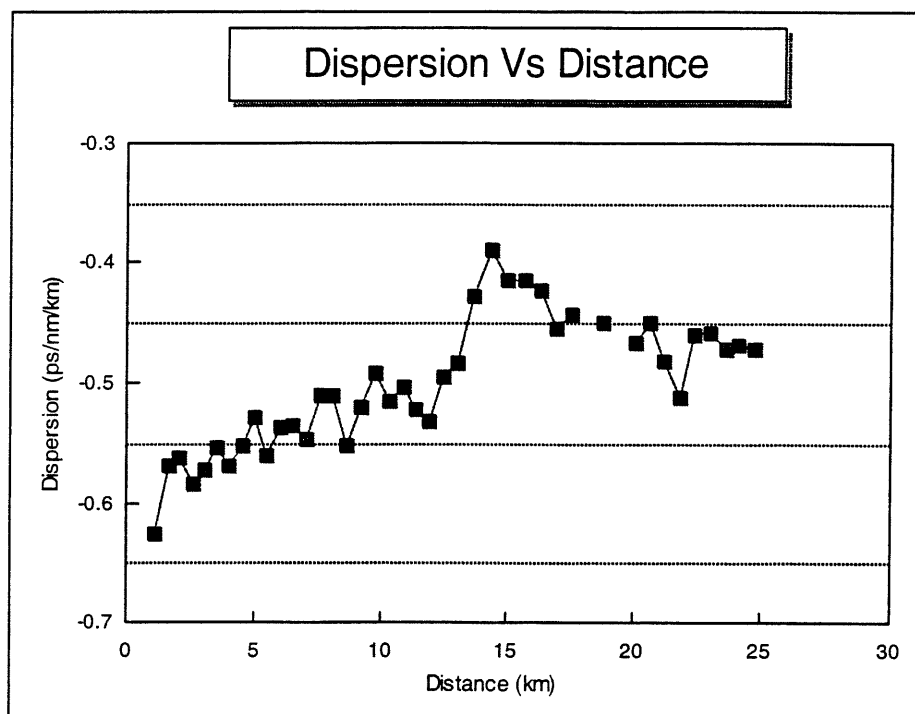


Figure 5. Dispersion as a function of position along the concatenated DSF calculated from the OTDR trace in Figure 4.

It can be clearly seen from Figure 5 that the dispersion of the concatenated fibre sample changes abruptly in the region where the two 12.8 km fibres are joined together. The two fibres were actually manufactured consecutively from the same preform and it is apparent that the dispersion was increasing or decreasing monotonically along it. Repeating the OTDR measurements by launching into the opposite end of the fibre gave the same characteristic dispersion profile - confirming that it was not an artefact of the measurement system. The measured dispersion shown in Figure 5 agrees qualitatively with the values from the phase shift technique shown in Table 1. However, the mean value of the distributed dispersion is typically 0.08 ps/nm.km lower than the aggregate value from the phase-shift technique. The most likely cause of this is that the pump pulse wavelength (λ_2 in 2-1a or λ_1 in 2-1b) was not exactly at 1500 nm. Figure 3 indicates that a wavelength shift of ~1 nm can give this level of discrepancy.

It can be appreciated from Figure 5 why the aggregate dispersions of the two fibres were significantly different. Clearly the dispersion of the parent fibre was changing monotonically and the phase shift method gives an average value for the dispersion.

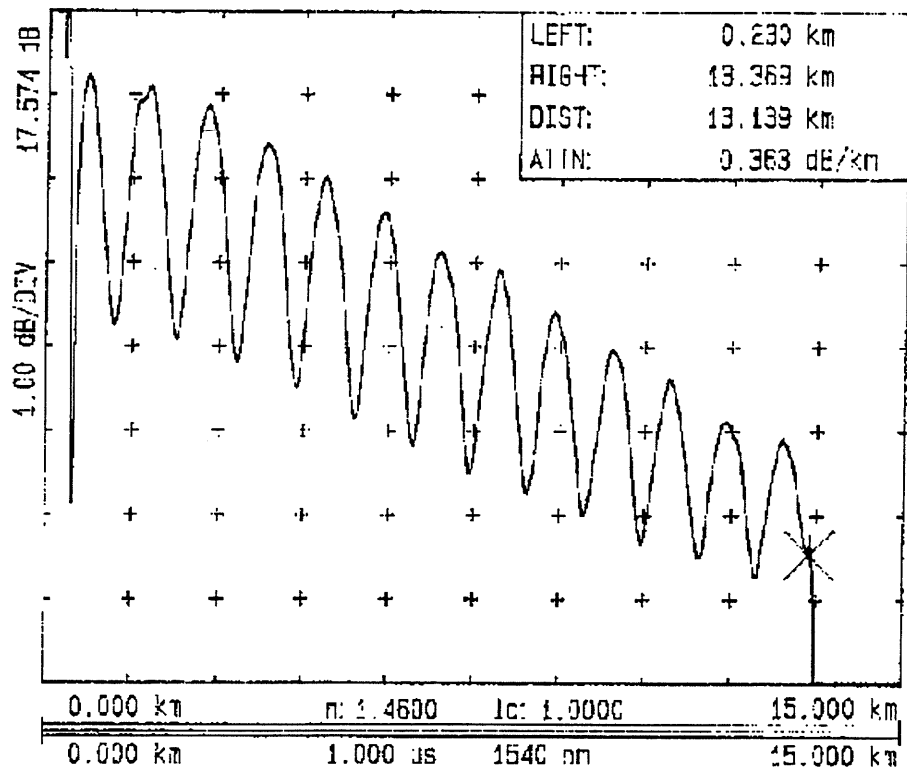


Figure 6. Measured OTDR trace of the FWM antiStokes product power in a 13.5 km length of non-zero dispersion-shifted large effective area fibre. $\Delta\lambda = 1.5$ nm

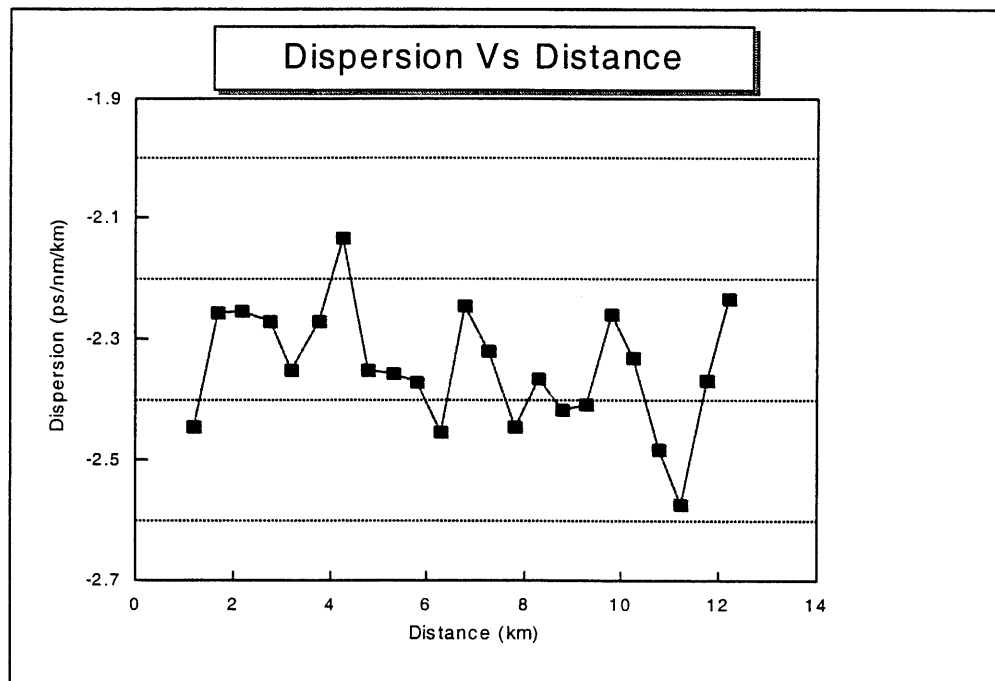


Figure 7. Dispersion as a function of position along the NZDS LEAF calculated from the OTDR trace in Figure 6.

The distributed dispersion profile for the LEA fibre indicates a random, rather than a consistent change in the dispersion with position. The absolute fluctuations in the calculated dispersion are considerably larger than in the dispersion shifted fibres, although in both cases the variations are within

approximately $\pm 12\%$ of the mean. The mean of the distributed values is approximately 0.5 ps/nm.km higher than the value from the phase-shift method.

It has been demonstrated that the FWM OTDR technique can be used to measure distributed dispersion in optical fibres. Work is continuing to increase the accuracy of the results.

3. Conclusions

Experimental investigations have been performed using four-wave mixing for distributed measurements in optical fibre. The measurements were able to identify changes in the dispersion of a 26 km length of dispersion-shifted fibre. More detailed data analysis techniques are needed to improve the accuracy of these measurements. Specifically, the local spatial frequency of the oscillations in FWM product power must be calculated with greater accuracy and the origin of the offset error in the calculated dispersion clarified.

4. Appendix A - Theory of Four-Wave Mixing

4.1 Frequencies of the Generated/Amplified Signals

Conservation of energy requirements dictate that the frequencies of the waves involved in FWM satisfy the relation:

$$\omega_{ijk} = \omega_i + \omega_j - \omega_k \quad 4-1$$

where ω_i , ω_j and ω_k are the angular frequencies of the input waves and ω_{ijk} is the angular frequency of the FWM product. In non-degenerate four-wave mixing (NDFWM), the three incident frequencies are different to each other. However partially degenerate four-wave mixing (PDFWM) also occurs in which one of the input signals is considered to act as both ω_i and ω_j . Examples of both of these cases are illustrated in Figure 8.

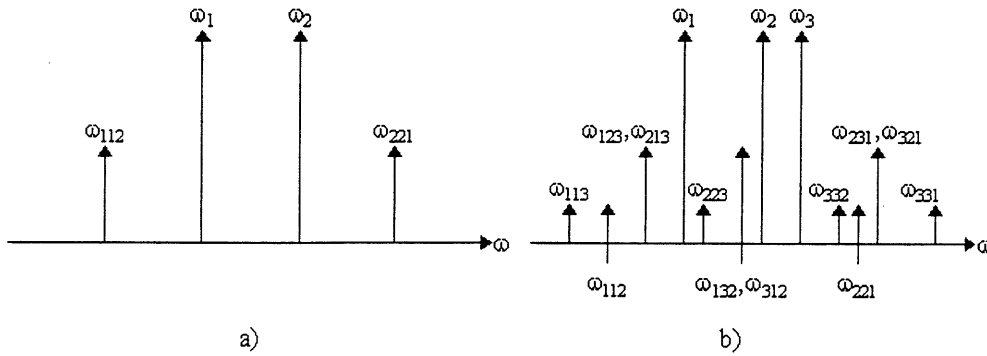


Figure 8. Additional frequencies generated through FWM in a) the partially degenerate and b) the non-degenerate case.

Equation 4-1 gives the frequency of radiation that would be generated by FWM, should the process actually occur. It does not provide any information as to how efficient the process will be therefore whether the power in the products will be significant. This is determined by the *phase matching condition*.

4.2 Phase Matching and FWM Efficiency

Phase matching is a requirement that the propagation constants of the waves involved in FWM satisfy a relation similar to 4-1 that ensures conservation of momentum before and after the interaction. The dispersion of the optical fibre causes signals at different wavelengths to propagate at different group velocities. Phase matching therefore depends on the dispersion of the fibre and the frequencies of the interacting waves. Four-wave mixing is maximised when the propagation constants of the interacting waves minimise the phase mismatch parameter, $\Delta\beta$, where:

$$\Delta\beta = \beta_1 + \beta_2 - \beta_3 - \beta_4. \quad 4-2$$

Expanding the propagation constants β as Taylor expansions around a central frequency, ω_m , gives the following expression for the phase mismatch [5]:

$$\Delta\beta = 2\pi c \frac{\Delta\omega_{13}\Delta\omega_{23}}{\omega_m^2} D(\omega_m), \quad 4-3$$

where:

$$\omega_m = \frac{(\omega_1 + \omega_2)}{2}$$

$$\Delta\omega_{ij} = \omega_i - \omega_j$$

$$D \text{ is the fibre dispersion, } D(\omega) = -\frac{\omega^2}{2\pi c} \frac{d^2\beta}{d\omega^2}.$$

At a given point along the fibre, the optical power of each FWM product is dependent on the degree of phase matching and the propagation length. In addition to determining the linear loss of the fibre, the propagation length affects the FWM output powers in a more complex way. This is due to the effects of interference between FWM products generated at different points. The result is an oscillatory growth and decay of the FWM products as they propagate along the fibre. The actual power in the FWM products at the output end of the fibre depends on where the length of the fibre intersects the oscillatory power vs. length curve.

The time-averaged optical power generated at frequency ω_4 for co-polarised input signals at ω_1 , ω_2 and ω_3 is given (in cgs units) by [6]:

$$P_4(L) = \eta \left(\frac{1024\pi^6}{n^4 \lambda^2 c^2} \right) \left(\frac{L_{eff}}{A_{eff}} \right) (D\chi_3)^2 P_1(0)P_2(0)P_3(0) \exp(-\alpha L) \quad 4-4$$

Here, η is the four-wave mixing efficiency, D is the degeneracy factor and A_{eff} and L_{eff} are the effective area and effective length of the fibre respectively. The degeneracy factor takes a value of 1, 3 or 6 depending on whether, respectively, all, two or none of the incident frequencies are the same.

The four-wave mixing efficiency parameter, η , is a function of phase mismatch, $\Delta\beta$, and is the most useful measurable quantity for describing four-wave mixing in optical fibres. The efficiency is normalised to 1 at zero phase mismatch and is given by [6]:

$$\eta = \frac{\alpha^2}{\alpha^2 + (\Delta\beta)^2} \left[1 + \frac{4 \exp(-\alpha L) \sin^2(\Delta\beta L / 2)}{(1 - \exp(-\alpha L))^2} \right] \quad 4-5$$

The $\sin^2(\Delta\beta L / 2)$ term accounts for the oscillatory nature of the FWM efficiency as a function of distance along the fibre. The spatial period of the oscillation depends on the phase mismatch and therefore on the fibre dispersion for fixed signal frequencies.

5. References

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